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Flavour Physics 1

International Workshop on High Energy Physics (TAE 2023)

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15 September 2023

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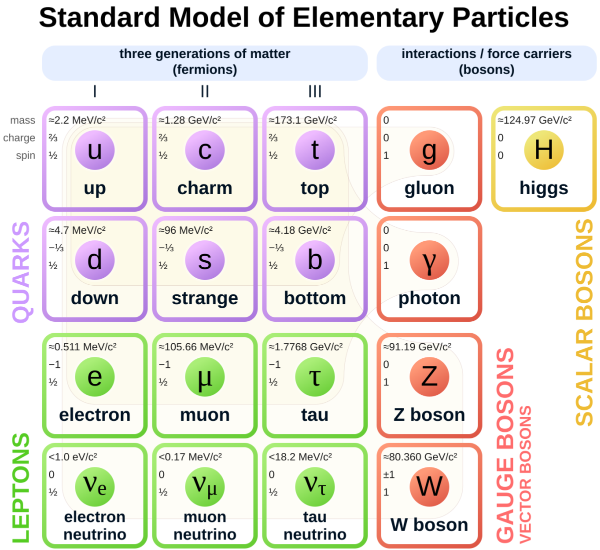
Introduction

Flavour physics: The study of the properties of quarks and leptons

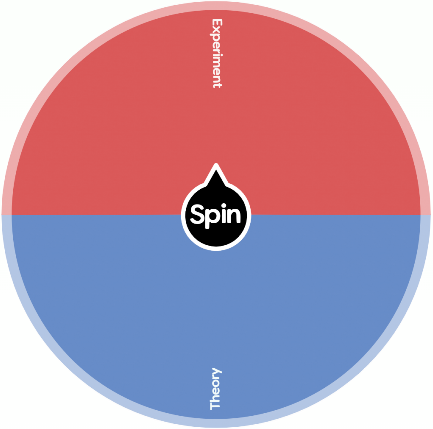


Standard Model of Elementary Particles

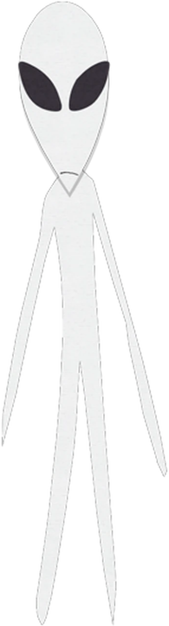
	three generations of matter (fermions)			interactions / force carriers (bosons)	
	I	II	III		
mass charge spin	≈2.2 MeV/c^2 ⅔ ½ u up	≈1.28 GeV/c^2 ⅔ ½ c charm	≈173.1 GeV/c^2 ⅔ ½ t top	0 0 1 g gluon	≈124.97 GeV/c^2 0 0 H higgs
QUARKS	≈4.7 MeV/c^2 −⅓ ½ d down	≈96 MeV/c^2 −⅓ ½ s strange	≈4.18 GeV/c^2 −⅓ ½ b bottom	0 0 1 γ photon	SCALAR BOSONS
	≈0.511 MeV/c^2 −1 ½ e electron	≈105.66 MeV/c^2 −1 ½ μ muon	≈1.7768 GeV/c^2 −1 ½ τ tau	≈91.19 GeV/c^2 0 1 Z Z boson	
LEPTONS	<1.0 eV/c^2 0 ½ ν_e electron neutrino	<0.17 MeV/c^2 0 ½ ν_μ muon neutrino	<18.2 MeV/c^2 0 ½ ν_τ tau neutrino	≈80.360 GeV/c^2 ±1 1 W W boson	GAUGE BOSONS VECTOR BOSONS

Introduction

Flavour physics: The study of the properties of quarks and leptons



Thought experiment



Suppose we came into contact with aliens

Communication via electromagnetic waves

- No exchange of charged particles

Is it safe to meet face to face?

Particles and antiparticles annihilate when they interact

Is the definition of matter/antimatter just convention?

- If so, then only a practical example can distinguish

Or is there a fundamental difference between matter and antimatter that we can describe beforehand?

Outline

Part I

1. Discrete symmetries
2. The CKM mechanism
3. B-physics experiments
   - B-meson properties
   - Belle II and LHCb
4. Standard model measurements
   - V_{cb} and V_{ub}
   - ϕ_3
5. Neutral meson mixing
   - Phenomenology
   - Experimental observables
6. Δm_d and Δm_s

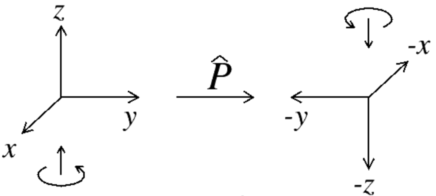
Part II

1. CP violation in mixing
   - A^d_{SL} and A^s_{SL}
2. Time-dependent CP violation
   - CP violation in decay
   - Mixing-induced CP violation
3. $\sin 2\phi_1$
4. Amplitude analysis
5. ϕ_s
6. Composite weak phases
   - ϕ_2
   - $\phi_s + \phi_3$
7. Constraining the CKM matrix
   - Statistical approaches
   - New Physics in B-$\bar{B}$ mixing

Discrete symmetries

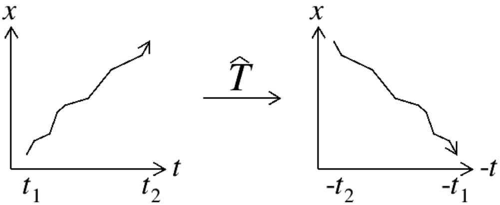
Discrete symmetries are important fundamental questions about Nature

Parity, $\hat{P}$



Is the event seen in the mirror allowed?

Time reversal, $\hat{T}$



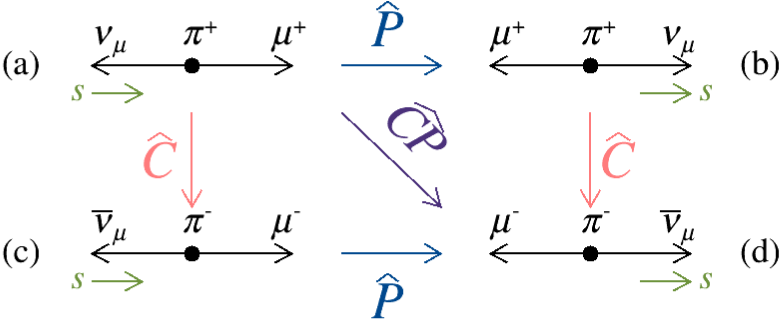
Does the movie played backwards make sense?

Charge conjugation, $\hat{C}$

Do particles and antiparticles have identical properties?

Pion decay

In general, C is violated with P in the weak interaction



(a) Dominant process for π^+ decay, neutrino helicity is negative

But ν_μ is always left-handed

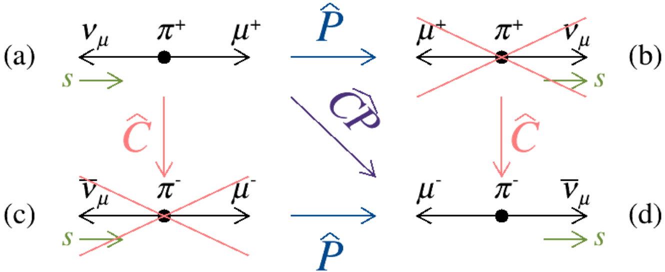
(b) P-conjugate process forbidden because ν_μ would be right-handed

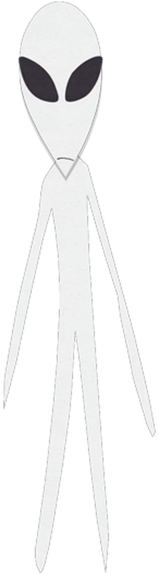
(c) C-conjugate process also forbidden because $\bar{\nu}_\mu$ always right-handed

(d) CP-conjugate process has same decay rate as (a)

Combined CP symmetry is preserved

Thought experiment





C and P maximally violated

- Still cannot explain what a π^+ is

If we say that π^+ decays to a neutral with negative helicity

Then they will ask how the sign of helicity is defined

- Left and right in helicity still convention

Not enough for C to be violated

$\boxed{C \text{ and } CP \text{ must be violated to distinguish matter from antimatter}}$

CP violation

Clearest evidence for CP violation

- Charge asymmetry in K_L^0 decays

K_L^0 is a neutral particle

- Well defined mass and lifetime
- There is no other particle with equal mass
- Therefore, K_L^0 is its own antiparticle

Can decay as $K_L^0 \to \pi^+ e^- \bar{\nu}_e$, or

$$K_L^0 \to \pi^- e^+ \nu_e$$

- the C-conjugate process

Consider decay rates

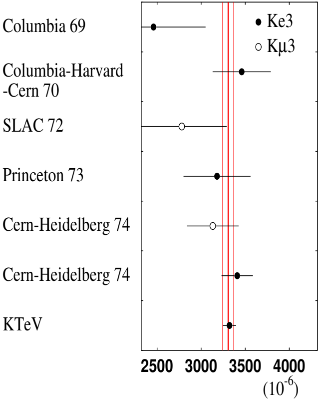
- Momenta and spin of all particles in the final state integrated out
- P eliminated from consideration

Different decay rates violate CP, not just C

A small CP asymmetry is observed

PRL **88** (2002) 181601

$$Kl3 \equiv K_L^0 \to \pi l \nu_l$$



$$\delta_L \equiv \frac{\Gamma(\pi^- l^+ \nu_l) - \Gamma(\pi^+ l^- \bar{\nu}_l)}{\Gamma(\pi^- l^+ \nu_l) + \Gamma(\pi^+ l^- \bar{\nu}_l)}$$

