Symmetries, broken or otherwise: soft theorems, black hole perturbation theory & wave dark matter

Lam Hui

Columbia University

a robust way to use highly nonlinear modes

use tidal deformation & nonlinear ringdown to test gravity / probe BH environment

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fuzzy or wavy?

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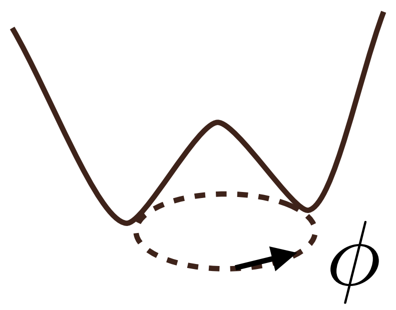
Soft theorems (aka consistency relations)

- They arise from spontaneously broken (nonlinearly realized) symmetries. Recall:



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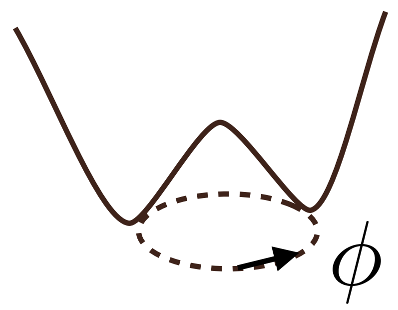

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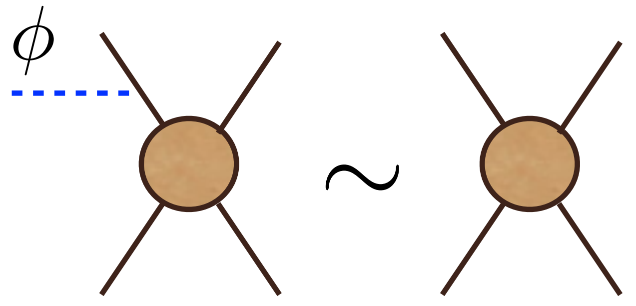

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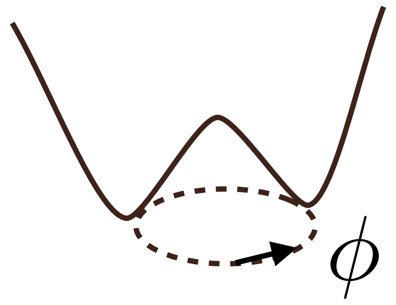

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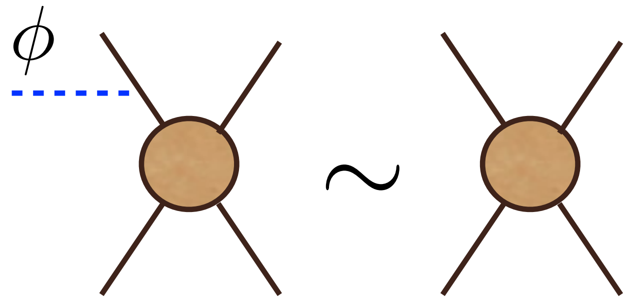

- Soft theorems hold even if 1. the hard modes are deep in the nonlinear regime,
  2. the hard mode observables are galaxies, 3. the observables are in redshift space.

To derive them, one needs to know how the initial conditions transform under the symmetry in question i.e. the form of the soft theorems is initial condition dependent. Thus, verifying soft theorems with data becomes a way to probe initial conditions.
It also can be thought of as checking the symmetries of the dynamics (e.g. equivalence principle for KRPP).

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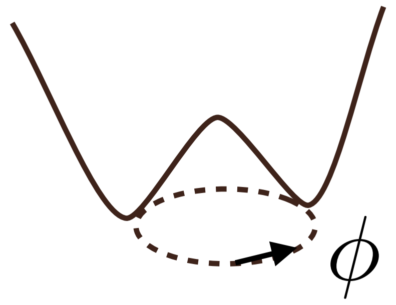

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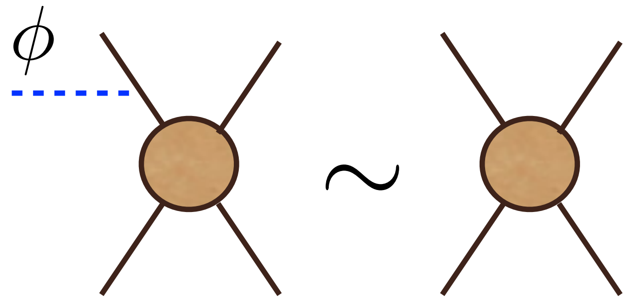

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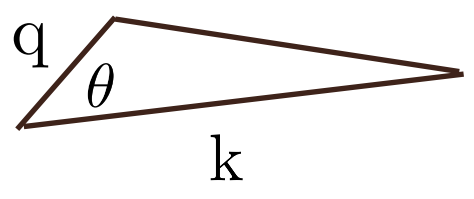
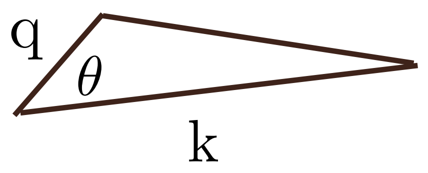
- More concretely: the squeezed bispectrum to power spectrum ratio should have no pole i.e.



$$B(q, k, \theta)/P(q) \sim \cancel{a_{-2}}\, q^{-2} + \cancel{a_{-1}}\, q^{-1} + a_0\, q^0 + ...$$

