Precision modelling for the cosmic large-scale structure

Oliver Hahn (Inst. f. Astrophysics, Inst. f. Mathematics, UVienna)

w/ Michaël Michaux, Florian List, Cornelius Rampf, Cora Uhlemann, and many others

Benasque 2023, Understanding Cosmological Observations

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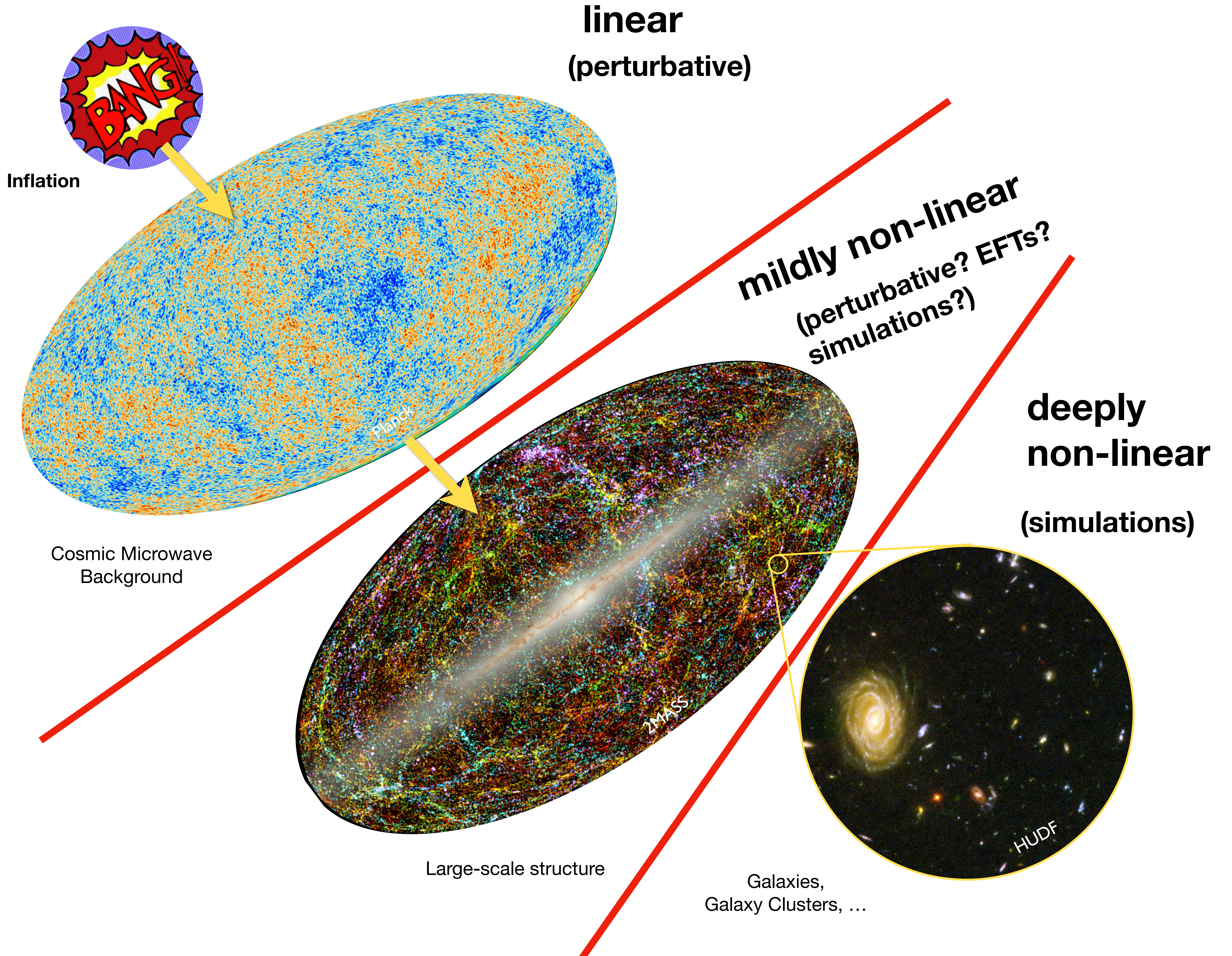
Cosmic evolution



Matching epochs in simulations

Early physics:

- GR effects (horizon+rel. species+aniso-stress)
- multi-species (CDM+baryon+photons+neutrinos)
- photon-baryon coupling + recombination
- perturbative quantity: δ and θ

Eulerian

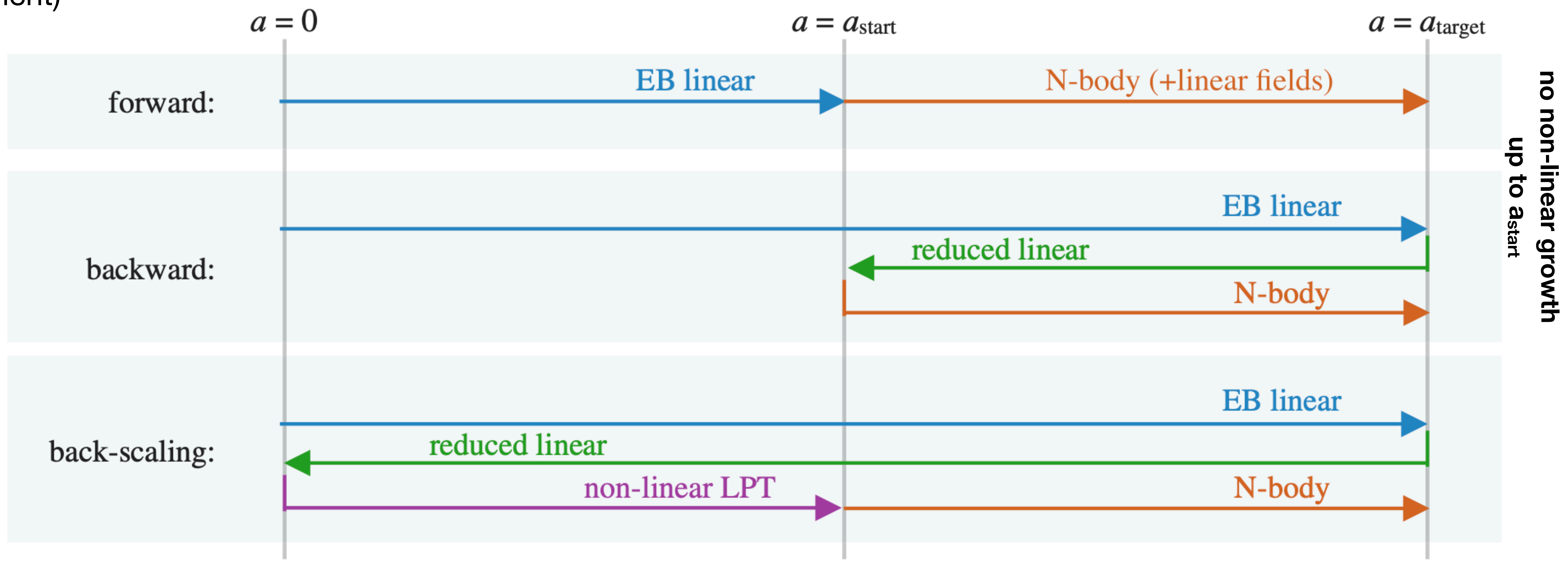
Matching problem!

Late physics:

- Newtonian gravity + small corrections
- mostly interested in mass distribution, CDM+baryons
- non-linear growth
- perturbative quantity: ψ (displacement)

Lagrangian

choices in the literature:



Angulo & Hahn 2022 review

Lagrangian Dynamics,

describing the motion of fluid elements in the continuum limit

Purely Gravitational Evolution: Vlasov-Poisson

Consider the (on-shell) phase-space density of non-interacting massive particles $f(\boldsymbol{x}, \boldsymbol{v}, t)$

Evolution given by non-relativistic limit of the relativistic Liouville equation = **Vlasov-Poisson**

$$\frac{\partial f}{\partial t} + \frac{\boldsymbol{v}}{a^2} \cdot \boldsymbol{\nabla}_x f - \boldsymbol{\nabla}_x \phi \cdot \boldsymbol{\nabla}_v f = 0$$

$$\nabla^2 \phi = \frac{3\Omega_m}{2a} \left(\frac{n}{\overline{n}} - 1 \right)$$

$$n(\boldsymbol{x}, t) := \int_{\mathbb{R}^d} \mathrm{d}^d v \; f \qquad \overline{n} := \int_{\mathscr{D}} \mathrm{d}^d x \; n.$$

+ let a(t=0)=0

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by chain rule

$$\frac{\mathrm{d} f_c}{\mathrm{d} t} = \frac{\partial f_c}{\partial t} \; + \; \boldsymbol{\nabla}_x f_c\big|_{\boldsymbol{x} = \boldsymbol{X}(t)} \cdot \frac{\mathrm{d} \boldsymbol{X}}{\mathrm{d} t} \; + \; \boldsymbol{\nabla}_v f_c\big|_{\boldsymbol{v} = \boldsymbol{V}(t)} \cdot \frac{\mathrm{d} \boldsymbol{V}}{\mathrm{d} t}.$$

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comparing with VP

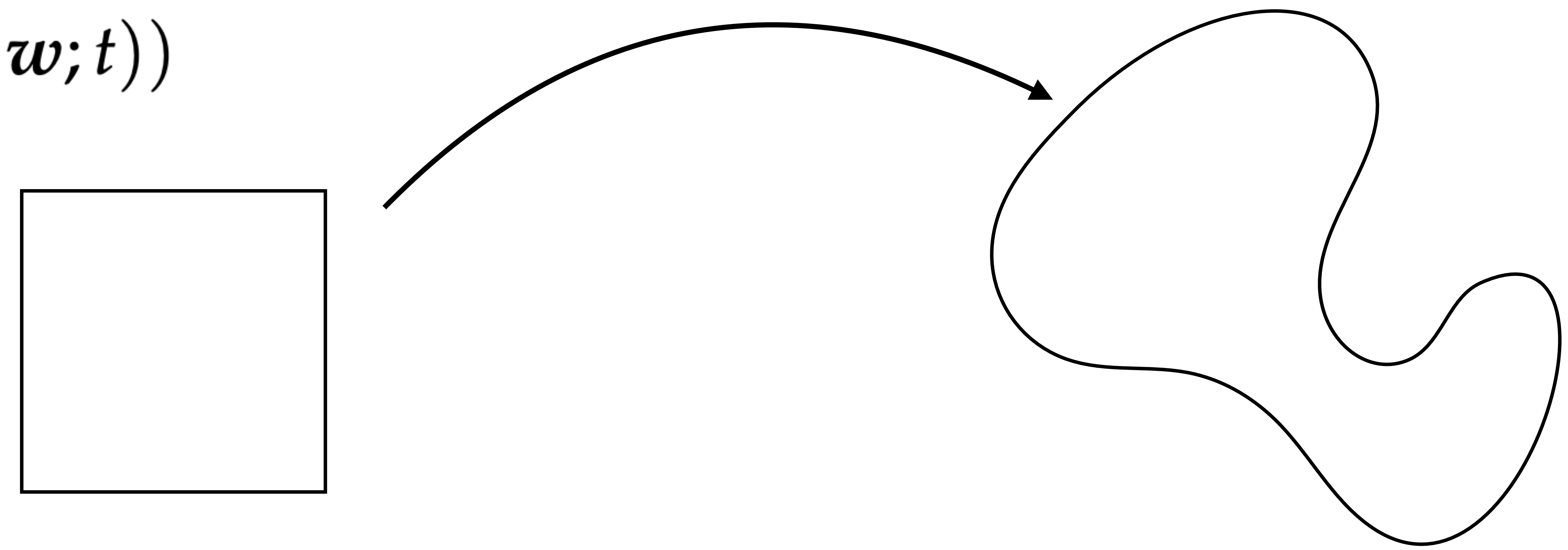
$$\frac{\mathrm{d}f_c}{\mathrm{d}t} = 0 \qquad \text{iff} \qquad \begin{cases} \dot{\boldsymbol{X}}(t) & = a^{-2}\, \boldsymbol{V}(t) \\ \dot{\boldsymbol{V}}(t) & = -\, \boldsymbol{\nabla}_x \phi|_{\boldsymbol{x}=\boldsymbol{X}(t)} \end{cases}$$

The fluid and the N-body model

Lagrangian description, evolution of fluid element

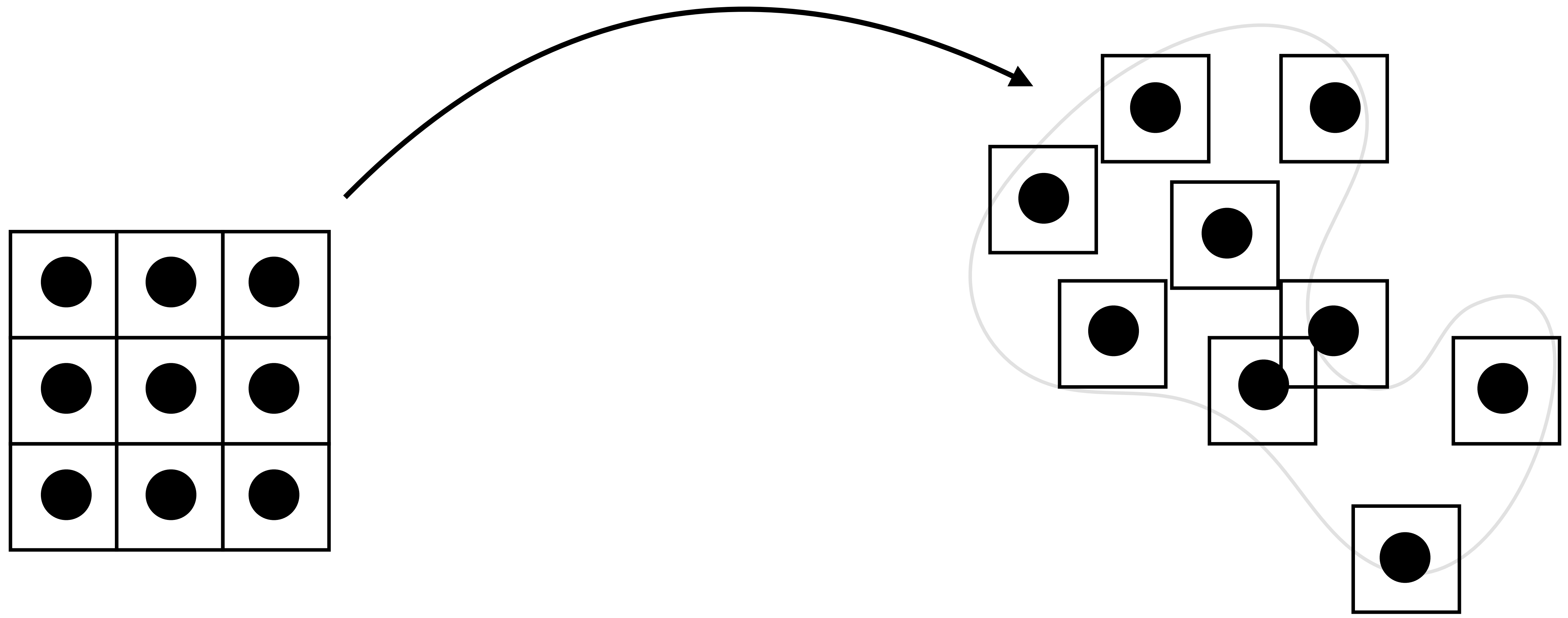
Lagrangian map of the $\mathbb{R}^{2d}$ phase space onto itself

$$\mathbb{R}^{2d} \to \mathbb{R}^{2d} : \quad (\boldsymbol{q}, \boldsymbol{w}) \mapsto (\boldsymbol{x}(\boldsymbol{q}, \boldsymbol{w}; t), \boldsymbol{v}(\boldsymbol{q}, \boldsymbol{w}; t))$$



The N-body approximation:
follow only *N* characteristics, use them to reconstruct the density field that sources the Poisson equation

$$\mathbb{R}^{N2d} \to \mathbb{R}^{N2d} : \quad (\boldsymbol{Q}_i, \boldsymbol{W}_i) \mapsto (\boldsymbol{X}_i(t), \boldsymbol{V}_i(t)) , \qquad i = 1, \ldots, N$$

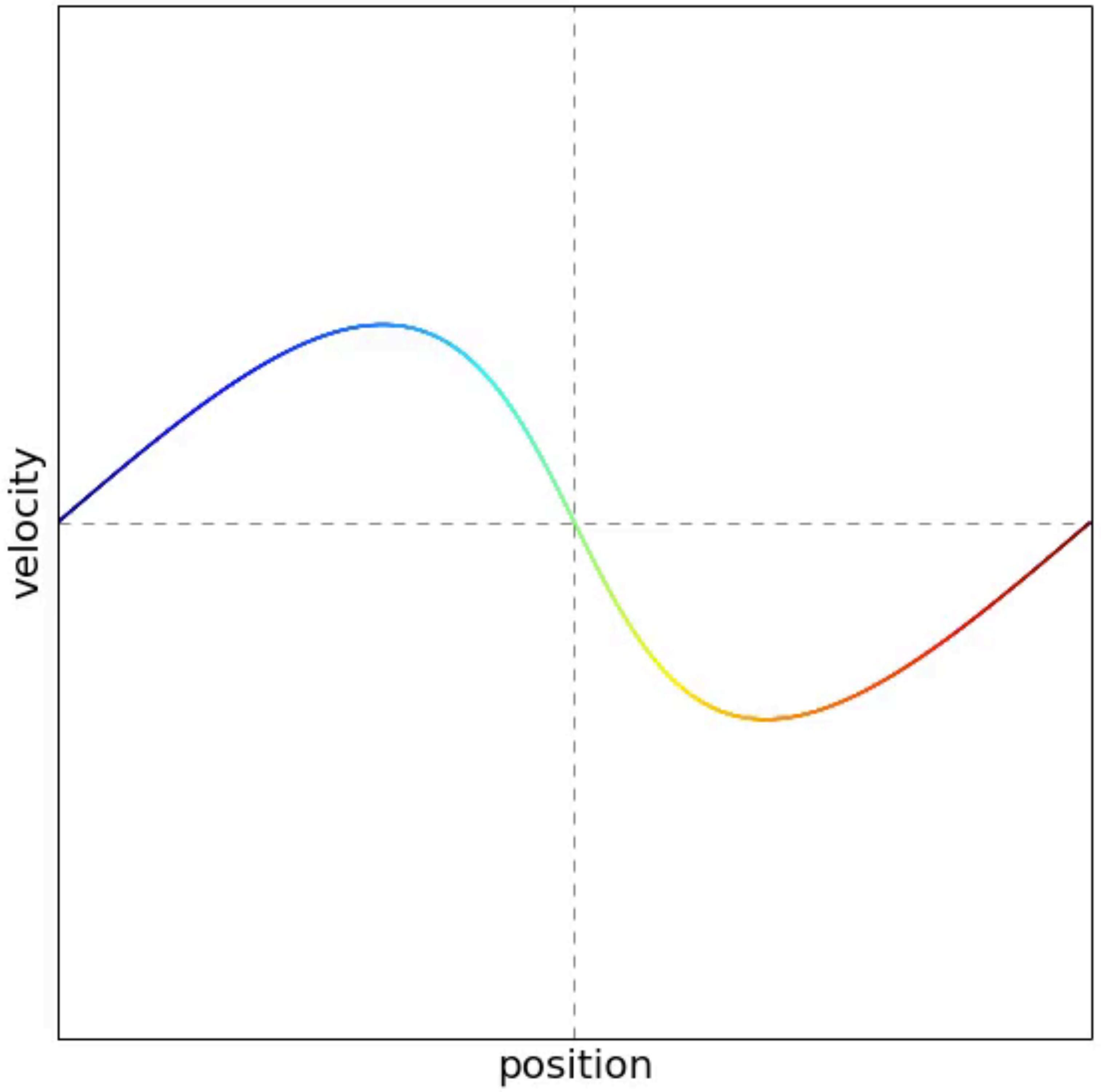


Non-Linear Evolution of Fluctuations

Cold Dark Matter lives on Lagrangian submanifold

Solve Vlasov-Poisson on submanifold characteristics $(\boldsymbol{q}, t) \mapsto (\boldsymbol{x}(\boldsymbol{q}, t), \boldsymbol{p}(\boldsymbol{q}, t))$

$$\frac{\partial f}{\partial t} + \frac{\boldsymbol{v}}{a^2} \cdot \boldsymbol{\nabla}_x f - \boldsymbol{\nabla}_x \phi \cdot \boldsymbol{\nabla}_v f = 0 \quad \Leftrightarrow \quad \mathbf{x}'' + \mathcal{H}\mathbf{x}' = -\nabla\phi$$



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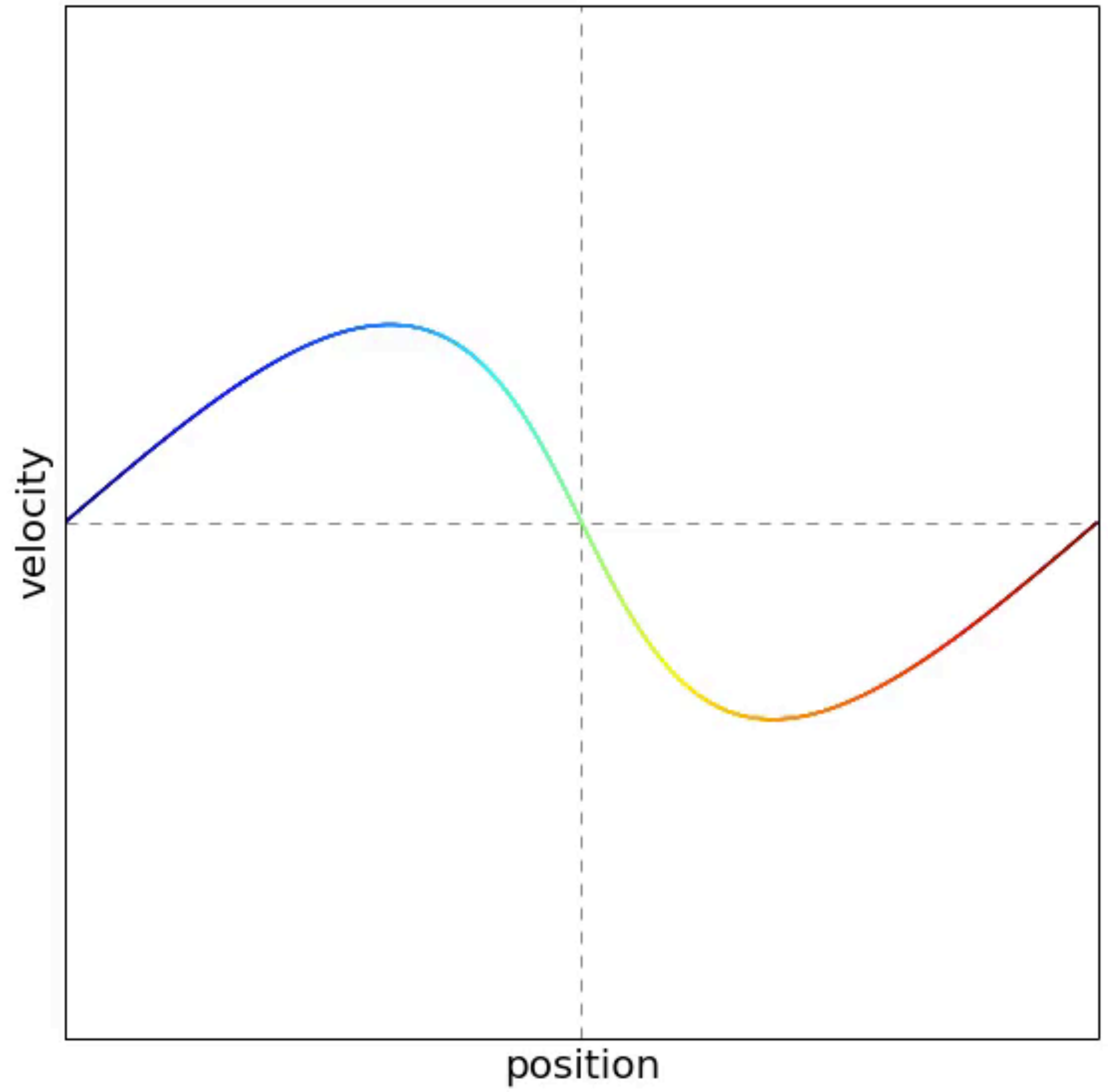

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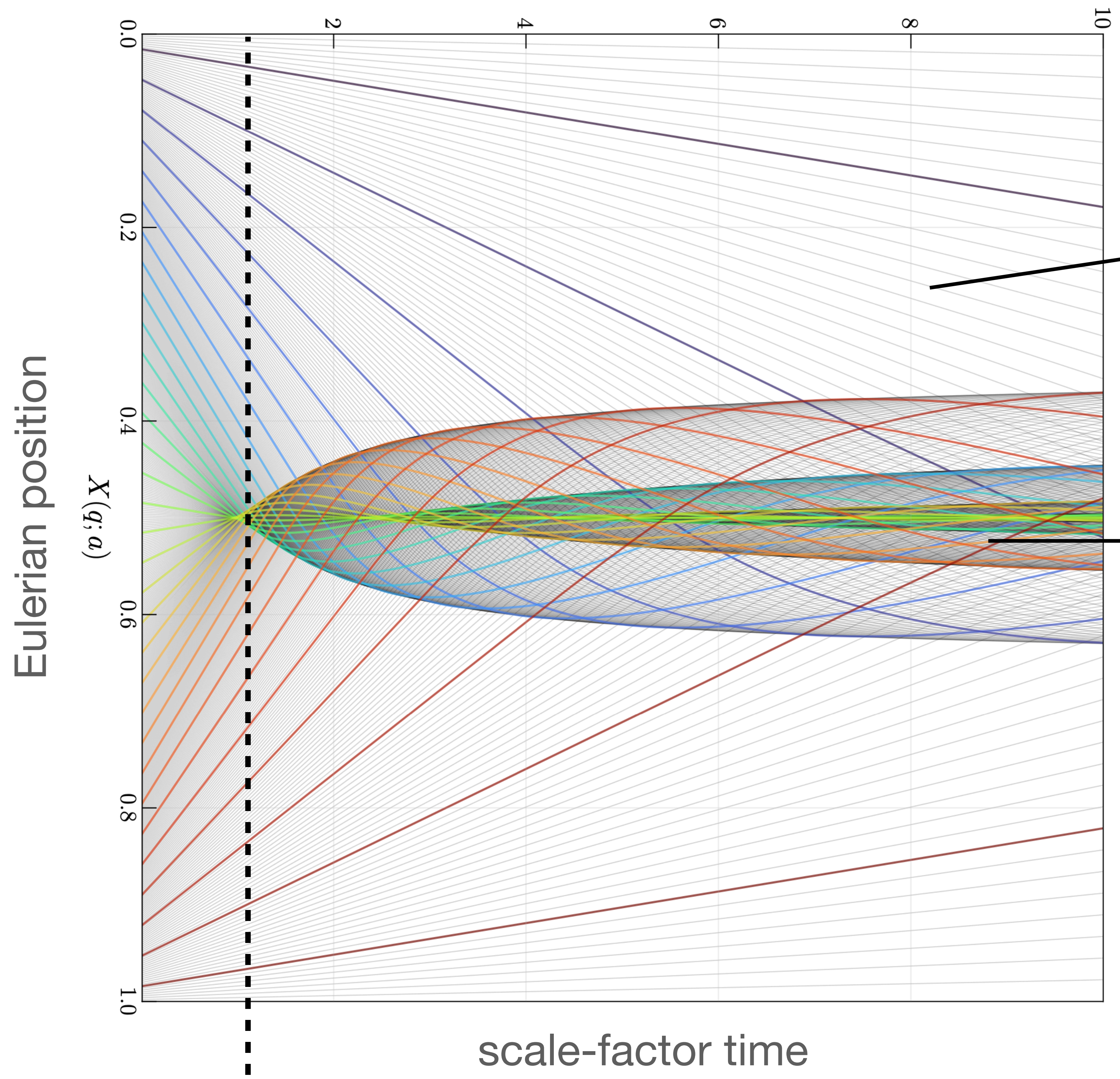
density + acceleration singularities



moment of shell-crossing

monokinetic, single-valued
(analytic treatment possible)

multikinetic, multi-valued
(simulations, EFTs [by integrating out])

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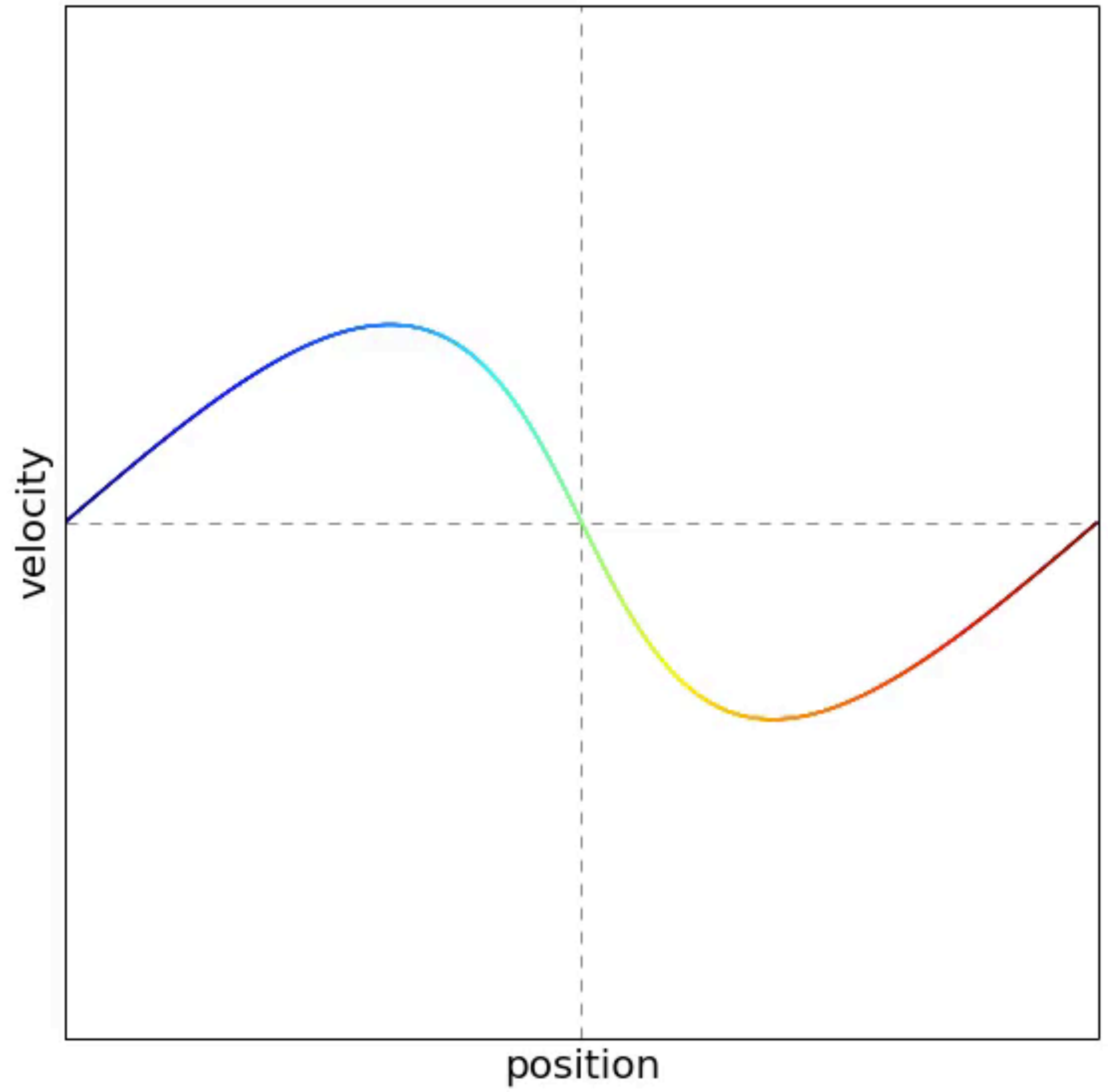

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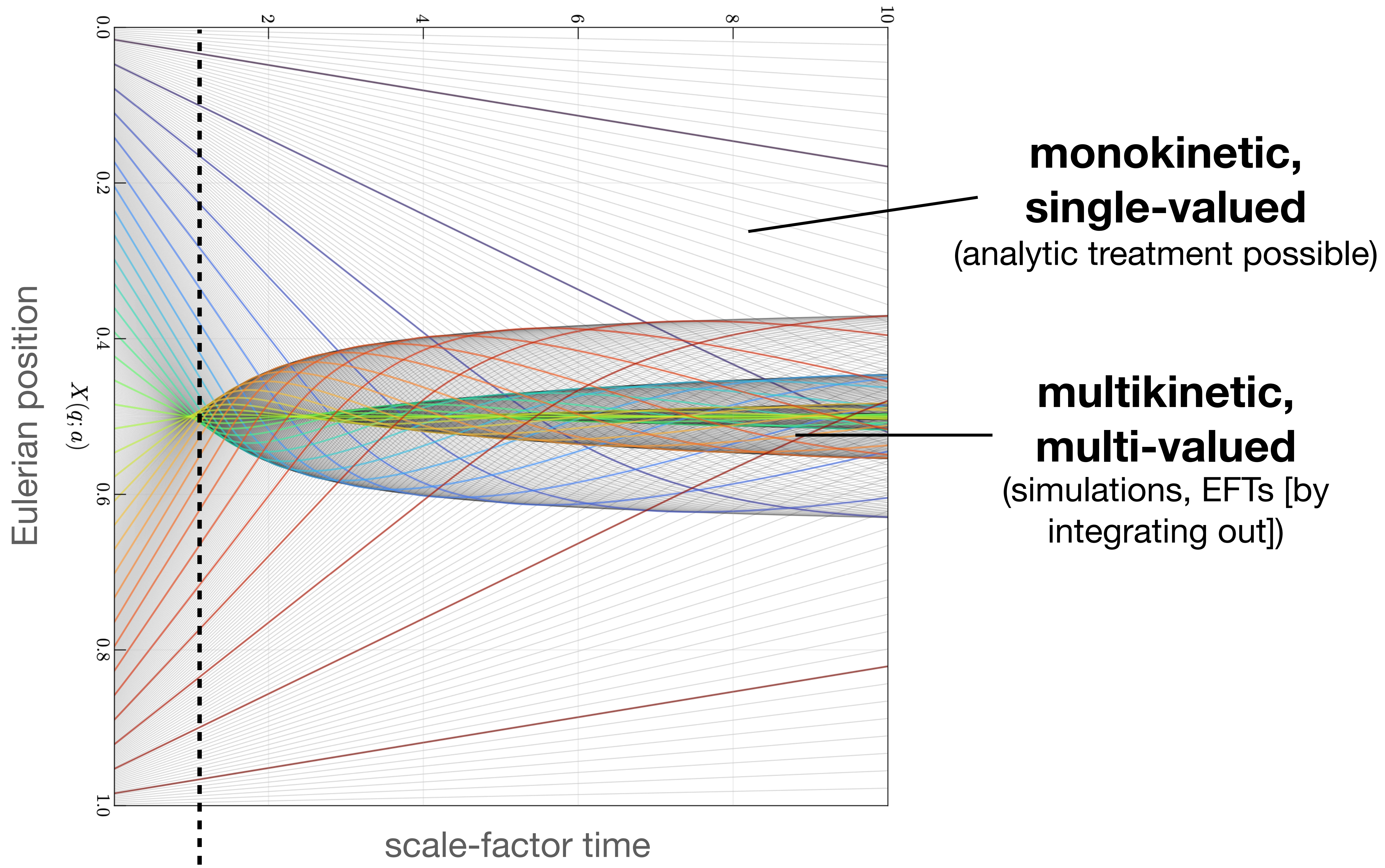