The flavour creed

Our communication problem is solved with $K_L^0 \to \pi^\pm l^\mp \nu_l$

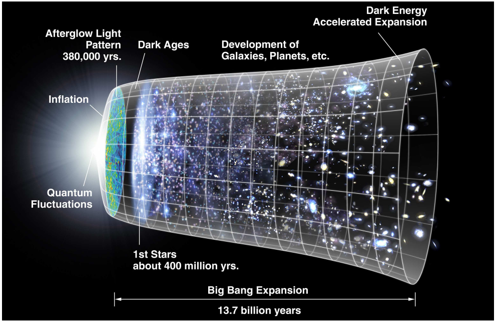
- The less frequent pion has the same charge as the proton

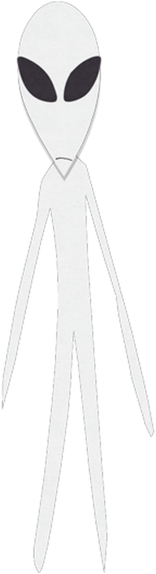
CP violation also important to understanding cosmology

The Standard Model of Particle Physics is incomplete

- Predicts almost no visible matter right after the Big Bang

Cosmological observations show this is incorrect by $\mathcal{O}(10^9)$





There must be new sources of matter-antimatter asymmetry

CKM mechanism

Recall the charged-current Lagrangian of the weak interaction

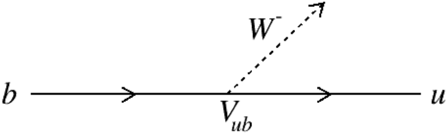
$$\mathcal{L}^{+}_{\mathrm{CC}} \propto \bar{u}_L \gamma_\mu d_L W^\mu$$

Transform from the weak basis to the physical mass basis

$$\mathcal{L}^{+}_{\mathrm{CC}} \propto \bar{u}'_L \gamma_\mu (V_u^\dagger V_d) d'_L W^\mu$$

CKM matrix, $V_{\mathrm{CKM}} \equiv V_u^\dagger V_d$

Describes strength of quark transitions mediated by W boson



To conserve overall probability, $V_{\mathrm{CKM}}^\dagger V_{\mathrm{CKM}} = \mathbb{1}$

For 3 quark generations, independent parameters in 3×3 matrix

- 9 mixing angles, 9 complex phases → 3 mixing angles, 1 complex phase

$\widehat{CP}\mathcal{L}^{+}_{\mathrm{CC}} \neq \mathcal{L}^{+}_{\mathrm{CC}}$ if V_{CKM} is complex

$$\boxed{\text{Single complex phase in } V_{\mathrm{CKM}} \text{ the source of all } CP \text{ violation in the SM}}$$

CKM mechanism

Written explicitly

$$V_{\text{CKM}} \equiv \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Only 4/18 parameters are independent

Perform an expansion around Cabibbo angle, $\lambda \equiv |V_{us}| = \sin\theta_C \approx 0.22$

$$V_{\text{CKM}} \simeq \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

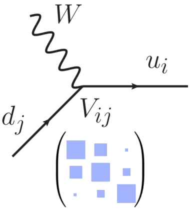
More intuitive view of transition strengths

Likely transitions on the diagonal

4 independent parameters remain to be measured

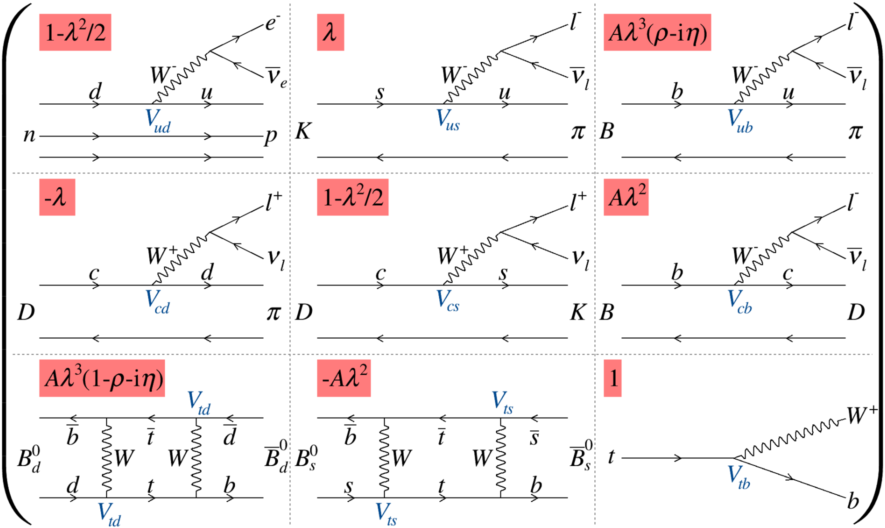
$V_{cb} = A\lambda^2$

$V_{ub} = A\lambda^3(\rho + i\eta)$, ρ and η parameterise CP violation



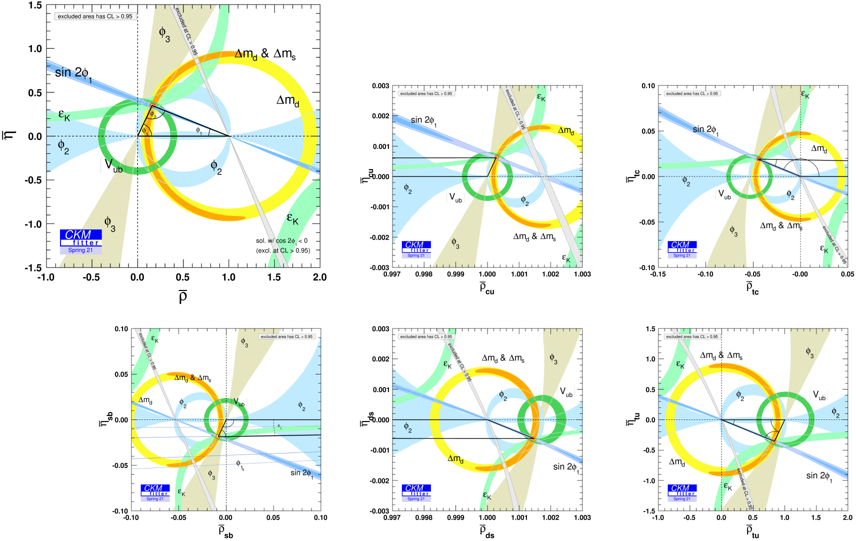
Wolfenstein parametrisation

Parameters accessible depend on the decay process being studied



Unitarity triangles

6 triangle relations $\sum_{j=1}^{3} V_{ij}^{*} V_{jk} = 0,\ i \neq k$, emerge from unitarity

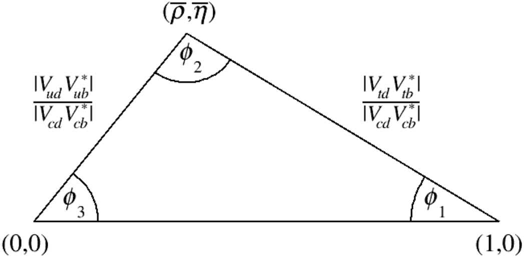


The unitarity triangle

Unitarity relation relevant to b-quark transitions

$$V_{\mathrm{CKM}} \equiv \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \simeq \mathcal{O}\begin{pmatrix} \boxed{\begin{matrix} 1 \\ \lambda \\ \lambda^3 \end{matrix}} & \begin{matrix} \lambda \\ 1 \\ \lambda^2 \end{matrix} & \boxed{\begin{matrix} \lambda^3 \\ \lambda^2 \\ 1 \end{matrix}} \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$$\begin{array}{ccccccc} V_{ud}V_{ub}^* & + & V_{cd}V_{cb}^* & + & V_{td}V_{tb}^* & = & 0 \\ \mathcal{O}(\lambda^3) & & \mathcal{O}(\lambda^3) & & \mathcal{O}(\lambda^3) & & \end{array}$$

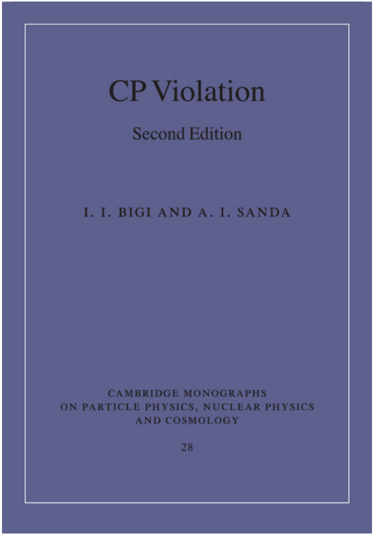


Similar lengths, $\mathcal{O}(\lambda^3) \Rightarrow$ large internal angles

- Large CP violation effects expected with b-quark mediation
- Great experimental environment to study CP-violating effects

Notation on CP-violating phases

ϕ_1, ϕ_2, ϕ_3



CP Violation

Second Edition

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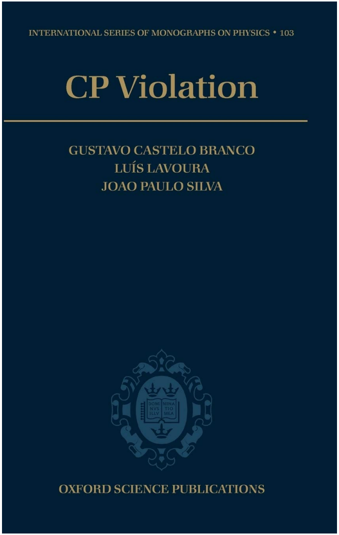
β, α, γ



