Flavour Physics 2

International Workshop on High Energy Physics (TAE 2023)

J. Dalseno
jeremypeter.dalseno {at} usc.es

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Outline

Part I

1. Discrete symmetries
2. The CKM mechanism
3. B-physics experiments
   - B-meson properties
   - Belle II and LHCb
4. Standard model measurements
   - V_{cb} and V_{ub}
   - ϕ_3
5. Neutral meson mixing
   - Phenomenology
   - Experimental observables
6. Δm_d and Δm_s

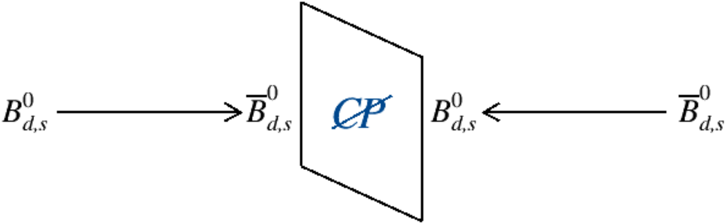
Part II

1. CP violation in mixing
   - A^d_{SL} and A^s_{SL}
2. Time-dependent CP violation
   - CP violation in decay
   - Mixing-induced CP violation
3. $\sin 2\phi_1$
4. Amplitude analysis
5. ϕ_s
6. Composite weak phases
   - ϕ_2
   - $\phi_s + \phi_3$
7. Constraining the CKM matrix
   - Statistical approaches
   - New Physics in B-$\bar{B}$ mixing

CP violation in mixing

3 types of CP violation

$$\lambda_{CP} \equiv \frac{\bar{A}}{A} = \left|\frac{q}{p}\right| e^{-i\phi_M} \left|\frac{\bar{A}_f}{A_f}\right| e^{-i\phi_D}$$



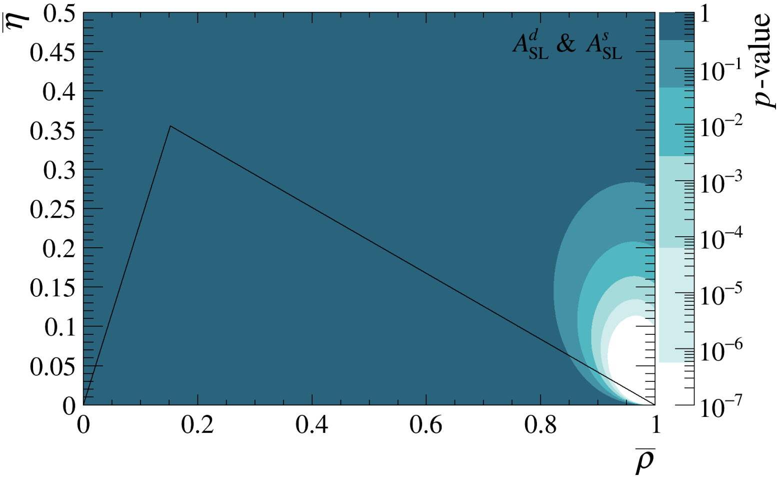
Mixing rate of $B \to \bar{B}$ differs from $\bar{B} \to B$

- $|q/p| \neq 1$

To date, CP asymmetry in B-$\bar{B}$ mixing has not been observed

- $|q/p| \sim 1$ still a good assumption

$A_{\rm SL}^{d,s}$



CP violation in B-$\bar{B}$ mixing

$A_{\rm SL}^{d,s} \simeq \Im(\Gamma_{12}^{d,s}/M_{12}^{d,s})$

$A_{\mathrm{SL}}^{d,s}$ theory

From solution to time-dependent Schrödinger equation for mixing

$$\frac{q_{d,s}}{p_{d,s}} = \sqrt{\frac{H_{21}^{d,s}}{H_{12}^{d,s}}} = -\frac{\Delta m_{d,s} + i\Delta\Gamma_{d,s}/2}{2M_{12}^{d,s} - i\Gamma_{12}^{d,s}}$$

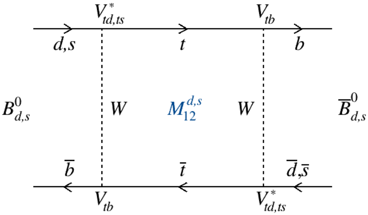
The CP asymmetry in B-$\bar{B}$ mixing given by

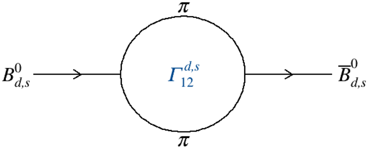
$$A_{\mathrm{SL}}^{d,s} \equiv \frac{|p_{d,s}/q_{d,s}|^2 - q_{d,s}/p_{d,s}|^2}{|p_{d,s}/q_{d,s}|^2 + q_{d,s}/p_{d,s}|^2} \simeq \Im\left(\frac{\Gamma_{12}^{d,s}}{M_{12}^{d,s}}\right) + \mathcal{O}\left(\left|\frac{\Gamma_{12}^{d,s}}{M_{12}^{d,s}}\right|^2\right)$$

Recall $|\Gamma_{12}^{d,s}/M_{12}^{d,s}|^2 \sim \mathcal{O}(m_b/m_t)^4$

$A_{\mathrm{SL}}^{d,s} \simeq \Im(\Gamma_{12}^{d,s}/M_{12}^{d,s})$

Interference between different ways to oscillate between B and $\bar{B}$





$A_{\mathrm{SL}}^{d,s}$ theory

Heavy-quark effective theory (HQET)

Simultaneous expansion in $\Lambda_{\mathrm{QCD}}/m_b$ and $\alpha_s(m_b)$

M. Beneke, G. Buchalla, A. Lenz and U. Nierste, Phys. Lett. **B576** (2003) 173

For CKM elements $\lambda_{id,is} = V_{id,is}^* V_{ib}$

$$M_{12}^{d,s} = \lambda_{td,ts}^2 \frac{G_F^2 m_W^2 m_{B_{d,s}}}{12\pi^2} \overbrace{\eta_{\mathrm{QCD}}}^{\text{Short-distance QCD correction}} f_{B_{d,s}}^2 \hat{B}_{B_{d,s}} \underbrace{S\left(\frac{m_t^2}{m_W^2}\right)}_{\text{Electroweak contribution}}$$

$$\Gamma_{12}^{d,s} = -(\lambda_{cd,cs}^2 \Gamma_{12}^{ds,cc} + 2\lambda_{cd,cs}\lambda_{ud,us}\Gamma_{12}^{ds,uc} + \lambda_{ud,us}^2 \Gamma_{12}^{ds,uu})$$

$$A_{\mathrm{SL}}^{d,s} \simeq \Im\left(\frac{\Gamma_{12}^{d,s}}{M_{12}^{d,s}}\right) = a\frac{\lambda_{ud,us}}{\lambda_{td,ts}} + b\left(\frac{\lambda_{ud,us}}{\lambda_{td,ts}}\right)^2$$

Parameters a and b calculated with HQET

T. Jubb *et al.*, Nucl. Phys. **B915** (2017) 431

$A_{\mathrm{SL}}^{d,s}$ results

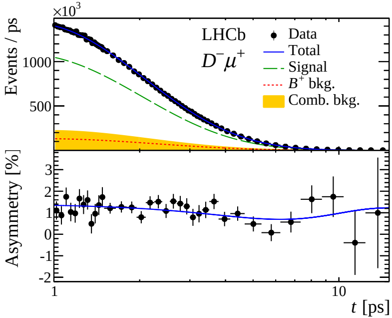
Consider flavour-specific decay $B \to f$ and conjugate $\bar{B} \to \bar{f}$

Decay f, should be CKM-favoured and governed by a single amplitude

- Avoids other types of CP violation and potential New Physics contributions
- $B^0_{(s)} \to D^{(*)-}_{(s)} l^+ \nu_l$, the semileptonic (SL) asymmetry

$$\frac{\bar{\Gamma}_{d,s}(t) - \Gamma_{d,s}(t)}{\bar{\Gamma}_{d,s}(t) + \Gamma_{d,s}(t)} = \frac{A_{\mathrm{SL}}^{d,s}}{2}\left(1 - \frac{\cos \Delta m_{d,s} t}{\cosh \Delta\Gamma_{m,s} t/2}\right)$$

A_{SL}^{d} [PRL]



A_{SL}^{s} [PRL]

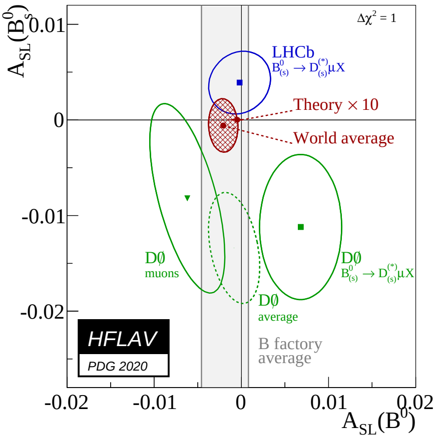
Δm_s is small

- Associated asymmetry $\mathcal{O}(10^{-3})$

Time-integrated analysis

- Rate asymmetry gives $A_{\mathrm{SL}}^{s}/2$

$A_{\rm SL}^{d,s}$ average



$$\boxed{\begin{aligned} A_{\rm SL}^{d} &= -0.0021 \pm 0.0017 \\ A_{\rm SL}^{s} &= -0.0006 \pm 0.0028 \end{aligned}}$$

No CP asymmetry in neutral B-$\bar{B}$ mixing observed yet

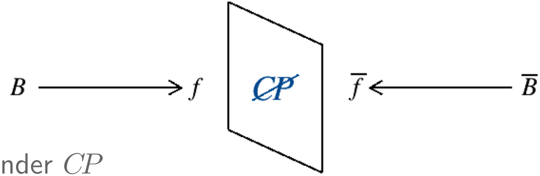
CP violation in decay

3 types of CP violation

$$\lambda_{CP} \equiv \frac{\bar{A}}{A} = \left|\frac{q}{p}\right| e^{-i\phi_M} \left|\frac{\bar{A}_f}{A_f}\right| e^{-i\phi_D}$$

$$A(B \to f) = \sum_i |A_i| e^{i(\delta_i + \phi_i)}$$

$$\bar{A}(\bar{B} \to \bar{f}) = \sum_i |A_i| e^{i(\delta_i - \phi_i)}$$



Strong phase (δ) invariant under CP

Weak phase (ϕ) changes sign under CP

$$\mathcal{A}_{CP}(B \to f) \equiv \frac{|\bar{A}|^2 - |A|^2}{|\bar{A}|^2 + |A|^2} \propto \sum_{i,j} |A_i||A_j| \sin(\delta_i - \delta_j) \sin(\phi_i - \phi_j)$$