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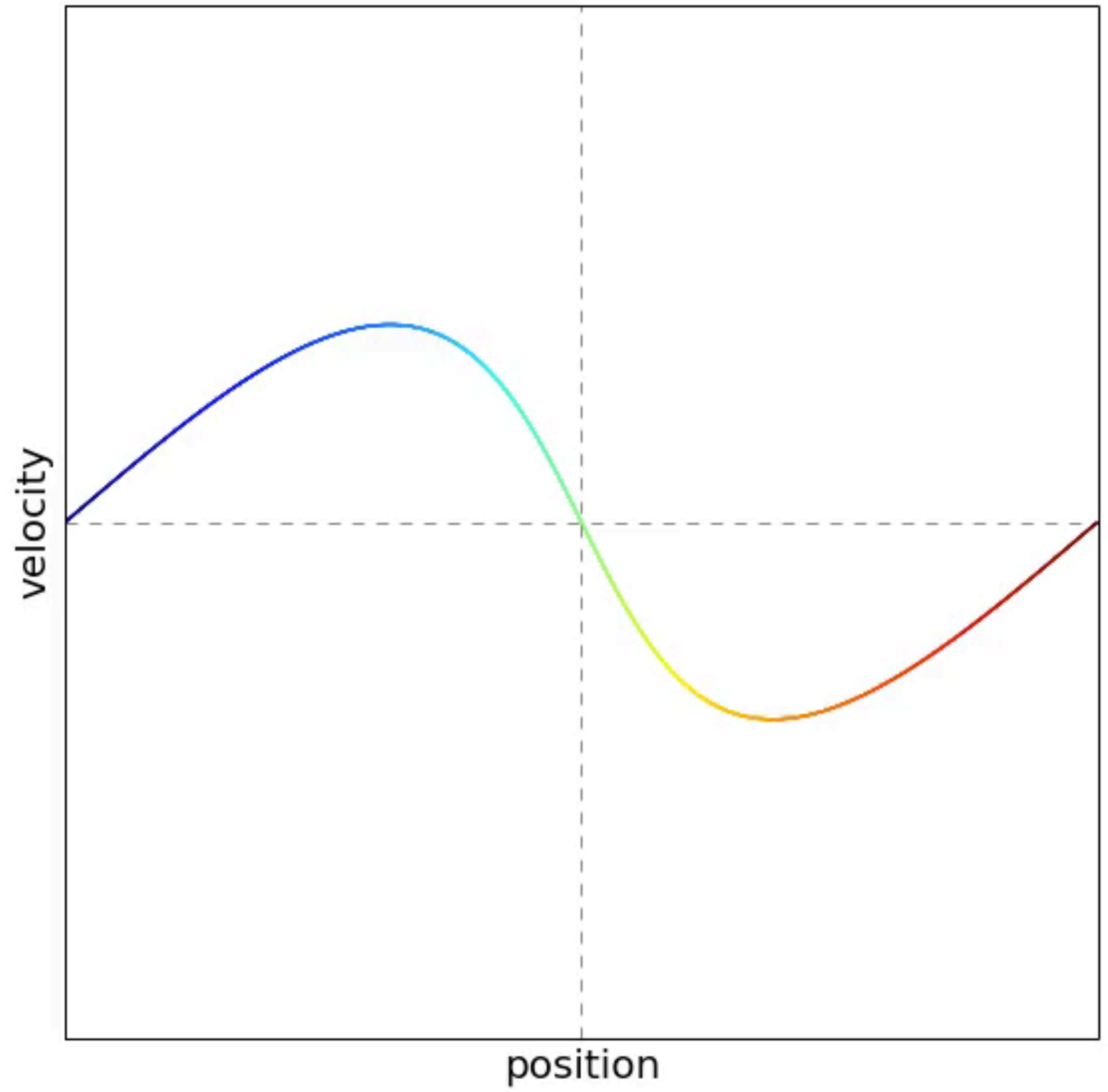

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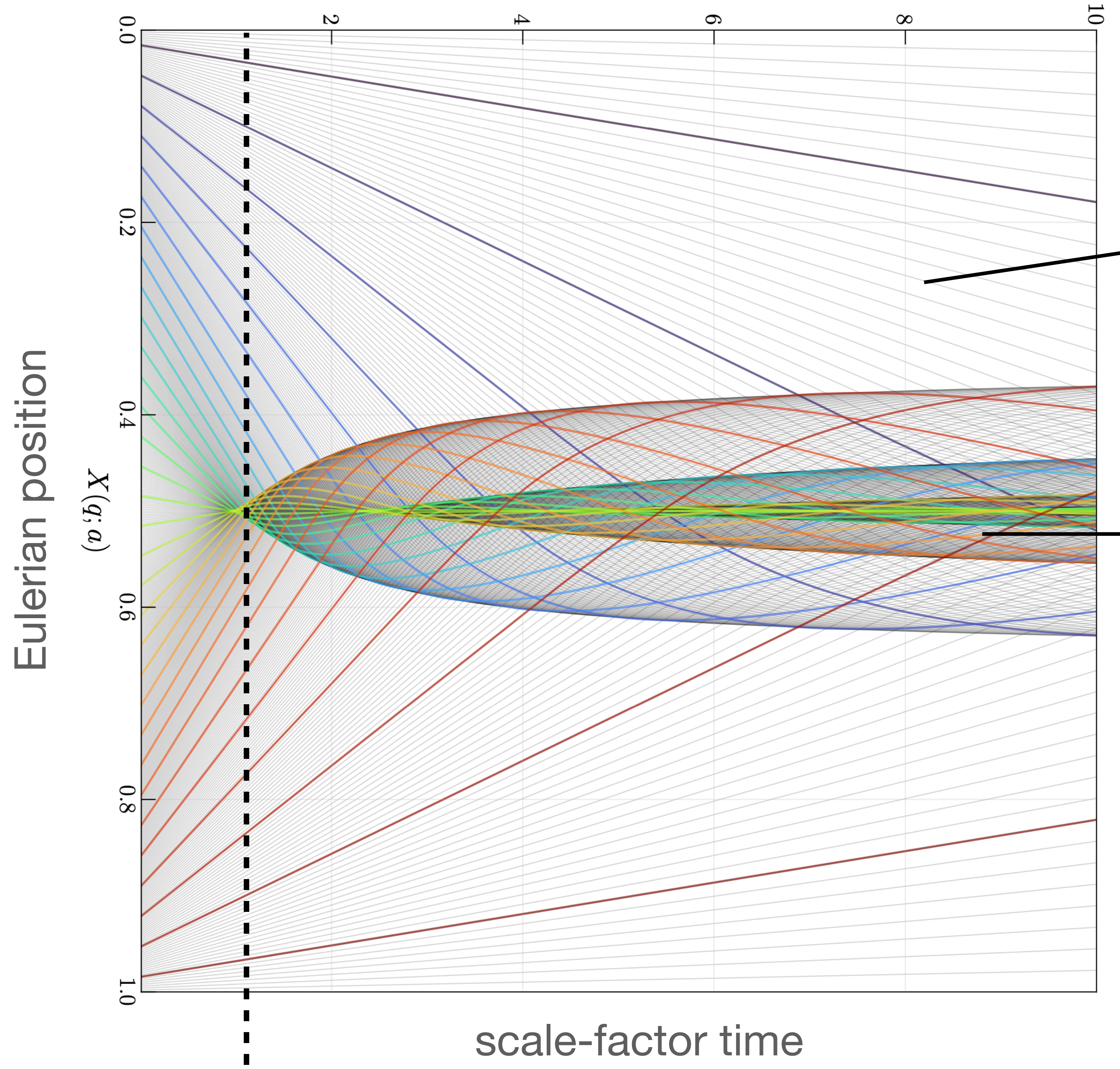

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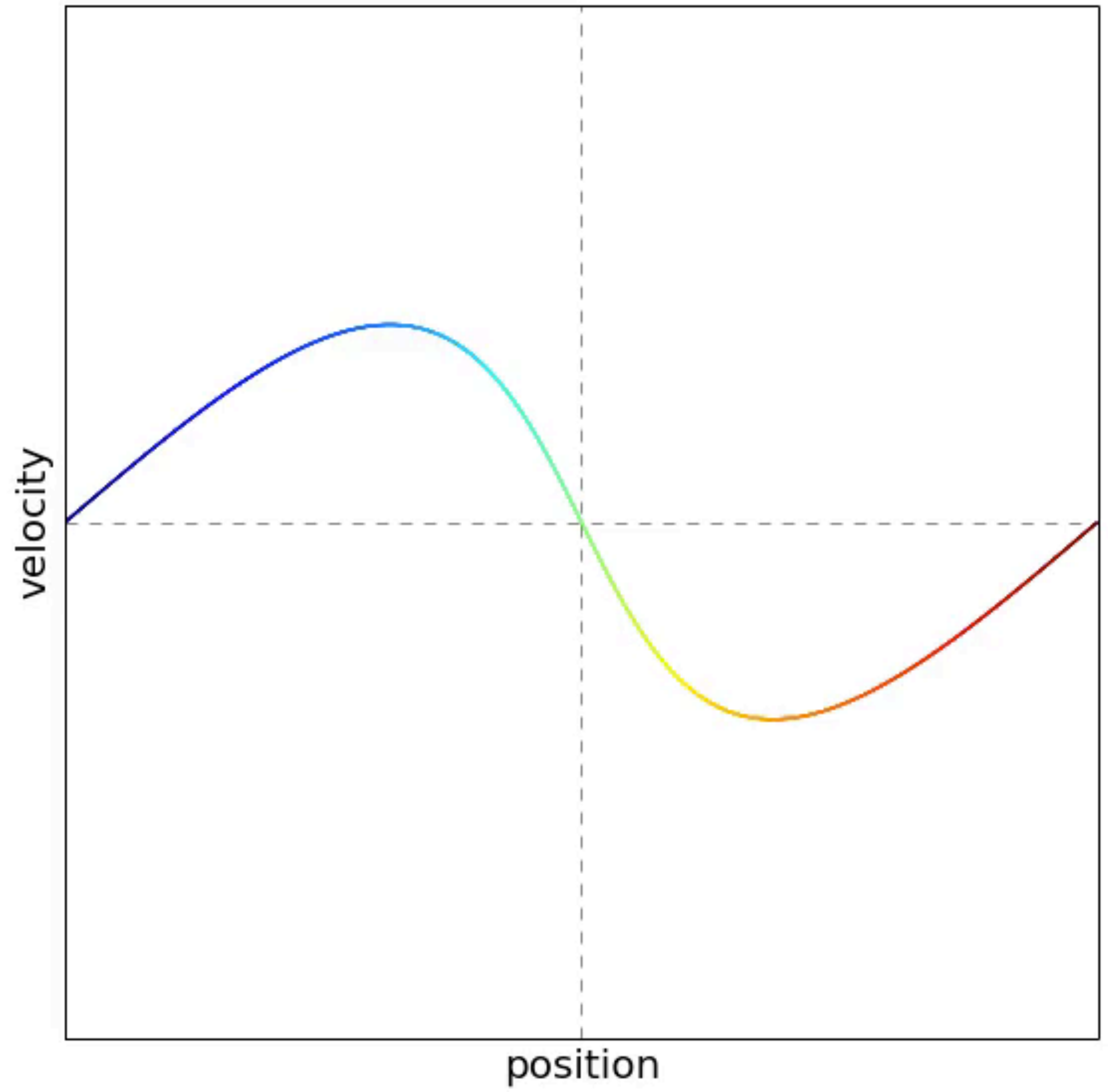
entering the multi-stream region is non-analytic (only finitely many bounded derivatives)



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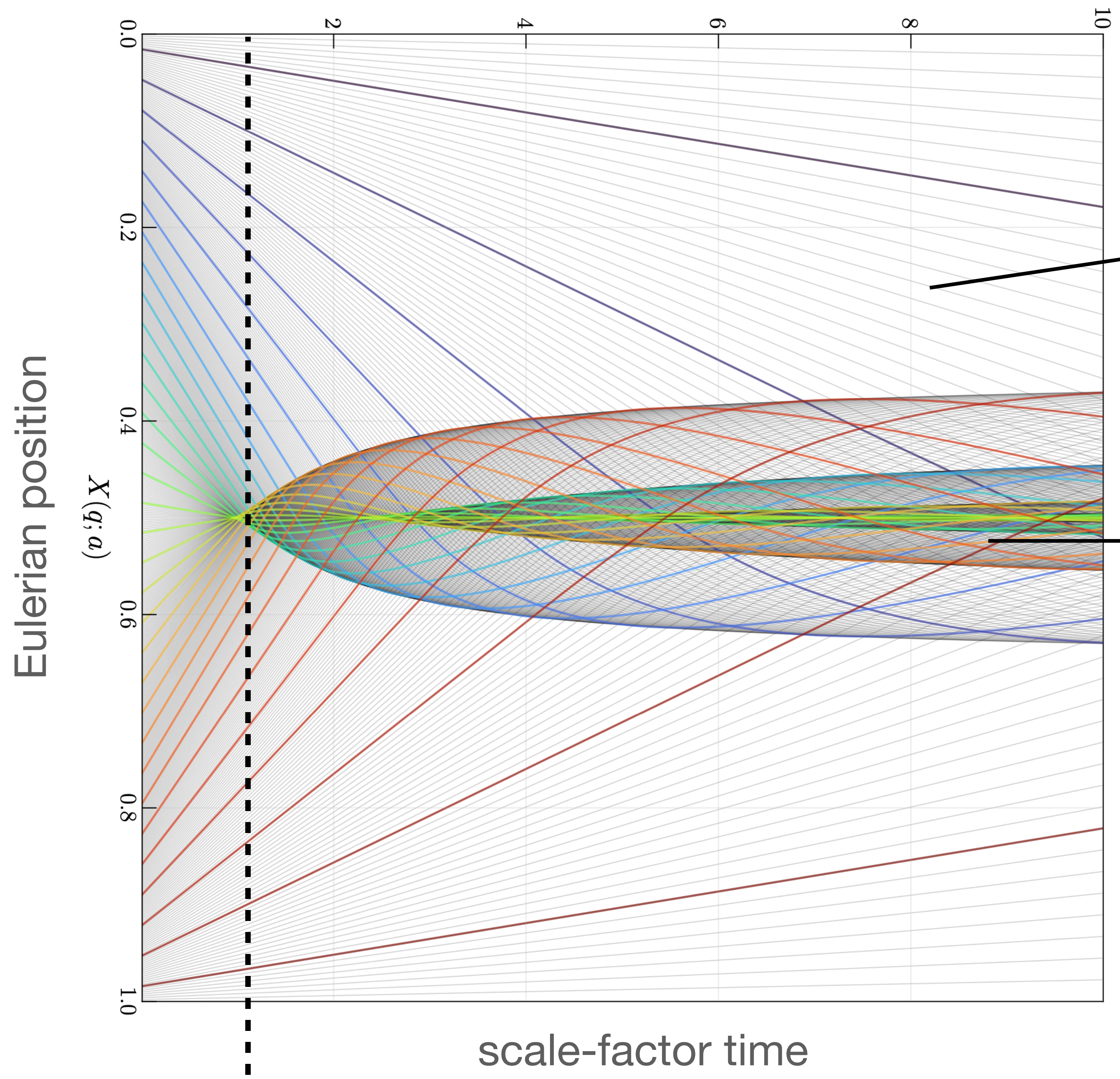

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Lagrangian Perturbation Theory
(for single fluid with cold initial data)

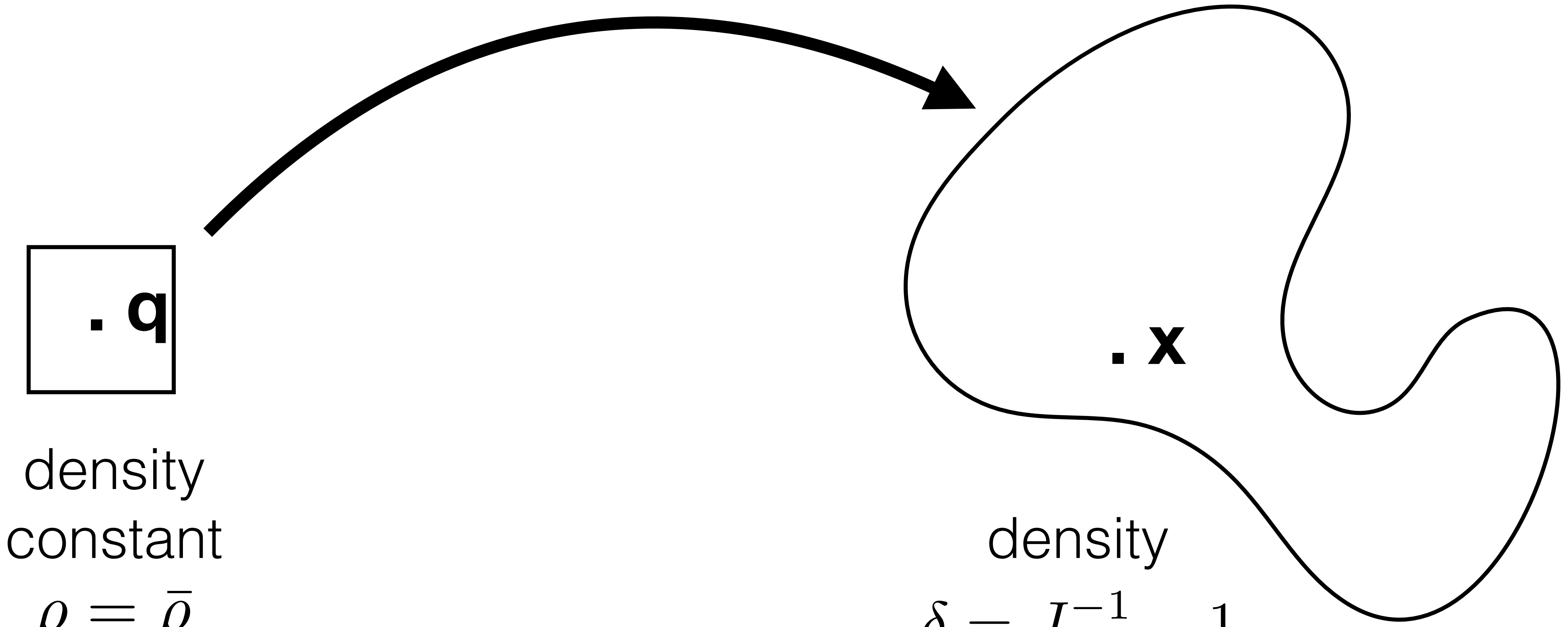
Lagrangian map

$$\boldsymbol{x}(\boldsymbol{q},t) = \boldsymbol{q} + \boldsymbol{\Psi}(\boldsymbol{q},t)$$

Overdensity given by Jacobian

$$\delta(\boldsymbol{x},t) = \frac{1}{J(\boldsymbol{q},t)} - 1$$

$$J := \det \, \frac{\partial \boldsymbol{x}}{\partial \boldsymbol{q}}$$



density
constant
$\rho = \bar{\rho}$

density
$\delta = J^{-1} - 1$

LPT eq: $$\mathrm{J}\,\left(\delta_{ij} + \Psi_{i,j}\right)^{-1}\left(\Psi''_{i,j} + \mathcal{H}\Psi'_{i,j}\right) = \frac{3}{2}\mathcal{H}^2\Omega_m(\mathrm{J}-1)$$

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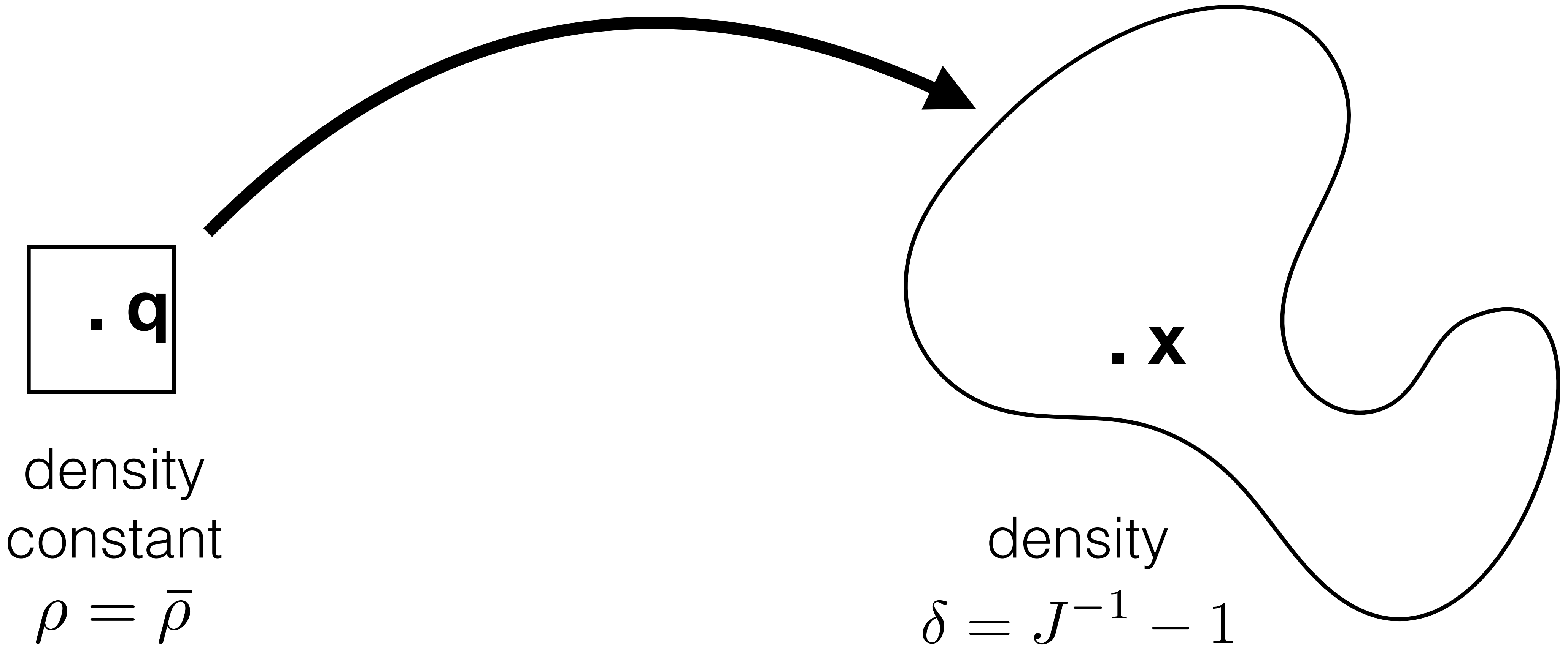

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We want to solve this as a perturbative series (D is small parameter)

$$\boldsymbol{\Psi}(\mathbf{q},\tau) = \sum_{n=1}^{\infty} D(\tau)^n\,\boldsymbol{\Psi}^{(n)}(\mathbf{q})$$

Buchert (1994), Catelan (1995), Bouchet+(1995), n=3
Rampf (2012), Zheligovsky&Frisch (2014), Matsubara (2015), all order

yields recursion relations to all orders.

For LCDM, actually $D^{(n)}(\tau) \neq D^n(\tau)$ **,** see Rampf, Schobesberger & OH(2022), Fasiello, Fujita, Vlah (2022)

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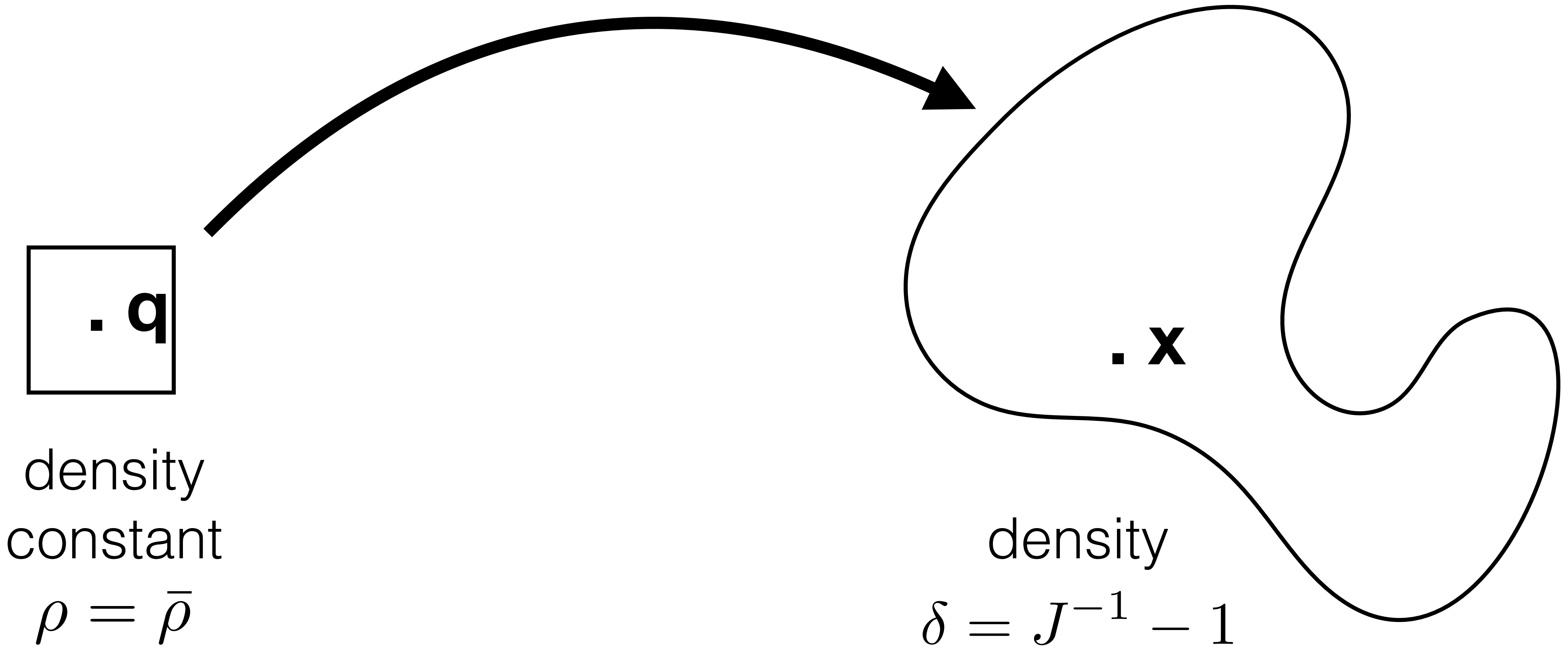

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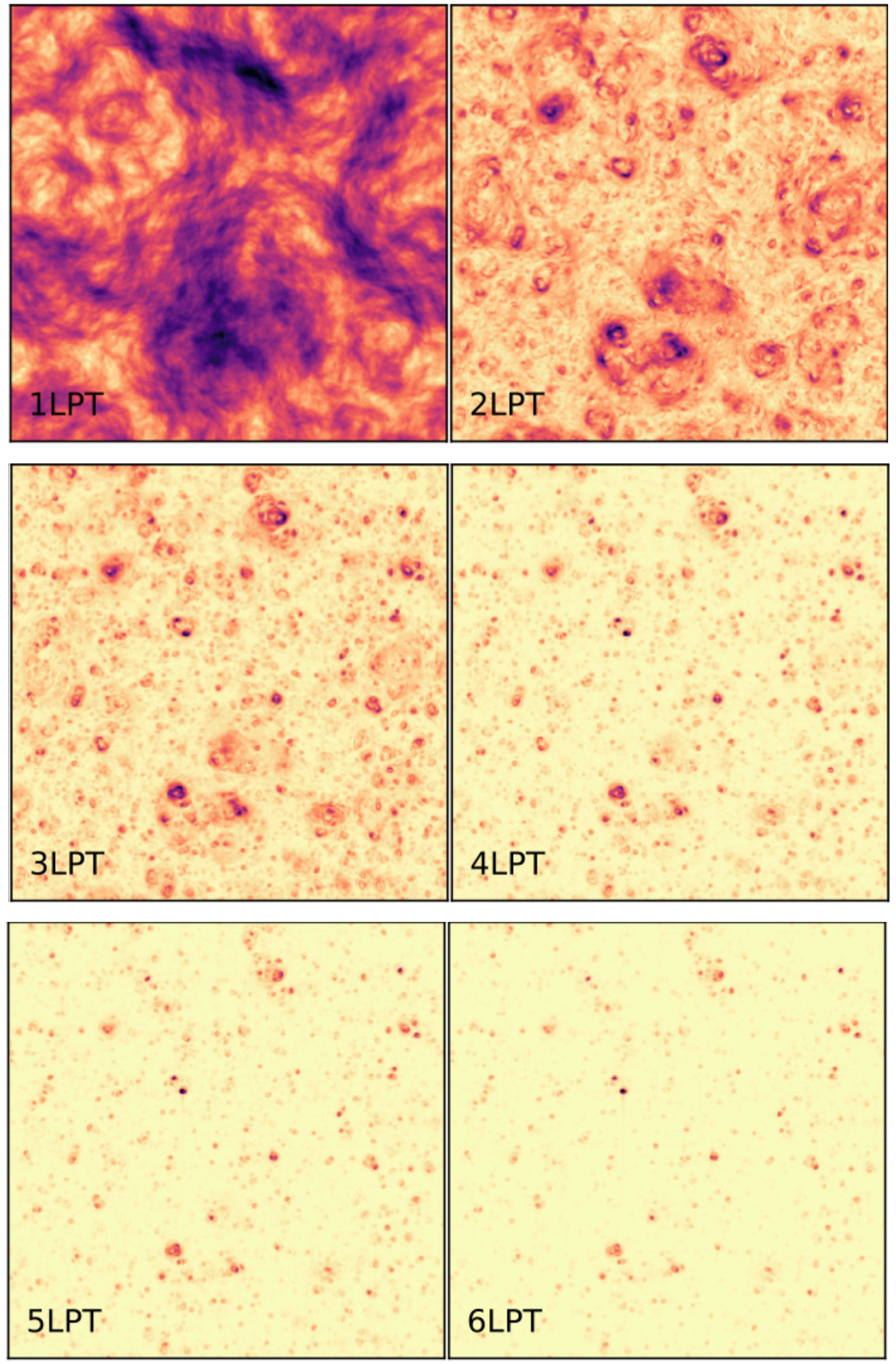

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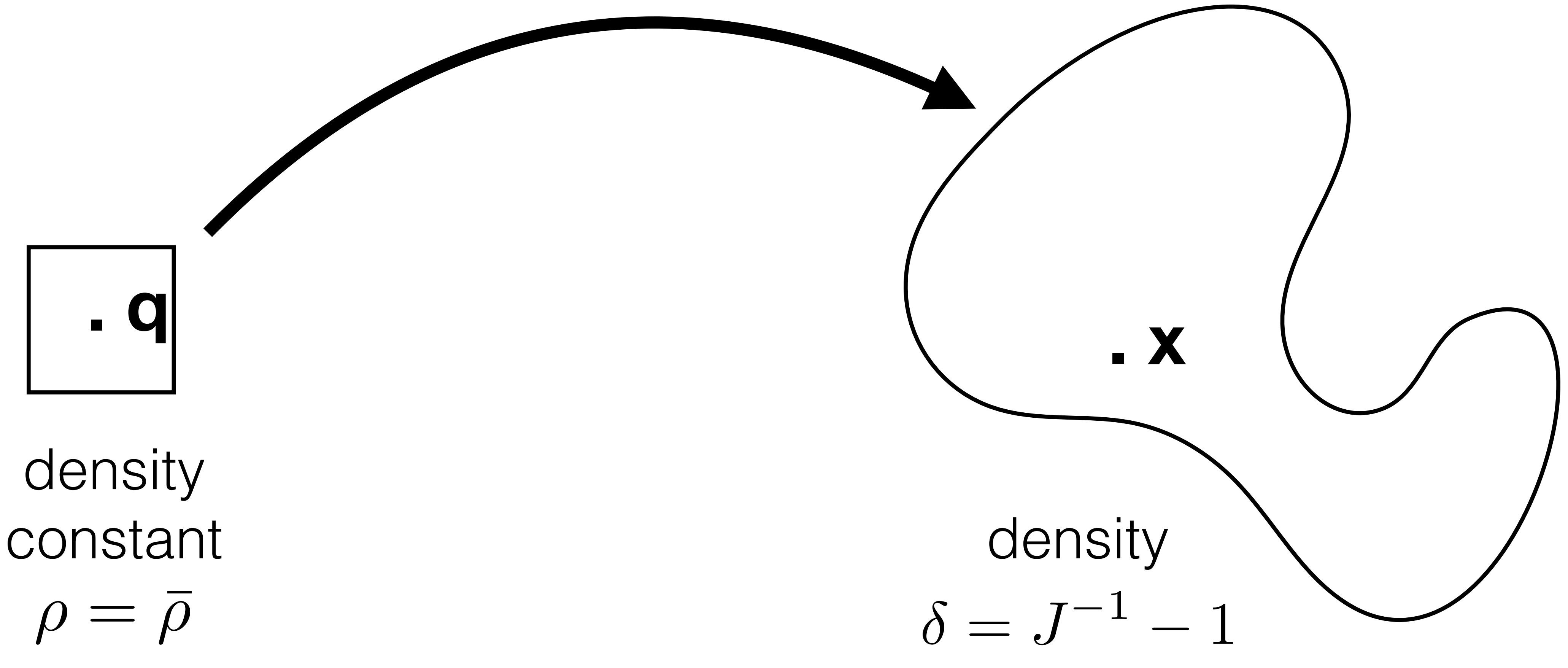

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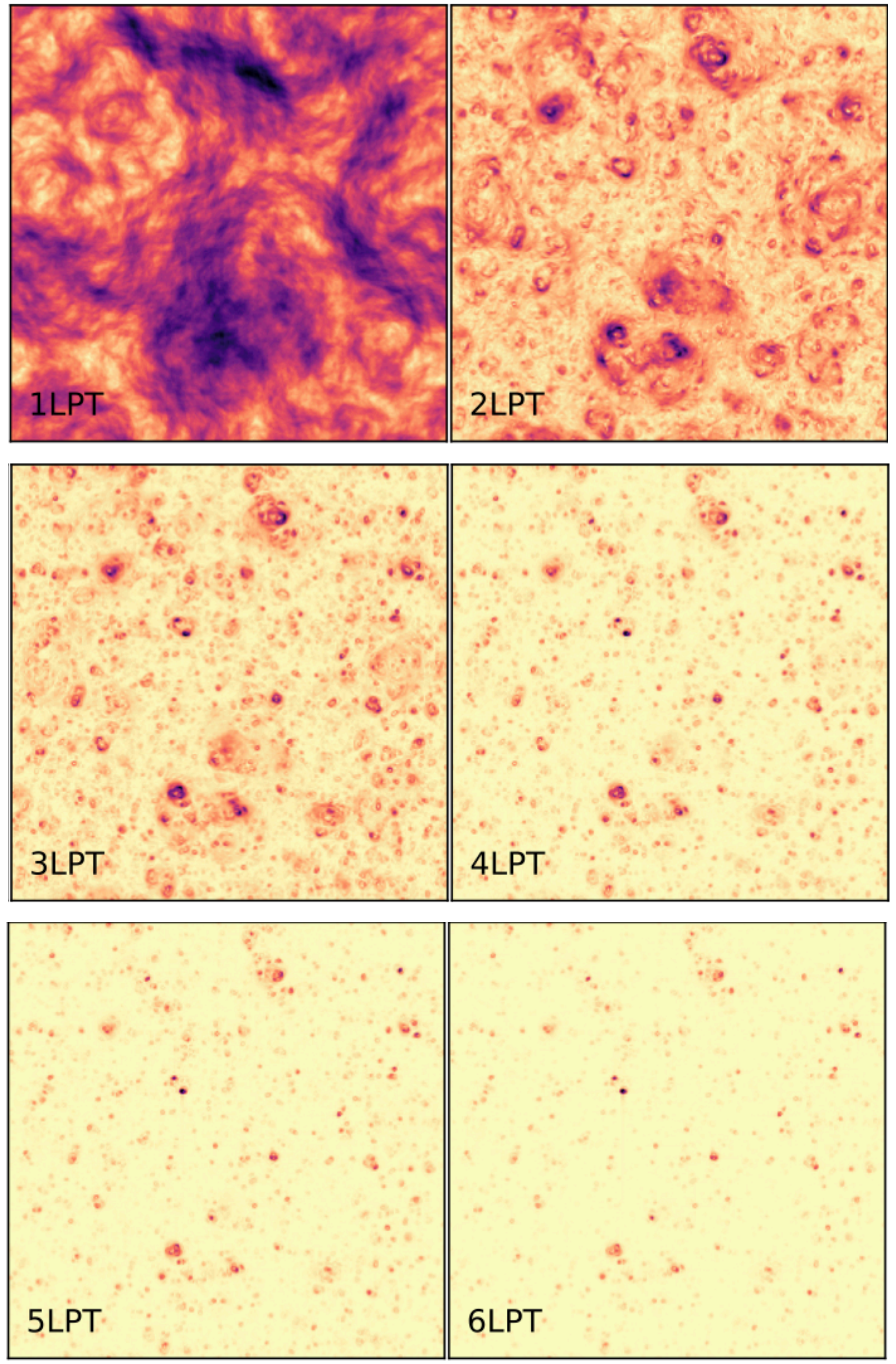
What is the range of applicability of such a theory (aka the ‘radius of convergence’)?



