IPARCOS

UNIVERSIDAD COMPLUTENSE MADRID

Cosmology

Jacobo Asorey

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CENTRO DE CIENCIAS DE BENASQUE PEDRO PASCUAL

Contents

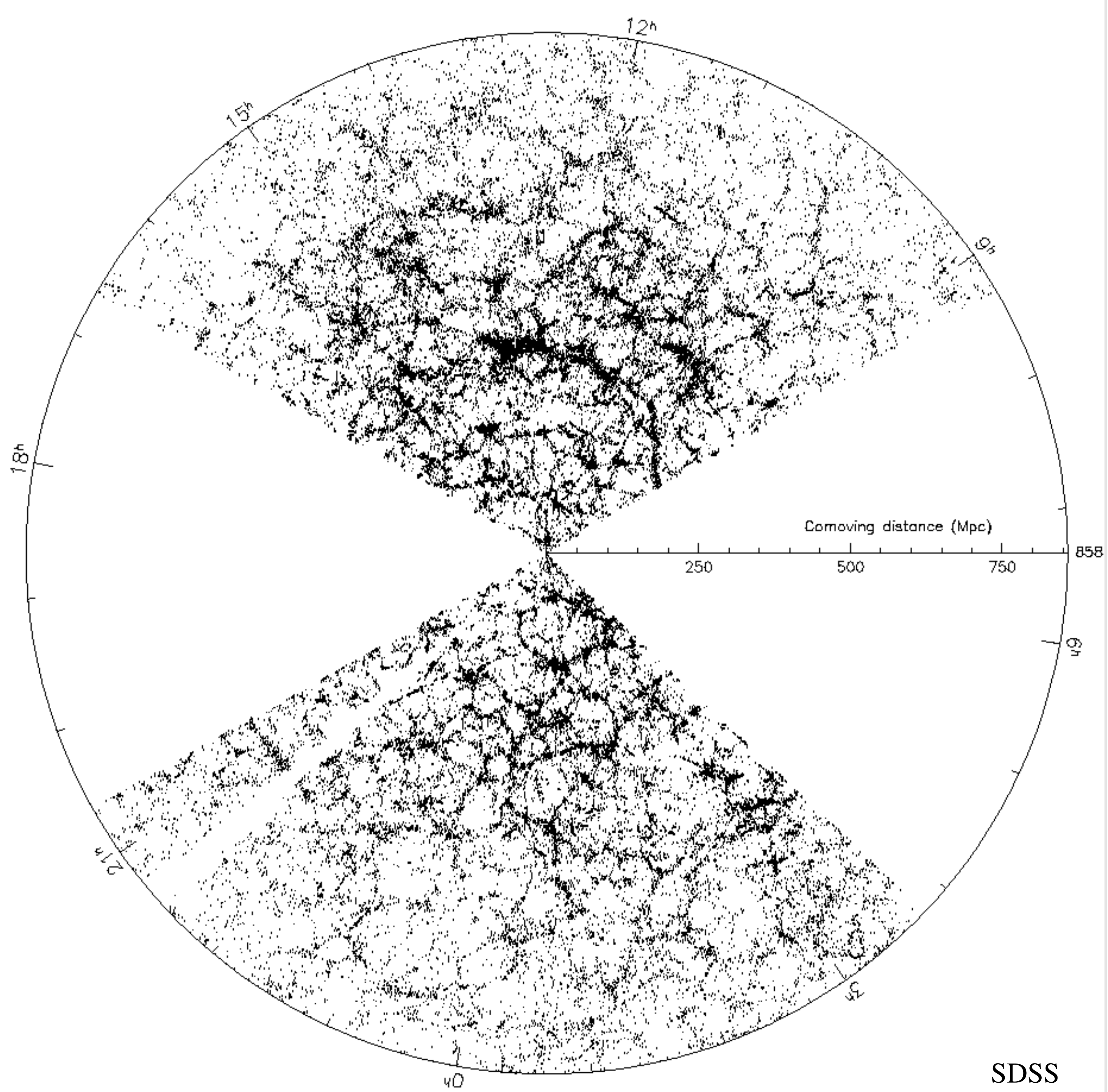
I) Rise of LCDM

II) Large-scale structure

III) Cosmological probes

Growth of structure (perturbations)

- In order to study the inhomogeneous Universe, we need to study the evolution of perturbations of the metric and the energy distributions + initial conditions



SDSS

Growth of structure (newtonian perturbations)

- By considering these 3 classical equations:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$ Continuity equation

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{1}{\rho}\nabla p - \nabla \phi$$ Euler equation

$$\nabla^2 \phi = 4\pi G \rho$$ Poisson equation

and the equation of state: $p = p(\rho)$

we can solve the equations to get $\rho, \mathbf{v}, \phi, p$

Growth of structure in a expanding Universe (newtonian perturbations)

- We define the background as the smooth and homogenous Universe, given by:

$$\rho = \rho(t) \qquad\qquad \mathbf{v} = a(t)\chi$$

- To perturbe the equations, we need to define the perturbed quantities:

Perturbed density: $\rho(\chi, t) = \bar{\rho}(t) + \delta\rho(\chi, t) = \bar{\rho}(1 + \delta(\chi, t))$

Perturbed pressure: $p = \bar{p} + \delta p$

Perturbed velocity: $\mathbf{v} = a(t)\chi + \delta\mathbf{v}$

Perturbed potential: $\phi = \phi + \delta\phi(\chi, t)$

Growth of structure in a expanding Universe (newtonian perturbations)

- Introducing the perturbations in the 3 dynamical equations:

$$\rho(\chi, t) = \bar{\rho}(1 + \delta(\chi, t))$$

$$p = \bar{p} + \delta p$$

$$\mathbf{v} = a(t)\chi + \delta\mathbf{v}$$

$$\phi = \phi + \delta\phi(\chi, t)$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{1}{\rho}\nabla p - \nabla\phi$$

$$\nabla^2\phi = 4\pi G\rho$$

we reach (considering only linear terms as the perturbations are small):

$$\frac{\partial \delta\rho}{\partial t} + \bar{\rho}\nabla\delta\mathbf{v} + \nabla(\delta\rho\mathbf{v}) = 0$$

$$\frac{\partial \delta\mathbf{v}}{\partial t} + \mathbf{v}(\nabla)\delta\mathbf{v} + (\delta\mathbf{v}\nabla)\mathbf{v} + \frac{c_s^2}{\rho}\nabla\delta\rho + \nabla\delta\phi = 0$$

$$\nabla^2\delta\phi = 4\pi G\delta\rho$$

Growth of structure in a expanding Universe (newtonian perturbations)

- Now, combining these equations and also changing to coordinates that move with the expansion we can obtain the growth history for density perturbations in an expanding Universe:

$$\ddot{\delta} + 2H\dot{\delta} - \frac{c_s^2}{a^2}\Delta\delta - 4\pi G\varepsilon_0\delta = 0,$$

In Fourier space:

$$\delta(\boldsymbol{x},t) = \sum_{\boldsymbol{k}} \delta_{\boldsymbol{k}}(t)\, e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$$

$$\delta_{\boldsymbol{k}}(t) = \frac{1}{V}\int \delta(\boldsymbol{x},t)\, e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}\, d^3x$$

$$\ddot{\delta}_k + 2H\dot{\delta}_k = \left(4\pi G\rho_0(t) - \frac{k^2 c_s^2}{a^2}\right)\delta_k.$$ For baryonic matter

$$\ddot{\delta}_k + 2H\dot{\delta}_k - 4\pi G\rho_m(t)\delta_k = 0$$ For dark matter

The full treatment

- If we want to do the full treatment, we need to do the perturbations (assumed small) in the frame of General Relativity and the flat FLRW metric:

$$ds^2 = \left[{}^{(0)}g_{\alpha\beta} + \delta g_{\alpha\beta}(x^\gamma)\right] dx^\alpha dx^\beta$$

- This produces a set of scalar, vector and tensor fluctuations. The tensor fluctuations are relevant for gravitational waves and polarisation of the CMB photons but we focus on the scalar one. We choose the Newtonian gauge in which:

$$ds^2 = a^2\left[(1+2\phi_l)d\eta^2 - (1-2\psi_l)\delta_{ij}dx^i dx^j\right]$$

The reason we choose this gauge is that because also the 2 potentials are the same, reducing to the standard Newtonian potential.