non − zero if $f_{\rm NL}^{\rm local} \neq 0$



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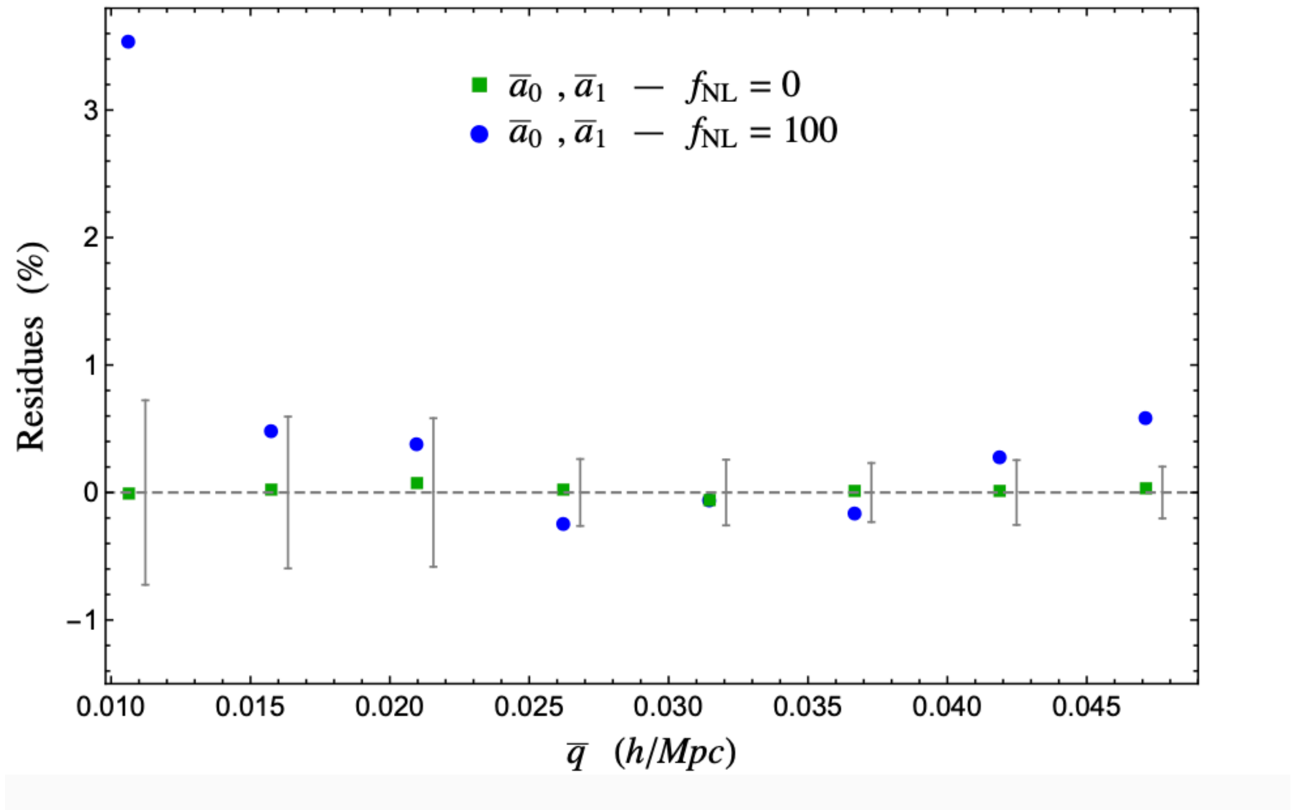
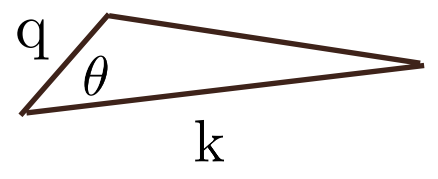


A model of the squeezed bispectrum to power spectrum with no pole fits the simulation results very well if $f_{\rm NL}$ ≈ 0, but not if $f_{\rm NL}$ ≈ 100.

Here, we use k up to 0.65 h/Mpc (where variance is about 10), and q up to 0.06 h/Mpc.

Collaboration with Esposito, Scoccimarro.



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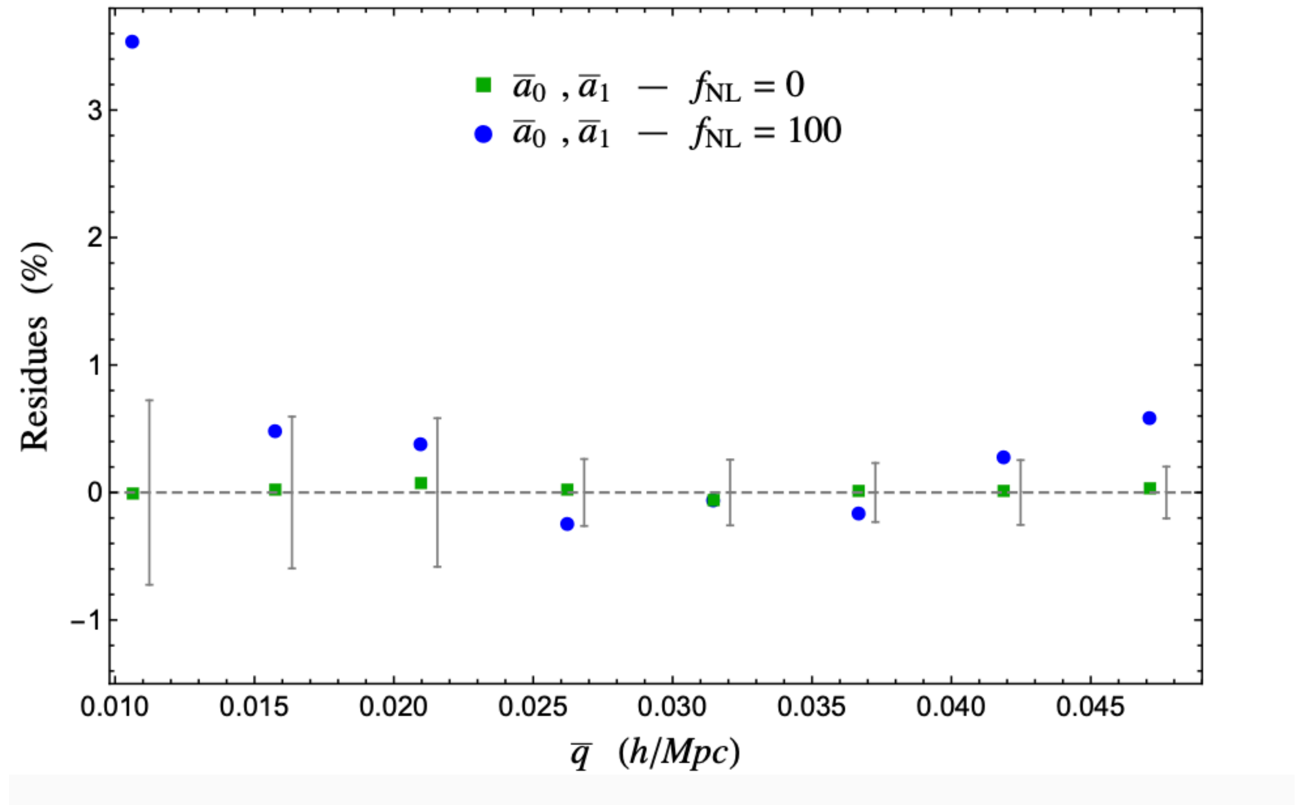

