

Neutrino Physics - Theory

Mariam Tórtola

IFIC, CSIC/Universitat de València



TAE 2023 - International Workshop on High Energy Physics

Benasque, September 2023

Lecture 1

- ◆ Historical introduction to neutrino physics

Neutrinos in the SM

- ◆ Neutrino interactions
- ◆ Neutrino mass in the SM

Neutrino oscillations

- ◆ First evidences and discovery
- ◆ Flavor oscillations in vacuum
- ◆ Flavor oscillations in matter

Lecture 2

Neutrino masses

- ◆ Current limits
- ◆ Neutrino mass models

Neutrino physics BSM

- ◆ Light sterile neutrinos
- ◆ Non-unitary neutrino mixing
- ◆ Non-standard interactions
- ◆ BSM searches with CEνNS

What is a neutrino?

- ◆ spin 1/2 particle
- ◆ massless particle (almost)
- ◆ neutral
- ◆ 3 flavors (mixing)

Elementary Particles

Quarks	u up	c charm	t top	γ photon
	d down	s strange	b bottom	
Leptons	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	Z Z boson
	e electron	μ muon	τ tau	
	I	II	III	

Three Families of Matter

Force Carriers

What is a neutrino?

- ◆ spin 1/2 particle
- ◆ massless particle (almost)
- ◆ neutral
- ◆ 3 flavors (mixing)

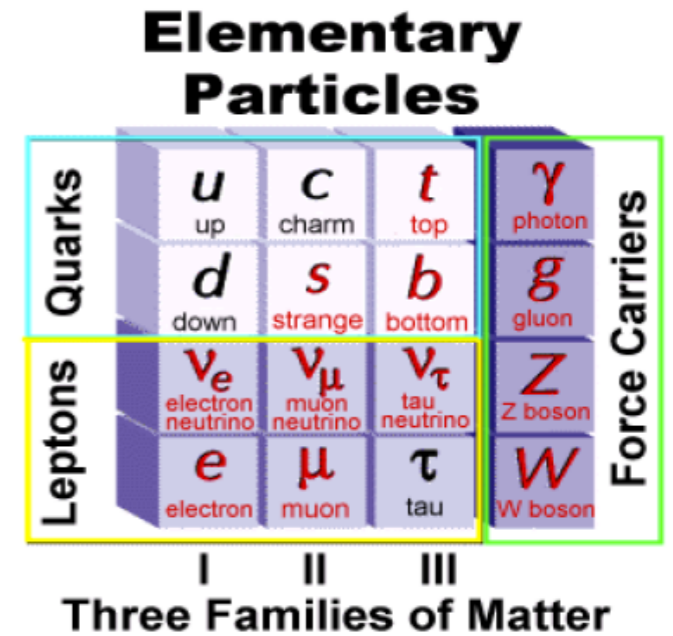
Anything else?

Every second we are traversed by:

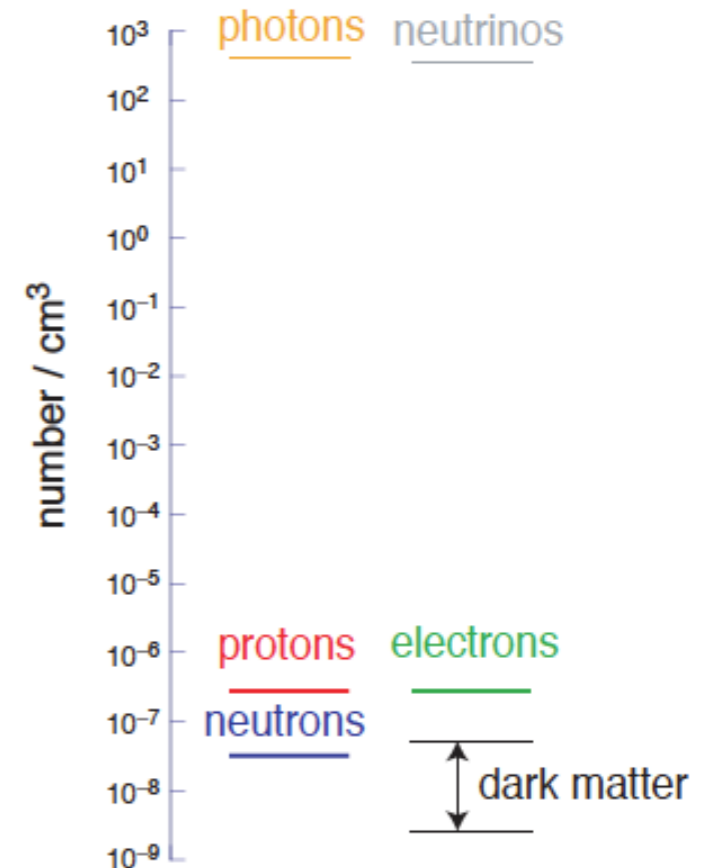
- ◆ 400×10^{12} neutrinos from the Sun
- ◆ 50×10^9 neutrinos from natural radioactivity
- ◆ 10×10^9 neutrinos from nuclear power plants

Moreover:

- ◆ our body emits 4000 neutrinos/s (^{40}K decay)
- ◆ the Universe contains $\sim 330 \times 10^6$ neutrinos/ m^3

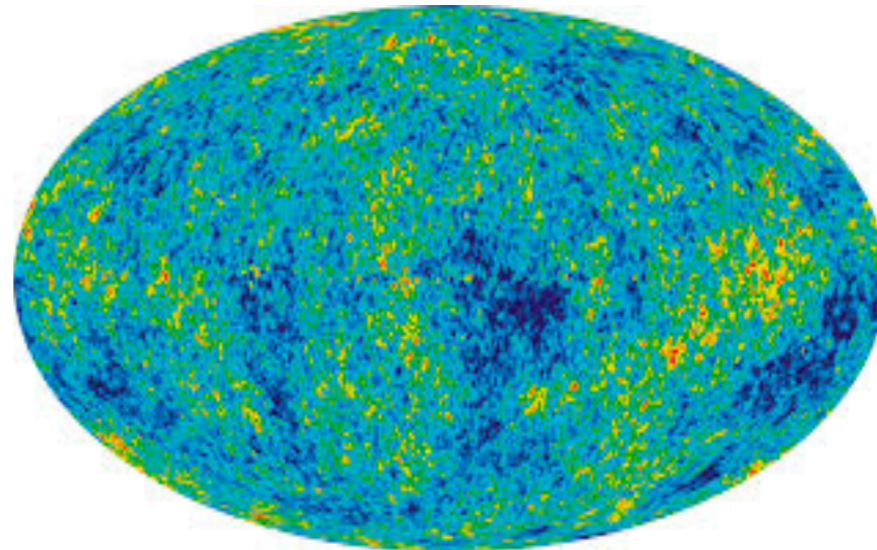
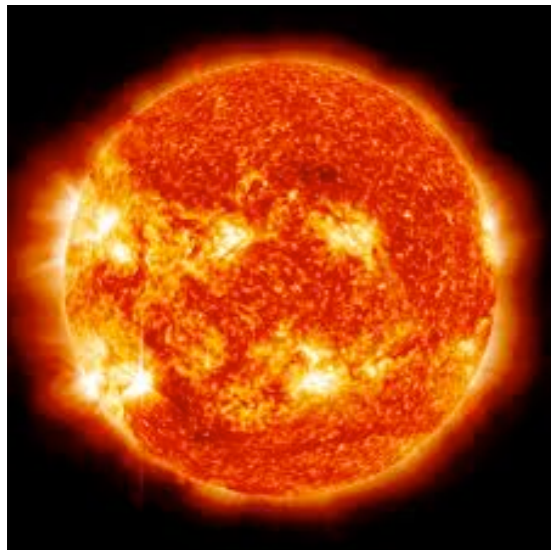


The Particle Universe



Why neutrinos are so important?

- ◆ They can probe environments that other techniques cannot: SN explosions, the core of the Sun,...
- ◆ Their role is crucial for the evolution of the universe (Big Bang Nucleosynthesis, structure formation).
- ◆ They could help explaining the matter-antimatter asymmetry of the Universe (leptogenesis mechanism).
- ◆ They could be a component of the dark matter of the universe.
- ◆ They provide the first evidence for physics beyond the SM!!!



Why neutrinos are so important?

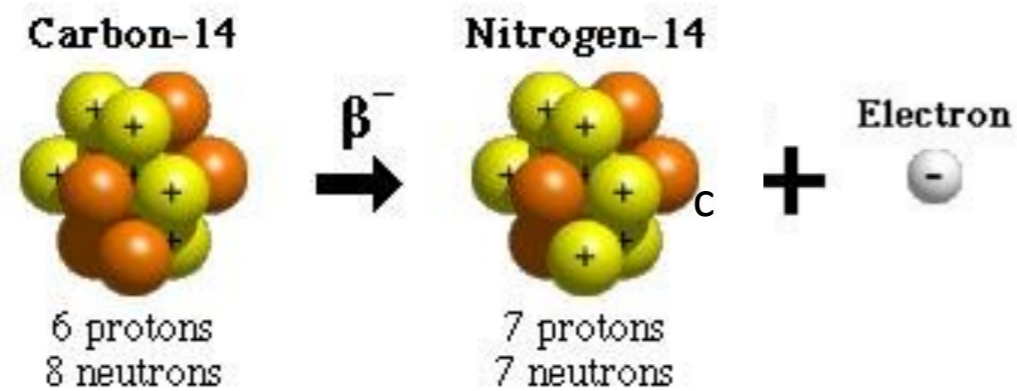
- ◆ They can probe environments that other techniques cannot: SN explosions, the core of the Sun,...
- ◆ Their role is crucial for the evolution of the universe (Big Bang Nucleosynthesis, structure formation)
- ◆ They could help explaining the matter-antimatter asymmetry of the Universe (leptogenesis mechanism)
- ◆ They could be a component of the dark matter of the universe.
- ◆ They provide the first evidence for physics beyond the SM!!!

However: there are still many open questions in neutrino physics

Historical introduction to neutrino physics

The proposal of the neutrino

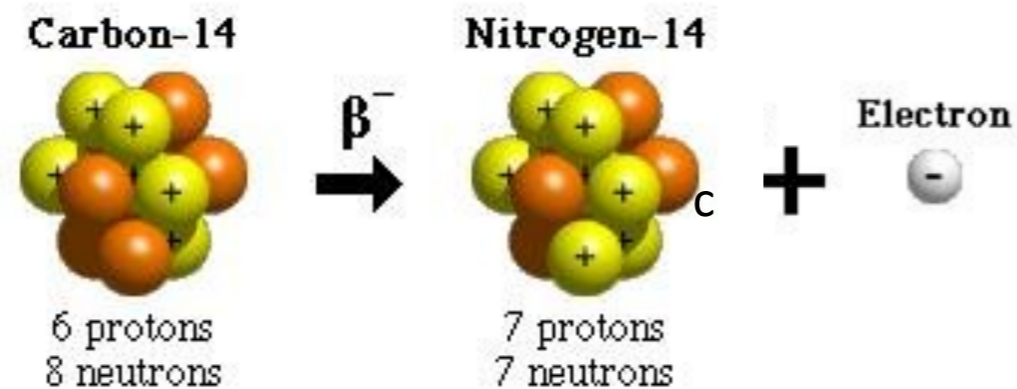
1910-1920: Experiments on radioactive decay of atomic nuclei



Lise Meitner and Otto Hahn

The proposal of the neutrino

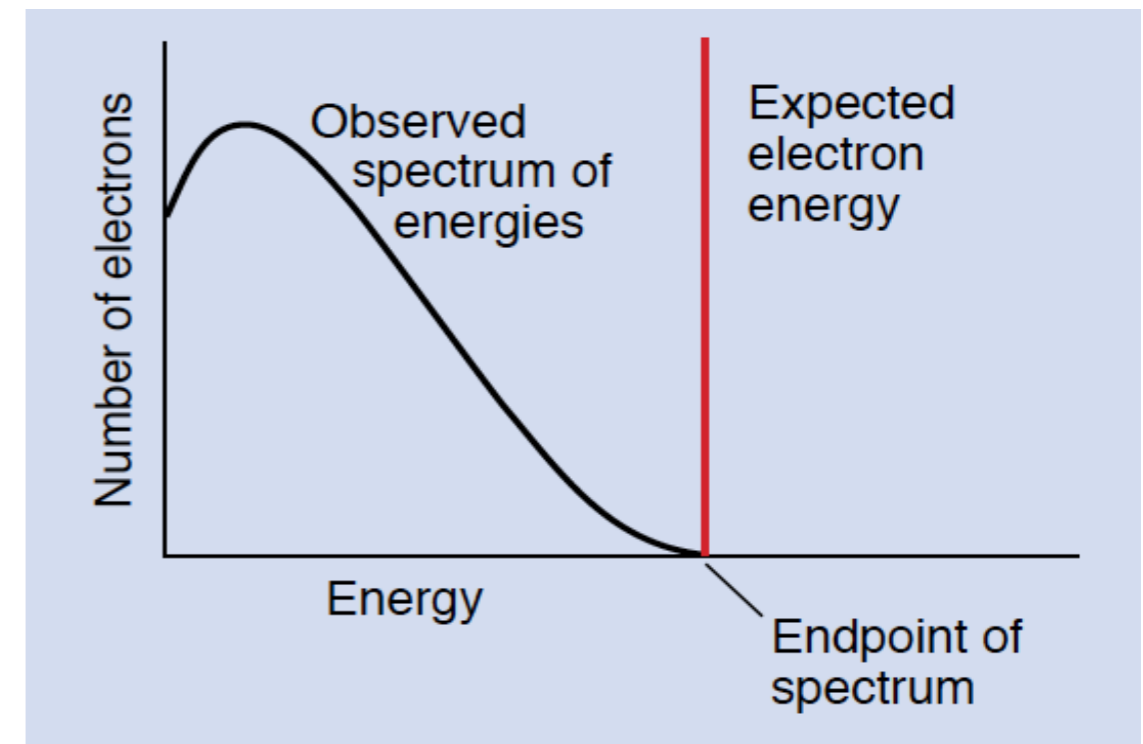
1910-1920: Experiments on radioactive decay of atomic nuclei



Lise Meitner and Otto Hahn

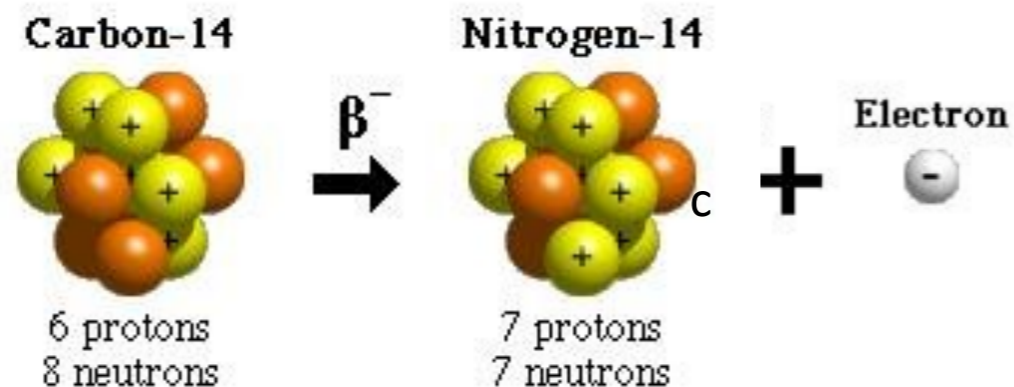
Energy and momentum conservation

⇒ emitted electrons should have a fixed energy



The proposal of the neutrino

1910-1920: Experiments on radioactive decay of atomic nuclei

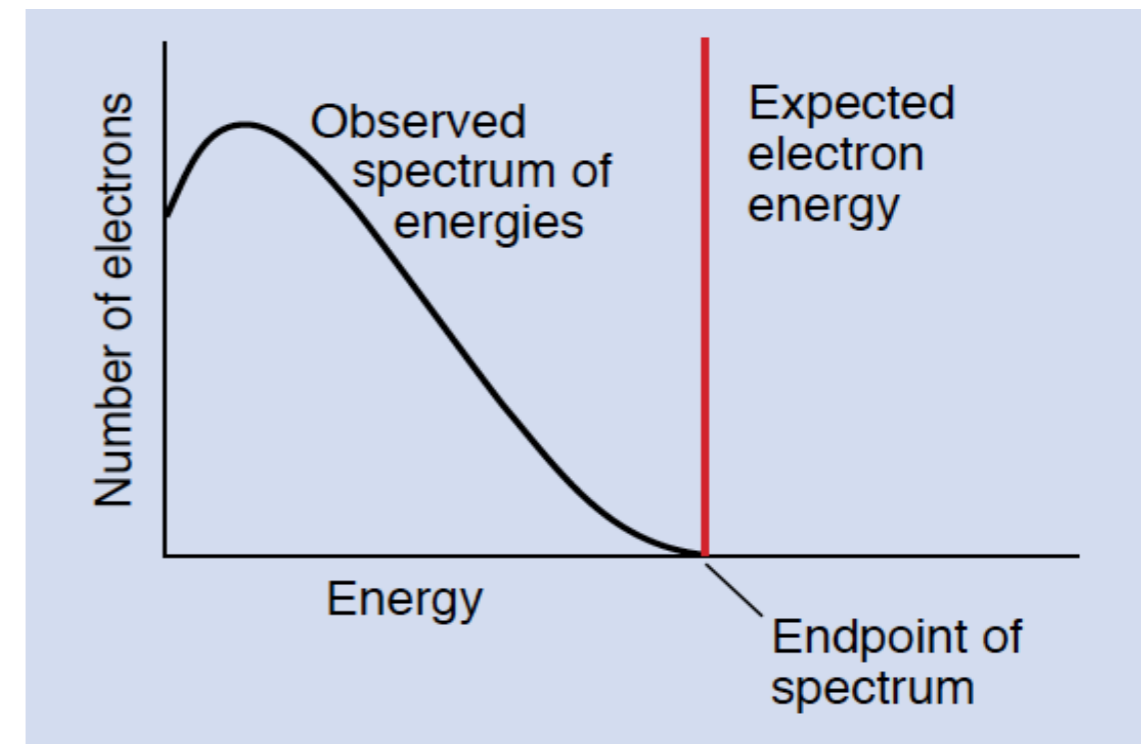


Lise Meitner and Otto Hahn

Energy and momentum conservation

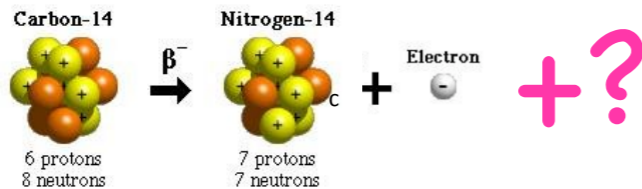
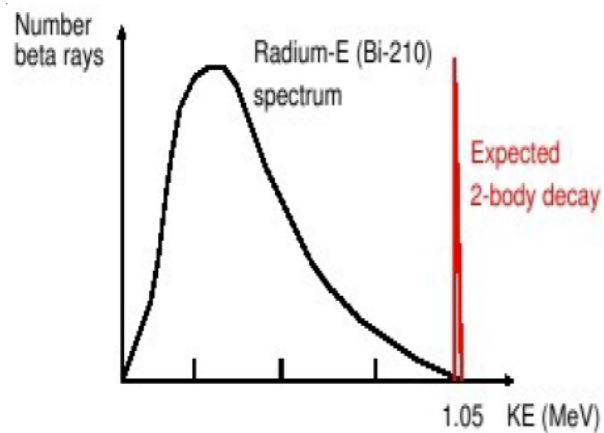
⇒ emitted electrons should have a fixed energy

Niels Bohr suggested that energy may not be conserved in individual nuclear processes



The proposal of the neutrino

1930: Pauli introduced the neutrino to explain the continuous electron spectrum



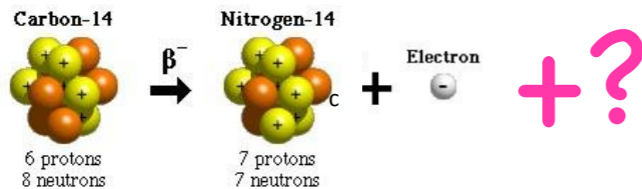
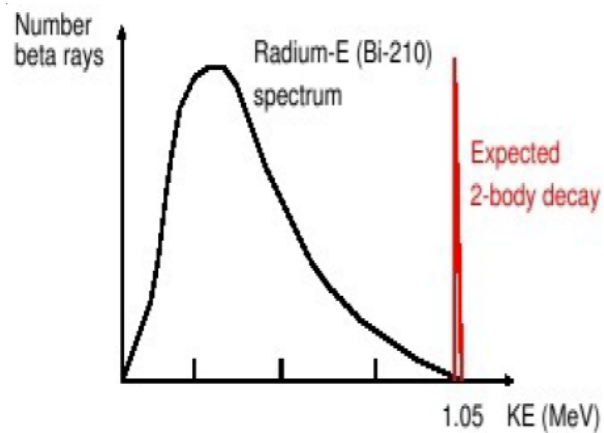
*"Dear radioactive ladies and gentlemen,
I have come upon a desperate way out regarding ... [some fairly obscure data], as well as to the continuous β -spectrum, in order to save ... The energy law. To wit, the possibility that there could exist in the nucleus electrically neutral particles which I shall call neutrons which have spin 1/2 and satisfy the exclusion principle and which are further distinct from light-quanta in that they do not move with light velocity. ... The continuous β -spectrum would then become understandable from the assumption that in β -decay a neutron is emitted along with the electron, in such a way that the sum of the energies of the neutron and the electron is constant."*

© CERN, Geneva


➔ from conservation of angular momentum: spin 1/2

The proposal of the neutrino

1930: Pauli introduced the neutrino to explain the continuous electron spectrum



"Dear radioactive ladies and gentlemen,
I have come upon a desperate way out regarding ... [some fairly obscure data], as well as to the continuous β -spectrum, in order to save ... The energy law. To wit, the possibility that there could exist in the nucleus electrically neutral particles which I shall call neutrons which have spin 1/2 and satisfy the exclusion principle and which are further distinct from light-quanta in that they do not move with light velocity. ... The continuous β -spectrum would then become understandable from the assumption that in β -decay a neutron is emitted along with the electron, in such a way that the sum of the energies of the neutron and the electron is constant."



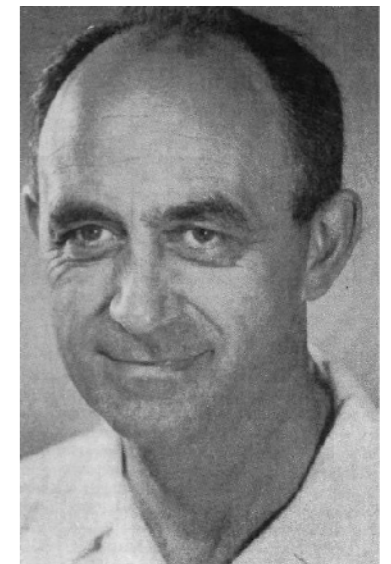
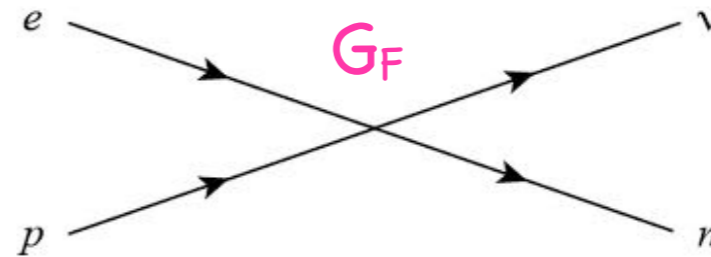
© CERN, Geneva

→ from conservation of angular momentum: spin 1/2

1933: Fermi postulated the first theory of nuclear beta decay, the theory of weak interactions

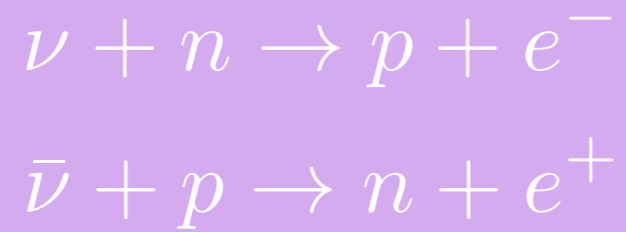
$$n \rightarrow p + e^- + \bar{\nu}_e$$

→ new name for particle: **neutrino**



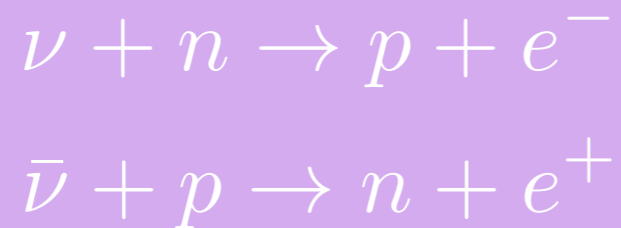
Where was the neutrino?

1934: Bethe and Peierls calculated the cross section σ for the processes:



Where was the neutrino?

1934: Bethe and Peierls calculated the cross section σ for the processes:



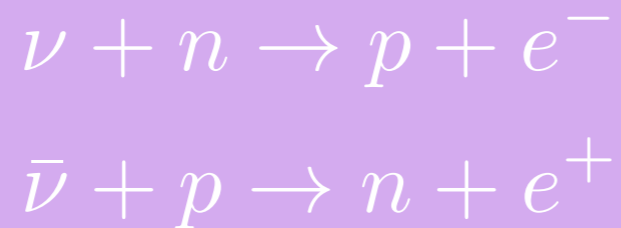
According to Fermi theory, they obtained (for antineutrino + proton):

$$\sigma \sim 5 \times 10^{-44} \text{ cm}^2 \text{ for a 2 MeV neutrino}$$

(to be compared with $\sigma_{\gamma p} \sim 10^{-25} \text{ cm}^2$) !!!

Where was the neutrino?

1934: Bethe and Peierls calculated the cross section σ for the processes:



According to Fermi theory, they obtained (for antineutrino + proton):

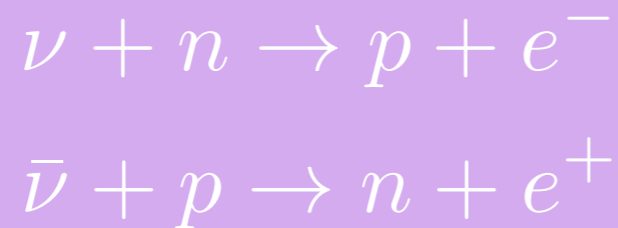
$$\sigma \sim 5 \times 10^{-44} \text{ cm}^2 \text{ for a 2 MeV neutrino}$$

factor 2 missing!

(to be compared with $\sigma_{\gamma p} \sim 10^{-25} \text{ cm}^2$) !!!

Where was the neutrino?