3 conditions required for CP violation in decay

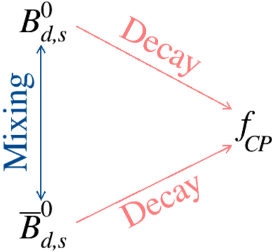
- At least 2 competitive amplitudes
- Non-zero strong phase difference, $\delta_i - \delta_j \neq 0$
- Non-zero weak phase difference, $\phi_i - \phi_j \neq 0$

Mixing-induced CP violation

3 types of CP violation

$$\lambda_{CP} \equiv \frac{\bar{A}}{A} = \left|\frac{q}{p}\right| e^{-i\phi_M} \left|\frac{\bar{A}_f}{A_f}\right| e^{-i\phi_D}$$

Interference between decays with and without mixing



$$\phi_M + \phi_D \neq 0$$

Impacts final term in time-dependent decay rate

$$\overset{(-)}{\Gamma}_{d,s}(t) = \frac{e^{-t/\tau_{d,s}}}{4\tau_{d,s}} \left[\cosh\frac{\Delta\Gamma_{d,s}t}{2} - \frac{2\Re(\lambda_{CP})}{|\lambda_{CP}|^2+1} \sinh\frac{\Delta\Gamma_{d,s}t}{2} \pm \frac{|\lambda_{CP}|^2-1}{|\lambda_{CP}|^2+1} \cos\Delta m_{d,s}t \pm \frac{2\Im(\lambda_{CP})}{|\lambda_{CP}|^2+1} \sin\Delta m_{d,s}t \right]$$

Mixing-induced CP violation

In (quasi-)two-body analyses, coefficients of time-dependent terms are free parameters of the model

CP violation in decay

$$\mathcal{A}_{CP} \equiv \frac{|\lambda_{CP}|^2 - 1}{|\lambda_{CP}|^2 + 1}$$

Mixing-induced CP violation

$$\mathcal{S}_{CP} \equiv \frac{2\Im(\lambda_{CP})}{|\lambda_{CP}|^2 + 1}$$

Mass-eigenstate decay rate asymmetry

$$\mathcal{A}_{\Delta\Gamma_{d,s}} \equiv -\frac{2\Re(\lambda_{CP})}{|\lambda_{CP}|^2 + 1}$$

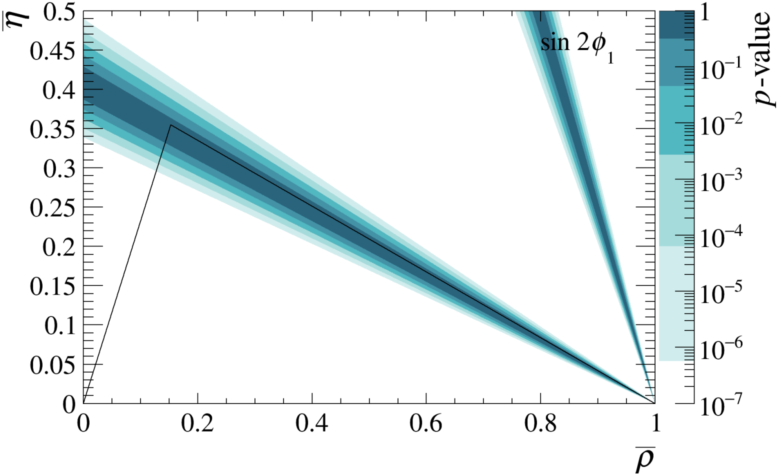
Values for each parameter expected to be in the range $[-1, +1]$

By definition, $(\mathcal{A}_{CP})^2 + (\mathcal{S}_{CP})^2 + (\mathcal{A}_{\Delta\Gamma_{d,s}})^2 = 1$

In practice, constraint not placed on parameters

Time distribution impacted differently

$\sin 2\phi_1$



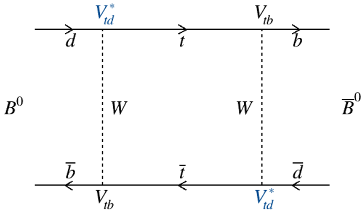
Phase of neutral B^0-$\bar{B}^0$ mixing

$\sin 2\phi_1 \equiv \sin(2 \arctan[\bar{\eta}/(1-\bar{\rho})])$

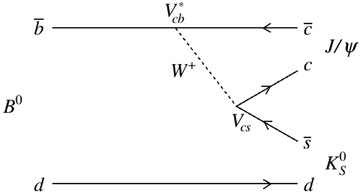
$\sin 2\phi_1$ theory

Consider the CP eigenstate $B^0 \to J/\psi K_S^0$

$$A(B^0 \to \bar{B}^0) \propto (V_{td}^* V_{tb})^2$$



$$A_f \propto V_{cb}^* V_{cs}$$



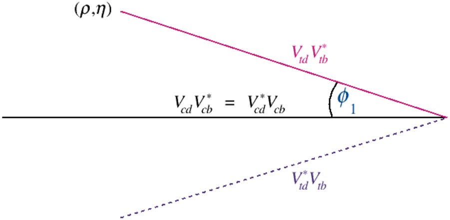
Calculate expectation for the CP-violating parameter, λ_{CP}

Note that neutral kaon mixing also present, $A(K^0 \to \bar{K}^0) \propto (V_{cs}^* V_{cd})^2$

$$\lambda_{CP} \equiv \frac{\bar{A}}{A} = \left(\frac{q}{p}\right)_{B^0} \left(\frac{\bar{A}_f}{A_f}\right) \left(\frac{q}{p}\right)_{K^0}$$

$$= \frac{V_{td} V_{tb}^*}{V_{td}^* V_{tb}} \; \frac{V_{cb} \cancel{V_{cs}^*}}{V_{cb}^* \cancel{V_{cs}}} \; \frac{\cancel{V_{cs}} V_{cd}^*}{\cancel{V_{cs}^*} V_{cd}}$$

$$= e^{-2i\phi_1}$$



$\sin 2\phi_1$ theory

Recall full time distribution

$$\overset{(-)}{\Gamma}_{d,s}(t) = \frac{e^{-t/\tau_{d,s}}}{4\tau_{d,s}} \left[\cosh \frac{\Delta\Gamma_{d,s}t}{2} - \frac{2\Re(\lambda_{CP})}{|\lambda_{CP}|^2+1} \sinh \frac{\Delta\Gamma_{d,s}t}{2} \pm \frac{|\lambda_{CP}|^2-1}{|\lambda_{CP}|^2+1} \cos \Delta m_{d,s}t \pm \frac{2\Im(\lambda_{CP})}{|\lambda_{CP}|^2+1} \sin \Delta m_{d,s}t \right]$$

Note that $\Delta\Gamma_d/\Gamma_d = 0.001 \pm 0.010$, therefore we take $\Delta\Gamma_d = 0$

$$\overset{(-)}{\Gamma}_{d}(t) = \frac{e^{-t/\tau_d}}{4\tau_d} \left[1 \pm \frac{|\lambda_{CP}|^2-1}{|\lambda_{CP}|^2+1} \cos \Delta m_d t \pm \frac{2\Im(\lambda_{CP})}{|\lambda_{CP}|^2+1} \sin \Delta m_d t \right]$$

Expectations formed from these relations using $\lambda_{CP} = \exp(-2i\phi_1)$

CP violation in decay

$$\mathcal{A}_{CP} \equiv \frac{|\lambda_{CP}|^2-1}{|\lambda_{CP}|^2+1} = 0$$

Mixing-induced CP violation

$$\mathcal{S}_{CP} \equiv \frac{2\Im(\lambda_{CP})}{|\lambda_{CP}|^2+1} = -\eta_{CP} \sin 2\phi_1$$

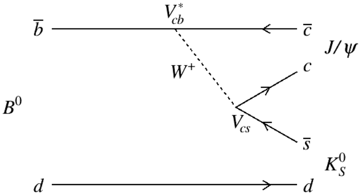
CP eigenvalue

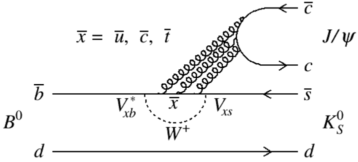
$$\eta_{CP} = \hat{CP}(J/\psi) \cdot \hat{CP}(K_S^0) \cdot (-1)^L = (+1) \cdot (+1) \cdot (-1) = -1$$

Depends on orbital angular momentum between J/ψ and K_S^0

$\sin 2\phi_1$ theory

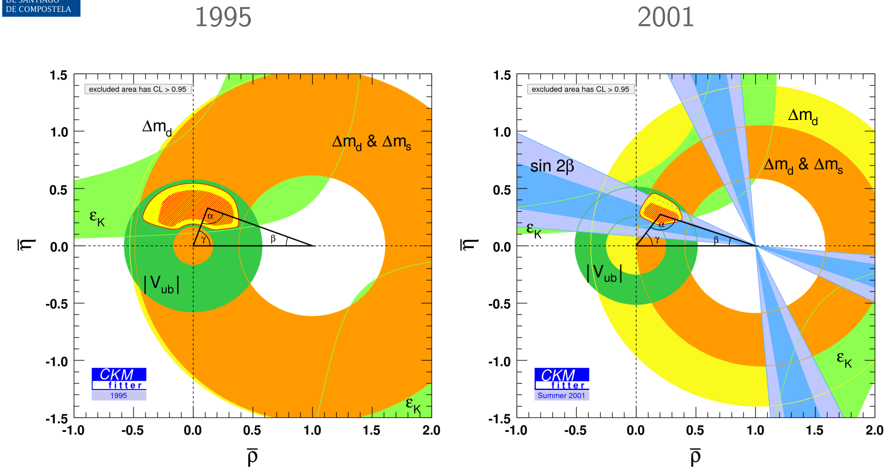
- Referred to as the “golden channel”
- Huge branching fraction, $\mathcal{B}(B^0 \to J/\psi K^0) = (8.91 \pm 0.21) \times 10^{-4}$
- Theoretically clean, relatively free from penguin contamination
    - $\bar{t}$ quark dominant, but CKM elements involving $\bar{t}$ and $\bar{c}$ don’t carry a phase
    - $\bar{u}$ quark contribution the most suppressed, but induces involvement from ϕ_3





- Important to note that measured $\mathcal{S}_{CP}$ is not strictly $\sin 2\phi_1$
- Non-zero measurement of $\mathcal{A}_{CP}$ becomes possible
- Recovery of $\sin 2\phi_1$ discussed in conjunction with ϕ_s from $B_s^0 \to J/\psi\phi$
- Experimentally clean, relatively free from background
    - J/ψ width ($\mathcal{O}(0.1)$ MeV) smaller than detector resolution ($\mathcal{O}(1)$ MeV)
    - K_S^0 has a long lifetime, separated vertex

Confirmation of the CKM mechanism



Before the B factories, ϵ_K from the neutral kaon sector was the only CP-violating parameter constraining the Unitarity Triangle