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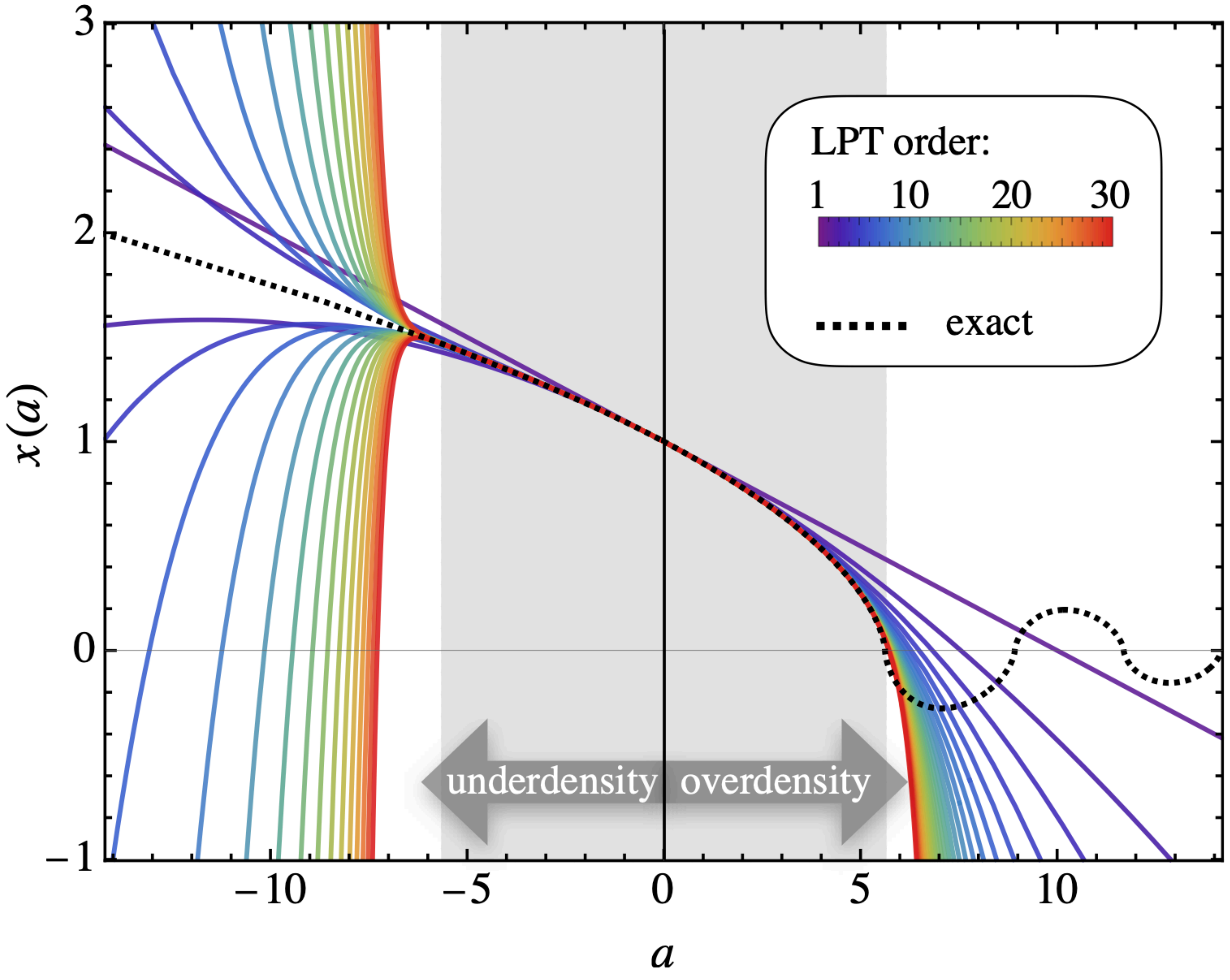
The LPT radius of convergence

worst case scenario: spherical collapse

Rampf&Hahn 23, see also Sahni&Shandarin (1996), Karakatsanis+(1997), Nadkarni-Ghosh&Chernoff (2011)



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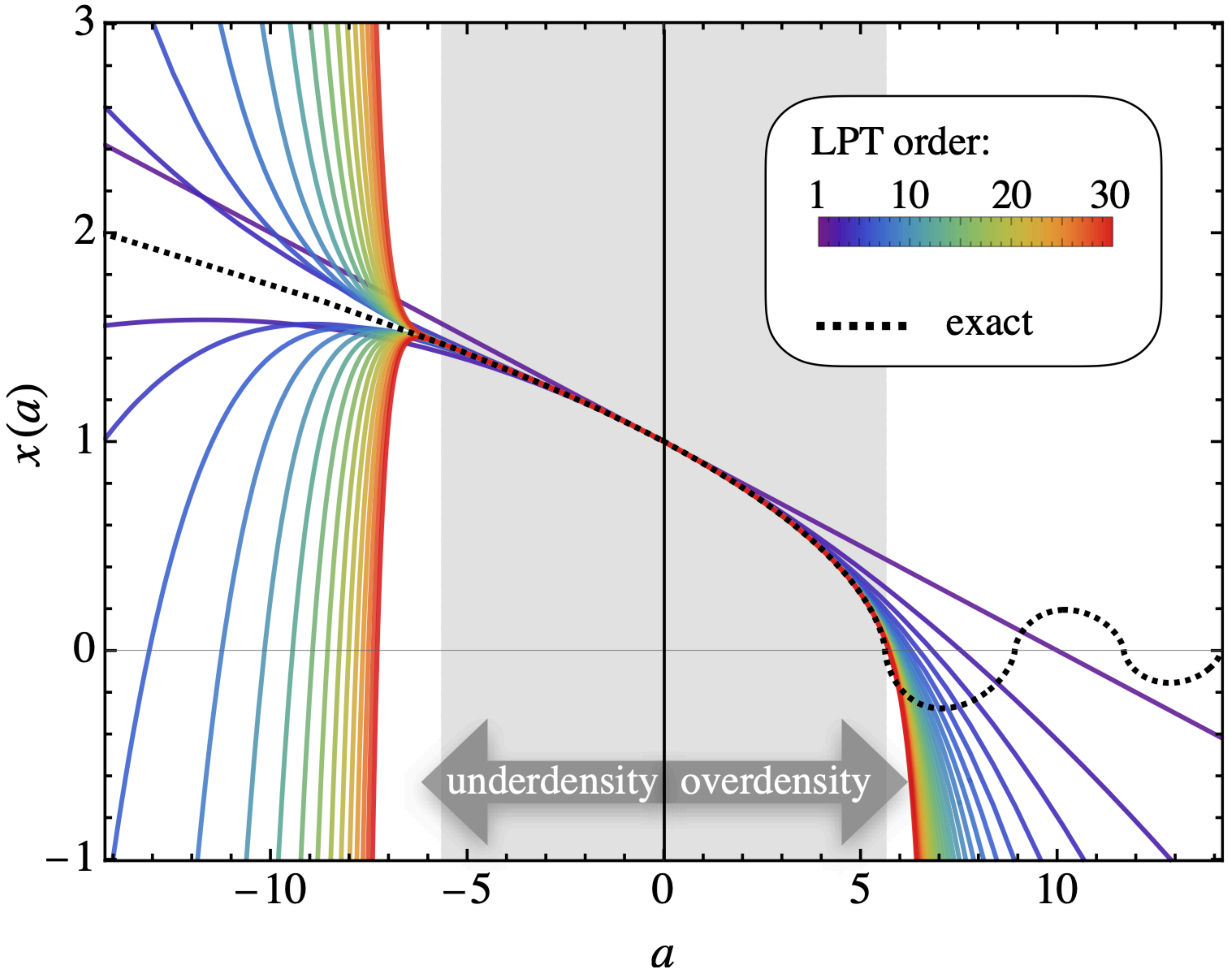

Spherical collapse has a physical singularity, acceleration is infinite

This leads to slow convergence of LPT, also no transition to “bound states”

numerous ad hoc UV completions in the literature
e.g. ALPT (Kitaura&Hess 2013),
MUSCLE (Neyrinck 2016),
PINOCCHIO (Monaco 2002,2013)

Problems less severe when collapse not exactly spherical, but PT still limited by shell-crossing.
—> EFT approaches (Baumann+2009, Carrasco+2010, …)
BUT: generally poor field-level convergence of SPT
—> Rampf, Frisch & Hahn 2022 (‘Eye of the Tyger’)

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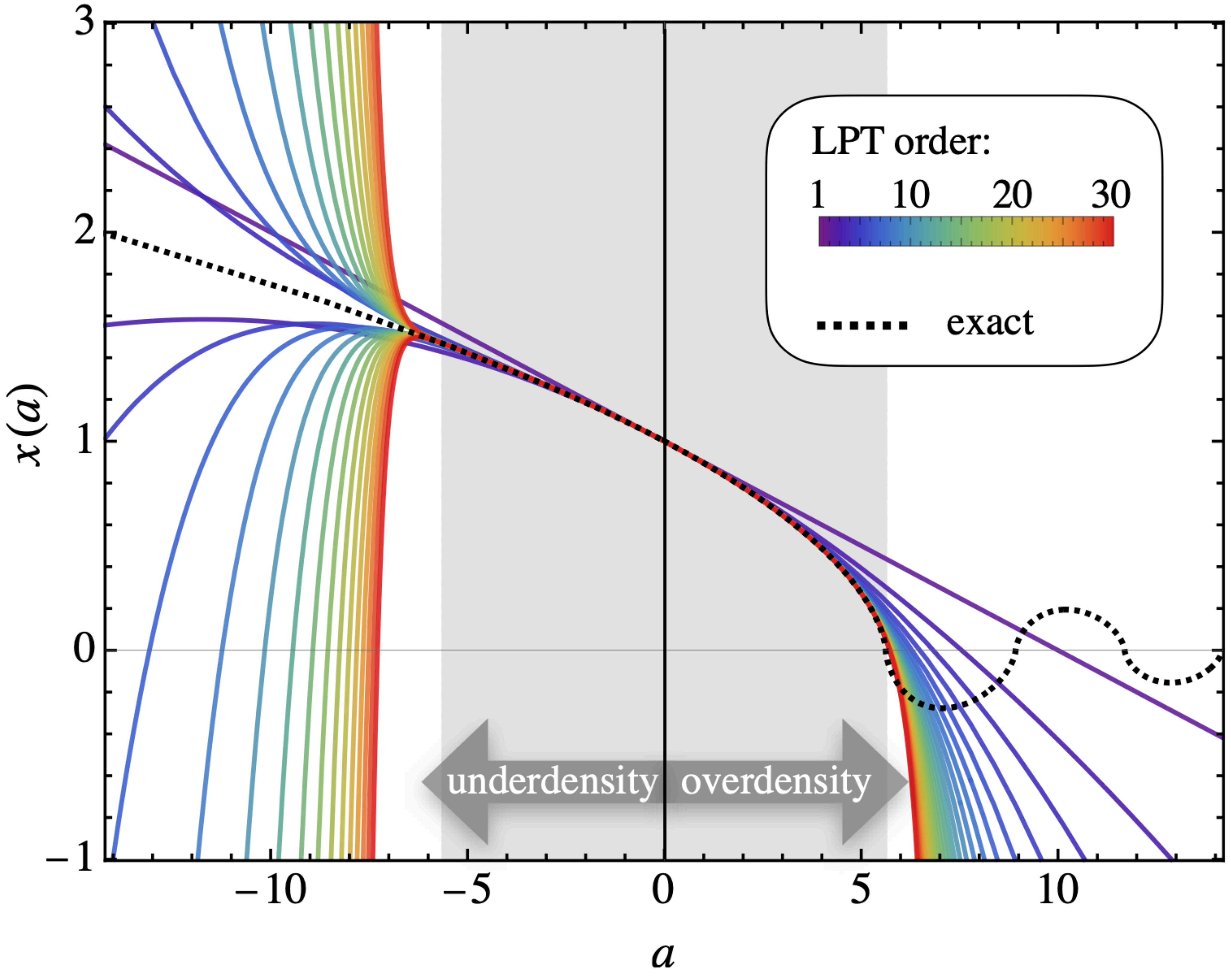

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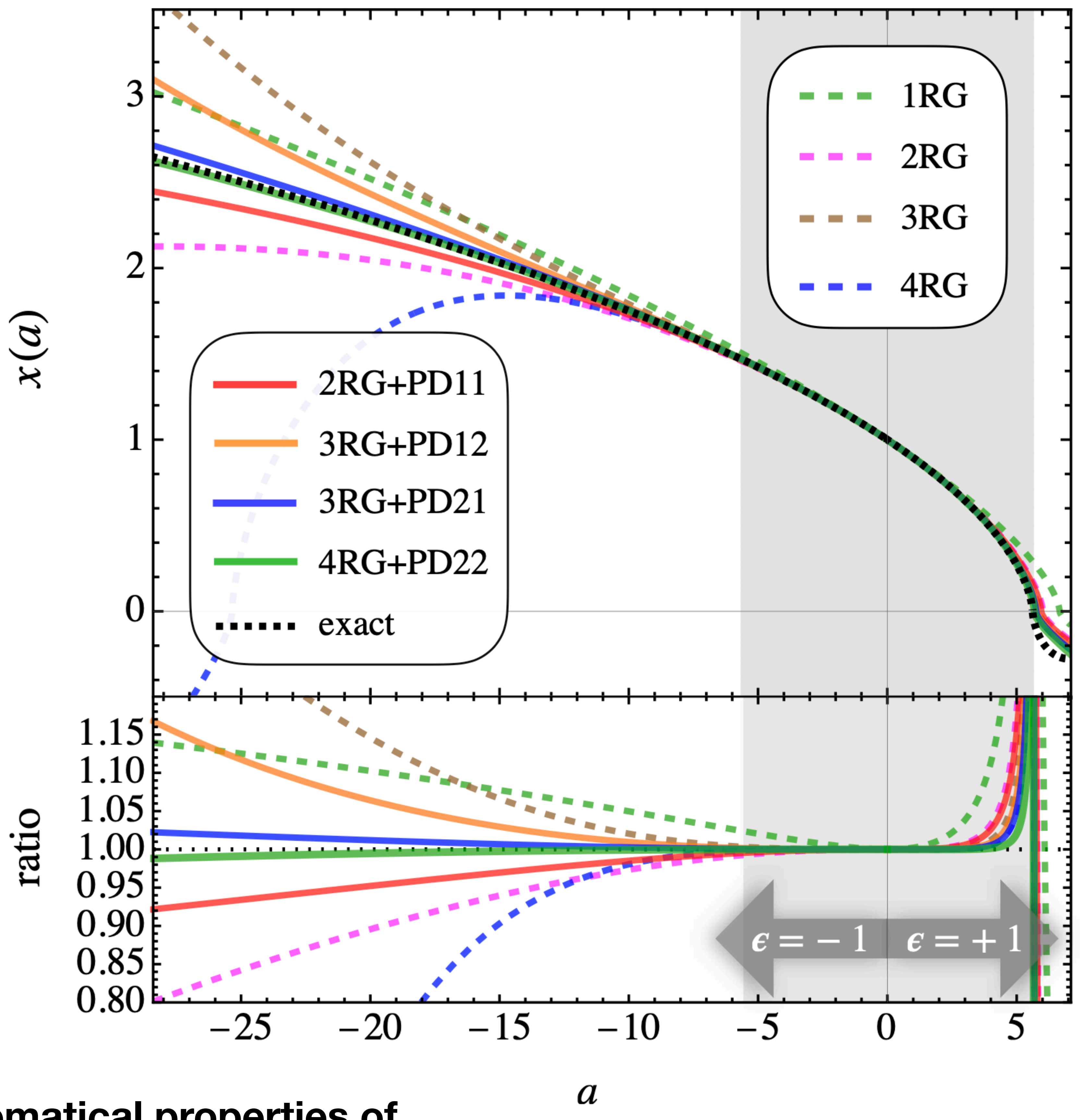

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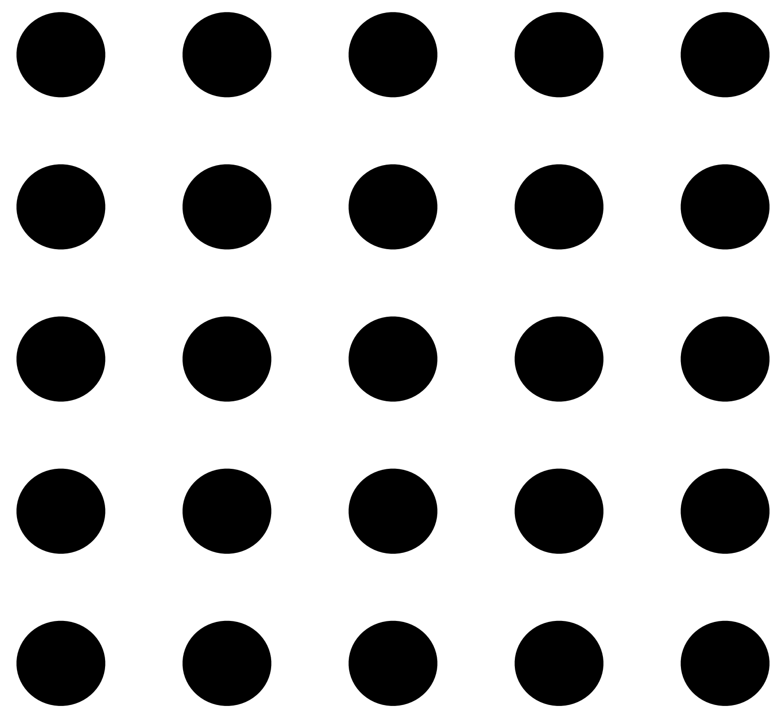
Singularity has mathematical properties of criticality / a phase transition (exponent 2/3) can be tamed using renormalisation group (we know how to in spherical 1D), or Cornelius Rampf's UV completion technique but unclear how to go to 3D GRF

Using N-body to go further:

use finite order, discretised LPT to set up an N-body simulation…

Setting up an N-body simulation

**(globally) isotropic
and homogeneous state**



system in (unstable) equilibrium,
velocities zero

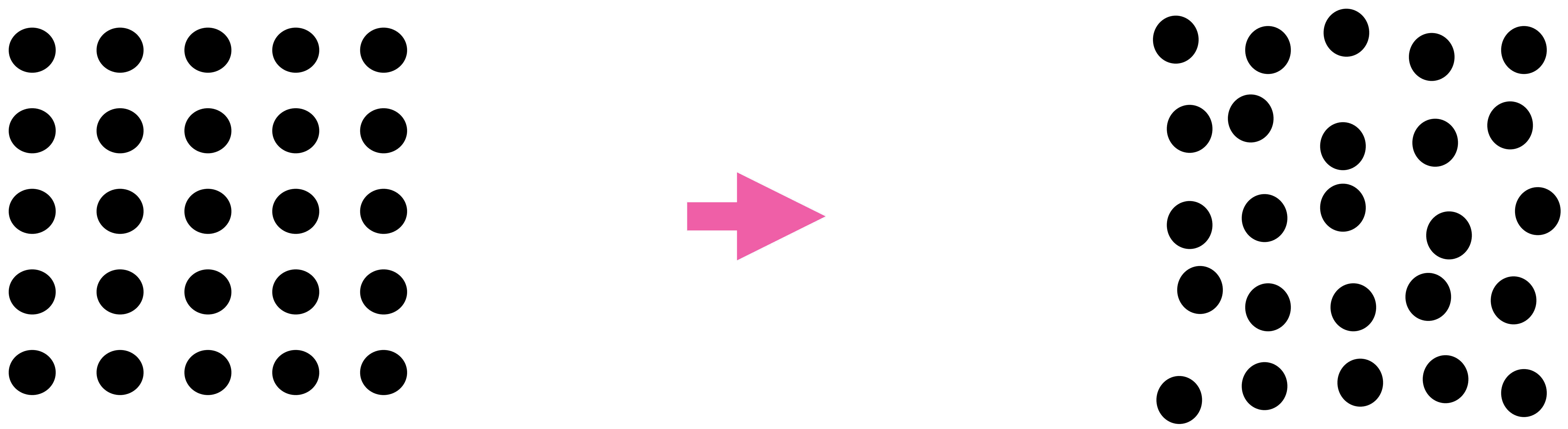
*symmetry always broken at
particle scale* -> *"discreteness"*

but global symmetry "ok" e.g. for:
- Bravais lattices
- glasses
- special tilings

Setting up an N-body simulation

(globally) isotropic and homogeneous state

perturbed state



system in (unstable) equilibrium,
velocities zero

system out of equilibrium,
positions and velocities set to
reproduce linear/non-linear modes

symmetry always broken at particle scale -> "discreteness"

but global symmetry "ok" e.g. for:

- Bravais lattices
- glasses
- special tilings

variant 1: SPT -> first order method:

$$\vec{\psi}_{\rm ini} = \vec{\nabla}\,\nabla^{-2}\delta_{\rm ini}$$

$$\dot{\vec{\psi}}_{\rm ini} = \vec{\nabla}\,\nabla^{-2}\theta_{\rm ini}$$

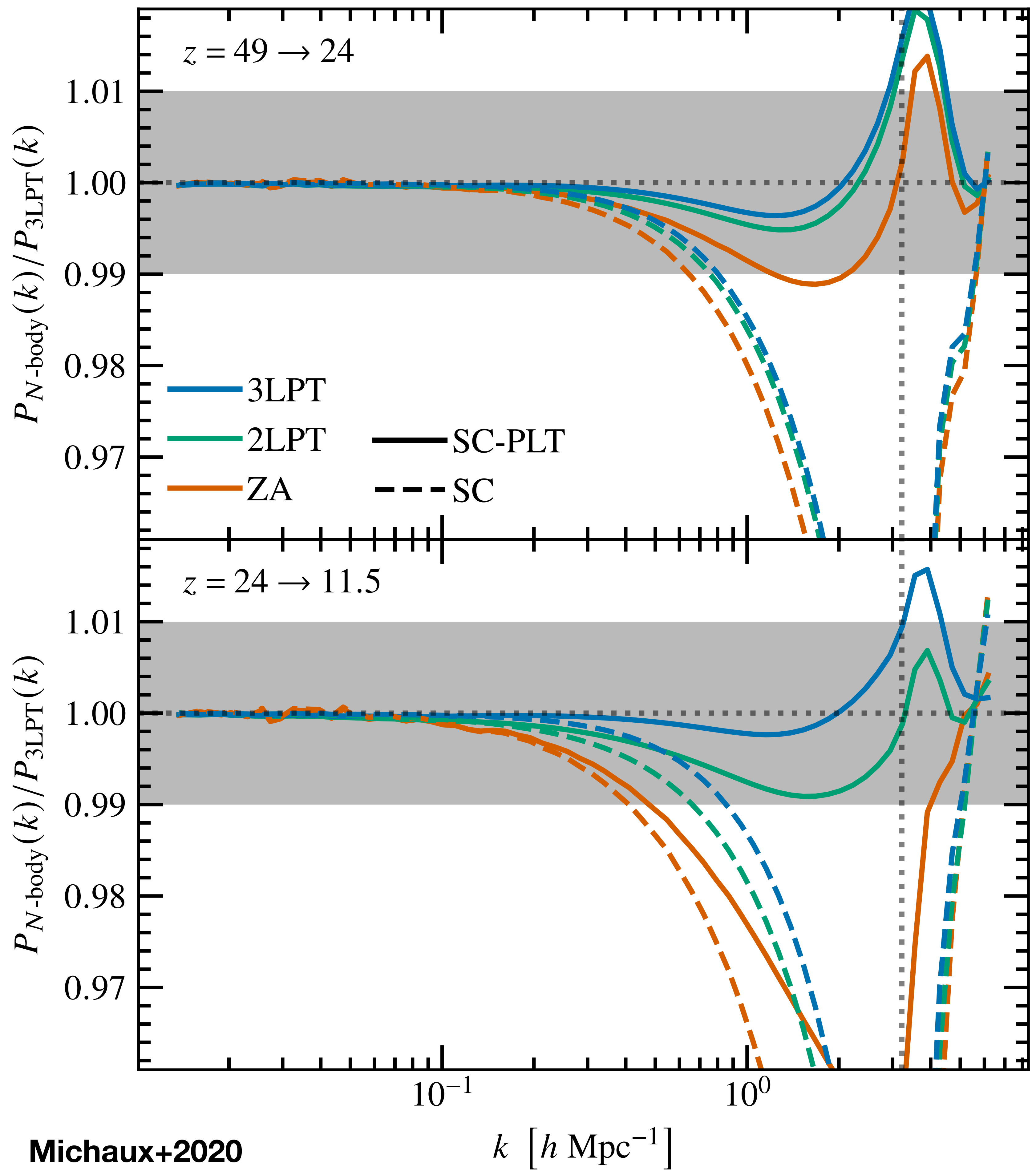
variant 2: LPT (nonlinear)

$$\vec{\psi}_{\rm ini} = \sum_{i=1}^{n} D_+^{(n)}\,\vec{\psi}^{(n)}$$

Agreement between N-body and LPT

Comparison of weakly evolved power spectra:

How is this possible? They model the same discrete set of modes



Michaux+2020

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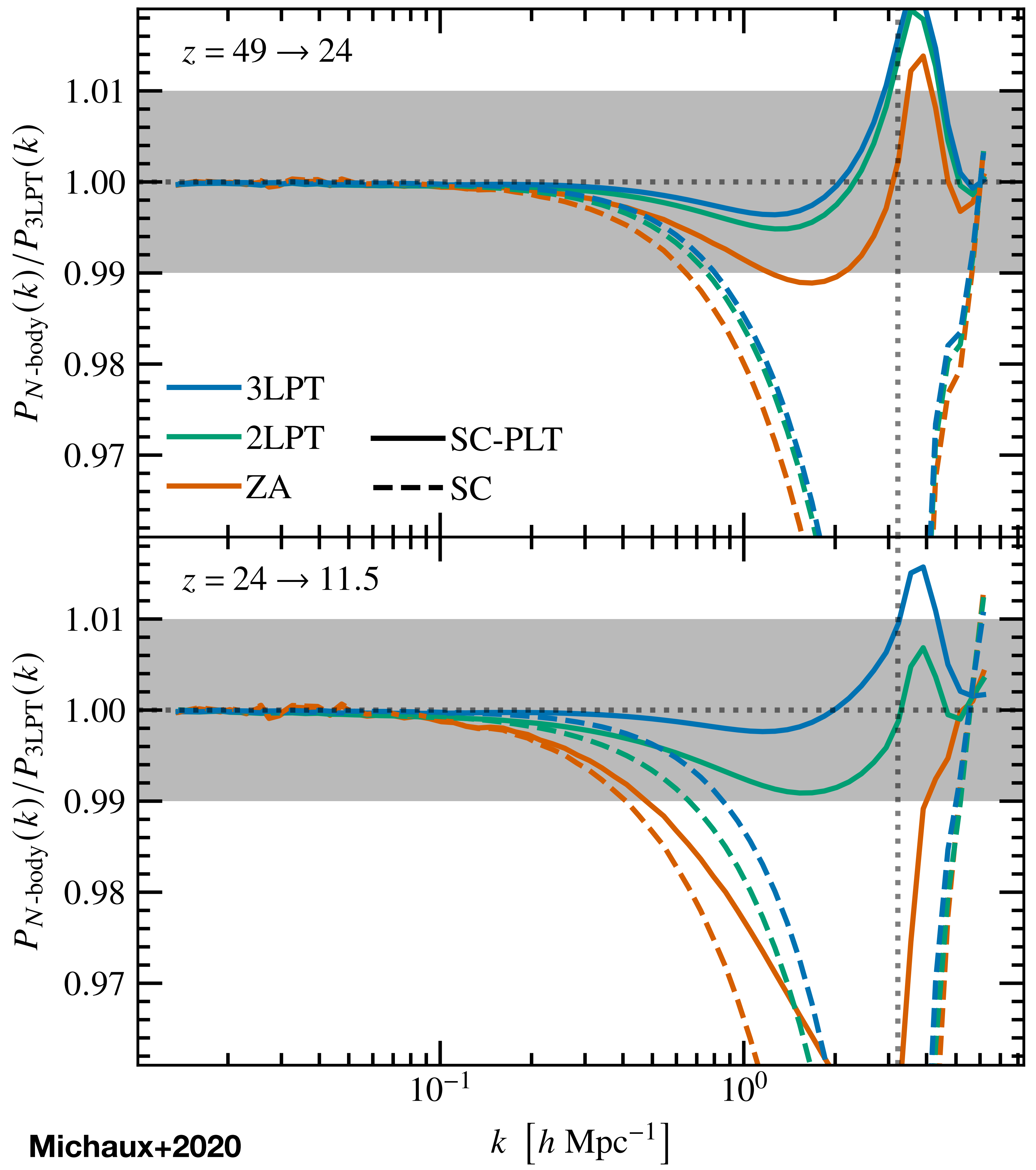

Michaux+2020

How is this possible? They model the same discrete set of modes

Two sources of error:

1) the nLPT truncation error (Scoccimarro 1998, Crocce+2006) aka ‘transients’

