



Cosmology

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Introduction

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3 hours of theory + 1 tutorial hour

Main bibliography:

- *Modern Cosmology*, S. Dodelson
- *Cosmological Physics*, J. Peacock
- *Physical foundations of Cosmology*, V. Mukhanov

Contents

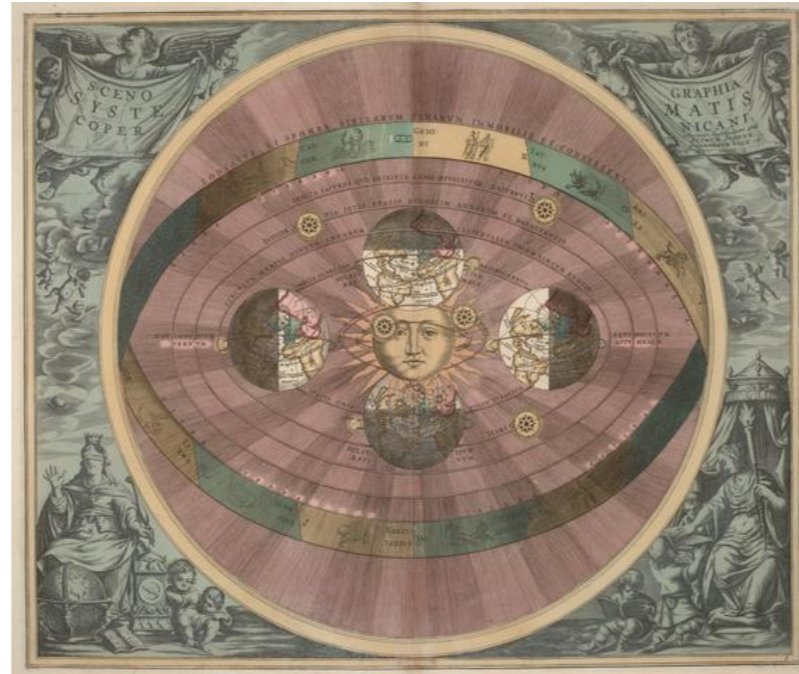
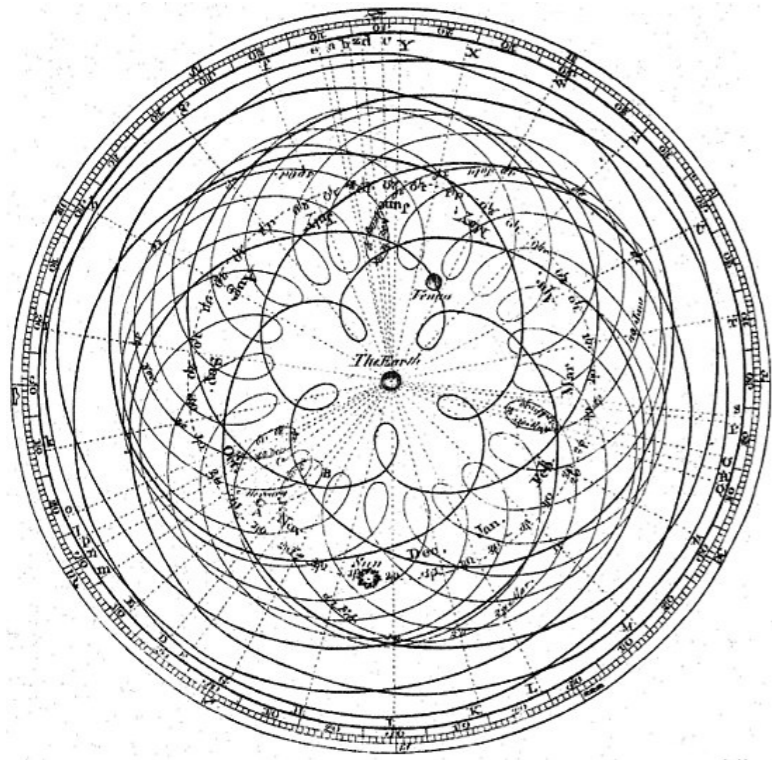
I) Rise of Λ CDM

II) Large-scale structure

III) Cosmological probes

Introduction

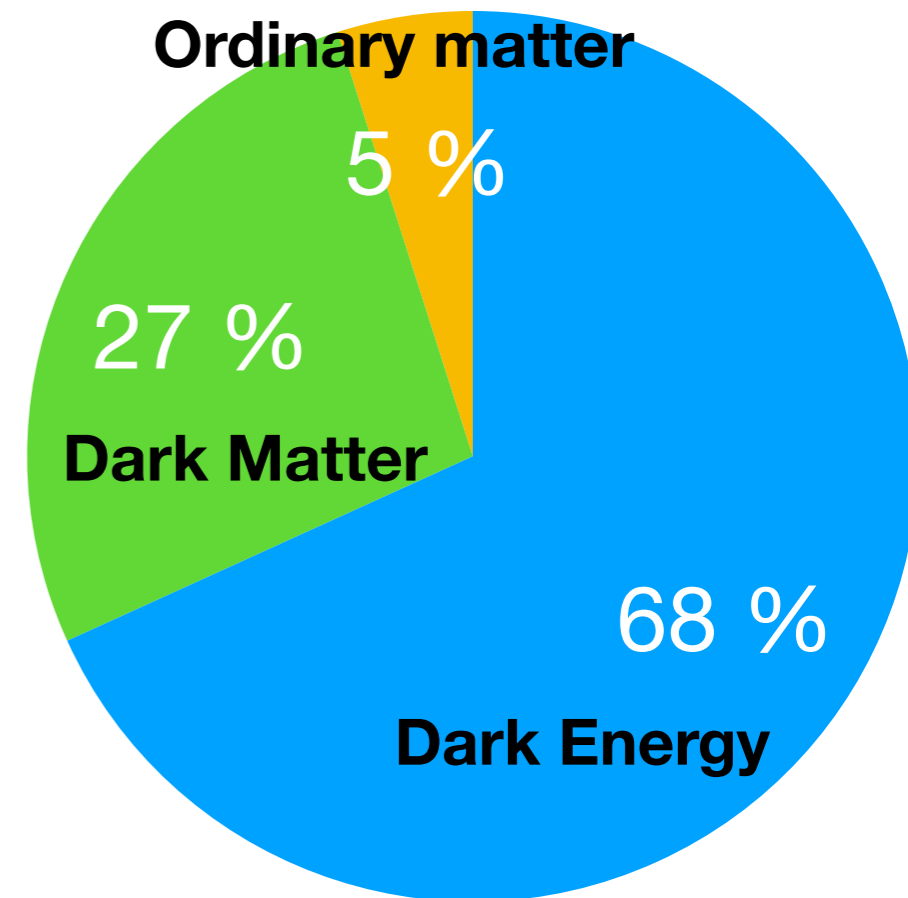
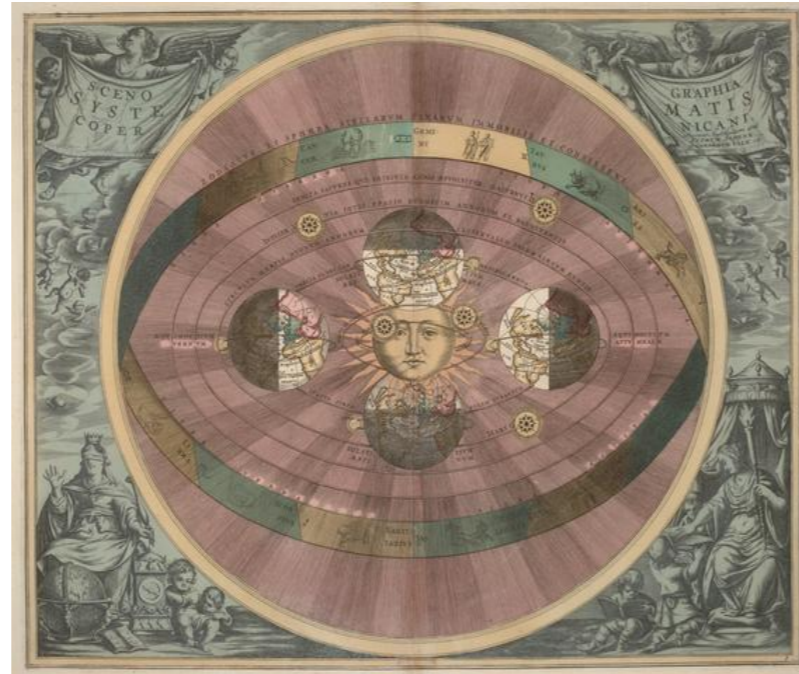
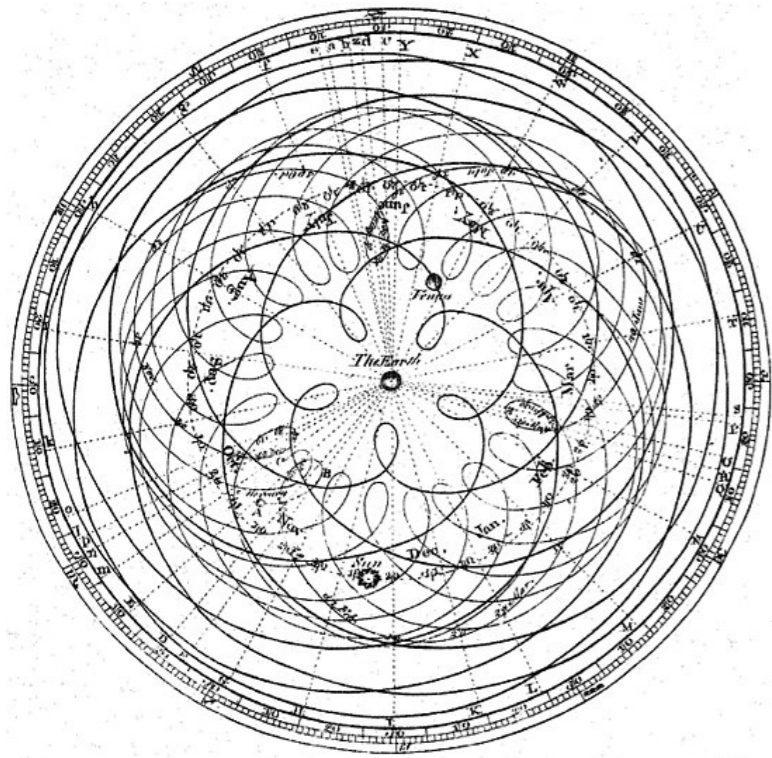
Cosmology: Study and explain the Universe as a whole (long history)



How did we get to the current standard model of cosmology

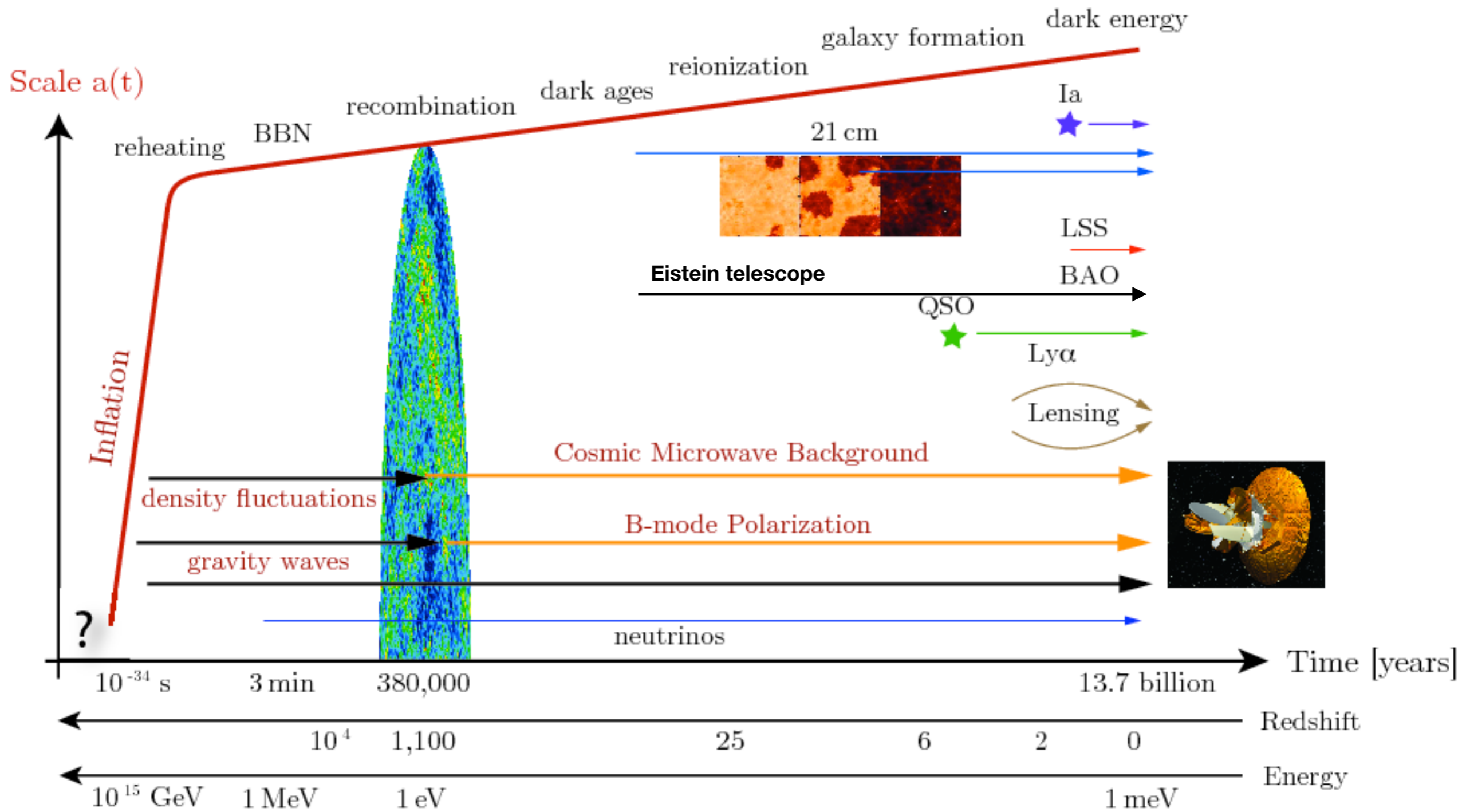
Introduction

Cosmology: Study and explain the Universe as a whole (long history)



How did we get to the current standard model of cosmology

Timeline



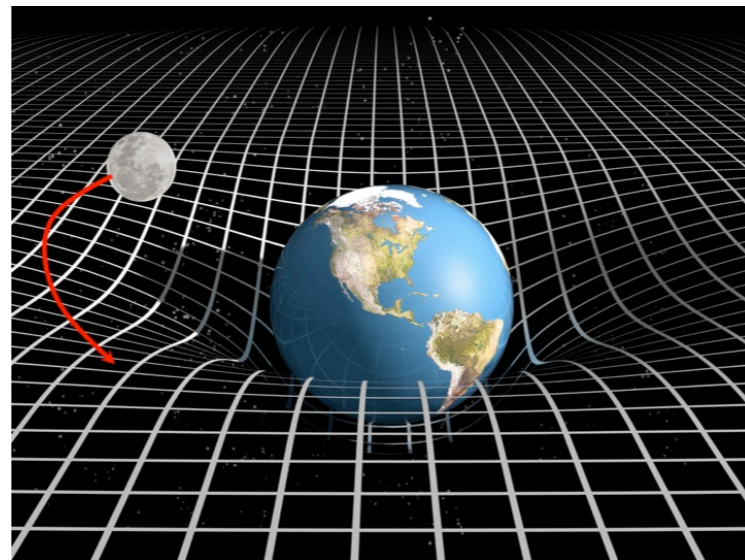
D. Baumann, 2009, arXiv:0907.5424

General relativity

Gravitation defined by Einstein's general relativity. Therefore, Einstein equations:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

We need to define the metric and the energy content of the Universe in order to proceed



FLRW metric

If we assume that at large-scales (> 100 Mpc) the Universe is:

- **Isotropic:** properties are the same in every direction
- **Homogeneous:** isotropic at every point.

Then we derive the Friedman-Lemaître-Robertson-Walker metric

$$ds^2 = c^2 dt^2 - a^2(t) \left[d\chi^2 + S_k^2(\chi) d^2\Omega \right].$$

$$S_k(\chi) = \begin{cases} \sin \chi & \text{si } k > 0 \text{ (} k = 1 \text{)} \\ \chi & \text{si } k = 0 \\ \sinh \chi & \text{si } k < 0 \text{ (} k = -1 \text{)} \end{cases}$$

$a(t)$: scale factor; χ : comoving distance;

Friedmann equations

From FLRW metric we derive the Friedmann equations:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left(\rho + \frac{3p}{c^2} \right) + \frac{\Lambda c^2}{3}$$
$$H^2 = \left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}$$

G: Newton constant
 ρ : energy density
P: pressure
 Λ : cosmological constant
H: Hubble parameter

We need to describe each fluid contributing to the energy-momentum tensor. Each fluid has a particular equation of state: $p = w\rho$

$$T_{\mu\nu} = \left(\rho + p/c^2 \right) u_\mu u_\nu + p g_{\mu\nu}$$

Energy-momentum tensor of a perfect fluid

Evolución del Universo

We define the critical density for the $k=0$ case and without cosmological constant, then we can describe three different curvature cases:

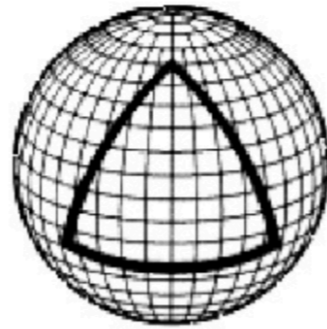
$$\rho_{crit} = \frac{3H^2}{8\pi G}$$

$$\Omega_X = \frac{\rho_X}{\rho_{crit}}$$

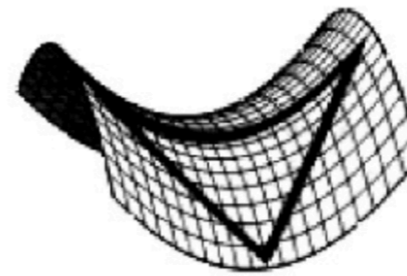
$\Omega > 1$: $k > 0$. Closed Universe

$\Omega = 1$: $k = 0$. Flat Universe

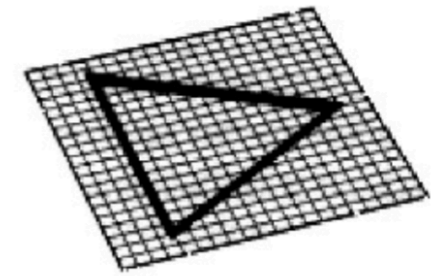
$\Omega < 1$: $k = 0$. Open Universe



Closed



Open



Flat

The evolution of each energy component depends on its equation of state.

$$p = w\rho \quad \left| \begin{array}{l} \longrightarrow \\ \rho \propto a^{-3(1+w)} \end{array} \right.$$

$$\dot{\rho} + 3\left(\rho + \frac{P}{c^2}\right)\frac{\dot{a}}{a} = 0$$

Matter: $w = 0$

Radiation: $w = 1/3$

Λ : $w = -1$

Dark Energy $w < -1/3$

Quintaessence: $w = w_0 + w_a(1+z)$

Universe expansion

Physical distance related with comoving distance: $\mathbf{r}(t) = a(t) \mathbf{x}$.