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CP Violation

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B mesons

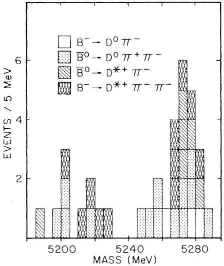
CKM physics and CP violation studies mostly revolve around B mesons

Antiparticle	Particle	Mass (GeV/c^2)
B^+: $\bar{b}u$	B^-: $b\bar{u}$	5.27934 ± 0.00012
B^0: $\bar{b}d$	$\bar{B}^0$: $b\bar{d}$	5.27966 ± 0.00012
B^0_s: $\bar{b}s$	$\bar{B}^0_s$: $b\bar{s}$	5.36692 ± 0.00010

B^+, B^0

Cornell Electron Storage Ring, USA

$e^+e^- \to \Upsilon(4S) \to B\bar{B}$

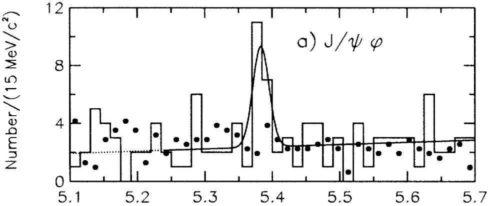


Phys. Rev. Lett. **50** (1983) 881

B^0_s

Tevatron at Fermilab, USA

$p\bar{p} \to b\bar{b}$



CDF, Phys. Rev. Lett. **71** (1993) 1685

B mesons

Neutral B mesons can transform from particle to antiparticle, $B \leftrightarrow \bar{B}$

Antiparticle	Particle	Mass (GeV/c^2)	Mixing frequency (ps^{-1})
B^+: $\bar{b}u$	B^-: $b\bar{u}$	5.27934 ± 0.00012	—
B^0: $\bar{b}d$	$\bar{B}^0$: $b\bar{d}$	5.27966 ± 0.00012	0.5065 ± 0.0019
B_s^0: $\bar{b}s$	$\bar{B}_s^0$: $b\bar{s}$	5.36692 ± 0.00010	17.765 ± 0.006

$B^0 \leftrightarrow \bar{B}^0$

Super $p\bar{p}$ Synchrotron, CERN

$p\bar{p} \to b\bar{b}$

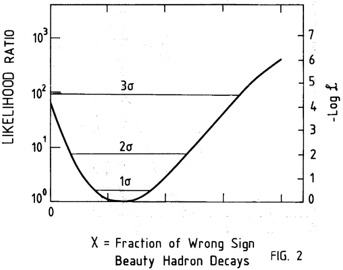


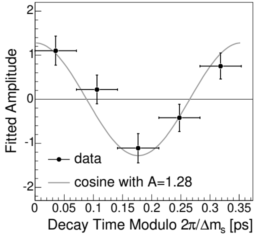
FIG. 2

UA1, Phys. Lett. **B186** (1987) 247

$B_s^0 \leftrightarrow \bar{B}_s^0$

Tevatron at Fermilab, USA

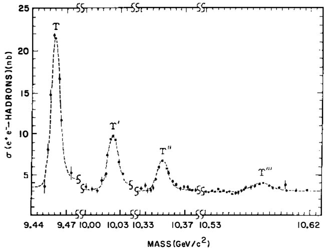
$p\bar{p} \to b\bar{b}$

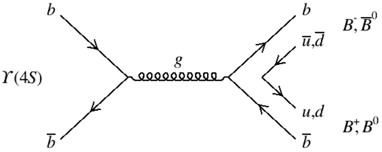


CDF, Phys. Rev. Lett. **97** (2006) 242003

B meson production

$e^+e^- \to \Upsilon(4S) \to B\bar{B}$





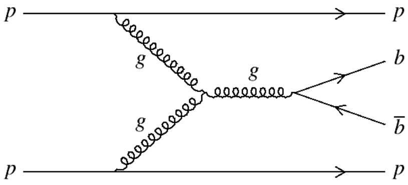
Υ resonances are $b\bar{b}$ bound states

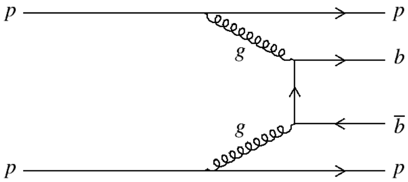
$\Upsilon(1S)$-$\Upsilon(3S)$ too light

$\Upsilon(4S)$ twice the B mass

B^+B^- ($\sim 50\%$) $B^0\bar{B}^0$ ($\sim 50\%$)

$pp \to b\bar{b}$





Parton scattering

$b\bar{b}$ quark pair produced

Hadronise with other quark pairs

Many b-hadrons produced, *eg* B_s^0

B meson production

SuperKEKB, Japan

$B\bar{B}$ pairs at rest in $\Upsilon(4S)$ frame

Decay time difference important

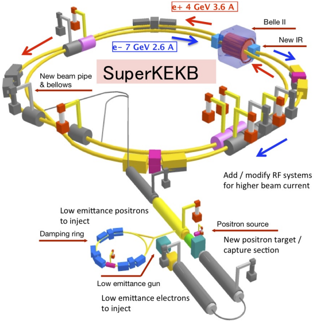
$\sim 1\ \mathrm{ps}$ too short to measure

Use asymmetric beam energies

$B\bar{B}$ pairs boosted

Measure separation instead

$\sim 0.1\ \mathrm{mm}$ easily measured



Large Hadron Collider, CERN

Operating energy $\mathcal{O}(10\ \mathrm{TeV})$

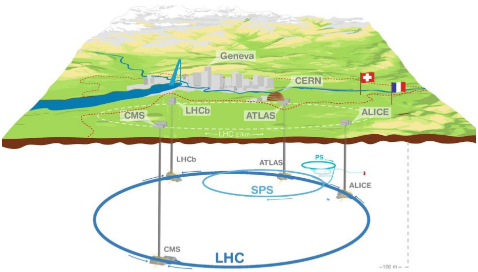
10^3 larger than $b\bar{b}$ threshold

Partons with significantly different momentum fractions can produce

$b\bar{b}$ greatly boosted along beamline

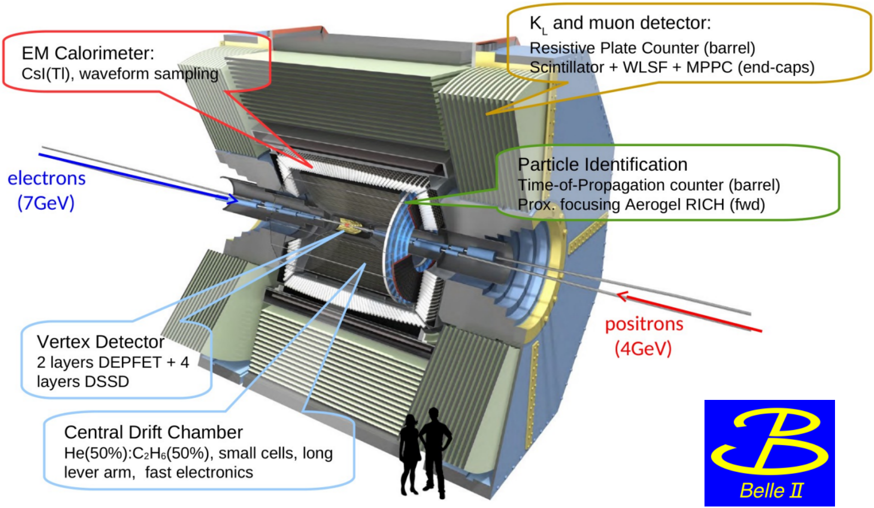
Measure distance from interaction

$\sim 10\ \mathrm{mm}$ very easily measured



Belle II detector

Hermetic detector



EM Calorimeter:
CsI(Tl), waveform sampling

K_L and muon detector:
Resistive Plate Counter (barrel)
Scintillator + WLSF + MPPC (end-caps)

electrons
(7GeV)

Particle Identification
Time-of-Propagation counter (barrel)
Prox. focusing Aerogel RICH (fwd)

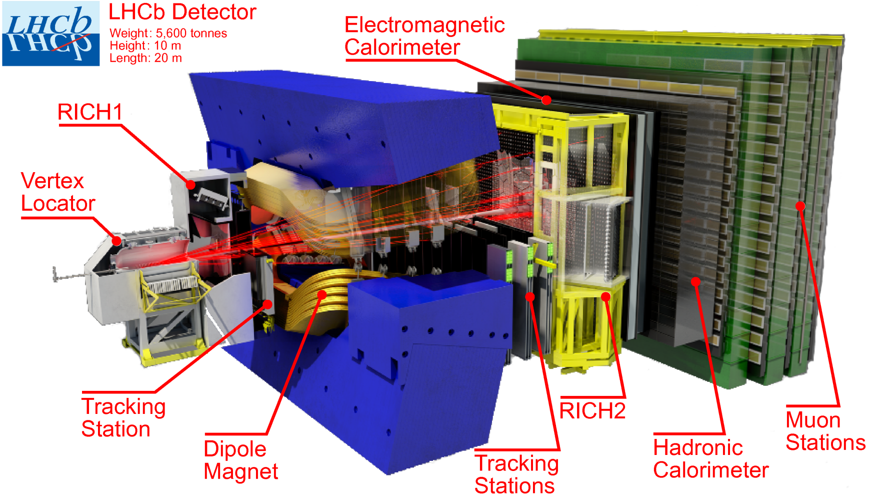
Vertex Detector
2 layers DEPFET + 4 layers DSSD

positrons
(4GeV)