The full treatment

- In this gauge, then the equations of scalar perturbations are:

$$\Delta\Phi - 3\mathcal{H}\left(\Phi' + \mathcal{H}\Phi\right) = 4\pi G a^2 \overline{\delta\varepsilon},$$

$$(a\Phi)'_{,i} = 4\pi G a^2 (\varepsilon_0 + p_0)\, \overline{\delta u}_{\parallel i},$$

$$\Phi'' + 3\mathcal{H}\Phi' + \left(2\mathcal{H}' + \mathcal{H}^2\right)\Phi = 4\pi G a^2 \overline{\delta p}.$$

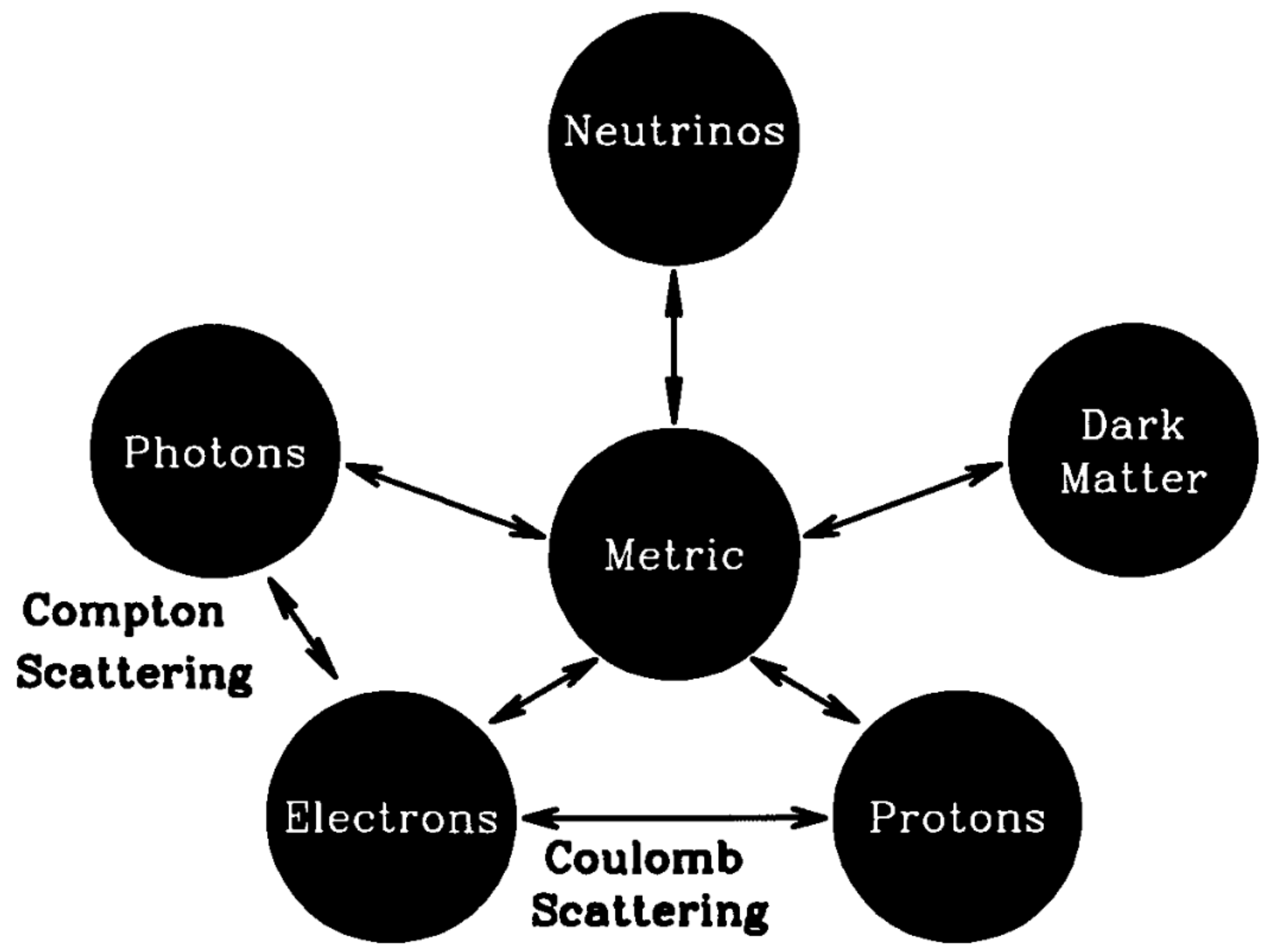
- Combining them we get:

$$\Phi'' + 3\left(1 + c_s^2\right)\mathcal{H}\Phi' - c_s^2 \Delta\Phi + \left(2\mathcal{H}' + \left(1 + 3c_s^2\right)\mathcal{H}^2\right)\Phi = 0$$

Boltzmann equations

- In order to study the cosmic distribution of photons and matter inhomogeneities, we can use the Boltzmann equations in the phase space

$$\frac{df}{dt} = C[f]$$



where C[f] accounts for the collisions, in case there are.

Boltzmann equations for photons

- We need to solve the Boltzmann equations for the different components of the Universe are for the zero-order and the perturbations on the equilibrium distribution

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x^i}\frac{dx^i}{dt} + \frac{\partial f}{\partial \hat{p}^i}\frac{d\hat{p}^i}{dt} + \frac{\partial f}{\partial p}\frac{dp}{dt}$$

Using the metric perturbations:

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\hat{p}^i}{a}\frac{\partial f}{\partial x^i} + \frac{\partial f}{\partial p}\frac{dp}{dt}$$

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\hat{p}^i}{a}\frac{\partial f}{\partial x^i} - p\frac{\partial f}{\partial p}\left[H + \frac{\partial \phi}{\partial t} + \frac{\hat{p}^i}{a}\frac{\partial \psi}{\partial x^i}\right]$$

Boltzmann equations for photons

- We need to address the photon distribution where the zero-th order is the Bose-Einstein distribution function:

$$f(\vec{x}, p, \vec{p}, t) = \left[exp\left(\frac{p}{T(t)[1 + \Theta(\vec{x}, \vec{p}, t)]} \right) \right]^{-1}$$

- Zero- order equation (collision less):

$$\left.\frac{df}{dt}\right|_0 = \frac{\partial f^{(0)}}{\partial t} - Hp\frac{\partial f^{(0)}}{\partial p} = 0 \longrightarrow T \propto \frac{1}{a}$$

- First- order equation:

(Compton scattering)

$$\frac{\partial \Theta}{\partial t} + \frac{\hat{p}^i}{a}\frac{\partial \Theta}{\partial x^i} + \frac{\partial \phi}{\partial t} + \frac{\hat{p}^i}{a}\frac{\partial \psi}{\partial x^i} = n_e \sigma_T [\Theta_0 - \Theta + \bar{p} \cdot \vec{v}_b]$$

- And in Fourier space and conformal time:

$$\dot{\tilde{\Theta}} + ik\mu\tilde{\Theta} + \dot{\tilde{\Phi}} + ik\mu\tilde{\Psi} = -\dot{\tau}\left[\tilde{\Theta}_0 - \tilde{\Theta} + \mu\tilde{v}_{\rm b}\right]$$

Boltzmann equations for dark matter and baryons

- Similar derivation but for DM no collision term while for baryons there is the Coulomb scattering proton-electron and the Compton scattering of the electron-photons coupling. For both components, the zero-order equation is just the same one as the density of both fluids in the background model

$$\dot{\Theta} + ik\mu\Theta = -\dot{\Phi} - ik\mu\Psi - \dot{\tau}\left[\Theta_0 - \Theta + \mu v_b - \frac{1}{2}\mathcal{P}_2(\mu)\Pi\right]$$

$$\Pi = \Theta_2 + \Theta_{P2} + \Theta_{P0}$$

Photons

$$\dot{\Theta}_P + ik\mu\Theta_P = -\dot{\tau}\left[-\Theta_P + \frac{1}{2}(1 - \mathcal{P}_2(\mu))\Pi\right]$$

Final set of eqs is:

$$\dot{\delta} + ikv = -3\dot{\Phi}$$

DM

$$\dot{v} + \frac{\dot{a}}{a}v = -ik\Psi$$

$$\dot{\delta}_b + ikv_b = -3\dot{\Phi}$$

Baryons

$$\dot{v}_b + \frac{\dot{a}}{a}v_b = -ik\Psi + \frac{\dot{\tau}}{R}\left[v_b + 3i\Theta_1\right]$$

Numerical solutions

- For the current model, the system of Einstein-Boltzmann equations have to be solved numerically.

- Most used public software:

    - CAMB: https://camb.info/ (Fortran)

      Python wrapper: https://camb.readthedocs.io/en/latest/

      Fast and widely used but difficult to modify from theoretical point of view

    - CLASS: https://lesgourg.github.io/class_public/class.html

      C++ code (also with python wrapper).

      Fast, modular and with several theoretical model implementations.

Matter power spectrum

- The theoretical cosmological function we mostly use for the analysis of large-scale structure is the power spectrum:

$$P(k) = T(k)^2 P_{ini}(k)$$

- We need to obtain the transfer function through the Boltzman - Einstein equations. For scales that cross the horizon at matter dominated time, there is an overall decrease in the potential but for the modes that enter during radiation dominated phase, the potential changes because of the interaction with the radiation (photons or neutrinos)

- The growth during matter domination is decoupled from this and grows only depending on the scale factor

Growth of structure

For scales larger than 10 Mpc, we can assume linear theory and estimate the growth for different times

Universe dominated by **matter**:

$$\delta = A(x)t^{2/3} + B(x)t^{-1}$$

Grows with scale factor $a(t) \sim t^{2/3}$.