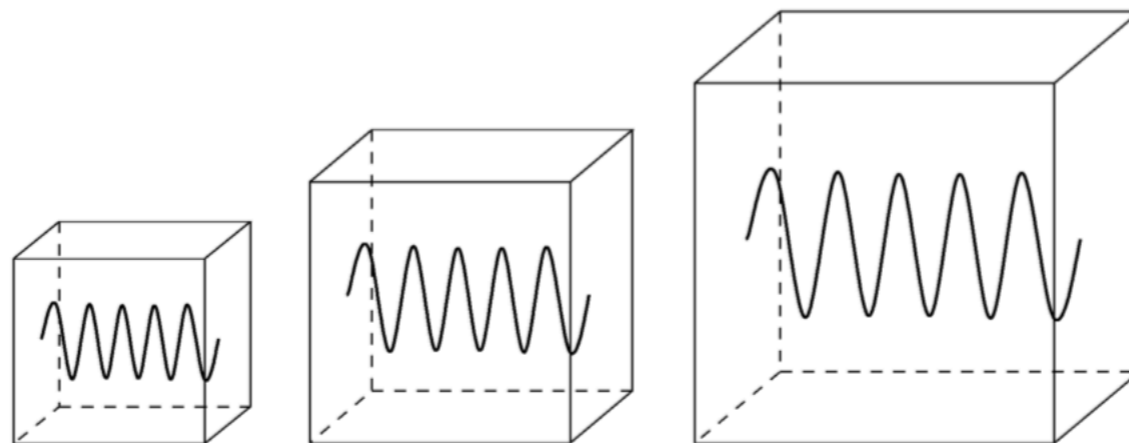
By setting today, $r=\chi$, then we normalise the value of a : $a(t_0) = 1$.

Velocity of a comoving particle: $\mathbf{v}(\mathbf{r}, t) = \frac{d}{dt} \mathbf{r}(t) = \frac{da}{dt} \mathbf{x} \equiv \dot{a} \mathbf{x} = \frac{\dot{a}}{a} \mathbf{r} \equiv H(t) \mathbf{r}$,

Due to the expansion, 2 comoving observers will measure the emitted light by the other with a different frequency than the emitted one, given by redshift z .

$$\frac{\lambda_e}{a(t_e)} = \frac{\lambda_o}{a(t_0)}$$

$$a(t_e) = 1/(1+z)$$



Cosmological distances

We can estimate the distances in the Universe using this formalism so we can directly study the energy content

$$H^2 = H_0^2 \left[\Omega_m a^{-3} + \Omega_r a^{-4} + \Omega_k a^{-2} + \Omega_{DE} a^{-3(1+w)} \right]$$

$$\chi(z) = \frac{c}{H_0} \int_{a=(1+z)^{-1}}^{a=1} \frac{da}{\sqrt{\Omega_m a + \Omega_r + \Omega_k a^2 + \Omega_\Lambda a^4}} = \frac{c}{H_0} \int_0^z \frac{dz}{E(z)}$$

$$d = cz/H_0$$

Distancia local

$$D = \chi(z)$$

Distancia comóvil

$$D_\theta = \chi(z)/(1+z)$$

Distancia tamaño angular

$$D_\theta \equiv \frac{d_p}{\theta_A}$$

$$D_L = \chi(z) (1+z) = D_\theta/a^2$$

Distancia de luminosidad

$$D_L \equiv \sqrt{\frac{L}{4\pi F_0}}$$

In optical photometry, we use magnitudes: $m_x = -2.5 \log_{10} \left(\frac{F_x}{F_{x,0}} \right)$

$$\mu \equiv m - M = 5 \log D_L(z) + 25$$

Distance modulus

magnitud aparente

magnitud absoluta

Universe history eras

The dynamics of the Universe and the growth of structure depend on which energy component is dominating at each time of history in the Universe

$$\rho \propto a^{-3(1+w)}$$

Matter: $w = 0$

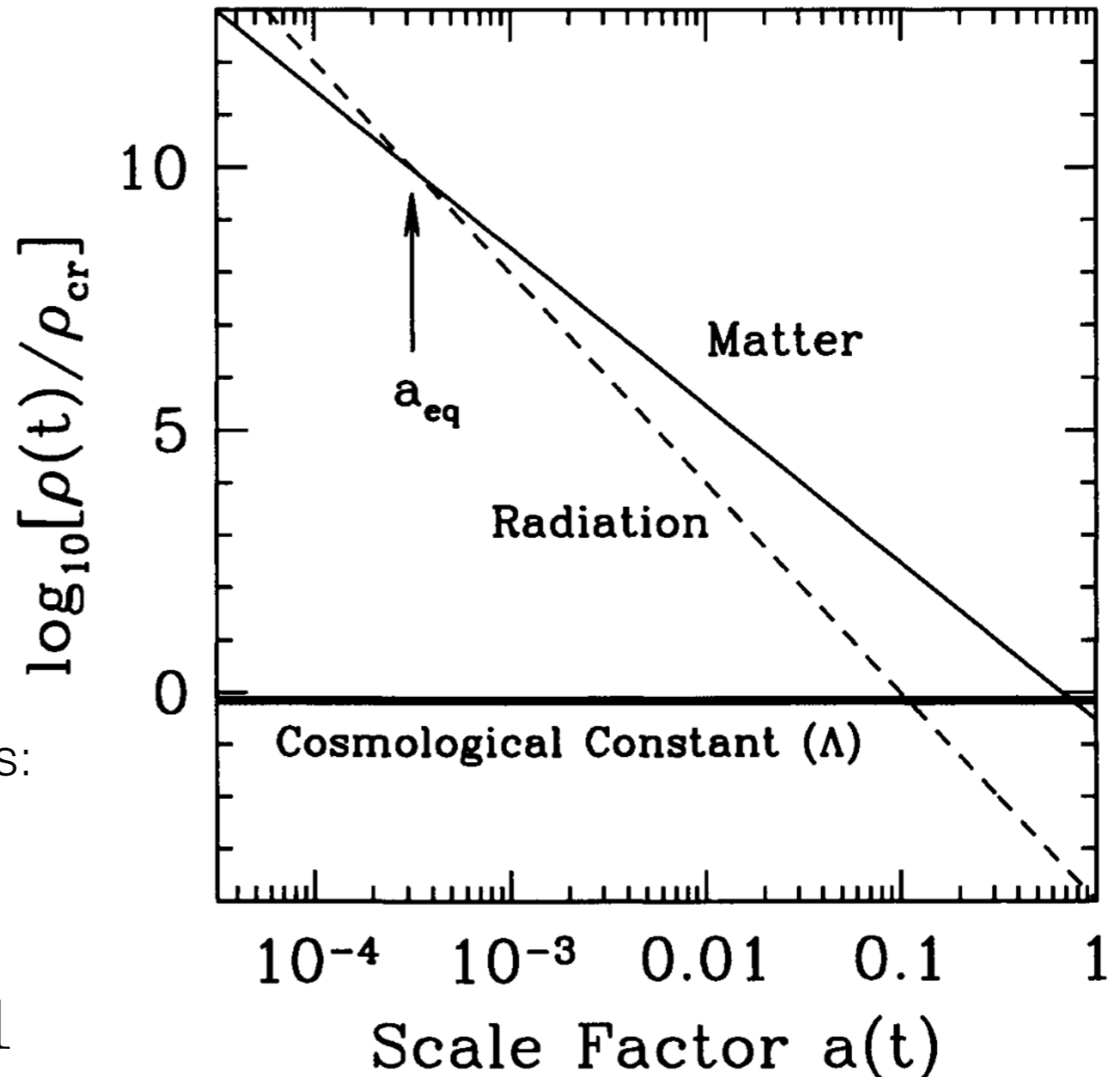
Radiation: $w = 1/3$

Λ : $w = -1$

If only one component dominates:

$$a \propto t^{\frac{2}{3(1+w)}}$$

$$a \propto e^{Ht} \quad \text{if} \quad w = -1$$



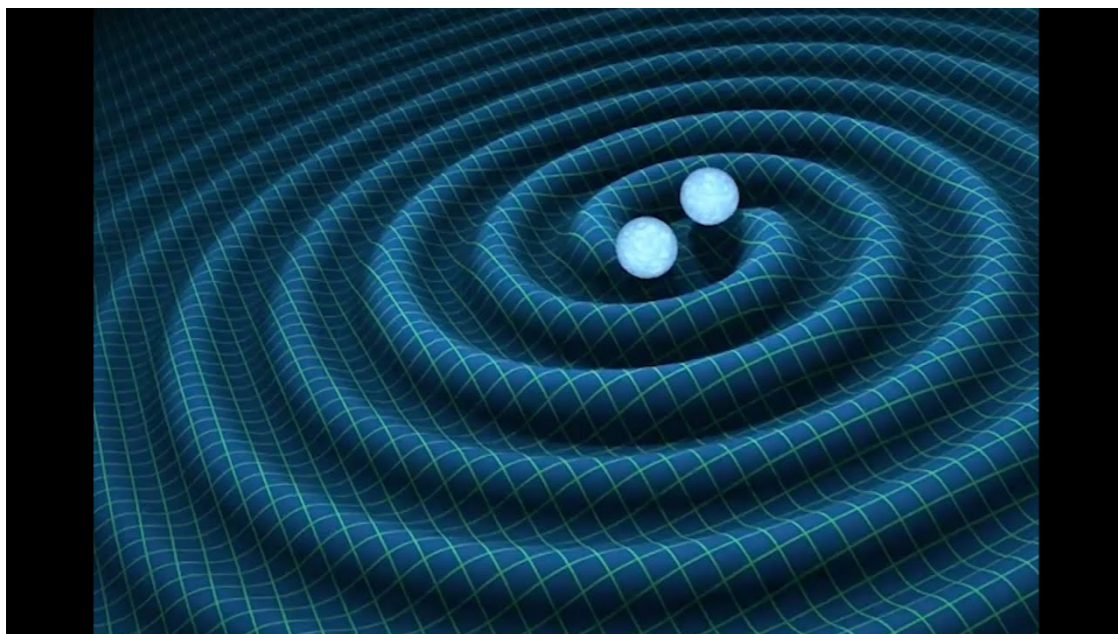
Observational tests of cosmology

We need to study **all cosmology** at **all times** and **all scales**.

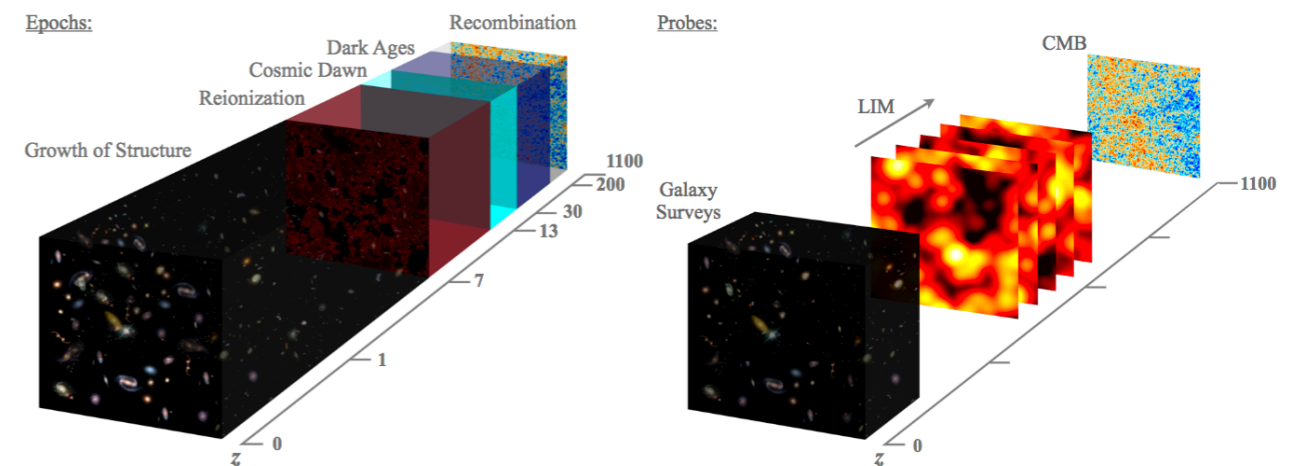
Study transient objects in the sky and the distribution of matter using different wavelengths.

Principal techniques

Time-domain cosmology



Large-scale structure of the Universe

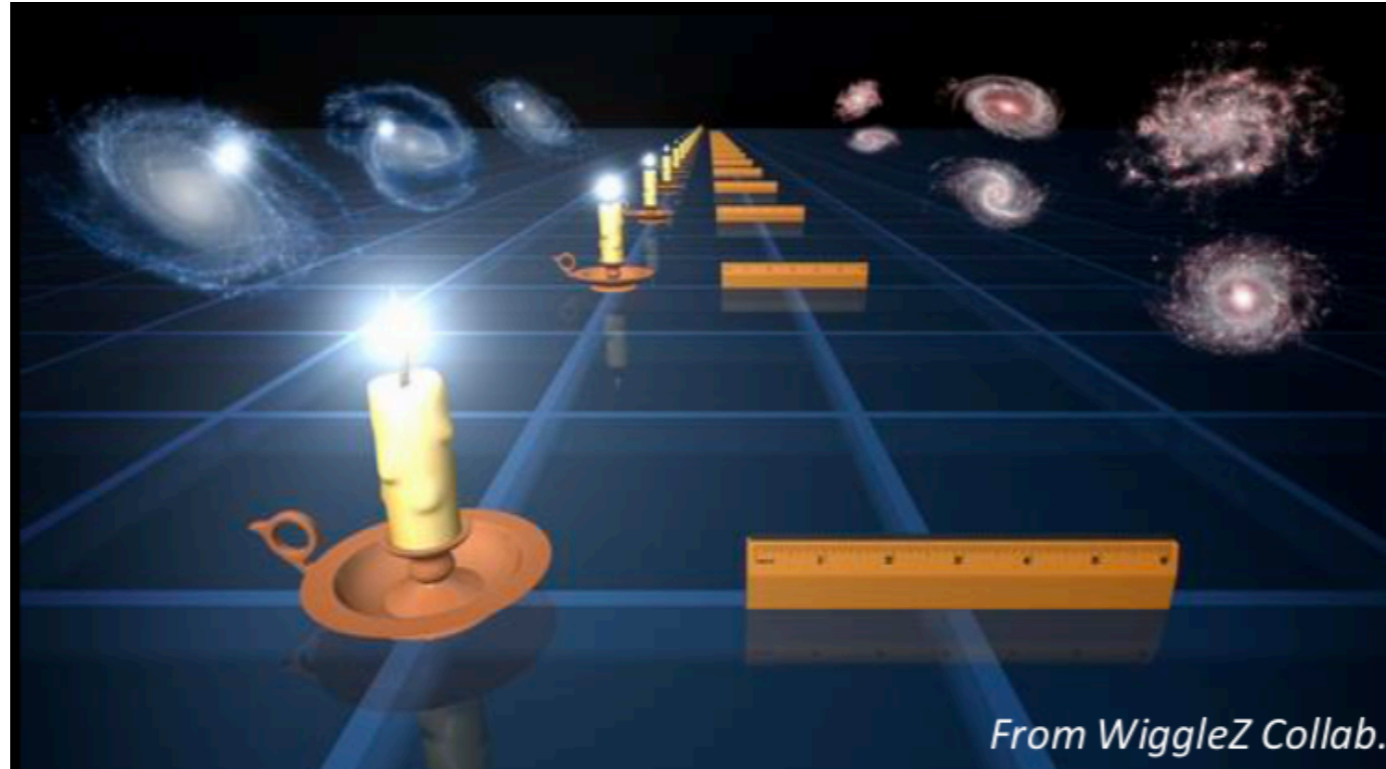


Cosmography

Using the luminosity or the angular diameter distance we can test the cosmological background model.