Central Drift Chamber
He(50%):C_2H_6(50%), small cells, long lever arm, fast electronics

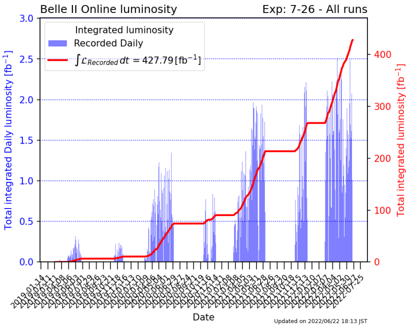
LHCb detector

Forward spectrometer



Detector performance

Belle II



Peak luminosity

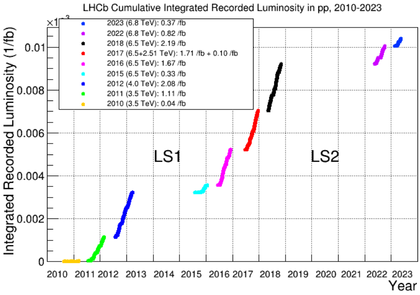
$$\mathcal{L} = 4.71 \times 10^{34}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$$

Data recorded

$$428\ \mathrm{fb}^{-1}\ (\text{Belle: } 832\ \mathrm{fb}^{-1})$$

$$N_{b\bar{b}} \sim 10^{9}$$

LHCb



Levelled luminosity

$$\mathcal{L} = 2 \times 10^{32}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$$

Data recorded

Run 1: $3\ \mathrm{fb}^{-1}$, Run 2: $6\ \mathrm{fb}^{-1}$

$$N_{b\bar{b}} \sim 10^{12}$$

Cross section of $b\bar{b}$ production in pp collisions $\sim \mathcal{O}(10^{5})$ greater

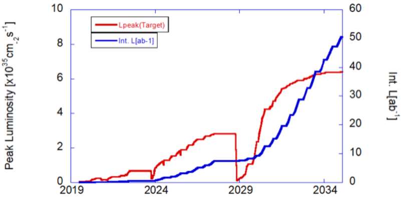
Detector comparison

Both approaches come with complementary strengths

- Hadronic environment
  - The hadronic background in pp collisions is extremely messy
    - Several trigger levels dedicated to specific topologies required to clean up
    - Electromagnetic calorimetry with e^- or γ remains relatively inefficient
  - In e^+e^- collisions, there is 1 lossless trigger for all B decays
  - Overwhelming advantage in $N_{b\bar{b}}$ pairs at LHCb rapidly diminished
  - $N_{b\bar{b}}$ can be calculated precisely for e^+e^-
    - LHCb cannot measure absolute branching fractions
- b-hadron production
  - e^+e^- limited to B^+, B^0 at $\Upsilon(4S)$, but can produce B_s^0 at $\Upsilon(5S)$
  - pp leads to every b-hadron, including extras like B_c^+ and Λ_b^0
- b-hadron boost
  - For pp, much larger boost means superior charged PID and time resolution
  - Belle-II has better coverage, but cannot resolve B_s^0-$\bar{B}_s^0$ oscillations
- Beam energy
  - Known CMS energy of e^+e^- allows reconstruction of undetectable K_L^0, ν_l
  - Partial reconstruction sometimes viable at LHCb

Future prospects

Belle II



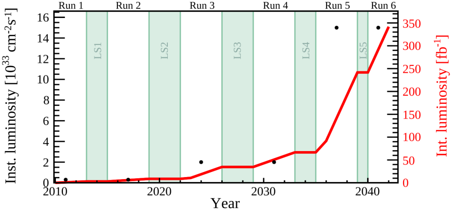
Target peak luminosity

$$\mathcal{L} = 6 \times 10^{35}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$$

Data target

$$50\ \mathrm{ab}^{-1}$$

LHCb



Target levelled luminosity

$$\mathcal{L} = 1.5 \times 10^{34}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$$

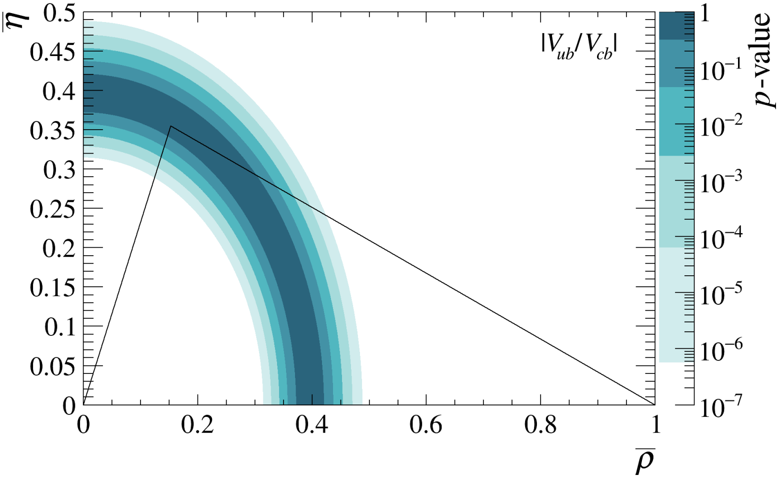
Data target

Run 6: $300\ \mathrm{fb}^{-1}$

Both experiments looking to increase data sample sizes by $\mathcal{O}(10^2)$

Naively expect uncertainties to drop by factor of 10

$|V_{ub}/V_{cb}|$

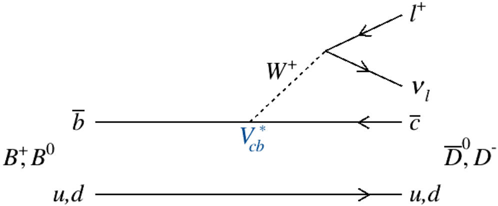


The smallest CKM element

$|V_{ub}/V_{cb}| \equiv \sqrt{\bar{\rho}^2 + \bar{\eta}^2}\lambda/(1-\lambda^2/2)$

V_{cb} theory

$|V_{cb}|$: Base of Unitarity Triangle



Generally measured from semileptonic $B \to D^{(*)} l^+ \nu_l$ decays

- Only 1 CKM element participates

For cleanest $B \to D l^+ \nu_l$, from heavy-quark effective theory (HQET)

$$\frac{d\Gamma}{dw} = \frac{G_F^2 m_D^3}{48\pi^3}(m_B + m_D)^2(w^2 - 1)^{3/2}\eta_{\mathrm{EW}}^2 \mathcal{G}^2(w)|V_{cb}|^2$$

- M. Neubert, Phys. Lett. **B264** (1991) 455

$\eta_{\mathrm{EW}} = 1.0066 \pm 0.0050$: Small electroweak, electromagnetic correction

$\mathcal{G}(w)$: Form factor depending on the recoil energy, $w \equiv p_B \cdot p_D$

V_{cb} theory

Phenomenological parameterisation (CLN)

$$\mathcal{G}(w) = \mathcal{G}(1)[1 - 8\rho^2 z + (51\rho^2 - 10)z^2 - (252\rho^2 - 84)z^3]$$

I. Caprini, L. Lellouch and M. Neubert, Nucl. Phys. **B530** (1998) 153

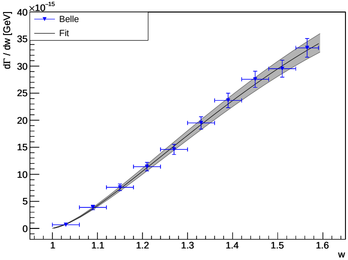
z: Linear transform of w, $z = 0$ at $w = 1$

ρ^2: Slope at at $w = 1$, free parameter of the model

$\mathcal{G}(1)$: Form factor at zero recoil $w = 1$, predicted to high precision

$\mathcal{G}(1) = 1.0541 \pm 0.0083$, Lattice QCD

Fermilab Lattice, MILC Collaborations, Phys. Rev. D **92** (2015) 034506

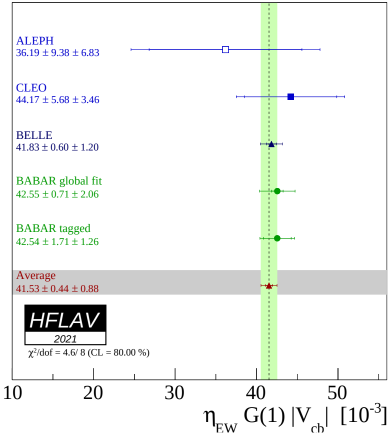


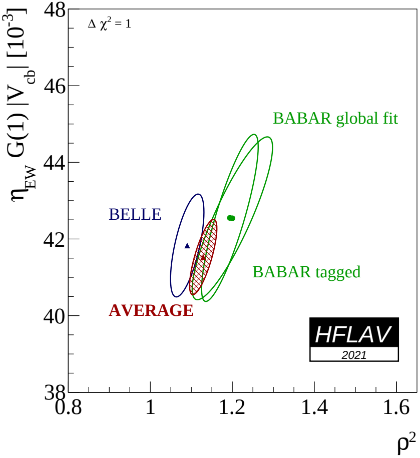
Measured $d\Gamma/dw$ fit with CLN model

Belle, Phys. Rev. D **93** (2016) 032006

V_{cb} measured by extrapolating differential decay rate to $w = 1$

$B \to \bar{D} l^+ \nu_l$ results





Including excited $B \to \bar{D}^{(*)} l^+ \nu_l$ results

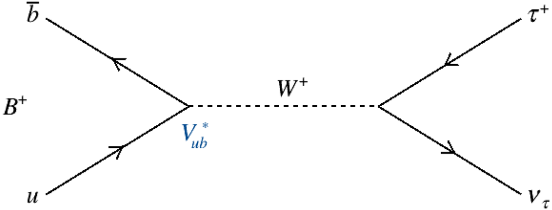
$$\boxed{|V_{cb}| = (38.90 \pm 0.53) \times 10^{-3}}$$