Agreement between N-body and LPT

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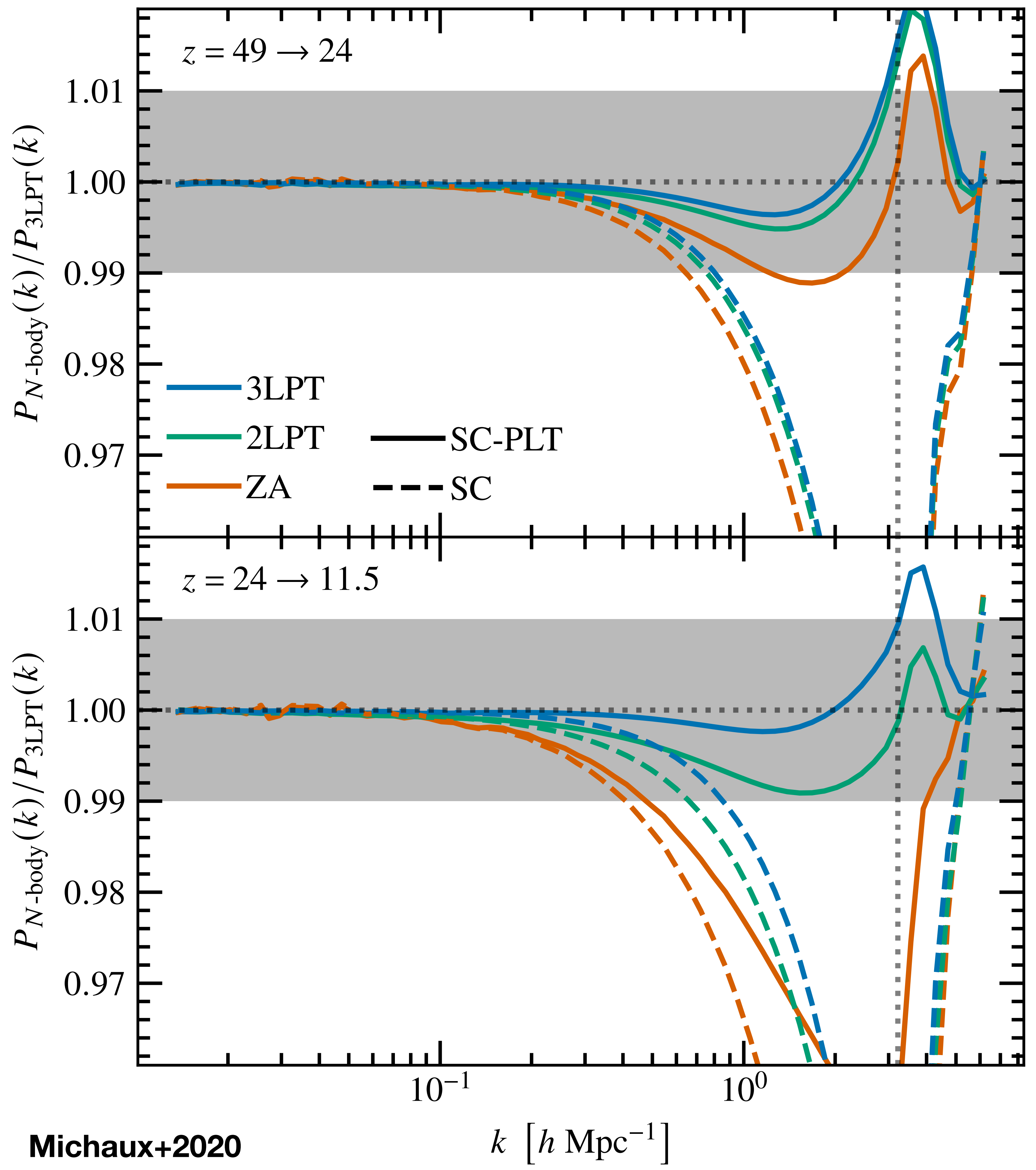
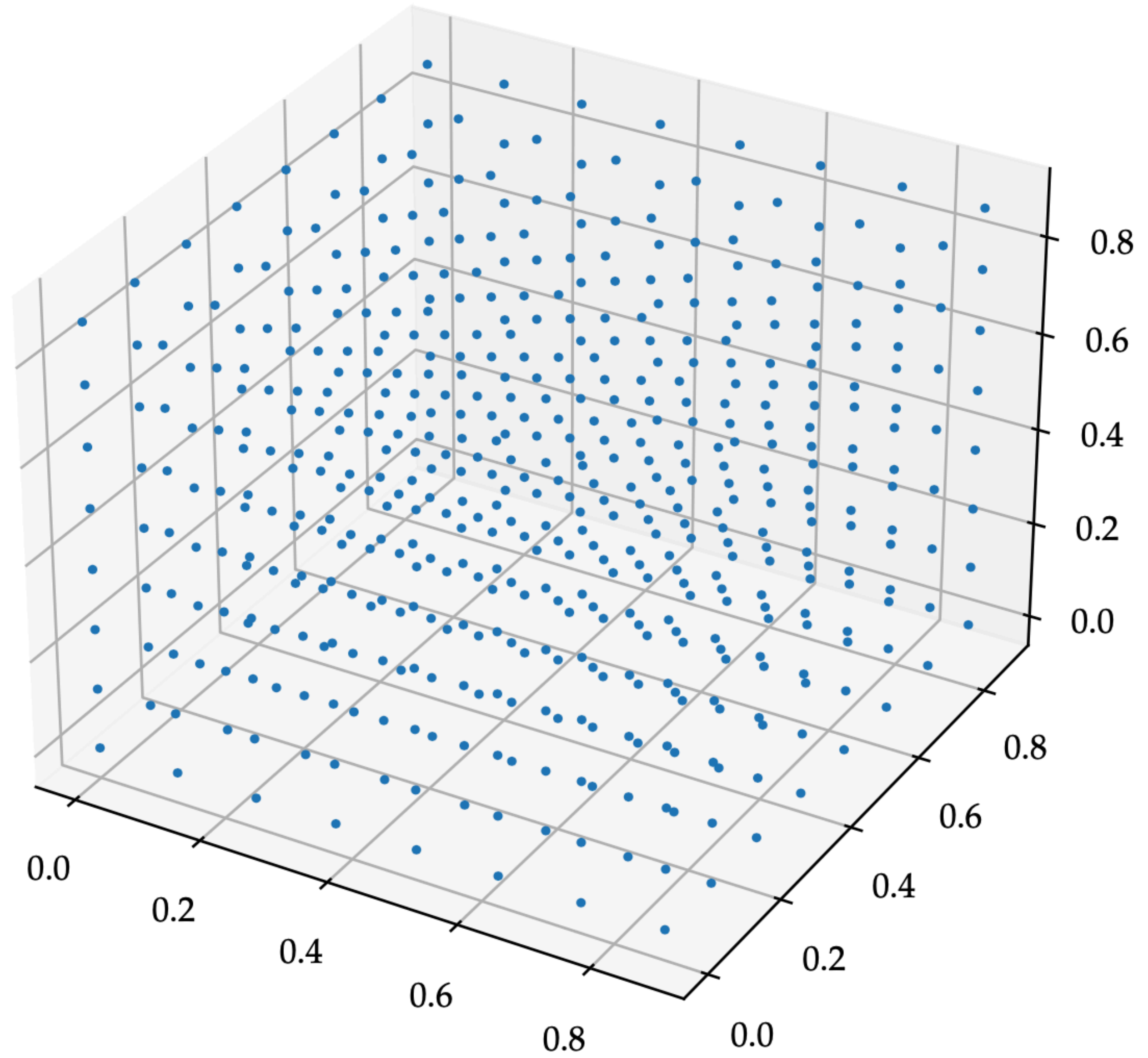


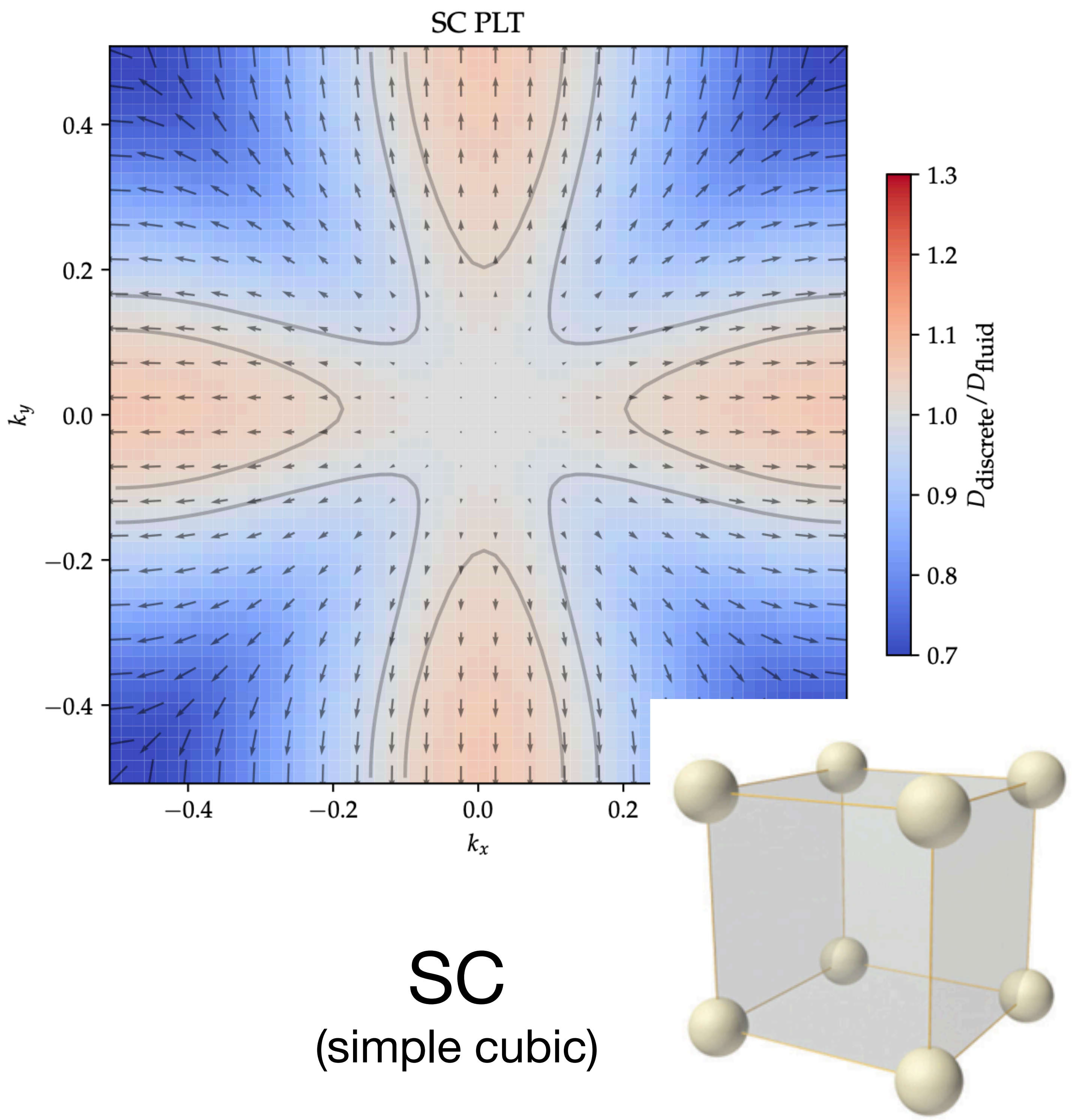
Michaux+2020

How is this possible? They model the same discrete set of modes

Two sources of error:

1) the nLPT truncation error (Scoccimarro 1998, Crocce+2006) aka ‘transients’
2) the N-body discreteness (and force) errors:





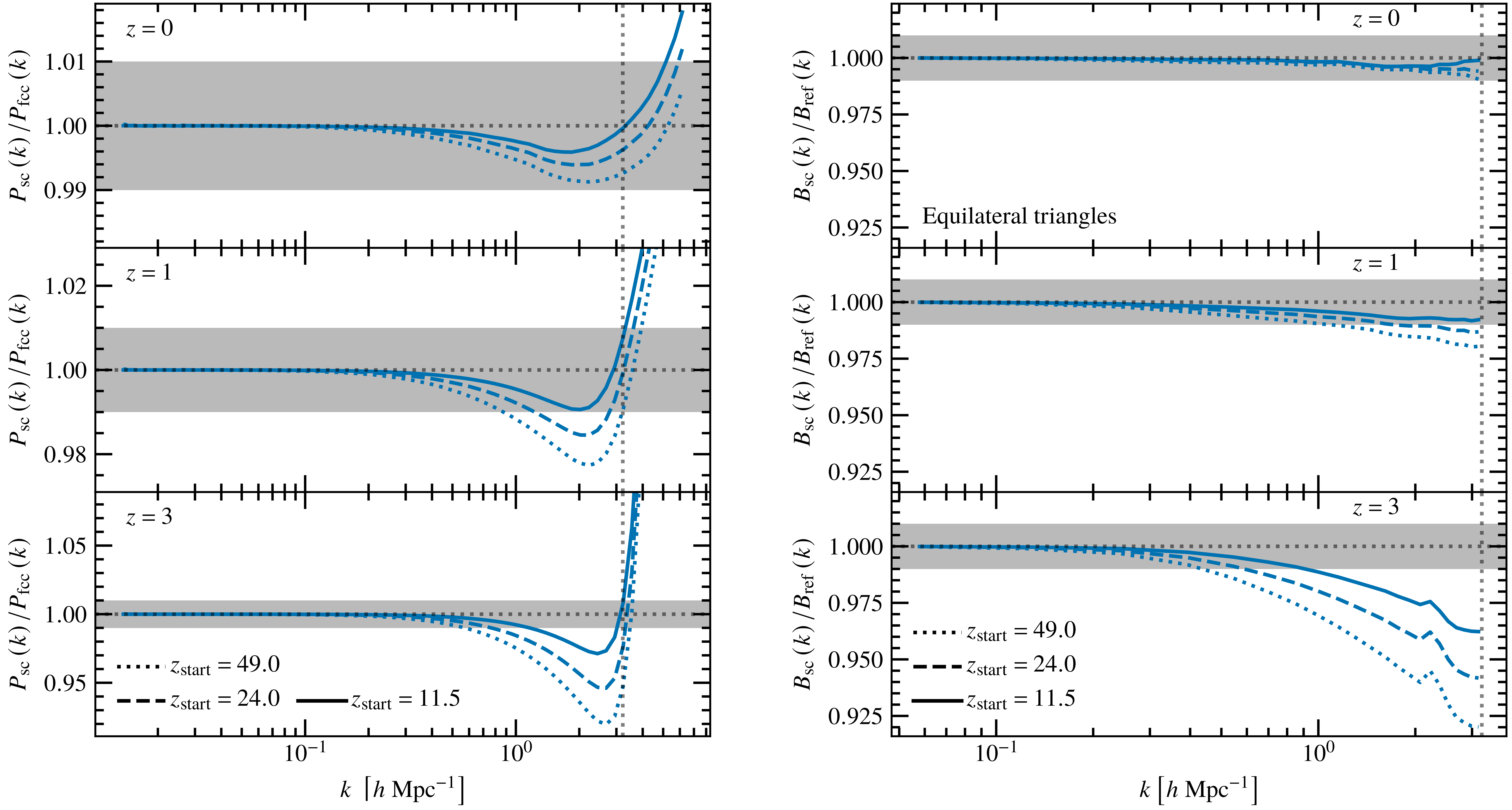
cf. Joyce+2005, Joyce&Marcos 2007, Marcos 2008, but also Garrison+2016

SC
(simple cubic)

Discreteness — impact on low-z power spectrum

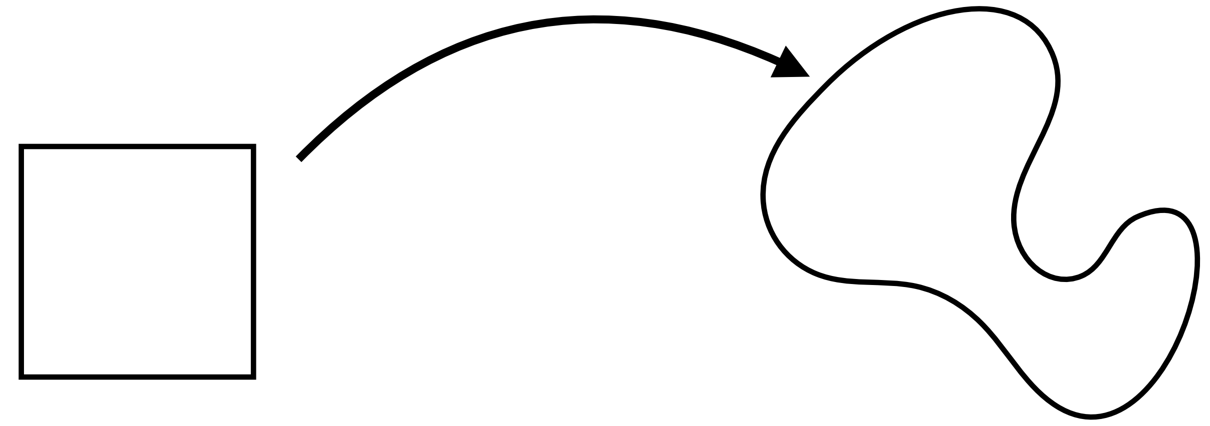
effect on power spectrum at z=0 wiped out by non-linearity (scale-mixing, asymptotic halo profiles), but not at higher z

Michaux+2020



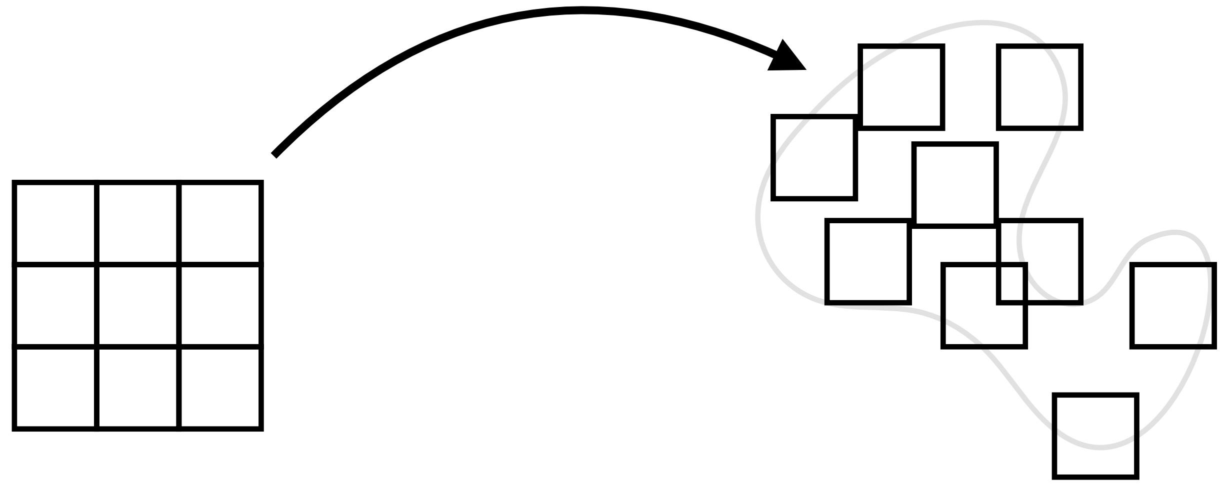
fluid-limit

$\mathbb{Q} \subset \mathbb{R}^3 \to \mathbb{R}^6\ :\ \mathbf{q} \mapsto (\mathbf{x_q}(t), \mathbf{v_q}(t))$



discrete case

$i \in \{1 \ldots N\} \mapsto (\mathbf{x}_i, \mathbf{v}_i)$



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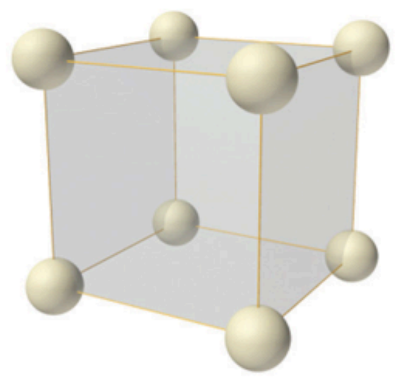
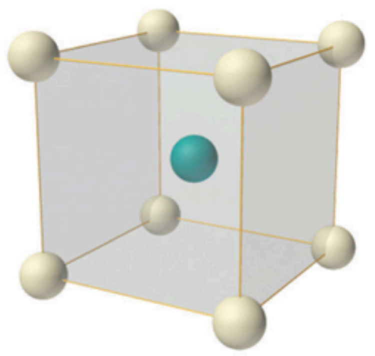
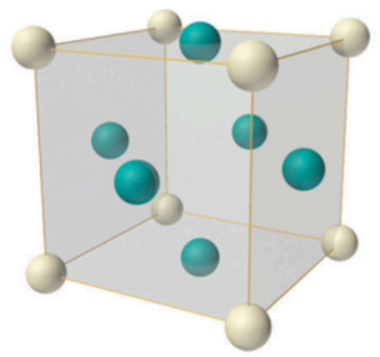


BCC
(body-centred cubic)

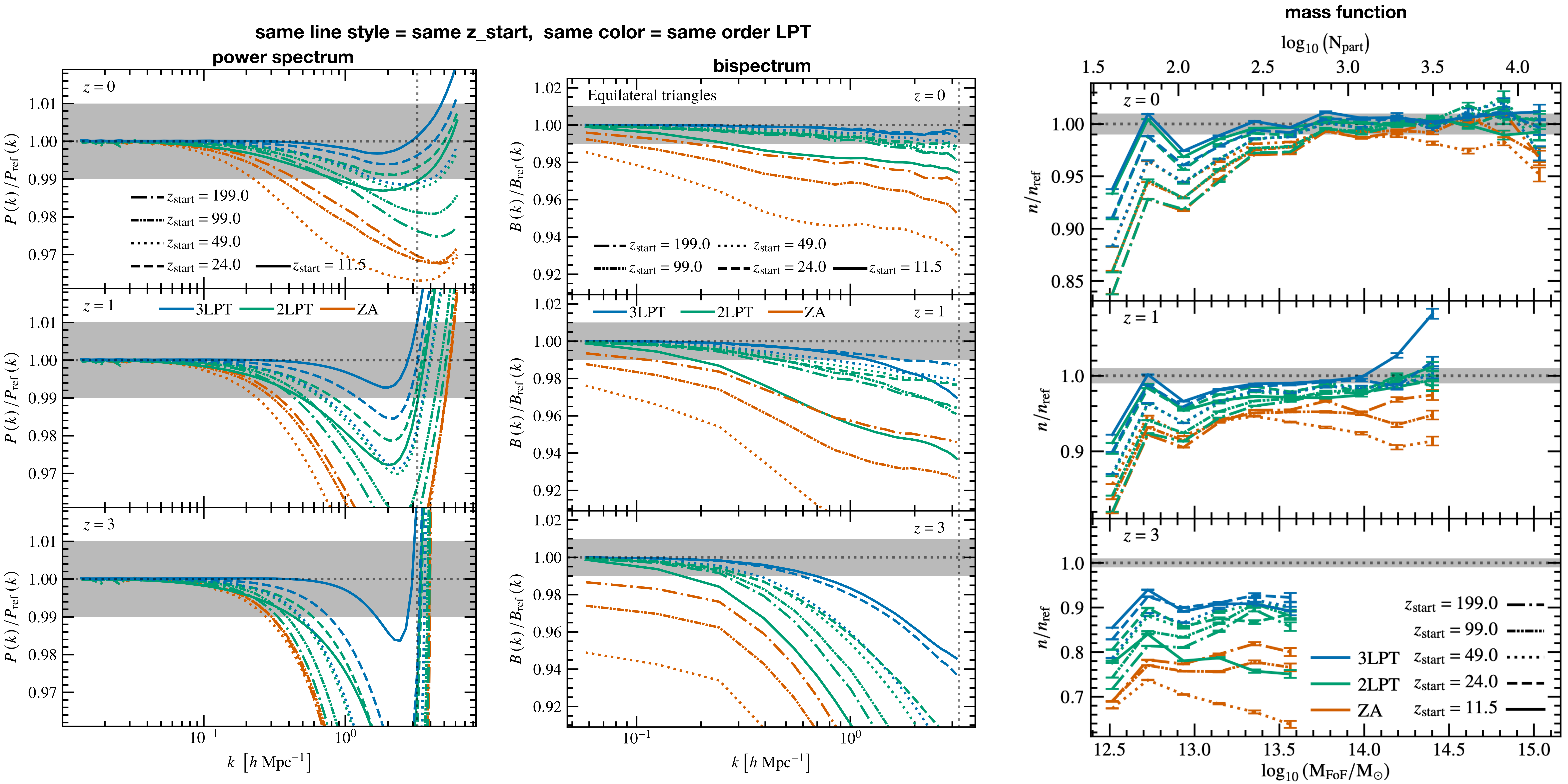


ref = FCC
(face-centred cubic)



(cf. also Marcos 2008)

Discreteness — impact on summary statistics



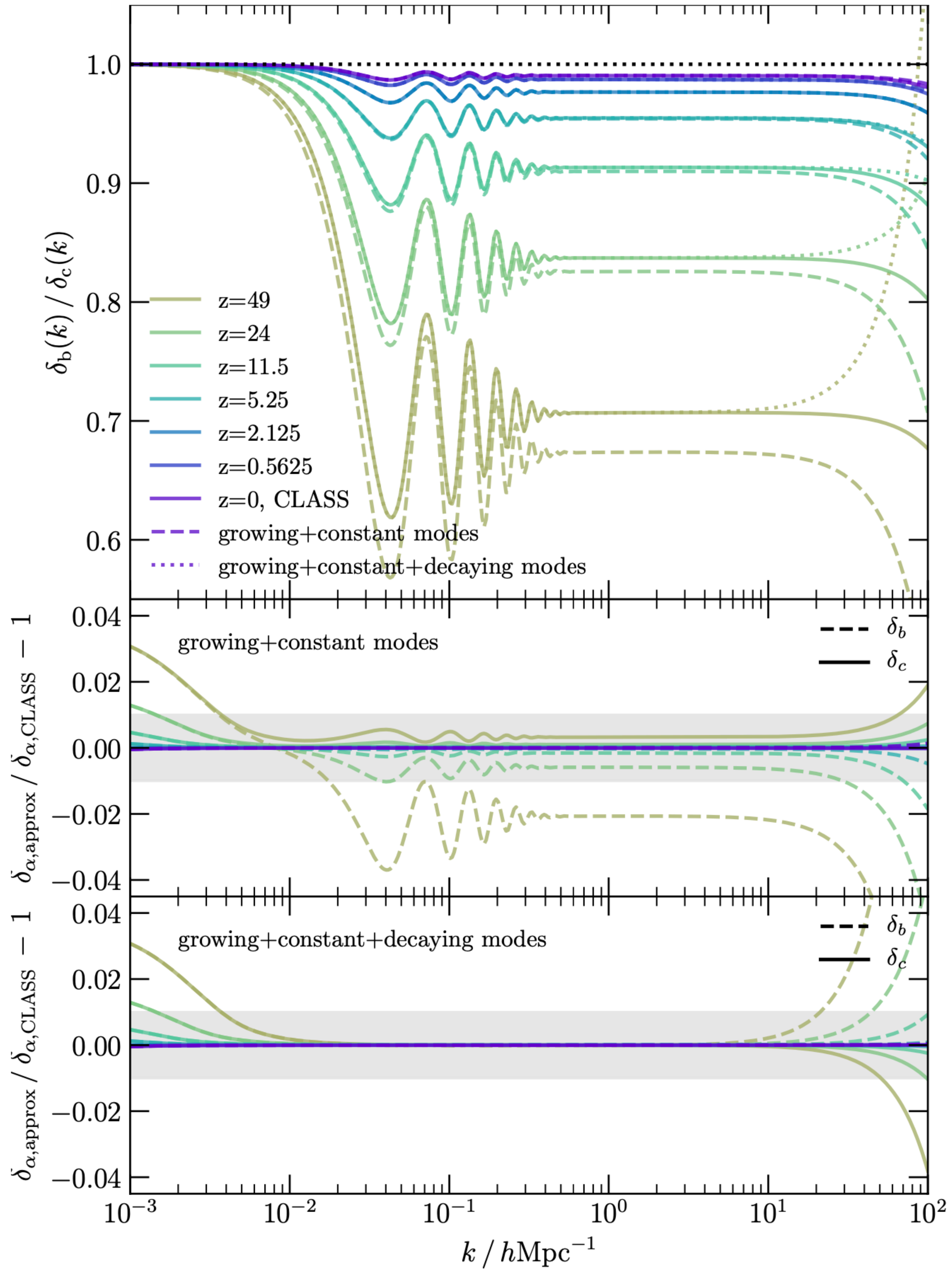
Michaux+2020

… discreteness effects large at early times. better to start as late as possible if you want to get the most out of your particles…

Including Baryons

Gravitational evolution of the baryon+CDM fluids

Linear Einstein-Boltzmann solvers predict baryon/CDM bias



Baryons:

- have Jeans scale ~ horizon scale until recombination
- therefore collapse on subhorizon only after
- have travelling acoustic waves that decay after recombination
  cf. Tseliakhovich&Hirata 2010

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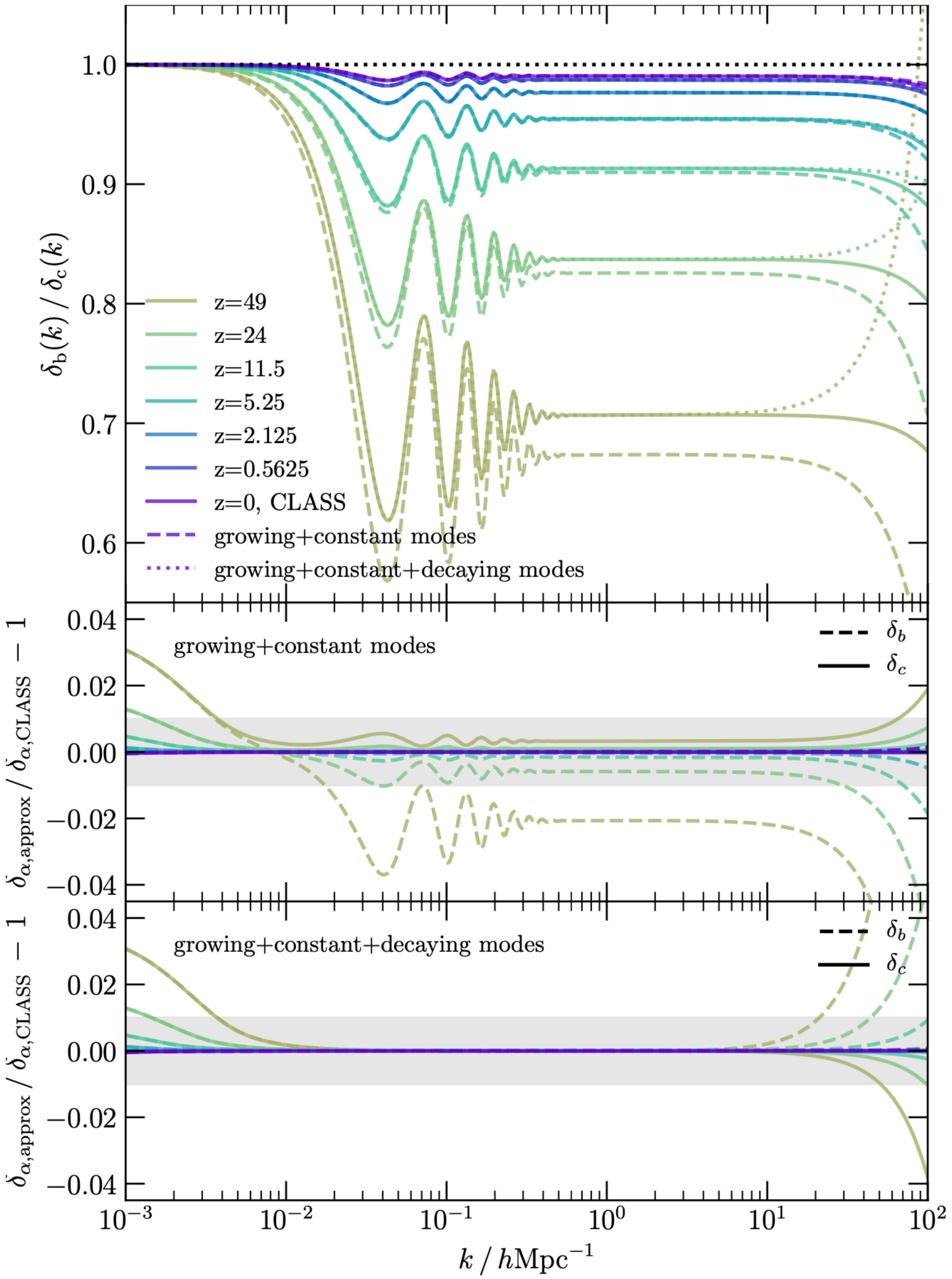

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Convenient variables (Schmidt 2016)

$$\delta_m = f_b \delta_b + f_c \delta_c \,, \qquad \theta_m = f_b \theta_b + f_c \theta_c$$
$$\delta_{bc} = \delta_b - \delta_c \,, \qquad \theta_{bc} = \theta_b - \theta_c$$