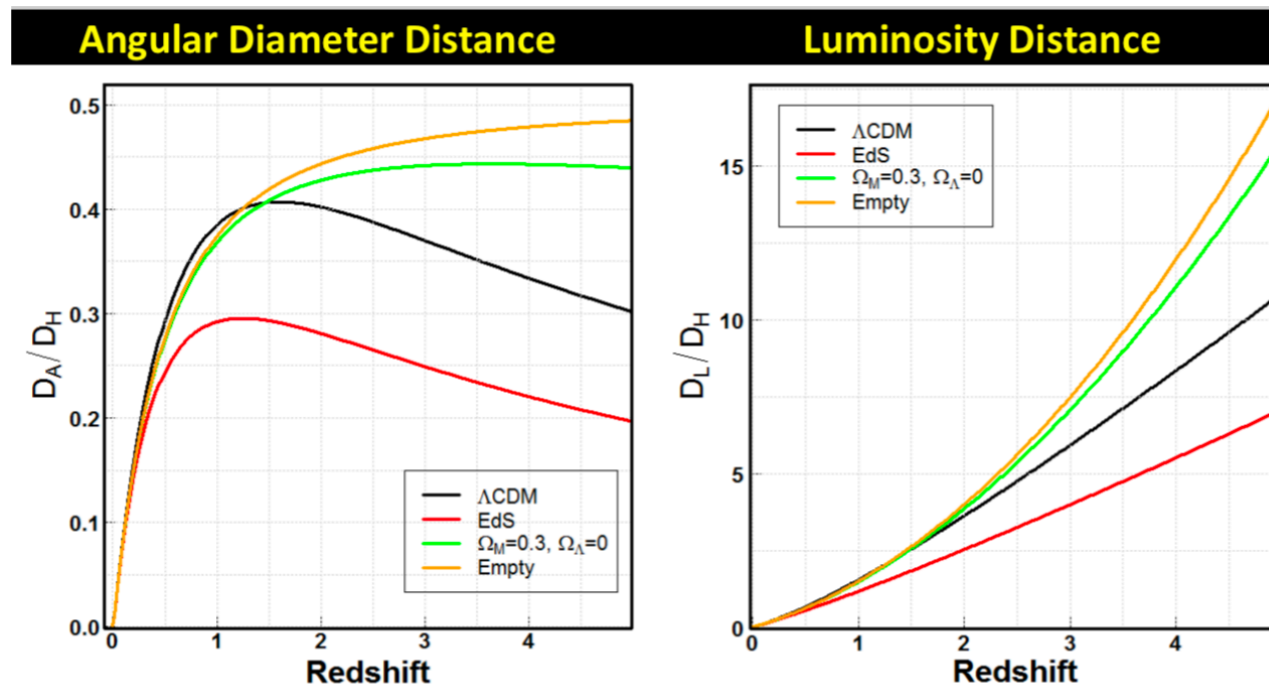
Standard candles

$$F = \frac{L}{4\pi D_L^2}$$



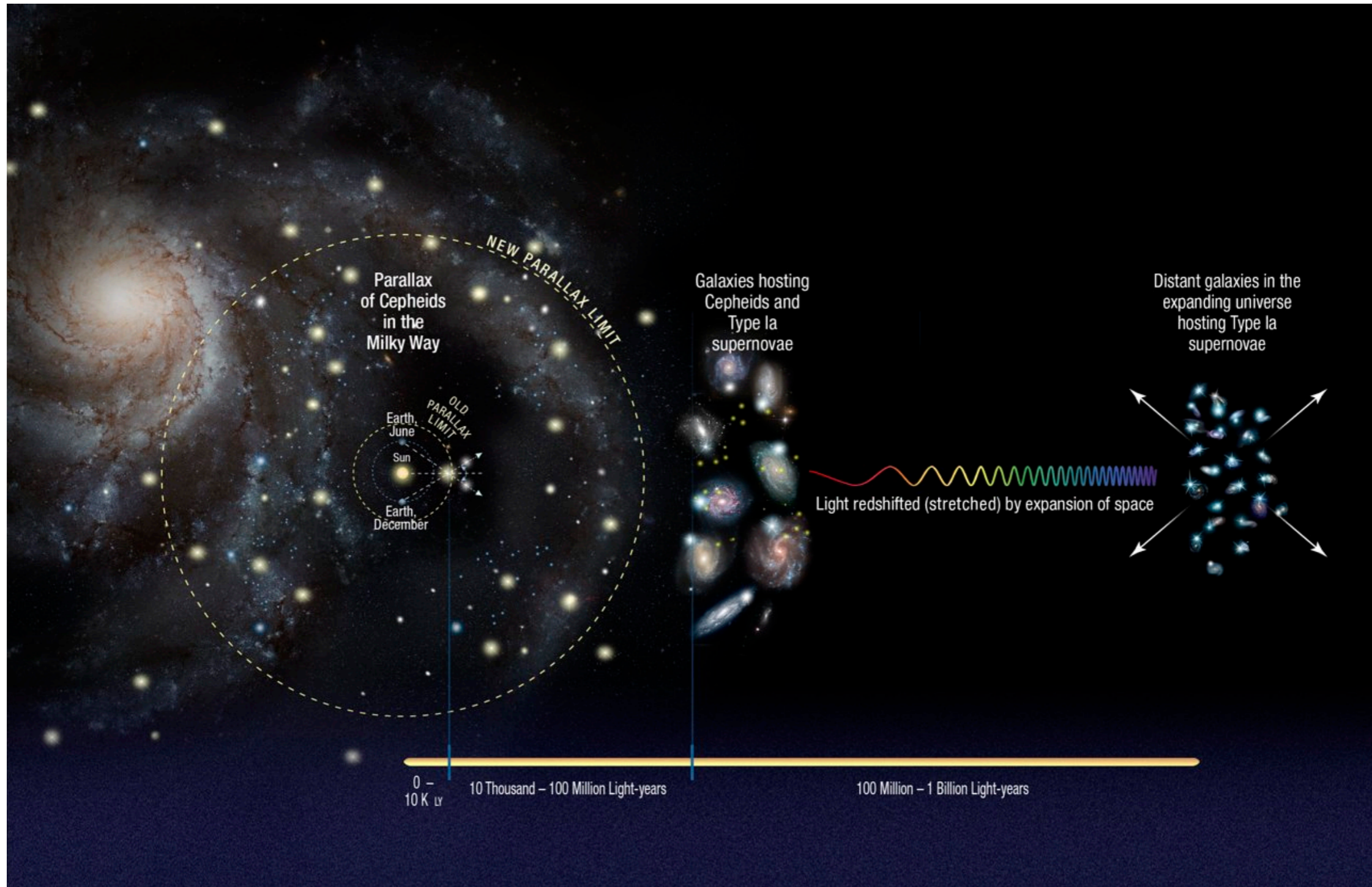
Standard rulers

$$D_A = \frac{R}{\theta}$$



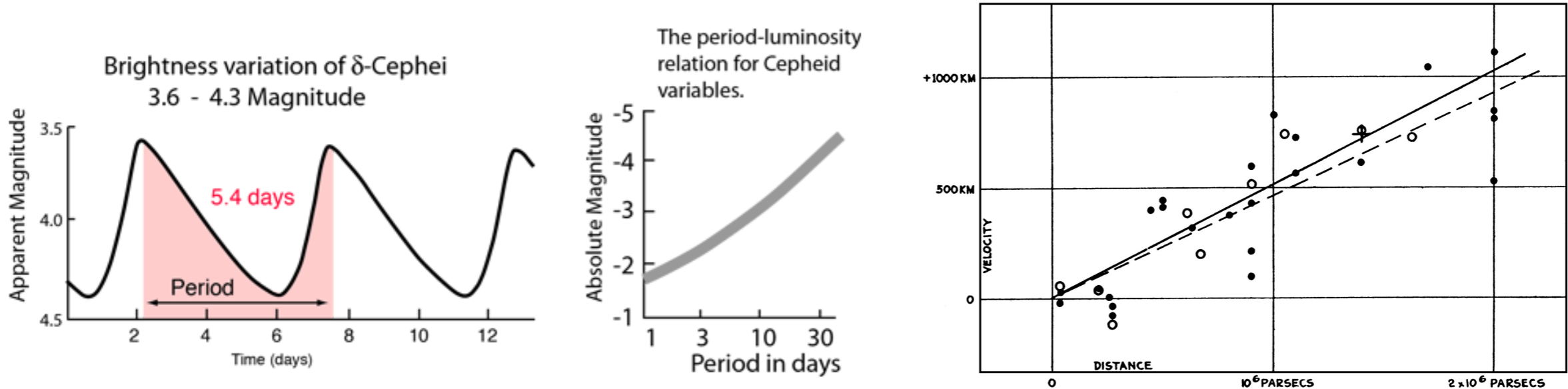
Cosmic distance ladder

Most distance measurements are carried using the distance ladder technique.



Hubble-Lemaître law

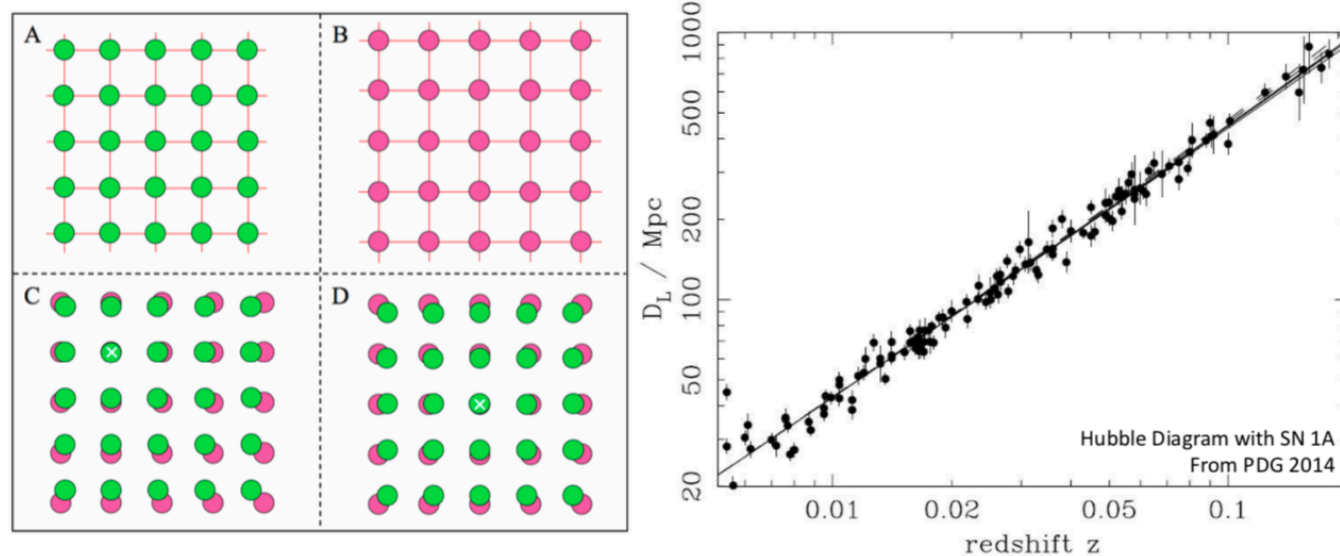
Standard candles: If we know luminosity -> We measure distance



We observe galaxies “receding” at a velocity proportional to the distance to them

$$cz \sim v = H d = \frac{\dot{a}}{a} d$$

Universe expands,
confirming FLRW metric



Expansion of the Universe (beginning)

During the 1920s, thanks to Vesta Slipher redshift measurements, Henrietta Leavitt cepheids period-luminosity relation, Georges Lemaître found Friedmann's solution and compared with data, funding the expansion but then with more data Edwin Hubble measured the distance-redshift relation.

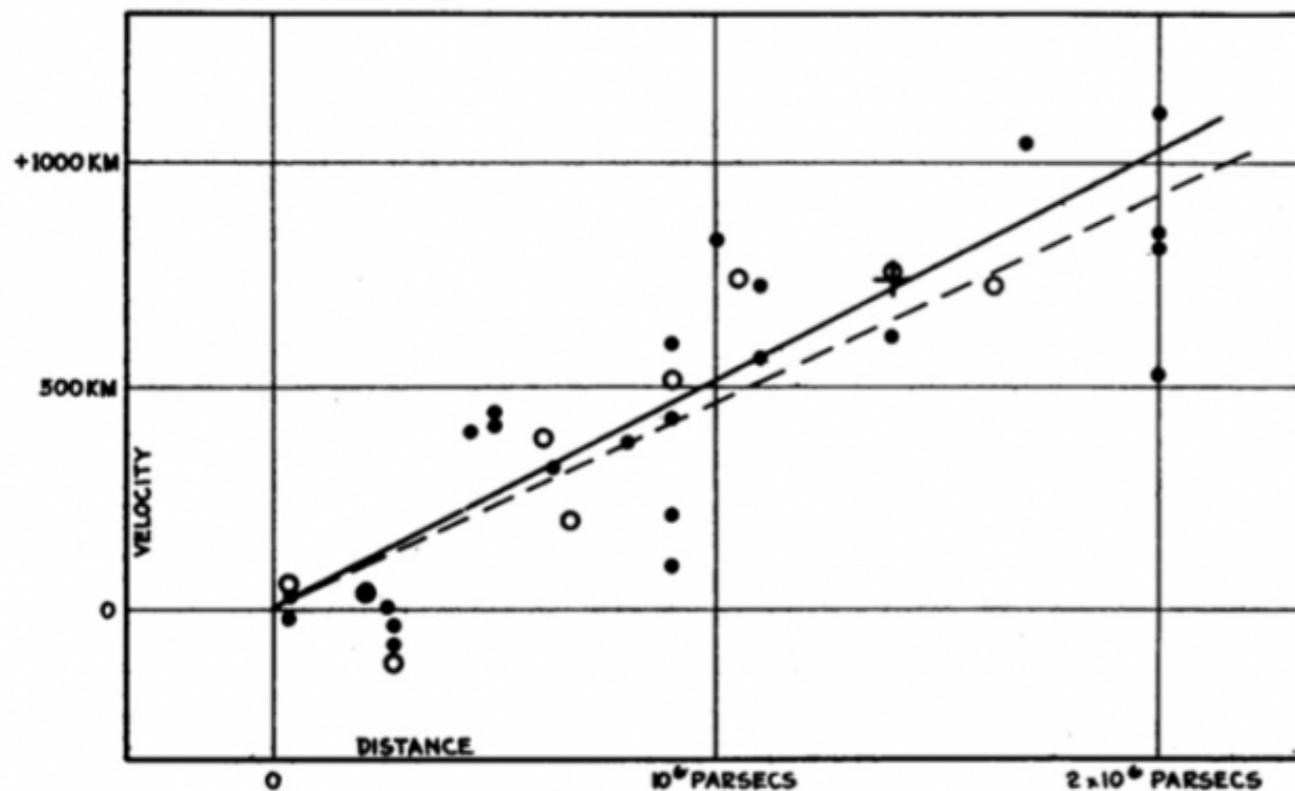
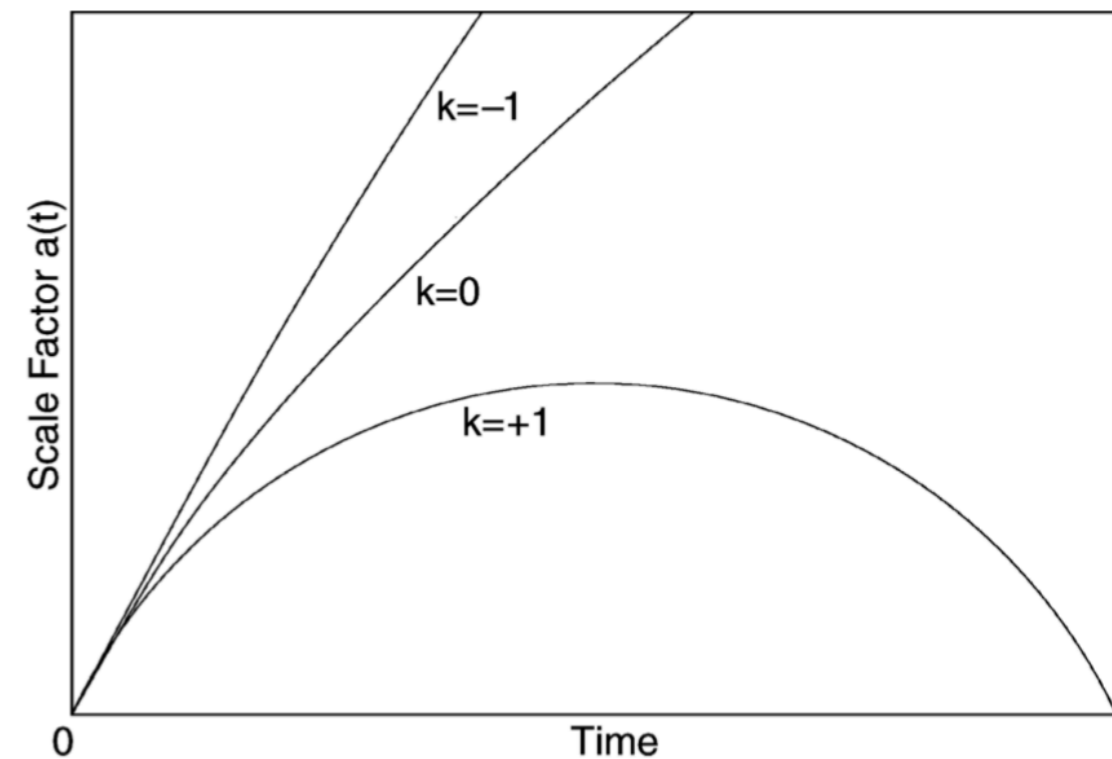