1934: Bethe and Peierls calculated the cross section σ for the processes:



According to Fermi theory, they obtained (for antineutrino + proton):

$$\sigma \sim 5 \times 10^{-44} \text{ cm}^2 \text{ for a 2 MeV neutrino}$$

factor 2 missing!

(to be compared with $\sigma_{\gamma p} \sim 10^{-25} \text{ cm}^2$) !!!

→ mean free path of neutrinos in **water**: $\lambda_{\text{water}} \approx 1.7 \times 10^{18} \text{ m} \sim 150 \text{ ly}$ (only free p)
(assuming interaction with all nucleons: $\lambda_{\text{water}} \approx 1.7 \times 10^{17} \text{ m} \sim 15 \text{ ly}$)

→ mean free path of neutrinos in **lead**: $\lambda_{\text{lead}} \approx 1.5 \times 10^{16} \text{ m} \sim 1.5 \text{ ly}$ (all nucleons)

Neutrino: impossible to detect?

"I have done something very bad today by proposing a particle that cannot be detected. It is something that no theorist should ever do."

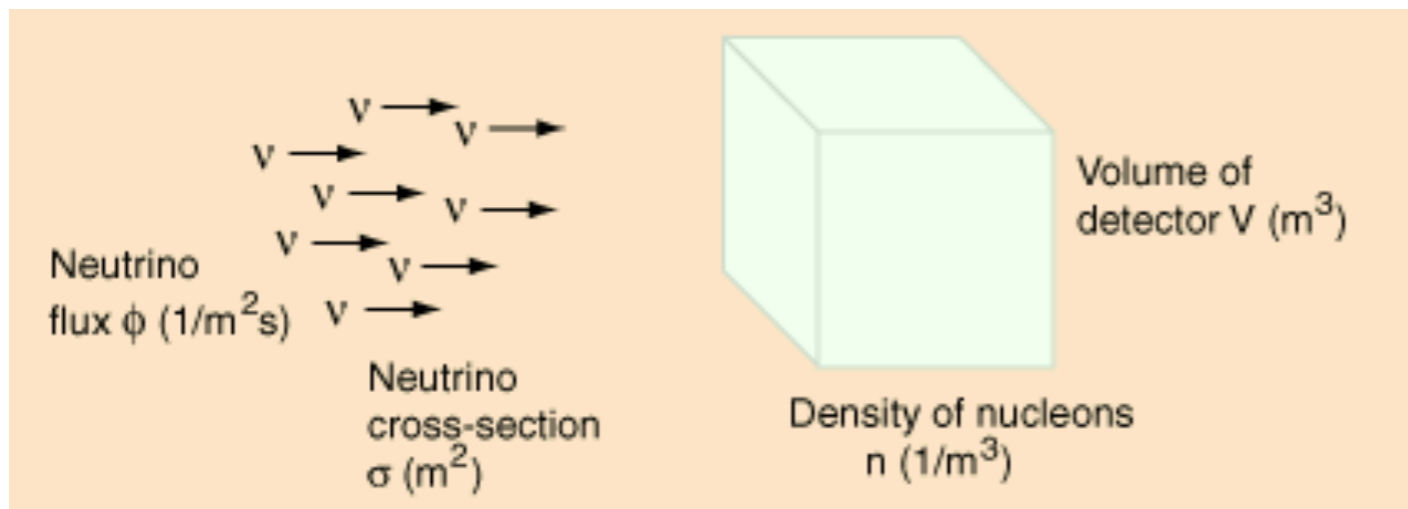
Pauli, 1930

Neutrino: impossible to detect?

"I have done something very bad today by proposing a particle that cannot be detected. It is something that no theorist should ever do."

Event number in a neutrino experiment:

Pauli, 1930



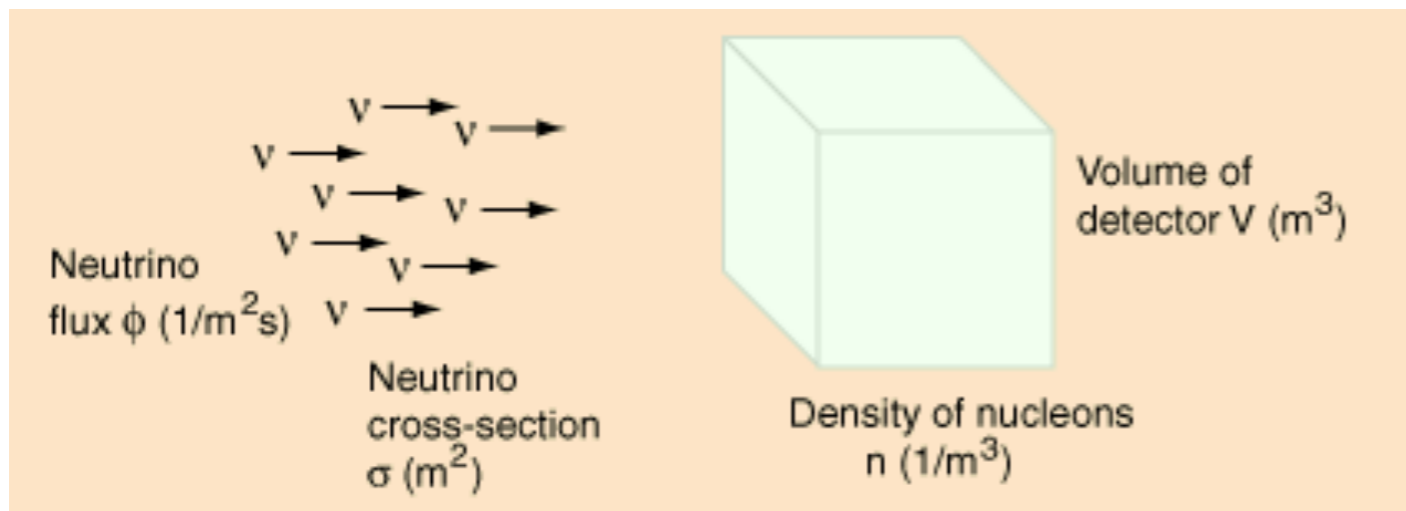
$$N = \phi \sigma N_{\text{targ}} \Delta t$$

Neutrino: impossible to detect?

"I have done something very bad today by proposing a particle that cannot be detected. It is something that no theorist should ever do."

Event number in a neutrino experiment:

Pauli, 1930



$$N = \phi \sigma N_{\text{targ}} \Delta t$$

with a 1000 kg detector and a flux of 10^{10} ν /s: few ν events/day

→ solar neutrino flux $\sim 7 \times 10^{10}$ $\nu/cm^2/s$

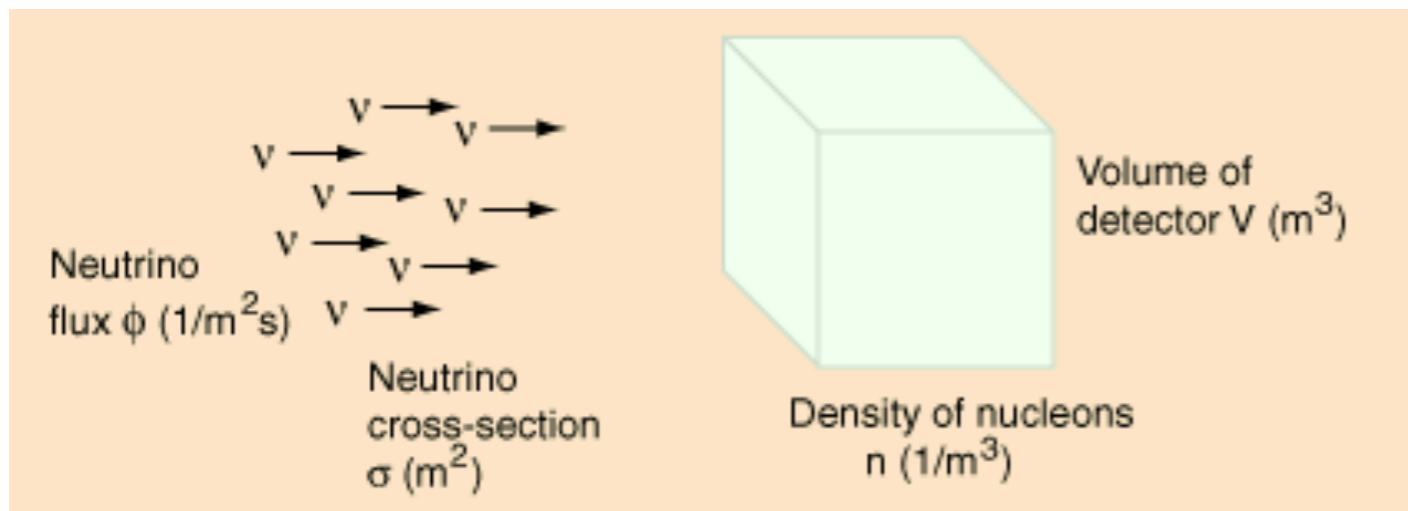
→ reactor neutrino flux $\sim 10^{20}$ ν/s

Neutrino: impossible to detect?

"I have done something very bad today by proposing a particle that cannot be detected. It is something that no theorist should ever do."

Event number in a neutrino experiment:

Pauli, 1930



$$N = \phi \sigma N_{\text{targ}} \Delta t$$

with a 1000 kg detector and a flux of 10^{10} ν /s: few ν events/day

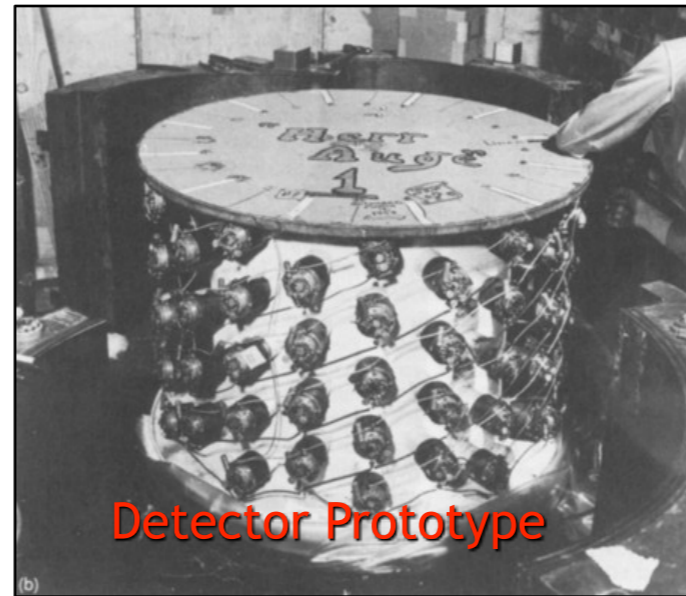
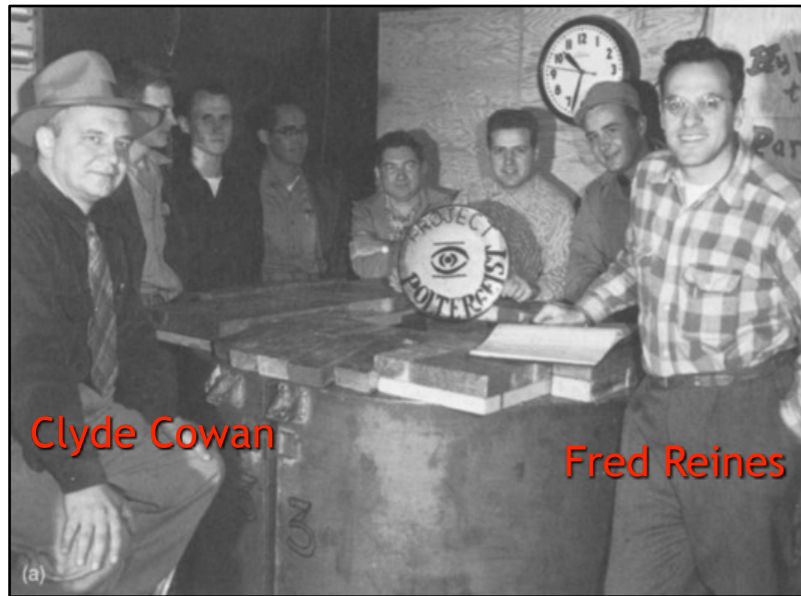
→ solar neutrino flux $\sim 7 \times 10^{10}$ $\nu/cm^2/s$

→ reactor neutrino flux $\sim 10^{20}$ ν/s

Difficult but not impossible!

Discovery of the neutrino

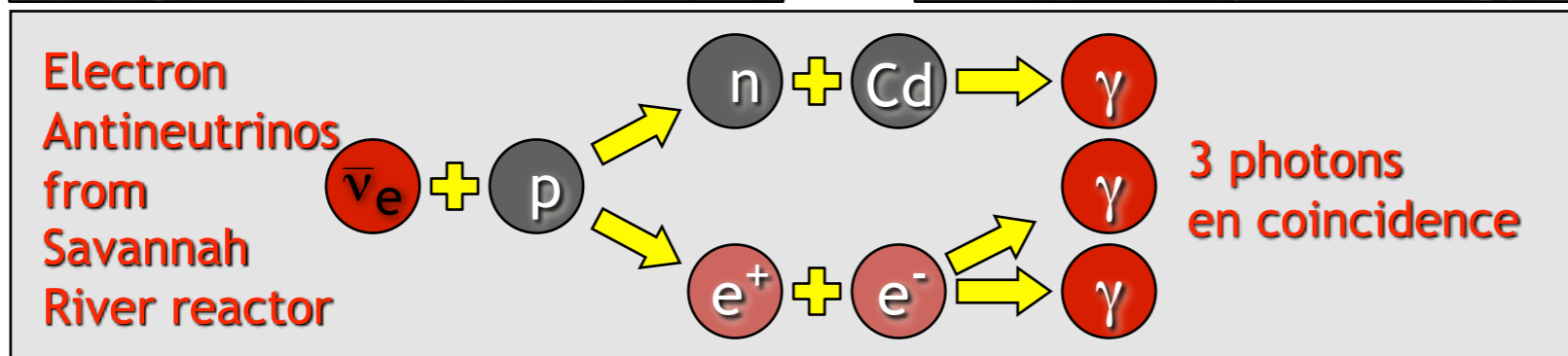
1956: First observation of reactor $\bar{\nu}_e$ by Reines and Cowan.



2 tanks with
200 liters H₂O
+
40 kg CdCl₂

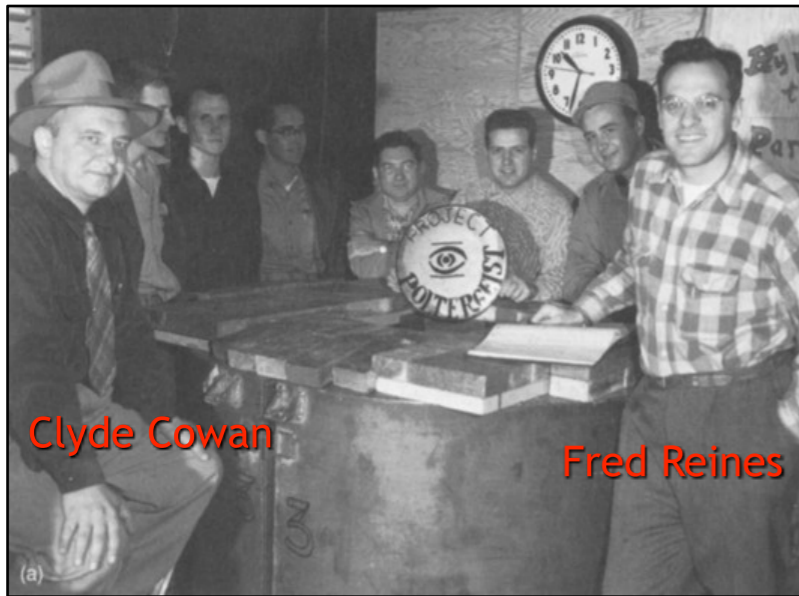
3 scintillator
layers with PMTs

1995: Nobel
Prize in Physics
to Reines



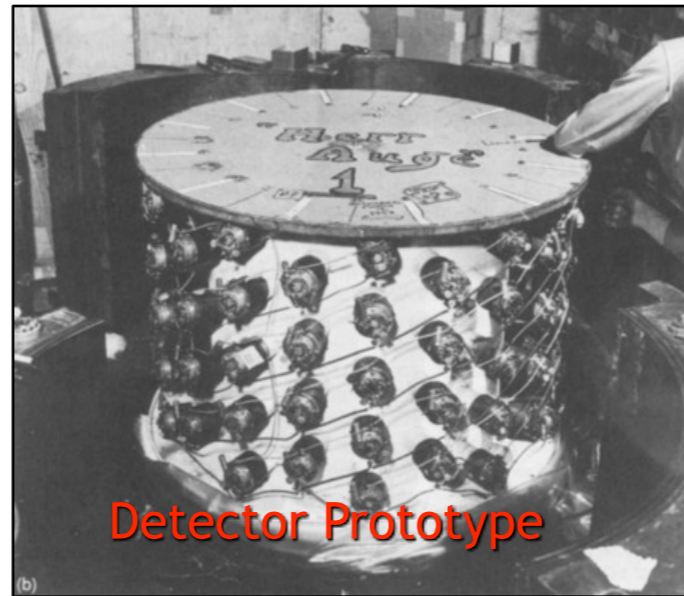
Discovery of the neutrino

1956: First observation of reactor $\bar{\nu}_e$ by Reines and Cowan.



Clyde Cowan

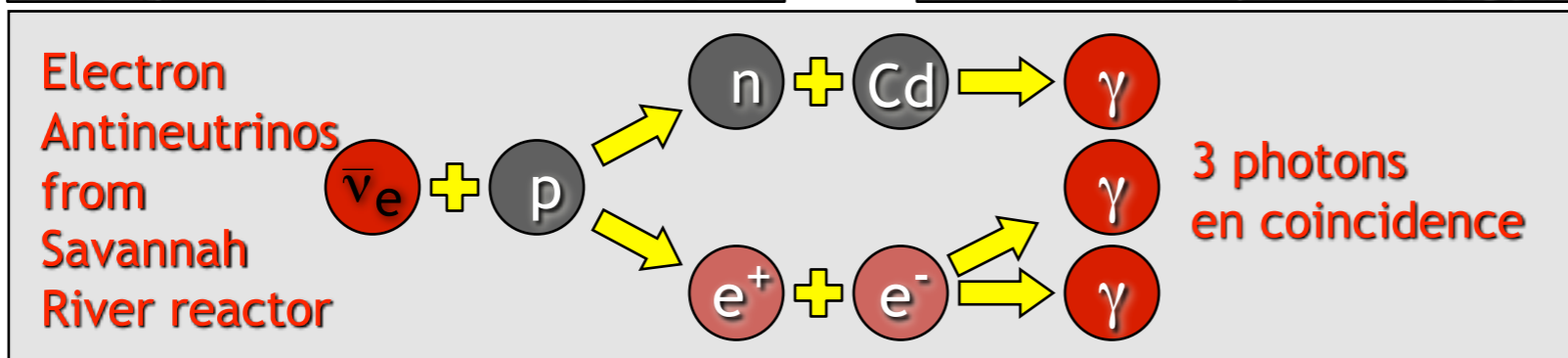
Fred Reines



Detector Prototype

2 tanks with
200 liters H_2O
+
40 kg $CdCl_2$

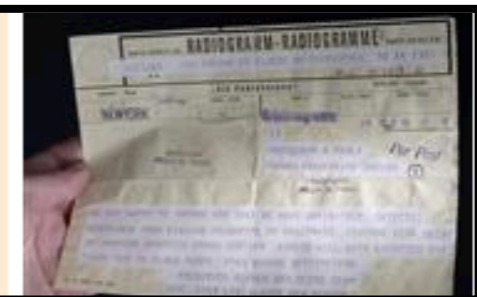
3 scintillator
layers with PMTs



1995: Nobel Prize in Physics to Reines

Telegram to Pauli on 12/06/1956

"We are happy to inform you that we have definitely detected neutrinos from fission fragments by observing inverse beta decay of protons. Observed cross section agrees well with expected six times ten to minus forty-four square centimeters"

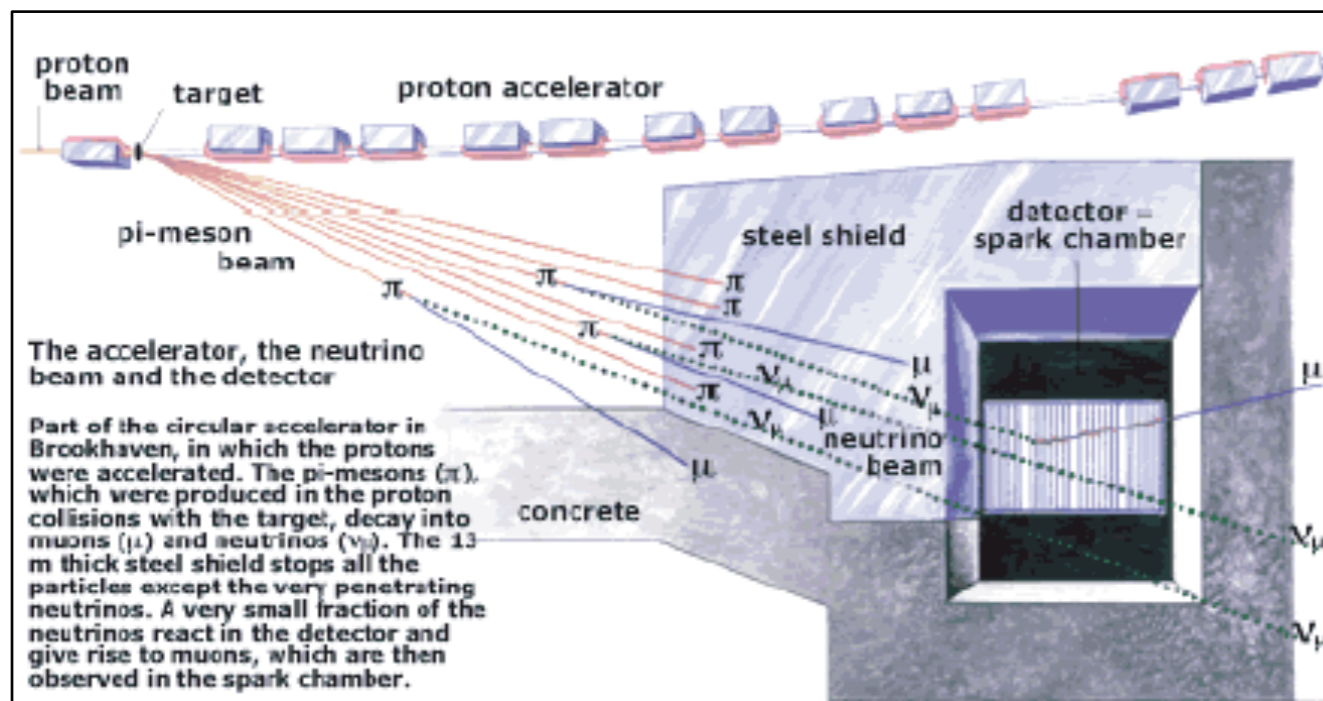


More than one neutrino flavour?

1959: Pontecorvo suggested the existence of a different neutrino, associated to muon decay and proposed an experiment to check it.



1962: Discovery of ν_{μ} by Lederman, Schwartz and Steinberger



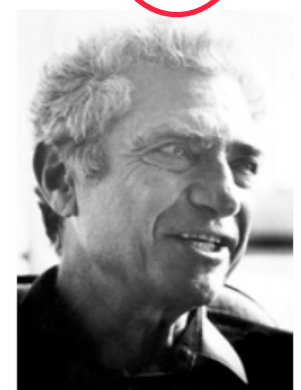
not e^-



Leon M. Lederman



Melvin Schwartz



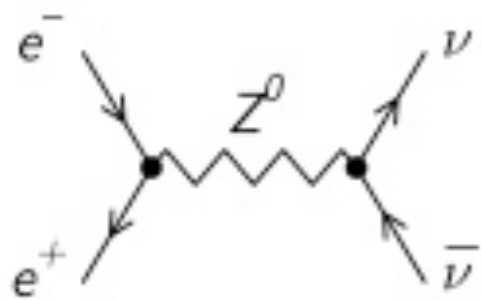
Jack Steinberger

1988: Nobel Prize in Physics

More than two neutrino flavours?

1978: Discovery of τ at SLAC \rightarrow imbalance of energy in τ decay suggests the existence of a third neutrino.

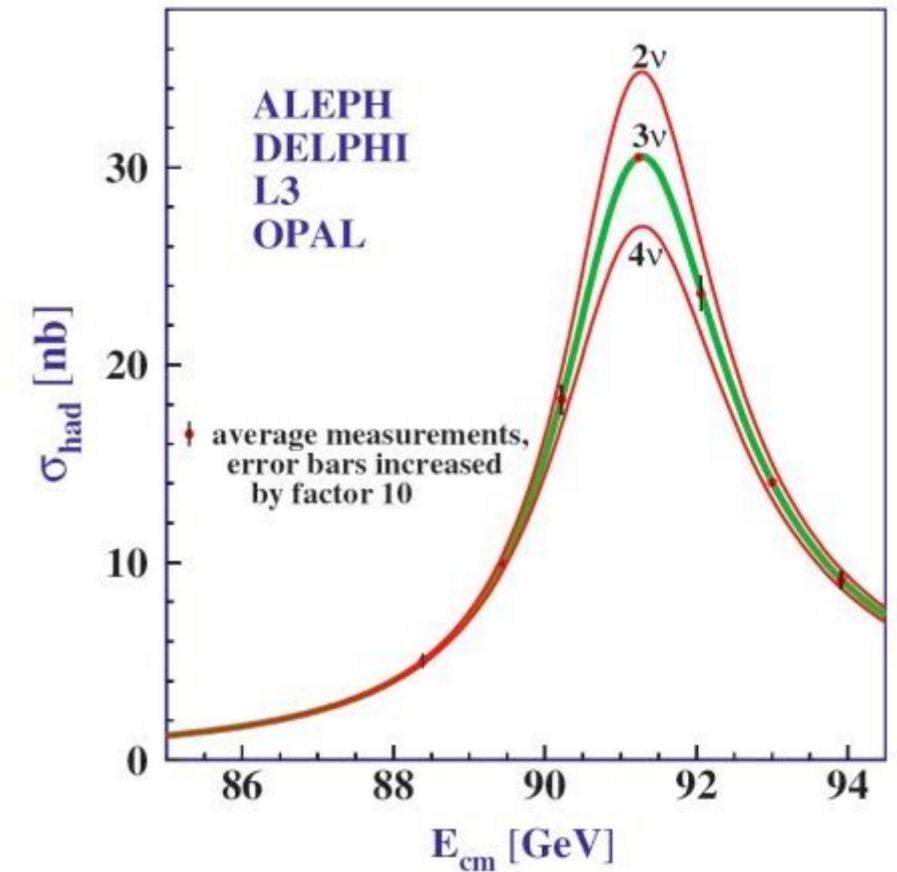
1989: LEP measurements of the invisible decay width of Z boson



$$\Gamma_{\text{inv}} \equiv \Gamma_Z - \Gamma_{\text{had}} - 3\Gamma_{\text{lep}}$$

$$N_\nu = \Gamma_{\text{inv}} / \Gamma_{\text{SM}}(Z \rightarrow \nu_i \bar{\nu}_i)$$

$$\rightarrow N_\nu = 2.984 \pm 0.008$$



2000: Discovery of ν_τ by the DONUT Collaboration.