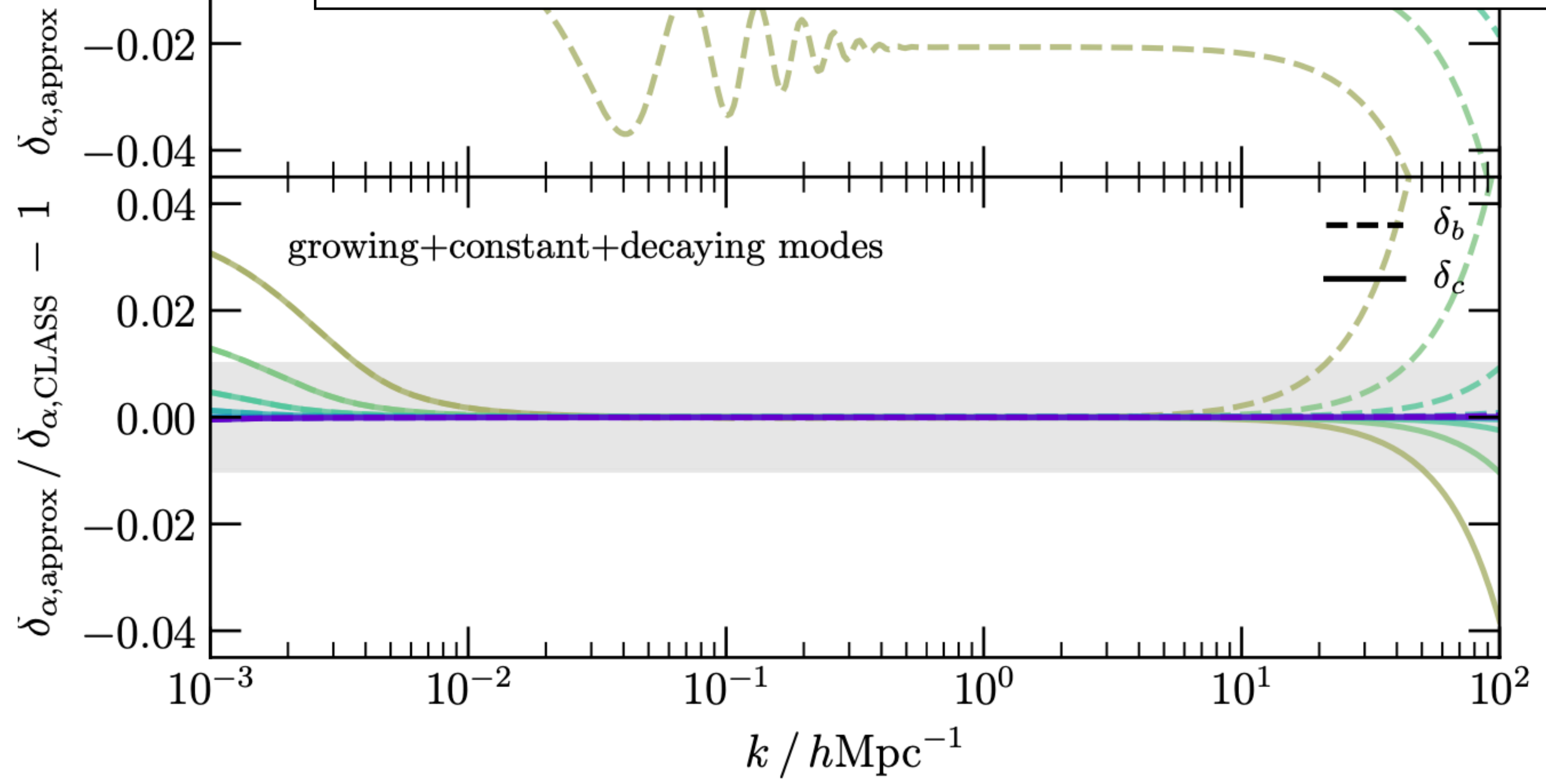
Non-decaying solutions (Rampf et al. 2020):

$$\delta_m = D\,\nabla^2 \varphi^{\mathrm{ini}} \,, \qquad \theta_m = -\nabla^2 \varphi^{\mathrm{ini}}$$
$$\delta_{bc} = \delta_{bc}^{\mathrm{ini}} \,, \qquad \theta_{bc} = 0 \,,$$

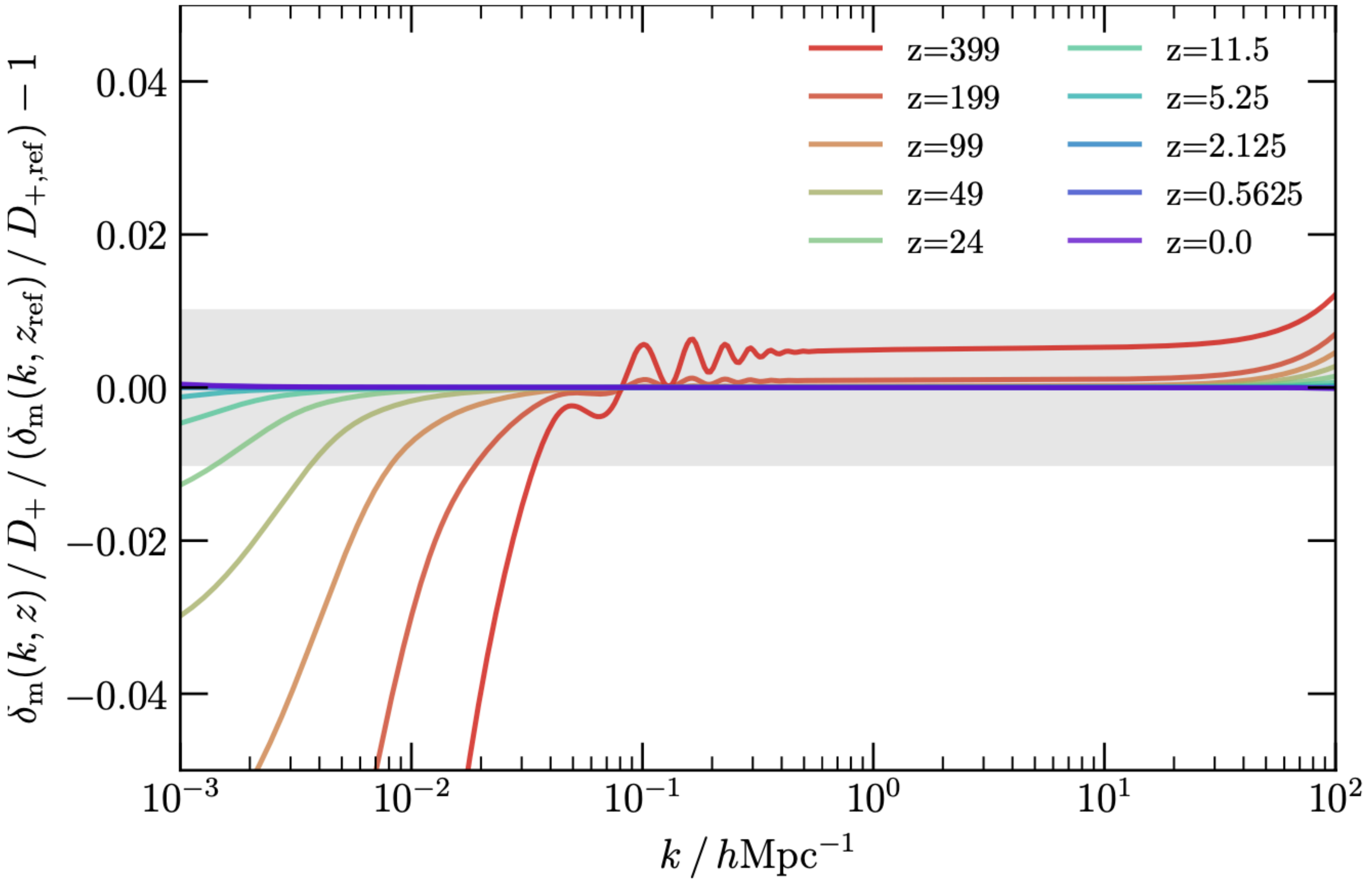
-> Neglecting decaying modes, we have growing mode + compensated isocurvature
-> standard LPT for m + Lagrangian bias for bc

Gravitational evolution of the baryon+CDM fluids

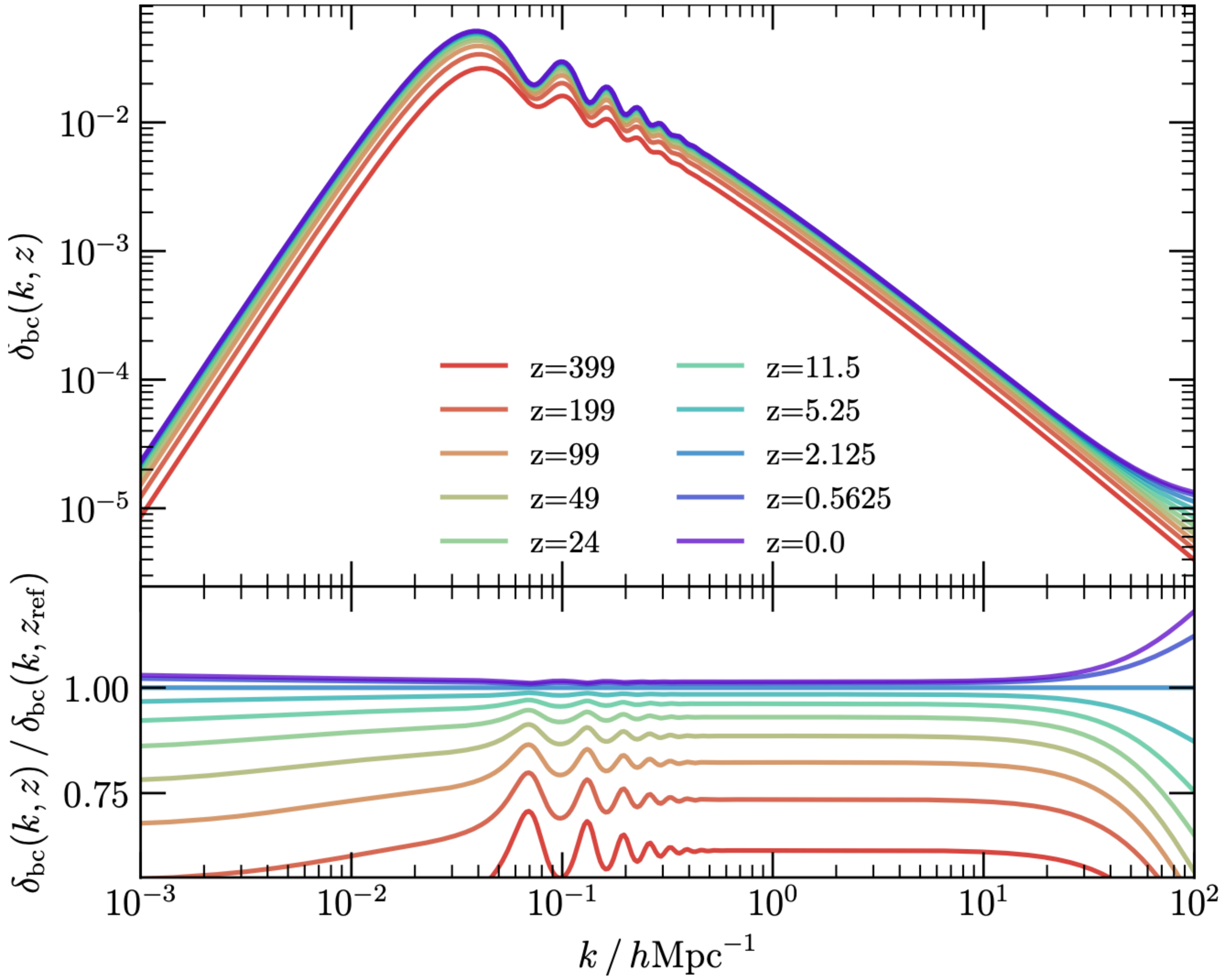
Linear Eins



Total matter: growing mode vs. scale-dependence



Isocurvature mode: constant only at low z



OH, Rampf & Uhlemann 2020

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Discreteness effects in multi-fluid simulations

From what we know now, expect spurious particle-level coupling

Simulations consistently could not reproduce the linear growth in two-fluid, purely gravitational simulations, with distinct b+c ICs

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cf. O'Leary&McQuinn 2012, Angulo,OH&Abel 2013, Valkenburg&Villaescusa-Navarro 2017, Bird et al. 2020

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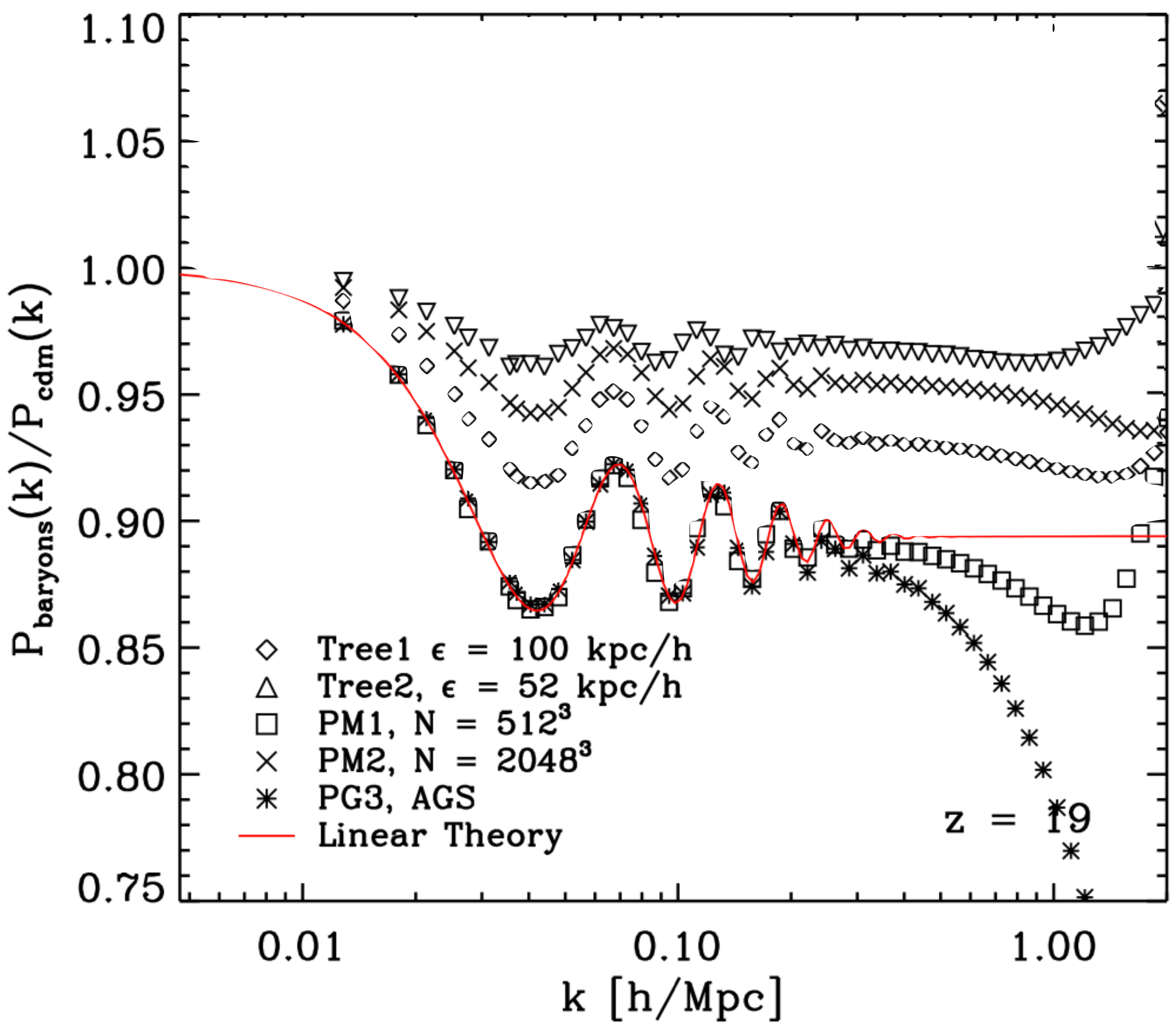

Angulo,OH,Abel 2013

(note that figure was fixed compared to paper)

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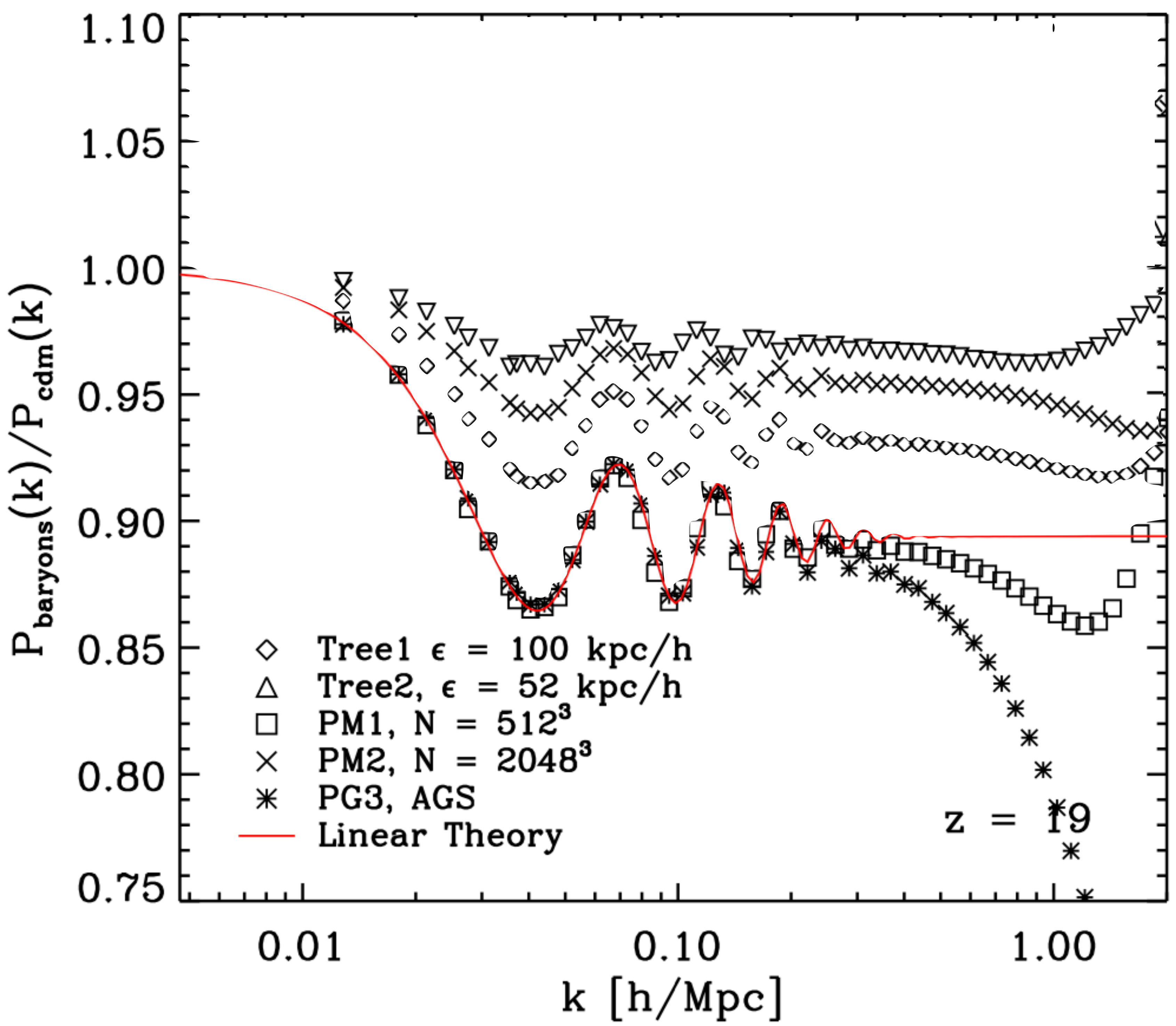
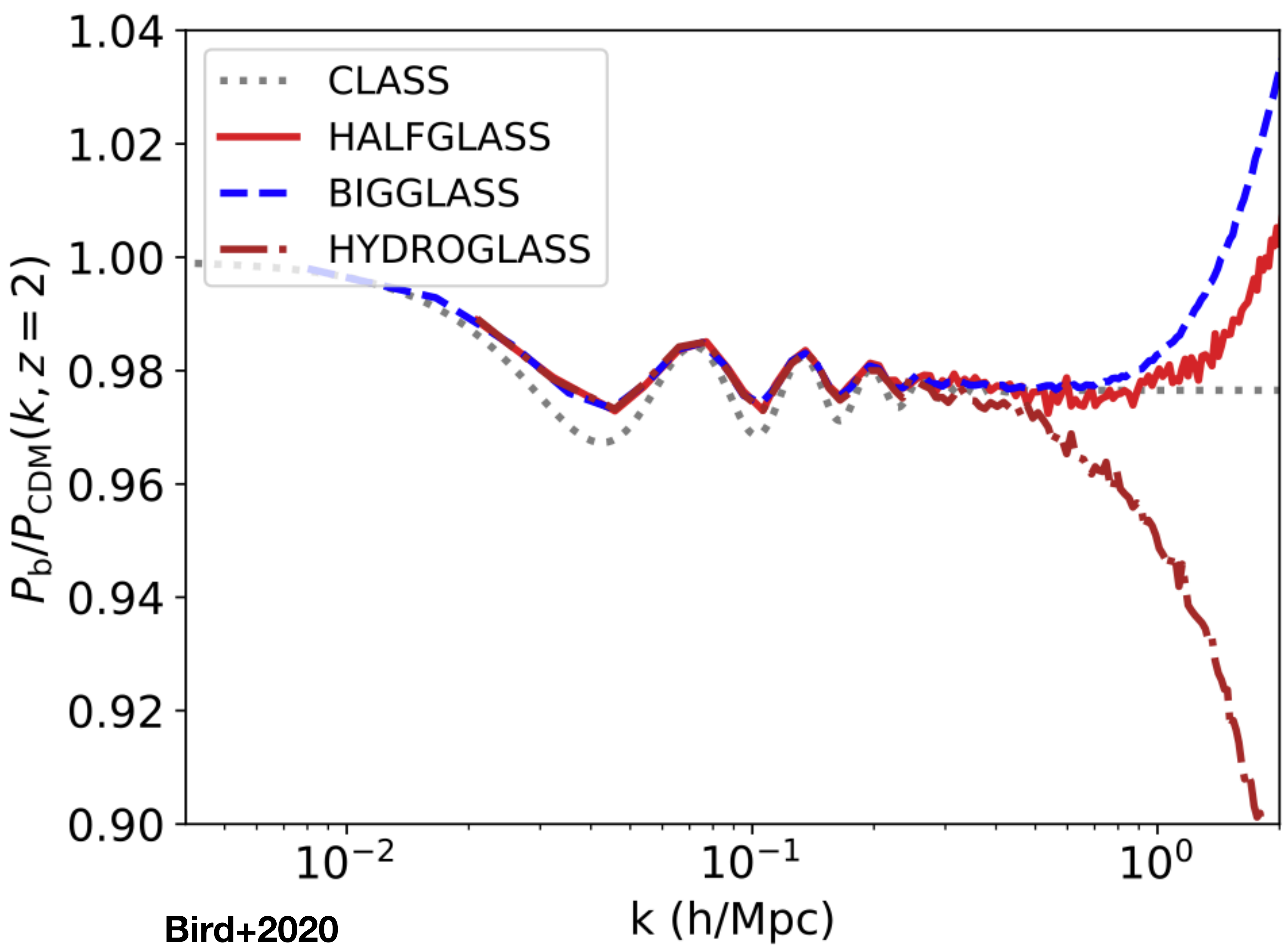


Angulo,OH,Abel 2013
(note that figure was fixed compared to paper)

or decorrelated particle arrangements…

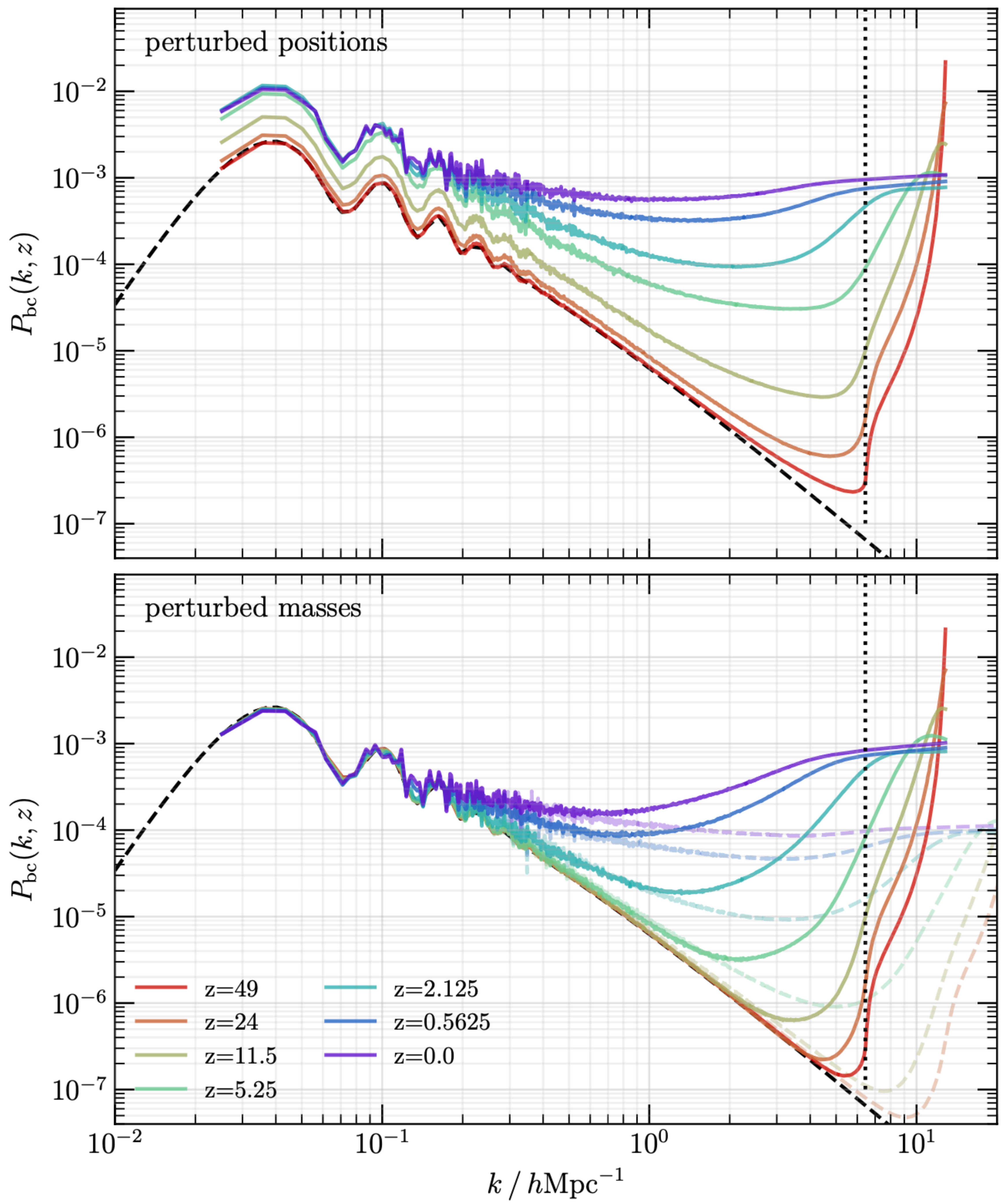


Bird+2020

Impact of discreteness on isocurvature mode

OH, Rampf & Uhlemann 2020: Isocurvature bias should be modelled as a perturbation in particle masses, not displacements

$$m_\alpha(\boldsymbol{q}) = \bar{m}_\alpha \left(1 + \delta_\alpha^{\mathrm{ini}}(\boldsymbol{q})\right), \qquad \bar{m}_\alpha := \Omega_\alpha \,/\, \Omega_{\mathrm{m}}$$



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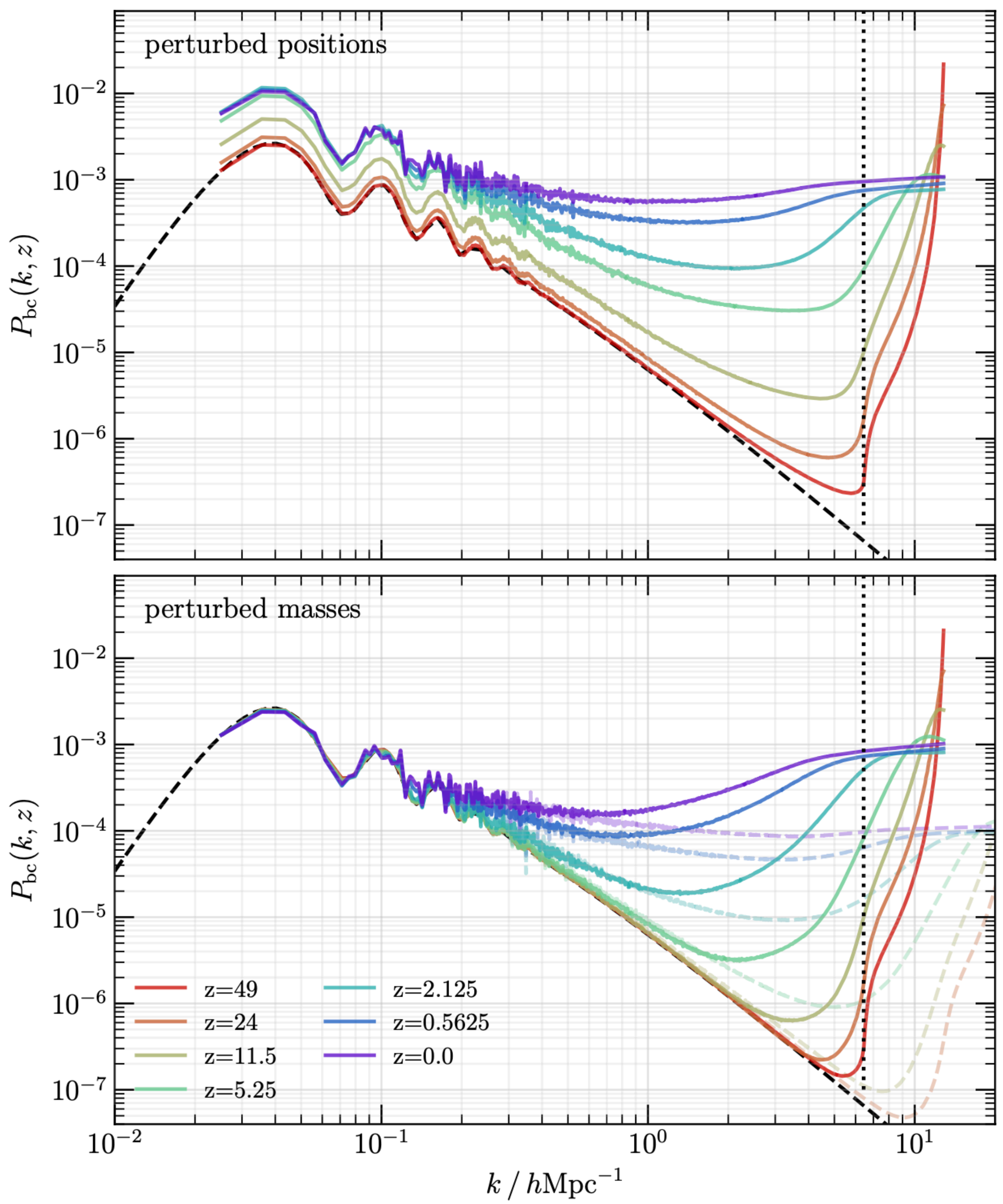
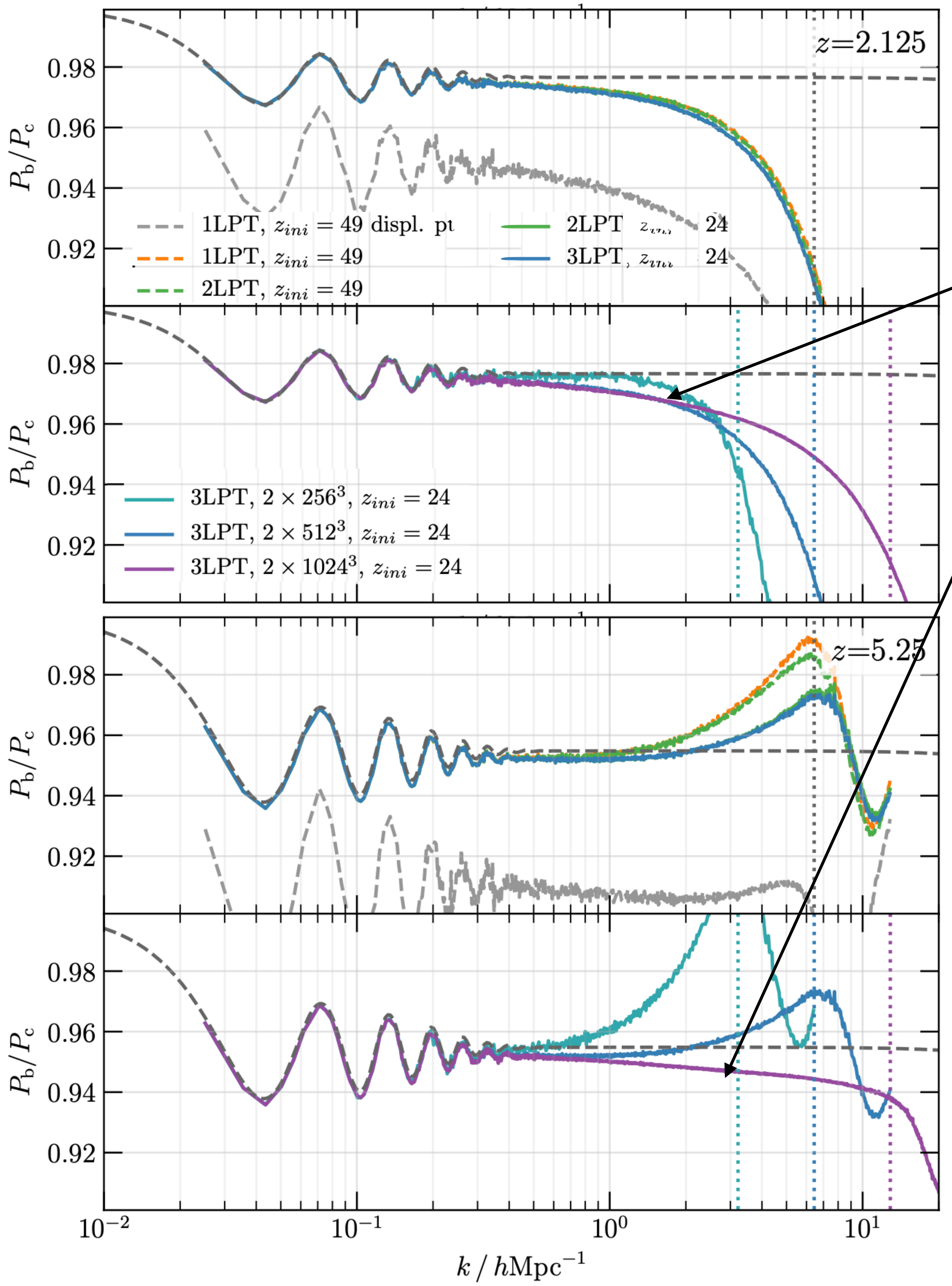




genuine non-linear suppression

Inclusion of decaying modes ➡ state of the art

Decaying modes can be put back at linear level (OH, Rampf & Uhlemann 2020)