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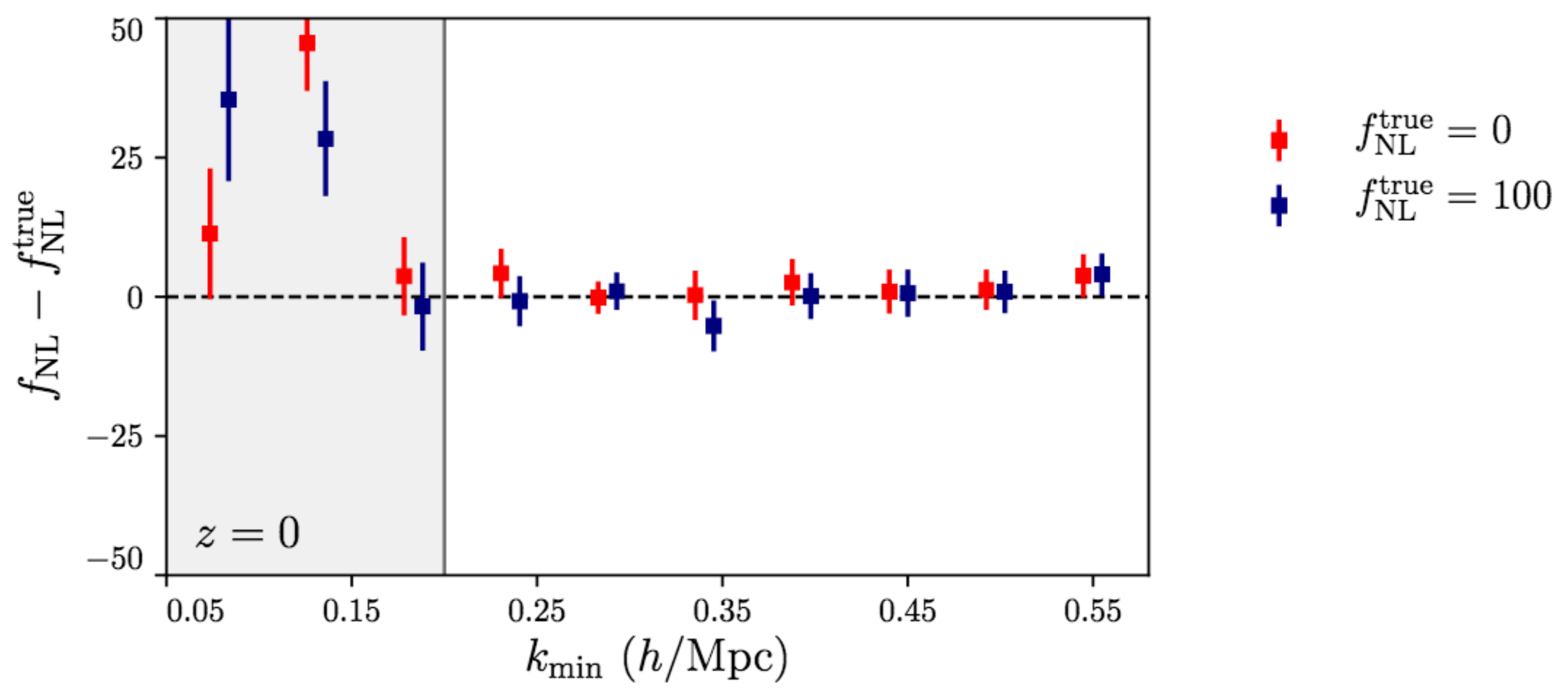

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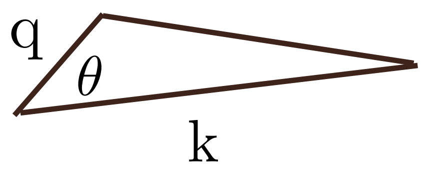

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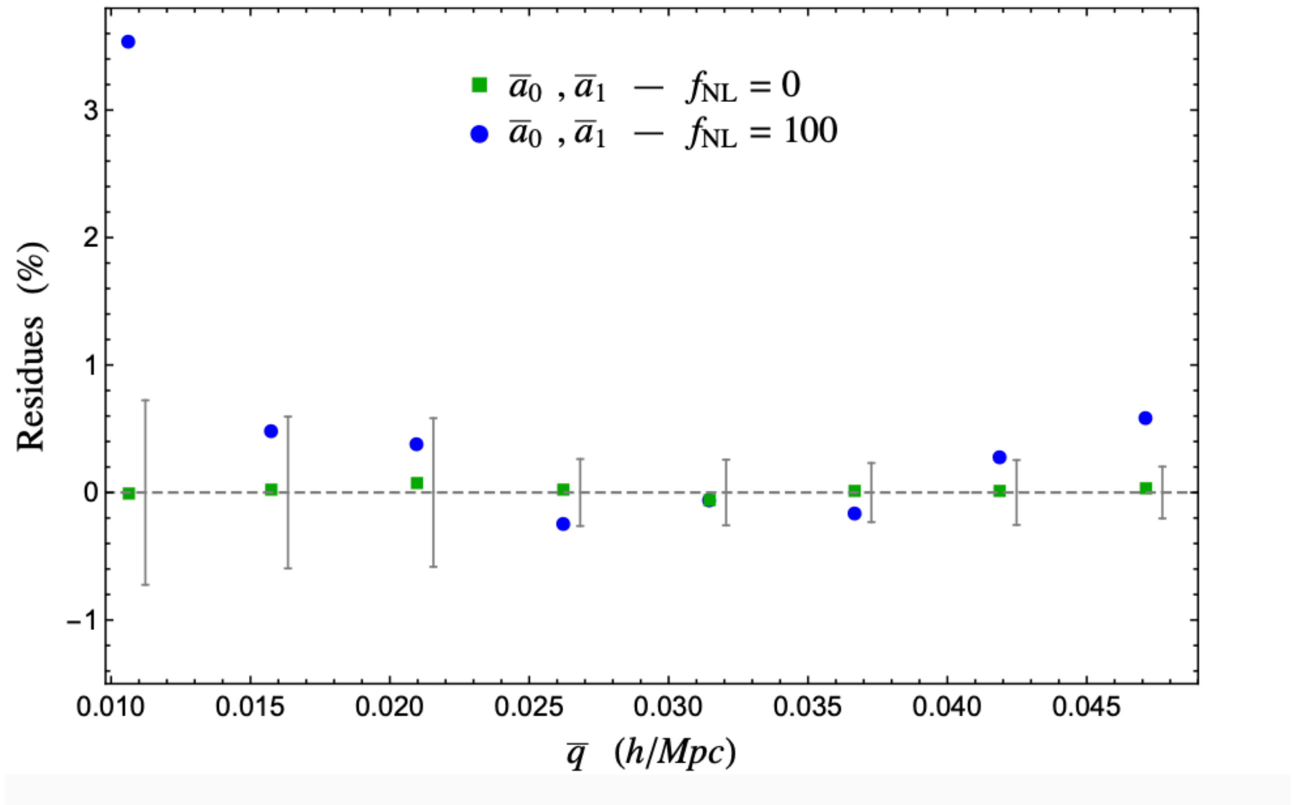