Universe dominated by **radiation**

$$\delta_{\boldsymbol{k}}(t) = A + B \ln t$$

Suppressed growth

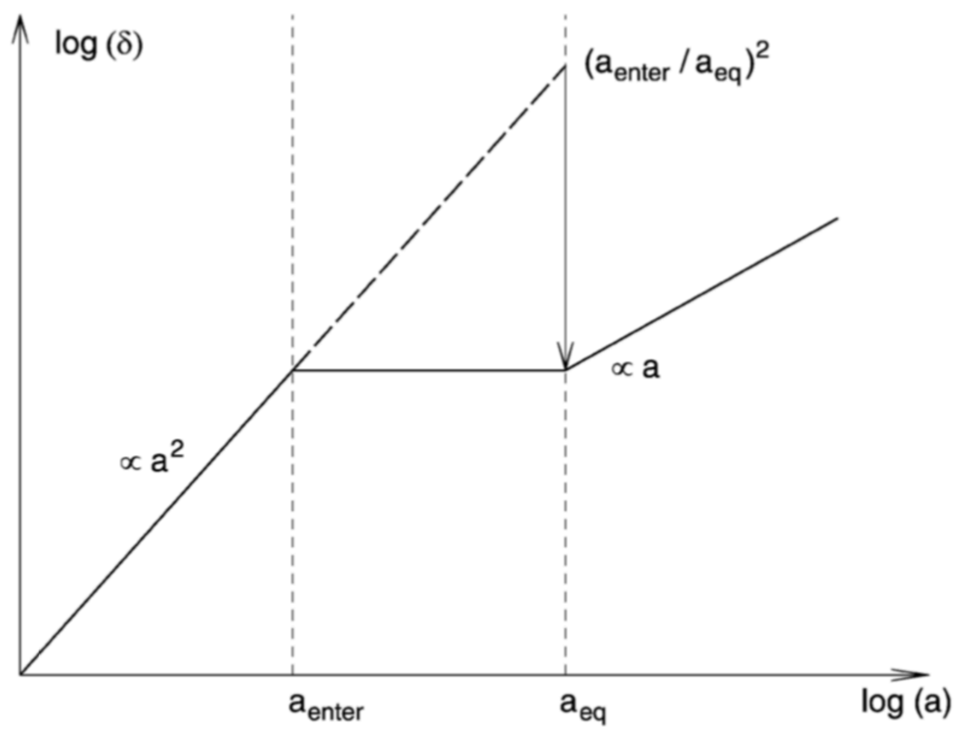


Fig. 7.5. A density perturbation that enters the horizon during the radiation-dominated epoch of the Universe ceases to grow until matter starts to dominate the energy content of the Universe. In comparison to a perturbation that enters the horizon later, during the matter-dominated epoch, the amplitude of the smaller perturbation is suppressed by a factor $(a_{eq}/a_{enter})^2$, which explains the qualitative behavior (7.29) of the transfer function

We can only predict the statistical properties of the distribution -> power spectrum P(k)

$$\left\langle \hat{\delta}(\vec{k})\hat{\delta}^*(\vec{k}')\right\rangle \equiv (2\pi)^3 P(k)\delta_D\left(\vec{k}-\vec{k}'\right)$$

We model P(k) and growth of structure with the transfer function T(k)

$$P_0(k) = \underline{A\, k^{n_s}}\, T^2(k)$$

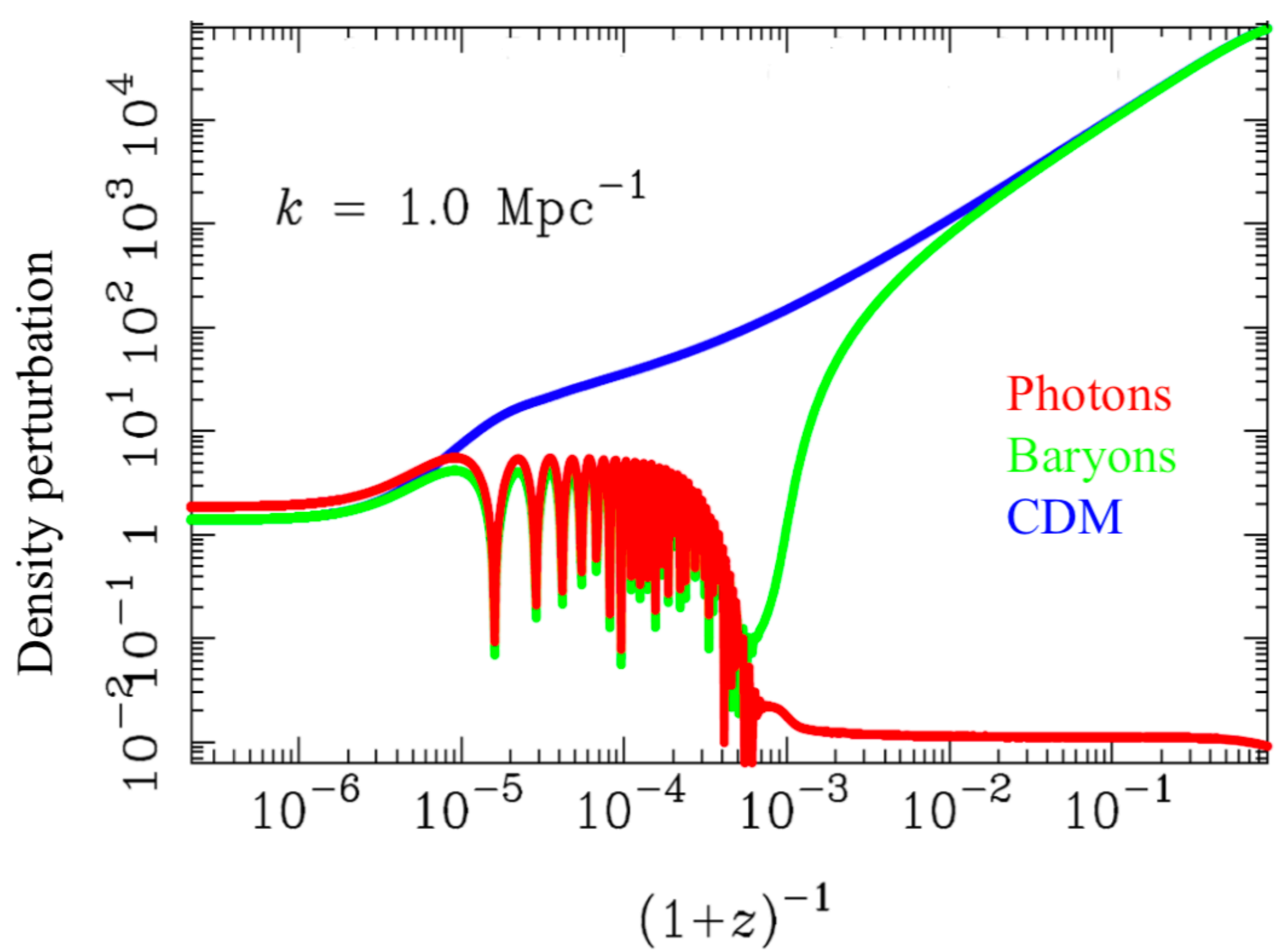
Espectro inicial

Role of DM

If $\delta \sim t^{2/3}$, and decoupling fluctuations size (z~1100) are of the order of 10^{-5}, we couldn't reach the current amplitude. We need DM

DM fluctuations start growing before decoupling at: z~3300

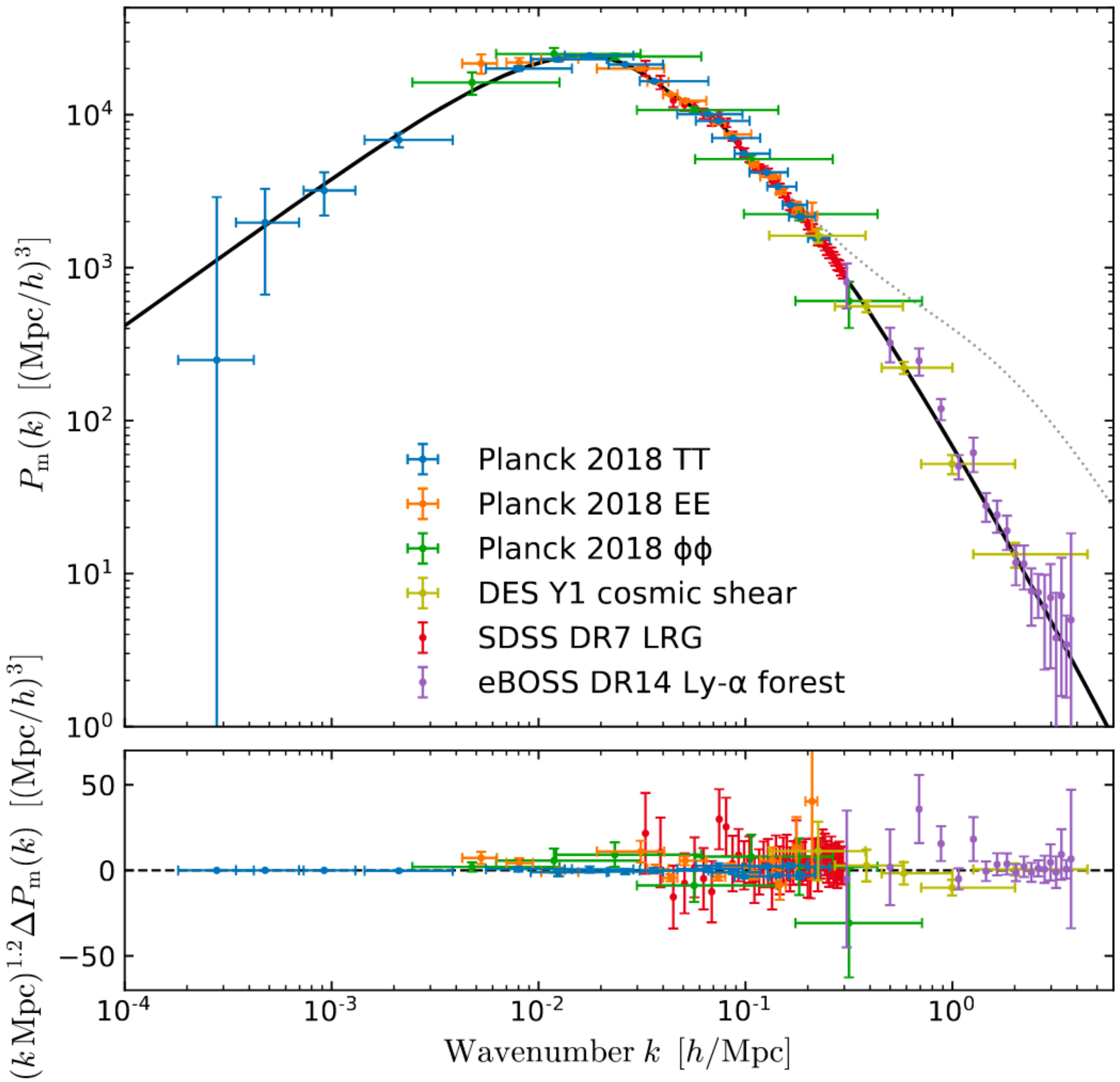
After decoupling, the baryons start following the DM fluctuations.