Need to measure another CP violation parameter influenced by a different quark generation to confirm the CKM mechanism

Confirmation of the CKM mechanism

Volume 87, Number 9 PHYSICAL REVIEW LETTERS 27 August 2001

Observation of *CP* Violation in the B^0 Meson System

B. Aubert,[1] D. Boutigny,[1] J.-M. Gaillard,[1] A. Hicheur,[1] Y. Karyotakis,[1] J. P. Lees,[1] P. Robbe,[1] V. Tisserand,[1]
A. Palano,[2] G. P. Chen,[3] J. C. Chen,[3] N. D. Qi,[3] G. Rong,[3] P. Wang,[3] Y. S. Zhu,[3] G. Eigen,[4] P. L. Reinertsen,[4] B. Stugu,[4]
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R. Kroeger,[45] J. Reidy,[45] D. A. Sanders,[45] D. J. Summers,[45] J. P. Martin,[46] J. Y. Nief,[46] R. Seitz,[46] P. Ta

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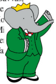

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Observation of Large *CP* Violation in the Neutral *B* Meson System

K. Abe,[9] K. Abe,[37] R. Abe,[27] I. Adachi,[9] Byoung Sup Ahn,[16] H. Aihara,[39] M. Akatsu,[20] G. Alimonti,[8] K. Asai,[21]
M. Asai,[10] Y. Asano,[44] T. Aso,[43] V. Aulchenko,[2] T. Aushev,[14] A. M. Bakich,[35] E. Banas,[25] S. Behari,[9] P. K. Behera,[45]
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(Belle Collaboration)

[1]*Aomori University, Aomori*
[2]*Budker Institute of Nuclear Physics, Novosibirsk*
[3]*Chiba University, Chiba*
[4]*Chuo University, Tokyo*
[5]*University of Cincinnati, Cincinnati, Ohio*
[6]*University of Frankfurt, Frankfurt*
[7]*Gyeongsang National University, Chinju*
[8]*University of Hawaii, Honolulu, Hawaii*
[9]*High Energy Accelerator Research Organization (KEK), Tsukuba*
[10]*Hiroshima Institute of Technology, Hiroshima*
[11]*Institute for Cosmic Ray Research, University of Tokyo, Tokyo*
[12]*Institute of High Energy Physics, Chinese Academy of Sciences, Beijing*
[13]*Institute of High Energy Physics, Vienna*
[14]*Institute for Theoretical and Experimental Physics, Moscow*
[15]*Kanagawa University, Yokohama*
[16]*Korea University, Seoul*
[17]*Kyoto University, Kyoto*
[18]*Kyungpook National University, Taegu*
[19]*University of Melbourne, Victoria*

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Confirmation of the CKM mechanism

M. Kobayashi

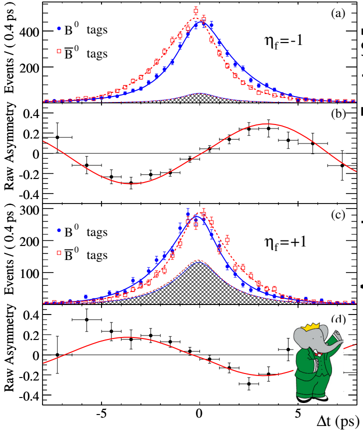
2008

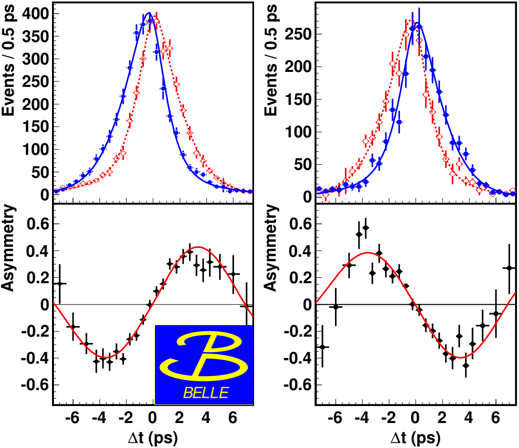
T. Maskawa

The CKM mechanism is the primary source of CP violation in the Standard Model

$\sin 2\phi_1$ results

Final results from the legacy BaBar and Belle experiments





BaBar: PRD **79** (2009) 072009

Belle: PRL **108** (2012) 171802

Note the impact of the CP eigenvalue on the raw candidate asymmetry

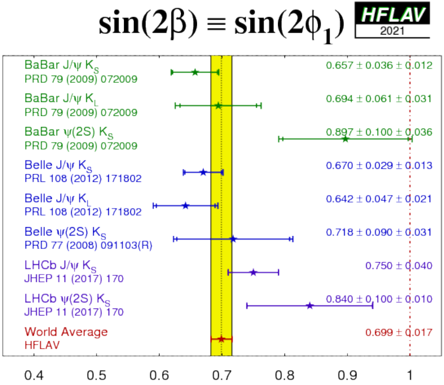
No hint of CP violation in decay, $\mathcal{A}_{CP} = +0.005 \pm 0.015$

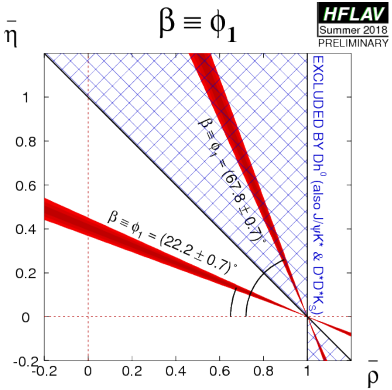
$\sin 2\phi_1$ average

Average includes contributions from other charmonium ($c\bar{c}$) states

Legacy B factory results from 2012 still dominate world average

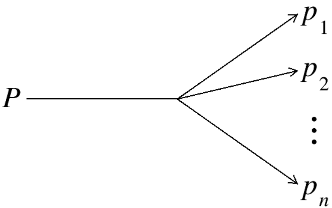
Multibody decays can rule out ambiguous solution





World average: $\boxed{\sin 2\phi_1 = +0.699 \pm 0.017}$

Amplitude analysis



Four-momenta	$4n$
Meson masses ($p_i^2 = m_i^2$)	$-n$
Total E, linear p conservation	-4
Arbitrary orientation	-3
Independent variables	$3n-7$

Fermi's Golden Rule leads to differential decay rate

$$\boxed{d\Gamma(s) = \frac{(2\pi)^4}{2\sqrt{s}} \sum_\lambda |A_\lambda(s)|^2 d\Phi_n(P; p_1, p_2, ..., p_n)}$$

where A is the decay amplitude and $d\Phi_n$ is n-body phase space density

$$d\Phi_n(P; p_1, p_2, ..., p_n) = \delta^4(P - \sum_{i=1}^{n} p_i) \prod_{i=1}^{n} \frac{d^4 p_i}{(2\pi)^3} \delta(p_i^2 - m_i^2)$$

Dalitz Plot defined as the scatter plot of the $3n-7$ variables

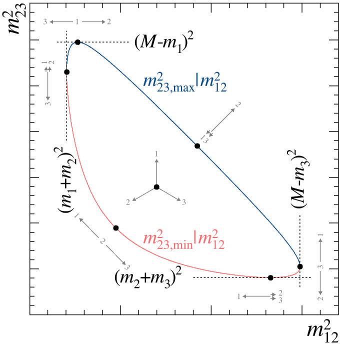
Contains the complete kinematic and dynamic information

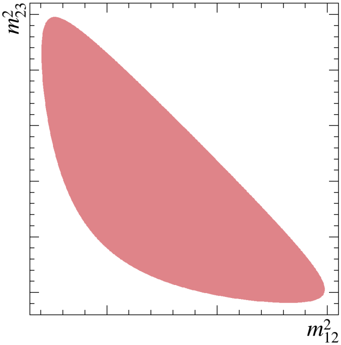
Amplitude analysis

For $P \to p_1 p_2 p_3$, Dalitz plot can be defined as

$m_{12}^2 = (p_1 + p_2)^2$ vs $m_{23}^2 = (p_2 + p_3)^2$

Kinematics completely understood from position in phase space





Flat distribution in phase space means Dalitz plot is uniformly populated

Any structure in the Dalitz plot is due to dynamic effects

Measurements at amplitude rather than at amplitude-squared level

Resonance dynamics

Multibody decays typically proceed by intermediate states

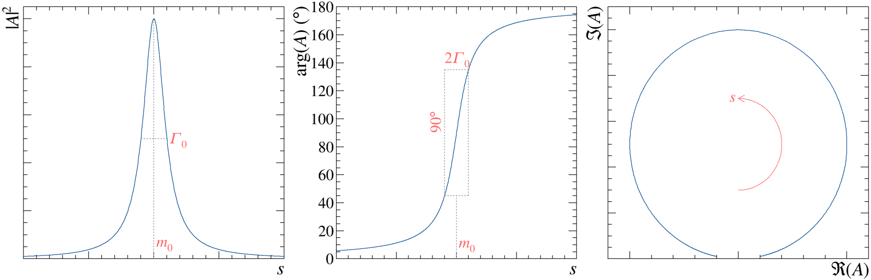
Extremely short-lived resonances

eg Lifetime: $\tau \sim \mathcal{O}(10^{-23}\ s) \Rightarrow$ Width: $\Gamma_0 \equiv 1/\tau \sim \mathcal{O}(100\ \mathrm{MeV})$

Heisenberg Uncertainty Principle implies significant uncertainty in energy

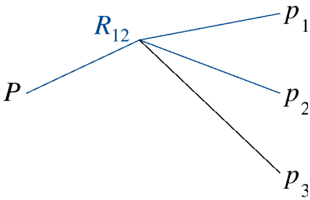
Manifests as peak in the scattering amplitude (Breit-Wigner propagator)

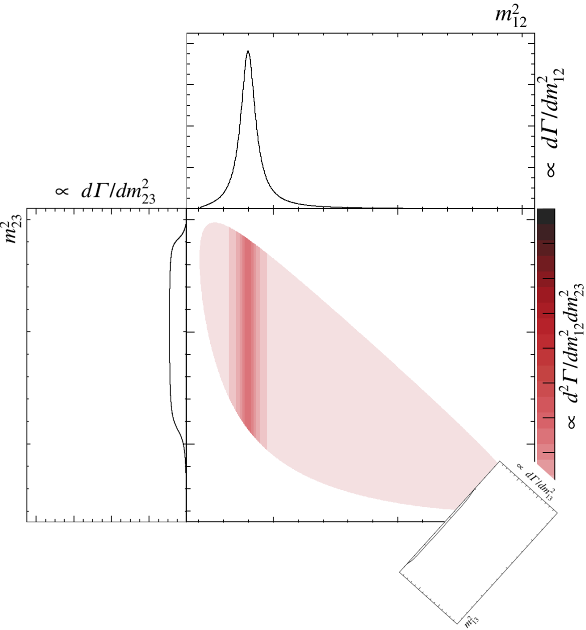
$$A(s \equiv m^2) = \frac{1}{m_0^2 - s - i\sqrt{s}\Gamma_0}$$