800 GeV $p \Rightarrow$ Ds meson ($\equiv c\bar{s}$) $\rightarrow \nu_\tau$ $\tau \Rightarrow \tau$ detected

Parity violation in weak interactions

τ - θ puzzle:

$$\Theta^+ \rightarrow \pi^+ + \pi^0$$

$$\tau^+ \rightarrow \pi^+ + \pi^+ + \pi^-$$

$$P = (-1)(-1) = +1$$

$$P = (-1)(-1)(-1) = -1$$

$$(\tau^+ = \theta^+ = K^+)$$

Parity violation in weak interactions

τ - θ puzzle:

$$\Theta^+ \rightarrow \pi^+ + \pi^0$$

$$\tau^+ \rightarrow \pi^+ + \pi^+ + \pi^-$$

$$P = (-1)(-1) = +1$$

$$(\tau^+ = \theta^+ = K^+)$$

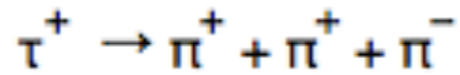
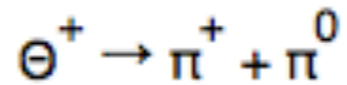
$$P = (-1)(-1)(-1) = -1$$

1956: Lee and Yang proposed parity violation in weak interactions

Lee and Yang, Phys. Rev. 104 (1956) 254.

Parity violation in weak interactions

τ - θ puzzle:



$$P = (-1)(-1) = +1$$

$$(\tau^+ = \theta^+ = K^+)$$

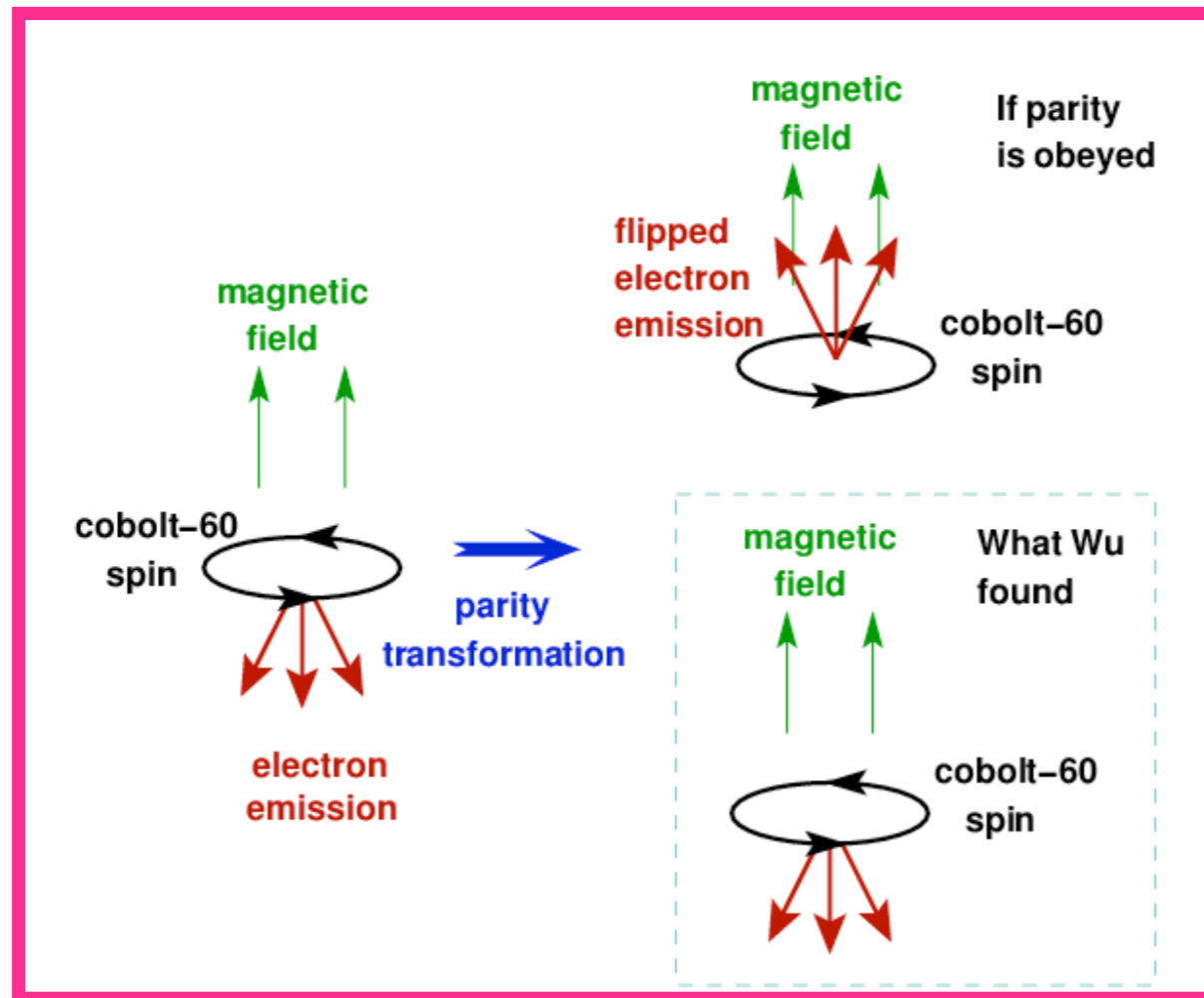
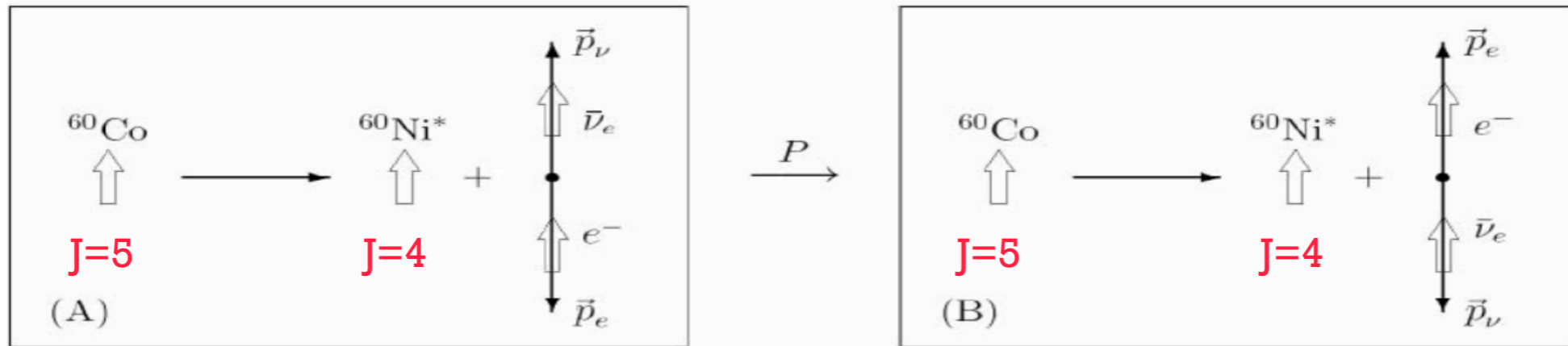
$$P = (-1)(-1)(-1) = -1$$

1956: Lee and Yang proposed parity violation in weak interactions

Lee and Yang, Phys. Rev. 104 (1956) 254.

1957: using a radioactive source of ^{60}Co , Chien-Shiung Wu et al. determined that weak interaction violates parity conservation maximally.

Parity violation in Wu experiment

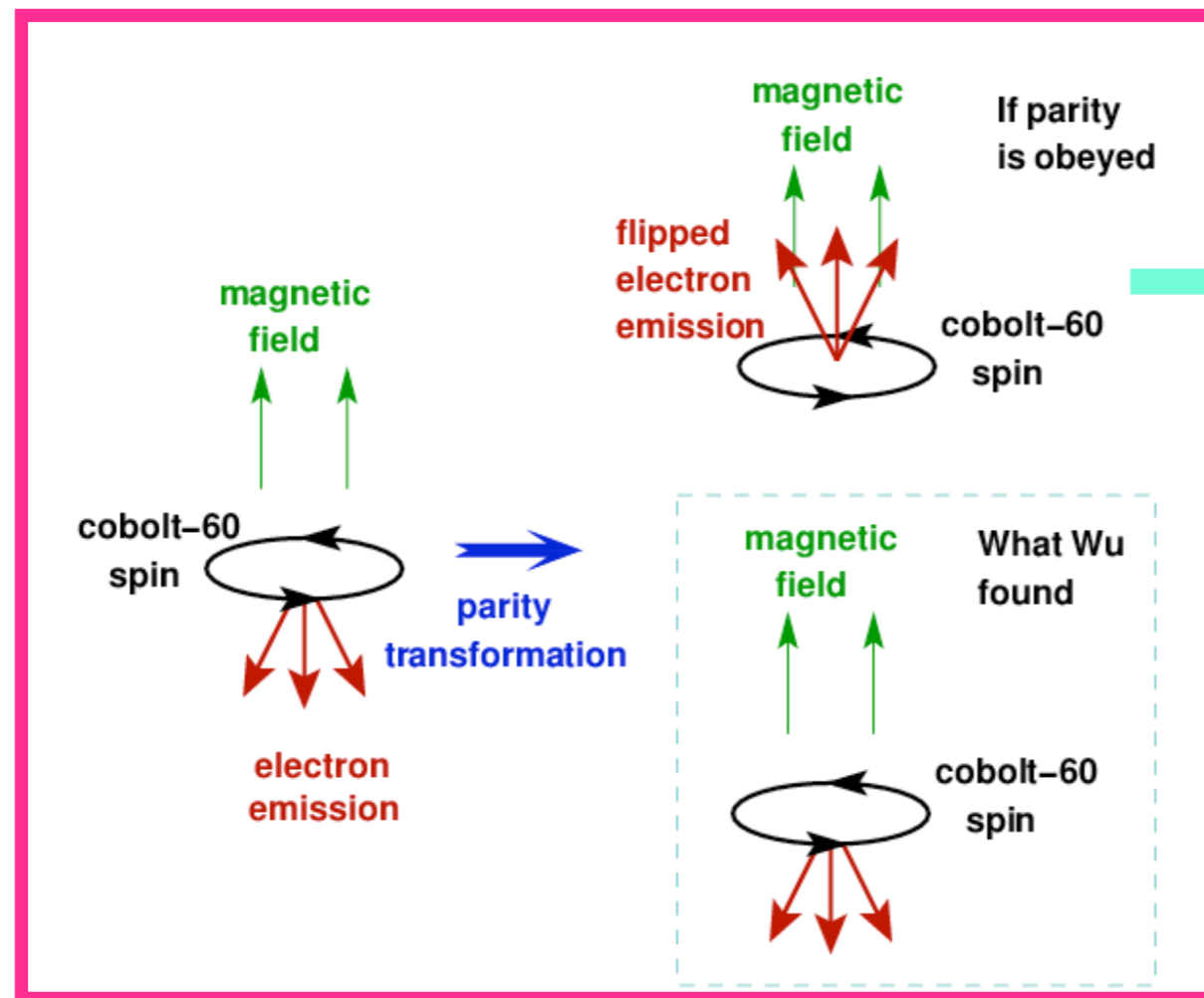
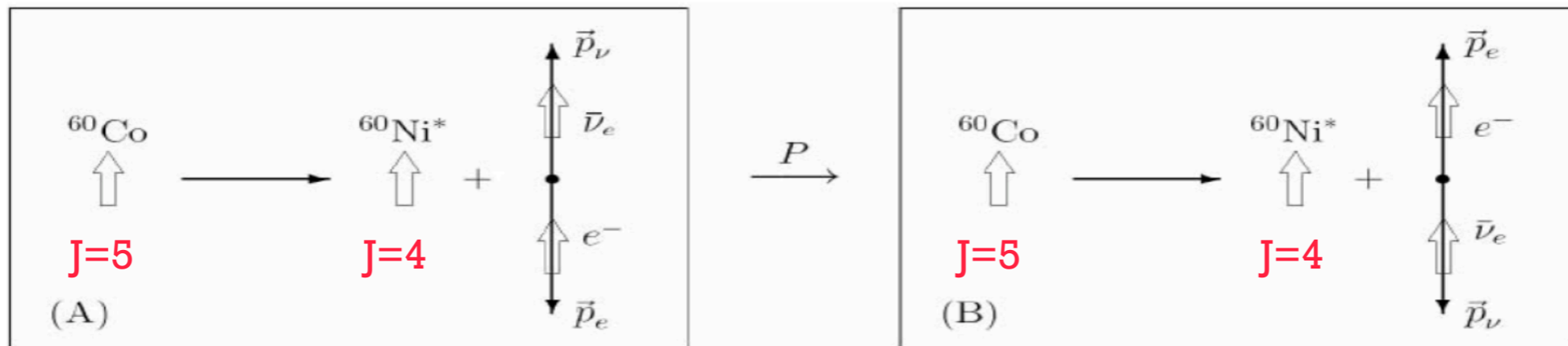


Fewer electrons emitted in the direction of the magnetic field!

Antineutrinos are preferably emitted in the direction of the field (in parallel to their spin)

Wu et al, Phys. Rev. 105 (1957)1413.

Parity violation in Wu experiment



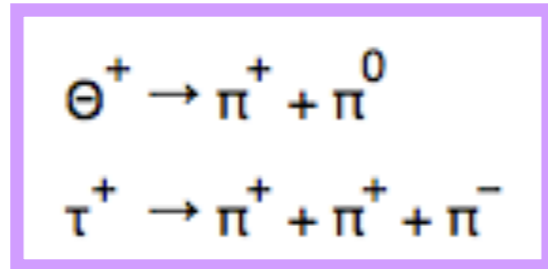
Fewer electrons emitted in the direction of the magnetic field!

Antineutrinos are preferably emitted in the direction of the field (in parallel to their spin)

Wu et al, Phys. Rev. 105 (1957)1413.

Parity violation in weak interactions

τ - θ puzzle:



$$P = (-1)(-1) = +1$$

$$(\tau^+ = \theta^+ = K^+)$$

$$P = (-1)(-1)(-1) = -1$$

1956: Lee and Yang proposed parity violation in weak interactions

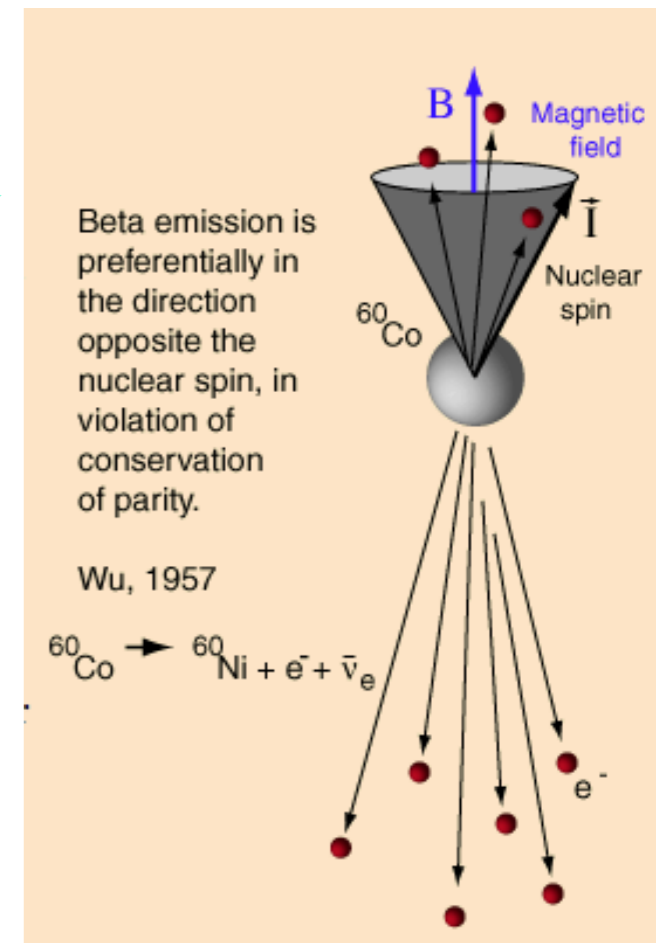
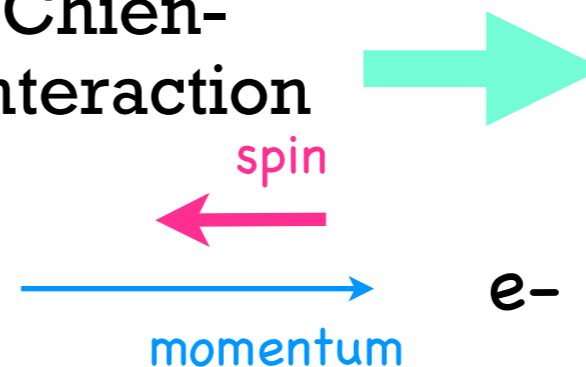
Lee and Yang, Phys. Rev. 104 (1956) 254.

1957: using a radioactive source of ^{60}Co , Chien-Shiung Wu et al. determined that weak interaction violates parity conservation maximally:

1957: Nobel Prize in Physics (Lee & Yang)

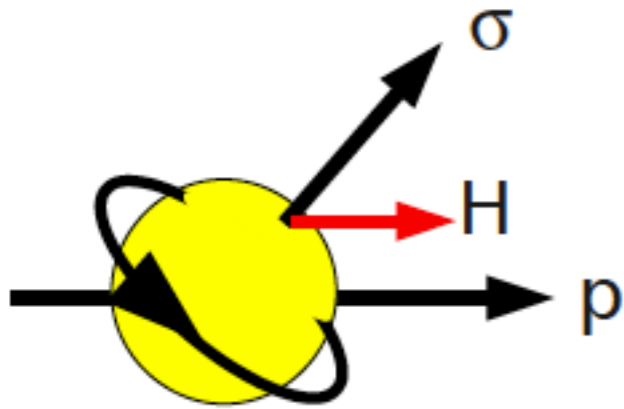
1978: Wolf Prize in Physics (C.S. Wu)

1958: Goldhaber et al. showed that neutrinos can only be emitted with their spin anti-parallel to their momentum direction.



Helicity and Chirality

Helicity is the projection of spin along the momentum direction

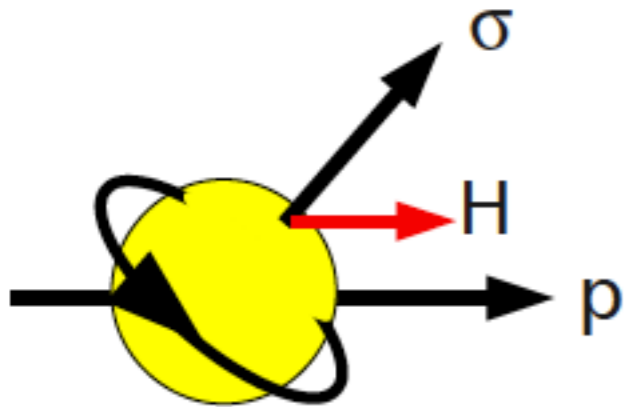


$$\hat{H} = \frac{\vec{\sigma} \cdot \vec{p}}{|\vec{p}|}$$

→ Lorentz-invariant only for massless particles

Helicity and Chirality

Helicity is the projection of spin along the momentum direction



$$\hat{H} = \frac{\vec{\sigma} \cdot \vec{p}}{|\vec{p}|}$$

→ Lorentz-invariant only for massless particles

Chirality is an asymmetry property: a chiral object is not identical to its mirror image, cannot be superimposed on it.

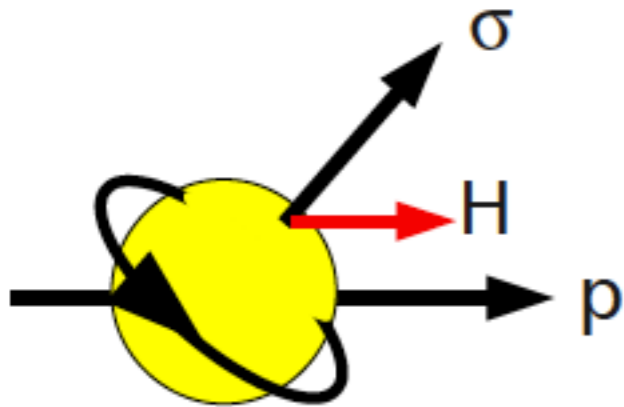
$$P_{L,R} = \frac{1 \mp \gamma_5}{2}, \quad \psi_{L,R} = P_{L,R} \psi$$

→ **Handedness = Chirality**

→ Lorentz-invariant although not directly measurable

Helicity and Chirality

Helicity is the projection of spin along the momentum direction



$$\hat{H} = \frac{\vec{\sigma} \cdot \vec{p}}{|\vec{p}|}$$

→ Lorentz-invariant only for massless particles

Massless particles: Helicity = Chirality

Massive particles: Chiral states contain contributions from both helicity states

Ultra-relativistic particles: LH (RH) chiral projection dominated by a - (+) helicity state

Chirality is an asymmetry property: a chiral object is not identical to its mirror image, cannot be superimposed on it.

$$P_{L,R} = \frac{1 \mp \gamma_5}{2}, \quad \psi_{L,R} = P_{L,R} \psi$$

→ **Handedness = Chirality**

→ Lorentz-invariant although not directly measurable

Neutrinos in the Standard Model

The Standard Model of particle physics

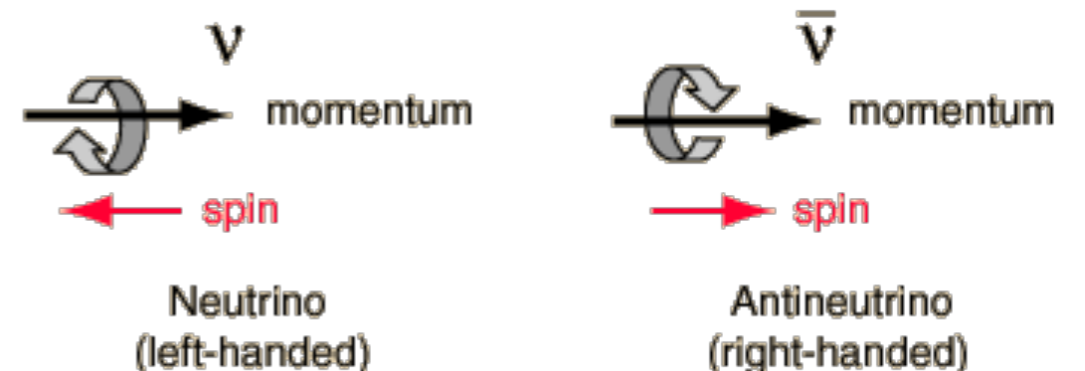
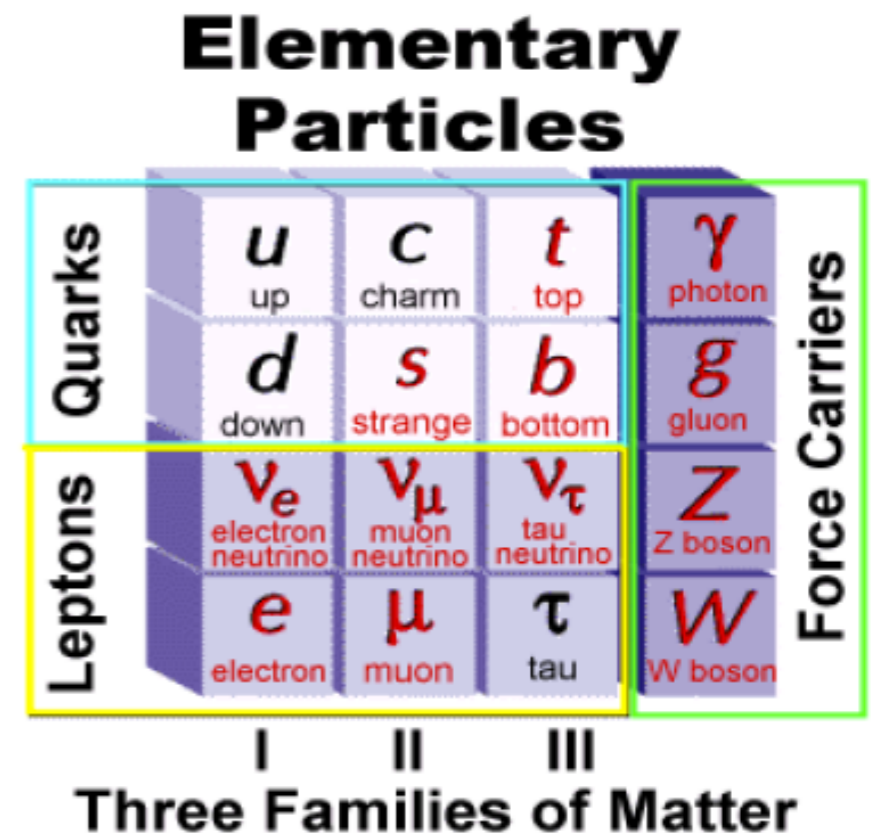
- ◆ Neutrinos come in 3 flavours, corresponding to the charged lepton associated
- ◆ Leptons are described as $SU(2)_L$ doublets

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

- ◆ Only two types of neutrinos have been observed in nature:

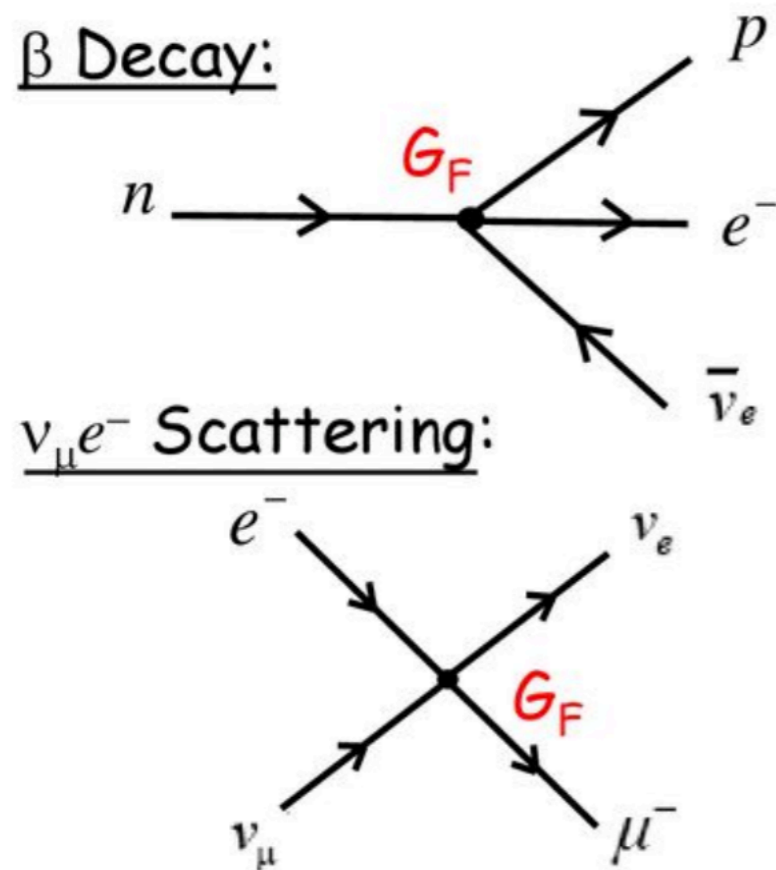
- left-handed neutrino
- right-handed antineutrino

[no $SU(2)$ neutrino singlets in the SM]



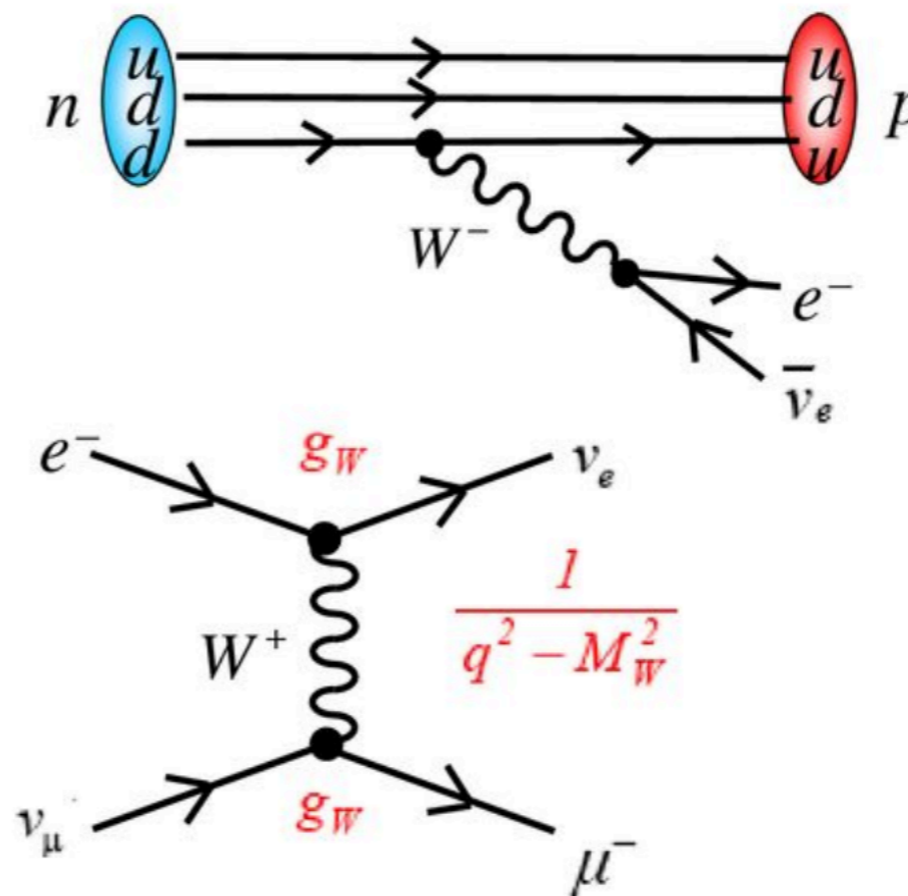
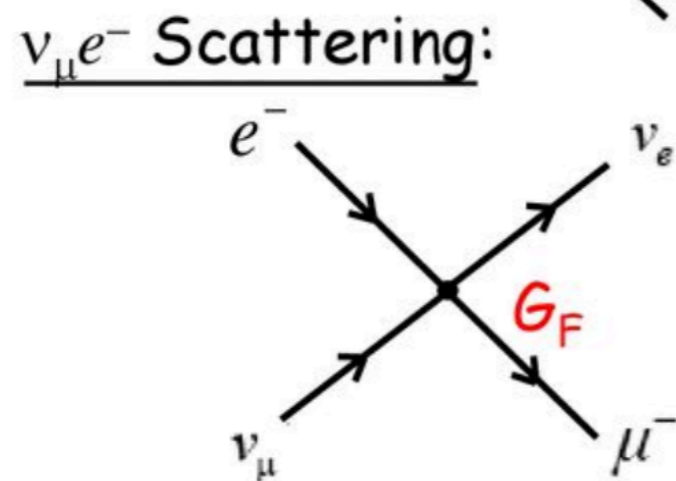
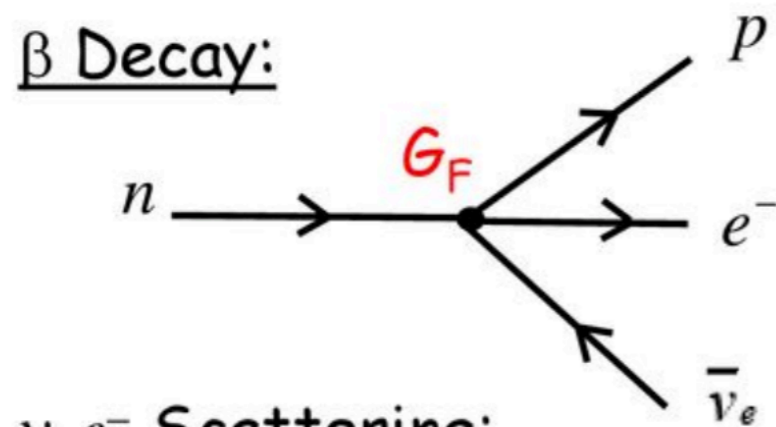
Neutrino interactions with charged leptons

- ◆ Neutrinos were first detected through their weak interactions with charged leptons, known as Charged Current (CC) interactions.
- ◆ CC weak interactions first described by Fermi as point-like 4-fermion vertex.



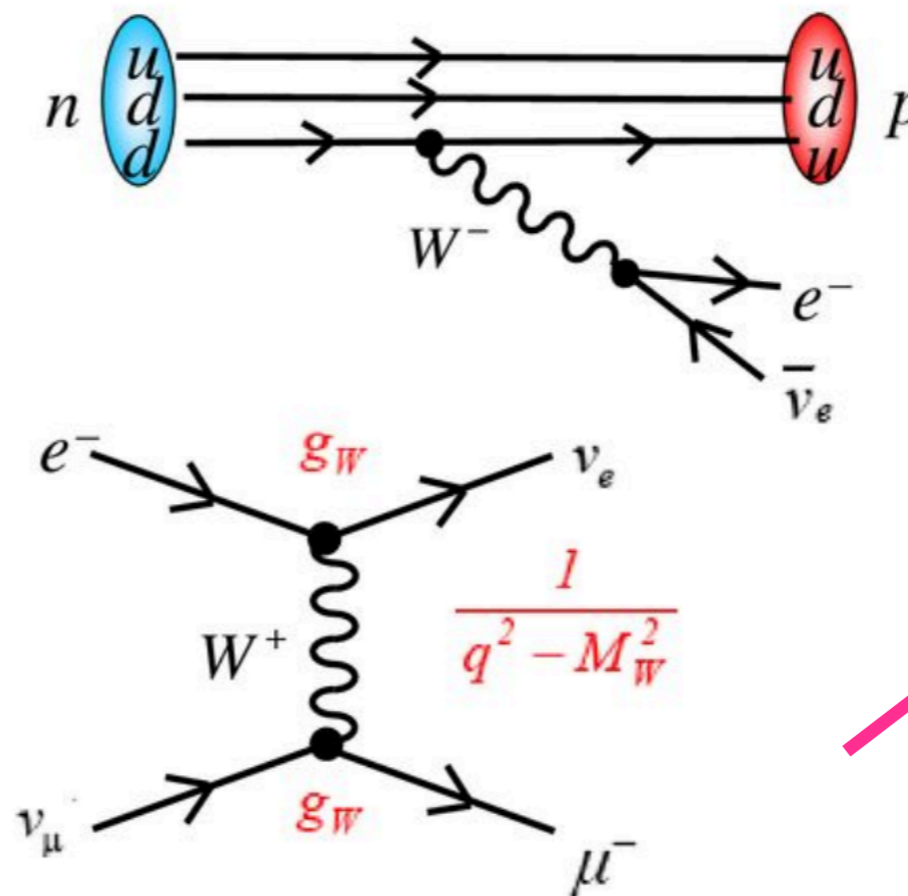
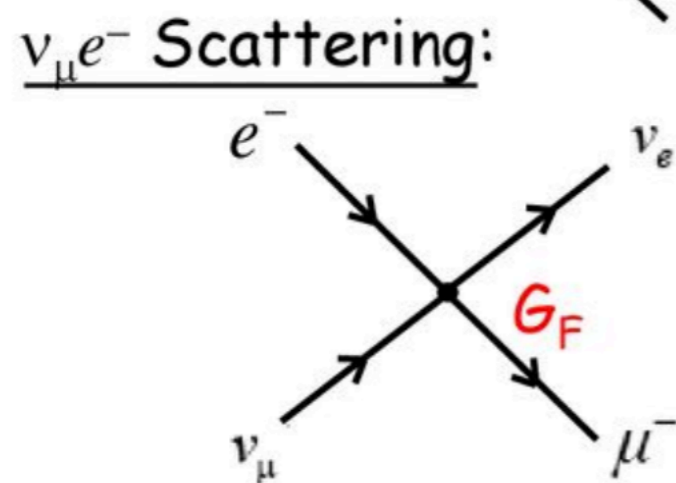
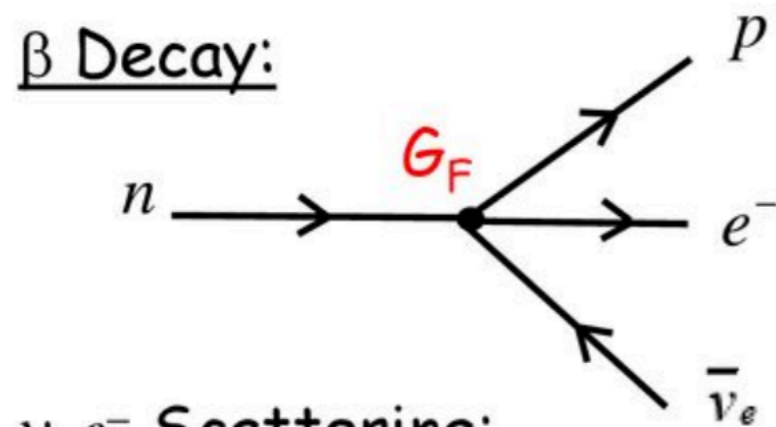
Neutrino interactions with charged leptons

- Neutrinos were first detected through their weak interactions with charged leptons, known as Charged Current (CC) interactions.
- CC weak interactions first described by Fermi as point-like 4-fermion vertex.
- SM: CC interactions are mediated by the vector boson W (W^- , W^+)



Neutrino interactions with charged leptons

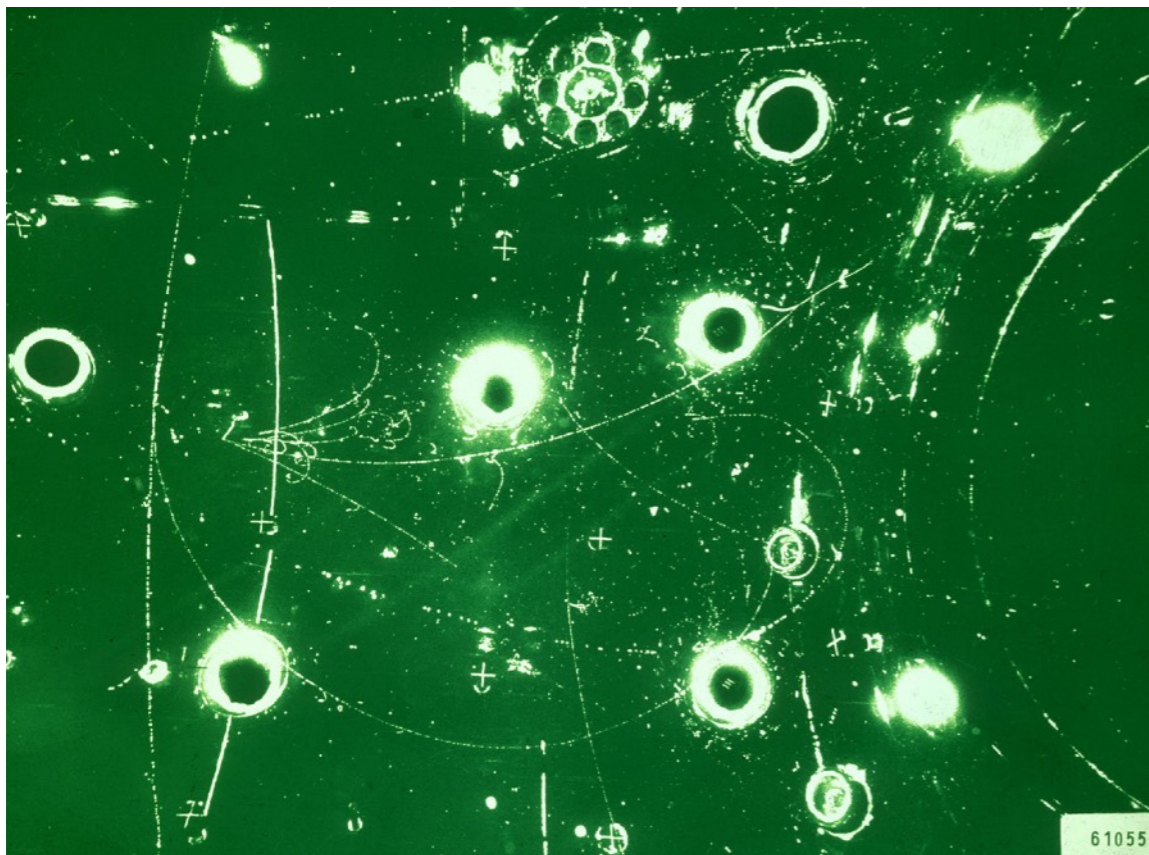
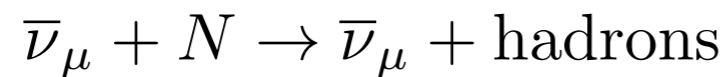
- Neutrinos were first detected through their weak interactions with charged leptons, known as Charged Current (CC) interactions.
- CC weak interactions first described by Fermi as point-like 4-fermion vertex.
- SM: CC interactions are mediated by the vector boson W (W^- , W^+)



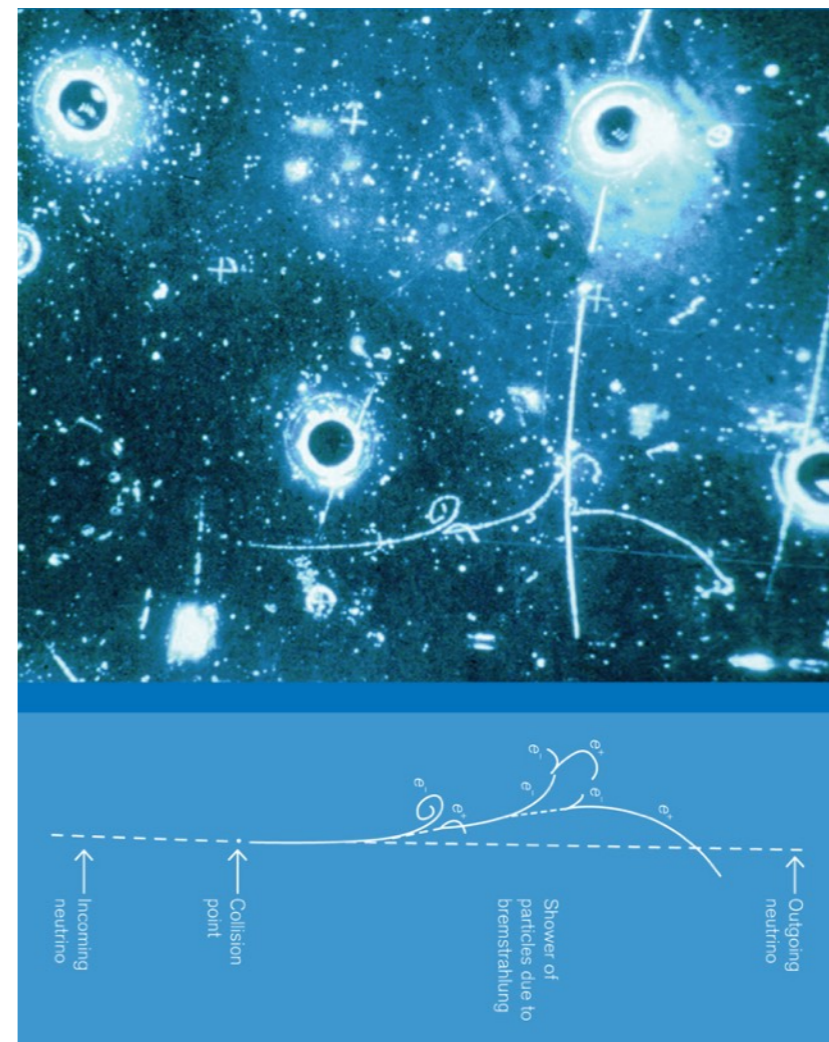
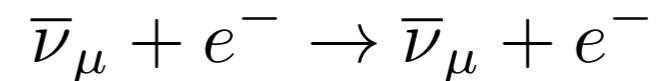
W 's couple to leptons in the same doublet

Discovery of Neutral Currents

- ◆ The Glashow-Weinberg-Salam model predicted the existence of weak interactions mediated by a neutral vector boson, the Z^0
- ◆ Neutral Current interactions were first observed in 1973 with Gargamelle bubble chamber



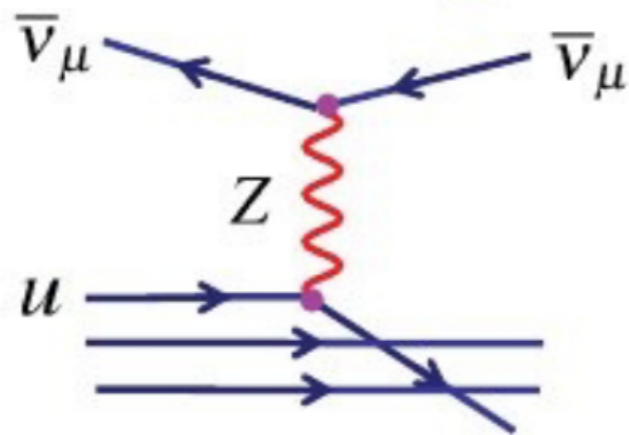
Hasert et al, Phys. Lett. B 46 (1973) 138.



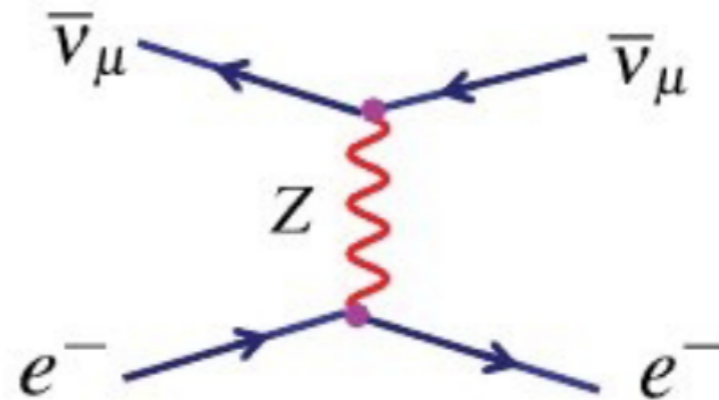
Discovery of Neutral Currents

- ◆ The Glashow-Weinberg-Salam model predicted the existence of weak interactions mediated by a neutral vector boson, the Z^0
- ◆ Neutral Current interactions were first observed in 1973 with Gargamelle bubble chamber

$$\bar{\nu}_\mu + N \rightarrow \bar{\nu}_\mu + \text{hadrons}$$



$$\bar{\nu}_\mu + e^- \rightarrow \bar{\nu}_\mu + e^-$$

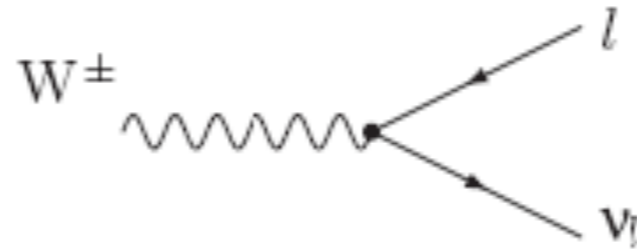


First evidence for Z boson

Hasert et al, Phys. Lett. B 46 (1973) 138.

Neutrino interactions in the SM

Charged Current (CC):



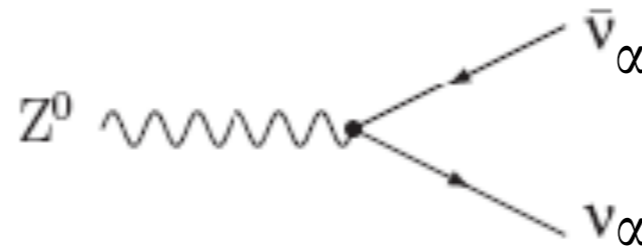
$$W^- \rightarrow l_{\alpha}^- + \bar{\nu}_{\alpha}$$

$$W^+ \rightarrow l_{\alpha}^+ + \nu_{\alpha}$$

$$(\alpha = e, \mu, \tau)$$

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}} \sum_{\alpha} \bar{\nu}_{\alpha L} \gamma^{\mu} l_{\alpha L} W_{\mu} + \text{h.c.}$$

Neutral Current (NC):



$$Z^0 \rightarrow \nu_{\alpha} \bar{\nu}_{\alpha}$$

$$\mathcal{L}_{NC} = -\frac{g}{2 \cos \theta_W} \sum_{\alpha} \bar{\nu}_{\alpha L} \gamma^{\mu} \nu_{\alpha L} Z_{\mu}^0$$

in the SM, only LH neutrinos and RH antineutrinos participate in weak interactions

◆ Interactions conserve total Lepton Number L : $L(l^-) = L(\nu) = -L(l^+) = -L(\bar{\nu}) = 1$

◆ Family lepton numbers L_e, L_{μ}, L_{τ} are also conserved (1998: nu oscill !!)

Fermion masses in the SM lagrangian

- ◆ In the SM, fermion masses appear in the lagrangian with terms like:

$$m\bar{\psi}\psi \quad \rightarrow \text{Dirac mass term}$$

decomposing into its chiral states: $\psi \equiv \psi_L + \psi_R$

$$\rightarrow m\bar{\psi}\psi = m\bar{\psi}_L\psi_R + m\bar{\psi}_R\psi_L$$

→ **forbidden**: not invariant under SU(2): it couples ψ_L with ψ_R ($I_W=1/2$)

→ solved by **Higgs mechanism**: after SSB, Dirac mass terms appear from Yukawa couplings:

$$\mathcal{L}_{\text{Yukawa}} = Y\bar{\psi}_L\phi\psi_R + \text{h.c.} \quad \langle\phi^0\rangle = v$$

→ OK for most of particles but SM neutrino has only a L-chiral state (no ψ_R)

Fermion masses in the SM lagrangian

- ◆ In the SM, fermion masses appear in the lagrangian with terms like:

$$m\bar{\psi}\psi \quad \rightarrow \text{Dirac mass term}$$

decomposing into its chiral states: $\psi \equiv \psi_L + \psi_R$

$$\rightarrow m\bar{\psi}\psi = m\bar{\psi}_L\psi_R + m\bar{\psi}_R\psi_L$$

→ **forbidden**: not invariant under SU(2): it couples ψ_L with ψ_R ($I_W=1/2$)

→ solved by **Higgs mechanism**: after SSB, Dirac mass terms appear from Yukawa couplings:

$$\mathcal{L}_{\text{Yukawa}} = Y\bar{\psi}_L\phi\psi_R + \text{h.c.} \quad \langle\phi^0\rangle = v$$

→ OK for most of particles but SM neutrino has only a L-chiral state (no ψ_R)

→ a Dirac mass term for neutrinos can not be built in the Standard Model

Fermion masses in the SM lagrangian

- ◆ In the SM, fermion masses appear in the lagrangian with terms like:

$$m\bar{\psi}\psi \quad \rightarrow \text{Dirac mass term}$$

decomposing into its chiral states: $\psi \equiv \psi_L + \psi_R$

$$\rightarrow m\bar{\psi}\psi = m\bar{\psi}_L\psi_R + m\bar{\psi}_R\psi_L$$

→ **forbidden**: not invariant under SU(2): it couples ψ_L with ψ_R ($I_W=1/2$)

→ solved by **Higgs mechanism**: after SSB, Dirac mass terms appear from Yukawa couplings:

$$\mathcal{L}_{\text{Yukawa}} = Y\bar{\psi}_L\phi\psi_R + \text{h.c.} \quad \langle\phi^0\rangle = v$$

→ OK for most of particles but SM neutrino has only a L-chiral state (no ψ_R)

→ a Dirac mass term for neutrinos can not be built in the Standard Model

What about a Majorana mass term??

Majorana neutrino mass

Majorana, ~1930

◆ We build a R-chiral field from a L-chiral field by charge conjugation:

$$\psi_R \equiv \psi_L^C = \hat{C} \bar{\psi}^T \quad \hat{C} = i\gamma^2 \gamma^0$$

→ the total neutrino field is: $\psi = \psi_L + \psi_R = \psi_L + \psi_L^C$

→ taking the charge conjugate: $\psi^C = (\psi_L + \psi_R)^C = \psi_L^C + \psi_L = \psi$

$$\psi = \nu = \nu_L + \nu_L^C$$

neutrino = antineutrino

Majorana mass term:

$$-\mathcal{L}_M = \frac{1}{2} m \left(\overline{\nu_L^C} \nu_L + \overline{\nu_L} \nu_L^C \right)$$

Not invariant under U(1) transformations

However: this mass term not invariant under weak isospin ($I_W=1$)

→ solved with a **Higgs triplet** BUT it is not included in the SM.