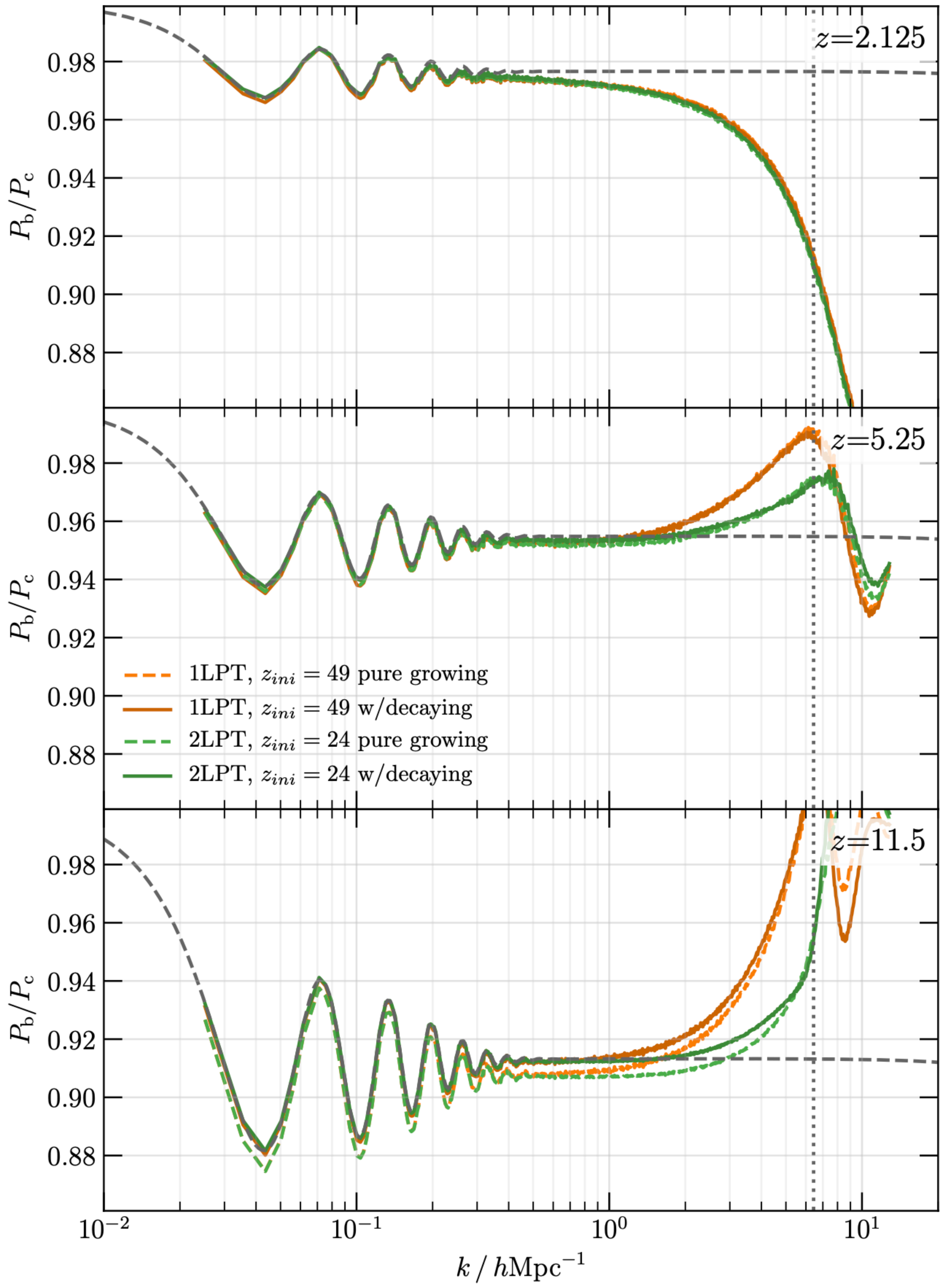
$$
\begin{aligned}
m_\alpha(\boldsymbol{q};\, a_{\text{start}}) &\to m_\alpha(\boldsymbol{q};\, a_{\text{start}}) \\
&\quad + 2\bar{m}_\alpha \left[\left(\frac{D_+(a_{\text{ref}})}{D_+(a_{\text{start}})} \right)^{1/2} - 1 \right] \theta_\alpha^{\text{ini}}(\boldsymbol{q}) \\
\boldsymbol{v}_\alpha(\boldsymbol{q};\, a_{\text{start}}) &\to \boldsymbol{v}_\alpha(\boldsymbol{q};\, a_{\text{start}}) \\
&\quad + \left(\frac{D_+(a_{\text{ref}})}{D_+(a_{\text{start}})} \right)^{1/2} \nabla^{-2} \boldsymbol{\nabla} \theta_\alpha^{\text{ini}}(\boldsymbol{q}).
\end{aligned}
$$

However, has a very limited range of applicability, since masses can become negative for very small astart

This is the current state of the art:

- nLPT for matter growing mode (all orders)
- mass perturbations to model constant isocurvature (all orders)
- additional linear mass+velocity perturbation for decaying mode (1st order)

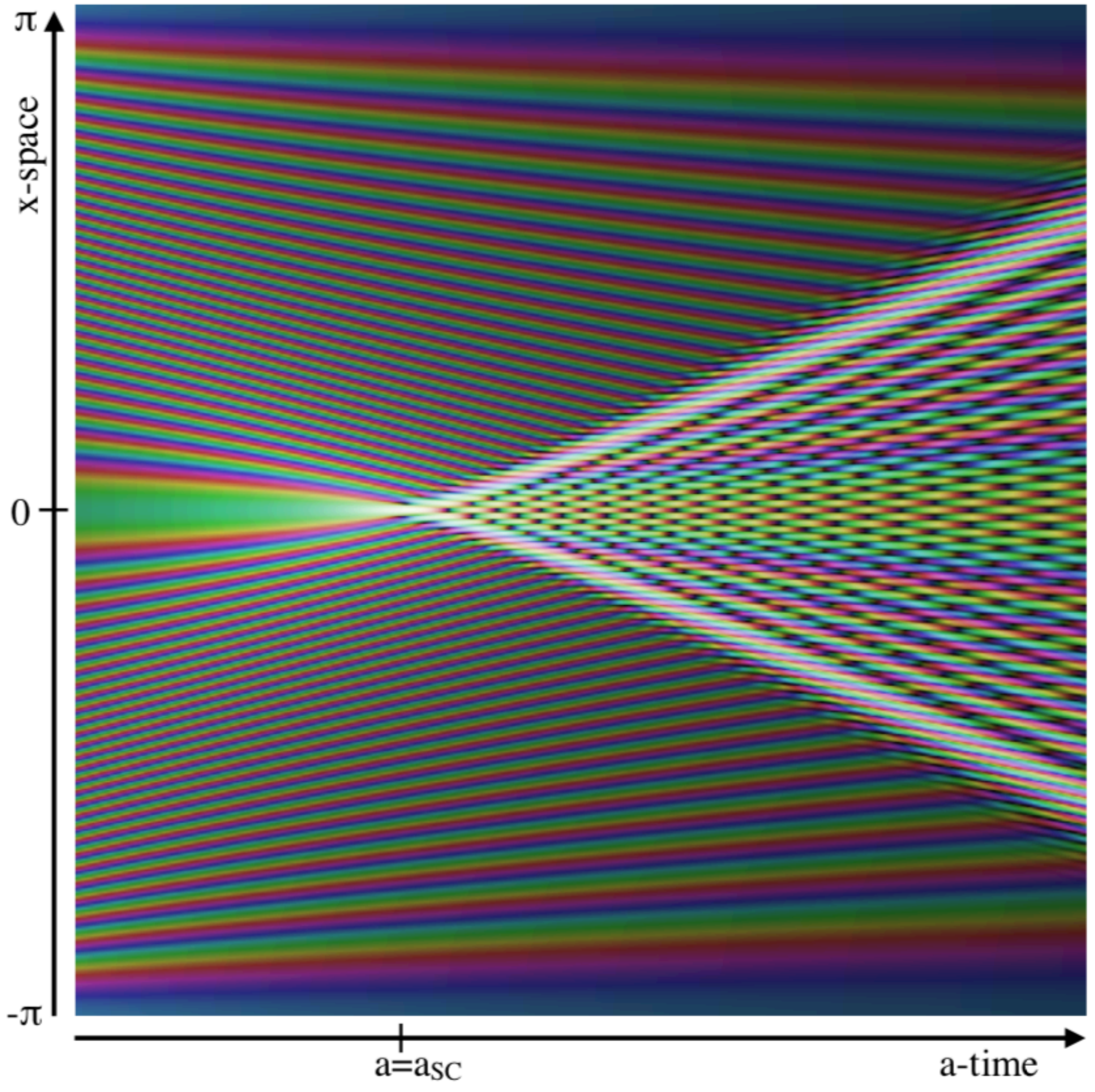
Recent extension to include massive neutrinos by Elbers et al. 2022
Used, e.g. in the FLAMINGO simulations (Schaye et al. 2023)



Lagrangian Perturbative Dynamics for Eulerian Fields

Use QM inspired transition matrix q->x to predict transition probabilities to go from Lagrangian to Eulerian space

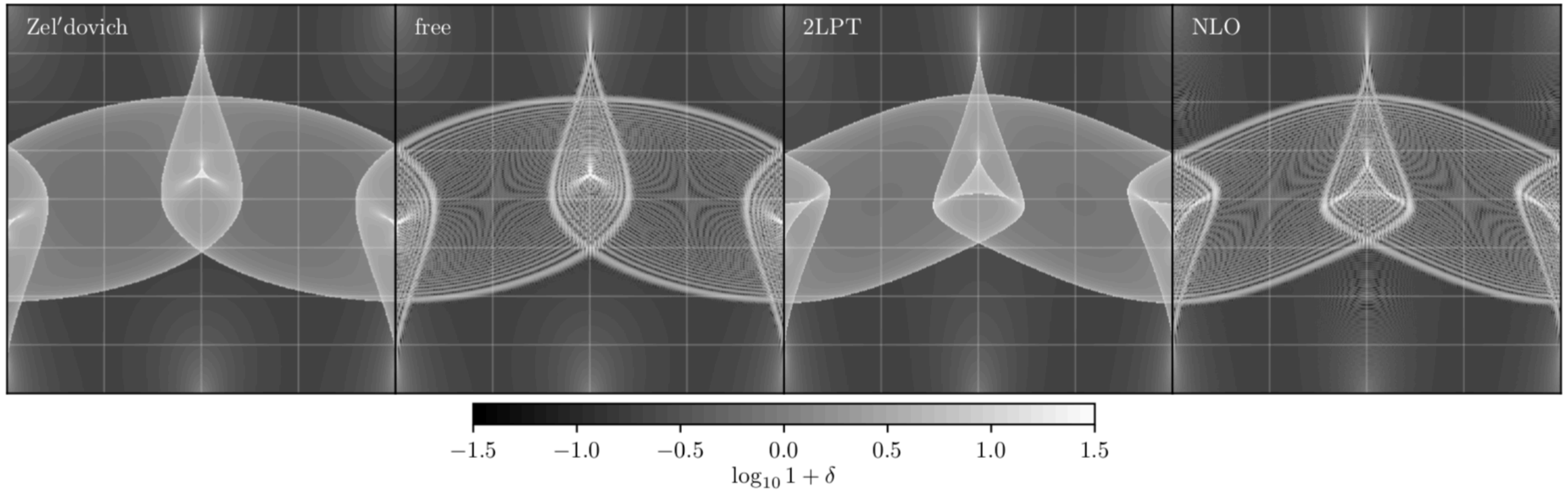
Obtain a Eulerian version of Zeldovich trajectories:



Uhlemann, Rampf, Gosenca, Hahn 2019

Interference = multi-streaming dynamics 'smoothed' by hbar scale

See also Short&Coles (2006)



also works trivially in redshift space, see Porqueres, OH et al. for use as a Ly-a forward model

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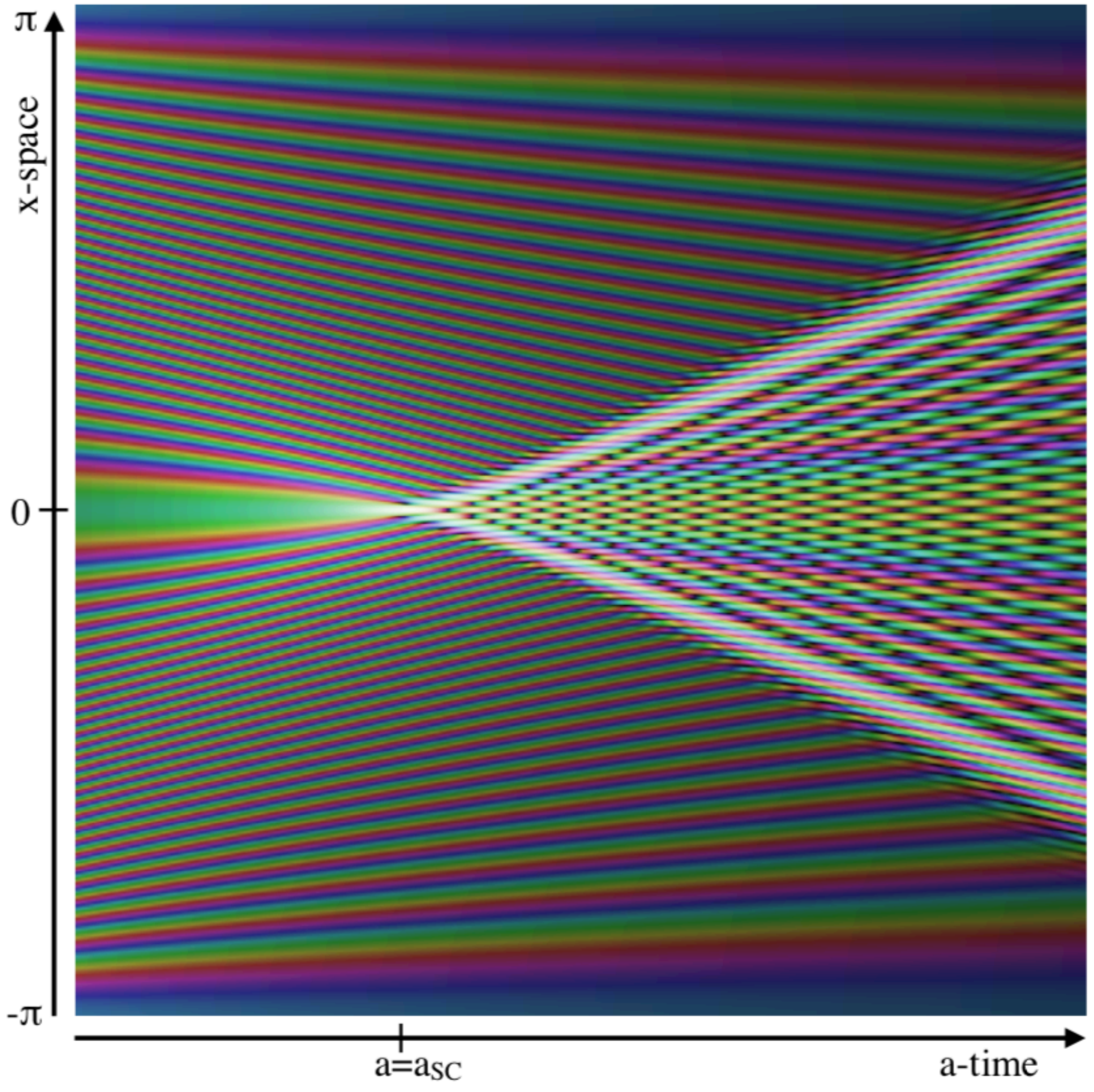

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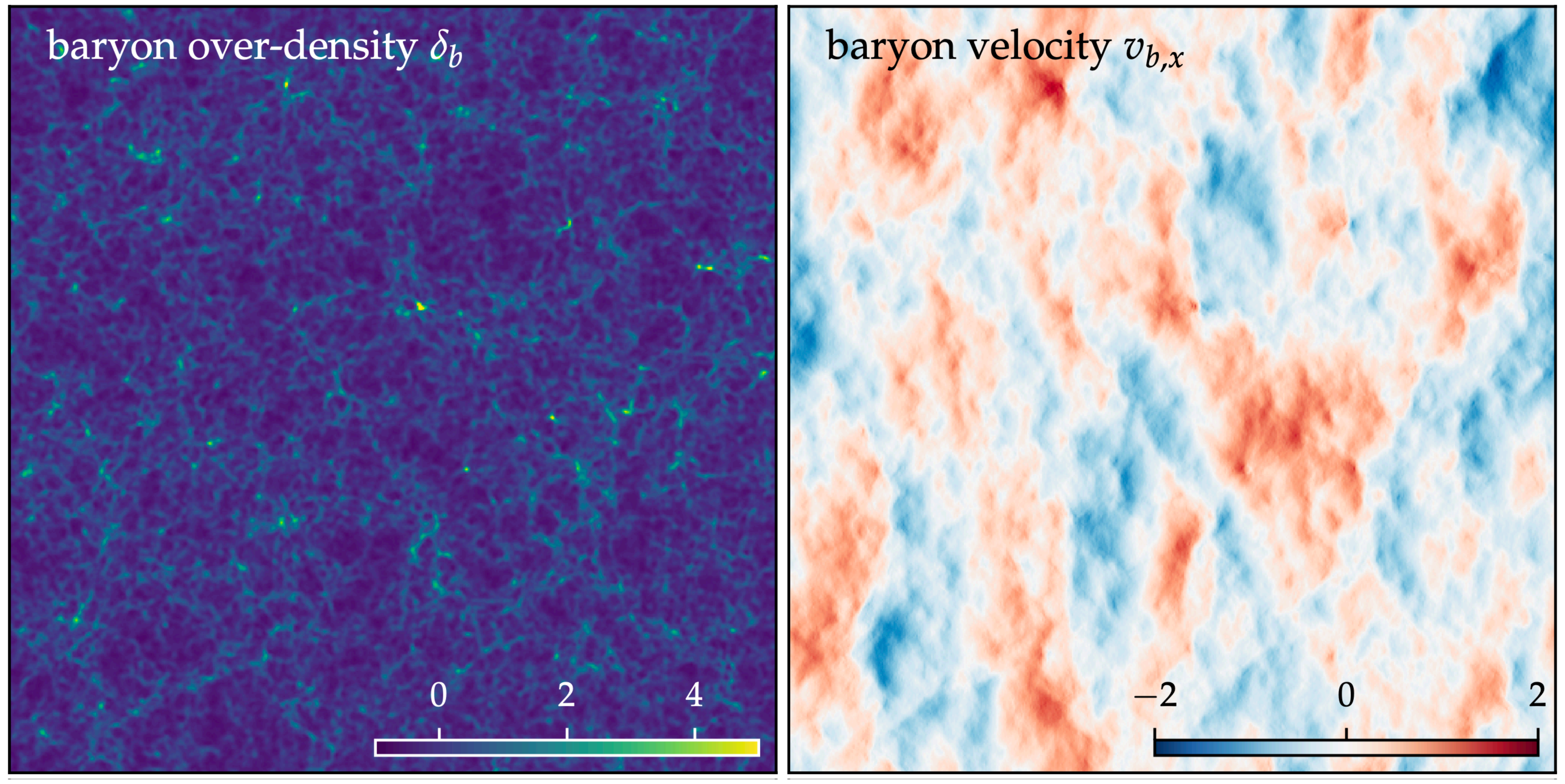
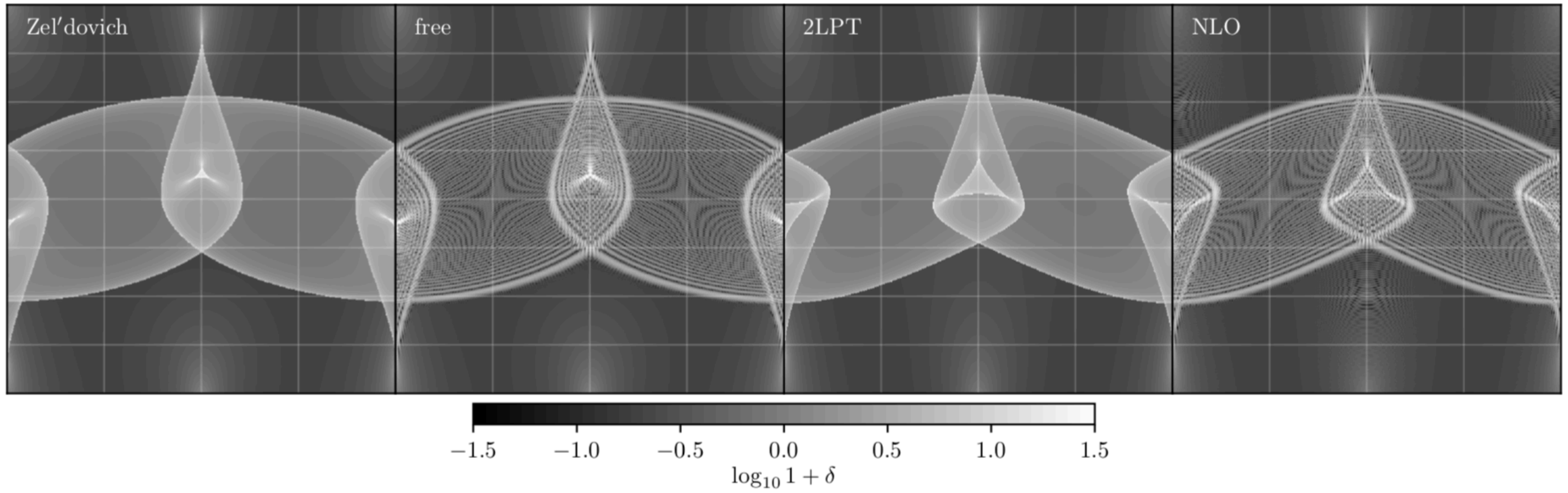
Perfect for ICs for Eulerian codes



Hahn, Rampf & Uhlemann 2021



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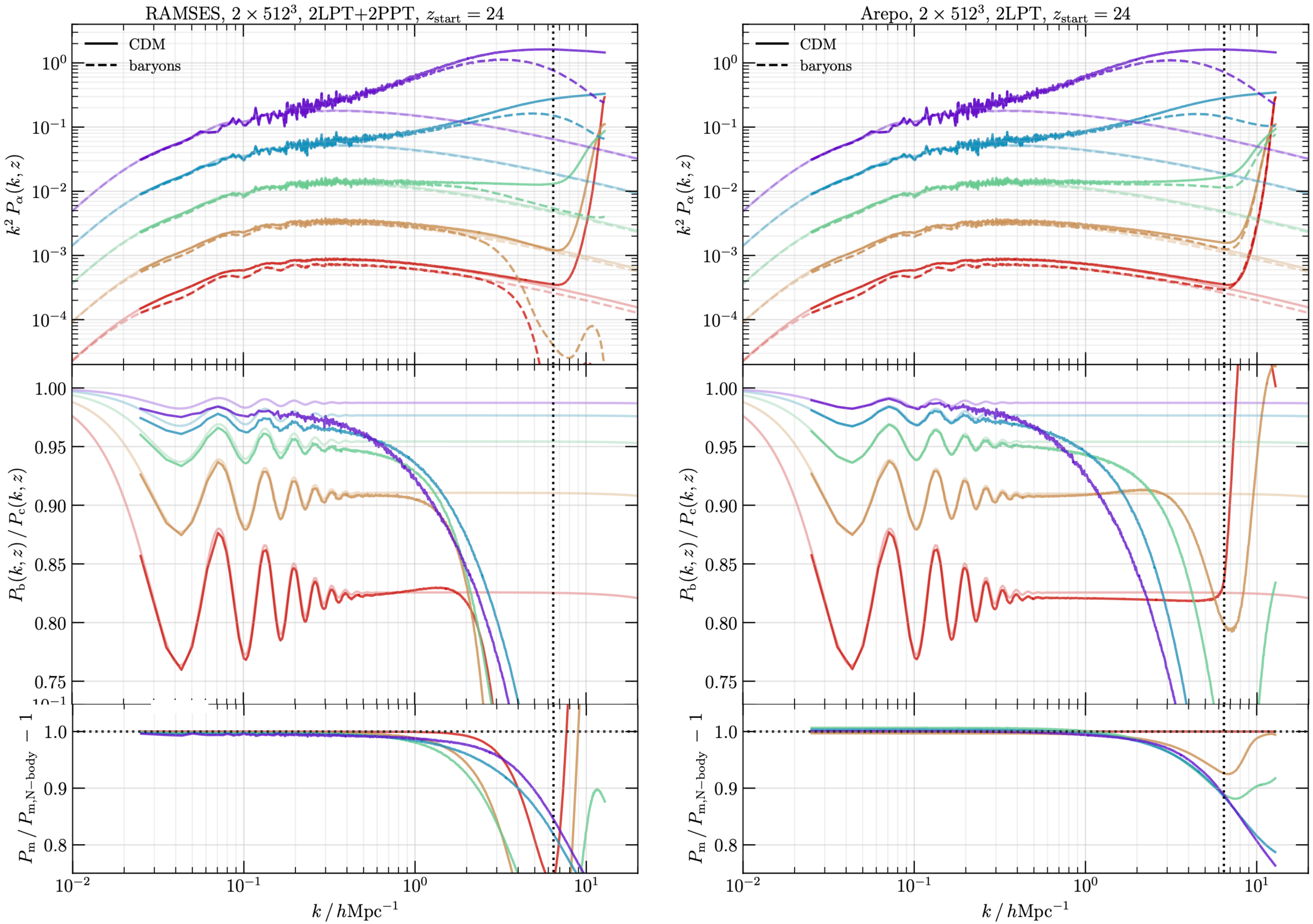
Astrophysics-free hydro simulations



Eulerian (AMR) codes require further care to provide Eulerian ICs (e.g. Propagator PT (PPT))

finite temperature effects on power spectrum purely due to collisionality of baryons

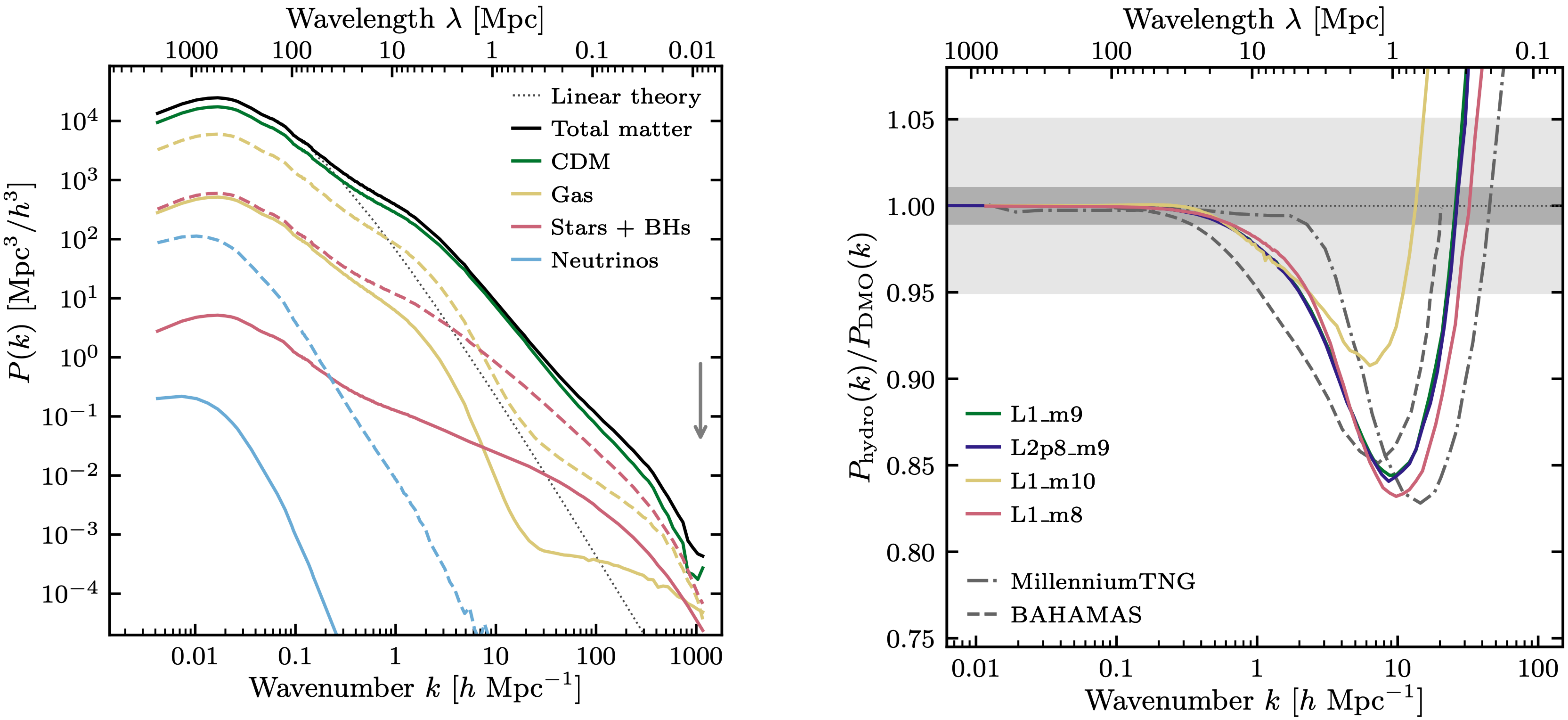
some numerical differences

OH, Rampf, Uhlemann (2020)

Full astrophysics simulations

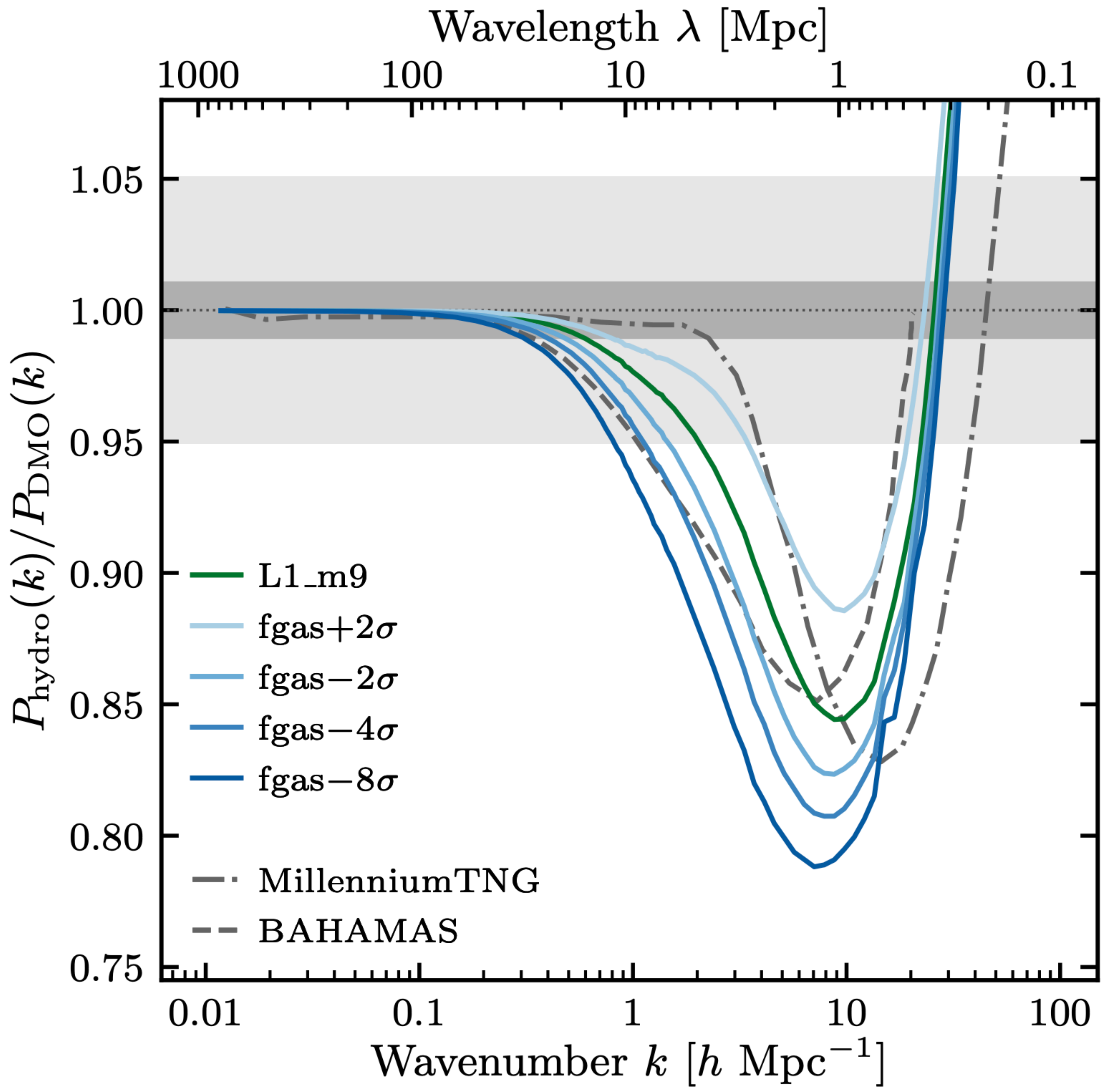
Next generation large-scale astrophysics simulations: Millennium-TNG (Pakmor+22), FLAMINGO (Schaye+23)