Resonance dynamics

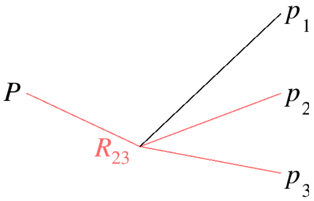
$R_{12} \to p_1 p_2$

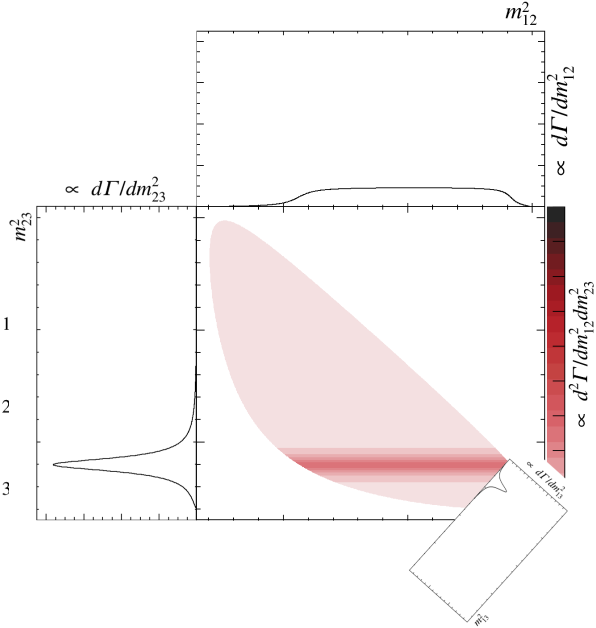




Resonance dynamics

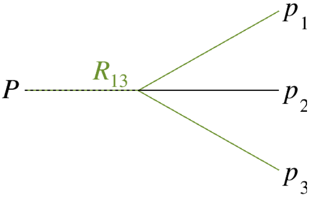
$R_{23} \to p_2 p_3$

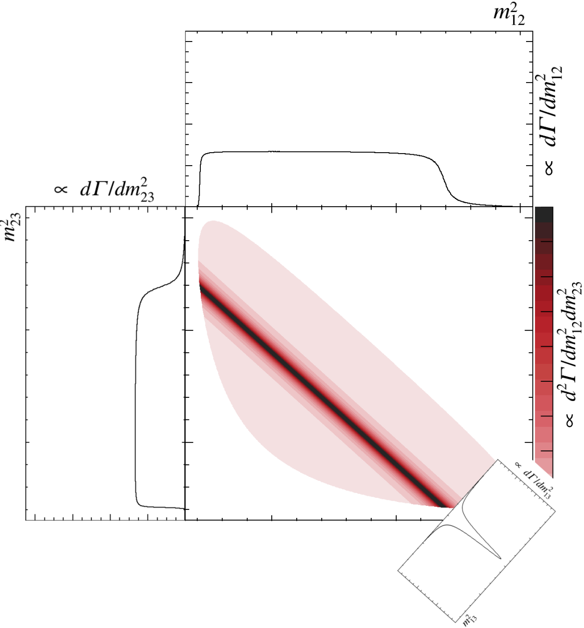




Resonance dynamics

$R_{13} \to p_1 p_3$

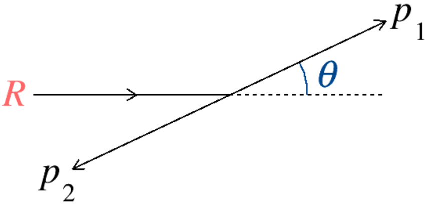




Spin density

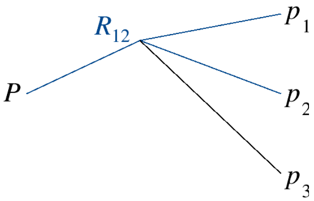
- If resonance R has spin, distribution of decay products are non-isotropic
- 1st Rarita-Schwinder condition for covariant formulation of spin
  - Polarisation vector $\epsilon^\mu(s_z)$, orthogonal to momentum
- Consider rest frame of R
- Spin represents how a particle at rest behaves under spatial rotations
  - In the rest frame of R, the time component must vanish
- Select z-axis by convention, 3 solutions to Rarita-Schwinger condition
  - Interpreted as the 3 independent spin projections along z-axis
  - Longitudinal: $\epsilon^\mu(0) = (0, 0, 0, 1)$
  - Transverse: $\epsilon^\mu(\pm 1) = \mp(0, 1, \pm i, 1)/\sqrt{2}$
- Decay products should preserve spin direction
  - Relative momentum of decay products: $q^\mu = p_1^\mu - p_2^\mu$

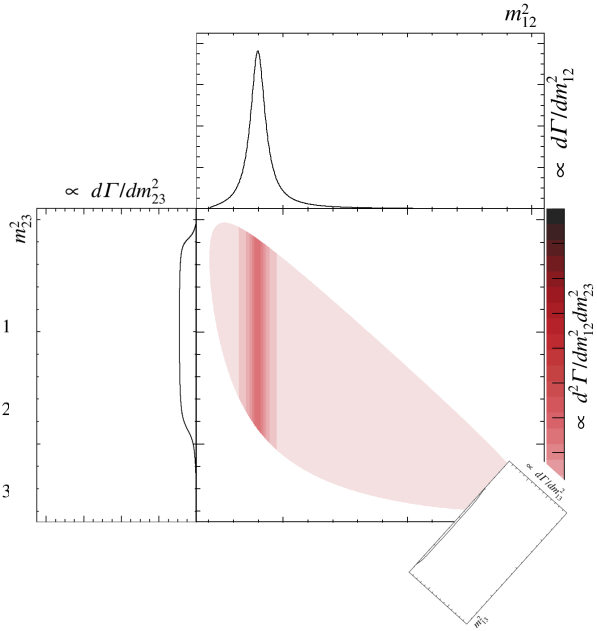
- For pseudo-scalar decay products
  - $J = 0$: $A = 1$
  - $J = 1$: $A = \epsilon_\mu q^\mu \propto \cos\theta$
  - $J = 2$: $A = \epsilon_{\mu\nu} q^\mu q^\nu \propto \cos^2\theta - 1/3$



Spin density

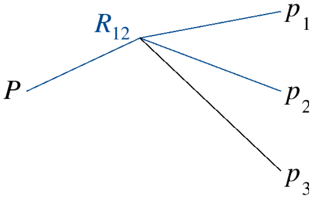
Spin-0: $R_{12} \to p_1 p_2$

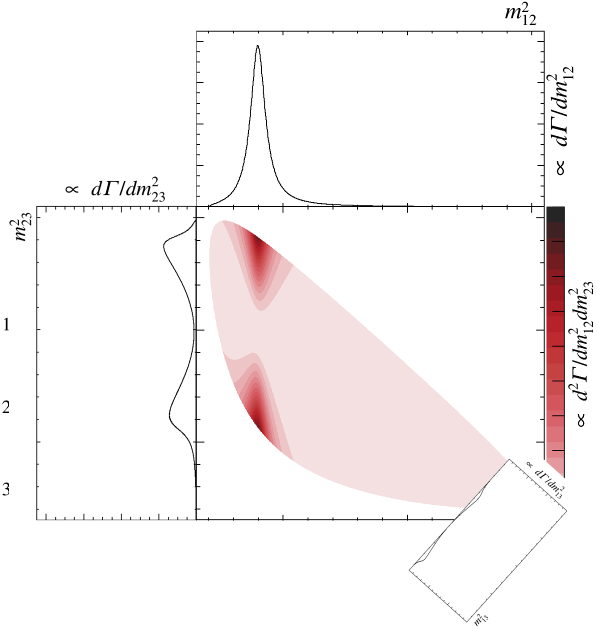




Spin density

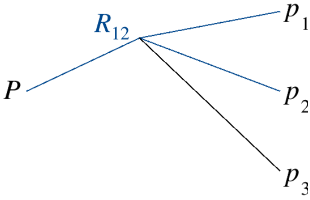
Spin-1: $R_{12} \to p_1 p_2$

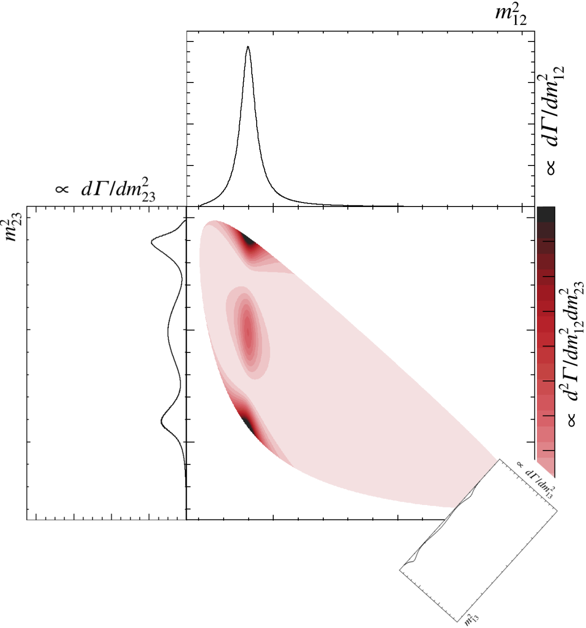




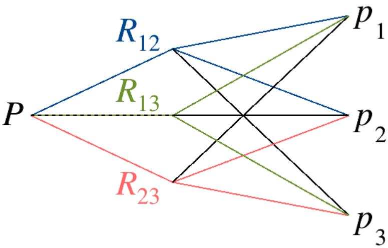
Spin density

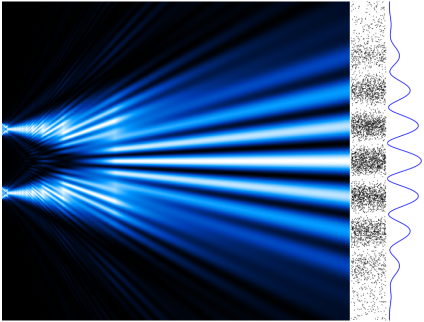
Spin-2: $R_{12} \to p_1 p_2$





Interference





Multiple resonances may contribute to the same final state

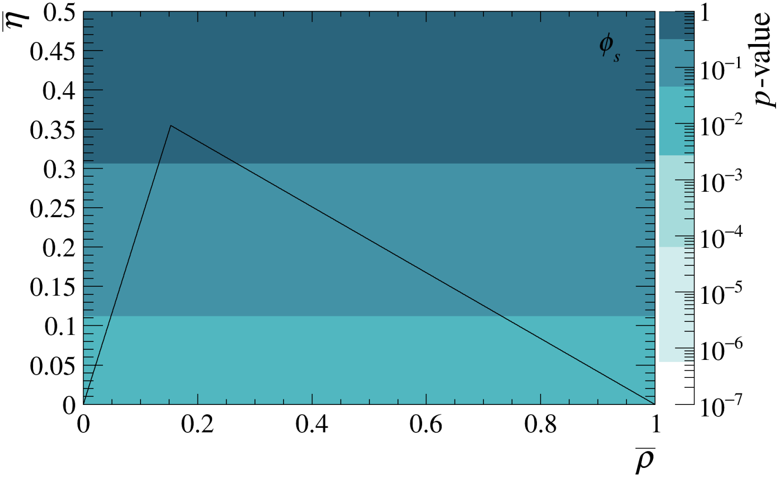
- Intermediate resonant states may interfere