This is another evidence for the existence of DM

Large-scale structure

- Universe filled with density fluctuations
- Structure only only visible through galaxies (distribution) and photons (weak lensing)
- Galaxies and photons here are functioning as test particles - tracing out the gravitational field
- Most low-redshift surveys have measured the transfer function.
- Need very large volumes to measure primordial power spectrum and determine initial conditions (independently from CMB)



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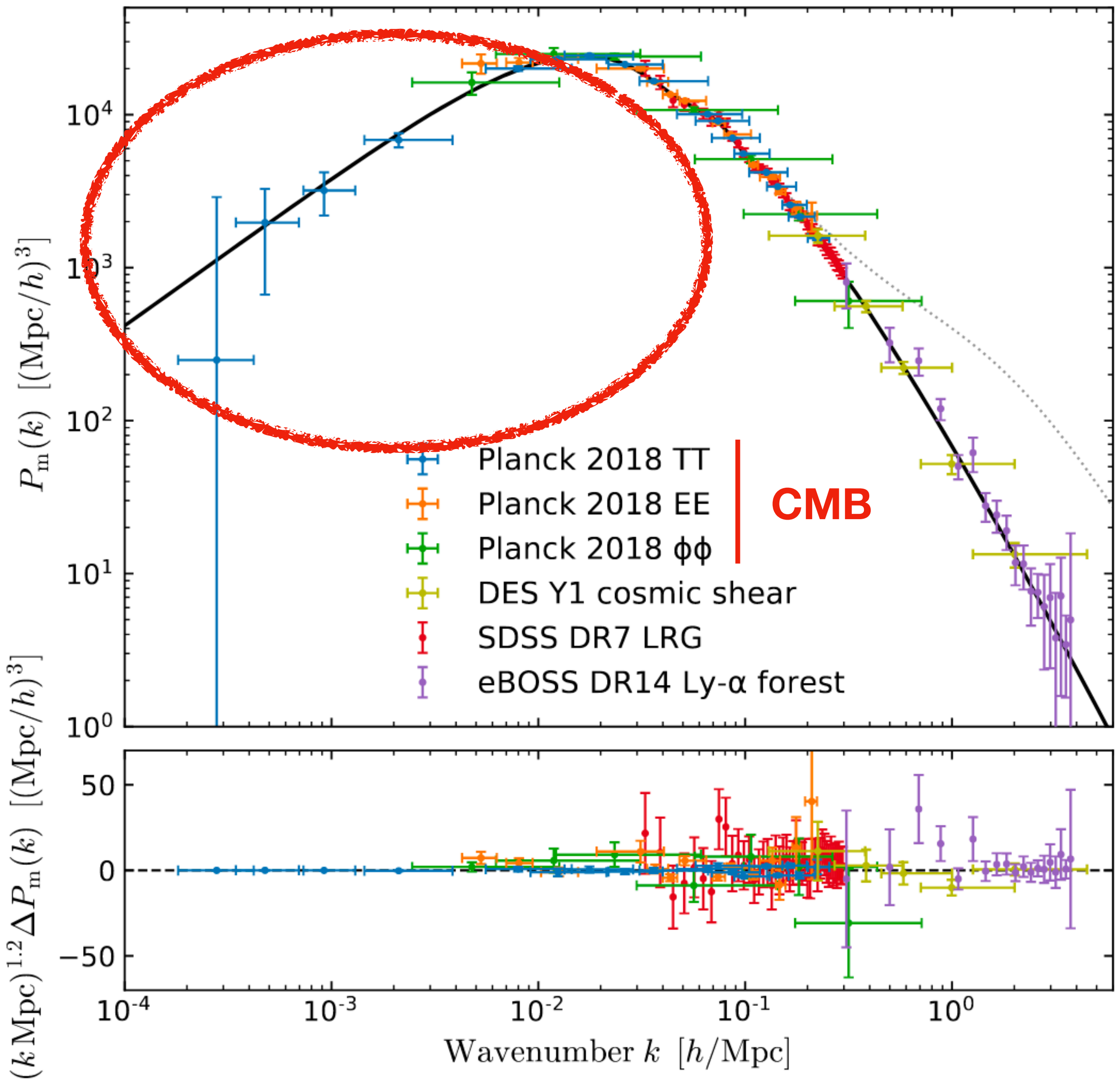

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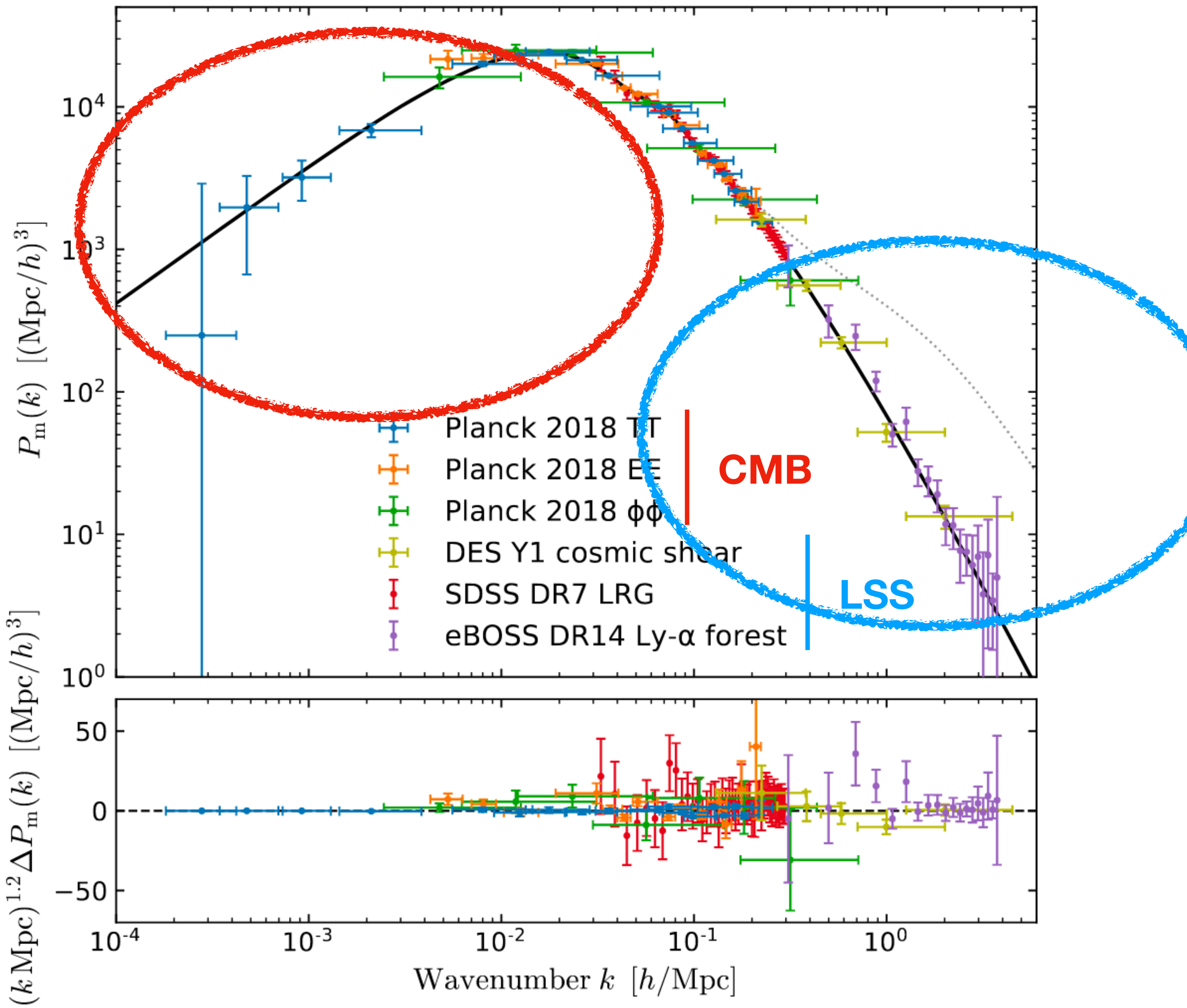