→ solved with a **dim-5 operator** (Weinberg operator) BUT non-renormalizable

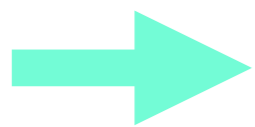
Neutrino mass in the SM

- ◆ Since the SM does not contain **right-handed neutrinos**: a Dirac mass term as for the rest of fermions is not allowed.
- ◆ The SM only contains one Higgs doublet: no **Higgs triplet** to build a Majorana mass term
- ◆ The SM is **renormalizable** and, therefore, dim-5 terms as the Weinberg operator are not allowed.

Neutrinos are strictly massless in the Standard Model!

Neutrino mass in the SM

- ◆ Since the SM does not contain **right-handed neutrinos**: a Dirac mass term as for the rest of fermions is not allowed.
- ◆ The SM only contains one Higgs doublet: no **Higgs triplet** to build a Majorana mass term
- ◆ The SM is **renormalizable** and, therefore, dim-5 terms as the Weinberg operator are not allowed.



Neutrinos are strictly massless in the Standard Model!

Neutrino oscillations



nobelprize.org

Neutrino oscillations

1957: Pontecorvo suggests oscillations between neutrinos & antineutrinos (only ν_e).

B. Pontecorvo, J. Exp. Theor. Phys. 33 (1957) 549.

B. Pontecorvo, J. Exp. Theor. Phys. 34 (1958) 247.



Бруно Понтекорво

1962: Maki, Nakagawa and Sakata propose neutrino mixing between flavor eigenstates

$$\begin{aligned}\nu_1 &= \nu_e \cos \delta + \nu_\mu \sin \delta, \\ \nu_2 &= -\nu_e \sin \delta + \nu_\mu \cos \delta.\end{aligned}$$

2 ν mixing

Z. Maki, M. Nakagawa, S. Sakata,
Prog. Theor. Phys. 28 (1962) 870.

true
neutrinos

weak
neutrinos



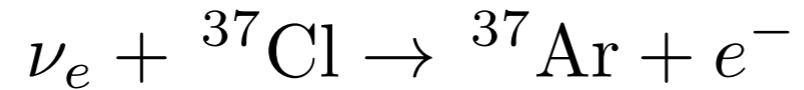
Shoichi Sakata

1969: Gribov & Pontecorvo calculated the neutrino oscillation probability (in vacuum) for the first time

V. Gribov, B. Pontecorvo, Phys. Lett. B28 (1969) 493.

First indication of ν oscillations

1968: First observation of solar neutrinos by R. Davis in Homestake.

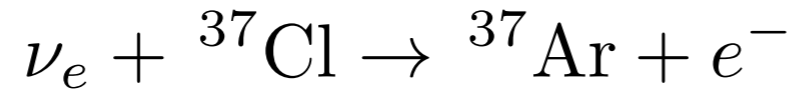


➔ 1/3 of the Standard Solar Model prediction !!

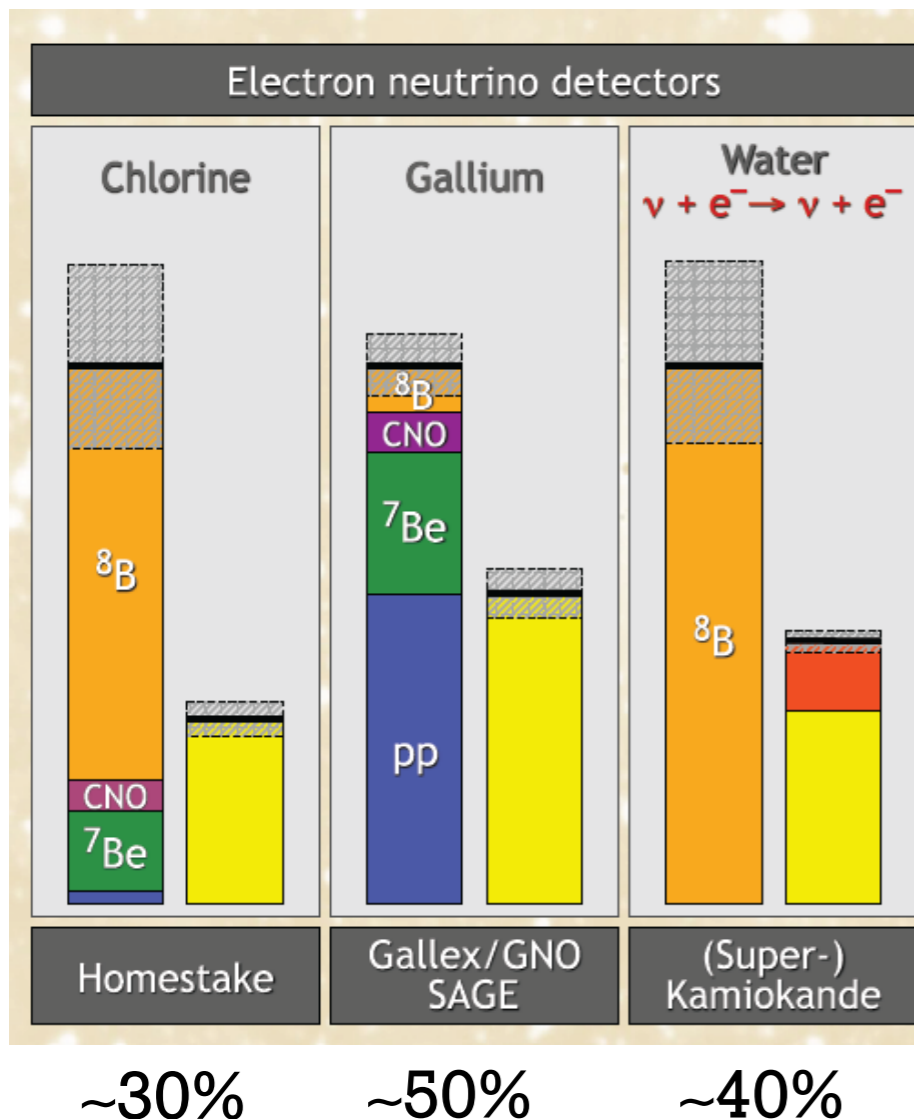


First indication of ν oscillations

1968: First observation of solar neutrinos by R. Davis in Homestake.



→ 1/3 of the Standard Solar Model prediction !!

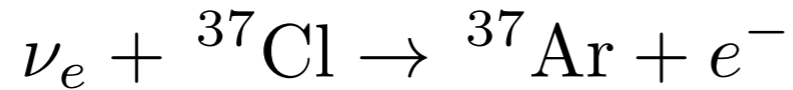


→ confirmed by the following solar experiments

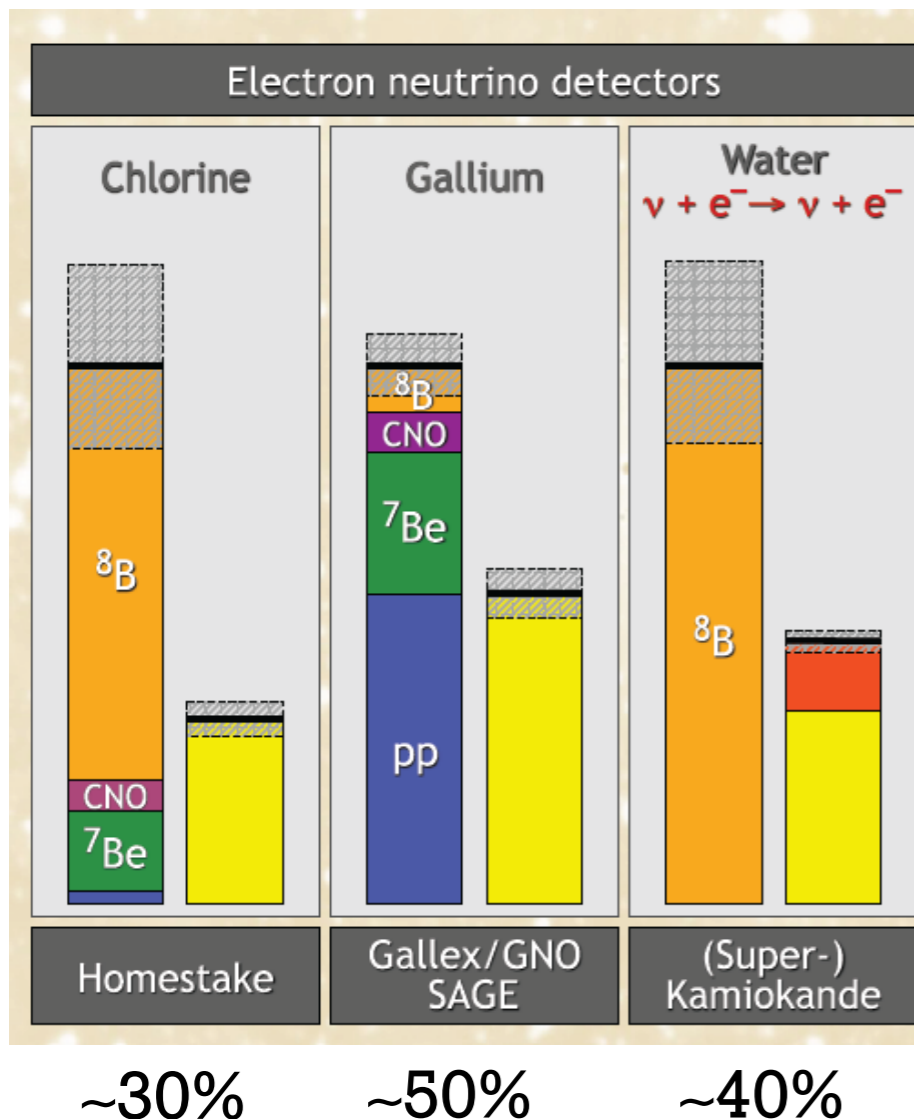


First indication of ν oscillations

1968: First observation of solar neutrinos by R. Davis in Homestake.



→ 1/3 of the Standard Solar Model prediction !!



→ confirmed by the following solar experiments

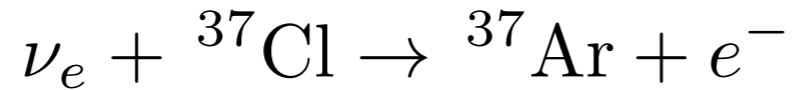
Explanation?

- theory (SM, SSM) was wrong
- experiments were wrong (all of them?)
- something was happening to neutrinos

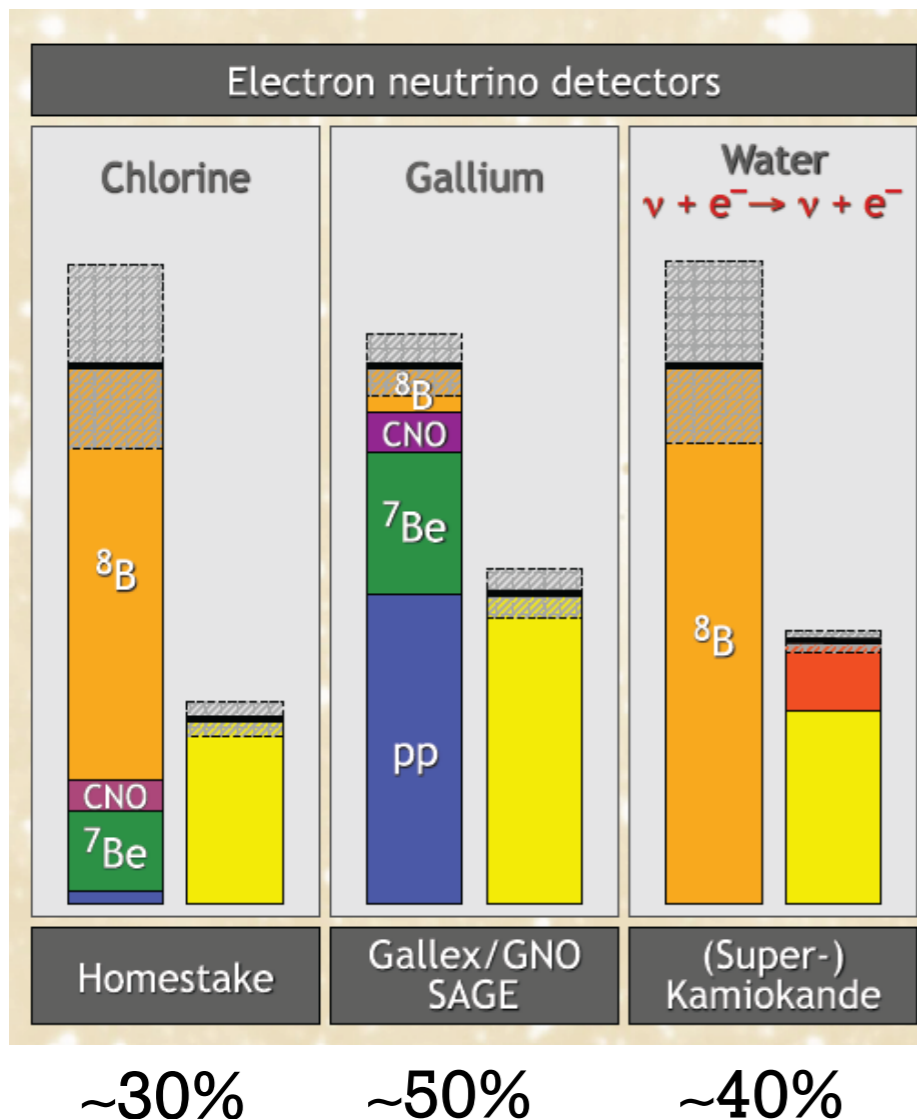


First indication of ν oscillations

1968: First observation of solar neutrinos by R. Davis in Homestake.



→ 1/3 of the Standard Solar Model prediction !!



→ confirmed by the following solar experiments

Explanation?

→ theory (SM, SSM) was wrong

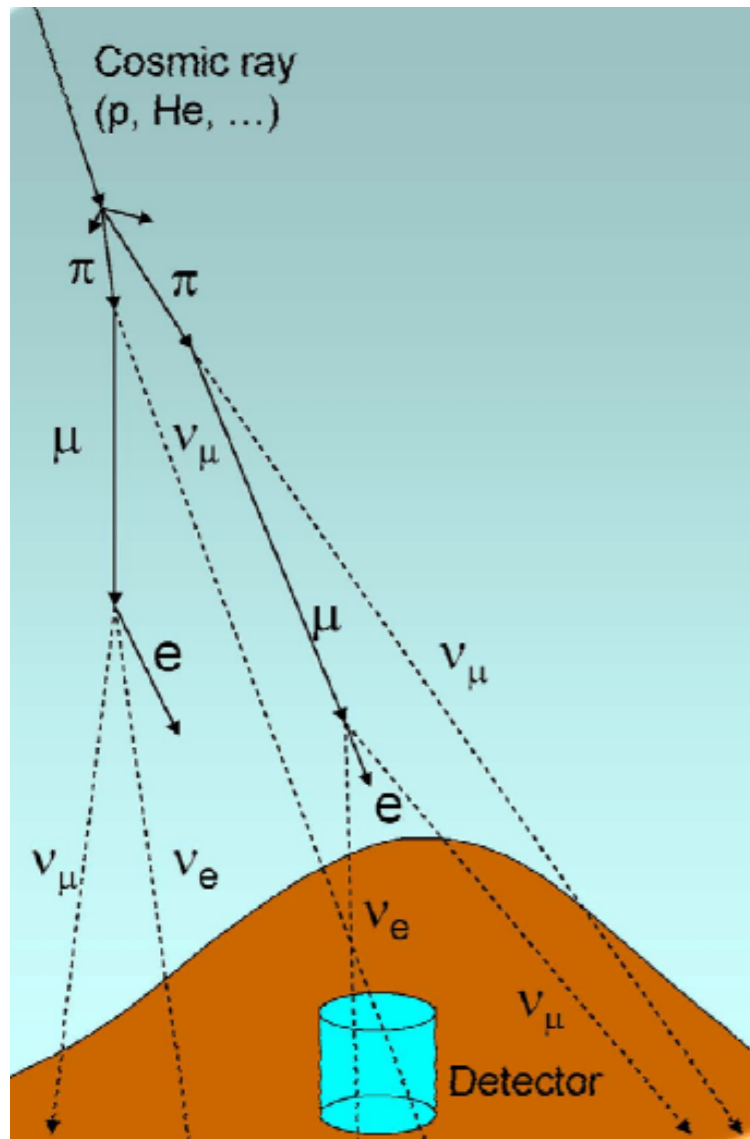
→ experiments were wrong (all of them?)

→ something was happening to neutrinos



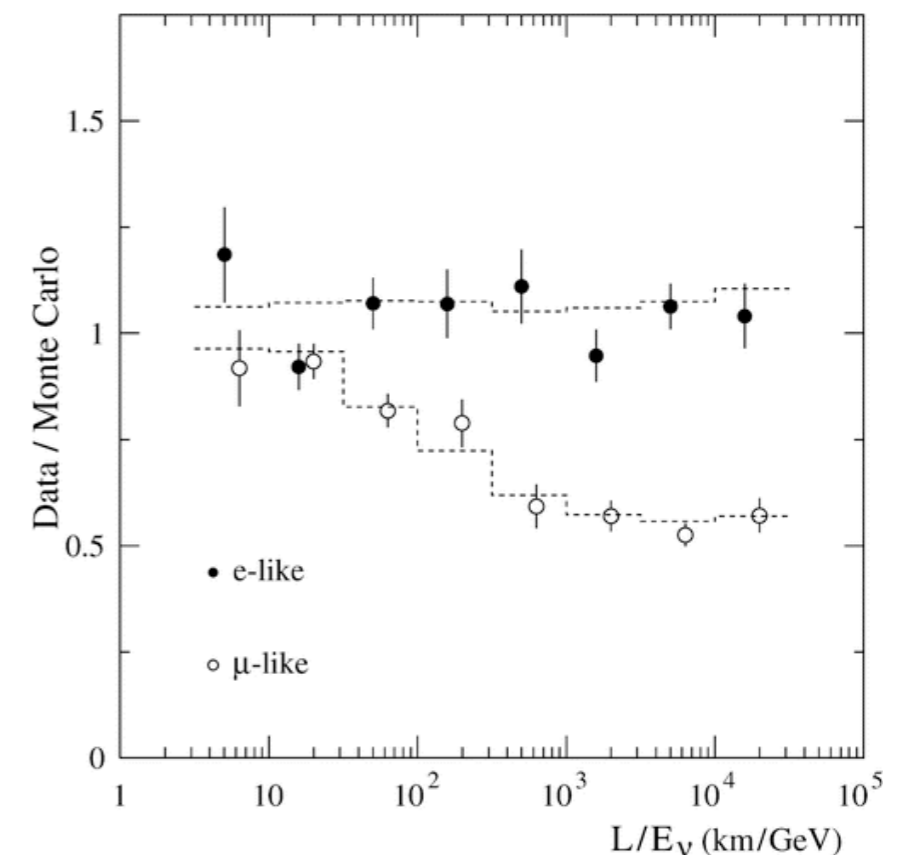
2002 Nobel Prize in Physics

The atmospheric neutrino anomaly



1985: First indications of a deficit in the observed number of atmospheric ν_μ at the IMB experiment.

1994: Kamiokande finds the ν_μ deficit depends on the distance travelled by the neutrino and its energy.

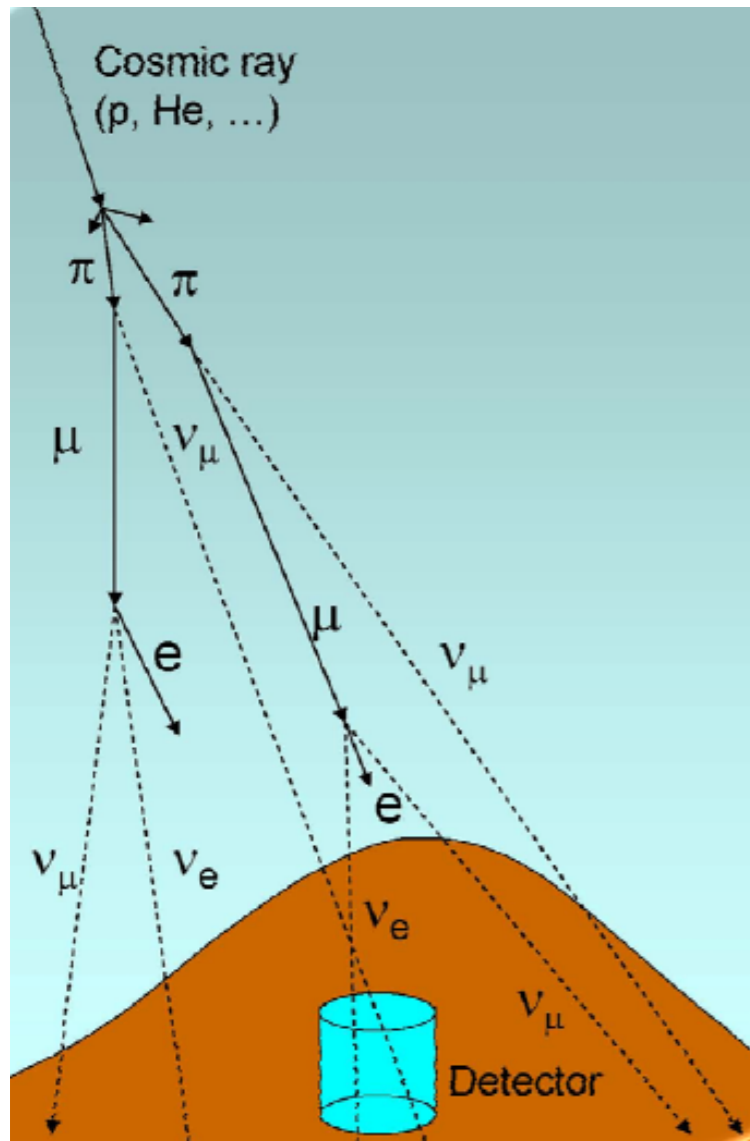


1998: Discovery of atmospheric neutrino oscillations in Super-Kamiokande.

oscillation channel $\nu_\mu \rightarrow \nu_\tau$

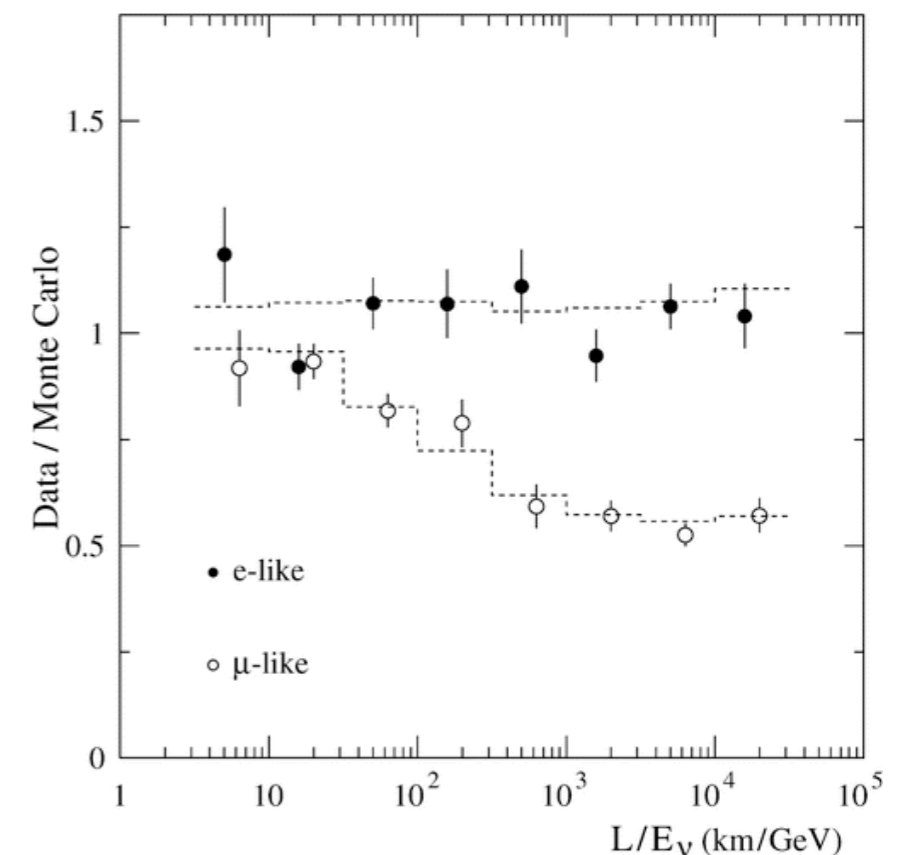
➔ first evidence for non-zero **neutrino masses**.

The atmospheric neutrino anomaly



1985: First indications of a deficit in the observed number of atmospheric ν_μ at the IMB experiment.

1994: Kamiokande finds the ν_μ deficit depends on the distance travelled by the neutrino and its energy.