For example, with just 2 interfering contributions,

$$|A|^2 = \underbrace{|A_1 + A_2|^2}_{\text{Coherent sum}} = \underbrace{|A_1|^2 + |A_1|^2}_{\text{Incoherent sum}} + \underbrace{2|A_1||A_2|\cos(\arg(A_2) - \arg(A_1))}_{\text{Interference term}}$$

Phase difference leaves unique signature in phase space

ϕ_s

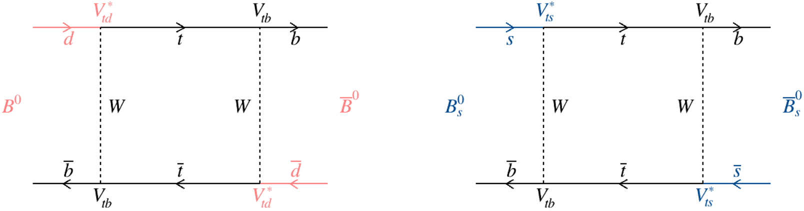


Phase of neutral B_s^0-$\bar{B}_s^0$ mixing

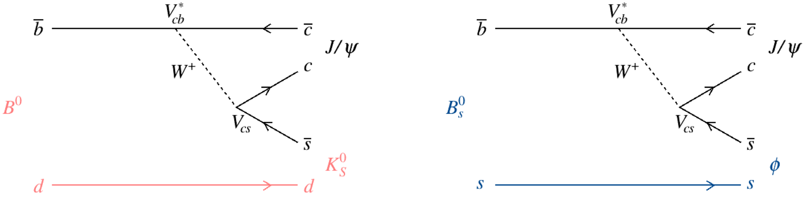
$\phi_s \equiv -2\lambda^2\bar{\eta}$

ϕ_s theory

Analogous to the measurement of $\sin 2\phi_1$ from $B^0 \to J/\psi K^0_S$



Mixing process differs only in the light quark, $d \to s$



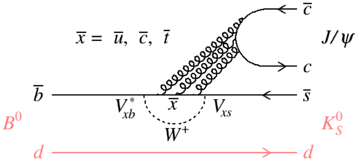
Decay differs only in the spectator interaction, $d \to s$

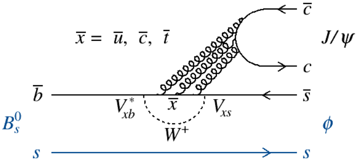
Best prediction of any CKM parameter based on other CKM input

$\phi_s = -36.8^{+0.9}_{-0.6}$ mrad [CKMfitter], $\phi_s = -37.0 \pm 1.0$ mrad [UTfit]

ϕ_s theory

Same Feynman topologies, same problems





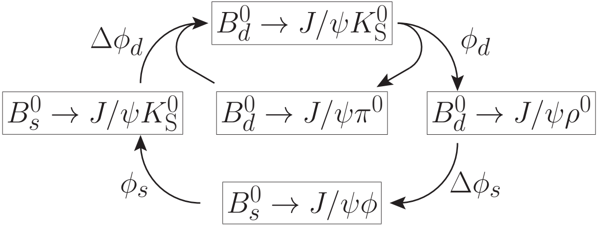
- Suppressed loop contribution carries a different weak phase
  - Experimentally sensitive to the sum, $\phi_{d,s} \to \phi_{d,s}^{\text{eff}} \equiv \phi_{d,s} + \Delta\phi_{d,s}$
- Hadronic phase shift $\Delta\phi_{d,s}$, cannot be calculated reliably within QCD
- Can be constrained from SU(3) flavour symmetry instead
  - Assumes u, d, s quarks have the same mass
  - Relies on further related experimental results
- Induces additional uncertainties
  - Factorisable SU(3)-breaking: *eg*. ratios of CKM elements, decay constants
  - Non-factorisable SU(3)-breaking: Unknown sources, dominant effect

ϕ_s theory

ϕ_d and ϕ_s treated simultaneously

M.Z. Barel, K. De Bruyn, R. Fleischer and E. Malami, J. Phys. **G48** (2021) 065002

Mixing phases $\phi_{d,s}$, and shifts $\Delta\phi_{d,s}$, interrelated through various decays



$B_d^0 \to J/\psi K_S^0$

$\Delta\phi_d$

ϕ_d

$B_s^0 \to J/\psi K_S^0$

$B_d^0 \to J/\psi \pi^0$

$B_d^0 \to J/\psi \rho^0$

ϕ_s

$\Delta\phi_s$

$B_s^0 \to J/\psi \phi$

Amplitudes constructed for each B flavour and final state, f

$A_f \to A_f[1 + a_f e^{i(\theta_f + \phi_3)}]$, $\bar{A}_f = \eta_{CP} A_f \to \eta_{CP} A_f [1 + a_f e^{i(\theta_f - \phi_3)}]$

Derive theoretical forms for experimental observables

$\lambda_{CP}^f \equiv \bar{A}_f / A_f \to \mathcal{A}_{CP}^f, \mathcal{S}_{CP}^f, \mathcal{A}_{\Delta\Gamma}^f$

Compare theoretical forms with each experimental measurement

- Statistical tool deployed such as a χ^2 test
- Constrain pollution amplitudes a_f, θ_f for the decays above

Not enough physical observables for the number of unknowns

ϕ_s theory

Parameter space reduced by SU(3) relations

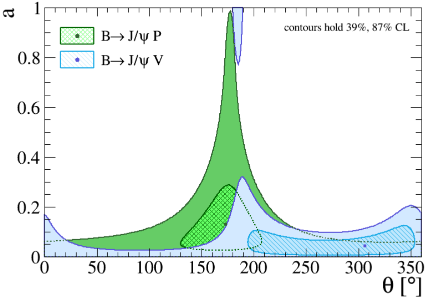
Account for factorisable SU(3)-breaking and rewrite amplitudes, *eg*

$$A(B^0 \to J/\psi K^0_S) \qquad\qquad A(B^0_s \to J/\psi K^0_S)$$

$$\left(1-\frac{\lambda^2}{2}\right) A_d \left[1+\frac{\lambda^2}{1-\lambda^2} a_d e^{i(\theta_d+\phi_3)}\right] \qquad -\lambda A_s[1 - a_s e^{i(\theta_s+\phi_3)}]$$

SU(3) symmetry then imposes $A_d = A_s$, $a_d = a_s$ and $\theta_d = \theta_s$

Conversion to phase shift via $\eta_{CP}\mathcal{S}^{d,s}_{CP} = \sqrt{1-(\mathcal{A}^{d,s}_{CP})^2}\,\sin(\phi_{d,s}+\Delta\phi_{d,s})$



$$\boxed{\begin{aligned}\Delta\phi_d &= (-0.73^{+0.60}_{-0.91})^\circ \\ \Delta\phi_s &= (+0.14^{+0.54}_{-0.70})^\circ\end{aligned}}$$

Same size as experimental error

ϕ_s theory

There are further complications for $B_s^0 \to J/\psi\phi$ *wrt* $B^0 \to J/\psi K_S^0$

$\Delta\Gamma_s/\Gamma_s = +0.128 \pm 0.007$

- Time-dependence more complicated than for B^0 decays, where $\Delta\Gamma_d \sim 0$

The ϕ resonance is a vector resonance as opposed to the scalar K_S^0

- J/ψ and K_S^0 in a pure relative orbital angular momentum $L = 1$ state
- J/ψ and ϕ may present in an $L = 0, 1, 2$ state

CP eigenvalue η_{CP}, goes like $(-1)^L$

- Admixture of CP-even $(L = 0, 2)$ and CP-odd $(L = 1)$ contributions

Quasi-two-body time-dependence not meaningful

$$\overset{(-)}{\Gamma}_s(t) = \frac{e^{-t/\tau_s}}{4\tau_s}\left[\cosh\frac{\Delta\Gamma_s t}{2} - \frac{2\Re(\lambda_{CP})}{|\lambda_{CP}|^2+1}\sinh\frac{\Delta\Gamma_s t}{2} \pm \frac{|\lambda_{CP}|^2-1}{|\lambda_{CP}|^2+1}\cos\Delta m_s t \pm \frac{2\Im(\lambda_{CP})}{|\lambda_{CP}|^2+1}\sin\Delta m_s t\right]$$

Coefficients of $\sinh$ and $\sin$ terms implicitly depend on sign of η_{CP}

- Valid only for a pure CP eigenstate, which $B_s^0 \to J/\psi\phi$ is not

ϕ_s theory

The CP-violation parameter $\lambda_{CP} = \bar{A}/A$, is an amplitude-level quantity

Return to more fundamental form by explicitly expanding out λ_{CP}

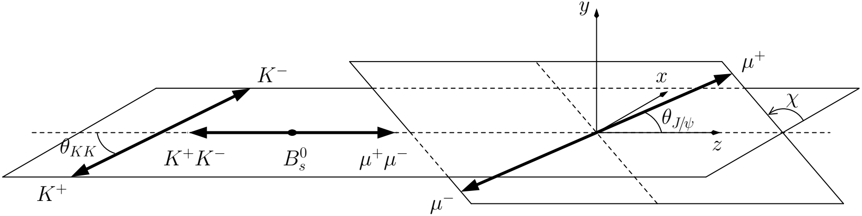
$$\overset{(-)}{\Gamma}_s(t) = \frac{e^{-t/\tau_s}}{4\tau_s}\left[(|A|^2 + |\bar{A}|^2)\cosh\frac{\Delta\Gamma_s t}{2} - 2\Re(A^*\bar{A})\sinh\frac{\Delta\Gamma_s t}{2}\right.$$
$$\left.\pm\,(|A|^2 - |\bar{A}|^2)\cos\Delta m_s t \pm 2\Im(A^*\bar{A})\sin\Delta m_s t\right]$$