FIGURE 1

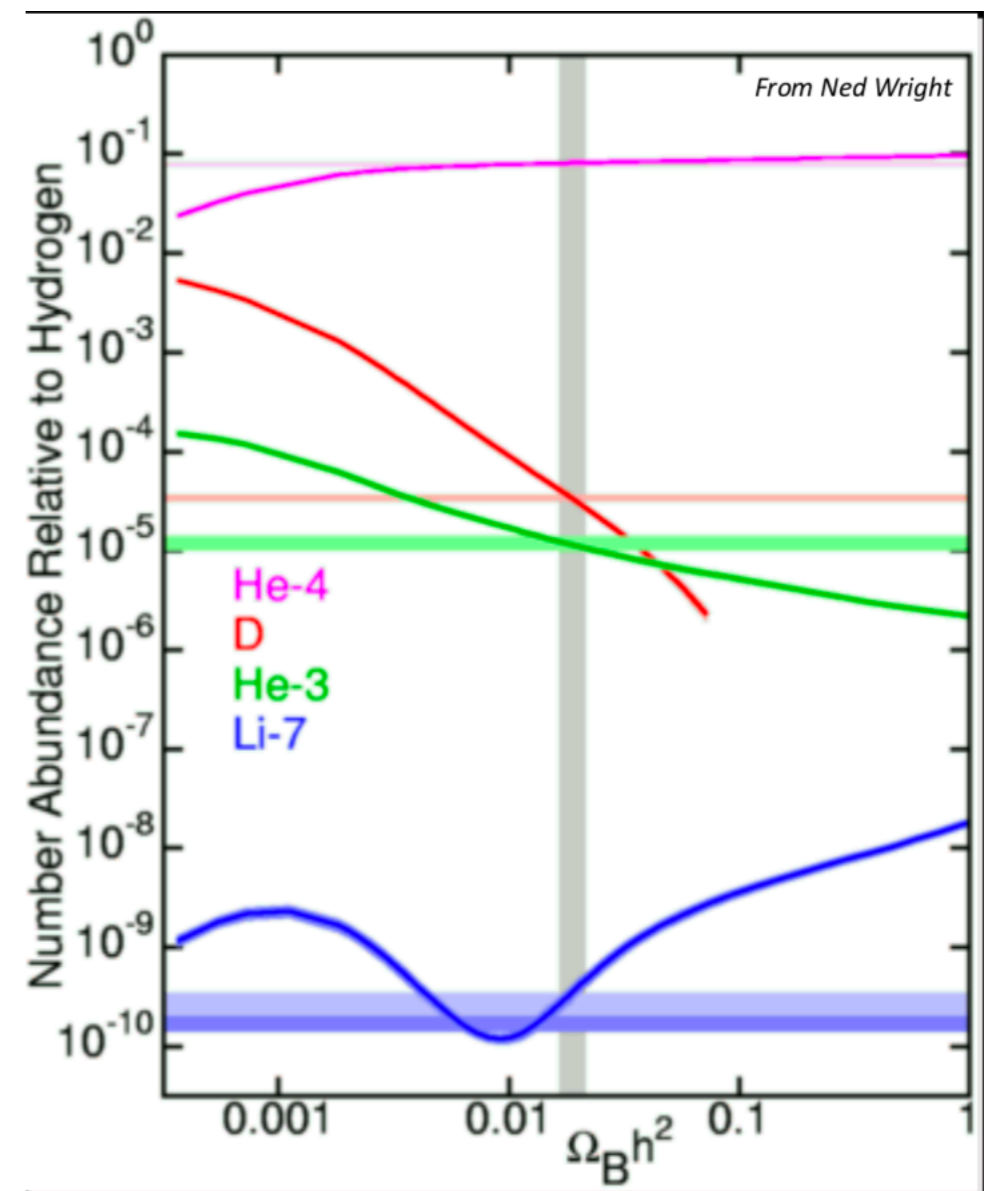
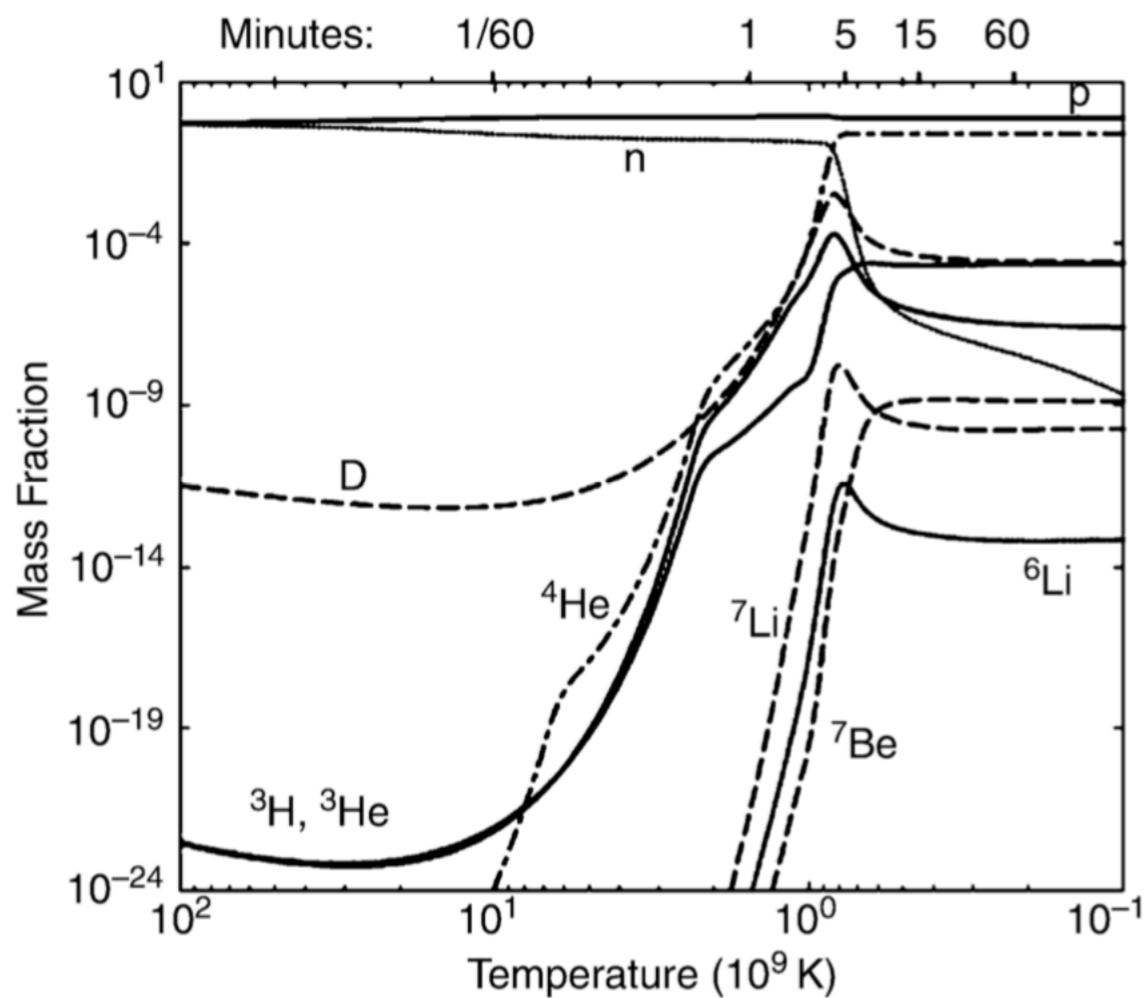
Velocity-Distance Relation among Extra-Galactic Nebulae.



One of Lemaître conclusions was to formulate the primeval atom of the Universe in which going back on time, the Universe must have been in a dense state.

Primordial nucleosynthesis

We can predict the abundances of light elements, depending on the amount of baryons in the Universe. The abundances given by the observed baryon density at present agree with astrophysical abundances of these elements.

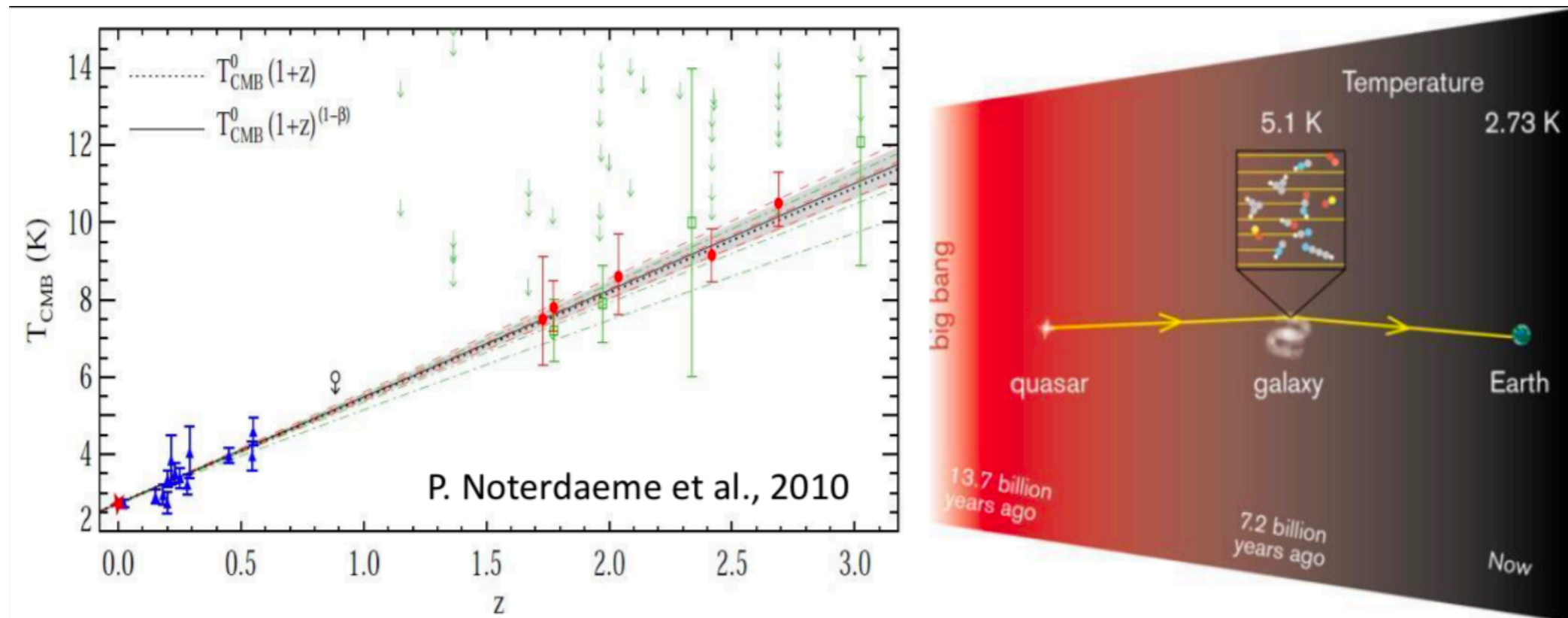


Thermodynamics of CMB

According to the Big Bang model, there should be a remnant radiation cooled down to a few K today.

Observing this radiation -> significant support of the model

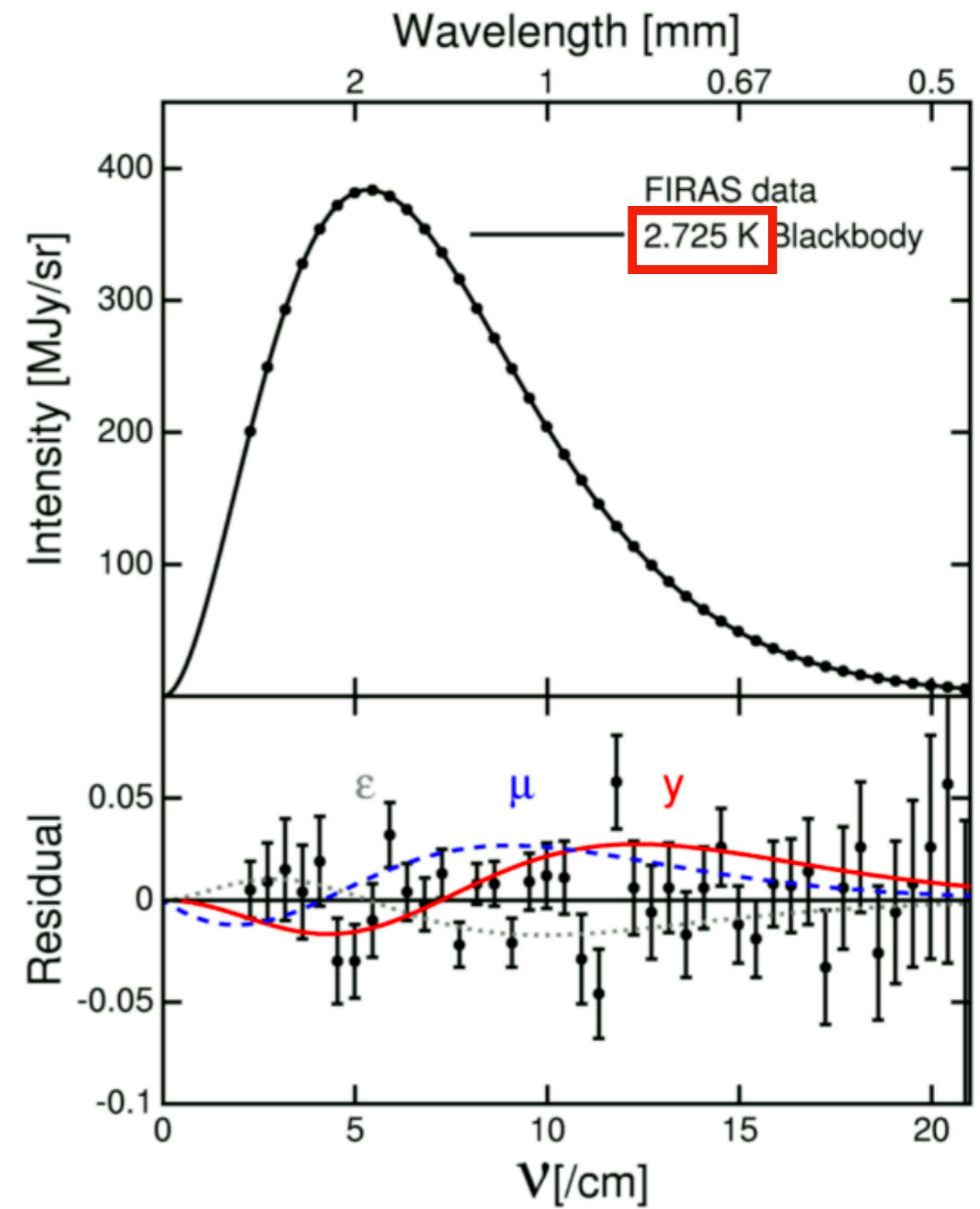
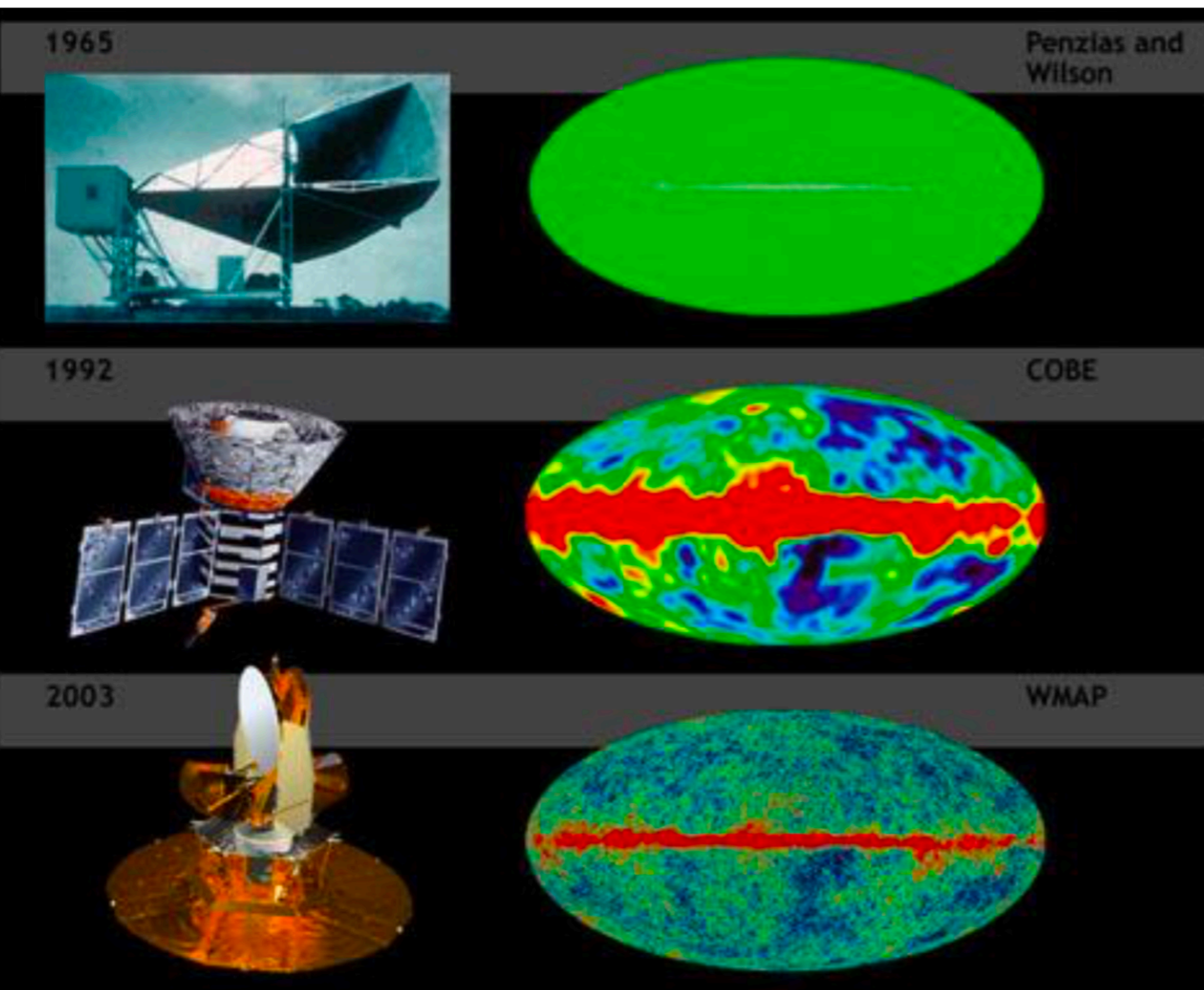
$$T = T_0(1 + z)$$



The evolution of the CMB temperature has been measured with redshift, agreeing with the expected result.