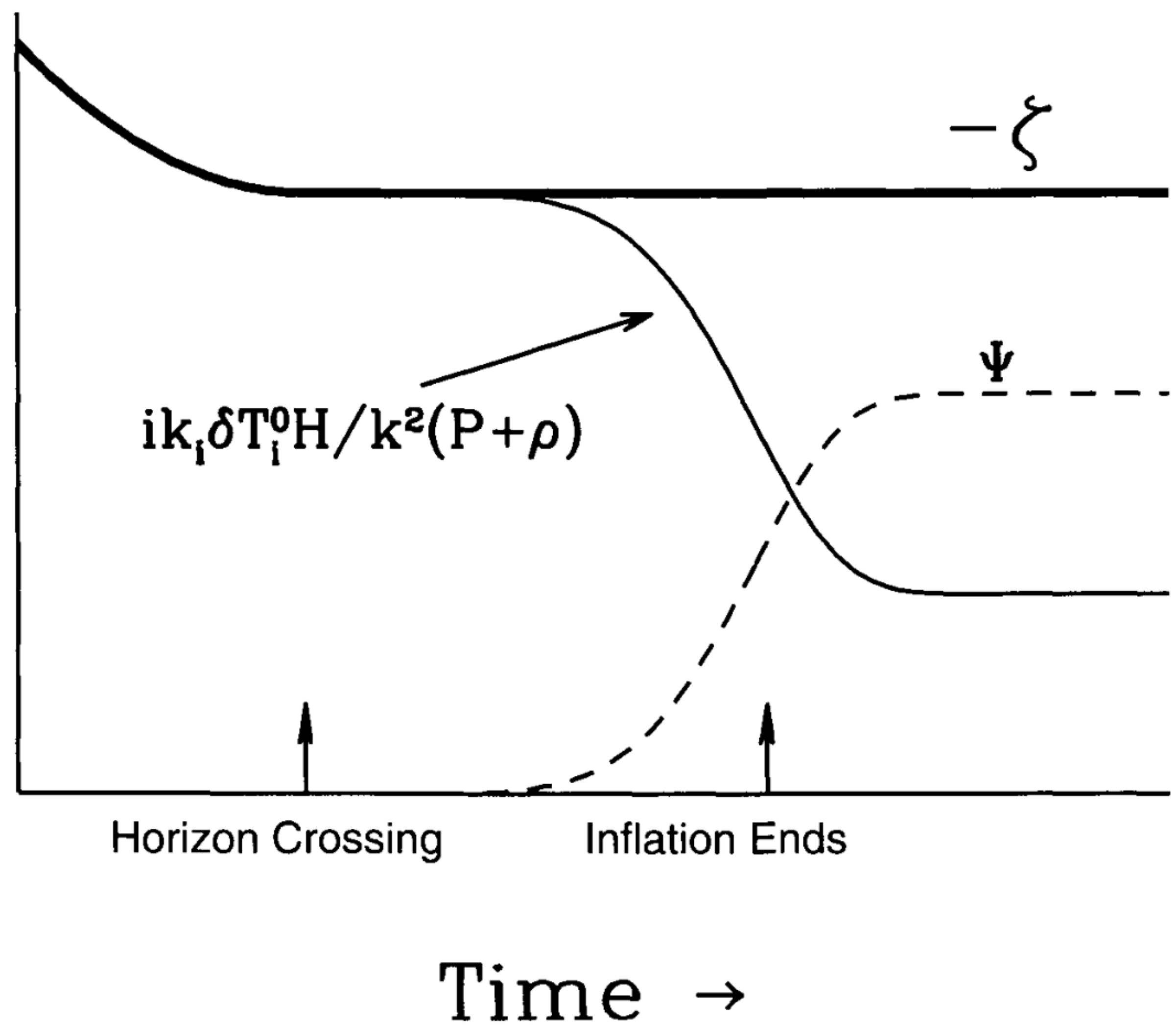
Initial conditions

- Besides the equations that describe the evolution of metric perturbations and energy perturbations, we need to set the initial conditions.

- The initial distribution of scalar perturbations is almost scale invariant and inflationary models tend to predict some deviations from the pure scale-free spectrum.

- These modes leave the horizon and then they enter the Universe later on in radiation and matter dominated phases depending on the scale.

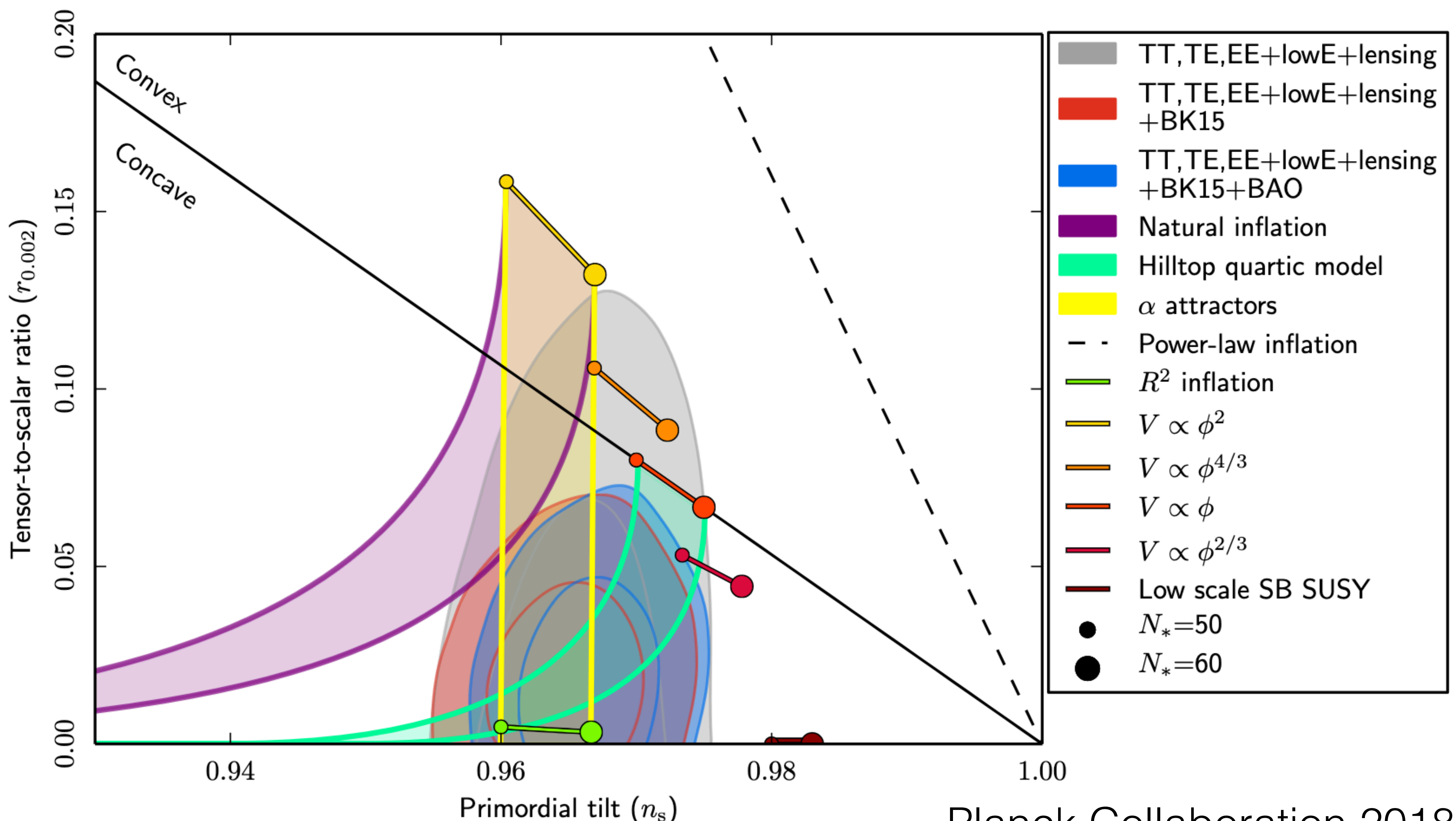
$$\mathcal{P}_\chi(k) = A_s \left(\frac{k}{k_{s0}}\right)^{n_s - 1}.$$



Inflationary constraints from CMB

- n_S=1 has been ruled out by Planck.

- Inflation also predicts an initial spectrum for tensor modes of the metric perturbations -> GW imprint and primordial polarisation of CMB would be a direct probe of inflation.