V_{ub} theory

Similarly for V_{ub}, semileptonic decays such as $B \to \pi l^+ \nu_l$ can be studied

However, semileptonic decays are 3-body decays at minimum

- Input from theory to model decay rates as functions of the hadron recoil
- Experimental uncertainty still dominant, but unclear if this will hold



$B^+ \to \tau^+ \nu_\tau$ annihilation has no hadrons in the final state

- Theoretically cleaner

$$\mathcal{B}(B^+ \to \tau^+ \nu_\tau) = \frac{G_F^2 m_B m_\tau^2 \tau_B}{8\pi^3}\left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 f_{B^+}^2 |V_{ub}|^2$$

$f_{B^+} = 189.4 \pm 1.4$ MeV [FLAG]: B decay constant from lattice QCD

Theory uncertainty in $B \to \pi l^+ \nu_l$ decays around 4 times larger

$B^+ \to \tau^+ \nu_\tau$ results

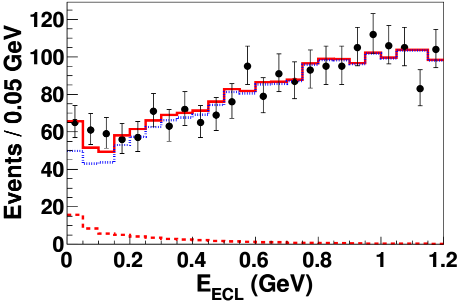
Belle Collaboration, Phys. Rev. Lett. **110** (2013) 131801

Difficult signal reconstruction

- Final state contains 2-3 neutrinos depending on how the τ^+ decays

However, Belle produces 2 B mesons with known CMS energy

- Fully reconstruct accompanying B_{Tag} in common channels



Account for all particles

- Signal and tag sides

Should be nothing left

Look at energy in calorimeter, E_{ECL}

- Signal peaks at 0

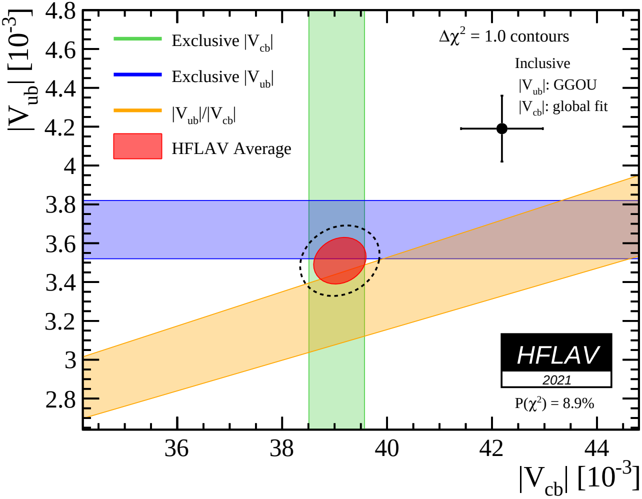
Dashed red: Signal

Dotted blue: Background

$\mathcal{B}(B^+ \to \tau^+ \nu_\tau) = [0.72^{+0.27}_{-0.25}\ (\text{stat}) \pm 0.11\ (\text{syst})] \times 10^{-4}$

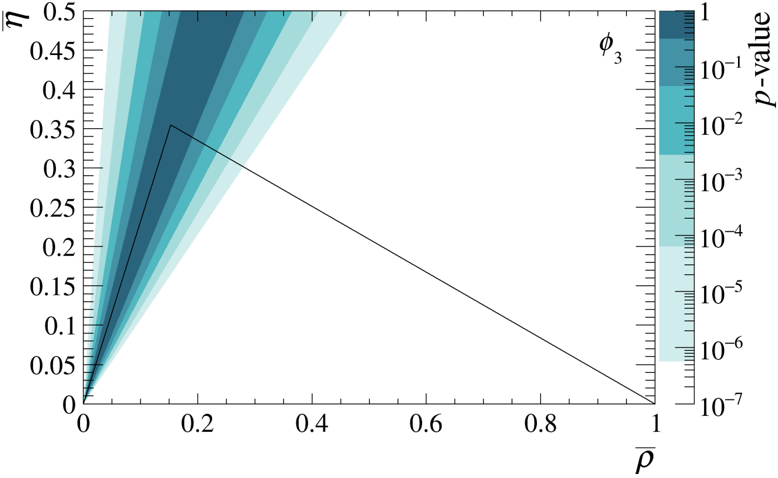
- 3σ significance

$|V_{ub}/V_{cb}|$ average



$$\boxed{\left|\frac{V_{ub}}{V_{cb}}\right| = 0.0838 \pm 0.0046}$$

ϕ_3



Phase of V_{ub}^*

$\phi_3 \equiv \arctan(\bar{\eta}/\bar{\rho})$

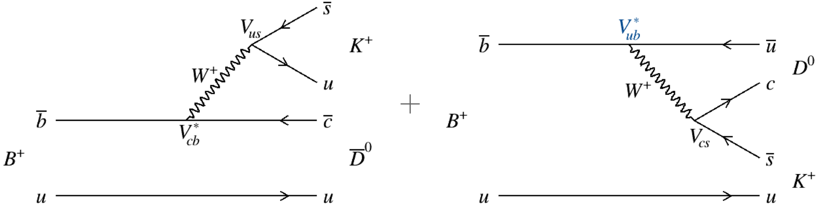
ϕ_3 theory

Consider $B^+ \to DK^+$ decays

- Neutral D represents D^0 or $\bar{D}^0$
- D decays to the same final state, $D^0 \to f$ and $\bar{D}^0 \to f$

Interference environment between the dominant $b \to c\bar{u}s$ with the corresponding doubly-Cabibbo and colour-suppressed $b \to u\bar{c}s$

$$A_{B^+} \propto A_{\bar{D}^0} \quad + \quad r_B e^{i\delta_B} e^{+i\phi_3} A_{D^0}$$



r_B: Strong ratio of colour-suppressed to colour-favoured amplitudes

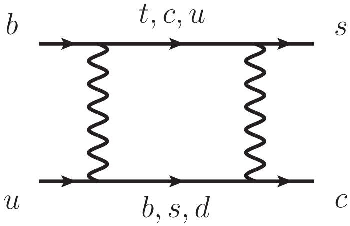
δ_B: Strong phase difference between both diagrams, blind to flavour

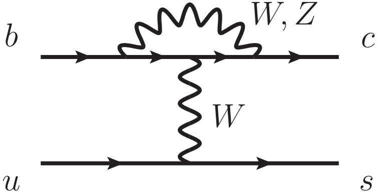
Strong parameters blind to flavour, *ie.* the B charge

Sensitivity to $2\phi_3$ comes from the inclusion of B^- decays

ϕ_3 theory

Pollution of experimental measurement arises from electroweak processes





Irreducible theory error calculated to be $|\delta\phi_3|/\phi_3 \lesssim \mathcal{O}(10^{-7})$

J. Brod and J. Zupan, JHEP **01** (2014) 051

Well beyond reach of any currently planned future experiment

Improving the experimental ϕ_3 measurement will always be relevant

3 approaches to ϕ_3 depending on the $D \to f$ decay

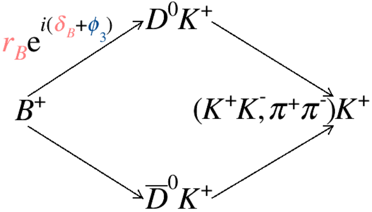
GLW method

The original method

M. Gronau and D. London, Phys. Lett. **B253** (1991) 483

M. Gronau and D. Wyler, Phys. Lett. **B265** (1991) 172

D decays to a CP eigenstate, $D_{CP} \to K^+K^-$, $\pi^+\pi^-$



CP-even D_{CP}: δ_B

CP-odd D_{CP}: $\delta_B \to \delta_B + \pi$

$\mathcal{R}_{CP}$: Sum of B^- and B^+ rates normalised by a flavour-specific D decay

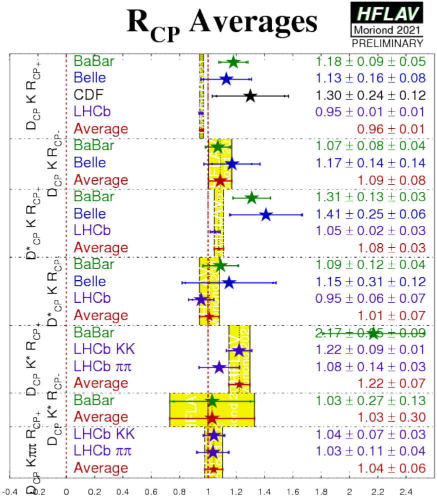
$$\frac{\Gamma(B^- \to D_{CP}K^-) + \Gamma(B^+ \to D_{CP}K^+)}{\Gamma(B^- \to D^0[K^-\pi^+]K^-) + \Gamma(B^+ \to \bar{D}^0[K^+\pi^-]K^+)} = 1+r_B^2+2r_B\cos\delta_B\cos\phi_3$$