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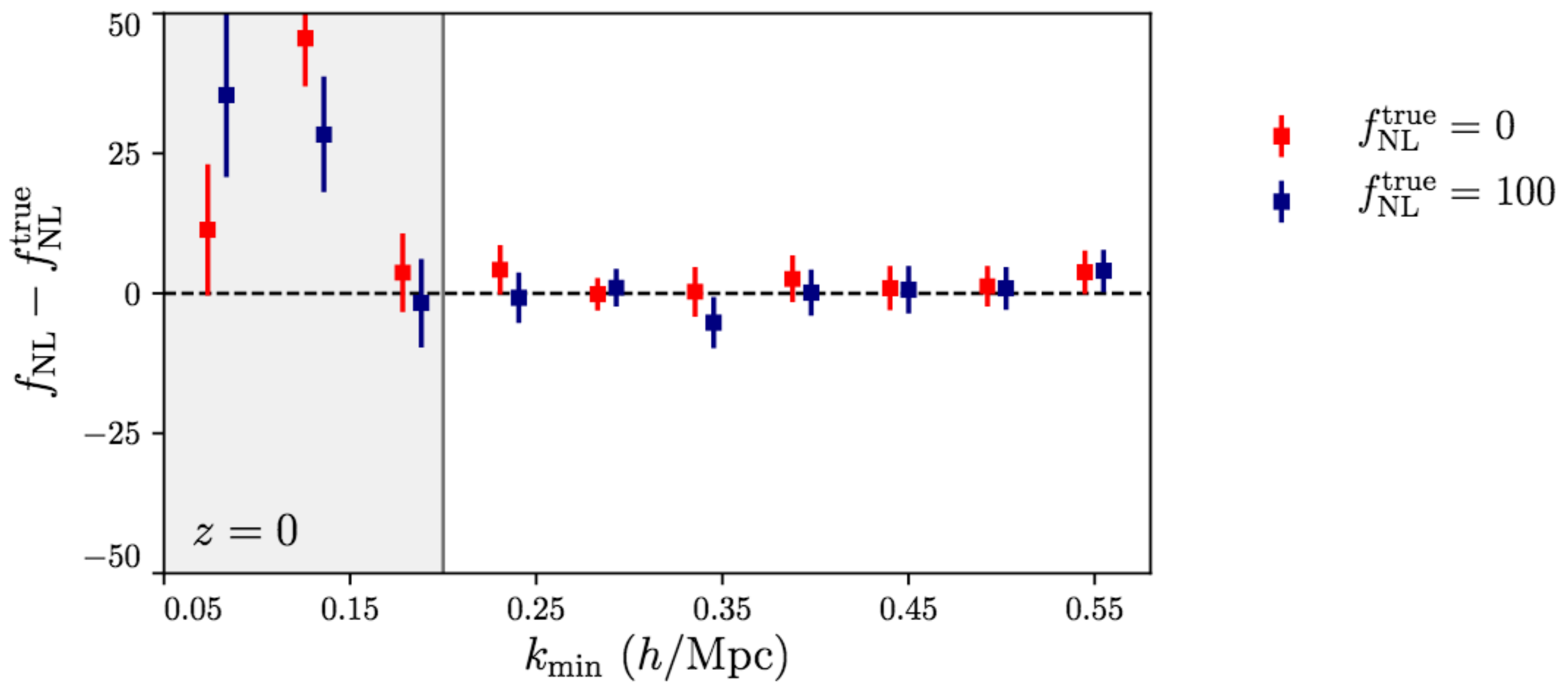

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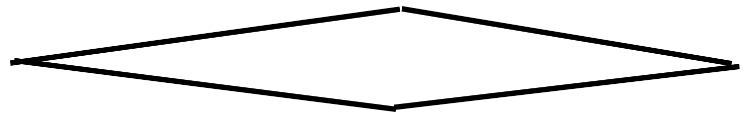
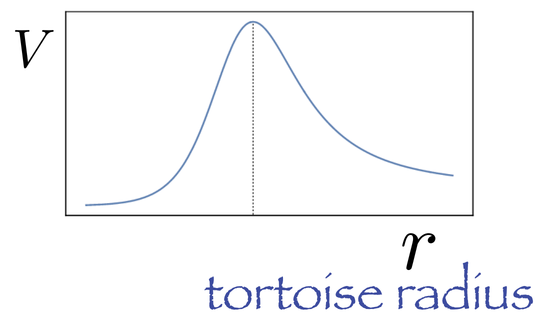
But: error bar on $f_{\rm NL}$ not competitive (i.e. around 10, for a (2.4 Gpc/h)^3 box). This uses mass. If we use halos, shot-noise will be important, but scale dependent bias will also become relevant. To be explored: collapsed trispectrum:



See also LH, Joyce, Komissarov, Parmentier, Santoni, Wong on spontaneously broken boost.

Black hole perturbation theory

- Consider small perturbations of a black hole: $g = g_{\mathrm{BH}} + \phi \longrightarrow [\partial^2 + V]\phi = 0$ (linearized Einstein equation)



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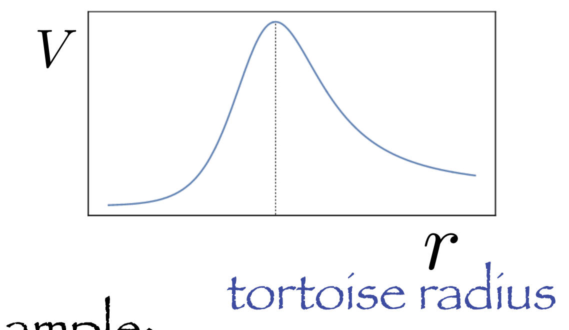
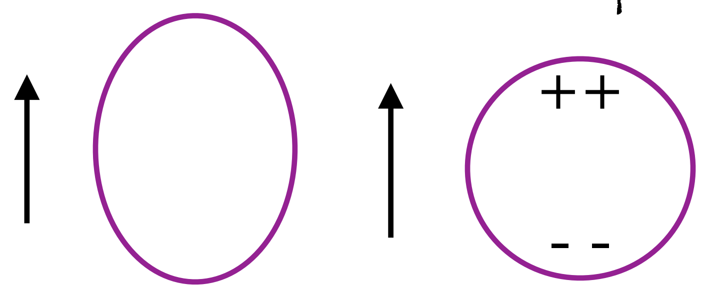


- We can use this to study tidal deformation i. e. imagine an object in a static, external tidal field $\phi \sim r^\ell$ (expanding in spherical harmonics), tidal deformation induces a response tail: $\phi \sim 1/r^{\ell+1}$

$$\phi \sim r^\ell + ... + \frac{\lambda}{r^{\ell+1}} + ... \quad , \quad \lambda \sim \text{Love number} \;\sim\; \text{size}^{2\ell+1}$$

EM example:



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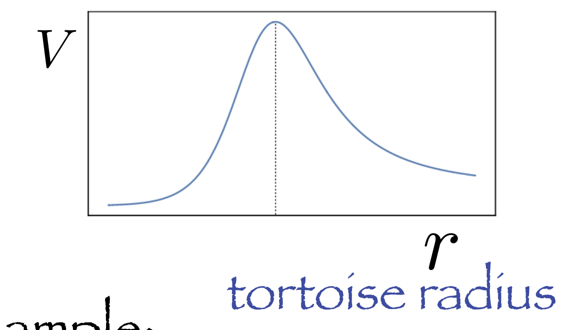

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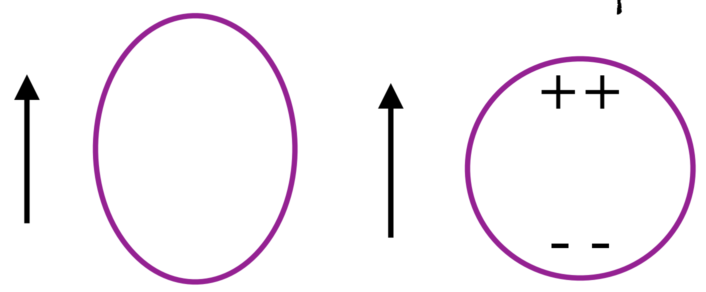