Perform amplitude analysis

- Model $A = \sum_i A_i$, $\bar{A} = \sum_i \eta^i_{CP} A_i$

Indices run over final state polarisations $L = 0, 1, 2$

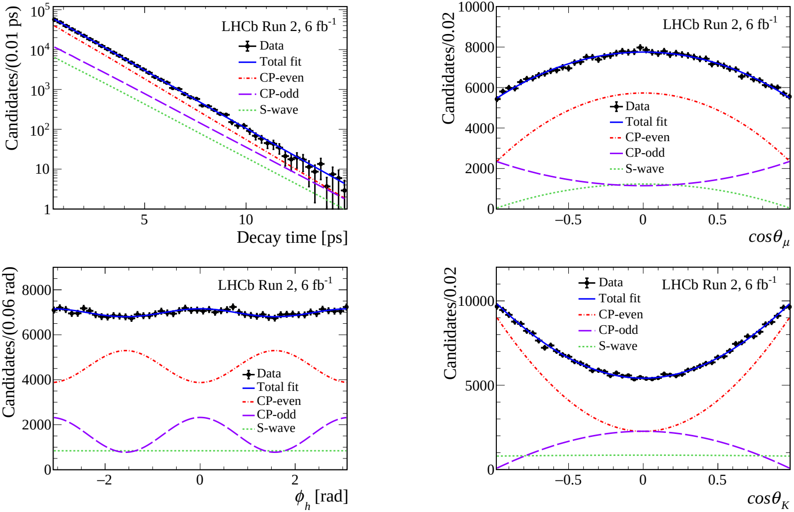
- Transversity basis preferred, $\Phi_4 \to m_{\mu\mu}$, m_{KK}, $\cos\theta_{\mu\mu}$, $\cos\theta_{KK}$ and χ
- Physical polarisations $L = 0, 1, 2$ (numerical) → $L = 0, \parallel, \perp$ (analytic)
- Abstract polarisations still CP eigenstates: $0, \parallel$ (CP-even), $\perp$ (CP-odd)



ϕ_s results

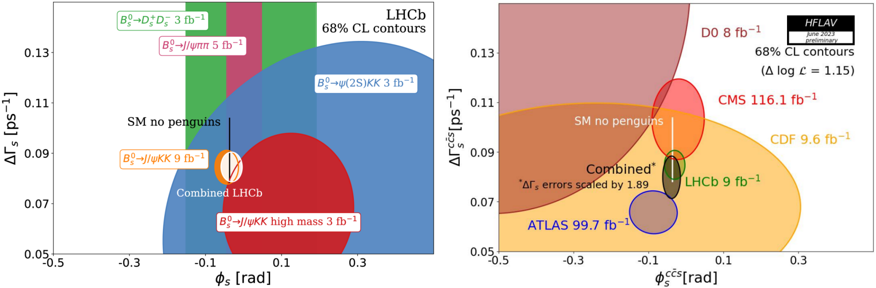
Latest results from LHCb, arXiv:2308.01468 [hep-ex] (2023)

Measures other fundamental parameters: $|\lambda_{CP}|$, $\Gamma_s - \Gamma_d$, $\Delta\Gamma_s$ and Δm_s



$$\phi_s = -0.039 \pm 0.022\ (\text{stat}) \pm 0.006\ (\text{syst})\ \text{rad}$$

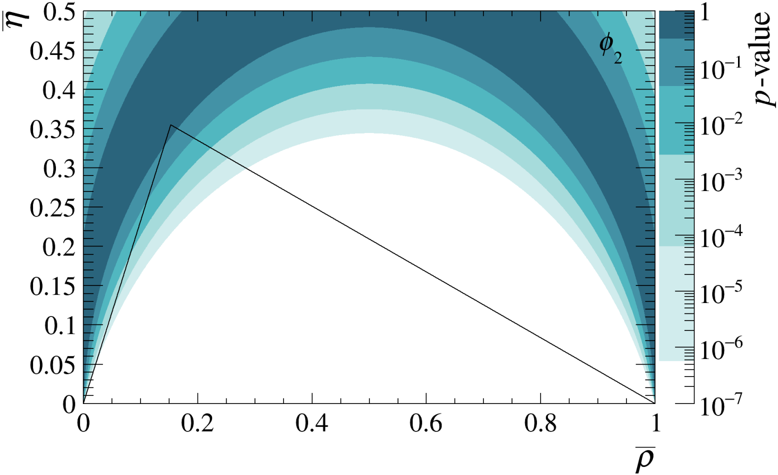
ϕ_s average



Experimental uncertainty still no where near that of the CKM prediction

$$\boxed{\phi_s = -0.039 \pm 0.016 \text{ rad}}$$

ϕ_2

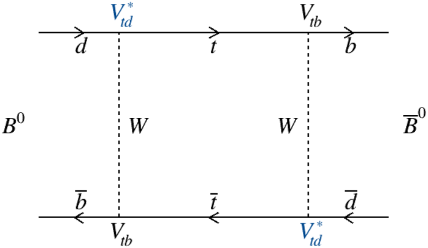


Composite phase

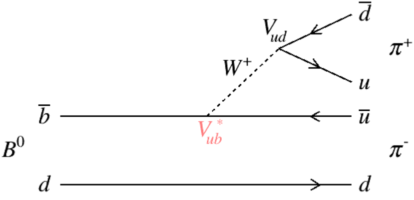
$\phi_2 \equiv \pi - \arctan[\bar{\eta}/(1-\bar{\rho})] - \arctan(\bar{\eta}/\bar{\rho})$

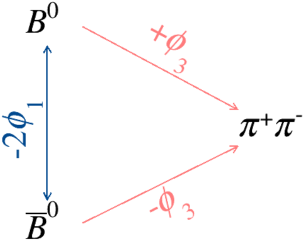
ϕ_2 theory

$A(B^0 \to \bar{B}^0) \propto (V_{td}^* V_{tb})^2$



$A_f \propto V_{ub}^* V_{ud}$





Phase difference, $-2\phi_1 - 2\phi_3$

Why ϕ_2? Add trivial period, 2π

$2\pi - 2\phi_1 - 2\phi_3 = 2(\pi - \phi_1 - \phi_3) = 2\phi_2$

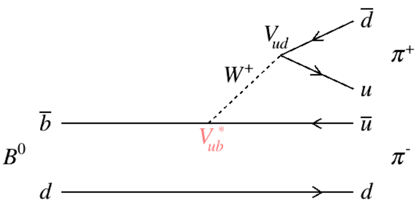
Common myth: Experiment imposes unitarity

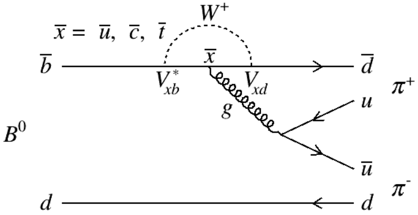
Truth: Experiment measures composite phase

CKM fit imposes unitarity

ϕ_2 the "least-known" input to fits of the CKM, greatest potential

ϕ_2 theory





Once again, gluonic penguin amplitudes pollute clean measurements

- Unlike the case for ϕ_1 and ϕ_s, stronger relative contribution in ϕ_2

Competing amplitudes give rise to CP violation in decay, $\mathcal{A}_{CP}^{+-}$

Mixing-induced CP violation, $\mathcal{S}_{CP}^{+-} = \sqrt{1-(\mathcal{A}_{CP}^{+-})^2}\sin(2\phi_2 + 2\Delta\phi_2^{+-})$

- Only sensitive to effective ϕ_2 biased by some phase shift, $\Delta\phi_2^{+-}$

Must again look to related decays to control penguin pollution

This time, $\Delta\phi_2^{+-}$ can be constrained by SU(2) isospin symmetry

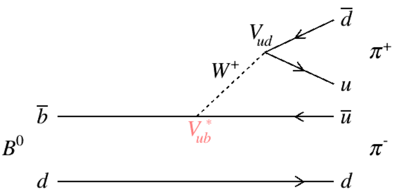
- Only assumes u and d quarks have the same mass, much better than SU(3)
- M. Gronau and D. London, Phys. Rev. Lett. **65** (1990) 3381

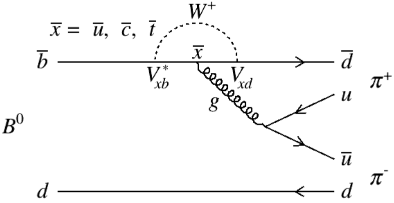
ϕ_2 theory

3 isospin-related decays: $B^0 \to \pi^+\pi^-$, $B^0 \to \pi^0\pi^0$ and $B^+ \to \pi^+\pi^0$

Isospin symmetry relates decay amplitudes,

$$B: \quad A^{+0} = \frac{1}{\sqrt{2}} A^{+-} + A^{00}, \qquad \bar{B}: \quad \bar{A}^{+0} = \frac{1}{\sqrt{2}} \bar{A}^{+-} + \bar{A}^{00}$$





Pions are isovectors, start with final state being $I = 0, 1, 2$ for each decay

Pions also identical in isospin, Bose-Einstein forbids antisymmetric state

$\Rightarrow I = 0, 2$ for each final state

Focus on $B^+ \to \pi^+\pi^0$, $I_3 = +1$, $\Rightarrow I = 2$ only

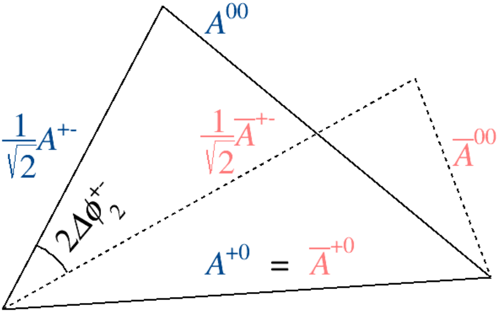
Gluon from penguin an isoscalar, remainder can only generate $I = 0, 1$

Already forbidden, $B^+ \to \pi^+\pi^0$ is pure tree

ϕ_2 theory

$$B: \quad A^{+0} = \frac{1}{\sqrt{2}}A^{+-} + A^{00}, \qquad \bar{B}: \quad \bar{A}^{+0} = \frac{1}{\sqrt{2}}\bar{A}^{+-} + \bar{A}^{00}$$

No penguin in $B^+ \to \pi^+\pi^0$ means $A^{+0} = \bar{A}^{+0}$



- Expressed as triangles, isospin relations must share the same base
- 5 sides, 5 observables, enough degrees of freedom to constrain $2\Delta\phi_2^{+-}$
  - Branching fractions: $\mathcal{B}^{+0}$, $\mathcal{B}^{+-}$, $\mathcal{B}^{00}$
  - CP asymmetries in decay: $\mathcal{A}_{CP}^{+-}$, $\mathcal{A}_{CP}^{00}$