Cosmic Microwave Background radiation

Discovery in 1965 by Penzias and Wilson. Decoupled from matter at 3000K (decoupling). Blackbody distribution with a temperature close to 3K with huge precision.



Cosmic Microwave Background radiation anisotropies

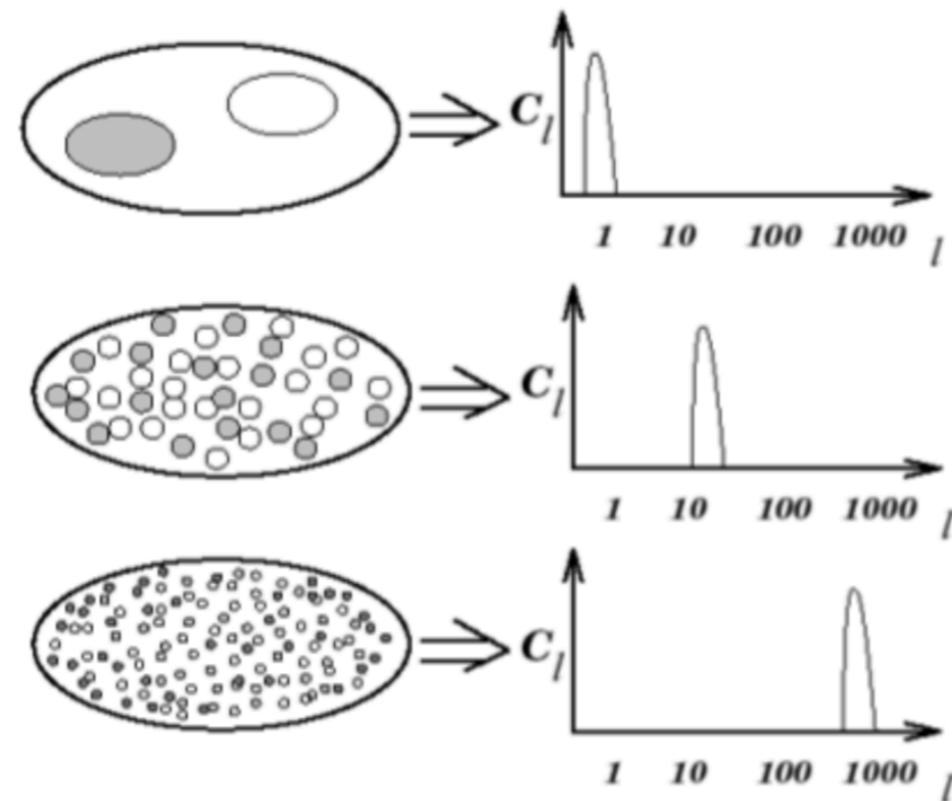
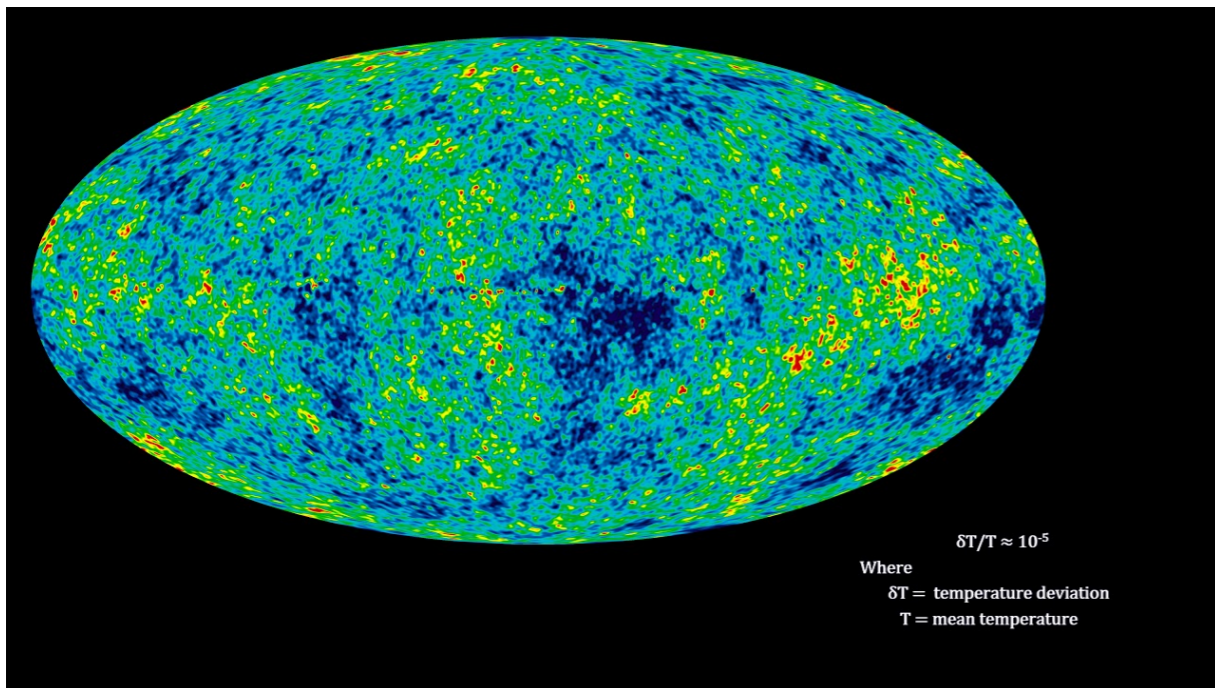
Power spectrum of CMB anisotropies contains cosmological information.
Basis to establish Λ CDM model

$$\frac{\Delta T}{T}(\theta, \phi) = \sum_{l,m} a_{lm} Y_{lm}(\theta, \phi) \quad C_l \equiv \langle |a_{lm}|^2 \rangle$$

Anisotropy

Weight

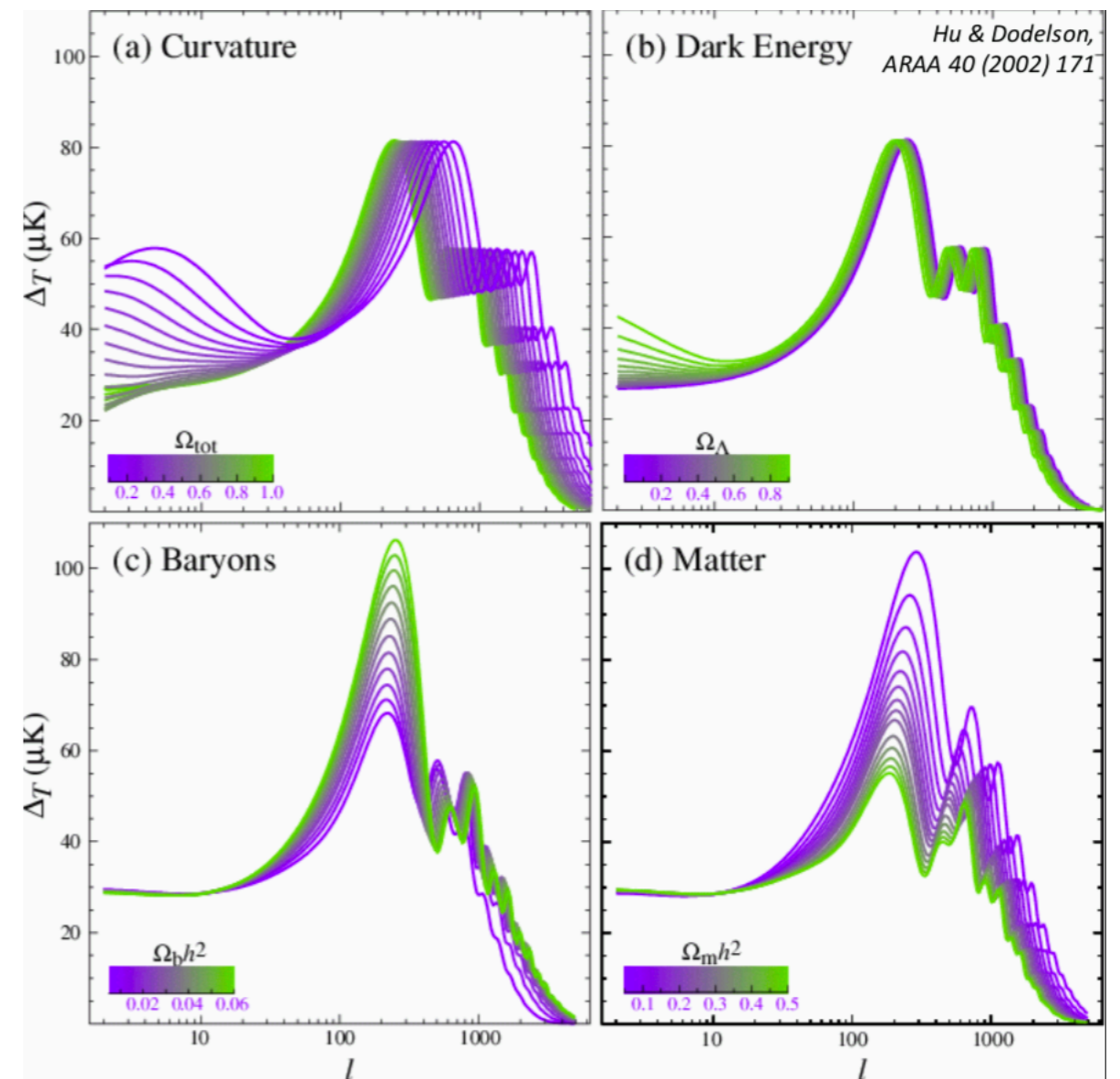
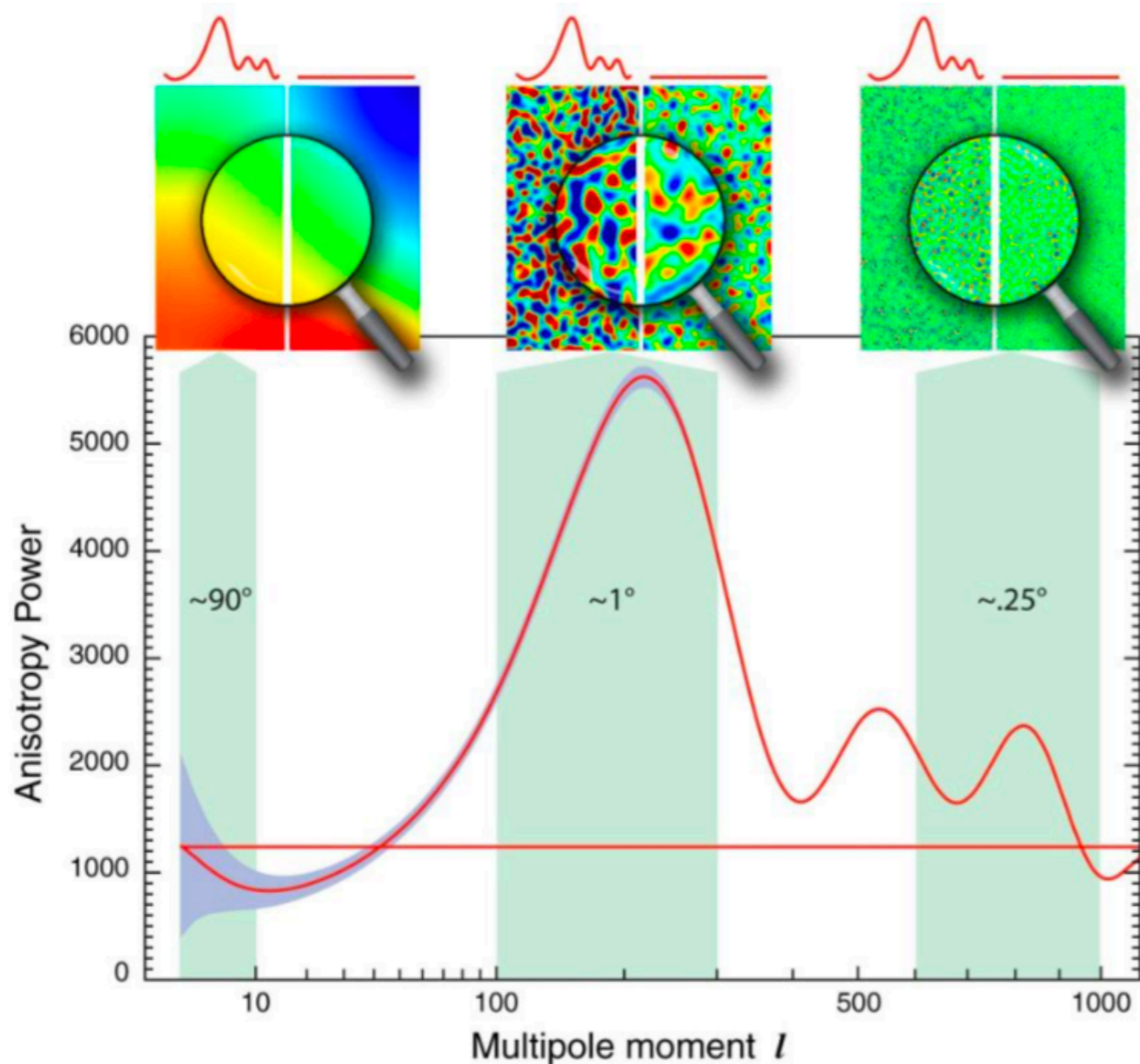
Spherical harmonics



CMB anisotropies power spectrum

Cosmological model success to predict the CMB power spectrum

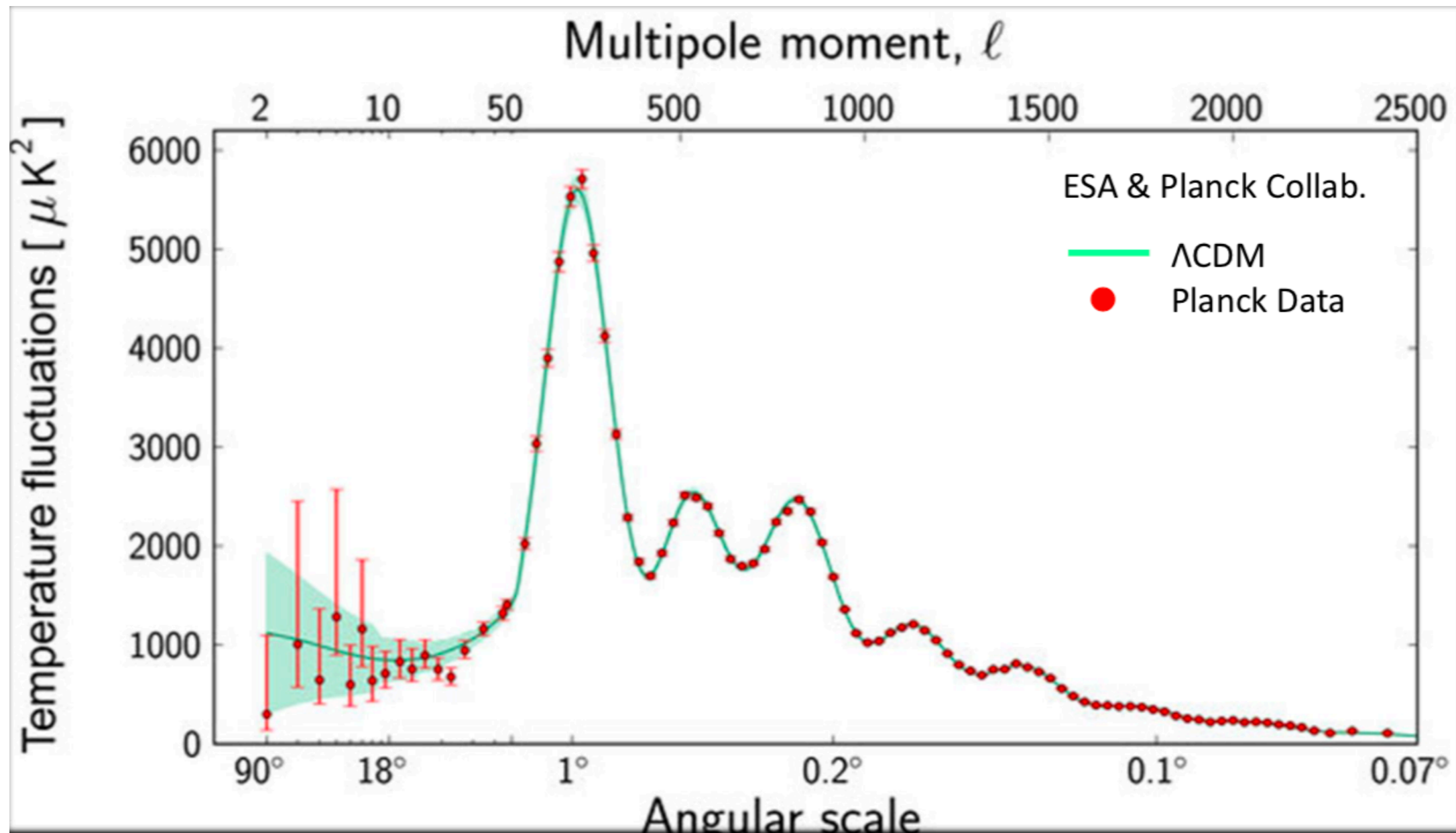
Role of dark energy at time on early Universe small, important for secondary effects on the photons (integrated Sachs-Wolfe effect)



CMB anisotropies as cosmological probe

Position of first peak clear indicator of flatness of the Universe ($k=0$)

Relation between peaks gives us relation between total matter density and baryons density.