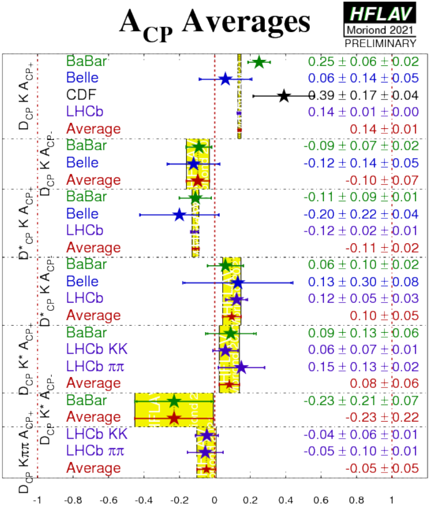
$\mathcal{A}_{CP}$: B^- and B^+ decay rate asymmetry

$$\frac{\Gamma(B^- \to D_{CP}K^-) - \Gamma(B^+ \to D_{CP}K^+)}{\Gamma(B^- \to D_{CP}K^-) + \Gamma(B^+ \to D_{CP}K^+)} = \frac{2r_B\sin\delta_B\sin\phi_3}{\mathcal{R}_{CP}}$$

GLW results

Method can be adapted for excited D and K states of $B^+ \to DK^+$





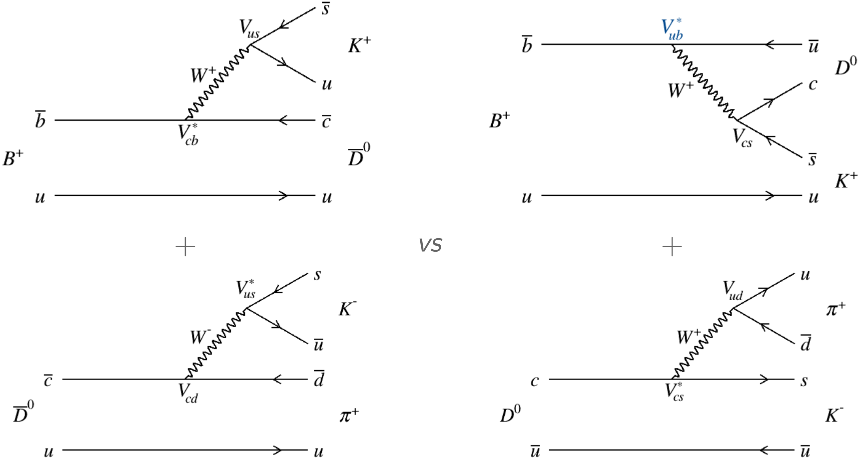
GLW approach to ϕ_3 dominated by LHCb

ADS method

Enhancing sensitivity to ϕ_3

D. Atwood, I. Dunietz and A. Soni, Phys. Rev. Lett. **78** (1997) 3257

Match Cabibbo-favoured B decay with Cabbibo-suppressed D decay



ADS method

Larger asymmetries at the cost of additional D hadronic parameters

Cabibbo-favoured

$$\begin{tikzcd}
 & D^0K^+ \arrow[dr, "r_De^{i\delta_D}"] & \\
B^+ \arrow[ur, "r_Be^{i(\delta_B+\phi_3)}"] \arrow[dr] & & (K^-\pi^+)K^+ \\
 & \overline{D}^0K^+ \arrow[ur] &
\end{tikzcd}$$

Cabibbo-suppressed

$$\begin{tikzcd}
 & D^0K^+ \arrow[dr] & \\
B^+ \arrow[ur, "r_Be^{i(\delta_B+\phi_3)}"] \arrow[dr] & & (K^+\pi^-)K^+ \\
 & \overline{D}^0K^+ \arrow[ur, swap, "r_De^{i\delta_D}"] &
\end{tikzcd}$$

$R_\pm$: Ratio of Cabibbo-suppressed to favoured decay rates for $B^\pm$

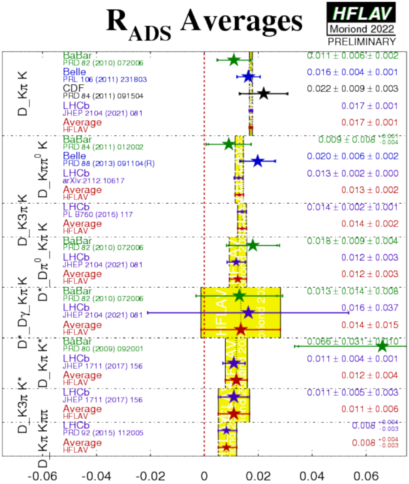
$$\frac{\Gamma(B^\pm \to D_{\text{Sup}}K^\pm)}{\Gamma(B^\pm \to D_{\text{Fav}}K^\pm)} = \frac{r_B^2 + r_D^2 + 2r_Br_D\cos(\delta_B+\delta_D\pm\phi_3)}{1 + r_B^2r_D^2 + 2r_Br_D\cos(\delta_B-\delta_D\pm\phi_3)}$$

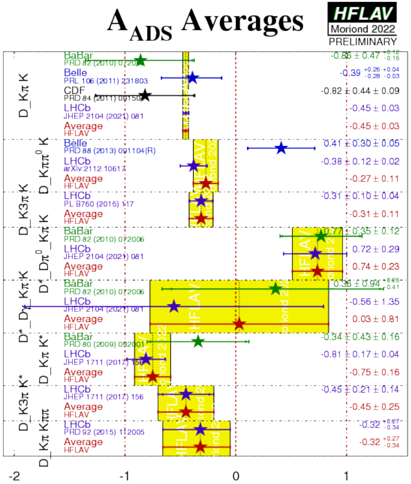
$$\mathcal{R}_{\text{ADS}} = \frac{R_- + R_+}{2}, \qquad \mathcal{A}_{\text{ADS}} = \frac{R_- - R_+}{R_- + R_+}$$

ADS results

Method can be adapted for excited D and K states of $B^+ \to DK^+$

Can also be adapted for $B \to D\pi$





ADS approach again dominated by LHCb

BPGGSZ method

Earlier approaches to measure ϕ_3 considered 2-body D decays

Experimentally much simpler to work with, but suppressed

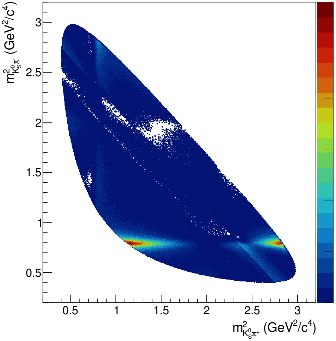
$\mathcal{B}(D_{CP} \to K^+K^-) \sim 4 \times 10^{-3}$
$\mathcal{B}(D_{\text{Sup}} \to K^+\pi^-) \sim 2 \times 10^{-4}$

What about $\mathcal{B}(D \to K_S^0\pi^+\pi^-) \sim 3\%$?

$|A(B^+ \to DK^+)|^2 =$

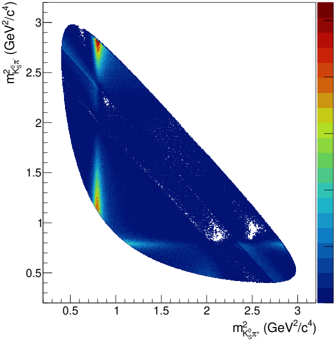
$A_{\bar{D}^0}$

$\Bigg|$



$+\, r_B e^{i(\delta_B + \phi_3)}$

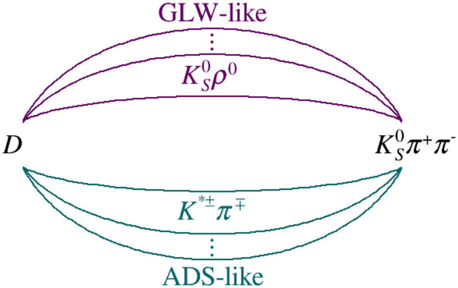
A_{D^0}



$\Bigg|^2$

BPGGSZ method

- GLW and ADS approaches also suffer from insufficient degrees of freedom to constrain ϕ_3
  - GLW: $\Gamma(B^{\pm}) \to r_B$, δ_B and ϕ_3
  - ADS: $\Gamma(B^{\pm}_{\mathrm{Fav}})$, $\Gamma(B^{\pm}_{\mathrm{Sup}}) \to r_B$, δ_B, r_D, δ_D and ϕ_3
- External input incurs additional theory error
- Alternatively, harness 3-body decays such as $D \to K^0_S\pi^+\pi^-$



- GLW- and ADS-like analysis
- Proceeds via excited $\pi^+\pi^-$ and $K^{\pm}\pi^{\mp}$ intermediate states
- Admixture of broad overlapping resonant states across phase space
- Amplitude A_D, known without ambiguity through interference

- Measures $\Gamma(B^{\pm})$ as each point in phase space $(m^2_{K^0_S\pi^+}, m^2_{K^0_S\pi^-})$
- r_B, δ_B and ϕ_3 independent of phase space, sufficient degrees of freedom

BPGGSZ method

D decay amplitude A_D, can be determined in 2 ways

1. Unbinned model-dependent amplitude analysis

   Unacceptable additional bias incurred through model systematic uncertainty

2. Binned model-independent amplitude analysis

   A. Bondar and A. Poluektov, Eur. Phys. J. **C55** (2008) 51

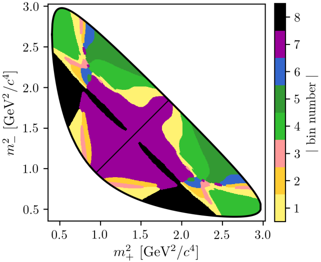
   A. Giri, Y. Grossman, A. Soffer and J. Zupan, Phys. Rev. D **68** (2003) 054018