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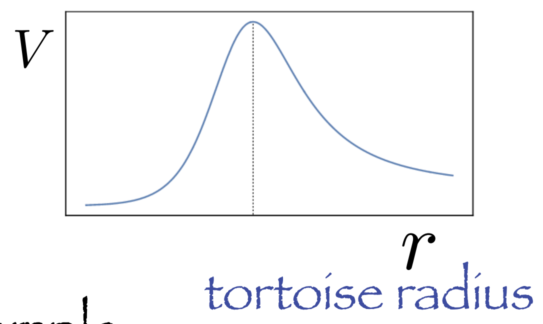

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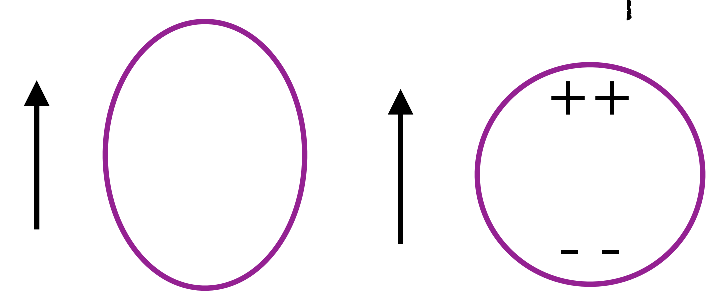

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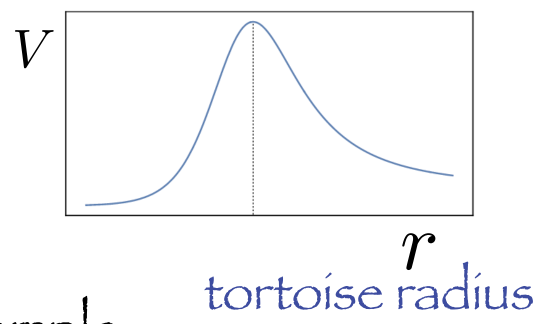

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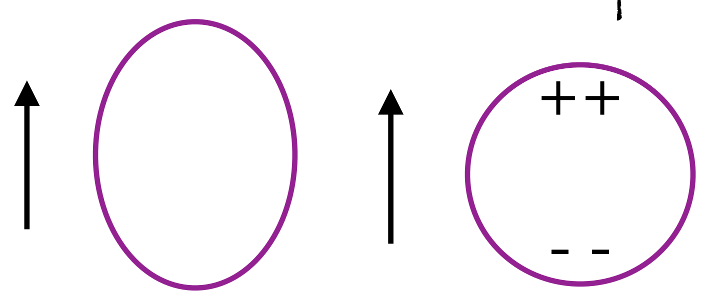

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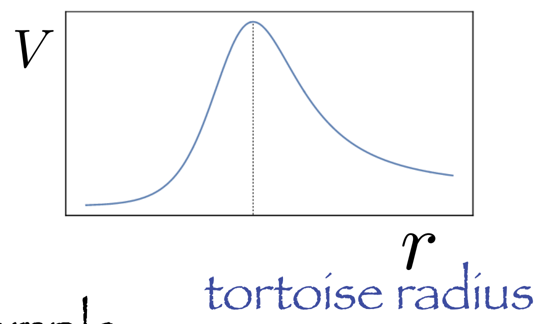

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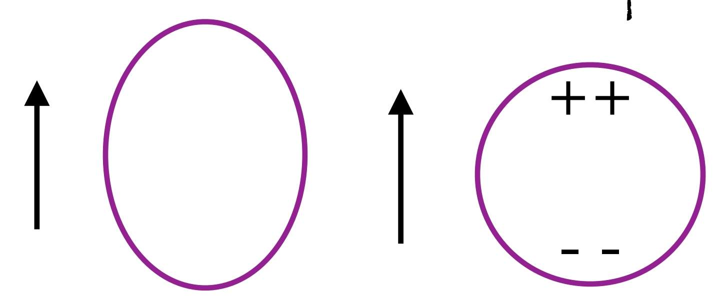

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- Interesting questions to explore: What are the symmetries for dynamical perturbations? For nonlinear perturbations? Can we understand I-Love-Q relations of neutron stars as consequence of weakly (or spontaneously) broken symmetries?

Collaboration with Joyce, Penco, Santoni, Solomon, and with Berens, Sun.

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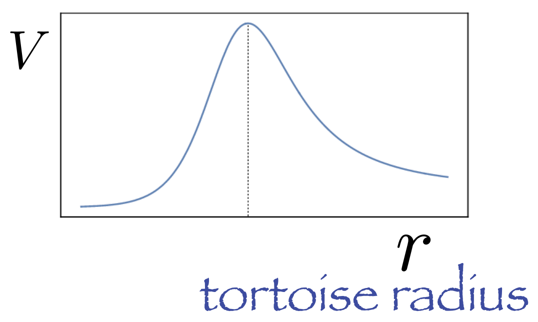

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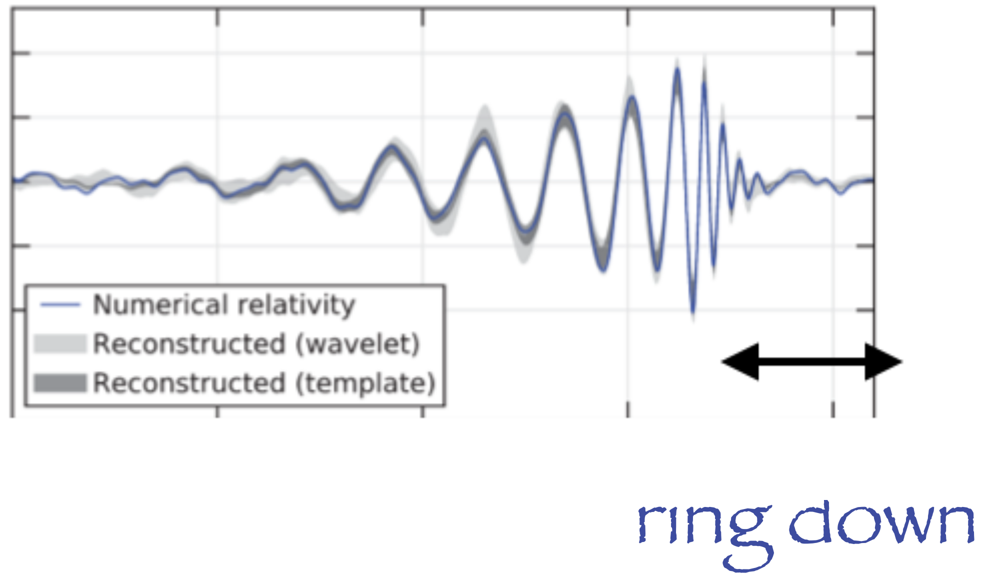

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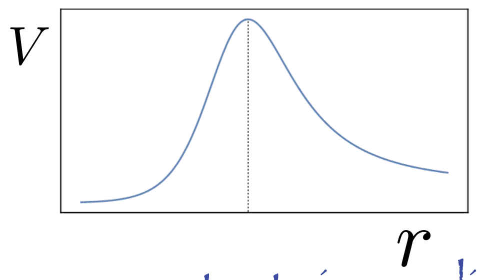