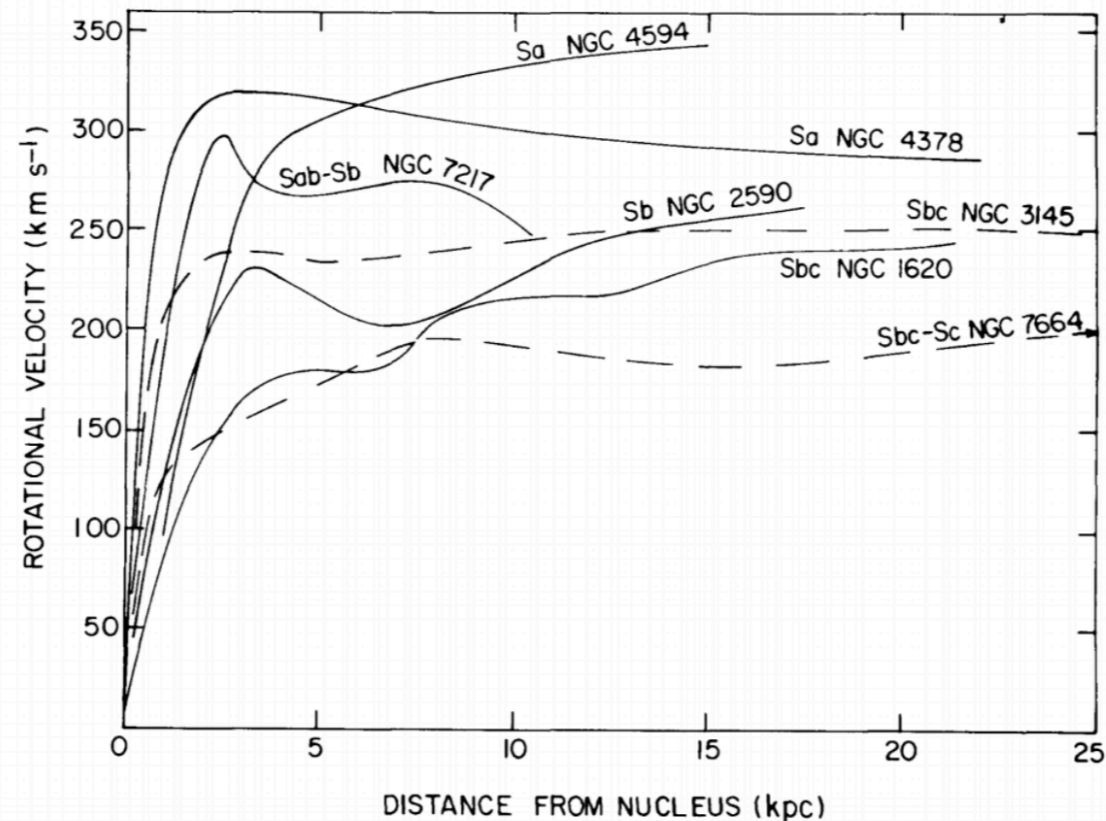


Dark matter

In parallel to all these, another mysterious component of the Universe has been proposed, dark matter.

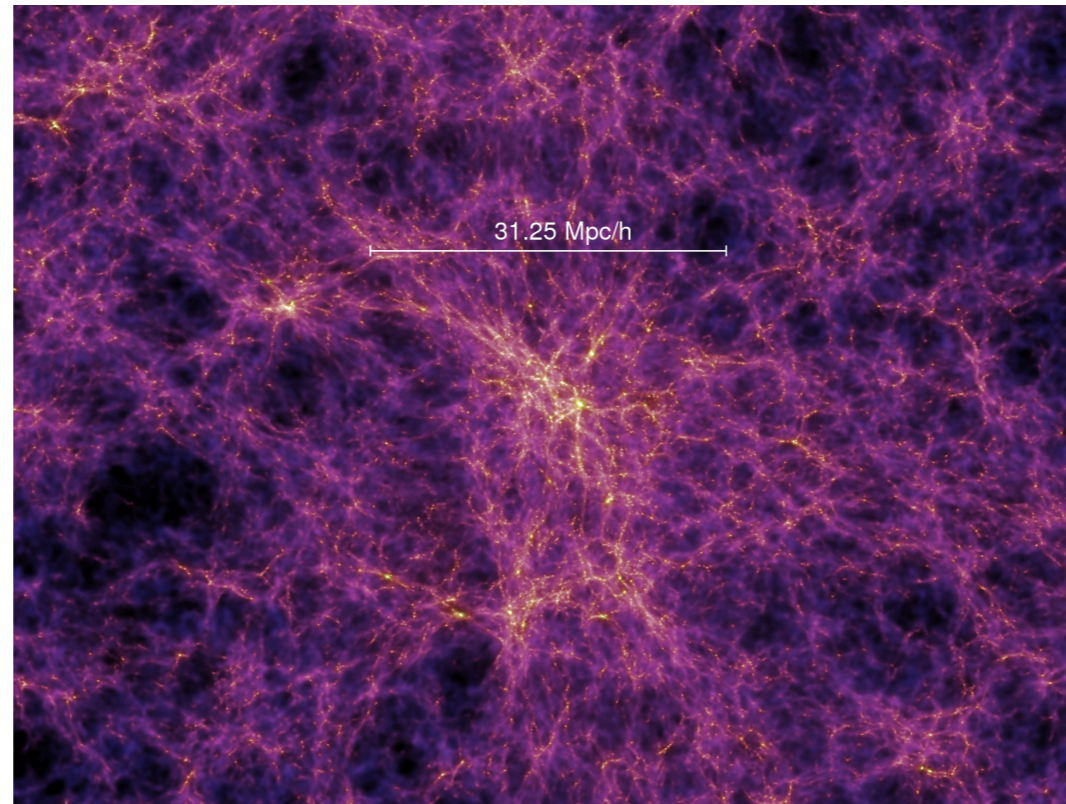
- In the 30s, F. Zwicky showed that the galaxies in the Coma cluster are moving much faster than they should according to the gravity from the visible matter in the cluster. He named the matter component holding the cluster “dunkle materie” (dark matter)

- Then in the 60s and 70s, Vera Rubin measured the rotational curves of spiral galaxies, finding a flat profile at large radii when according to the distribution of matter in the center of the galaxy the profile should fall.



Dark matter

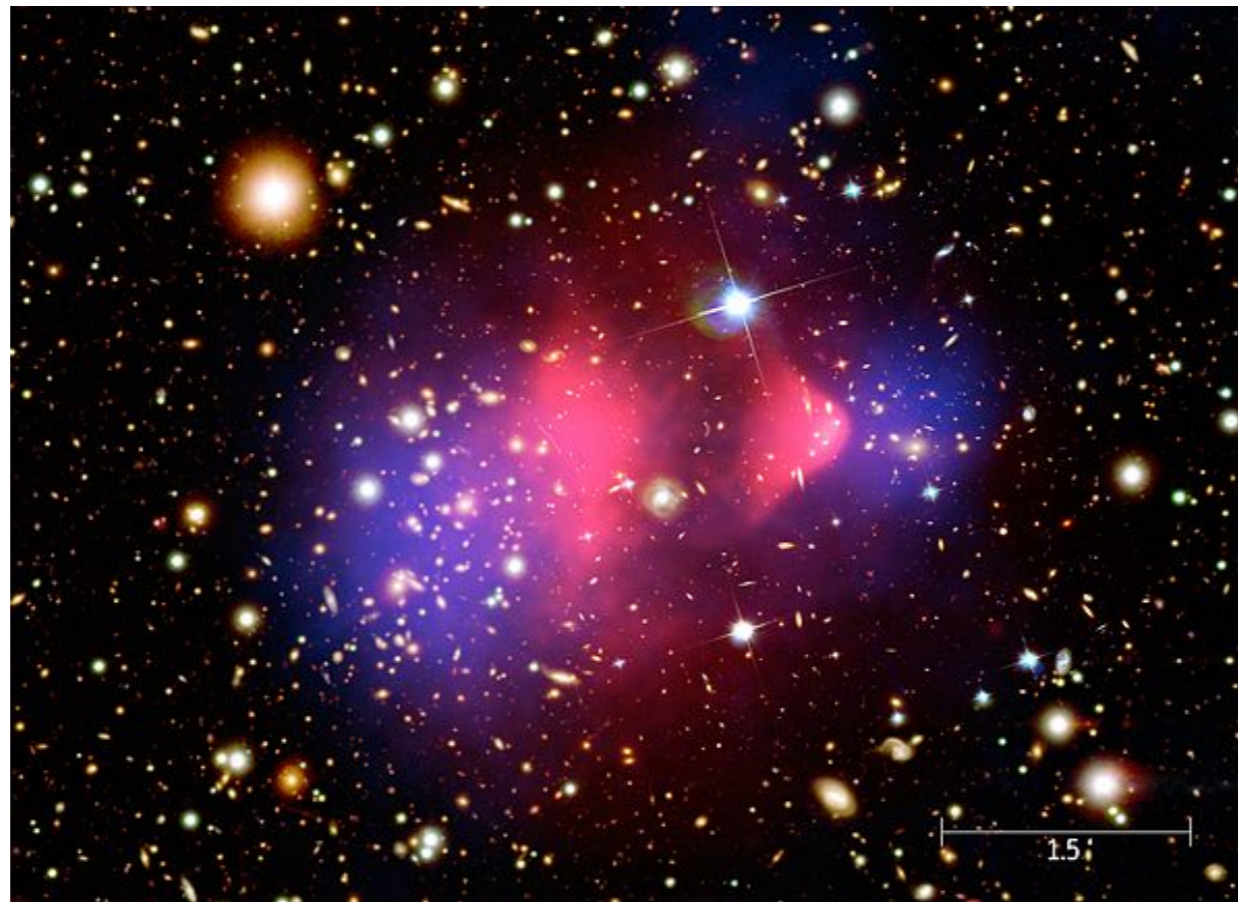
Once we started observing the cosmic web, also the presence of DM is needed to glue it and get the structures to growth up to what we see today.



This same evidence is why the theoretical fit to the CMB anisotropies need the existence of DM as the largest component of matter to

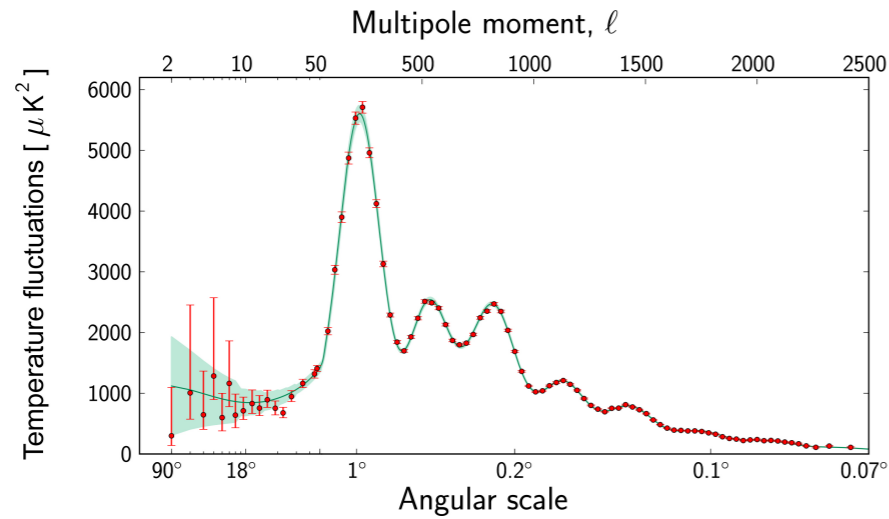
Dark matter

And also we can see the dark matter component is collisionless (cold) by comparing the distribution of a weak lensing image of the Bullet cluster (mapping all matter) with the hot gas distribution after the collision of the 2 clusters

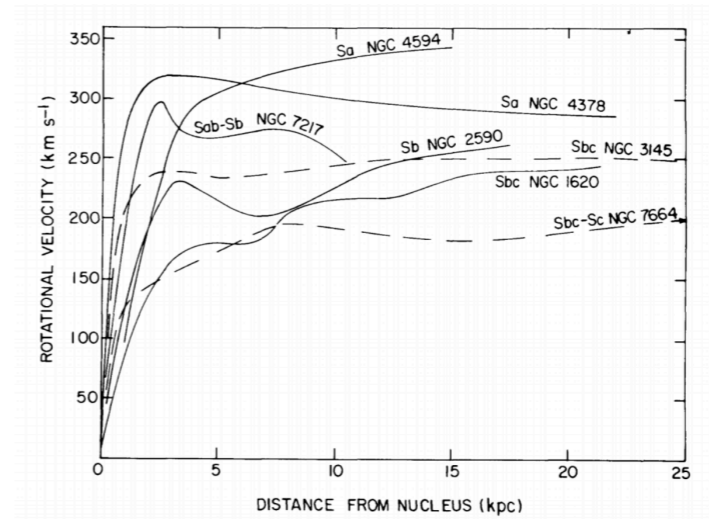


Dark matter

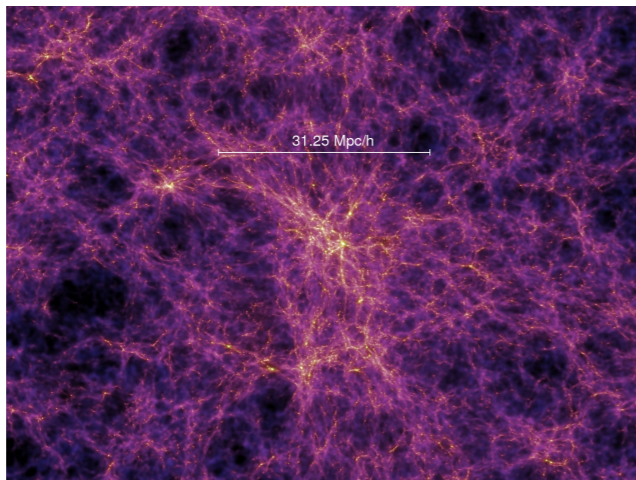
In summary, there are several astrophysical indirect evidences of DM but we still need to find the nature of this component.



Planck Collaboration



V. Rubin et al. 1978



Cold Dark Matter en Estructura a gran escala

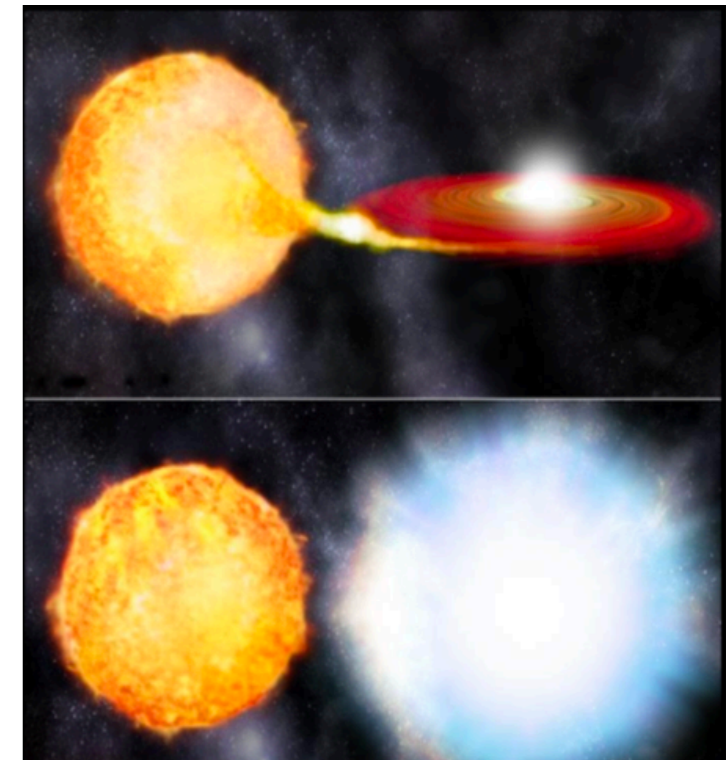


Bullet Cluster

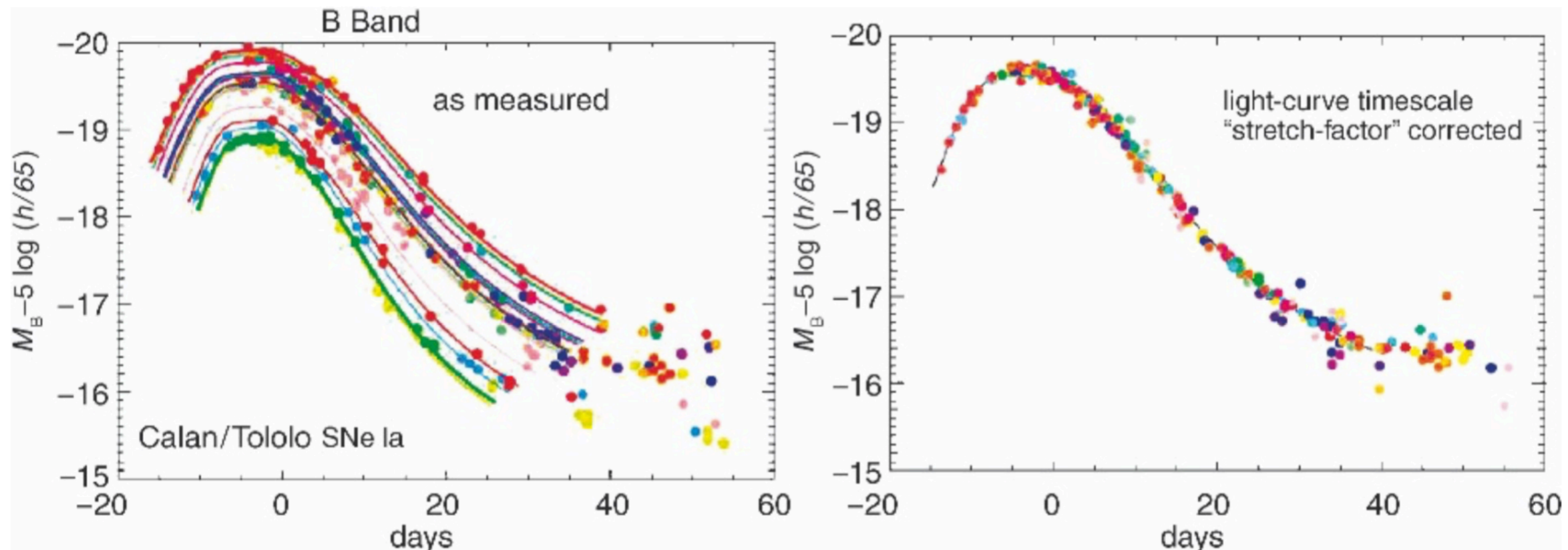
Standard candles at high redshift

The most successful standard candle in the last decades have been the type Ia SN (produced by a red giant-white dwarf binary)