FLAMINGO z=0 power spectrum and baryon effects on PS (Schaye et al. 2023)

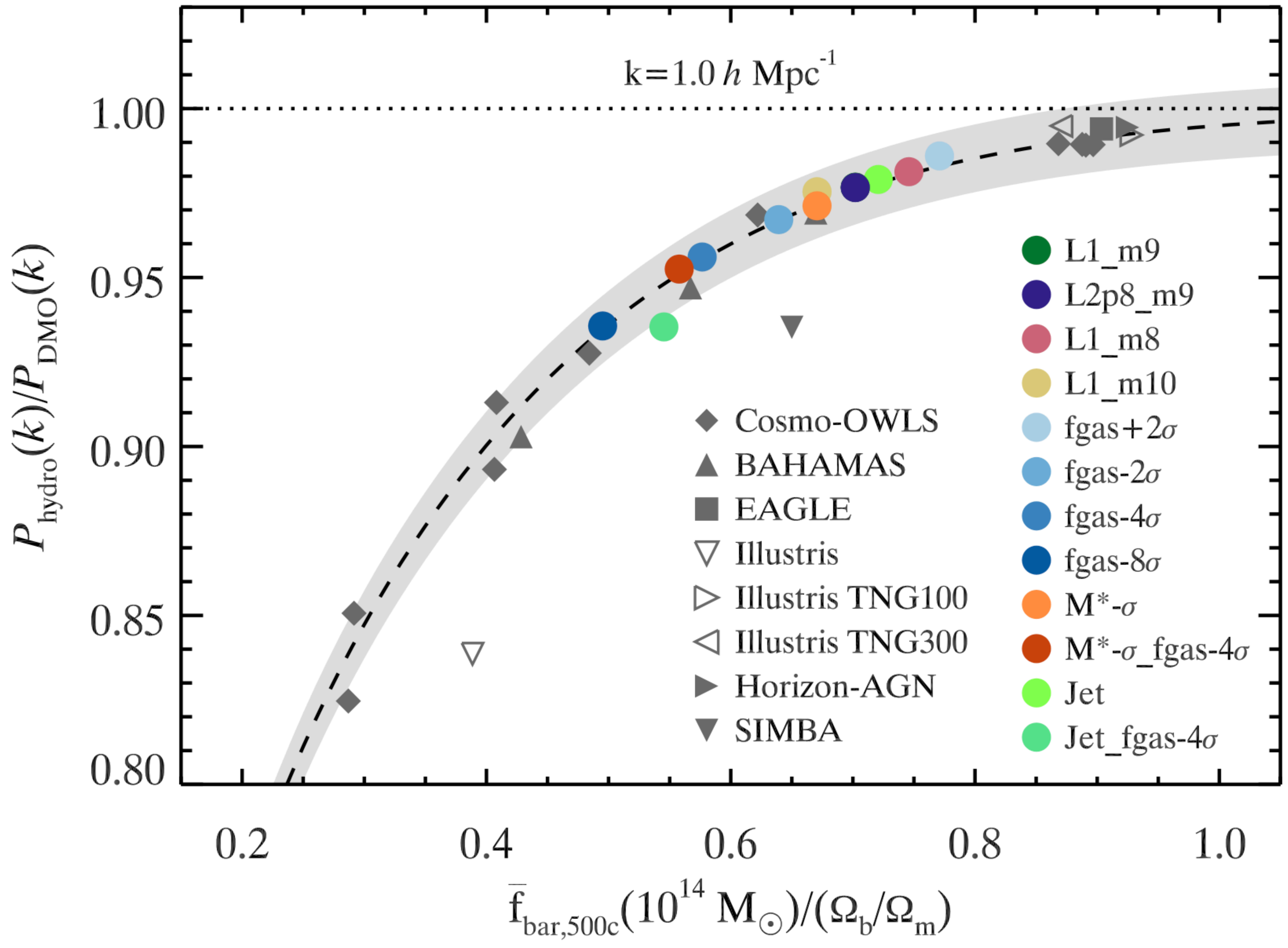


Baryon suppression in FLAMINGO

Schaye et al. (2023)



van Daalen+2020 plot:



AGN, SF, SN parameters tuned using emulator of observables from suite of calibration simulations (Kugel et al. 2023)

Closing the gap between N-body and LPT…

Hamiltonian Structure of Cosmological N-body

EoM came with Hamiltonian (e.g. in superconformal time)

$$\tilde{\mathscr{H}} = \sum_{i=1}^{N} \frac{\|\boldsymbol{V}_i\|^2}{2} + a^2 \sum_i \sum_{j \neq i} I(\boldsymbol{X}_i, \boldsymbol{X}_j; a) =: T(\{\boldsymbol{P}\}) + U(\{\boldsymbol{X}\}, a).$$

Hamiltonian systems have the symplectic property, i.e. they can be written

$$\frac{\mathrm{d}}{\mathrm{d}\tilde{t}}(\boldsymbol{X}_i, \boldsymbol{V}_i) = \begin{bmatrix} 0 & \mathbb{I} \\ -\mathbb{I} & 0 \end{bmatrix} \cdot \begin{pmatrix} \partial\tilde{\mathscr{H}}/\partial \boldsymbol{X}_i \\ \partial\tilde{\mathscr{H}}/\partial \boldsymbol{V}_i \end{pmatrix} \qquad i = 1, \ldots, 3N.$$

This structure guarantees area conservation in phase space.

A time-integrator is a map advancing the state vector from time t to time $t+\epsilon$, i.e.

$$\boldsymbol{F}_\epsilon : \boldsymbol{\xi}_t \mapsto \boldsymbol{\xi}_{t+\epsilon}.$$

A *symplectic* time-integrator is the sub-class of integrators for which

$$\det \frac{\partial \boldsymbol{F}}{\partial \boldsymbol{\xi}} = 1,$$

which guarantees ‘energy conservation’ and implies phase space volume conservation.

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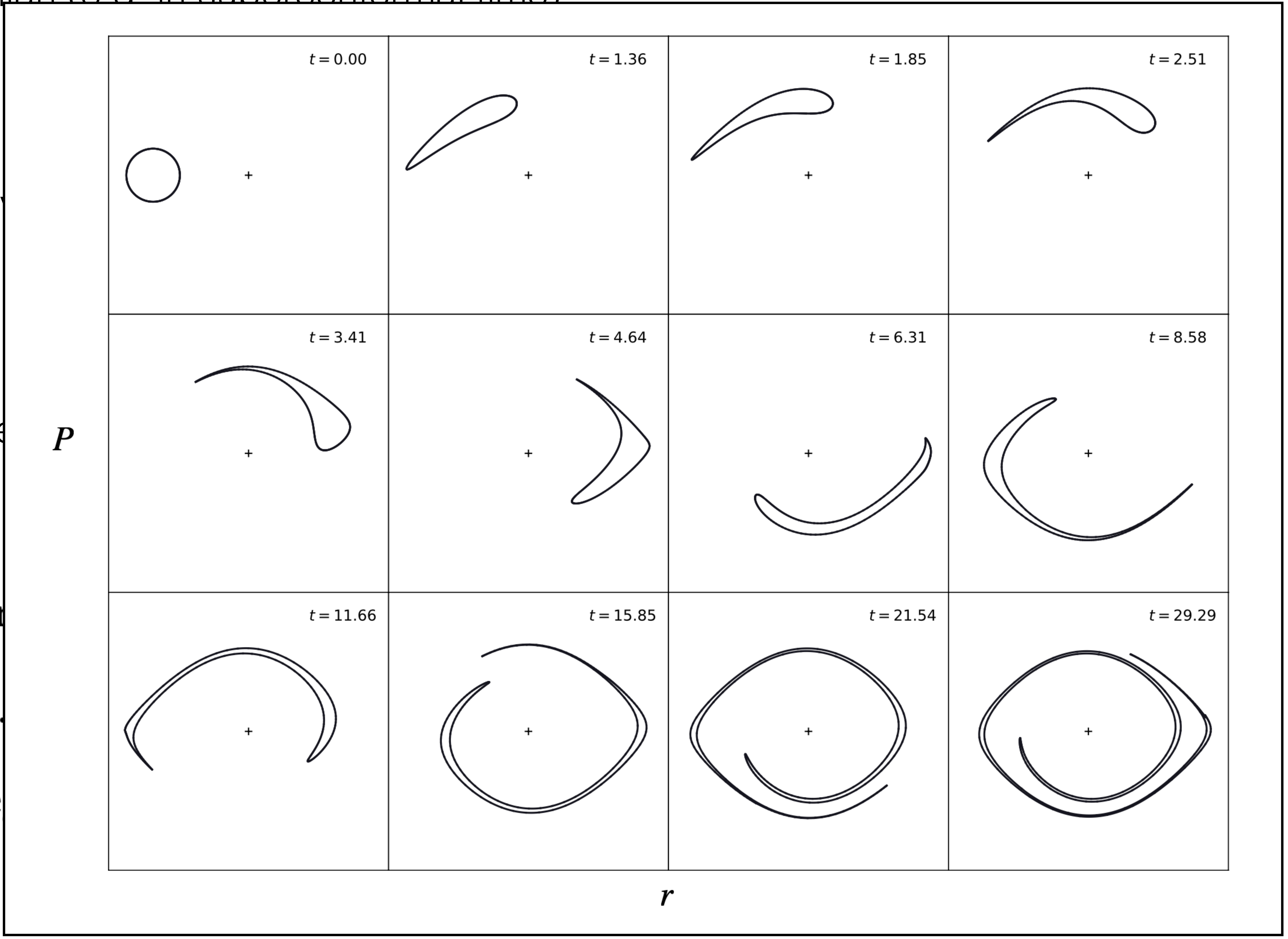

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which guarantees ‘energy conservation’ and implies phase space volume conservation.

Time integrators, what do they do

Lump all phase space coordinates together $\boldsymbol{\xi}_j := (\boldsymbol{X}, \boldsymbol{P})_j$

Then Hamiltonian EoMs are a first order operator equation

$$\dot{\boldsymbol{\xi}}_j = \hat{\mathscr{H}}(t)\,\boldsymbol{\xi}_j \qquad \text{with} \qquad \hat{\mathscr{H}}(t) := \{\cdot,\, \mathscr{H}(t)\} = \{\cdot,\, \alpha T\} + \{\cdot,\, \beta V\} =: \hat{D}(t) + \hat{K}(t).$$

With formal solution

$$\boldsymbol{\xi}_j(t) = \mathcal{T}\exp\left[\int_0^t \mathrm{d}t'\,\hat{\mathscr{H}}(t')\right]\boldsymbol{\xi}_j(0)$$

Now apply Strang operator splitting to find coefficients consistent with an expansion to order m

$$\mathcal{T}\exp\left[\int_t^{t+\epsilon} \mathrm{d}t'\,\hat{\mathscr{H}}(t')\right] \simeq \exp\left[\epsilon_n \hat{K}\right]\cdots\exp\left[\epsilon_3 \hat{D}\right]\,\exp\left[\epsilon_2 \hat{K}\right]\,\exp\left[\epsilon_1 \hat{D}\right] + \mathcal{O}(\epsilon^m).$$

Finally expand operator exponentials into generators

$$\boldsymbol{\xi}_j(\tilde{t}+\epsilon) = \left(I + \frac{\epsilon}{2}\hat{D}\right)\left(I + \epsilon\hat{K}\right)\left(I + \frac{\epsilon}{2}\hat{D}\right)\boldsymbol{\xi}_j(\tilde{t})$$

Better time integrators for LSS studies

Definition 1 (Canonical DKD integrator)**.**

$$
\begin{aligned}
\boldsymbol{X}_i^{n+1/2} &= \boldsymbol{X}_i^n + \alpha(\tau_n, \tau_{n+1})\boldsymbol{P}_i^n, \\
\boldsymbol{P}_i^{n+1} &= \boldsymbol{P}_i^n + \beta(\tau_n, \tau_{n+1})\boldsymbol{A}\left(\boldsymbol{X}_i^{n+1/2}\right), \\
\boldsymbol{X}_i^{n+1} &= \boldsymbol{X}_i^{n+1/2} + \gamma(\tau_n, \tau_{n+1})\boldsymbol{P}_i^{n+1},
\end{aligned}
$$

Proposition 1. *Let the acceleration field $\boldsymbol{A} = \boldsymbol{A}(\boldsymbol{X})$ be a differentiable function of the spatial coordinates. Then, any canonical DKD integrator is symplectic.*

Given some conditions on the coefficients, the integrator is globally 2nd order, unless $\boldsymbol{A}$ has singularities

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Consider also less restrictive form (List&OH 23)

Definition 2 (Π-integrator)**.**

$$\begin{aligned}
\boldsymbol{X}_i^{n+1/2} &= \boldsymbol{X}_i^n + \frac{\Delta D}{2}\boldsymbol{\Pi}_i^n, \\
\boldsymbol{\Pi}_i^{n+1} &= p(\Delta D, D_n)\boldsymbol{\Pi}_i^n + q(\Delta D, D_n)\boldsymbol{A}(\boldsymbol{X}_i^{n+1/2}), \\
\boldsymbol{X}_i^{n+1} &= \boldsymbol{X}_i^{n+1/2} + \frac{\Delta D}{2}\boldsymbol{\Pi}_i^{n+1},
\end{aligned}$$

Do N-body integrators reproduce LPT?

Should be able to get Zeldovich in **one** time step

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Example (FastPM). FastPM method of Feng et al. (2016), but this is the DKD version, while they used KDK

$$\begin{aligned}
\boldsymbol{X}_i^{n+1/2} &= \boldsymbol{X}_i^n + \frac{D_{n+1/2} - D_n}{F(D_n)}\, \boldsymbol{P}_i^n && \overset{\text{EdS}}{\asymp}\ \boldsymbol{X}_i^n + \frac{\Delta a}{2 a_n^{3/2}}, \\
\boldsymbol{P}_i^{n+1} &= \boldsymbol{P}_i^n + \frac{F(D_{n+1}) - F(D_n)}{G(D_{n+1/2})}\, \boldsymbol{A}(\boldsymbol{X}^{n+1/2}) && \overset{\text{EdS}}{\asymp}\ \boldsymbol{P}_i^n + \frac{2}{3}\frac{a_{n+1}^{3/2} - a_n^{3/2}}{a_{n+1/2}} \boldsymbol{A}(\boldsymbol{X}^{n+1/2}), \\
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\end{aligned}$$

FastPM is the unique Π*-integrator that is both Zel'dovich consistent and symplectic.*

Proof of symplecticity, consistency, or convergence order see List&OH 23 (not done int Feng+12)

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Counterexample. All other integrators used for N-body simulations are not Zel'dovich consistent.

Can we do even better?

Spoiler: yes, but have to give up symplecticity

Second order LPT (2LPT) can be written

$$
\begin{aligned}
\boldsymbol{X}_i(D) &= \boldsymbol{X}_i^n + [D - D_n]\boldsymbol{\psi}_i^{n,(1)} + [D - D_n]^2\boldsymbol{\psi}_i^{n,(2)} \\
\mathrm{d}_D \boldsymbol{X}_i(D) &= \boldsymbol{\psi}_i^{n,(1)} + 2[D - D_n]\boldsymbol{\psi}_i^{n,(2)}, \\
\mathrm{d}_D^2 \boldsymbol{X}_i(D) &= 2\boldsymbol{\psi}_i^{n,(2)} = \text{const.}
\end{aligned}
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i.e. acceleration is constant

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This can be matched in various ways to yield new integrators that should reproduce 2LPT, not only Zeldovich.

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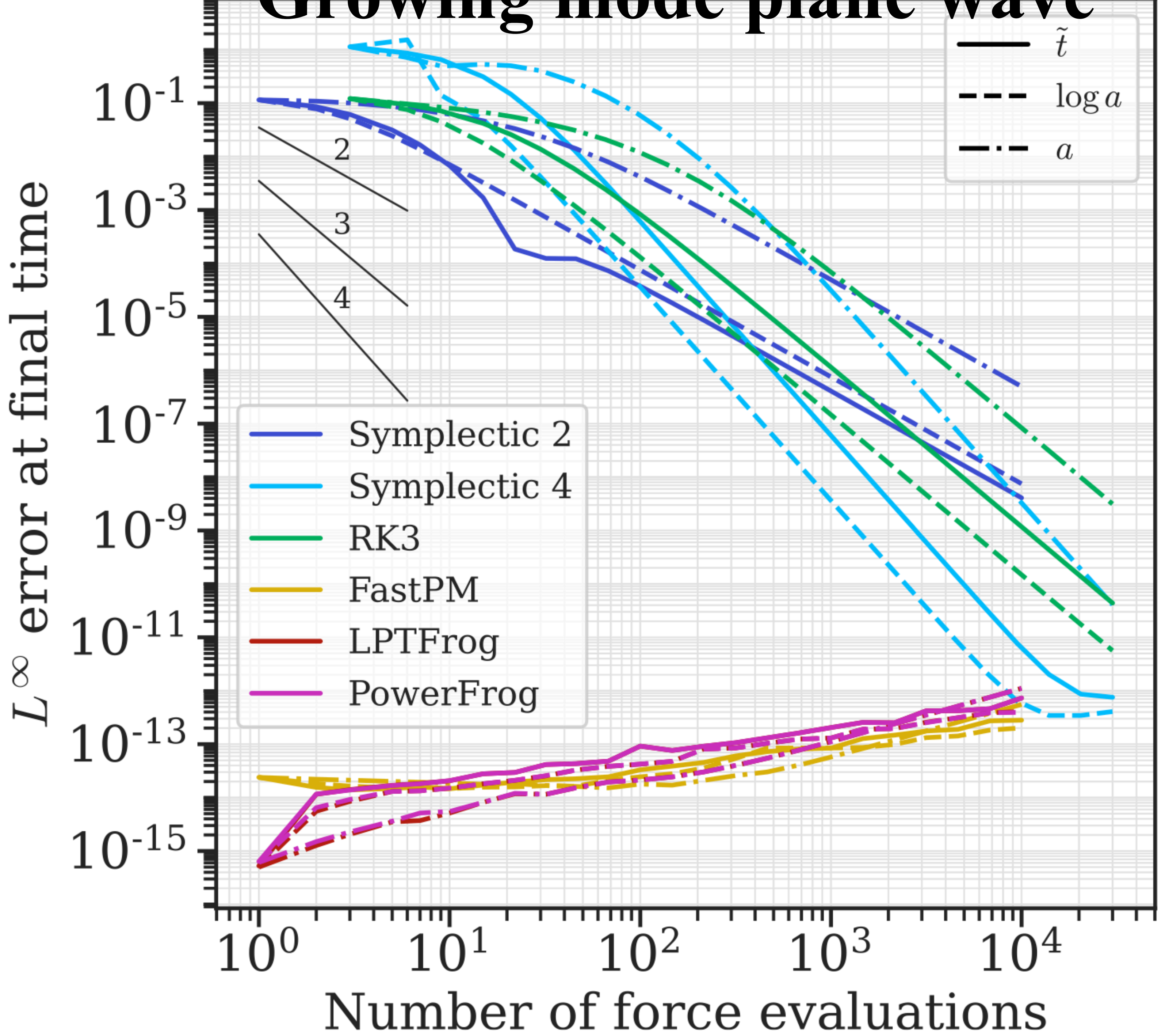
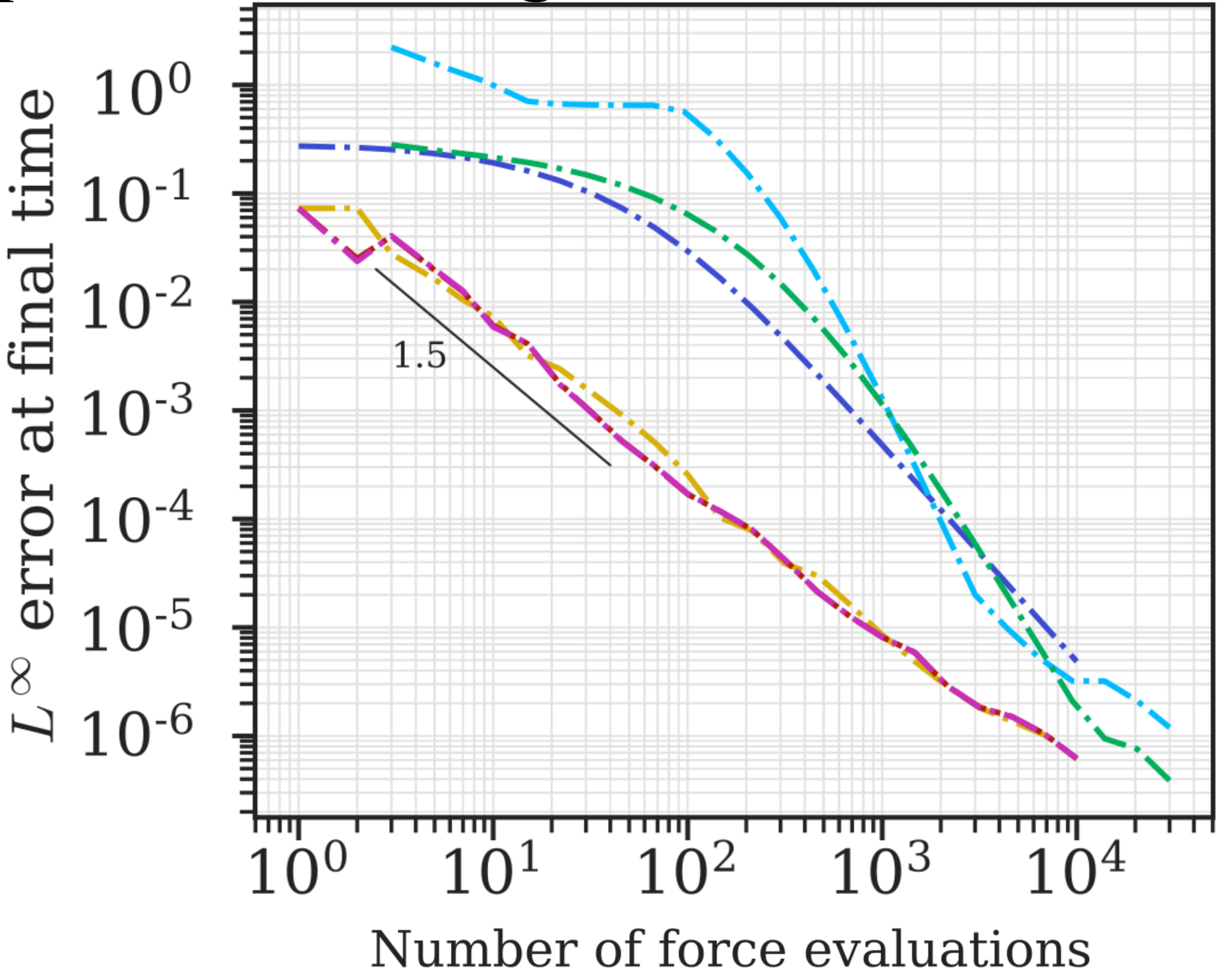
Tests for 1D collapse

Growing mode plane wave

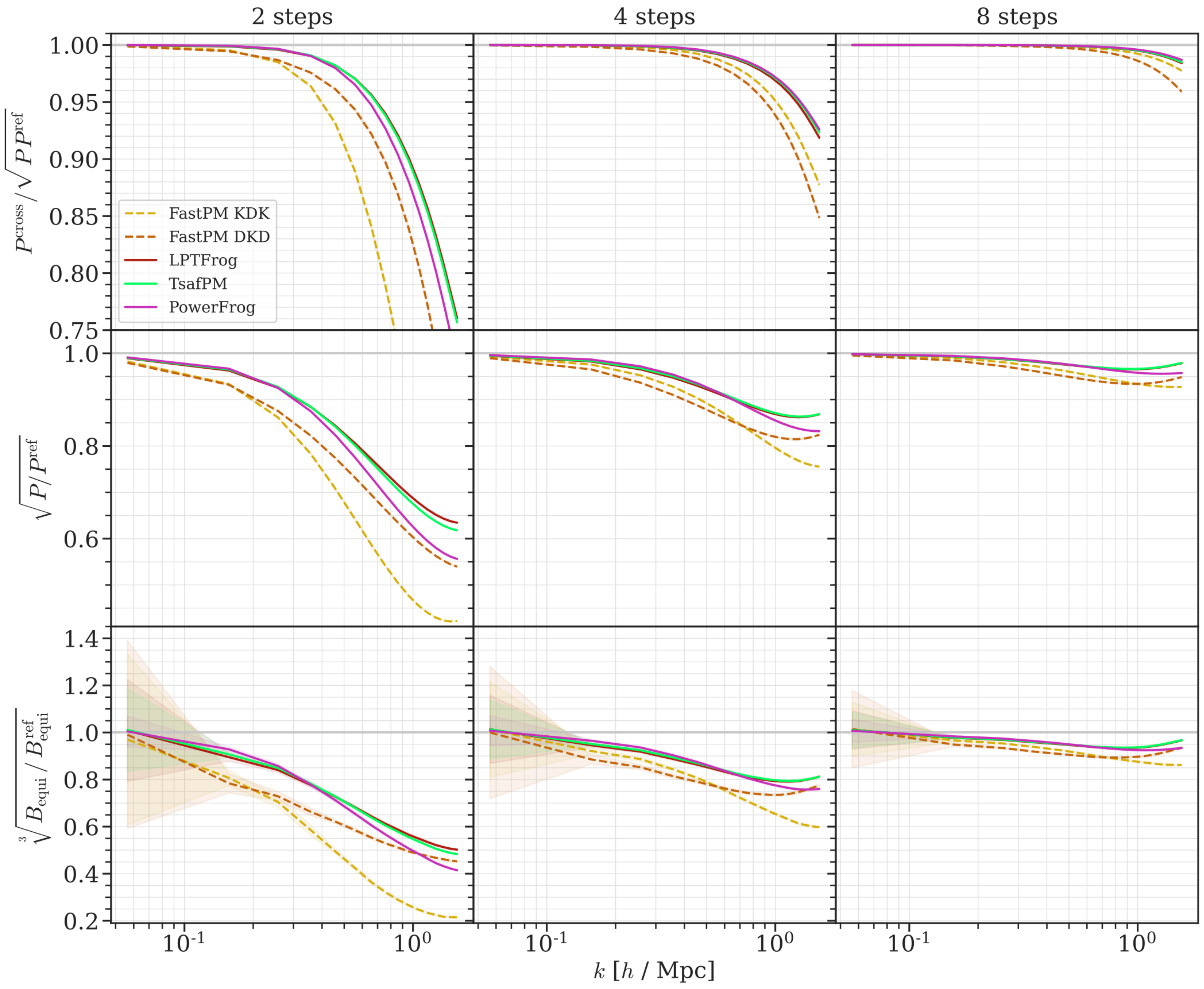


post shell-crossing: a

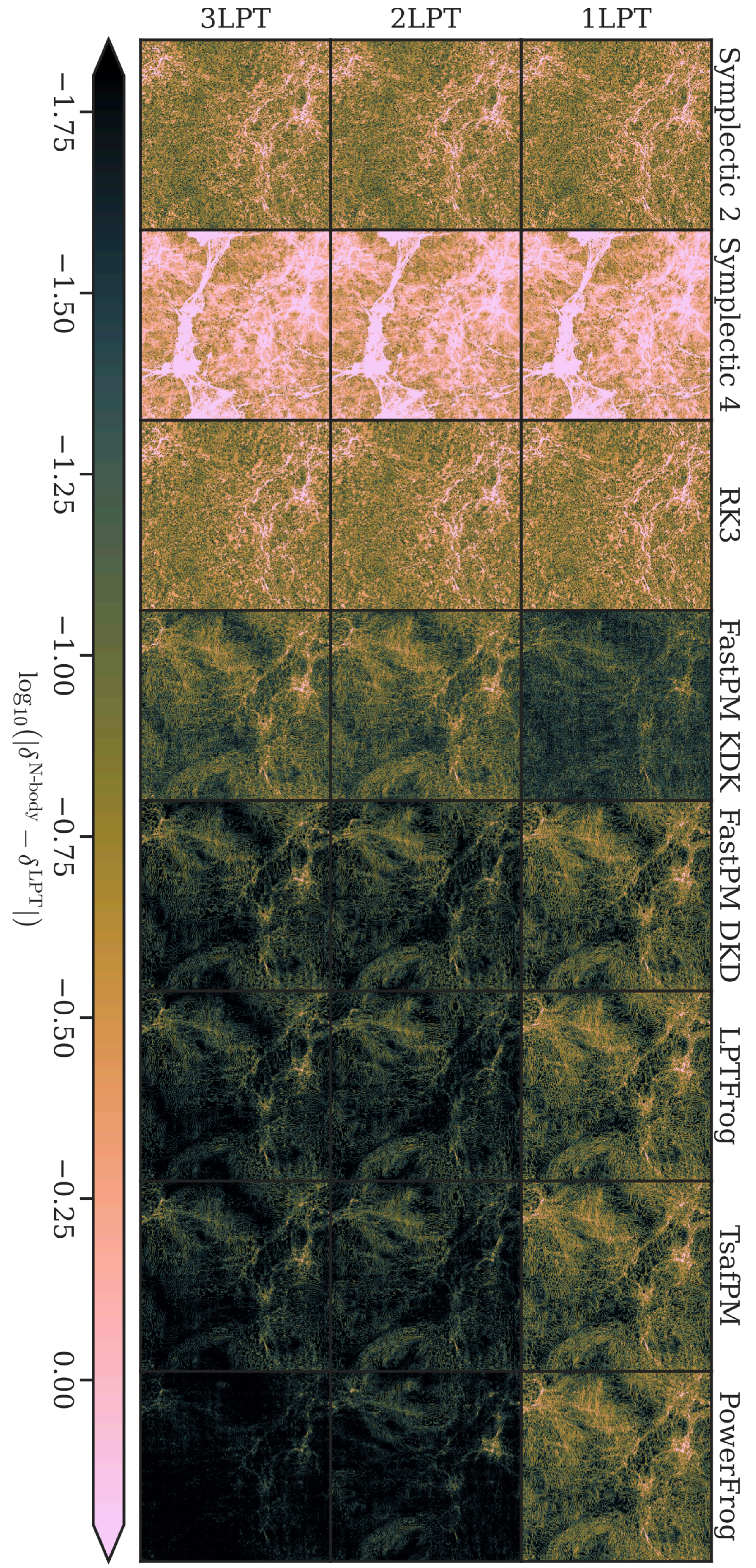


Tests in 2D and 3D

Quijote simulation at z=0



Comparison to nLPT



List&Hahn23

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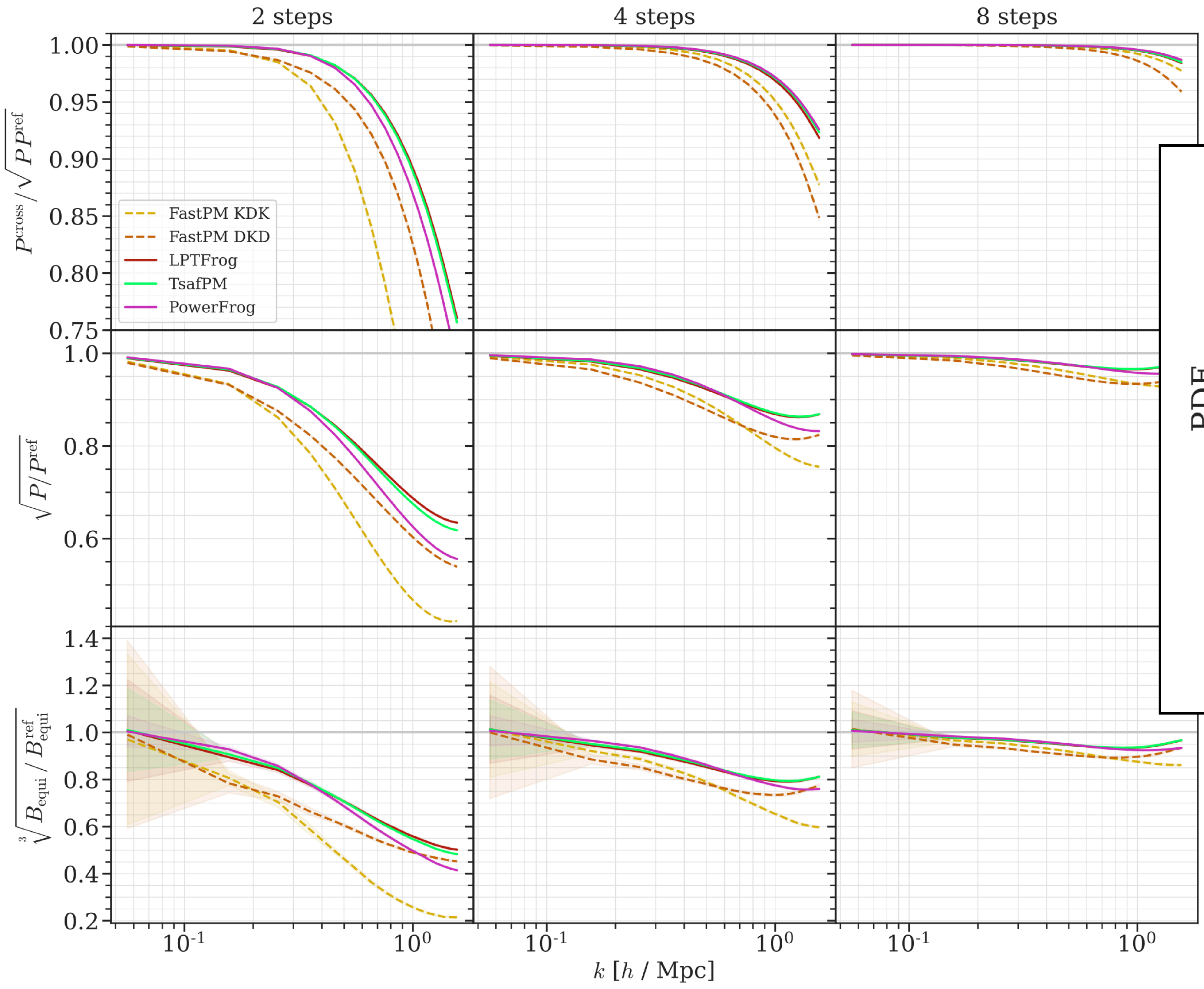