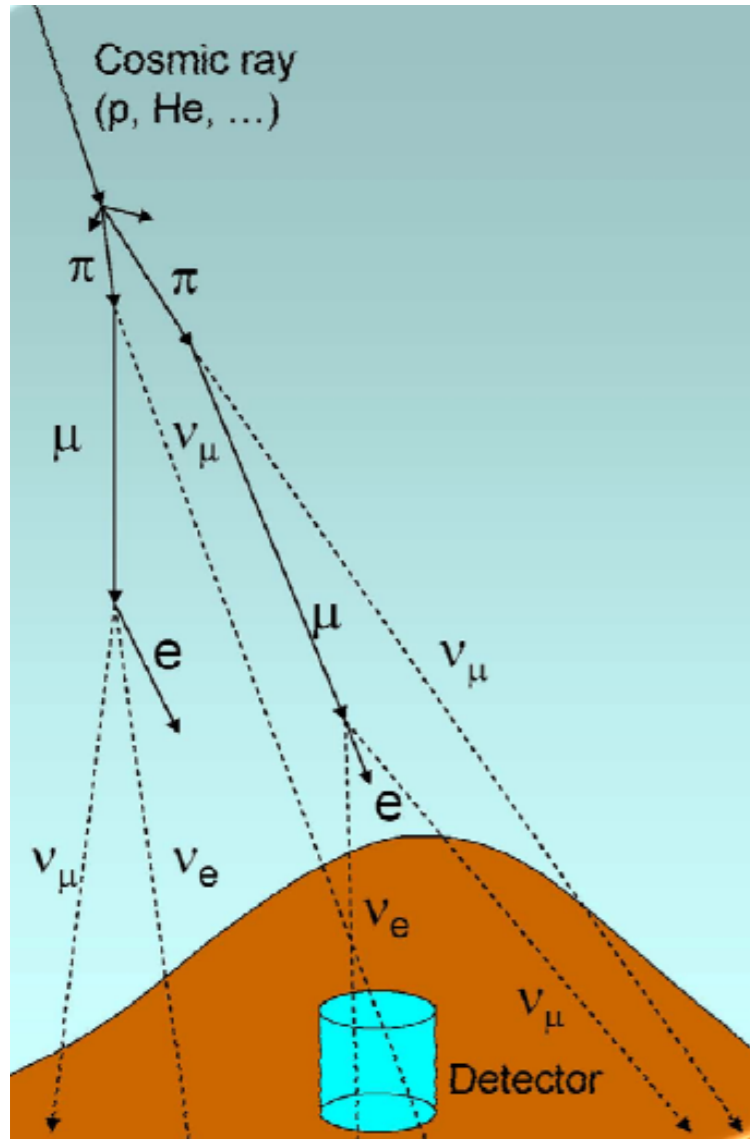


1998: Discovery of atmospheric neutrino oscillations in Super-Kamiokande.

oscillation channel $\nu_\mu \rightarrow \nu_\tau$

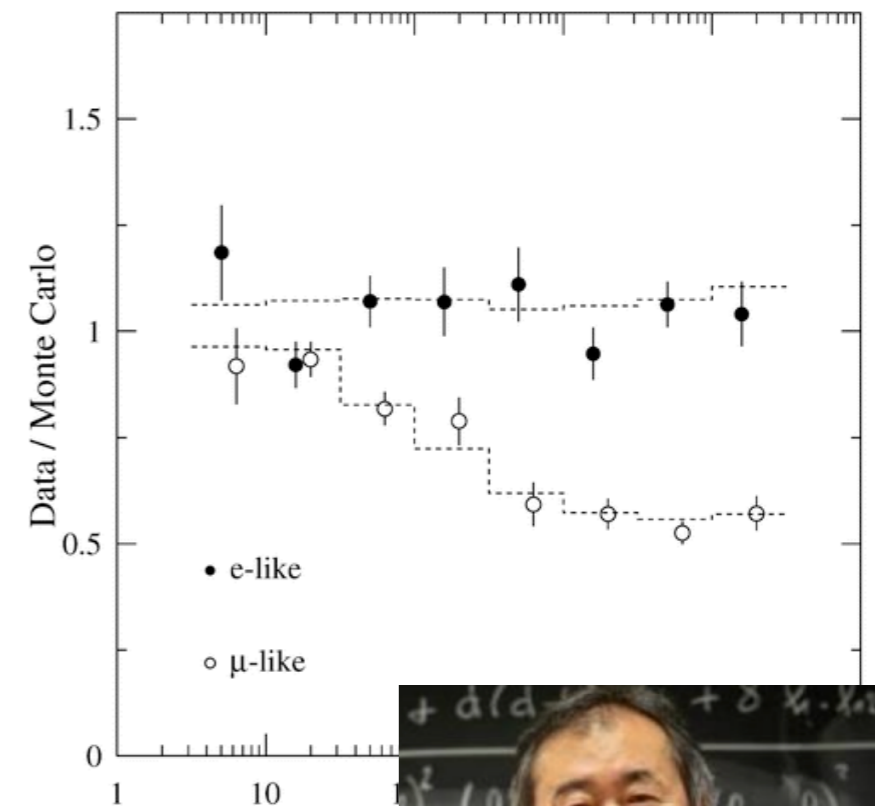
➔ first evidence for non-zero **neutrino masses**.

The atmospheric neutrino anomaly



1985: First indications of a deficit in the observed number of atmospheric ν_μ at the IMB experiment.

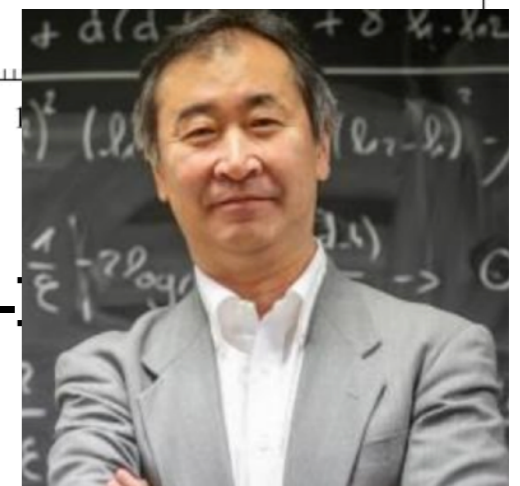
1994: Kamiokande finds the ν_μ deficit depends on the distance travelled by the neutrino and its energy.



1998: Discovery of atmospheric neutrino oscillations in Super-

oscillation channel $\nu_\mu \rightarrow \nu_\tau$

→ first evidence for non-zero **neutrino masses**.

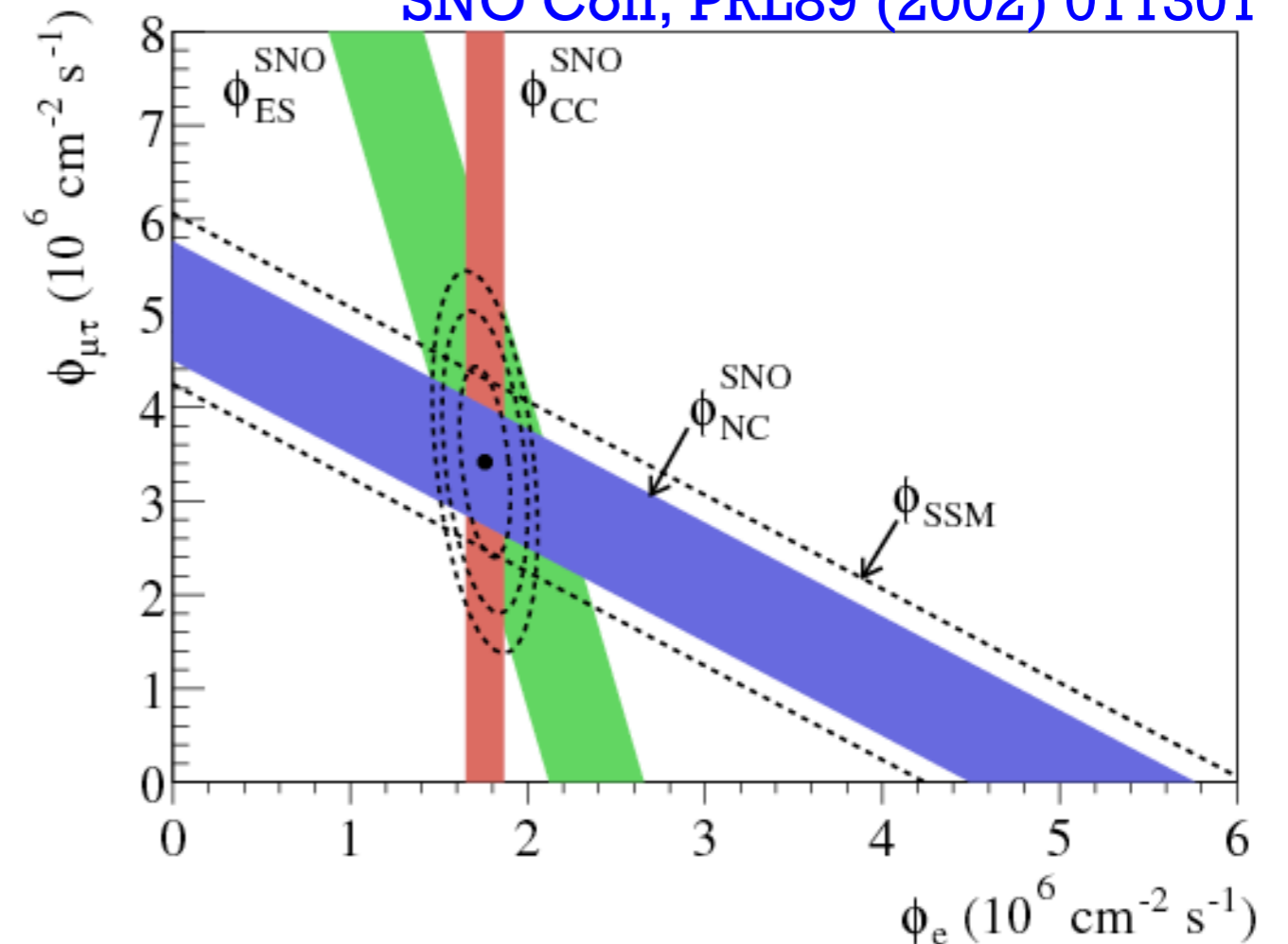
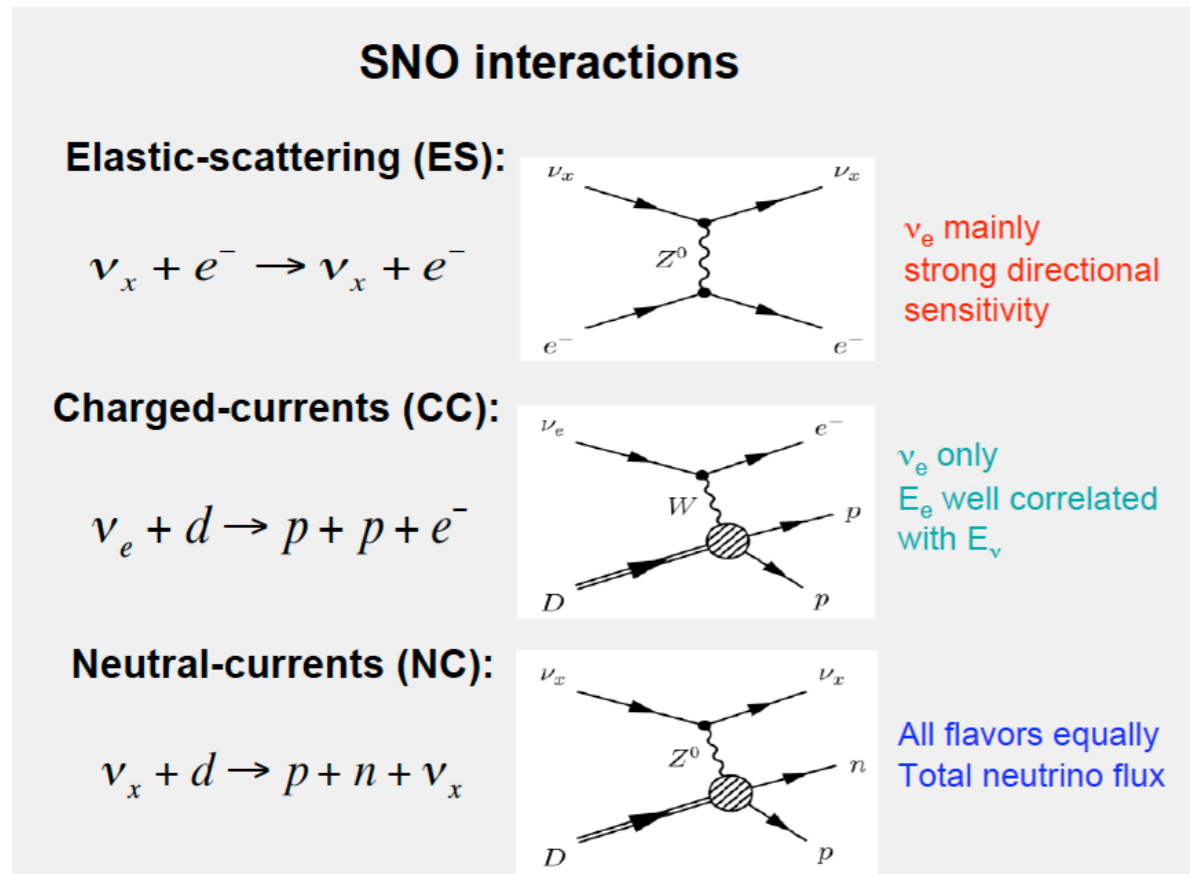


2015 Nobel Prize in Physics

The Sudbury Neutrino Observatory

2001: Confirmation of flavor conversion in solar neutrinos in SNO.

SNO Coll, PRL89 (2002) 011301



$$\frac{\phi_{\text{CC}}^{\text{SNO}}}{\phi_{\text{NC}}^{\text{SNO}}} = 0.301 \pm 0.033 \quad \rightarrow \quad \text{30\% of solar neutrinos are detected as } \nu_e$$

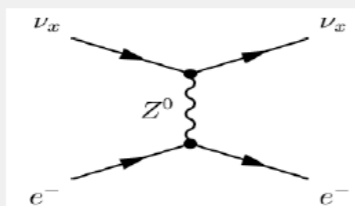
$$\phi_{\text{NC}}^{\text{SNO}} \simeq \phi_{8\text{B}}^{\text{SSM}} \quad \rightarrow \quad \boxed{\text{conversion } \nu_e \rightarrow \nu_{\mu\tau}}$$

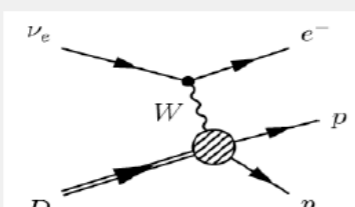
The Sudbury Neutrino Observatory

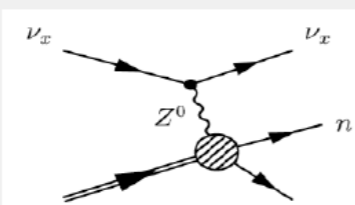
2001: Confirmation of flavor conversion in solar neutrinos in SNO.

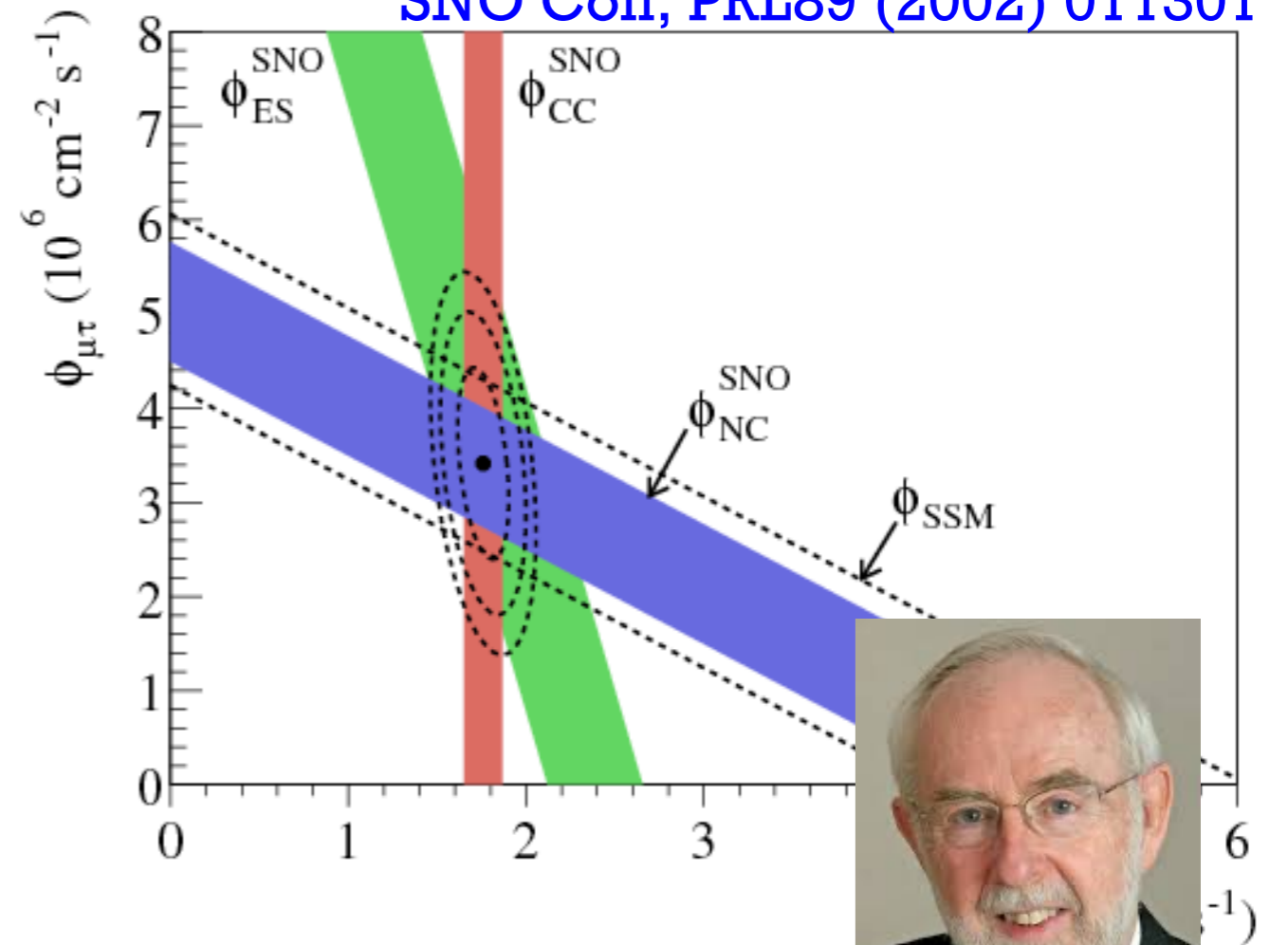
SNO Coll, PRL89 (2002) 011301

SNO interactions

Elastic-scattering (ES):
 $\nu_x + e^- \rightarrow \nu_x + e^-$

 ν_e mainly strong directional sensitivity

Charged-currents (CC):
 $\nu_e + d \rightarrow p + p + e^-$

 *ν_e only
 E_e well correlated with E_ν*

Neutral-currents (NC):
 $\nu_x + d \rightarrow p + n + \nu_x$

*All flavors equally
 Total neutrino flux*



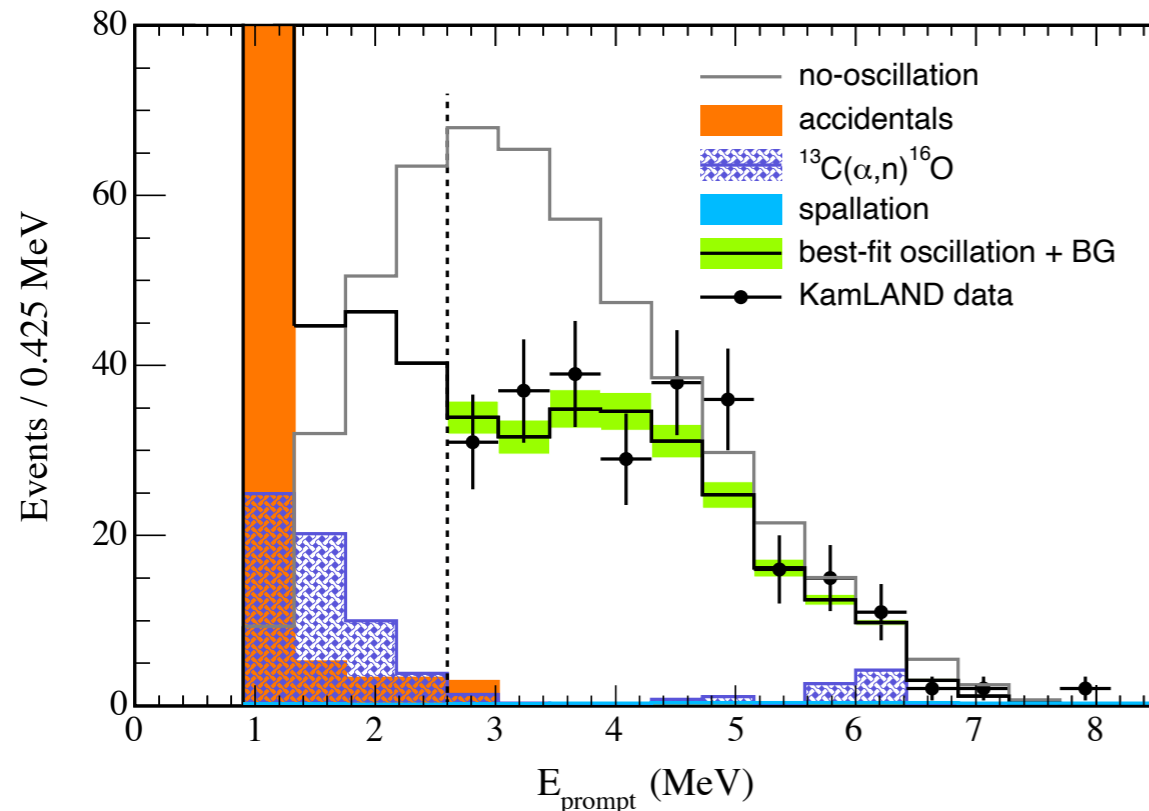
2015 Nobel Prize in Physics

$$\frac{\phi_{CC}^{SNO}}{\phi_{NC}^{SNO}} = 0.301 \pm 0.033 \rightarrow 30\% \text{ of solar neutrinos are detected as } \nu_e$$

$$\phi_{NC}^{SNO} \simeq \phi_{8B}^{SSM} \rightarrow \text{conversion } \nu_e \rightarrow \nu_{\mu\tau}$$

Other important results

2002: The reactor experiment **KamLAND** observed neutrino oscillations consistent with the solar anomaly.



KamLAND Coll, PRL 90 (2003) 021802

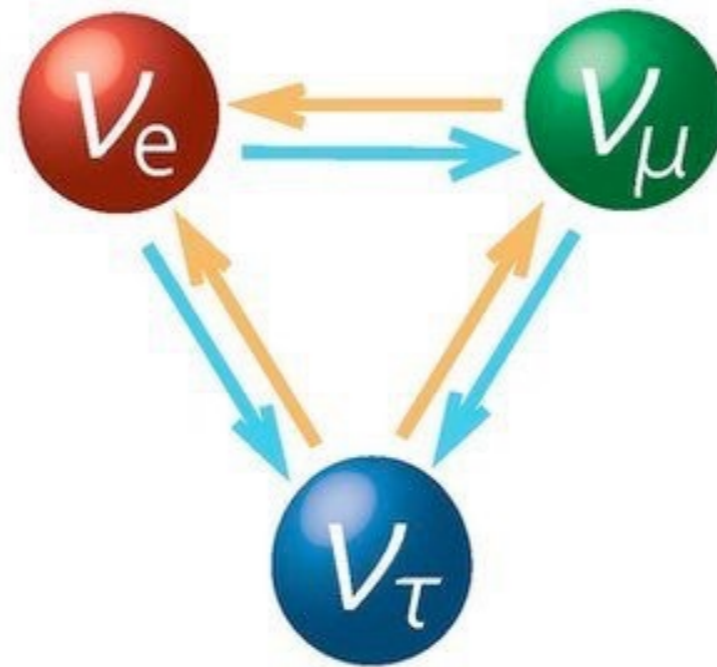
2002: Results of the accelerator experiment **K2K** consistent with ν_{μ} oscillations as in the atmospheric anomaly (**MINOS, T2K, NO ν A**).

2011: $\nu_{\mu} \rightarrow \nu_e$ oscillations observed in long-baseline accelerator experiments.

2011: Double Chooz confirmed reactor antineutrino oscillations in a baseline of ~ 1 km (**Daya Bay, RENO**).

neutrino oscillations have been observed in solar, atmospheric, reactor and accelerator neutrino experiments.

Neutrino oscillations: formalism



Neutrino mixing

- ◆ Mixing described by the **Pontecorvo-Maki-Nakagawa-Sakata** (PMNS) matrix:

$$\nu_{\alpha L} = \sum_k U_{\alpha k} \nu_{k L}$$

Neutrino mixing

- ◆ Mixing described by the **Pontecorvo-Maki-Nakagawa-Sakata** (PMNS) matrix:

$$\nu_{\alpha L} = \sum_k U_{\alpha k} \nu_{k L}$$

- ◆ NxN unitary matrix: NxN real parameters

→ $N(N-1)/2$ **mixing angles** + $N(N+1)/2$ **phases** (not all observables!)

Neutrino mixing

- ♦ Mixing described by the **Pontecorvo-Maki-Nakagawa-Sakata** (PMNS) matrix:

$$\nu_{\alpha L} = \sum_k U_{\alpha k} \nu_{k L}$$

- ♦ NxN unitary matrix: NxN real parameters

→ $N(N-1)/2$ **mixing angles** + $N(N+1)/2$ **phases** (not all observables!)

- ♦ Leptonic weak charged current:

$$j_{\rho}^{\text{CC}\dagger} = 2 \sum_{\alpha} \bar{\alpha}_L \gamma_{\rho} \nu_{\alpha L} = 2 \sum_{\alpha} \sum_k \bar{\alpha}_L \gamma_{\rho} U_{\alpha k} \nu_{k L}$$

$U = U_l^{\dagger} U_{\nu}$

Neutrino mixing

- ◆ Mixing described by the **Pontecorvo-Maki-Nakagawa-Sakata** (PMNS) matrix:

$$\nu_{\alpha L} = \sum_k U_{\alpha k} \nu_{k L}$$

- ◆ NxN unitary matrix: NxN real parameters

→ $N(N-1)/2$ **mixing angles** + $N(N+1)/2$ **phases** (not all observables!)

- ◆ Leptonic weak charged current:

$$j_{\rho}^{\text{CC}\dagger} = 2 \sum_{\alpha} \overline{\alpha}_L \gamma_{\rho} \nu_{\alpha L} = 2 \sum_{\alpha} \sum_k \overline{\alpha}_L \gamma_{\rho} U_{\alpha k} \nu_{k L}$$

$$U = U_l^{\dagger} U_{\nu}$$

- ◆ Lagrangian invariant under global phase transformations of Dirac fields:

$$\alpha \rightarrow e^{i\theta_{\alpha}} \alpha, \quad \nu_k \rightarrow e^{i\phi_k} \nu_k$$

+ common rephasing of fields leaves current unchanged

Neutrino mixing

- ◆ Mixing described by the **Pontecorvo-Maki-Nakagawa-Sakata** (PMNS) matrix:

$$\nu_{\alpha L} = \sum_k U_{\alpha k} \nu_{k L}$$

- ◆ NxN unitary matrix: NxN real parameters

→ N(N-1)/2 **mixing angles** + N(N+1)/2 **phases** (not all observables!)