   Harness quantum-correlated $e^+e^- \to \psi(3770) \to D^0\bar{D}^0$ decays

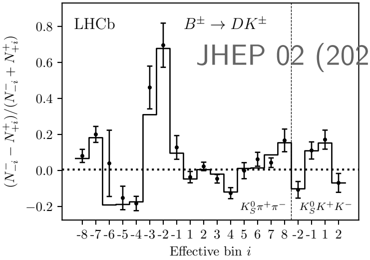
   Data from legacy CLEO (USA) and BESIII (China) experiments

   Gives average strong phase in each bin of D decay phase space, $\bar{\delta}_D$

Binning scheme



CP-violating asymmetry due to impact of ϕ_3



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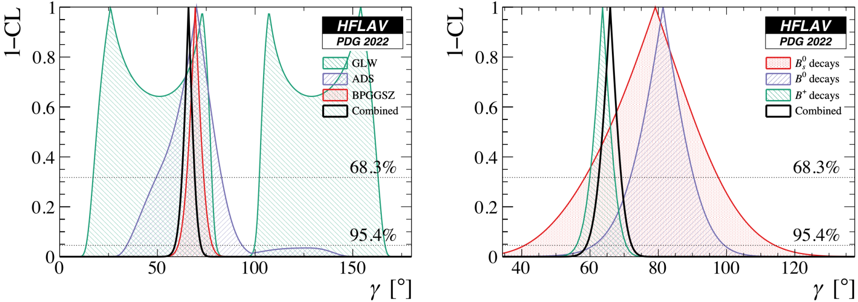
ϕ_3 average

Constraint dominated by BPGGSZ approach

Other approaches involving B^0 and B_s^0 decays also possible



World average: $\boxed{\phi_3 = (65.9^{+3.3}_{-3.5})^\circ}$

Neutral meson mixing

There is no symmetry that forbids neutral meson mixing

Arises because flavour eigenstates are not the physical mass eigenstates

Express light (L) and heavy (H) eigenstates in terms of flavour states

$$|B_{L,H}\rangle = p|B^0\rangle \pm q|\bar{B}^0\rangle$$

Effective Hamiltonian of 2-state system in flavour basis

$$\boldsymbol{H} = \boldsymbol{M} - \frac{i}{2}\boldsymbol{\Gamma} = \begin{pmatrix} M_{11} - i\Gamma_{11}/2 & M_{12} - i\Gamma_{12}/2 \\ M_{21} - i\Gamma_{21}/2 & M_{22} - i\Gamma_{22}/2 \end{pmatrix}$$

$\boldsymbol{M}$ is the Hermitian mass matrix, $M_{21} = M_{12}^*$

$\boldsymbol{\Gamma}$ is the Hermitian decay matrix, $\Gamma_{21} = \Gamma_{12}^*$

$-\frac{i}{2}\boldsymbol{\Gamma}$ is the anti-Hermitian part

- Flavour not conserved in weak interaction due to particle decay

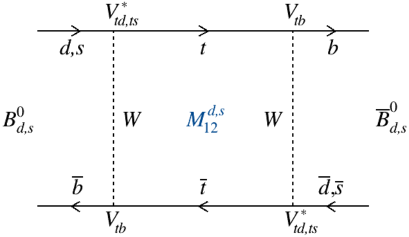
CPT conserved, $M_{11} = M_{22} = m$, $\Gamma_{11} = \Gamma_{22} = \Gamma$

- Average mass and decay width

Neutral meson mixing

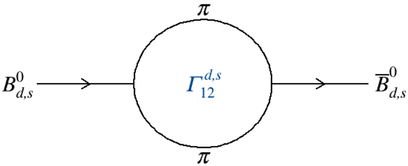
Off-diagonal terms represent mixing

Dispersive (off-shell)



Virtual intermediate states

Absorptive (on-shell)



Real intermediate states

$$\boldsymbol{H} = \begin{pmatrix} m - i\Gamma/2 & M_{12} - i\Gamma_{12}/2 \\ M^*_{12} - i\Gamma^*_{12}/2 & m - i\Gamma/2 \end{pmatrix}$$

Obtain physical eigenstates

Solve the time-dependent Schrödinger equation

$$i\hbar \frac{d}{dt} \begin{pmatrix} |B(t)\rangle \\ |\bar{B}(t)\rangle \end{pmatrix} = \boldsymbol{H} \begin{pmatrix} |B(t)\rangle \\ |\bar{B}(t)\rangle \end{pmatrix}$$

Neutral meson mixing

Diagonalise Hamiltonian to obtain eigenvalues for physical mass states

$$\lambda_{L,H} = H \pm \sqrt{H_{12}H_{21}}$$

Equate directly to Hamiltonian eigenvalues for the mass basis

$$\lambda_{L,H} = m_{L,H} - i\Gamma_{L,H}/2 = m - i\Gamma/2 \pm (\Delta m - i\Delta\Gamma/2)/2$$

where $\Delta m \equiv m_H - m_L$ and $\Delta\Gamma \equiv \Gamma_L - \Gamma_H$

From this, relations between the mass and flavour eigenstates are derived

$$(\Delta m - i\Delta\Gamma/2)^2 = 4(M_{12} - i\Gamma_{12}/2)(M_{12}^* - i\Gamma_{12}^*/2)$$

$$\Rightarrow \boxed{(\Delta m)^2 - (\Delta\Gamma/2)^2 = 4|M_{12}|^2 - |\Gamma_{12}|^2 \text{ and } \Delta m\Delta\Gamma = 4\Re(M_{12}\Gamma_{12}^*)}$$

The eigenvectors of the Hamiltonian for the physical mass states are

$$|B_{L,H}\rangle = \sqrt{H_{12}H_{21}}|B^0\rangle \pm H_{21}|\bar{B}^0\rangle$$

$$\textit{cf} \quad |B_{L,H}\rangle = p|B^0\rangle \pm q|\bar{B}^0\rangle$$

$$\Rightarrow \boxed{\frac{q}{p} = \pm\sqrt{\frac{M_{12}^* - i\Gamma_{12}^*/2}{M_{12} - i\Gamma_{12}/2}} = -\frac{\Delta m + i\Delta\Gamma/2}{2M_{12} - i\Gamma_{12}}}$$

Neutral meson mixing

Solving time-dependent Schrodinger equation now trivial

$$i\hbar\frac{d}{dt}|B_{L,H}(t)\rangle = m_{L,H} - i\frac{\Gamma_{L,H}}{2}|B_{L,H}\rangle$$

$$\Rightarrow |B_{L,H}(t)\rangle = e^{-im_{L,H}t}e^{-\Gamma_{L,H}t/2}|B_{L,H}\rangle$$

Substitute $|B_{L,H}\rangle = p|B^0\rangle \pm q|\bar{B}^0\rangle$ and invert

Gives expressions for the time-evolution of the flavour states

$$|B(t)\rangle = \; g_+(t)|B\rangle + \frac{q}{p}g_-(t)|\bar{B}\rangle$$
$$|\bar{B}(t)\rangle = \; g_+(t)|B\rangle + \frac{p}{q}g_-(t)|\bar{B}\rangle$$

$$\text{where} \quad g_\pm \equiv \frac{1}{2}(e^{-im_Ht}e^{-\Gamma_Ht/2} \pm e^{-im_Lt}e^{-\Gamma_Lt/2})$$

Neutral meson mixing

Consider decay into CP-conjugate final state, f

Static decay amplitude, $\overset{(-)}{A} \equiv \langle f | \overset{(-)}{B} \rangle$

Time dependent decay rate given by

$$\overset{(-)}{\Gamma}(t) = |\langle f | \overset{(-)}{B}(t) \rangle|^2$$

$$= \frac{e^{-t/\tau}}{4\tau} \left[\cosh \frac{\Delta\Gamma t}{2} - \frac{2\Re(\lambda_{CP})}{|\lambda_{CP}|^2 + 1} \sinh \frac{\Delta\Gamma t}{2} \right. \\ \left. \pm \frac{|\lambda_{CP}|^2 - 1}{|\lambda_{CP}|^2 + 1} \cos \Delta m t \pm \frac{2\Im(\lambda_{CP})}{|\lambda_{CP}|^2 + 1} \sin \Delta m t \right]$$

Lifetime, $\tau = 1/\Gamma$

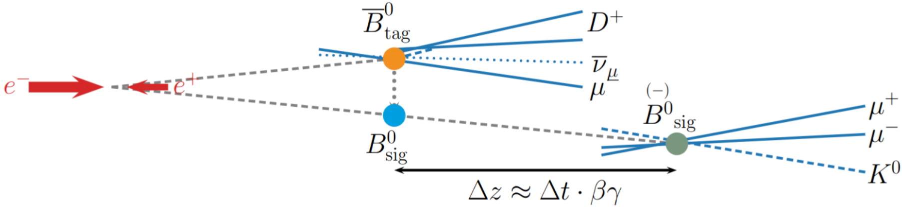
Physical observables are decay time t and flavour of the neutral B decay

Time-dependent decay rates differ only if CP-violation parameter not 1

$$\boxed{\lambda_{CP} \equiv \frac{q}{p} \frac{\bar{A}}{A}}$$

Time-dependent analysis

Belle II



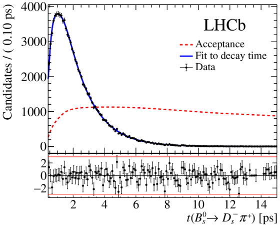
Two B mesons must be produced, partially reconstruct B_{tag} as well

- Measure difference Δt instead, resolution $\sigma_{\Delta t} \sim \mathcal{O}(1)$ ps *cf* $\tau_B \sim 1.5$ ps
- B_s^0 oscillates $\sim 20\ \mathrm{ps}^{-1}$, insufficient resolution to resolve
- No knowledge of absolute position in detector, no efficiency effects