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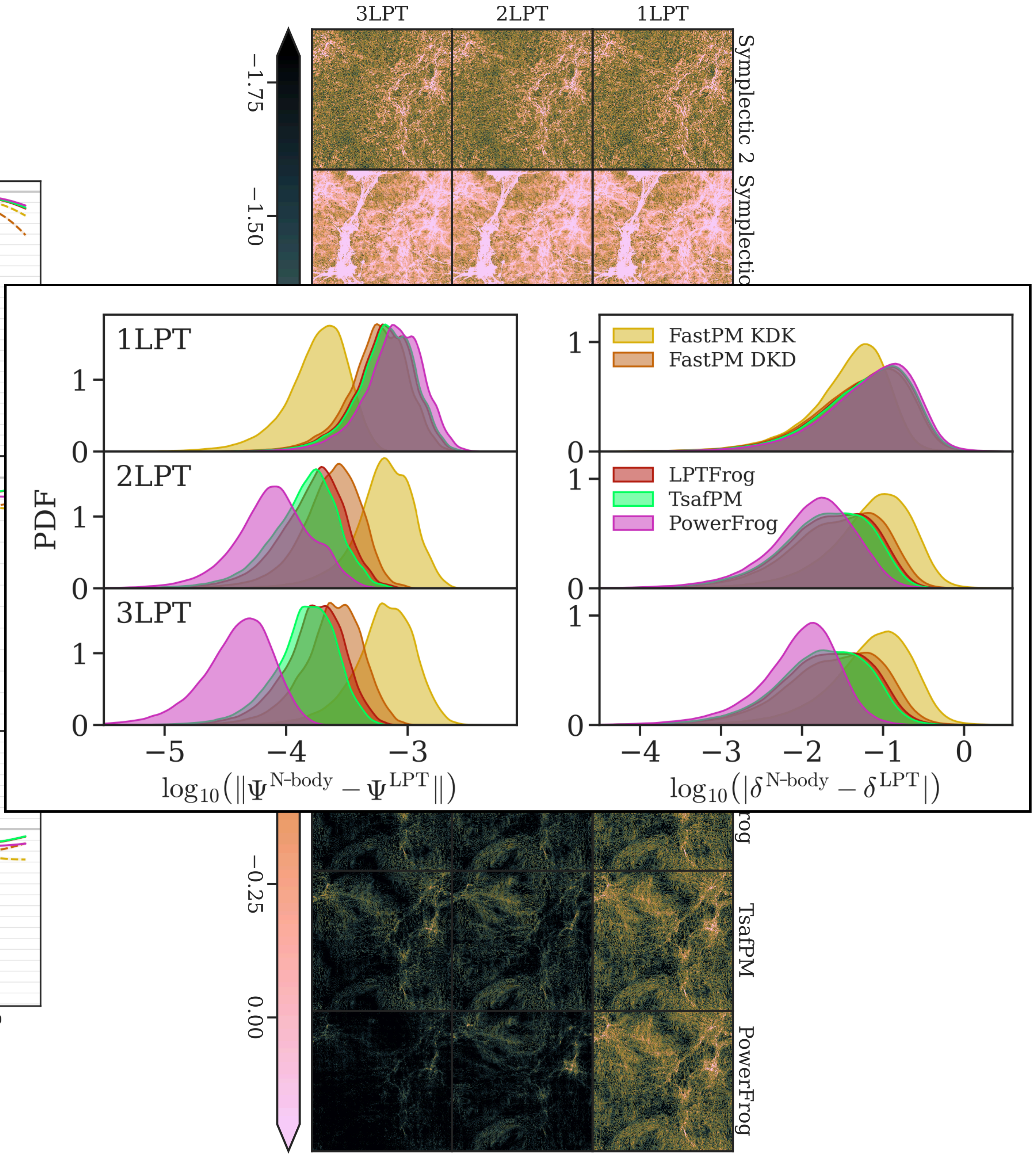
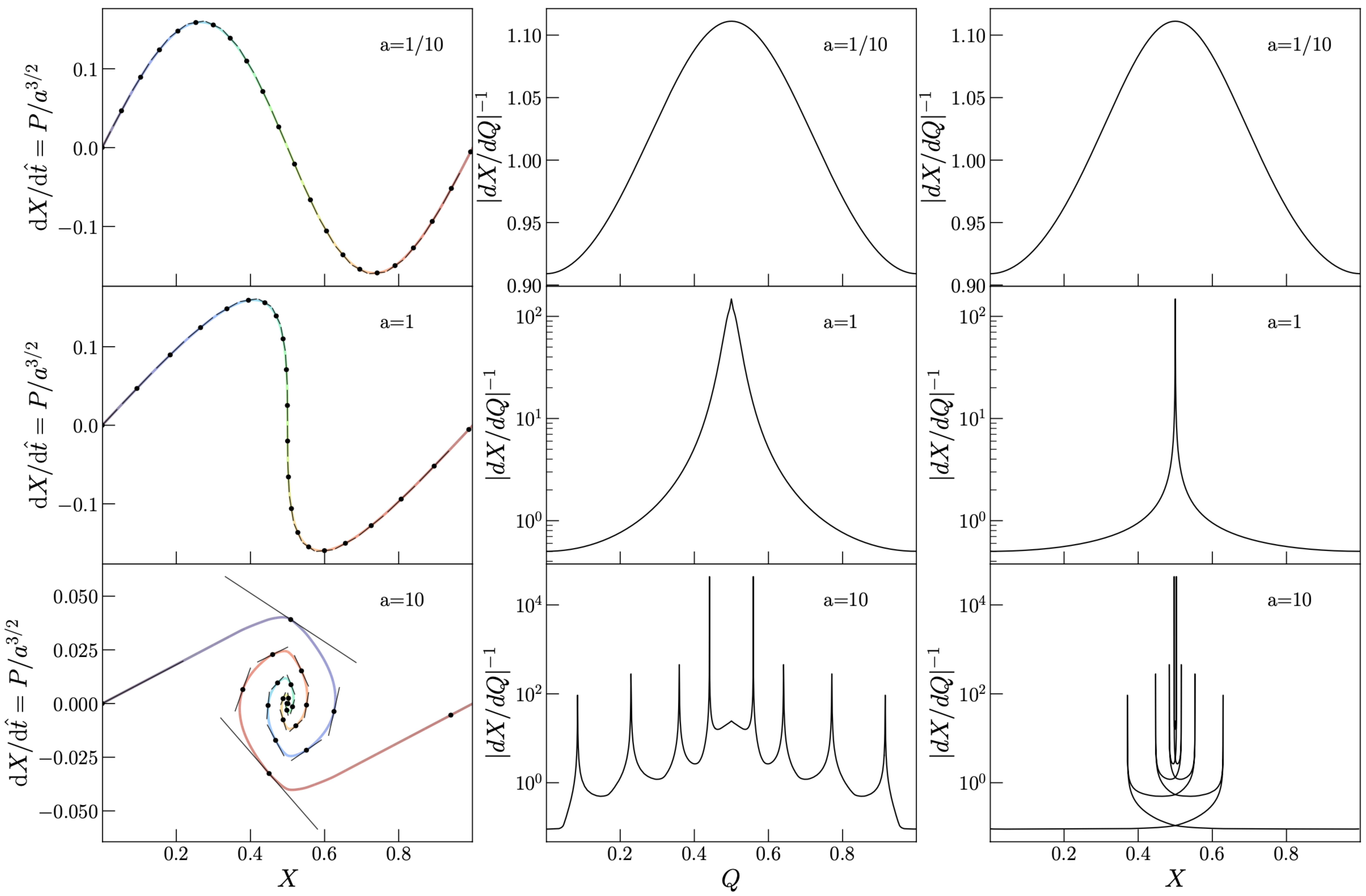
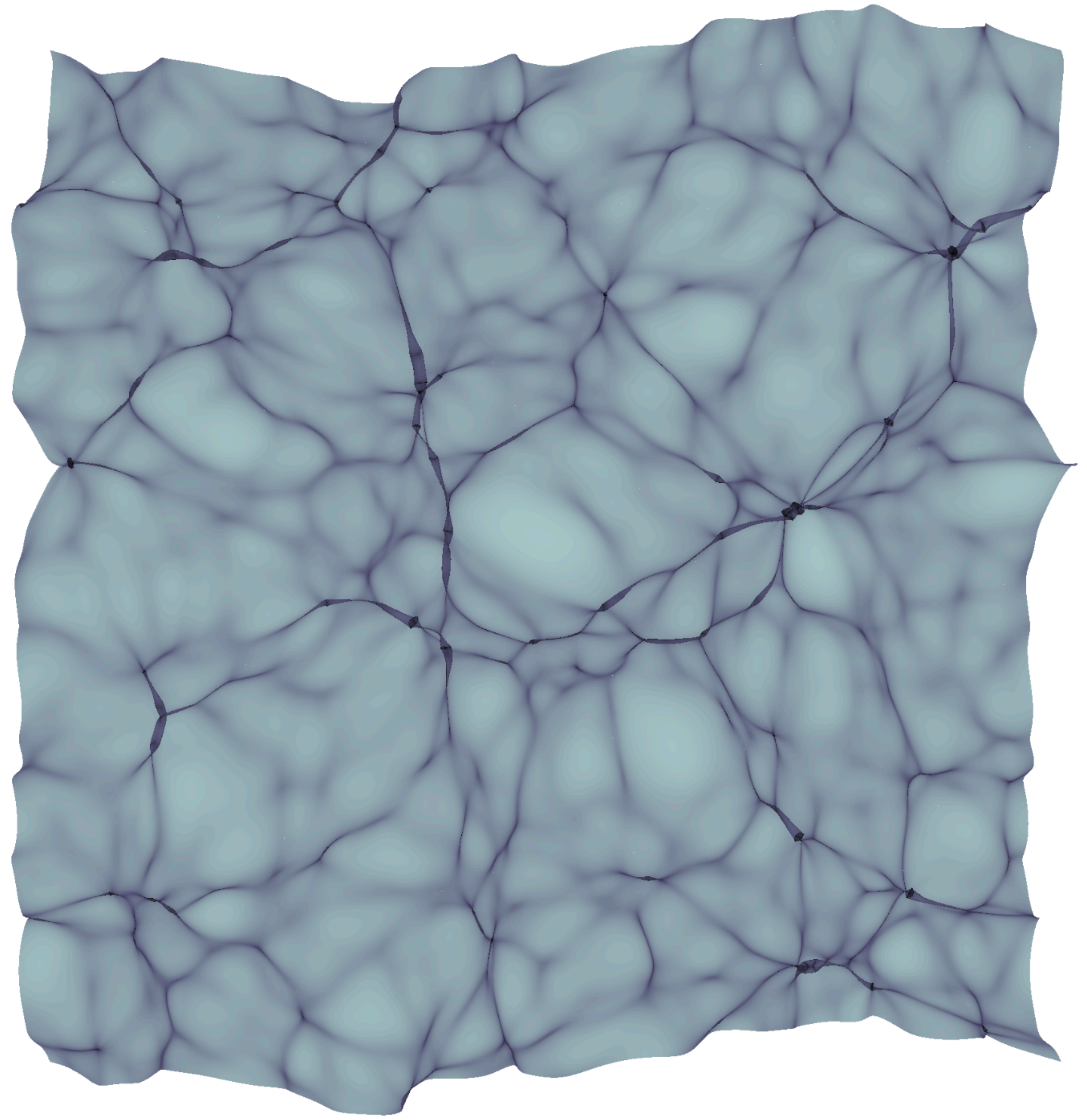


List&Hahn23

Beating discreteness with the DM sheet

N-body just have particles, but there is an underlying manifold structure





Resampling schemes

Abel, Hahn & Kaehler 2012 ***Schandarin et al. 2012***

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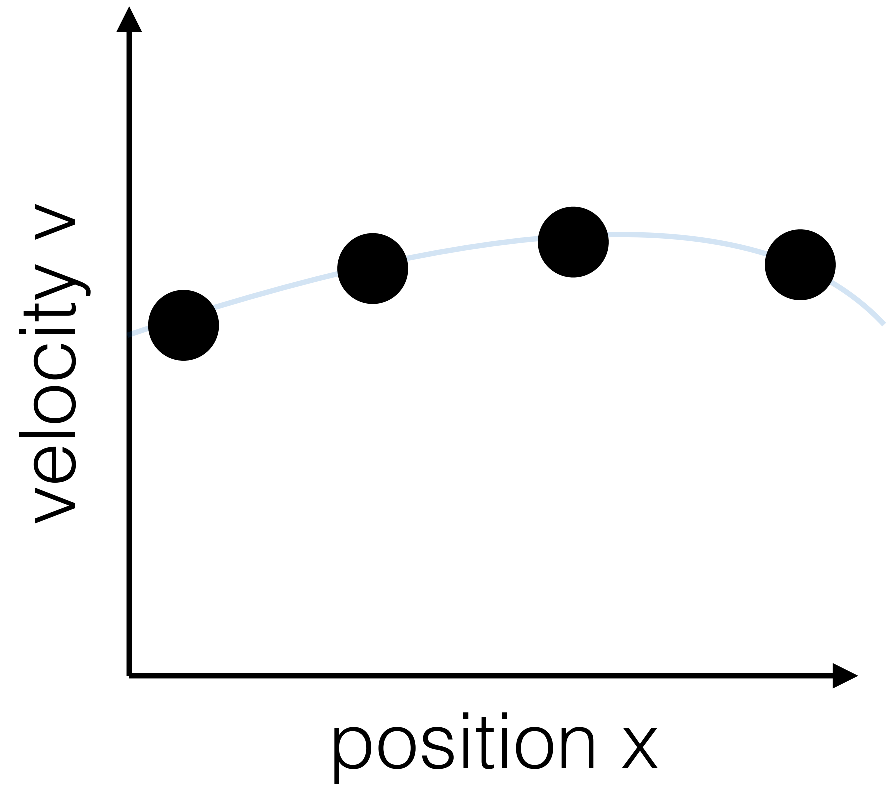
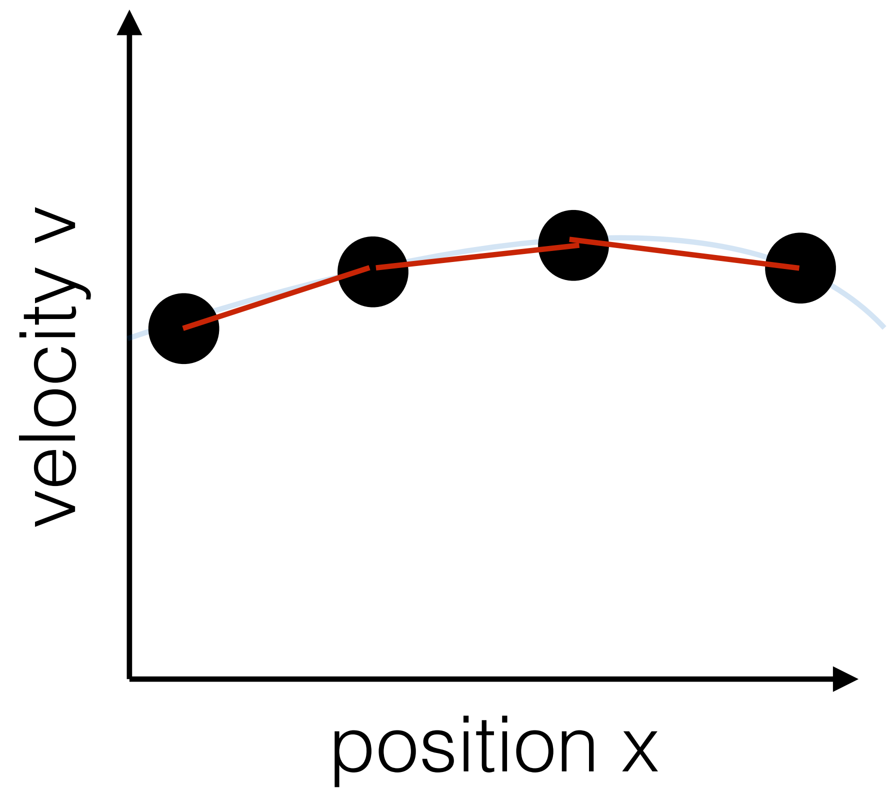
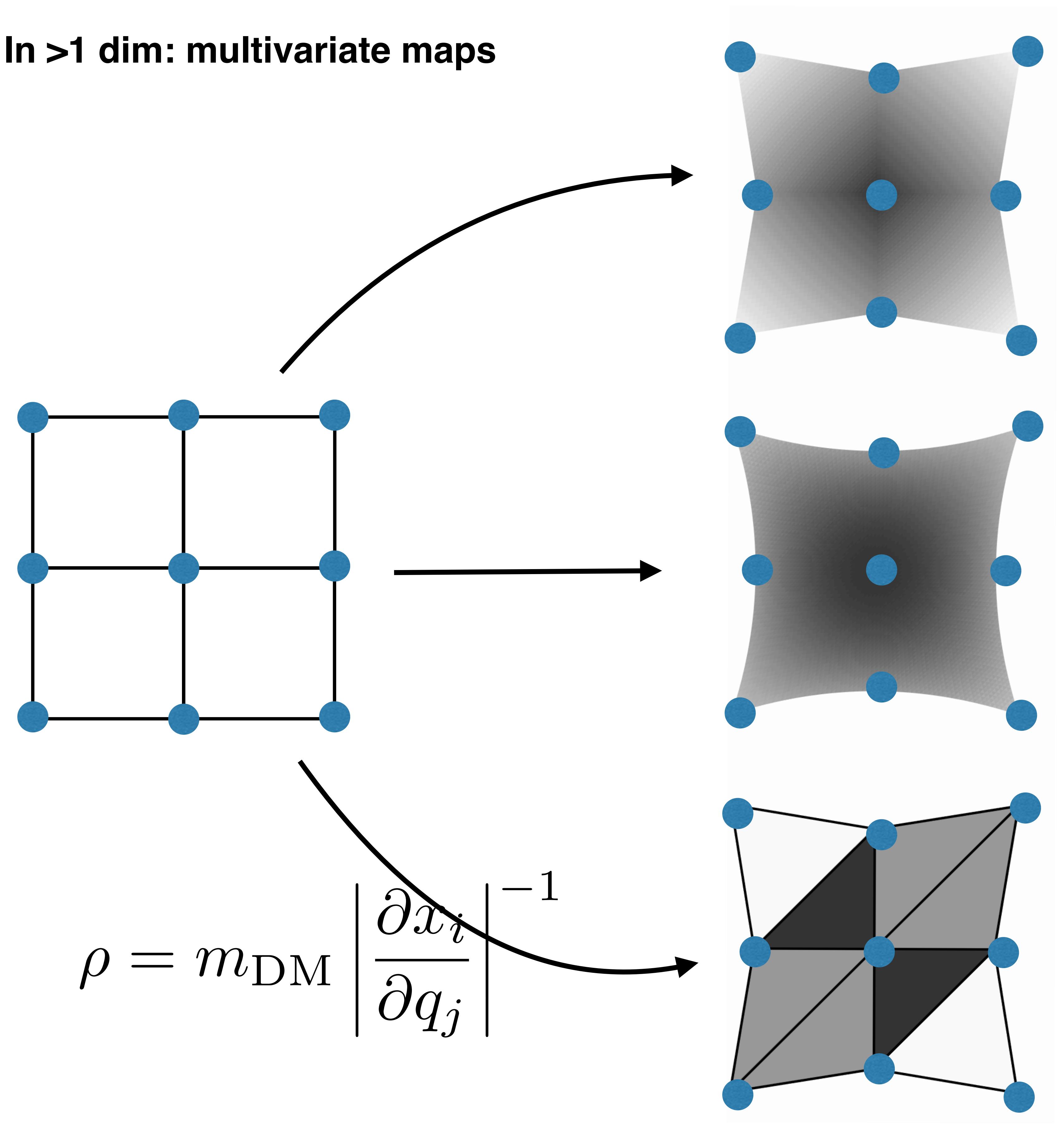


now: connect particles by interpolating functions



In >1 dim: multivariate maps



$$\rho = m_{\mathrm{DM}} \left| \frac{\partial x_i}{\partial q_j} \right|^{-1}$$

k=1

bi/tri-linear

Hahn&Angulo 2016

k=2

bi/tri-quadratic

Hahn&Angulo 2016

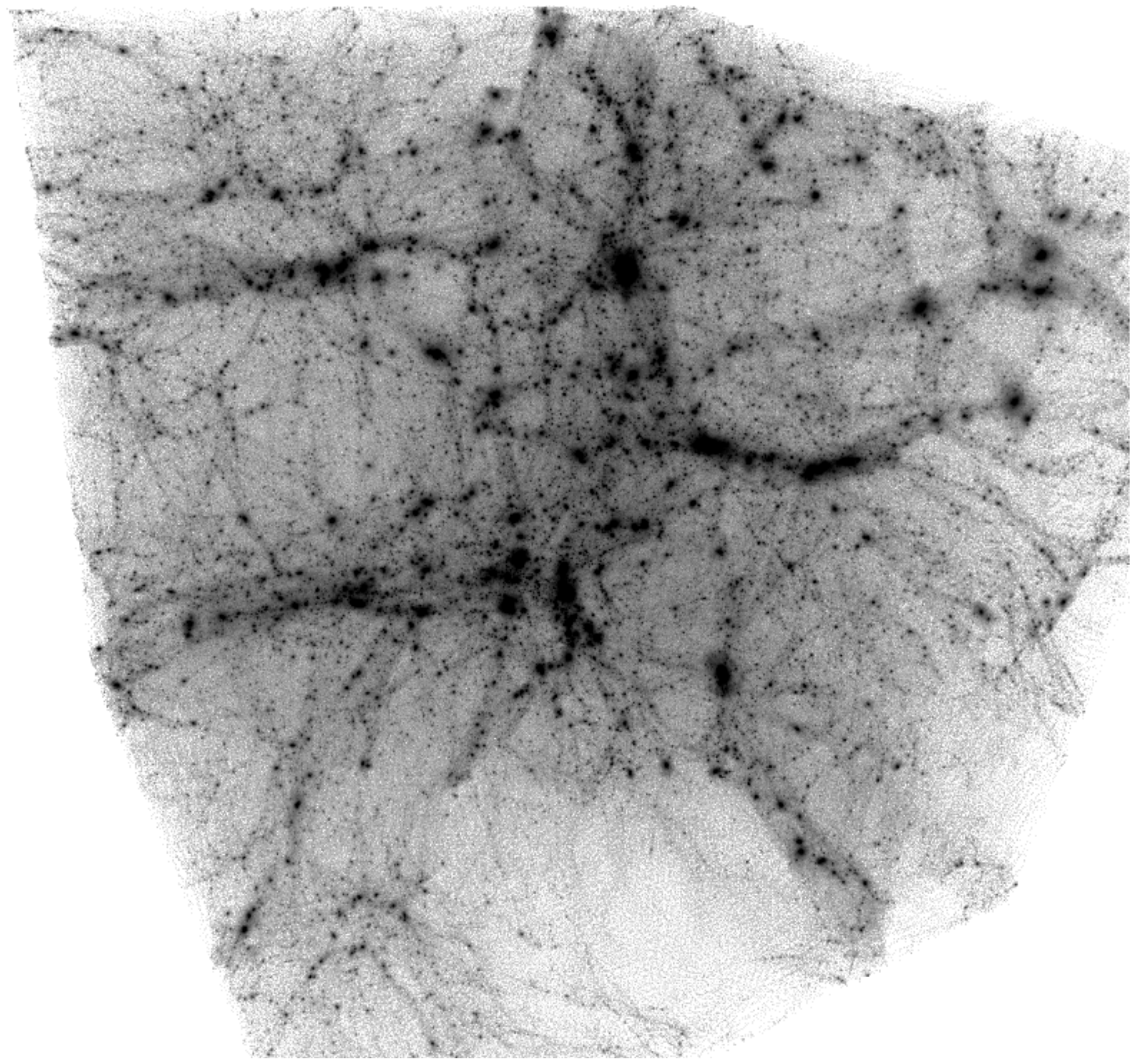
affine map/
simplicial complex

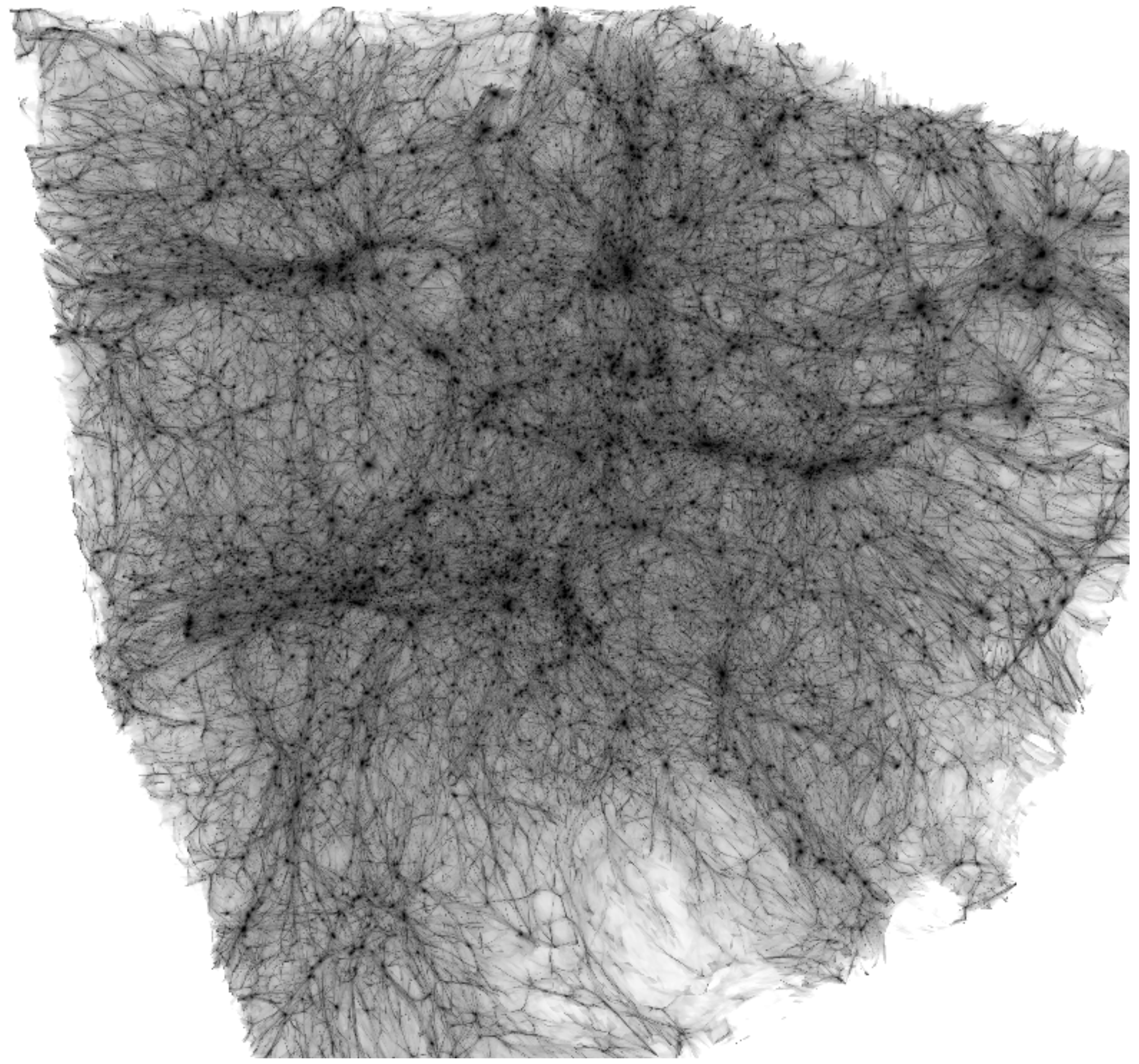
tetrahedral

Abel, Hahn & Kaehler 2012

Schandarin et al. 2012

Sousbie&Colombi 2015
*(also **quadratic** tetrahedra)*





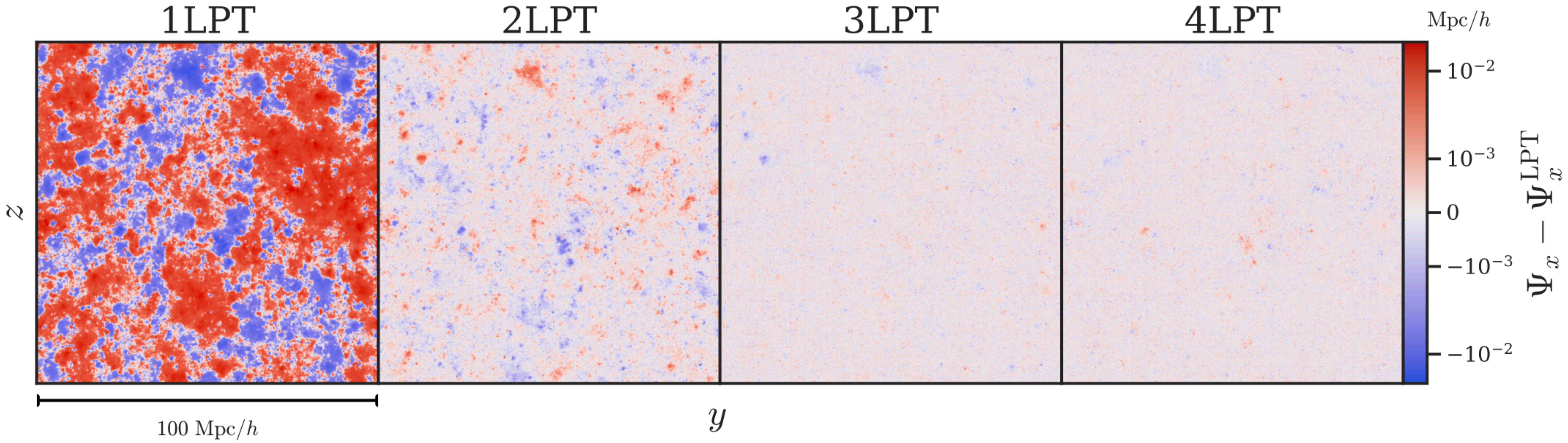
Defines density etc. everywhere in space…

We actually need spectral accuracy for the new method! (List&Hahn 2023b, to be submitted)

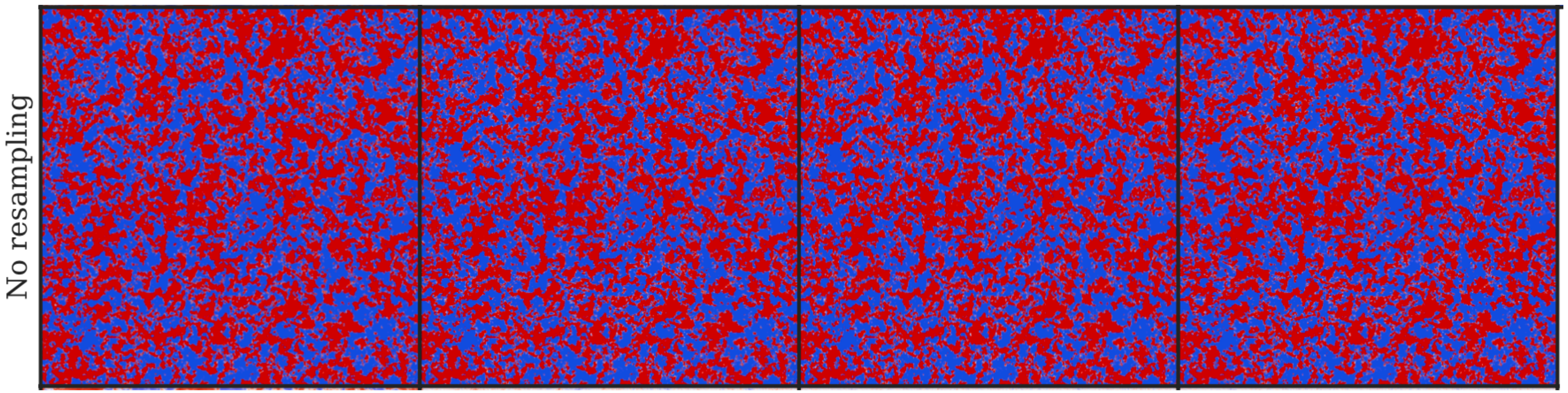
Towards unifying LPT and N-body…

PowerFrog integrator is asymptotically consistent with 2LPT for a→0, can start at a=0 as we do in LPT

Residual of single PowerFrog step from a=∞ to a=0.05, wrt. nLPT:



This is only possible after controlling **all** discreteness effects in the N-body simulation, otherwise:



SUMMARY

- LPT has key role in ICs for cosmological simulations, or as a field-level forward model
- Demonstrated convergence of LPT beyond shell-crossing, slow convergence due to SC singularities
- 3LPT needed for precision era N-body simulations,
  push to late starts to reduce errors
- new way to include multi-fluid perturbations (isocurvature as Lagrangian bias)
- new LPT inspired integrators (beyond ‘FastPM’) for fast simulations
- might ultimately be able to replace LPT initial conditions or LPT in general as they are also UV complete

MUSIC2 monofonIC **https://bitbucket.org/ohahn/monofonic**

single resolution (=only full cosmological volume) version

- direct integration of CLASS
- up to 3LPT, (nLPT exists already, not public just yet)
- PLT corrections
- more accurate treatment of baryons
- new propagator approach for Eulerian baryons
- modular architecture: multi code, easily extensible
- MPI+threads (no more memory limits)
- neutrino version exists (Willem Elbers)
- primordial non-Gaussianity and relativistic 2nd order version exist (Thomas Montandon)