- ◆ Leptonic weak charged current:

$$j_{\rho}^{\text{CC}\dagger} = 2 \sum_{\alpha} \bar{\alpha}_L \gamma_{\rho} \nu_{\alpha L} = 2 \sum_{\alpha} \sum_k \bar{\alpha}_L \gamma_{\rho} U_{\alpha k} \nu_{k L}$$

$$U = U_l^{\dagger} U_{\nu}$$

- ◆ Lagrangian invariant under global phase transformations of Dirac fields:

$$\alpha \rightarrow e^{i\theta_{\alpha}} \alpha, \quad \nu_k \rightarrow e^{i\phi_k} \nu_k$$

$$j_{\rho}^{\text{CC}\dagger} \rightarrow 2 \sum_{\alpha, k} \bar{\alpha}_L e^{-i(\theta_{\alpha} - \phi_1)} \gamma_{\rho} U_{\alpha k} e^{i(\phi_k - \phi_1)} \nu_{k L}$$

N **N-1**

+ common rephasing of fields leaves current unchanged

2N-1 arbitrary phases can be eliminated from U: (N-1)(N-2)/2 physical phases

Neutrino mixing

◆ For **Majorana neutrinos**, the lagrangian is NOT invariant under global phase transformations of the Majorana fields:

$$\nu_k \rightarrow e^{i\phi_k} \nu_k \quad \longrightarrow \quad \nu_{kL}^T \mathcal{C}^\dagger \nu_{kL} \rightarrow e^{2i\phi_k} \nu_{kL}^T \mathcal{C}^\dagger \nu_{kL}$$

Neutrino mixing

◆ For **Majorana neutrinos**, the lagrangian is NOT invariant under global phase transformations of the Majorana fields:

$$\nu_k \rightarrow e^{i\phi_k} \nu_k \quad \rightarrow \quad \nu_{kL}^T \mathcal{C}^\dagger \nu_{kL} \rightarrow e^{2i\phi_k} \nu_{kL}^T \mathcal{C}^\dagger \nu_{kL}$$

→ only N phases can be eliminated by rephasing charged lepton fields (neutrino fields can not be rephased!!):

$$j_\rho^{\text{CC}^\dagger} \rightarrow 2 \sum_{\alpha,k} \overline{\alpha_L} e^{-i\theta_\alpha} \gamma_\rho U_{\alpha k} \nu_{kL}$$

\mathbf{N}

Neutrino mixing

♦ For **Majorana neutrinos**, the lagrangian is NOT invariant under global phase transformations of the Majorana fields:

$$\nu_k \rightarrow e^{i\phi_k} \nu_k \quad \longrightarrow \quad \nu_{kL}^T \mathcal{C}^\dagger \nu_{kL} \rightarrow e^{2i\phi_k} \nu_{kL}^T \mathcal{C}^\dagger \nu_{kL}$$

→ only N phases can be eliminated by rephasing charged lepton fields (neutrino fields can not be rephased!!):

$$j_\rho^{\text{CC}^\dagger} \rightarrow 2 \sum_{\alpha, k} \overline{\alpha_L} e^{-i\theta_\alpha} \gamma_\rho U_{\alpha k} \nu_{kL}$$

$N(N+1)/2 - N = N(N-1)/2$ physical phases for Majorana neutrinos

Neutrino mixing

◆ For **Majorana neutrinos**, the lagrangian is NOT invariant under global phase transformations of the Majorana fields:

$$\nu_k \rightarrow e^{i\phi_k} \nu_k \quad \longrightarrow \quad \nu_{kL}^T \mathcal{C}^\dagger \nu_{kL} \rightarrow e^{2i\phi_k} \nu_{kL}^T \mathcal{C}^\dagger \nu_{kL}$$

→ only N phases can be eliminated by rephasing charged lepton fields (neutrino fields can not be rephased!!):

$$j_\rho^{\text{CC}^\dagger} \rightarrow 2 \sum_{\alpha,k} \overline{\alpha_L} e^{-i\theta_\alpha} \gamma_\rho U_{\alpha k} \nu_{kL}$$

$N(N+1)/2 - N = N(N-1)/2$ physical phases for Majorana neutrinos

→ $N(N-1)/2$ **physical phases**: $(N-1)(N-2)/2$ Dirac phases

$(N-1)$ Majorana phases

Neutrino mixing

◆ For **Majorana neutrinos**, the lagrangian is NOT invariant under global phase transformations of the Majorana fields:

$$\nu_k \rightarrow e^{i\phi_k} \nu_k \quad \longrightarrow \quad \nu_{kL}^T \mathcal{C}^\dagger \nu_{kL} \rightarrow e^{2i\phi_k} \nu_{kL}^T \mathcal{C}^\dagger \nu_{kL}$$

→ only N phases can be eliminated by rephasing charged lepton fields (neutrino fields can not be rephased!!):

$$j_\rho^{\text{CC}^\dagger} \rightarrow 2 \sum_{\alpha, k} \overline{\alpha_L} e^{-i\theta_\alpha} \gamma_\rho U_{\alpha k} \nu_{kL}$$

$N(N+1)/2 - N = N(N-1)/2$ physical phases for Majorana neutrinos

→ $N(N-1)/2$ **physical phases**: $(N-1)(N-2)/2$ Dirac phases → effect in ν oscil.

$(N-1)$ Majorana phases

Neutrino mixing

◆ For **Majorana neutrinos**, the lagrangian is NOT invariant under global phase transformations of the Majorana fields:

$$\nu_k \rightarrow e^{i\phi_k} \nu_k \quad \rightarrow \quad \nu_{kL}^T \mathcal{C}^\dagger \nu_{kL} \rightarrow e^{2i\phi_k} \nu_{kL}^T \mathcal{C}^\dagger \nu_{kL}$$

→ only N phases can be eliminated by rephasing charged lepton fields (neutrino fields can not be rephased!!):

$$j_\rho^{\text{CC}^\dagger} \rightarrow 2 \sum_{\alpha, k} \overline{\alpha_L} e^{-i\theta_\alpha} \gamma_\rho U_{\alpha k} \nu_{kL}$$

$N(N+1)/2 - N = N(N-1)/2$ physical phases for Majorana neutrinos

→ $N(N-1)/2$ **physical phases**: $(N-1)(N-2)/2$ Dirac phases → effect in ν oscil.

$(N-1)$ Majorana phases → relevant for $0\nu\beta\beta$

Neutrino mixing

- ◆ 2-neutrino mixing depends on 1 angle only (+1 Majorana phase)

$$\begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$$

- ◆ 3-neutrino mixing is described by 3 angles and 1 Dirac (+2 Majorana) CP violating phases.

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

atmospheric + LBL

reactor + LBL

solar + KamLAND

Neutrino oscillations

◆ Flavour states are admixtures of mass eigenstates: $\nu_{\alpha L} = \sum_k U_{\alpha k} \nu_{k L}$

◆ Neutrino evolution equation: $-i \frac{d}{dt} |\nu\rangle = H |\nu\rangle$

in the neutrino mass eigenstates basis ν_j :

$$H = \begin{pmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_3 \end{pmatrix}$$

neutrino mass eigenstates evolve as plane waves *:

$$|\nu_j(t)\rangle = e^{-iE_j t} |\nu_j\rangle$$

◆ For ultrarelativistic neutrinos:

$$E_j \simeq E + \frac{m_j^2}{2E}$$

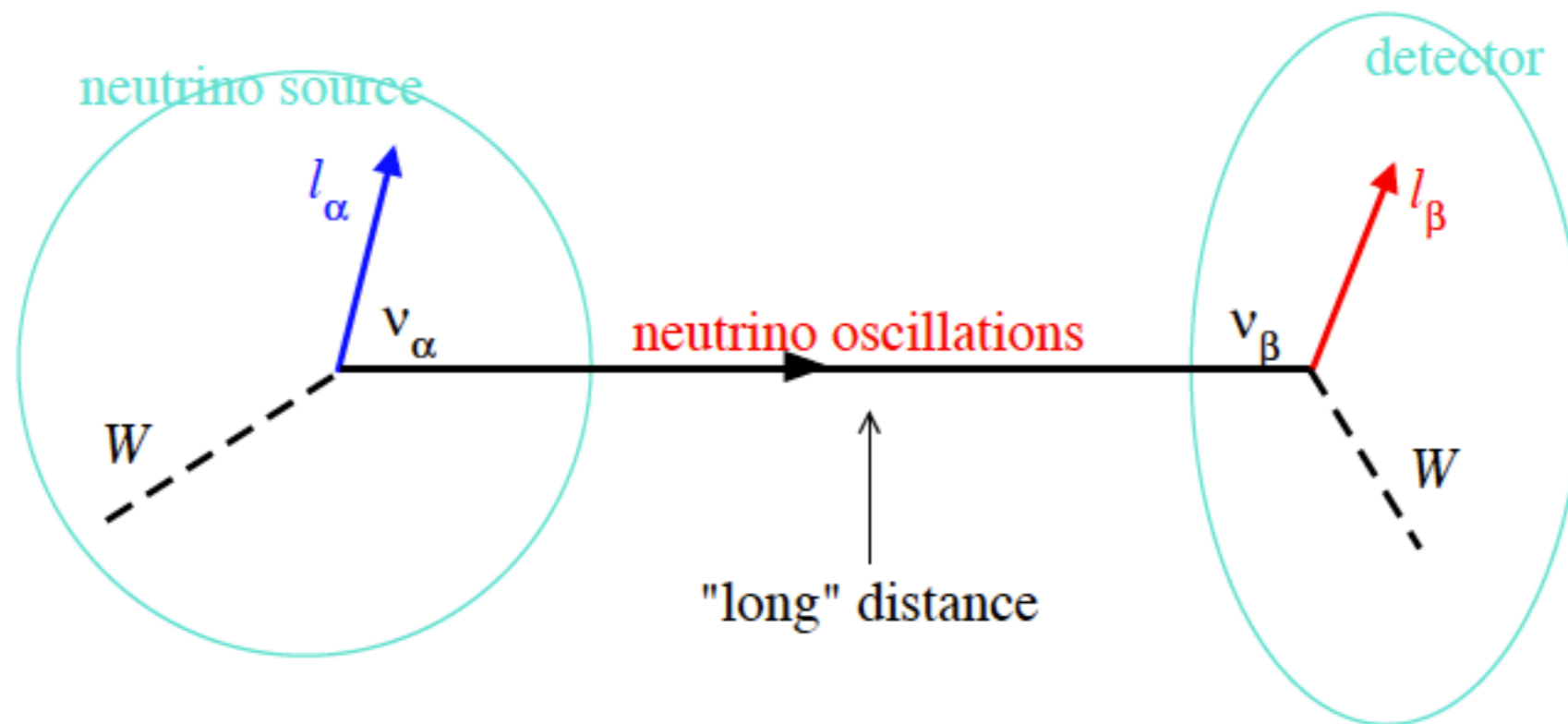
and $t = L$:

$$\longrightarrow |\nu_j(t)\rangle = e^{-iEL} e^{-i\frac{m_j^2 L}{2E}} |\nu_j\rangle \longrightarrow e^{-i\frac{m_j^2 L}{2E}} |\nu_j\rangle$$

* For a wave-packet treatment see:

Giunti & Kim, Fundamentals of Neutrino Physics and Astrophysics. Oxford University Press, 2007.

Neutrino oscillations picture



Production

$$|\nu_\alpha\rangle = \sum_j U_{\alpha j}^* |\nu_j\rangle$$

coherent superposition
of massive states

Propagation

$$\nu_j : e^{-i \frac{m_j^2 L}{2E}}$$

different propagation
phases change ν_j
composition

Detection

$$\langle \nu_\beta | = \sum_j \langle \nu_j | U_{\beta j}$$

projection over
flavour eigenstates

General properties of neutrino oscillations

◆ Conservation of probability: $\sum_{\beta} P(\nu_{\alpha} \rightarrow \nu_{\beta}) = 1$

◆ For **antineutrinos**: $U \rightarrow U^*$

◆ Neutrino oscillations violate flavour **lepton number conservation** but conserve total lepton number.

◆ Phases in the mixing matrix induce **CP violation**:

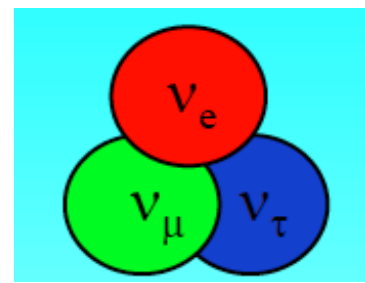
$$P(\nu_{\alpha} \rightarrow \nu_{\beta}) \neq P(\bar{\nu}_{\alpha} \rightarrow \bar{\nu}_{\beta})$$

◆ Neutrino oscillations do not depend on the absolute neutrino mass scale and Majorana phases.

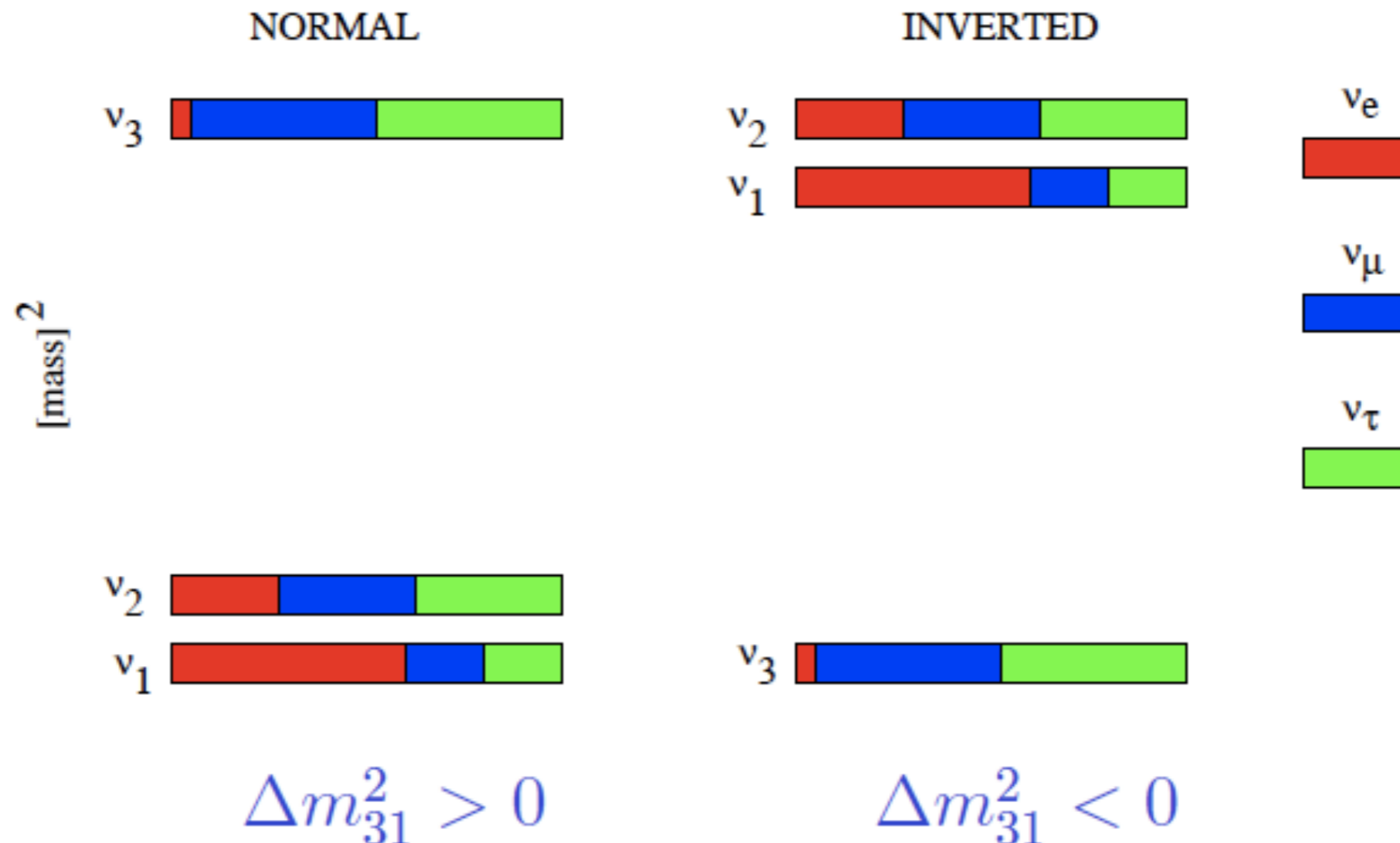
◆ Neutrino oscillations are sensitive only to **mass squared differences**:

$$\Delta m_{kj}^2 = m_k^2 - m_j^2$$

Two possible mass orderings



- ◆ Δm^2_{21} : solar + KamLAND (positive)
- ◆ Δm^2_{31} : atmospheric + LBL accelerator + SBL reactor (sign?)



Two-neutrino oscillations

◆ Two-neutrino mixing matrix:
$$\begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$$

◆ Two-neutrino oscillation probability ($\alpha \neq \beta$):

$$P(\nu_\alpha \rightarrow \nu_\beta) = \left| U_{\alpha 1} U_{\beta 1}^* + U_{\alpha 2} U_{\beta 2}^* e^{-i \frac{\Delta m_{21}^2 L}{2E}} \right|^2 = \sin^2 2\theta \sin^2 \left(\frac{\Delta m_{21}^2 L}{4E} \right)$$

◆ The **oscillation phase**:

$$\phi = \frac{\Delta m_{21}^2 L}{4E} = 1.27 \frac{\Delta m_{21}^2 [\text{eV}^2] L [\text{km}]}{E [\text{GeV}]}$$

→ short distances, $\phi \ll 1$: oscillations do not develop, $P_{\alpha\beta} = 0$

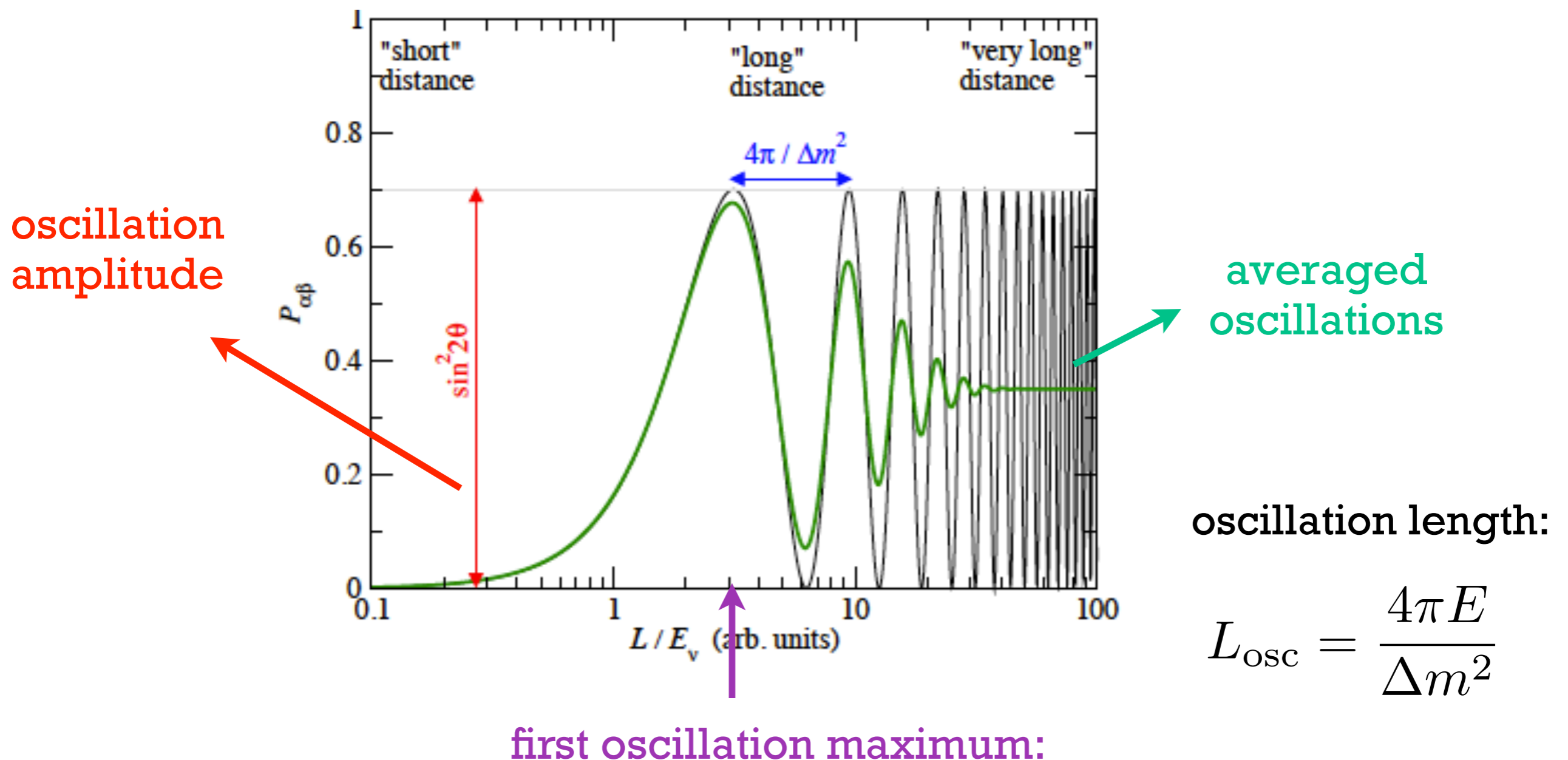
→ long distance, $\phi \sim 1$: oscillations are observable

→ very long distances, $\phi \gg 1$: oscillations are averaged out:

$$P_{\alpha\beta} \simeq \frac{1}{2} \sin^2 2\theta$$

2-neutrino oscillation probability

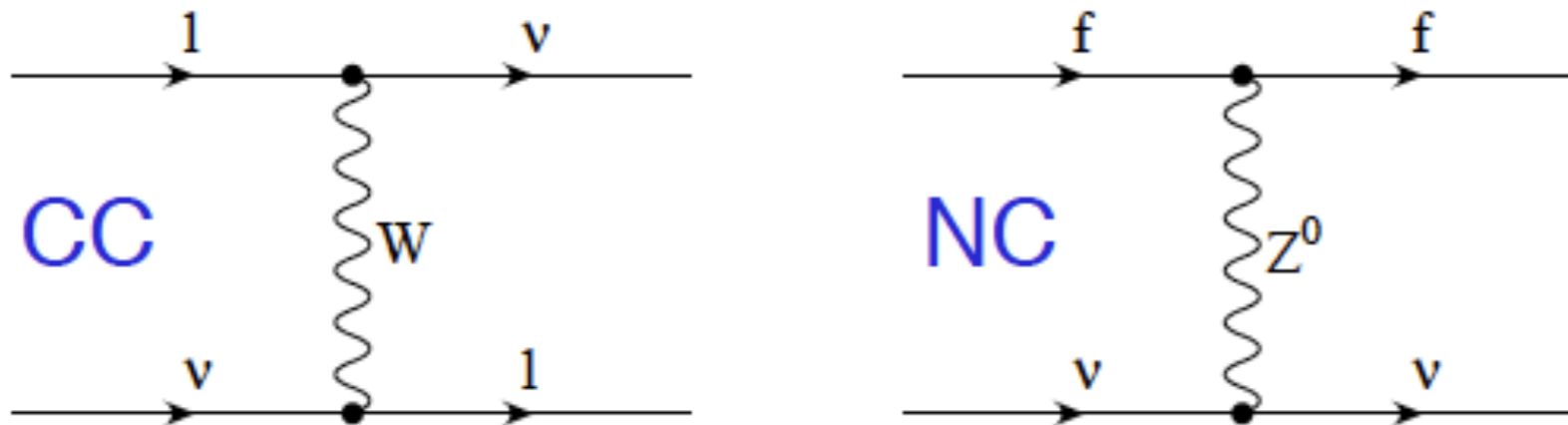
$$P_{\alpha\beta} = \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2 L}{4E} \right)$$



Matter effects on neutrino oscillations

◆ When neutrinos pass through matter, the interactions with the particles in the medium induce an **effective potential** for neutrinos.

[→ the coherent forward scattering amplitude leads to an index of refraction for neutrinos. **L. Wolfenstein, 1978**]



→ modifies the **mixing between flavor states and mass eigenstates** as well as the eigenvalues of the Hamiltonian, leading to a different oscillation probability with respect to vacuum oscillations.