We need first to classify the SN by using the SN spectrum (which can also give us the redshift) and we also need the photometric like curves

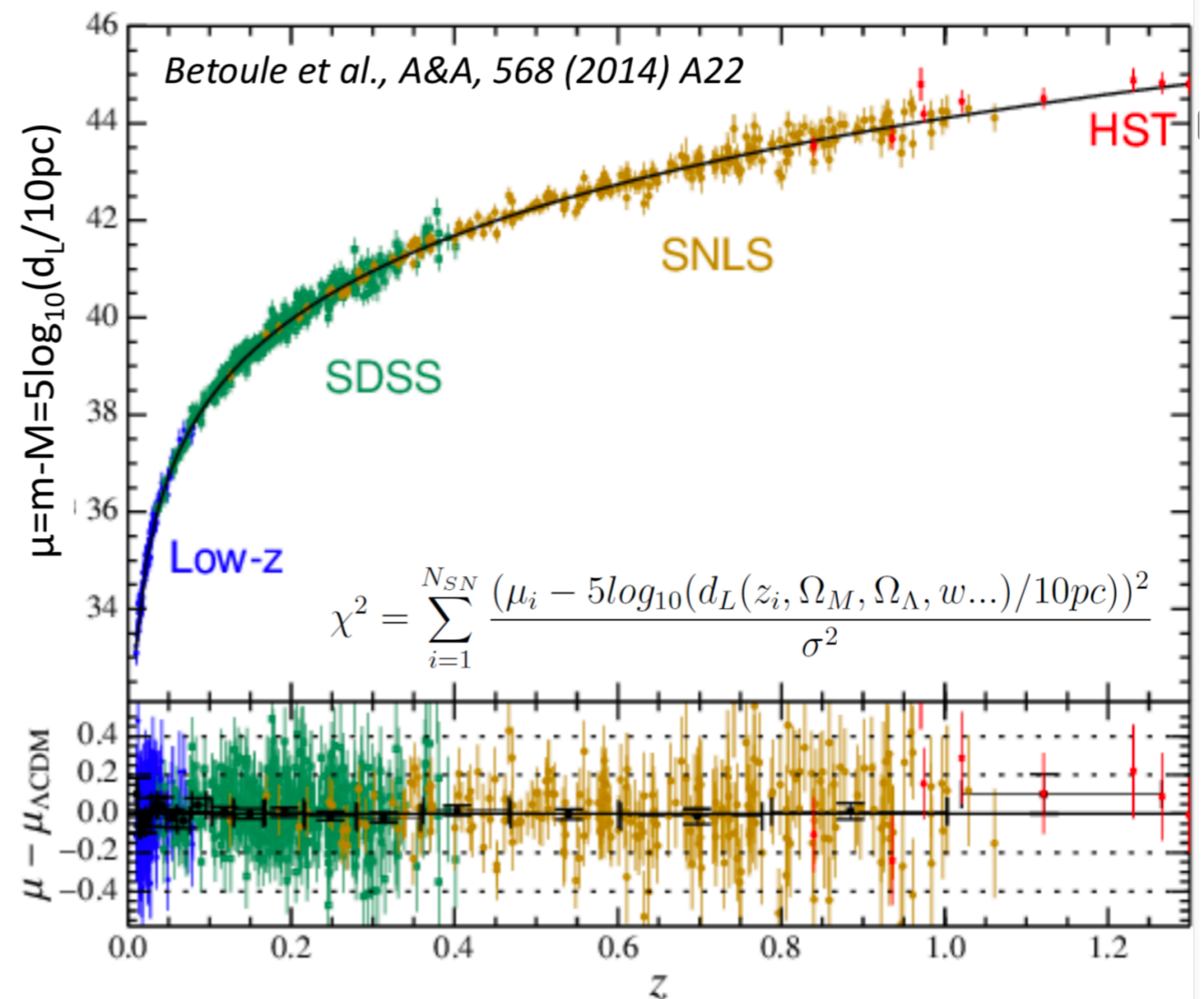
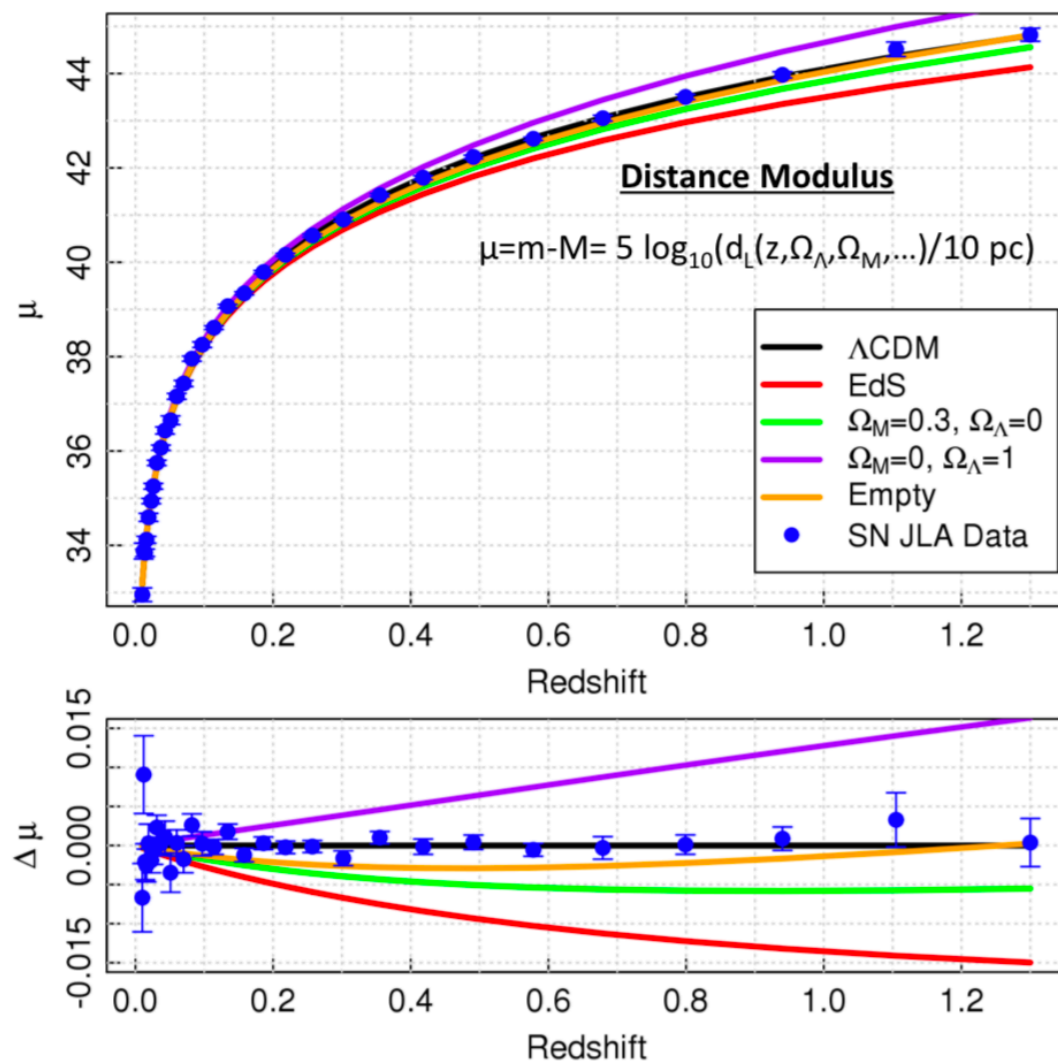


$$\mu = m_B^* - (M_B - \alpha \times X_1 + \beta \times C)$$

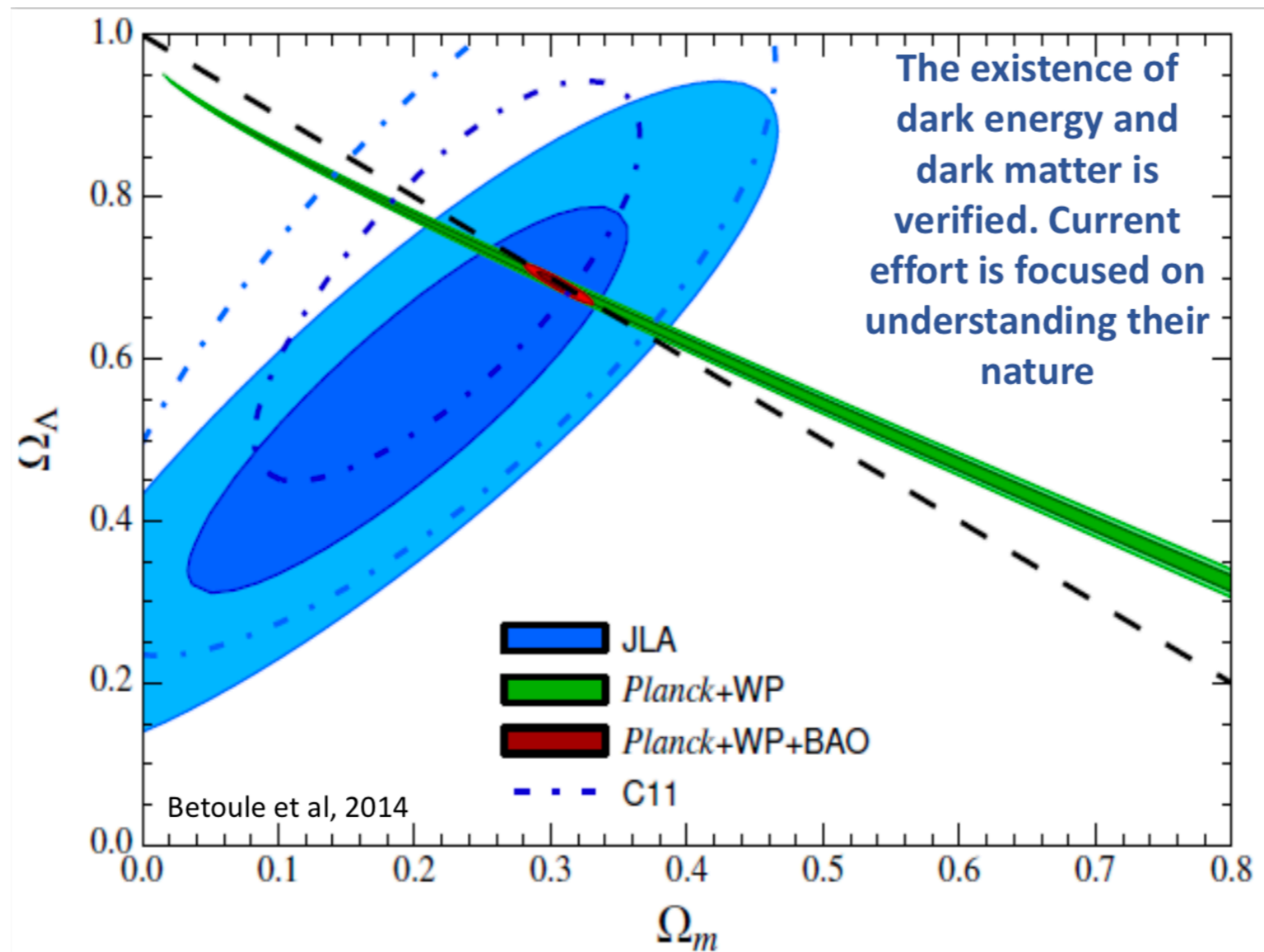


Expansion of the Universe (standard candles)

Comparing the distance modulus (measured from light curves) and the redshift, we can determine which geometry of the Universe explains the data better.



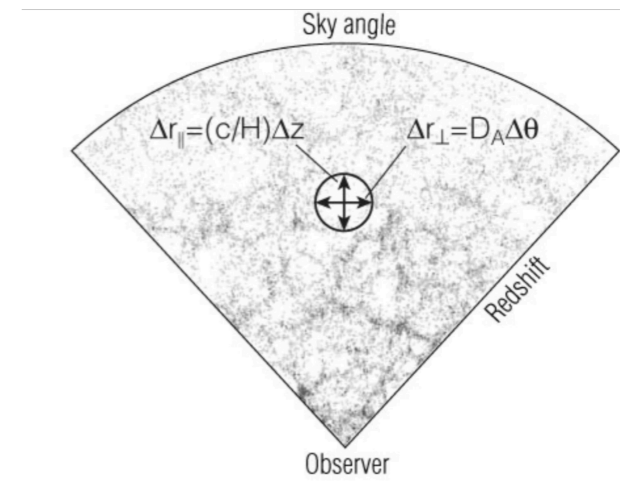
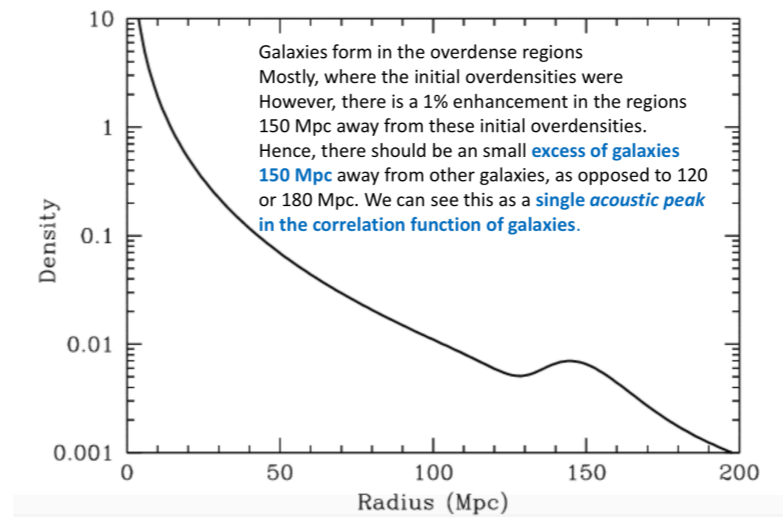
Universe accelerated expansion



Type Ia SN showed (with 2sigma) that the expansion of the Universe is accelerating. This implies either the Einstein equations have to include the cosmological constant or that we need to modify gravitation theory. If the acceleration is produced by a university fluid, then we denominate it **dark energy**.

Baryonic acoustic oscillations (BAO)

If we measure the BAO overdensity peak at different times, we can trace the expansion rate.



$$r_s(z_{dec}) = \frac{c}{\sqrt{3}} \int_0^{1/(1+z_{dec})} \frac{da}{a^2 H(a) \sqrt{1 + (3\Omega_b/4\Omega_\gamma)a}} \text{ Mpc } h^{-1}$$

CMB o BBN

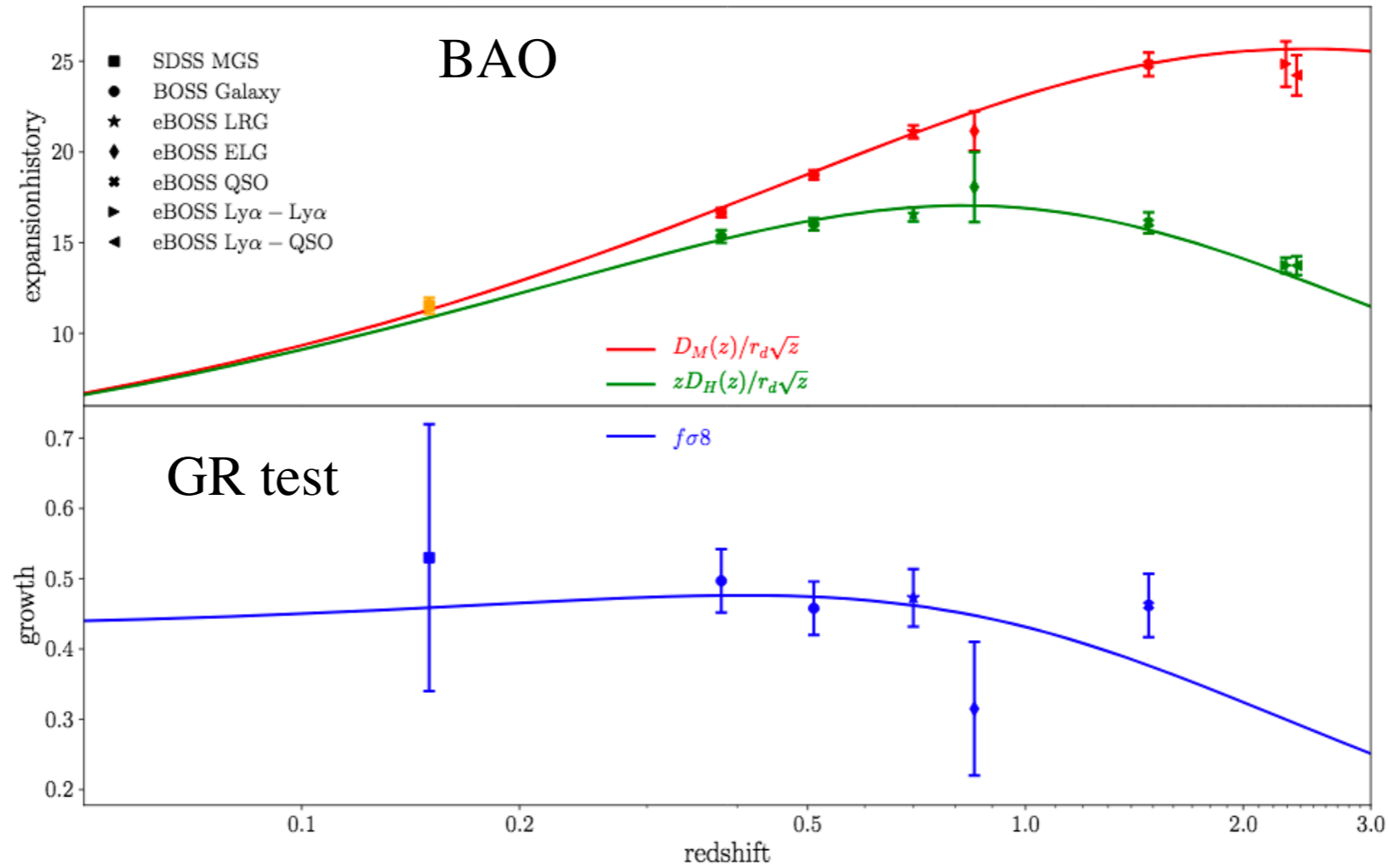
$$\theta_{BAO} = \frac{r_s(\Omega_M, w_0, w_a \dots)}{(1+z) d_A(z, \Omega_M, w_0, w_a \dots)}$$

$$\Delta z_{BAO} = H(z) r_s$$

We can determine r_s either with CMB or BBN and then compare with extragalactic surveys measurements