Planck Collaboration 2018

Correlation function

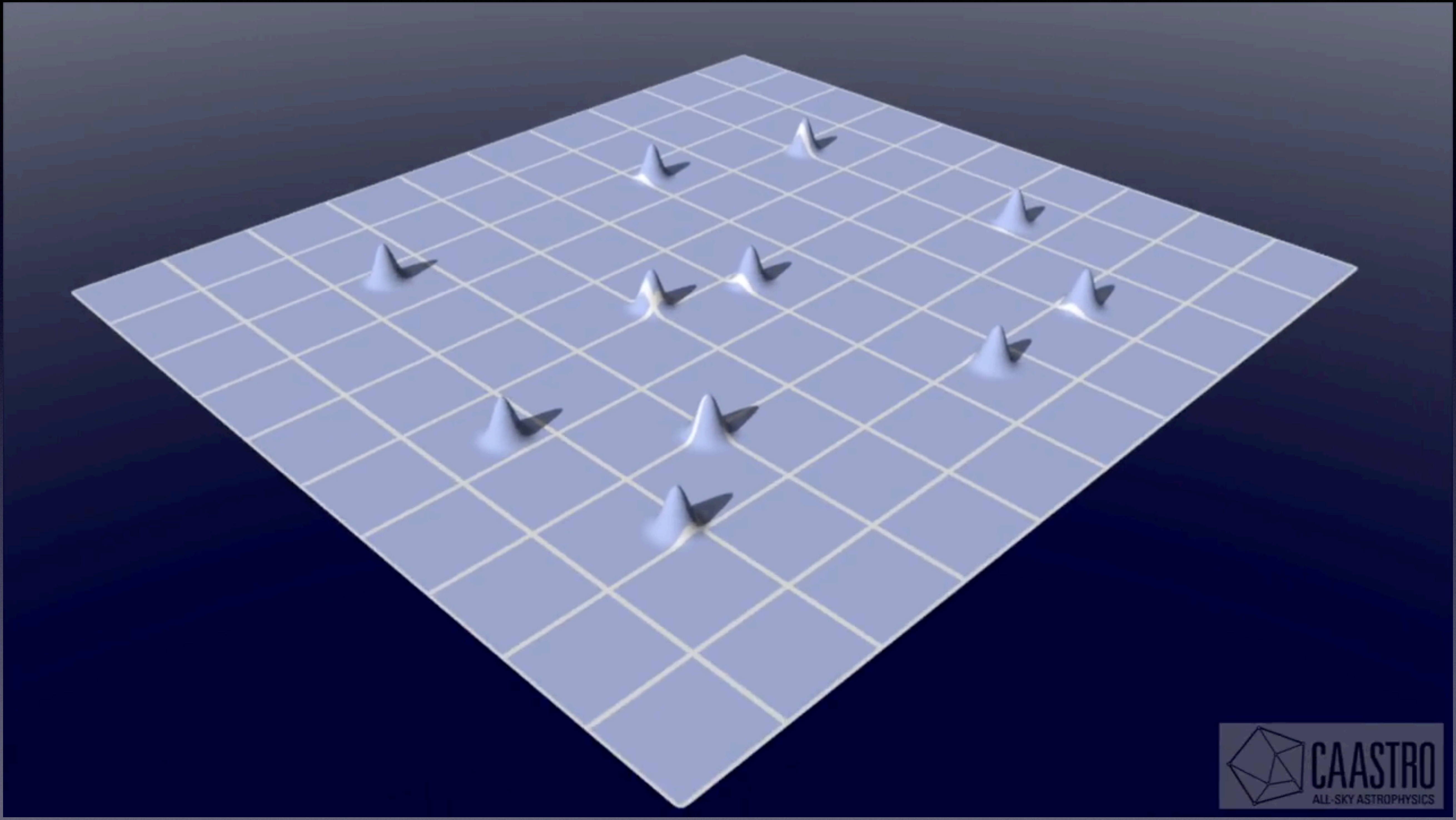
- The Fourier transform of the power spectrum is given by the correlation function:

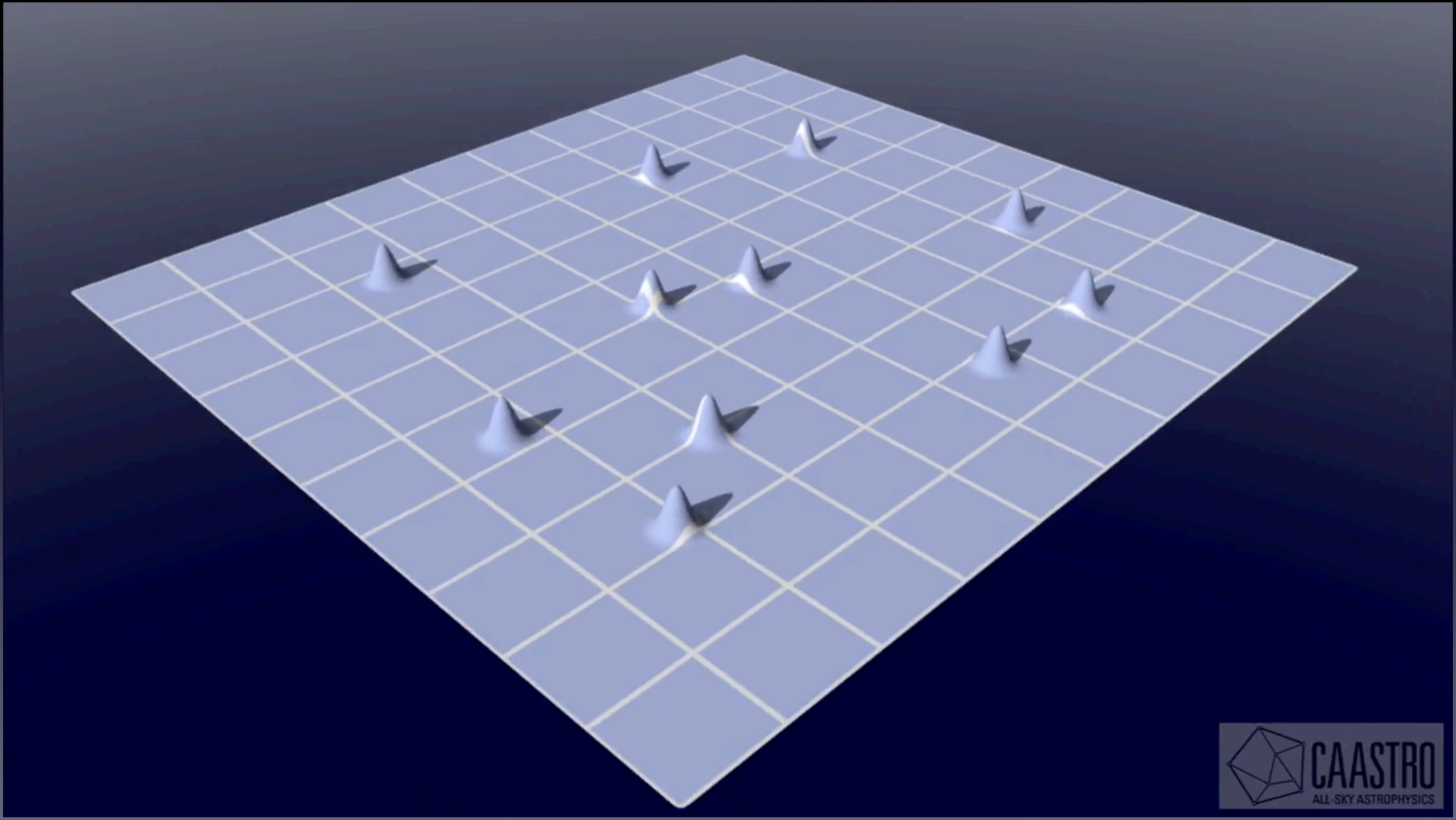
$$\xi(\mathbf{r}) = \frac{V}{(2\pi)^3} \int |\delta_k|^2 e^{-i\mathbf{k}\cdot\mathbf{r}} d^3k$$

- For an isotropic Universe this is:

$$\xi(\mathbf{r}) = \frac{V}{(2\pi)^3} \int P(k) \frac{\sin kr}{kr} 4\pi k^2 dk$$

- The physical meaning is that it measures the excess with respect to a uniform distribution.


CAASTRO
ALL-SKY ASTROPHYSICS


CAASTRO
ALL-SKY ASTROPHYSICS

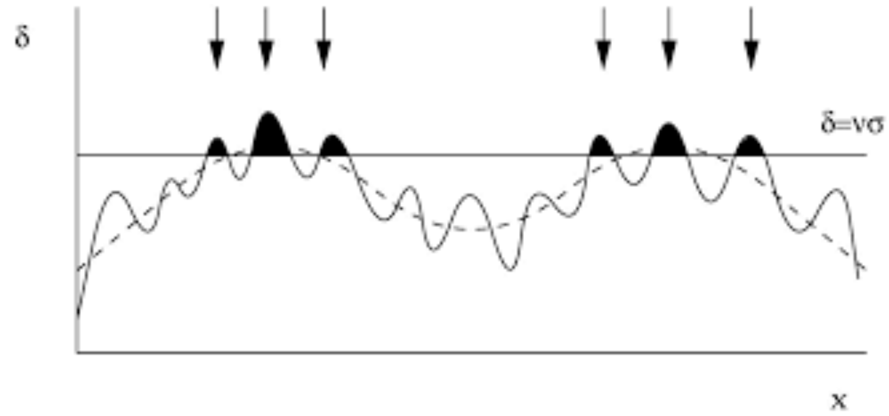
Galaxy bias

A particular problem is that we observe galaxies as tracers of the matter field, but the distribution of baryonic matter is biased with respect to the total matter field (dominated by dark matter). Galaxies grow in the peaks of the density field.