LHCb

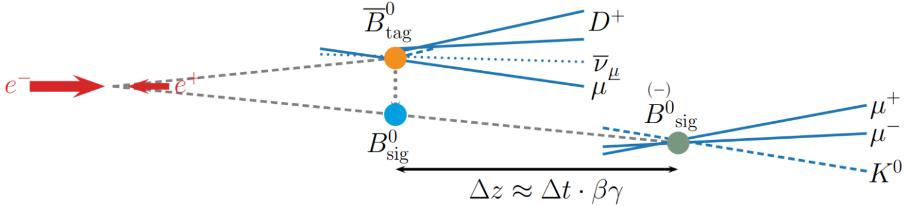
Measure flight length from primary vertex, L

- $t = m_B L / |\vec{p}_B|$
- $\sigma_t \sim \mathcal{O}(1/\Delta m_s)$ ps, negligible otherwise
- Heavily impacted by efficiency effects
- Lifetime bias most critical to study



Flavour tagging

Belle II

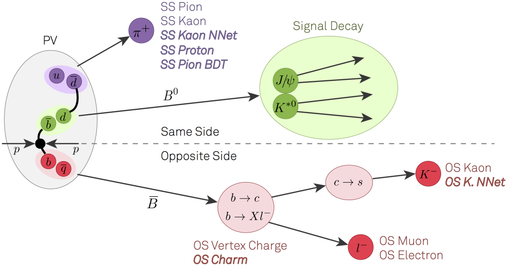


Exploits $C=-1$ eigenvalue of $\Upsilon(4S)$, B^0-$\bar{B}^0$ oscillations are correlated

Search for flavours-specific signatures on the tag side *eg* lepton charge

- Tagging efficiency: $\epsilon_{\text{tag}} = 31.7 \pm 0.4\%$

LHCb



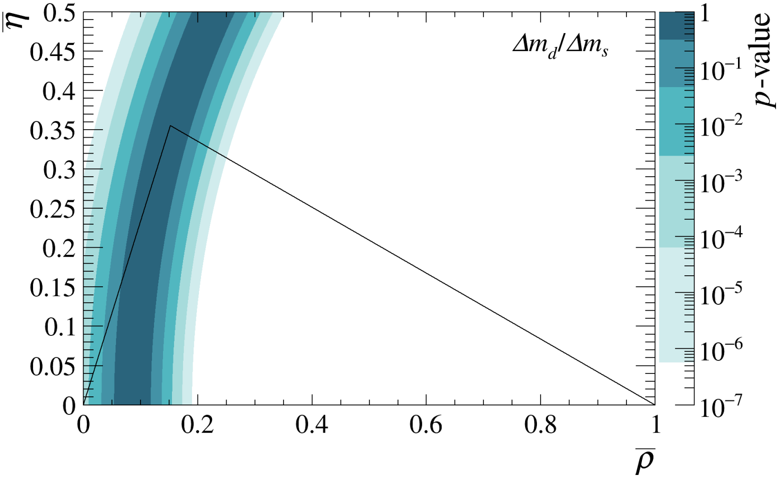
Opposite side similar to Belle II

Same side cascade down to B_{sig}

Much more difficult environment

- Highly signal-dependent
- $\epsilon_{\text{tag}} \sim 3-8\%$

$\Delta m_d/\Delta m_s$



Frequency of B-$\bar{B}$ oscillations

$\Delta m_d/\Delta m_s \simeq |M_{12}^d/M_{12}^s|$

$\Delta m_d/\Delta m_s$ theory

Flavour eigenstates are not the same as the mass eigenstates

- Neutral meson mixing $B^0_{d,s} \leftrightarrow \bar{B}^0_{d,s}$ in consequence

Mass difference sets the oscillation frequency

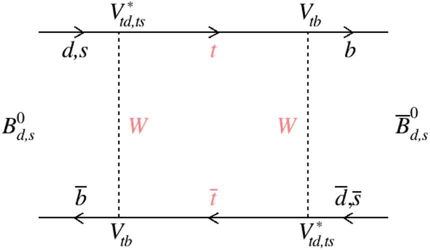
$$\Delta m_{d,s} = m_H^{d,s} - m_L^{d,s} = 2\Re(\sqrt{H_{12}^{d,s} H_{21}^{d,s}})$$

$$\simeq 2|M_{12}^{d,s}|\left(1 - \frac{|\Gamma_{12}^{d,s}|^2}{8|M_{12}^{d,s}|^2}\sin^2\phi_{12}^{d,s} + ...\right)$$

Note that $|\Gamma_{12}^{d,s}/M_{12}^{d,s}|^2 \sim \mathcal{O}(m_b/m_t)^4$

$\Delta m_{d,s} \simeq 2|M_{12}^{d,s}|$

- Dispersive "off-shell" contributions dominate mixing



$\Delta m_d/\Delta m_s$ theory

Calculation of $\Delta m_{d,s}$ fairly complicated

Theory uncertainties much greater than experimental uncertainties

Situation improves with the ratio

$$\frac{\Delta m_d}{\Delta m_s} = \underbrace{[(1-\bar{\rho})^2+\bar{\eta}^2]\lambda^2\left[1+\lambda^2\left(\frac{1}{2}-\bar{\rho}\right)\right]^2}_{|V_{td}/V_{ts}|^2} \frac{m_{B_d}}{m_{B_s}} \frac{f_{B_d}^2}{f_{B_s}^2} \frac{\hat{B}_{B_d}}{\hat{B}_{B_s}}$$

Cancellation of all short-distance QCD effects

Non-perturbative effects of the bound quarks

- B decay constants: $f_{B_{d,s}}$
- Quark confinement bag model factors: $\hat{B}_{B_{d,s}}$
- Calculated within Lattice QCD, HQET sum rule

$$\sqrt{\frac{f_{B_s}^2}{f_{B_d}^2}\frac{\hat{B}_{B_s}}{\hat{B}_{B_d}}} = 1.2014^{+0.0065}_{-0.0072}$$

[JHEP **05** (2019) 034]

$\Delta m_d/\Delta m_s$ results

For flavour-specific decay $B \to f$ or $\bar{B} \to \bar{f}$, two rates can be measured

$$B \to f: \quad \Gamma_{\text{Unmix}}(t) \propto e^{-\Gamma t}[\cosh(\Delta\Gamma t/2) + \cos(\Delta m t)]$$

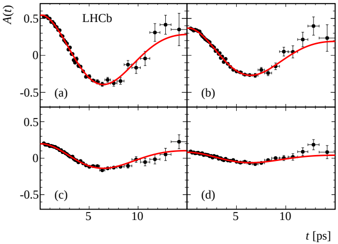
$$\bar{B} \to B \to f: \Gamma_{\text{Mix}}(t) \propto e^{-\Gamma t}[\cosh(\Delta\Gamma t/2) - \cos(\Delta m t)]$$

Flavour-tagging algorithm to check if B or $\bar{B}$ correctly matches f or $\bar{f}$

Decay rate asymmetry, $$\frac{\Gamma_{\text{Unmix}}(t) - \Gamma_{\text{Mix}}(t)}{\Gamma_{\text{Unmix}}(t) + \Gamma_{\text{Mix}}(t)} = \cos(\Delta m t)$$

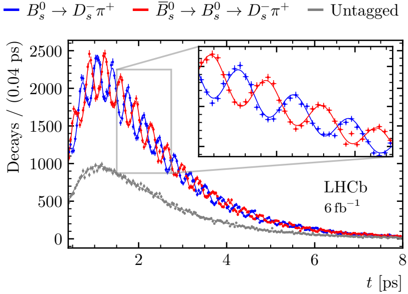
Δm_d

$B^0 \to D^{(*)-}\mu^+\nu_\mu X$ [EPJC]

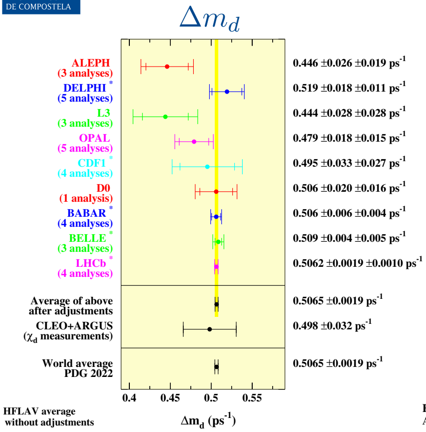


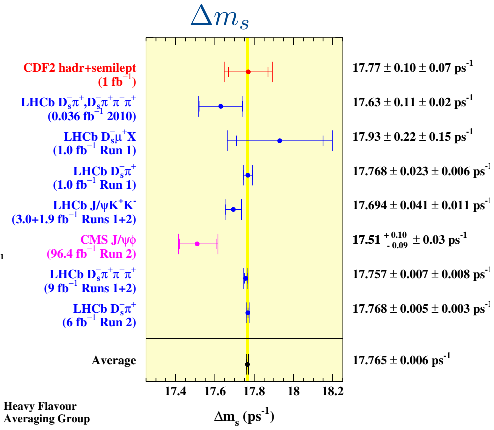
Δm_s

$B_s^0 \to D_s^-\pi^+$ [Nature]



$\Delta m_d/\Delta m_s$ average





$$\Delta m_d = 0.5065 \pm 0.0019\ \mathrm{ps}^{-1} \qquad \Delta m_s = 17.765 \pm 0.006\ \mathrm{ps}^{-1}$$

$$\boxed{\frac{\Delta m_d}{\Delta m_s} = 0.02851 \pm 0.00011}$$

Experimental uncertainty 0.4%, while theory at 0.6%