ϕ_2 theory

SU(2)-breaking $m_u \neq m_d$, leaves residual $I = 1$ amplitudes

Currently ignored, but not for much longer, $\Delta\phi_2 \sim 1^\circ$

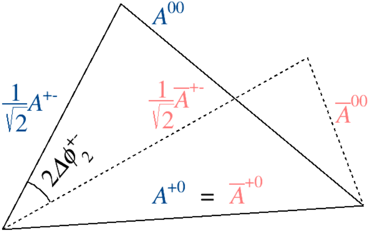
Some can be eliminated experimentally, others maybe not

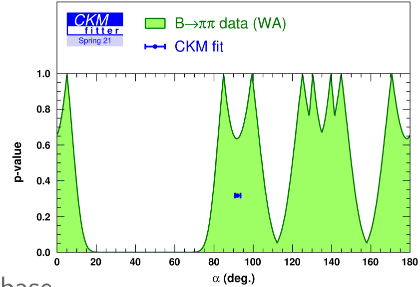
Electroweak penguin amplitudes
- $\Delta\phi_2 \sim 1^\circ$ in $B \to \pi\pi$, $\rho\rho$ and $B^0 \to (\rho\pi)^0$
  - M. Gronau and J. Zupan, Phys. Rev. D **71** (2005) 074017

π^0–η–η' mixing and ρ^0–ω–ϕ mixing
- $\Delta\phi_2 \sim 1^\circ$ in $B \to \pi\pi$, order of magnitude smaller in $B^0 \to (\rho\pi)^0$
  - M. Gronau and J. Zupan, Phys. Rev. D **71** (2005) 074017
- Experimentally manageable in $B \to \rho\rho$
  - M. Gronau and J. Rosner, Phys. Lett. B **766** (2017) 345

Invariant mass difference in $B \to \rho\rho$ from finite ρ width
- Experimentally manageable
  - F. Falk, Z. Ligeti, Y. Nir and H. Quinn, Phys.Rev. D **69** (2004) 011502(R)

ϕ_2 results





Each isospin triangles can flip about base

- 4-fold degeneracy in $\Delta\phi_2^{+-}$

$$\mathcal{S}_{CP}^{+-} = \sqrt{1-(\mathcal{A}_{CP}^{+-})^2}\,\sin(2\phi_2 + 2\Delta\phi_2^{+-})$$

- 2-fold ambiguity in argument of $\sin$

Therefore, 8 solutions expected for ϕ_2

- Clearly seen in $B \to \pi\pi$ system, can only state where ϕ_2 is not

$B^0 \to \pi^0\pi^0$ tree colour-suppressed relative to $B^0 \to \pi^+\pi^-$

- Penguin contribution much larger than anticipated

The golden channel that was not, weak CKM constraint, look elsewhere

ϕ_2 results

Field would be saved by $B \rightarrow \rho\rho$

Like $B_s^0 \rightarrow J/\psi\phi$, two spin-1 resonances in the final state

- States of definite CP need to be extracted, but with a key difference

QCD factorisation for B decays to light vector meson resonance pairs

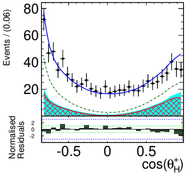
- Predicts ρ is spin perpendicular to flight direction, longitudinal polarisation $f_L = 1 - \mathcal{O}(1/m_b^2) \sim 1$ (CP-even)
- A. Kagan, Phys. Lett. **B601** (2004) 151

Reduces complexity of angular analysis, depends only on ρ helicities, $\theta_{1,2}$

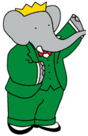
$$d^2\Gamma/d\cos\theta_1 d\cos\theta_2 = 4 f_L \cos^2\theta_1 \cos^2\theta_2 + (1 - f_L)\sin^2\theta_1 \sin^2\theta_2$$

Various f_L measurements dominated by different experiments

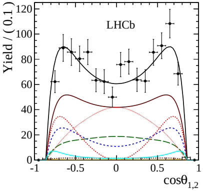
$B^0 \rightarrow \rho^+\rho^-$



$B^+ \rightarrow \rho^+\rho^0$

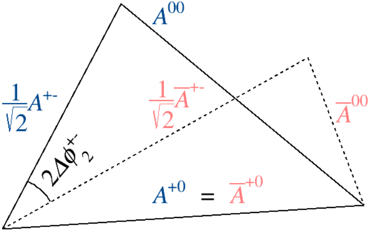


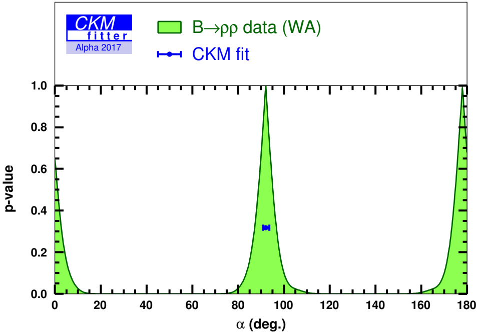
$B^0 \rightarrow \rho^0\rho^0$



Belle: PRD **93** (2016) 032010 BaBar: PRL **102** (2009) 141802 LHCb: PLB **747** (2015) 468

ϕ_2 results





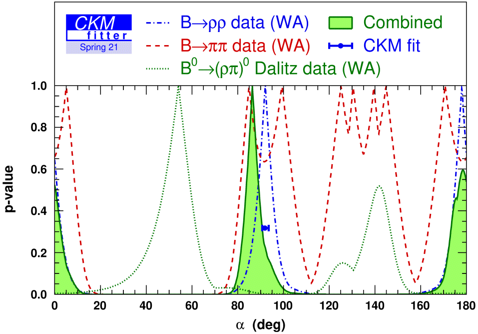
8-fold degeneracy in ϕ_2 collapses to 2 solutions

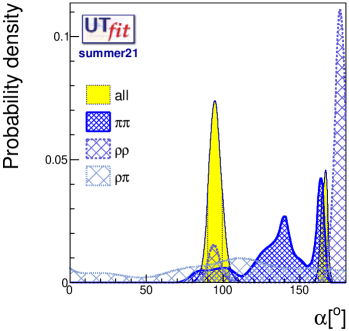
- Penguin amplitude is much more suppressed, $\Delta\phi_2^{+-} \sim 0$
- Isospin triangles flattened

$B \to \rho\rho$ is the best system in which to measure ϕ_2

- Superior likelihood profile, more meaningful contribution to CKM fit
- Potential to model/eliminate SU(2)-breaking effects in amplitude analysis

ϕ_2 average



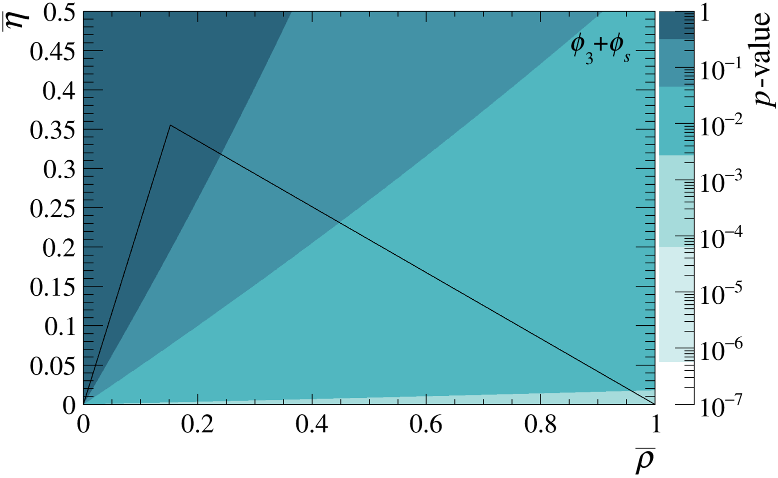


Additional information from $B^0 \to (\rho\pi)^0$ currently problematic

- Possibility to constrain ϕ_2 in a single time-dependent amplitude analysis
- Model parametrisation broken, research needed to fix

$$\boxed{\phi_2 = (86.4^{+4.3}_{-4.0})^\circ \text{ [CKMFitter]}, \qquad \phi_2 = (94.9 \pm 4.7)^\circ \text{ [UTfit]}}$$

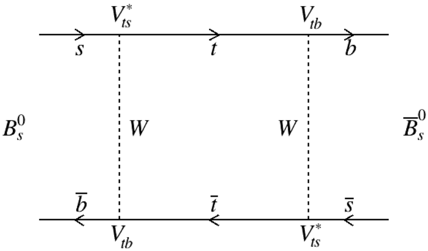
$\phi_s + \phi_3$

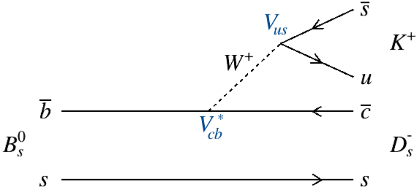


$\phi_s + \phi_3 \equiv -2\lambda^2\bar{\eta} + \arctan(\bar{\eta}/\bar{\rho})$

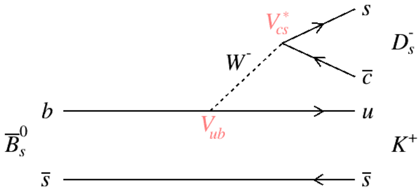
Composite phase

$\phi_s + \phi_3$ theory





$V_{cb}^* V_{us} \approx \lambda^3$



$V_{ub}^* V_{cs} \approx \lambda^3$

B_s^0-$\bar{B}_s^0$ oscillations are also fast, $\tau_s \gg \Delta m_s$

No colour suppression involved, like in $B^+ \to DK^+$

$\phi_s + \phi_3$ theory

Interfering processes expected to be of similar order

Final state flavour-non-specific, not CP eigenstate, $\lambda_{CP} \to \xi^{\pm} \equiv \bar{A}^{\pm}/A^{\pm}$

$$D_s^+ K^- : \xi^+ = \frac{-1}{r_B \delta_B e^{+i(\phi_s+\phi_3)}}, \quad D_s^- K^+ : \xi^- = \frac{-r_B \delta_B e^{-i(\phi_s+\phi_3)}}{1}$$

Time-dependent decay rate asymmetry

$$\frac{\Gamma(B_s^0(t) \to D_s^{\pm} K^{\mp}) - \Gamma(B_s^0(t) \to D_s^{\pm} K^{\mp})}{\Gamma(B_s^0(t) \to D_s^{\pm} K^{\mp}) + \Gamma(B_s^0(t) \to D_s^{\pm} K^{\mp})} = \frac{-\mathcal{C}^{\pm}\cos(\Delta m_s t) + \mathcal{S}^{\pm}\sin(\Delta m_s t)}{\cosh(\Delta\Gamma_s t/2) + \mathcal{A}^{\pm}_{\Delta\Gamma}\sinh(\Delta_s\Gamma t/2)}$$