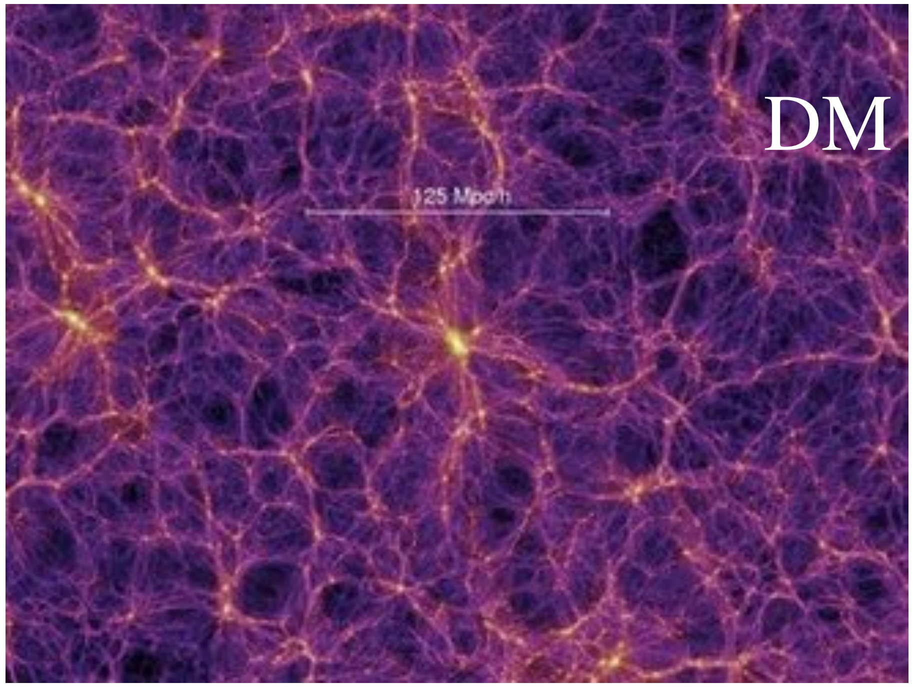
We parametrize linearly the bias with:

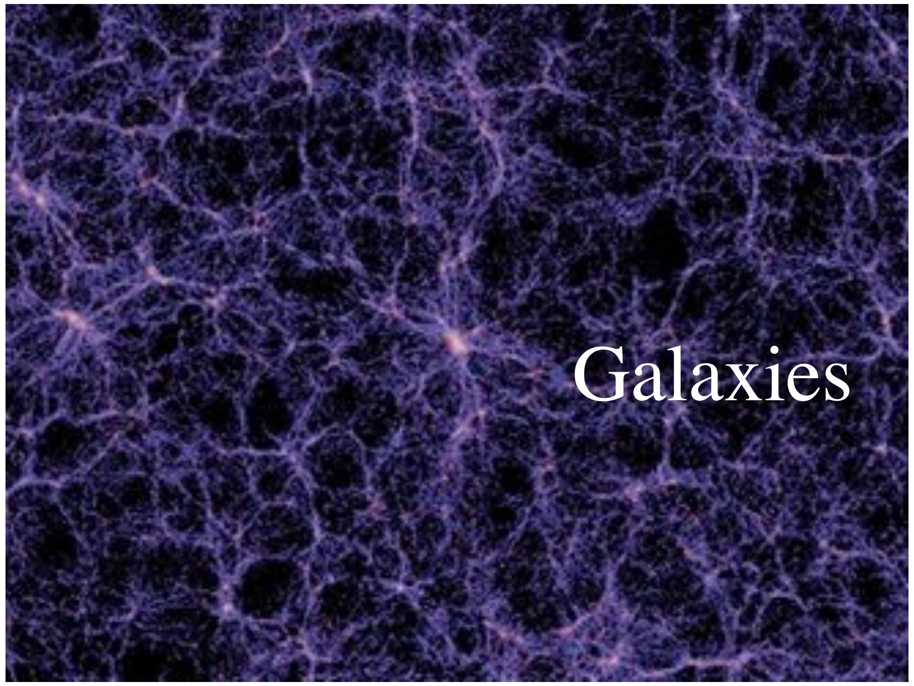
$$\delta_g(k,z) = b(k,z)D(z)\delta(k)$$

Bias is degenerated with the growth factor D(z).



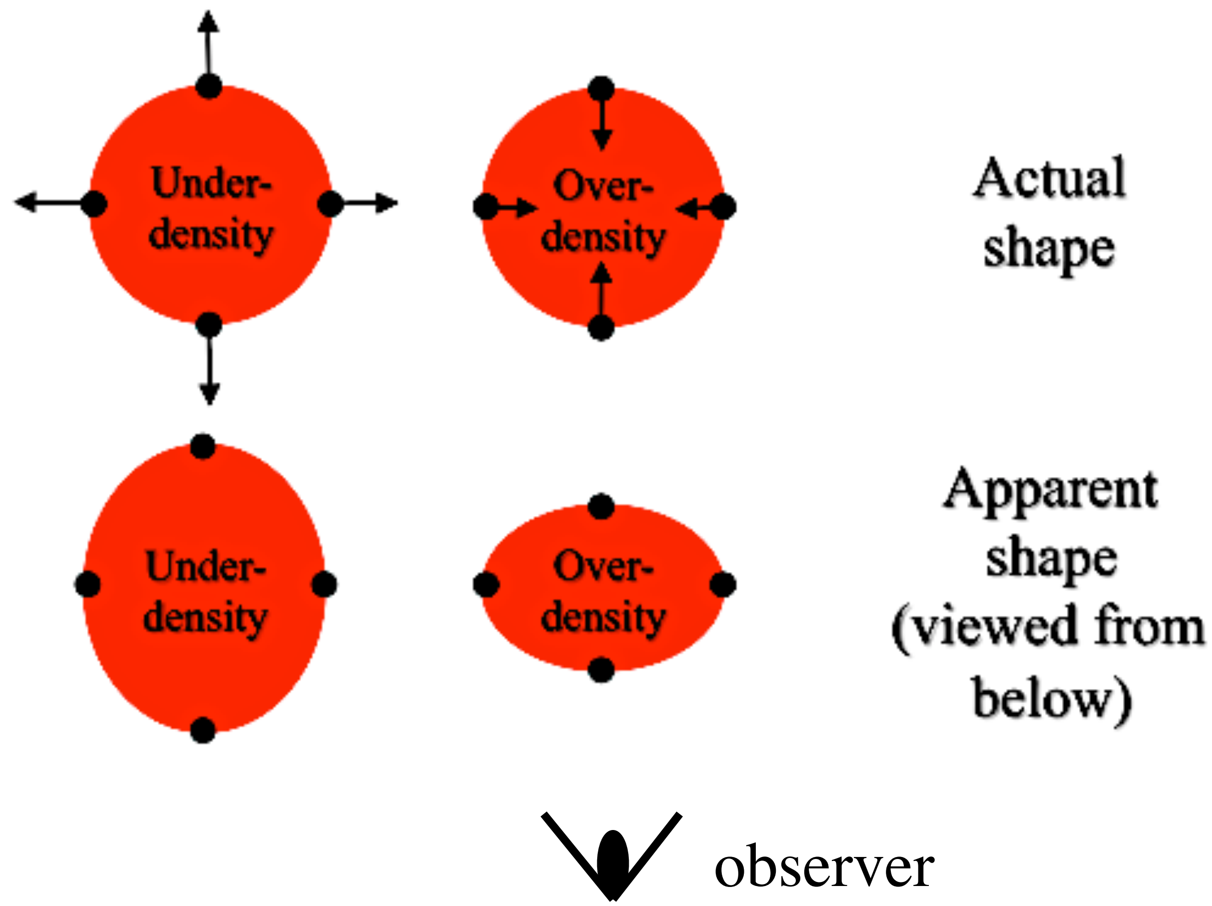
Millennium Simulation





Linear redshift space distortions

- 3D maps of the Universe are in redshift space where galaxy redshift positions differ from the real space positions due to their peculiar velocities.



Kaiser 1987

$$\delta^s_{gal}(k,\mu) = \underbrace{b\,\delta_{mass}(k)}_{\delta^r_{gal}} + \mu^2 \underbrace{\theta_{mass}(k)}_{\theta_{gal}}$$

$$\theta = -\nabla\cdot\mathbf{v}/\mathcal{H}$$

$$\frac{\partial\delta}{\partial\tau} + \nabla\cdot\mathbf{v} = 0$$

$$\theta_{mass} = f(z)\delta_{mass}$$

RSD break degeneracy between growth and bias

RSD is a test of Growth History : how does structure form and grow within the background evolution (Modified Grav. vs DE models (GR))

$$\delta^s(k,\mu) = (b+\mu^2 f)\delta_{mass}$$

Anisotropic clustering

Linear Growth factor: $D(z) = \delta(z)/\delta(z=0)$

Linear Growth rate: $f(z) = \frac{\partial \ln D(z)}{\partial \ln a}$

$$f(z) = \Omega_m(z)^\gamma$$

γ growth rate index

Linder 2005

Linear redshift space distortions

- RSD introduce an anisotropy we should include in the power spectrum or correlation function:

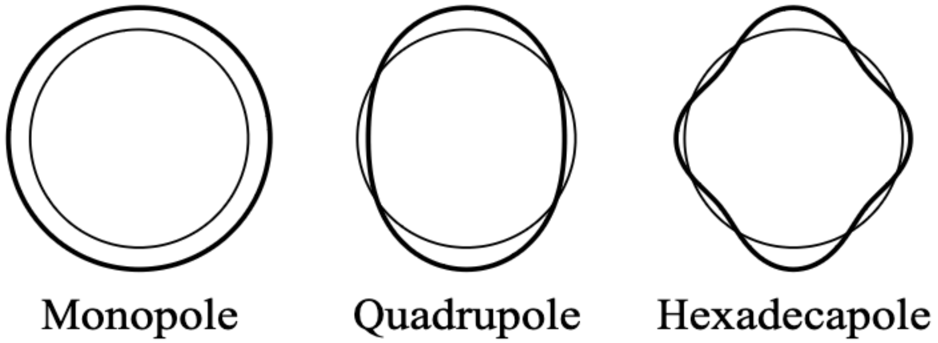
$$\xi(\sigma, \pi) = \xi_0(s) P_0(\mu) + \xi_2(s) P_2(\mu) + \xi_4(s) P_4(\mu)$$

where

$$\xi_\ell(s) = \frac{2\ell + 1}{2} \int_{-1}^{+1} \xi(\pi, \sigma) P_\ell(\mu) d\mu$$