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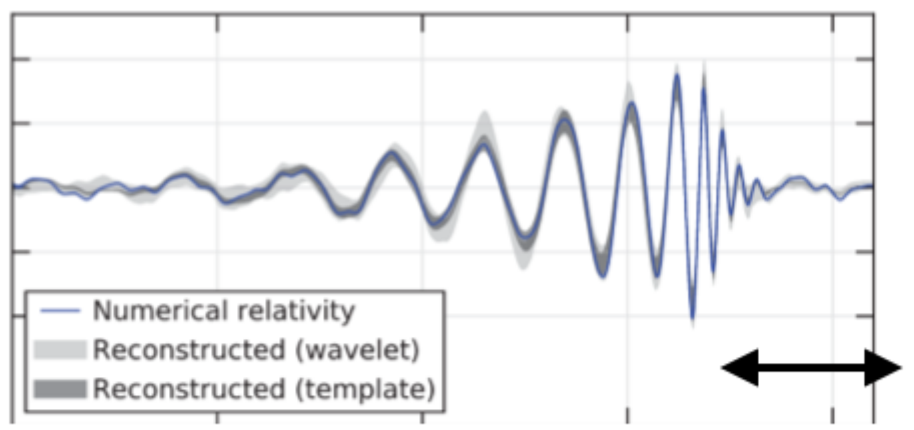

ring down

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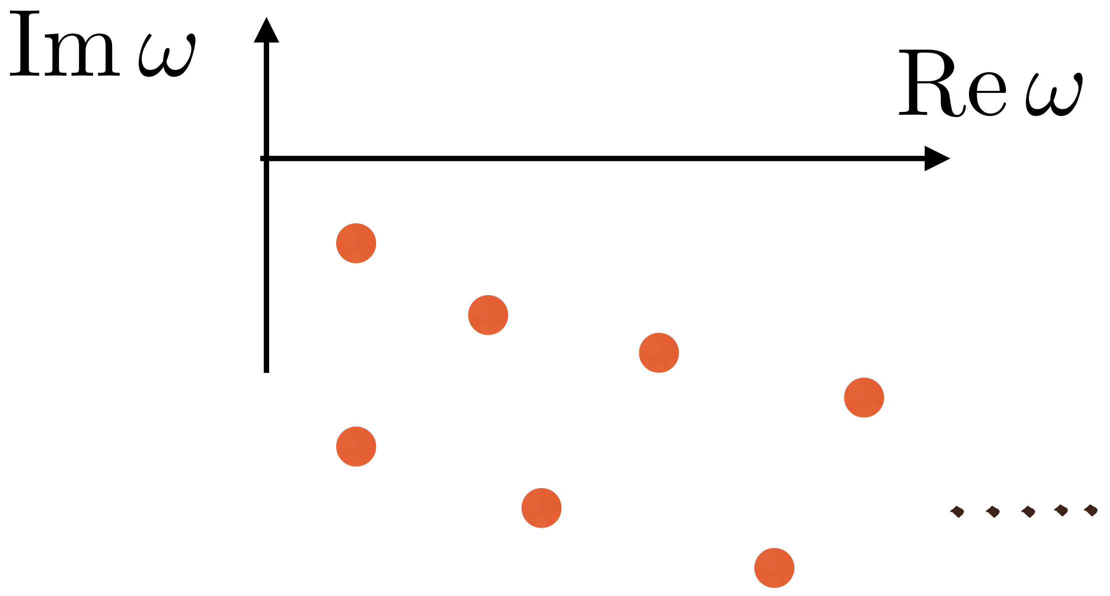

ω labeled by ℓ, m, n, e.g. ω_{220}

Typically focus on frequencies rather than amplitudes. The quasi-normal mode (QNM) spectrum tells us a lot about the BH space-time, analogous to seismology. See Chandrasekhar.

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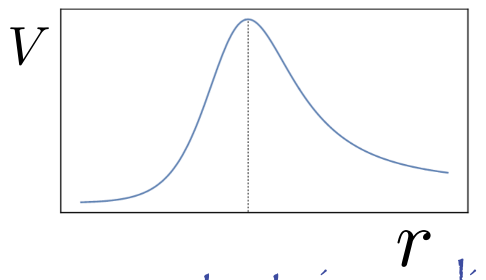

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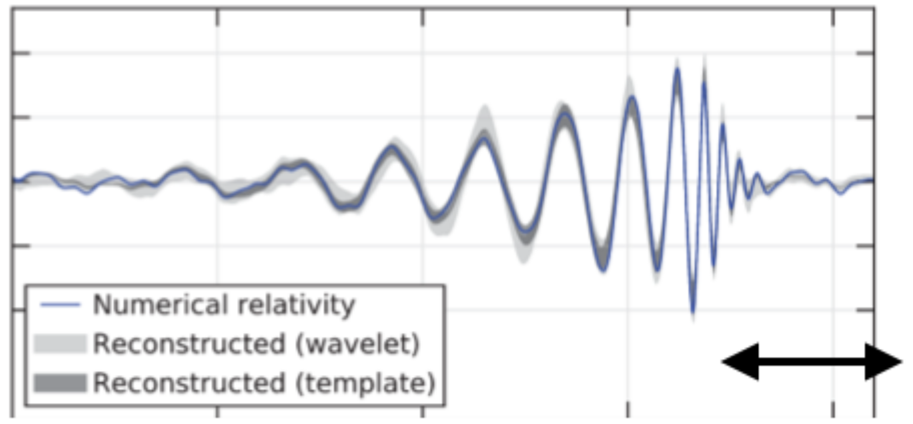

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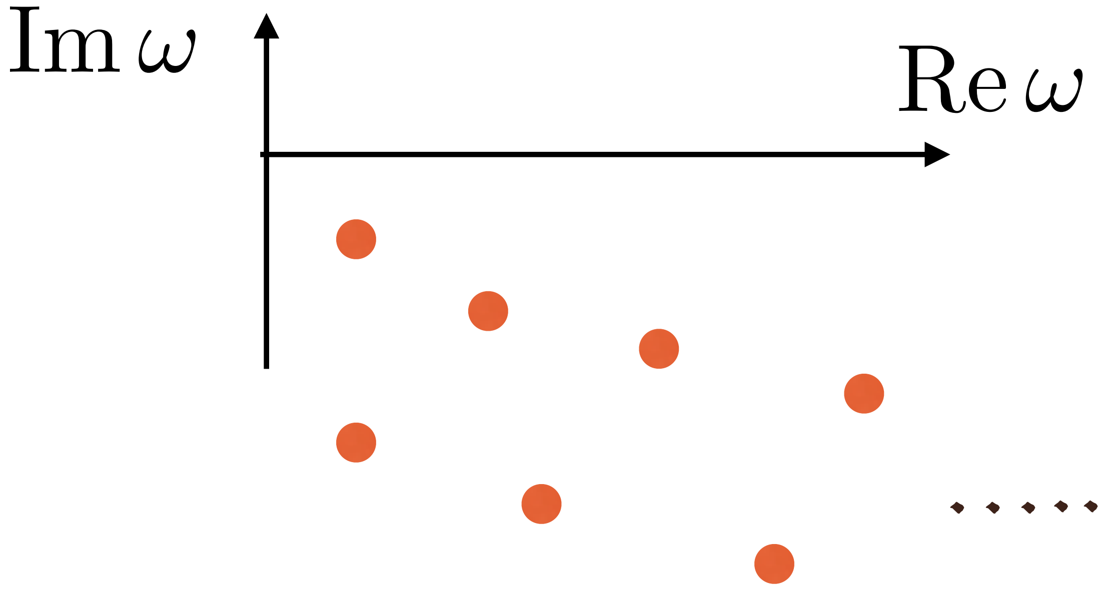