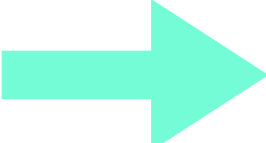
Effective matter potential

- ◆ Effective four-fermion interaction Hamiltonian (CC+NC)

$$H_{\text{int}}^{\nu_\alpha} = \frac{G_F}{\sqrt{2}} \bar{\nu}_\alpha \gamma_\mu (1 - \gamma_5) \nu_\alpha \sum_j \bar{f} \gamma_\mu (g_V^{\alpha,f} - g_A^{\alpha,f} \gamma_5) f$$

in ordinary matter: $f=e, p, n$

To obtain the **matter-induced potential** we integrate over f -variables,
For a non-relativistic unpolarised neutral medium


$$V_{\text{matt}} = \sqrt{2} G_F \text{diag} \left(N_e - \frac{1}{2} N_n, -\frac{1}{2} N_n, -\frac{1}{2} N_n \right)$$

- ◆ only ν_e are sensitive to CC (no μ, τ in ordinary matter)
- ◆ **NC** has the same effect for all flavours \rightarrow it has **no effect on evolution**
(however it can be important in presence of sterile neutrinos)
- ◆ for **antineutrinos** the potential has opposite sign

2-neutrino oscillations in matter

- ◆ Hamiltonian in **vacuum** in the flavour basis:

$$H_f^{\text{vac}} = U H_m U^\dagger = \frac{\Delta m^2}{4E} \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}$$

- ◆ Effective hamiltonian in **matter**

$$H_f^{\text{matt}} = H_f^{\text{vac}} + V_{\text{eff}} = \begin{pmatrix} -\frac{\Delta m^2}{4E} \cos 2\theta + V_{CC} & \frac{\Delta m^2}{4E} \sin 2\theta \\ \frac{\Delta m^2}{4E} \sin 2\theta & \frac{\Delta m^2}{4E} \cos 2\theta \end{pmatrix}$$
$$V_{CC} = \sqrt{2} G_F N_e$$

Diagonalizing the Hamiltonian, we identify the mixing angle and mass splitting in matter:

$$H_f^{\text{matt}} = \frac{\Delta M^2}{4E} \begin{pmatrix} -\cos 2\theta_M & \sin 2\theta_M \\ \sin 2\theta_M & \cos 2\theta_M \end{pmatrix}$$

In general: $N_e = N_e(\mathbf{x})$, so θ_M and ΔM^2 will be function of \mathbf{x} as well

→ however, in some cases analytical solutions can be obtained

2-ν oscillations in constant matter

◆ If N_e is constant (good approximation for oscillations in the Earth crust):

→ θ_M and ΔM^2 are constant as well

→ we can use vacuum expression for oscillation probability, replacing “vacuum” parameters by “matter” parameters:

$$P_{\alpha\beta} = \sin^2 2\theta_M \sin^2 \left(\frac{\Delta M^2 L}{4E} \right)$$

$$\sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - A)^2}$$

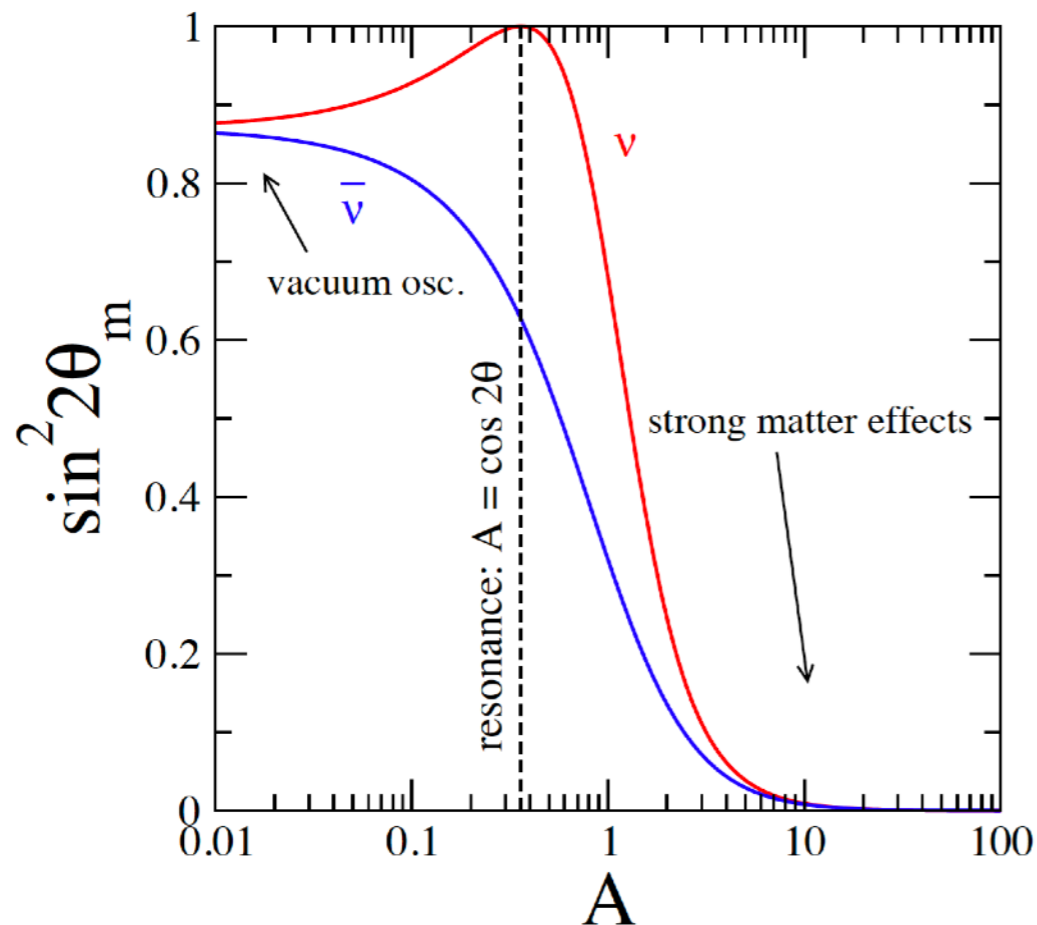
$$A = \frac{2EV}{\Delta m^2}$$

$$\Delta M^2 = \Delta m^2 \sqrt{\sin^2 2\theta + (\cos 2\theta - A)^2}$$

There is a **resonance** effect for $A = \cos 2\theta \rightarrow$ **MSW effect**

Wolfenstein, 1978. Mikheyev & Smirnov, 1986

2-ν oscillations in constant matter



mixing angle in matter:

$$\sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - A)^2}$$

$$A = \frac{2EV}{\Delta m^2}$$

- ◆ $A \ll \cos 2\theta$, small matter effect \rightarrow vacuum oscillations: $\theta_M = \theta$
- ◆ $A \gg \cos 2\theta$, matter effects dominate \rightarrow oscillations suppressed: $\theta_M \approx \pi/2$
- ◆ $A = \cos 2\theta$, resonance takes place \rightarrow maximal mixing $\theta_M \approx \pi/4$

\rightarrow **resonance condition** is satisfied for neutrinos for $\Delta m^2 > 0$

for antineutrinos for $\Delta m^2 < 0$

Solar neutrinos: the MSW effect

◆ neutrino oscillations in matter were first discussed by Wolfenstein, Mikheyev and Smirnov (MSW effect)

◆ electron neutrino is born at the center of the Sun as:

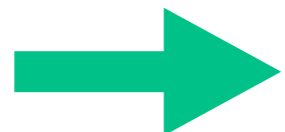
$$|\nu_e\rangle = \cos \theta_M |\nu_1^m\rangle + \sin \theta_M |\nu_2^m\rangle$$

→ ν_1^m and ν_2^m evolve adiabatically until the solar surface and propagate in vacuum from the Sun to the Earth:

$$P(\nu_e \rightarrow \nu_e) = P_{e1}^{\text{prod}} P_{1e}^{\text{det}} + P_{e2}^{\text{prod}} P_{2e}^{\text{det}}$$

$$P_{e1}^{\text{prod}} = \cos^2 \theta_M, \quad P_{1e}^{\text{det}} = \cos^2 \theta$$

$$P_{e2}^{\text{prod}} = \sin^2 \theta_M, \quad P_{2e}^{\text{det}} = \sin^2 \theta$$



$$P_{ee} = \cos^2 \theta_M \cos^2 \theta + \sin^2 \theta_M \sin^2 \theta$$

Solar neutrinos: the MSW effect

$$P_{ee} = \cos^2 \theta_M \cos^2 \theta + \sin^2 \theta_M \sin^2 \theta$$

◆ In the center of the Sun:

$$A = \frac{2EV}{\Delta m^2} \simeq 0.2 \left(\frac{E}{\text{MeV}} \right) \left(\frac{8 \times 10^{-5} \text{eV}^2}{\Delta m^2} \right)$$

and resonance occurs for $A = \cos(2\theta) = 0.4$

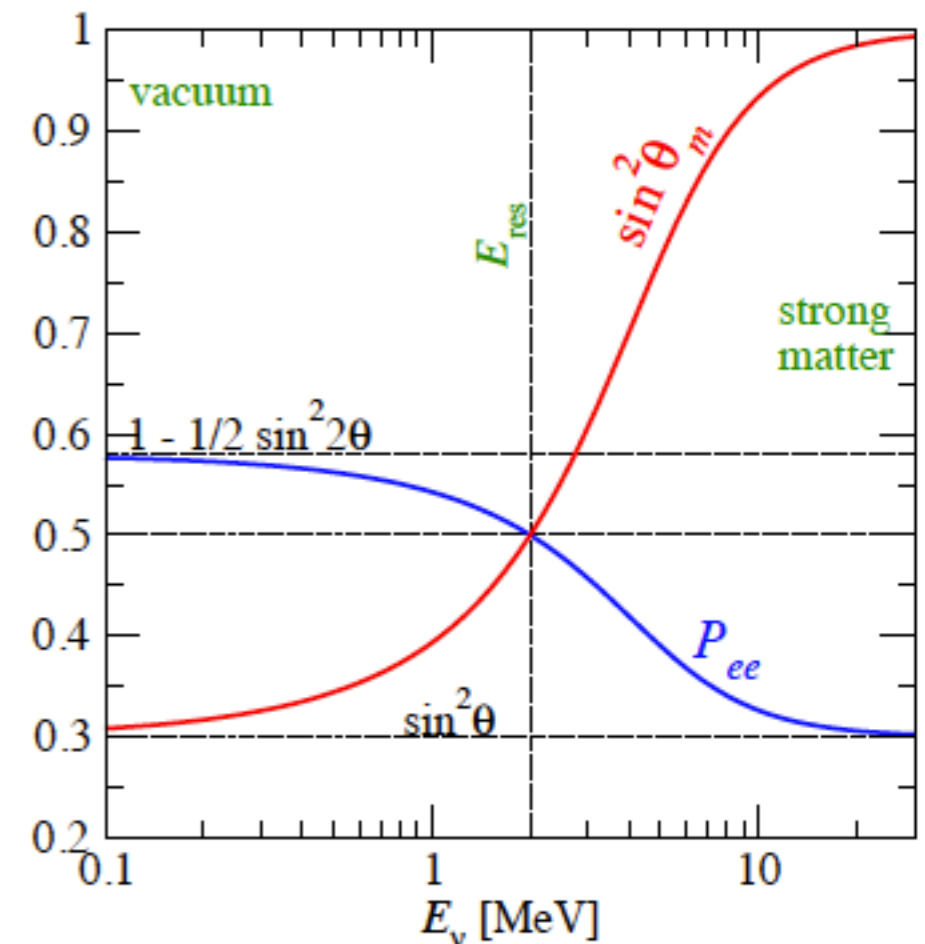
$$\rightarrow E_{\text{res}} \approx 2 \text{ MeV}$$

◆ For $E < 2 \text{ MeV} \rightarrow$ vacuum osc: $\theta_M = \theta$:

$$P_{ee} = 1 - \frac{1}{2} \sin^2 2\theta$$

◆ For $E > 2 \text{ MeV} \rightarrow$ strong matter effect: $\theta_M = \pi/2$: $P_{ee} = \sin^2 \theta$

$\rightarrow P_{ee}(E)$ will be crucial to understand solar neutrino data



Mass hierarchy in solar neutrinos

- ◆ Mixing angle in matter:

$$\sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - A)^2}$$

→ **resonance condition** $A = \cos 2\theta$ is satisfied for neutrinos for $\Delta m^2 > 0$ and for antineutrinos for $\Delta m^2 < 0$ (change of sign in V_{cc})

- ◆ Matter effects observed in solar neutrino data are in agreement with the presence of a resonance as predicted above:

→ since solar neutrinos are ν_e :

$$\Delta m_{21}^2 > 0 \rightarrow m_2^2 > m_1^2$$

Earth regeneration effect

- ◆ Neutrinos observed at night are also affected by Earth matter effects
- ◆ If neutrinos cross only the Earth mantle, P_{2e}^{det} is well approximated by the evolution of a constant potential:

$$P_{2e}^{\text{det}} = \underbrace{\sin^2 \theta}_{\text{prob. during day}} + \underbrace{f_{\text{reg}}}_{\text{regeneration term}}$$

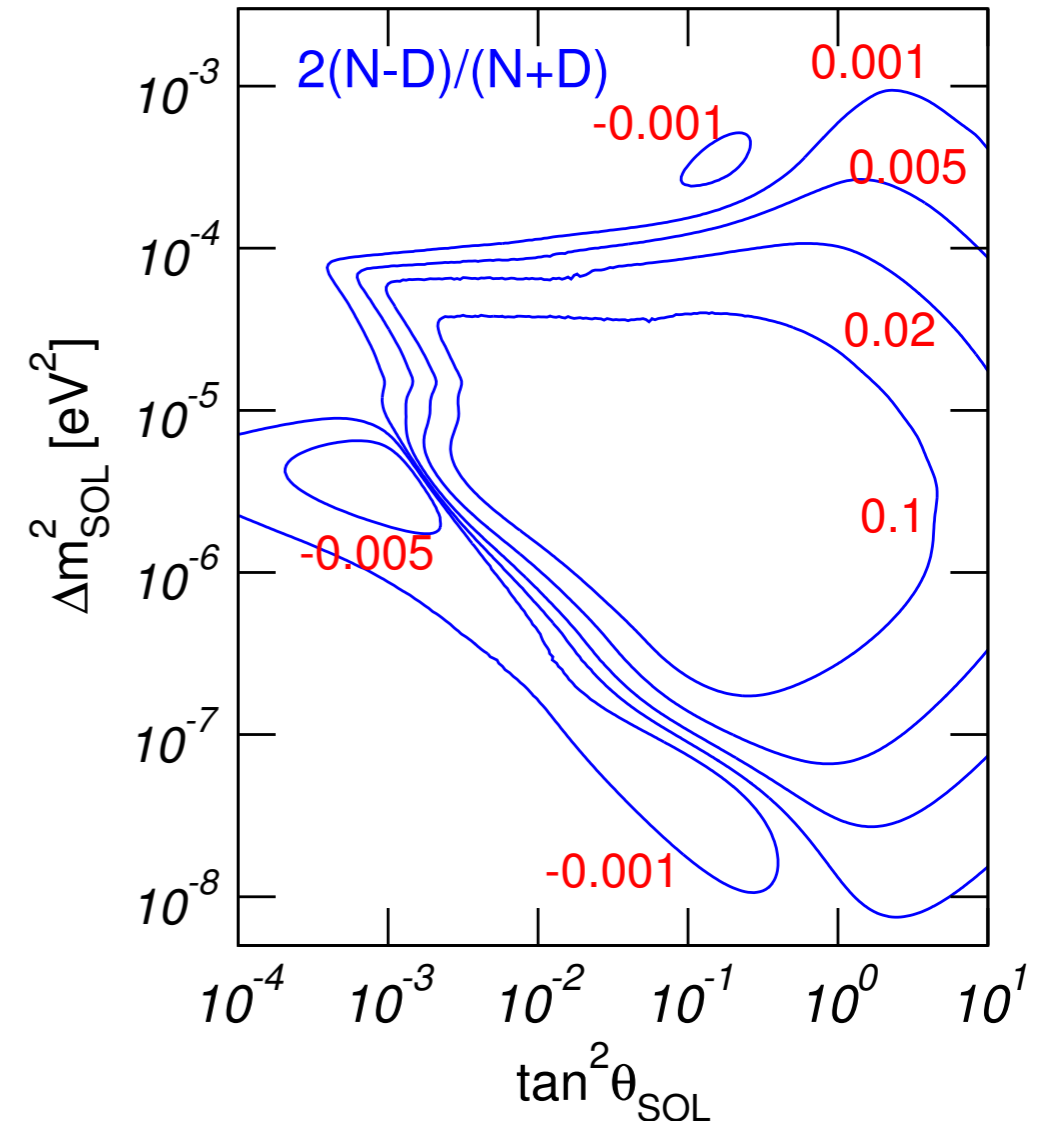
$$f_{\text{reg}} = \frac{4EV_{CC}}{\Delta m^2} \sin^2 2\theta_E \sin^2 \frac{\pi L}{L_{\text{osc}}}$$

$$P_{ee}^{\text{night}} = P_{ee}^{\text{day}} - \cos 2\theta_M f_{\text{reg}}$$

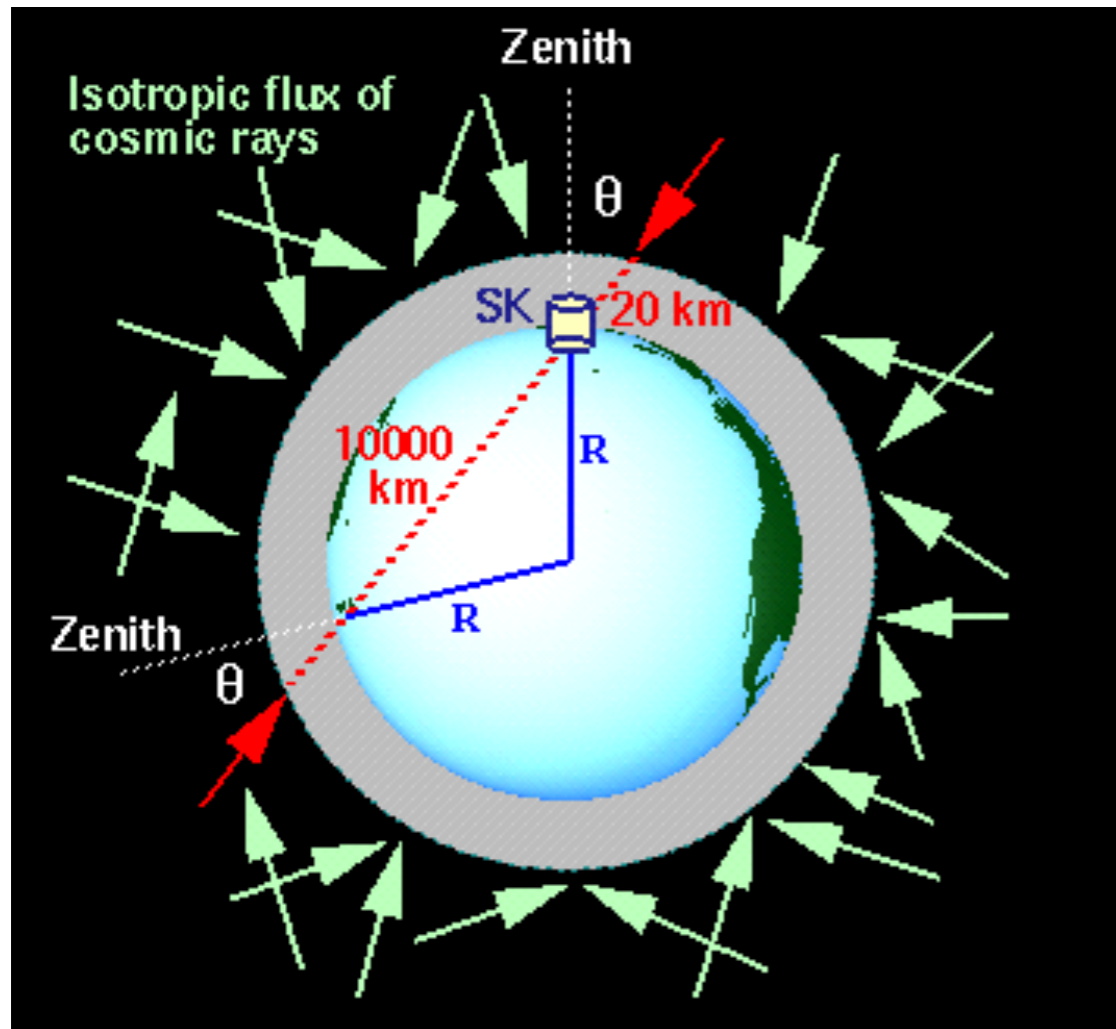
→ day-night asymmetry:

$$A_{\text{DN}} \equiv 2 \frac{(P_N - P_D)}{P_N + P_D}$$

- ◆ For the measured solar neutrino parameters $f_{\text{reg}} \sim +1\%$



Matter effects in atmospheric ν 's



- ◆ Atmospheric neutrinos interact with the Earth mantle and core
- ✓ no matter effects in $\nu_\mu \rightarrow \nu_\tau$ channel
- ✓ MSW resonance in $\nu_\mu \rightarrow \nu_e$ channel

$$\tan 2\theta_m = \frac{\frac{\Delta m^2}{4E} \sin 2\theta}{\frac{\Delta m^2}{4E} \cos 2\theta \mp \sqrt{2}G_F N_e}$$

(-) neutrinos (+)antineutrinos

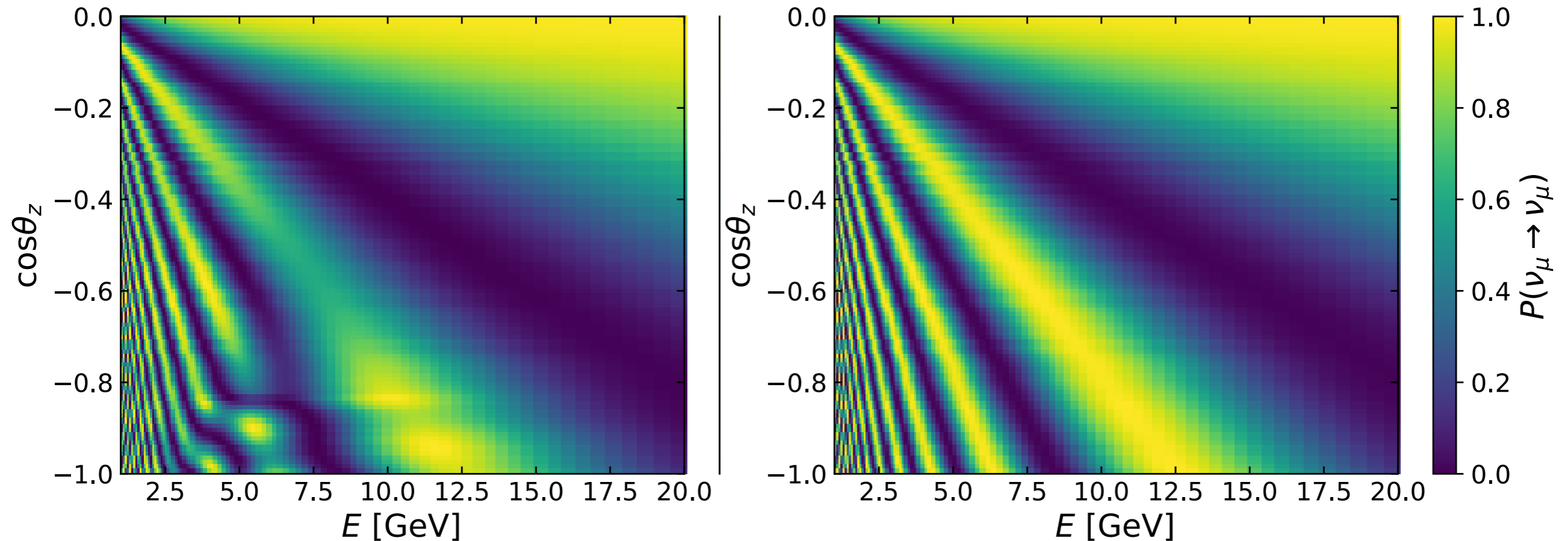
→ Matter effects on the atmospheric neutrino flux are sensitive to the **mass ordering**.

→ they are harder to observe since $P_{\mu e} \propto \theta_{13}$

Matter effects in atmospheric ν 's

NO

IO

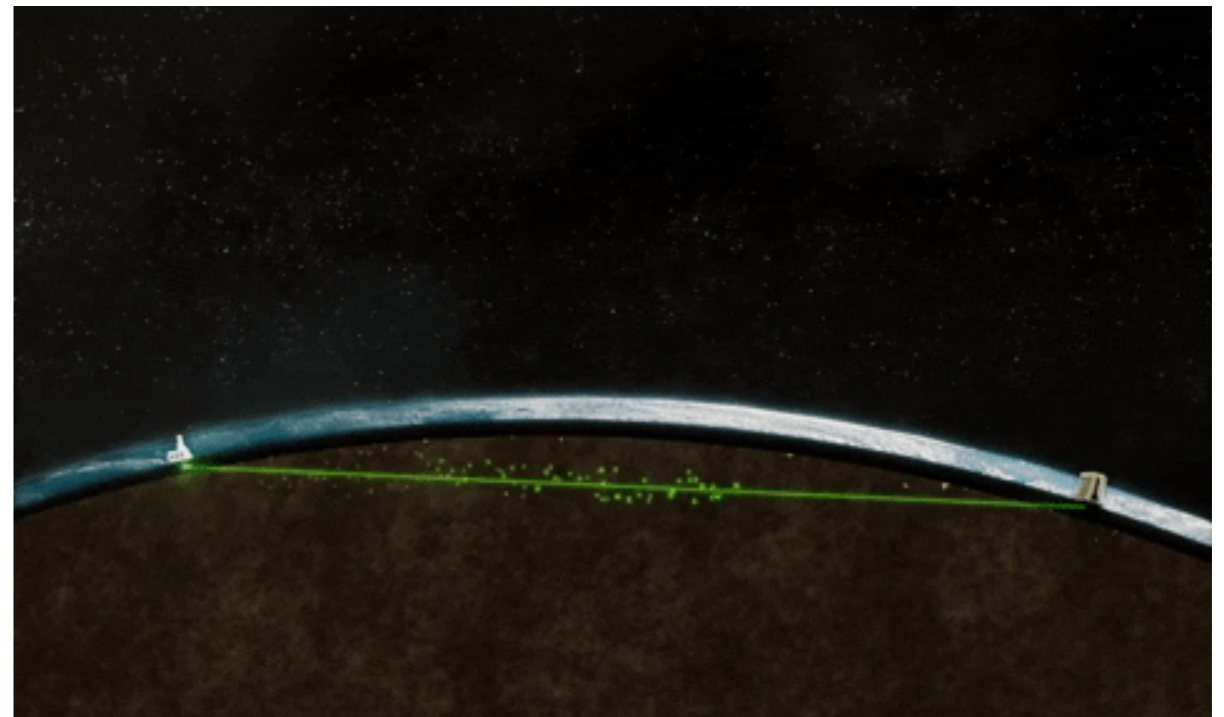
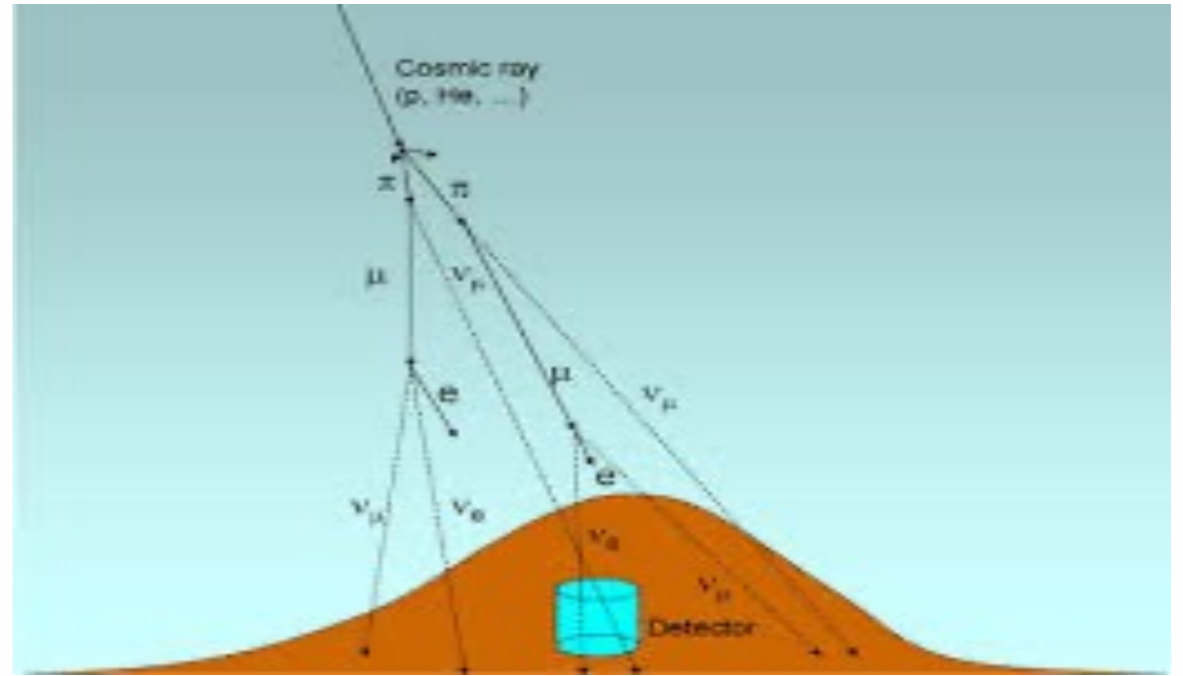


de Salas et al, arXiv:1806.11051

At $E \sim 3-8$ GeV: MSW resonance for neutrinos and NO mass spectrum.

For antineutrinos \Rightarrow the resonance appears in IO

Neutrino oscillation experiments



Neutrino oscillation experiments