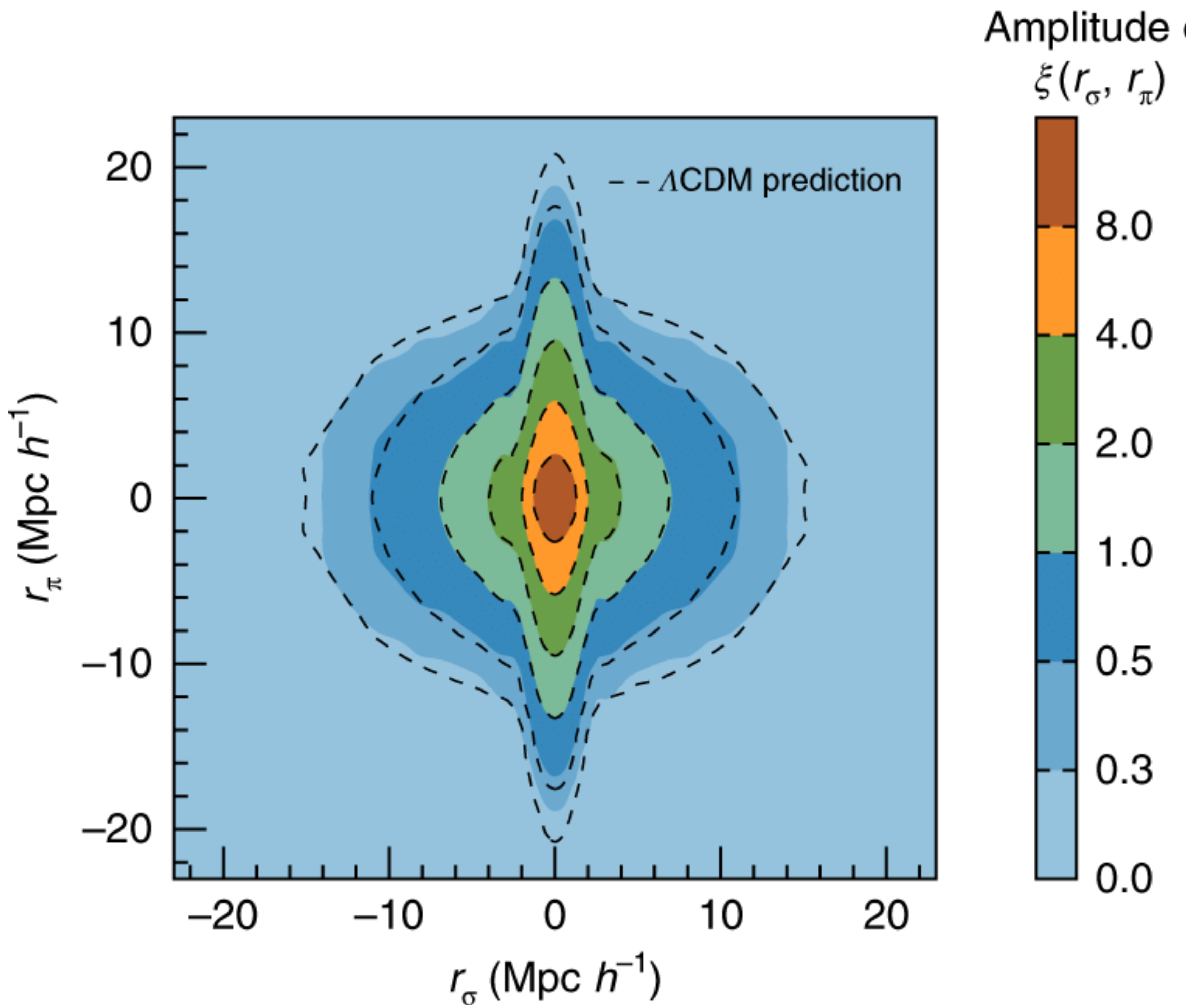
$$\begin{aligned}
\xi_0(s) &= b^2 \left(1 + \frac{2\beta}{3} + \frac{\beta^2}{5}\right) \xi(s) \\
\xi_2(s) &= b^2 \left(\frac{4\beta}{3} + \frac{4\beta^2}{7}\right) \left[\xi(s) - \bar{\xi}(s)\right] \\
\xi_4(s) &= b^2 \frac{8\beta^2}{35} \left[\xi(s) + \frac{5}{2}\bar{\xi}(s) - \frac{7}{2}\bar{\bar{\xi}}(s)\right]
\end{aligned}$$

$$\begin{aligned}
\bar{\xi}(r) &= \frac{3}{r^3} \int_0^r \xi(r') r'^2 dr', \\
\bar{\bar{\xi}}(r) &= \frac{5}{r^5} \int_0^r \xi(r') r'^4 dr'.
\end{aligned}$$



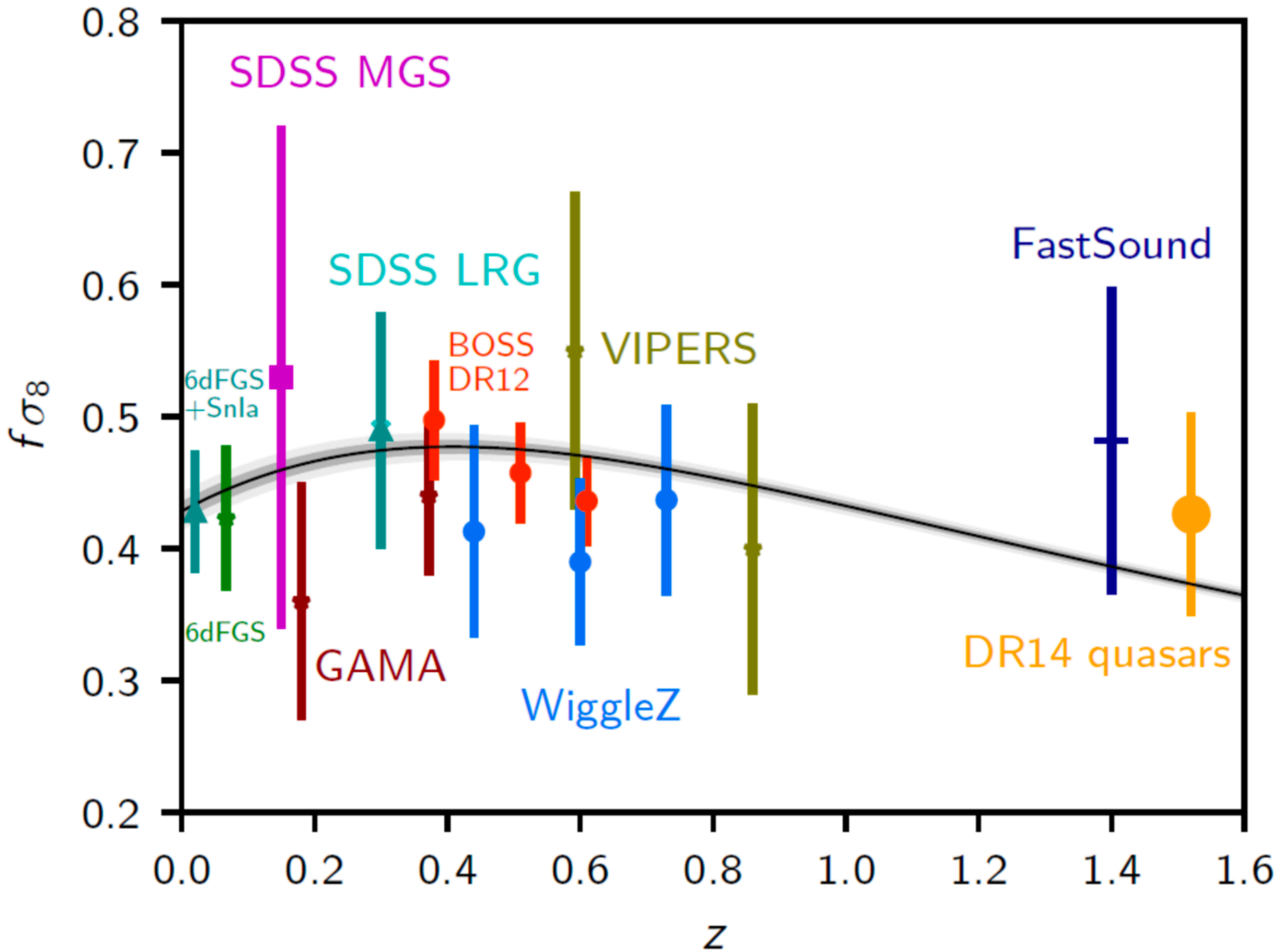
Linear redshift space distortions

- We can decompose the radial and transverse directions in order to measure the redshift space spectrum



Linear redshift space distortions

- RSD offer us a great GR test as we can measure the growth rate of structure for several populations.



Non-linear evolution

- When using the information from small scales, we need to include the information from non-linear evolution of the growth of structures.

- This can be done with non-perturbative methods but usually done with N-body simulations.

- Once the simulation is done, we can try to produce fitting formulas to include in our theory (e.g, Halofit).

- Also, ensemble of simulations for different cosmological and astrophysical parameters allows us to create emulators as we sample the space of simulations

- Simulations + Artificial intelligence can allow us to determine the best model without the need of fitting -> likelihood free inference