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- What if we go to second order: $[\partial^2 + V]\phi \sim \partial^2\phi^2$ $\qquad$ Write $\phi = \phi^{(1)} + \phi^{(2)} + \ldots \longrightarrow$

$$[\partial^2 + V]\phi^{(1)} \sim 0$$
$$[\partial^2 + V]\phi^{(2)} \sim \partial^2\phi^{(1)2}$$

Not surprising that pairs of linear QNMs source a quadratic QNM e.g.:

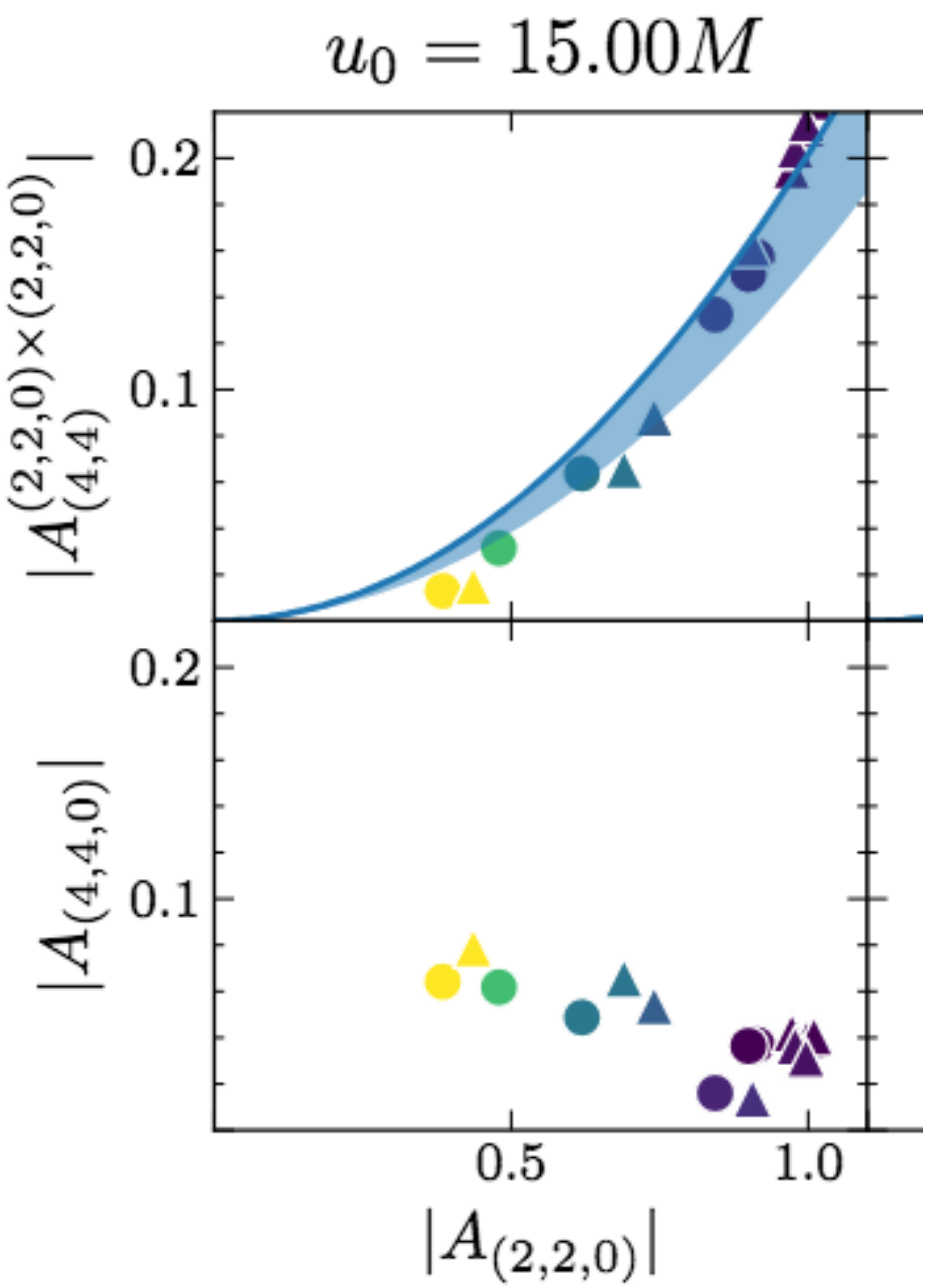
$$\phi^{(1)} \text{ contains } e^{-i\omega_{220}t} \longrightarrow \phi^{(2)} \text{ contains } e^{-i\,\underline{2}\,\omega_{220}t}$$

linear QNM $\qquad$ quadratic QNM

Expect quadratic quasi-normal modes from pairs of linear quasi-normal modes.

e.g. from pairs of ω_{220} we get quadratic $\omega = 2\omega_{220}$

We searched for quadratic QNM in black hole merger simulations, and verified (220)x(220) amplitude goes as square of (220) amplitude.



Colllaboration with Mitman, Lagos, Stein, Ma et al.

Colllaboration with Lagos.

See also Cheung et al.