Measure 5 observables for 3 unknowns, r_B, δ_B and composite $\phi_s + \phi_3$

$$\mathcal{C}^+ = -\mathcal{C}^- = -\frac{1 - r_B^2}{1 + r_B^2}$$

$$\mathcal{S}^{\pm} = \frac{2 r_B \sin(\phi_s + \phi_3 \pm \delta_B)}{1 + r_B^2}$$

$$\mathcal{A}^{\pm}_{\Delta\Gamma} = \frac{-2 r_B \cos(\phi_s + \phi_3 \pm \delta_B)}{1 + r_B^2}$$

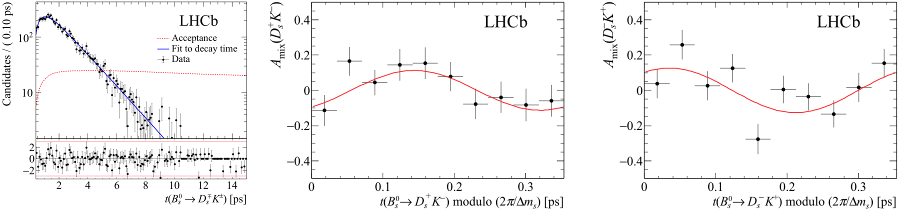
Analogous system $B^0 \to D^{\pm}\pi^{\mp}$ gives $\sin(2\phi_1 + \phi_3)$ but r_B suppressed

Input from theory needed, disfavoured approach for CKM fits

$\phi_s + \phi_3$ results and average

Analysis only performed at LHCb

JHEP **03** (2018) 059



Despite measurements, no formal average as yet

Still considered part of ϕ_3 average by subtracting ϕ_s contribution

Not ideal as correlation between ϕ_3 and ϕ_s measurements introduced

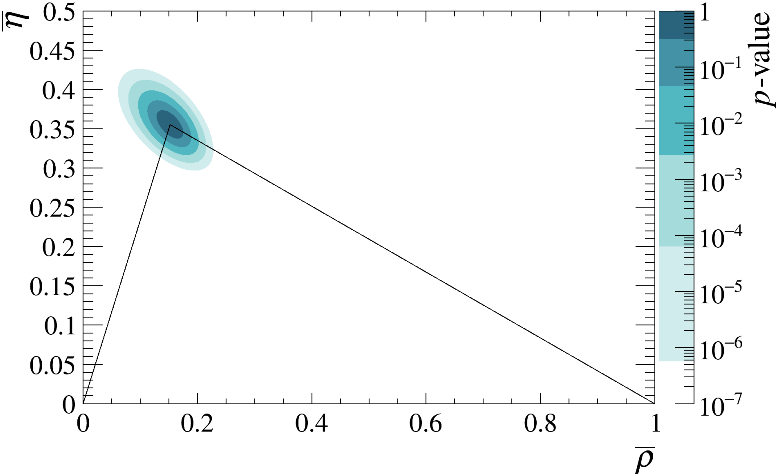
Different view into CKM fit lost

Hopefully changes in the future

Naively, $\phi_s + \phi_3 = (76 + 25)^\circ$

Great opportunity for improvement

Fitting the CKM matrix



Final goal to test unitarity in the CKM mechanism

Evaluate p-value from the combination of all measurements

Statistical approach

Experimental measurements: $\boldsymbol{x}$

Model parameters: $\boldsymbol{\mu}$

Frequentist

Construct test-statistic

$$\chi^2 = [\boldsymbol{x} - \boldsymbol{x}(\boldsymbol{\mu})]^T \boldsymbol{\Sigma}^{-1} [\boldsymbol{x} - \boldsymbol{x}(\boldsymbol{\mu})]$$

Fix parameter(s) of interest, x_i

Minimise χ^2 over range of x_i

Convert to p-value scan

Bayesian

Invoke Bayes Theorem

$$\mathcal{P}(\bar{\rho}, \bar{\eta}, \boldsymbol{\mu} | \boldsymbol{x}) = \prod_i \mathcal{P}(x_i | \bar{\rho}, \bar{\eta}, \boldsymbol{\mu}) \prod_j \mathcal{P}(\mu_j) \mathcal{P}_0(\bar{\rho}, \bar{\eta})$$

$\mathcal{P}_0(\bar{\rho}, \bar{\eta})$: Flat prior

Posterior probability density

$$\mathcal{P}(\bar{\rho}, \bar{\eta}) \propto \int \prod_i \mathcal{P}(x_i | \bar{\rho}, \bar{\eta}, \boldsymbol{\mu}) \prod_j \mathcal{P}(\mu_j) \mathcal{P}_0(\bar{\rho}, \bar{\eta}) d\boldsymbol{\mu}$$

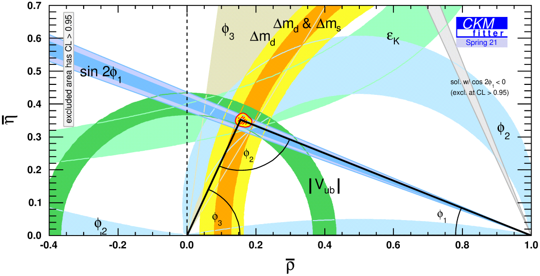
Difference in treatment of uncertainty: Gaussian *vs* arbitrary

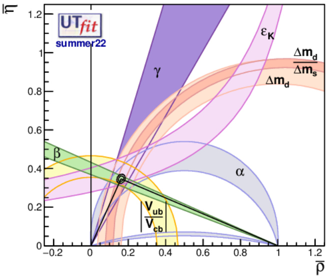
CKM fit results

A lot more input than those discussed here

CKMfitter - Frequentist

UTfit - Bayesian



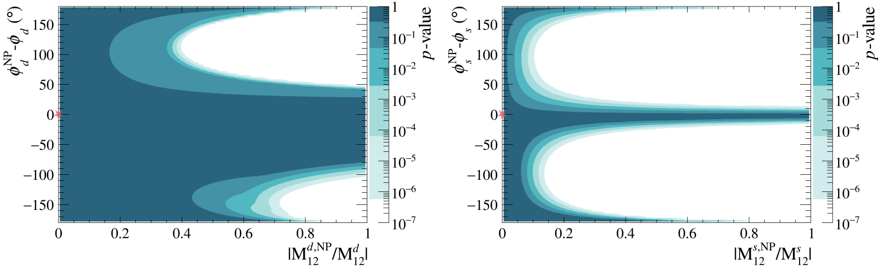


$$\bar{\rho} = 0.1566^{+0.0085}_{-0.0048}$$
$$\bar{\eta} = 0.3475^{+0.0118}_{-0.0054}$$

$$\bar{\rho} = 0.1609 \pm 0.0095$$
$$\bar{\eta} = 0.3470 \pm 0.0100$$

Both approaches in agreement

New Physics in neutral B-$\bar{B}$ mixing

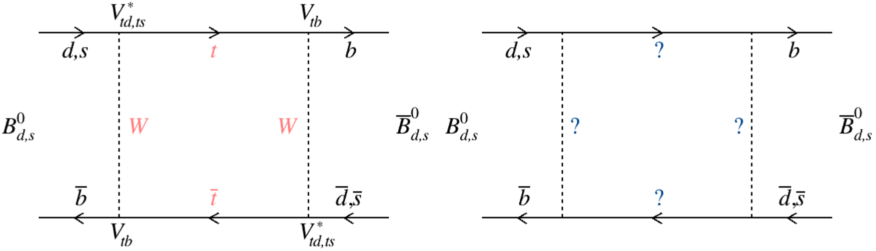


Most measurements constraining CKM involve tree-level processes

- Testing unitarity is primary goal
- No impact from New Physics amplitudes expected

Notable exception: B-$\bar{B}$ mixing is a loop-mediated process

New Physics in neutral B-$\bar{B}$ mixing



Loop processes potentially sensitive to New Physics

Unknown heavy particle (?) could present in the loop

- May carry a new CP violating phase

Short time-scale in loop the key to accessing arbitrarily high energies

- Heisenberg Uncertainty Principle: $\Delta E \Delta t \geq \hbar/2$

Can reach higher mass scales than possible with direct searches at LHC

Mass scale of New Physics through loop processes a matter of statistics rather than brute-force centre-of-mass energy

New Physics in neutral B-$\bar{B}$ mixing results

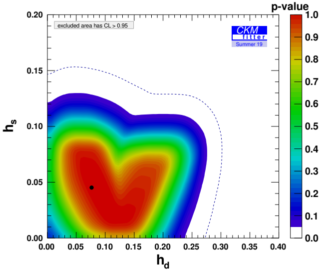
Include new model-independent amplitude relative to mixing amplitude

$$M_{12}^{d,s} \to M_{12}^{d,s,\mathrm{SM}}(1 + M_{12}^{d,s,\mathrm{NP}}/M_{12}^{d,s,\mathrm{SM}})$$

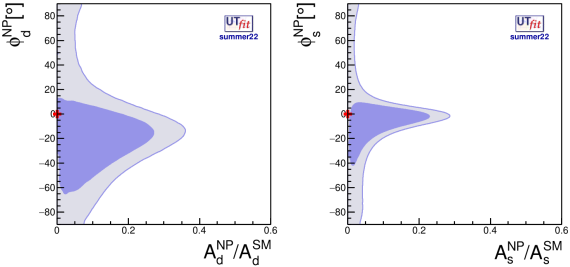
Reparameterise all physics observables that depend on $M_{12}^{d,s}$

- New expressions for $\Delta m_d/\Delta m_s$, $A_{SL}^{d,s}$, ϕ_1, ϕ_2, ϕ_s and $\phi_s + \phi_3$
- $|V_{ub}/V_{cb}|$ and ϕ_3 unaffected
- Recall $\Delta m_{d,s} \simeq 2|M_{12}^{d,s}|$, sets the scale of New Physics amplitude
- $A_{\mathrm{SL}} \simeq \Im(\Gamma_{12}^{d,s}/M_{12}^{d,s})$ extremely sensitive, depends on $\Re(\Gamma_{12}^{d,s,\mathrm{SM}}/M_{12}^{d,s,\mathrm{SM}})$

CKMfitter



UTfit



Standard Model model cannot be ruled out at this stage

Primary motivation to greatly improve precision in flavour sector

Summary

- Flavour physics is a very diverse field
- Rich history of discovery
  - High statistics indirect searches → SM amendment → Direct observation ↩ Reasonable to expect this sequence of events to continue and in this order
- Only discussed CKM mechanism within the context of B meson decays
- Only discussed useful experimental measurements where theory is good
  - This means there are great opportunities for theory development as well
- Challenges lie ahead for heavy flavour physics
  - Anomalies are disappearing
- Easy to get distracted
- Never lose sight of the big picture
- Identify connection to the cosmological matter-antimatter asymmetry

- CKM physics has the highest statistics when measurements combined
- Identify which uncertainties can be improved the most for real impact
- Don't run from the grind