Expansion of the Universe (standard rulers)

Current results agree quite well with Λ CDM model



eBOSS collaboration 2020

3x2pt probes

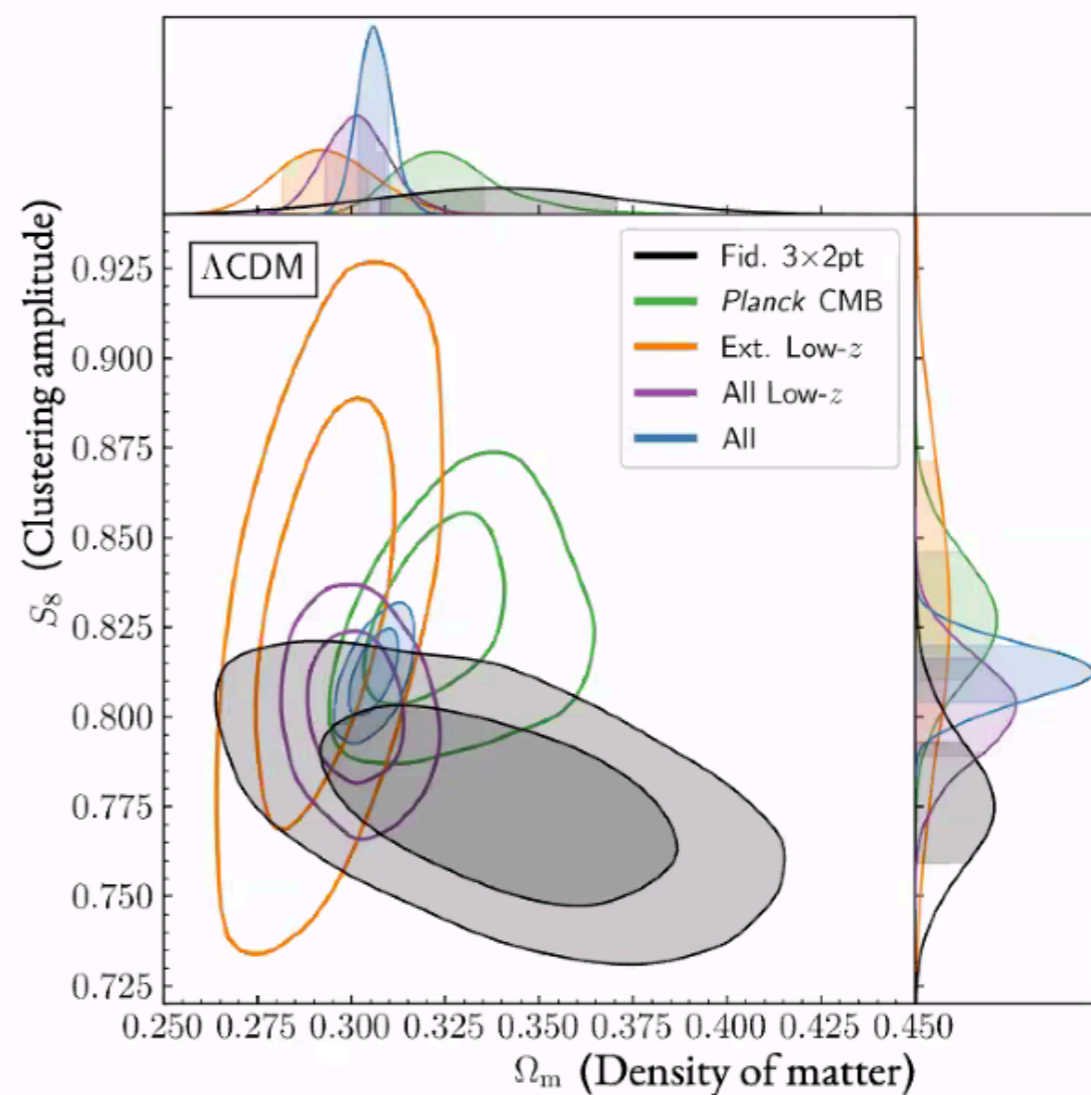
- Even new probes like weak lensing ones seem to confirm Λ CDM model

Joint constraints

Combining all these data sets we find:

$$\begin{aligned} S_8 &= 0.812^{+0.008}_{-0.008} \quad (0.815) \\ \text{In } \Lambda\text{CDM: } \Omega_m &= 0.306^{+0.004}_{-0.005} \quad (0.306) \\ \sigma_8 &= 0.804^{+0.008}_{-0.008} \quad (0.807) \\ h &= 0.680^{+0.004}_{-0.003} \quad (0.681) \\ \sum m_\nu &< 0.13 \text{ eV (95\% CL)} \end{aligned}$$

$$\begin{aligned} \sigma_8 &= 0.810^{+0.010}_{-0.009} \quad (0.804), \\ \text{In } w\text{CDM: } \Omega_m &= 0.302^{+0.006}_{-0.006} \quad (0.298), \\ w &= -1.03^{+0.03}_{-0.03} \quad (-1.00) \end{aligned}$$



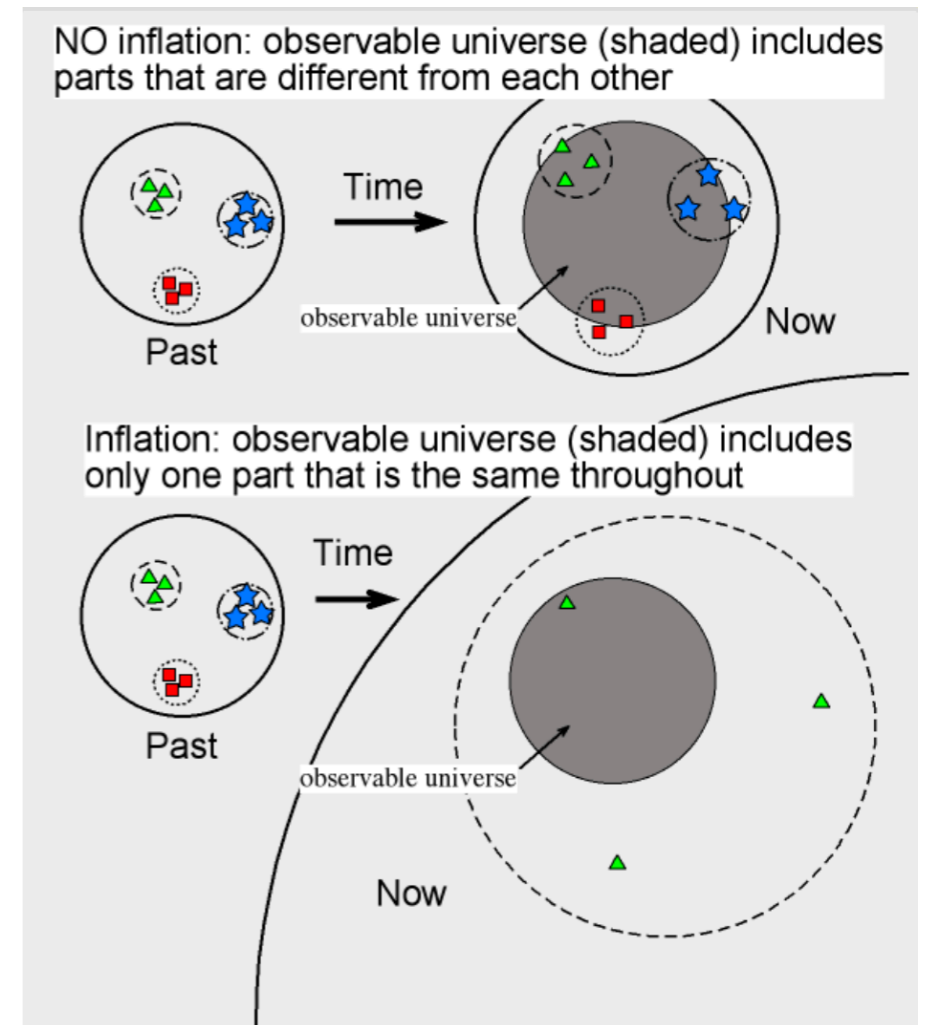
Initial conditions (aka inflation)

Currently, the theory more used by the community to explain the initial conditions is cosmic **Inflation**

It consists on an accelerated phase at the beginning of the Universe which allows us to explain the smoothness of the Universe.

Indirectly, it also provides a prediction for the initial distribution of matter:

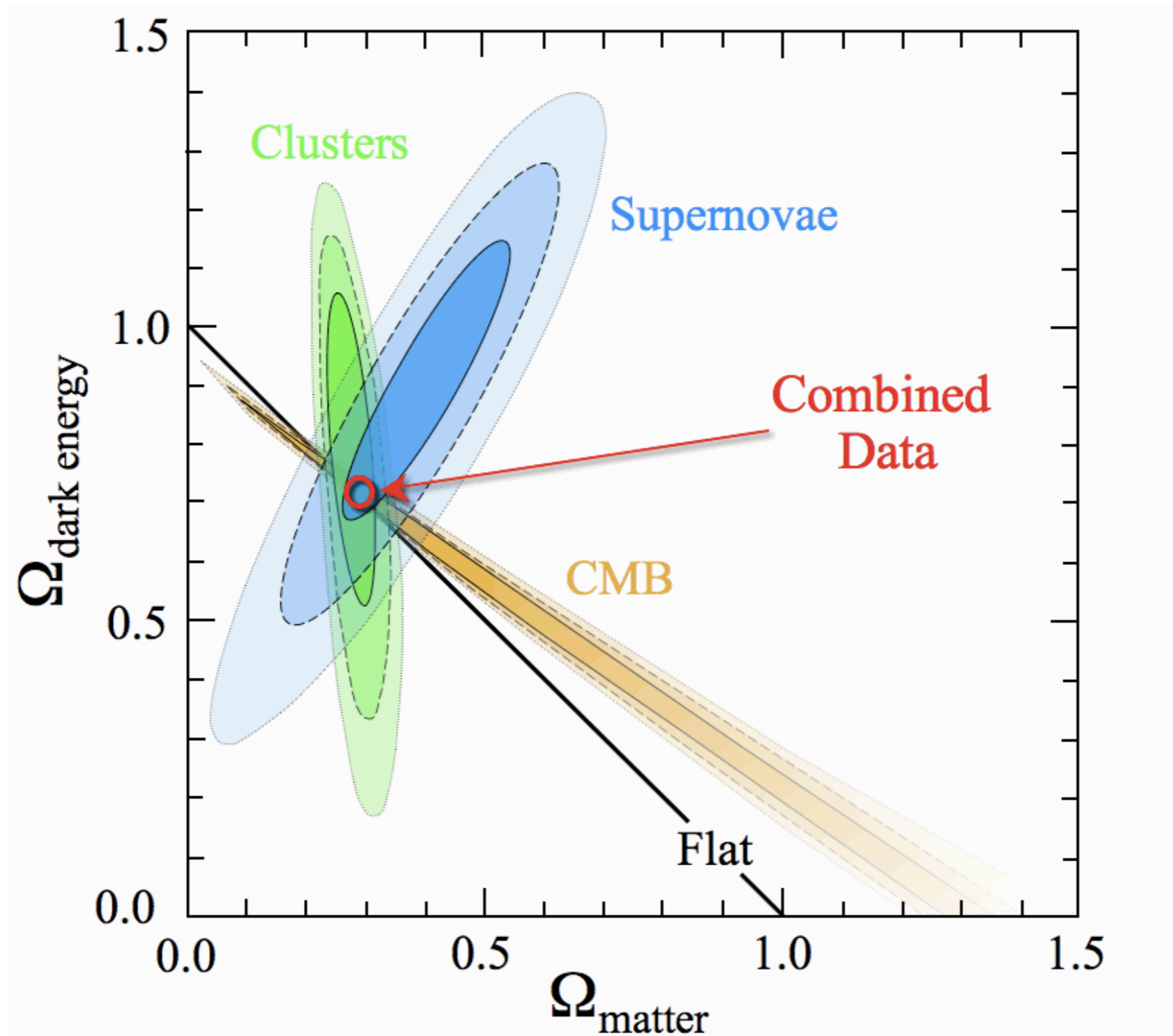
$$\mathcal{P}_\chi(k) = A_s \left(\frac{k}{k_{s0}} \right)^{n_s - 1} .$$



Not directly proven, this spectrum is one of the most successful predictions from inflation. A detection of the primordial B-modes of the CMB polarisation would be a much better detection.

Concordance model

- The combination of the different datasets gives us a great significance of the model

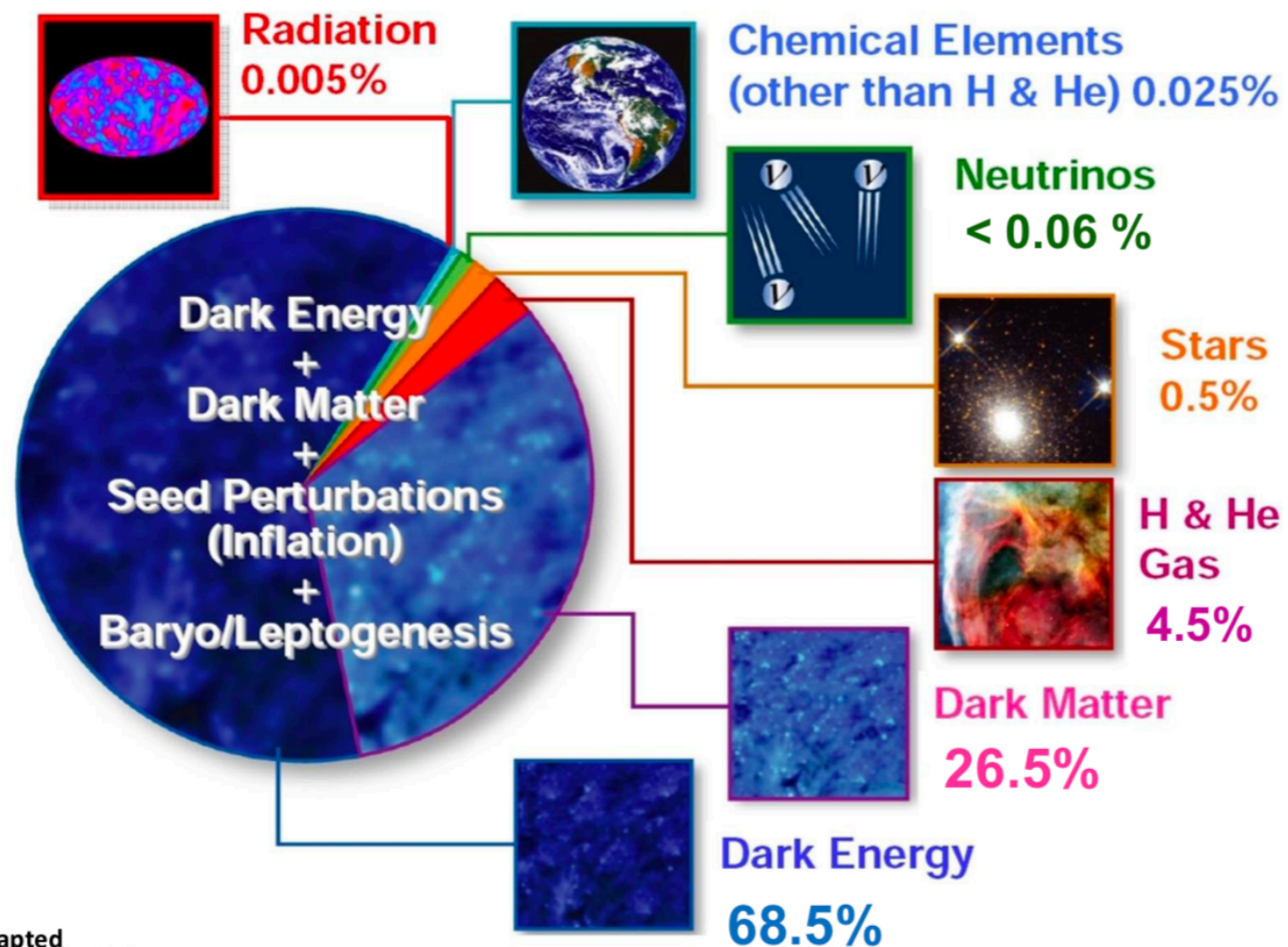


Λ CDM model

We can describe the Universe with only 6 parameters:

$$(\Omega_b, \Omega_m, n_s, A_s, \tau, H_0)$$

At least 2 of these components are still known unknowns.



Adapted
From Rocky Kolb