Black hole perturbation theory: tidal deformation and (nonlinear) ring-down

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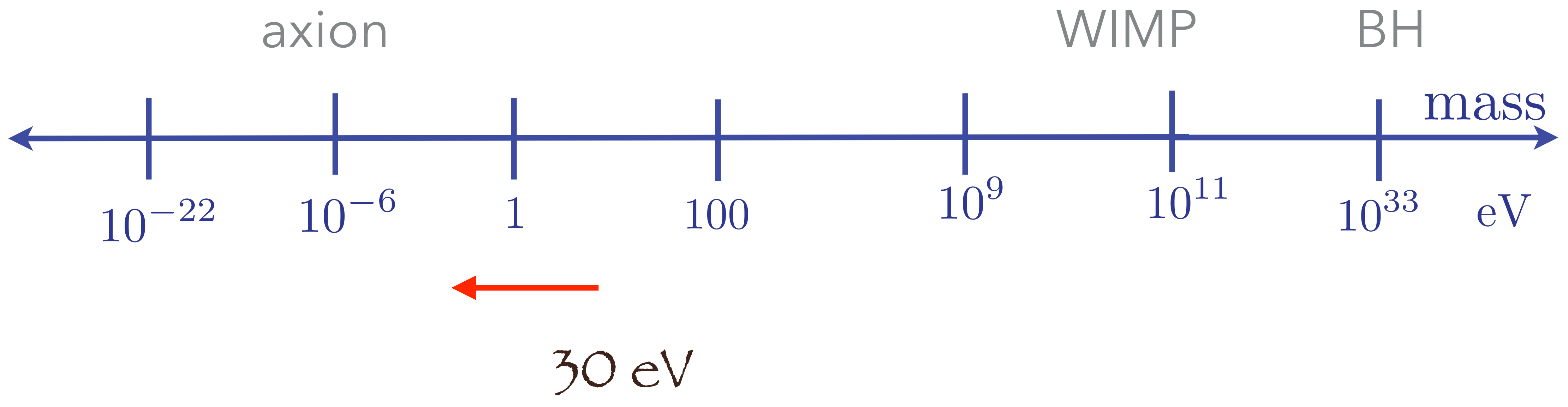
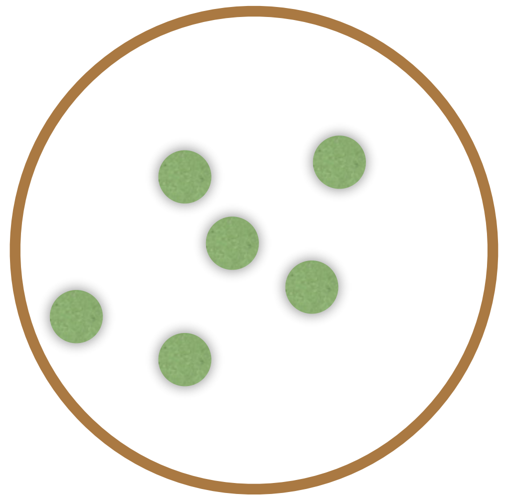
- Understanding nonlinear ringdown helps (a) provide a more accurate modeling of the ring-down, (b) get us closer to the merger time where signal to noise is highest, (c) test the nonlinear structure of perturbations around BH (the relative amplitude between $\phi^{(2)}$ and $\phi^{(1)}$ becomes useful to look at).

Wave dark matter

(1/mv)

- When dark matter mass m is below about 30 eV, its de Broglie wavelength > inter particle separation.





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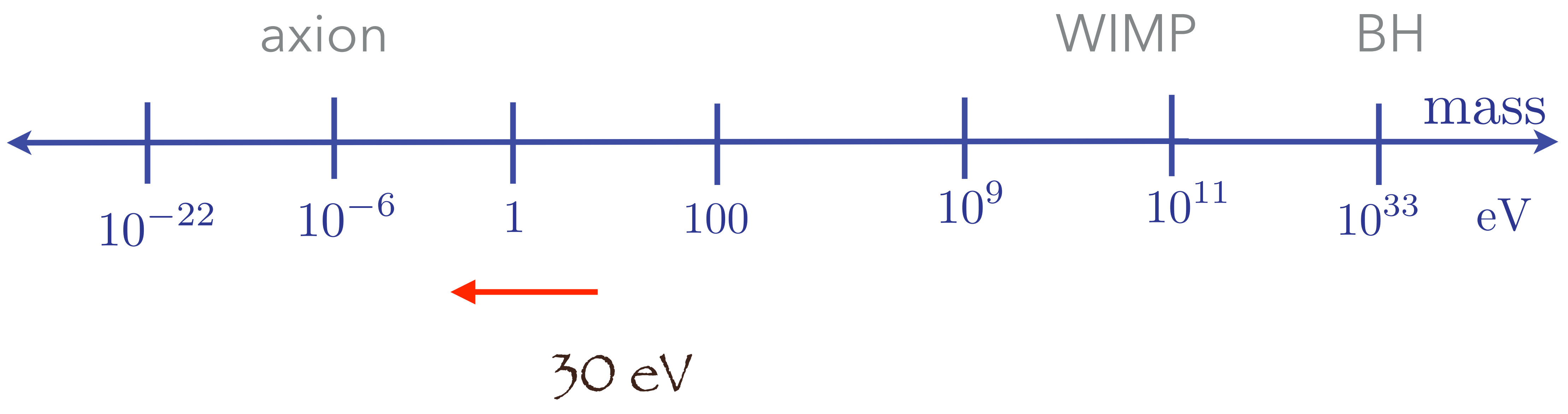
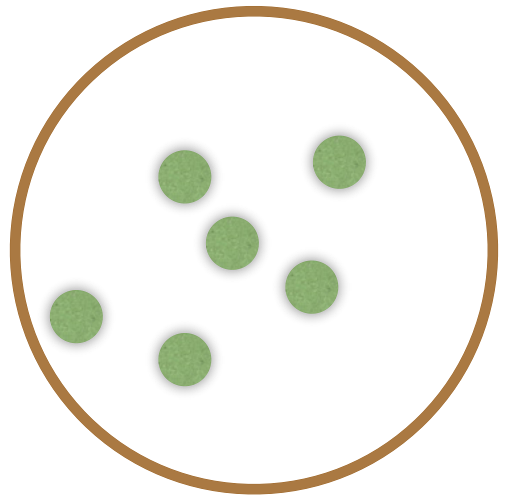
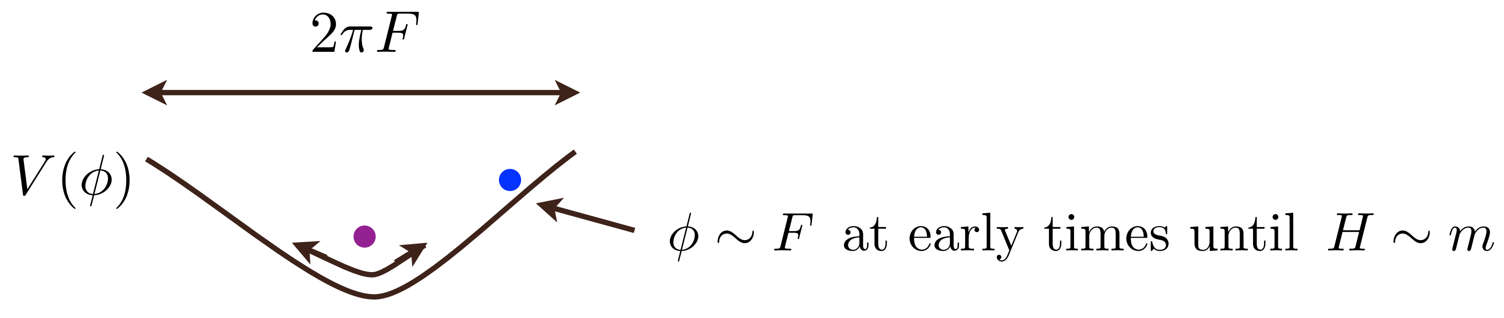




- The fuzzy limit (Hu, Barkana, Gruzinov) is 10^{-22} eV . An interesting motivation is relic abundance of axion/axion-like particle from misalignment mechanism (Preskill, Wise, Wilczek; Abbot, Sikivie; Dine, Fischler):

$$\Omega_{\rm matter} \sim 0.1 \left(\frac{F}{10^{17}\,{\rm GeV}}\right)^2 \left(\frac{m}{10^{-22}\,{\rm eV}}\right)^{1/2}$$

m = axion mass, F = axion decay constant



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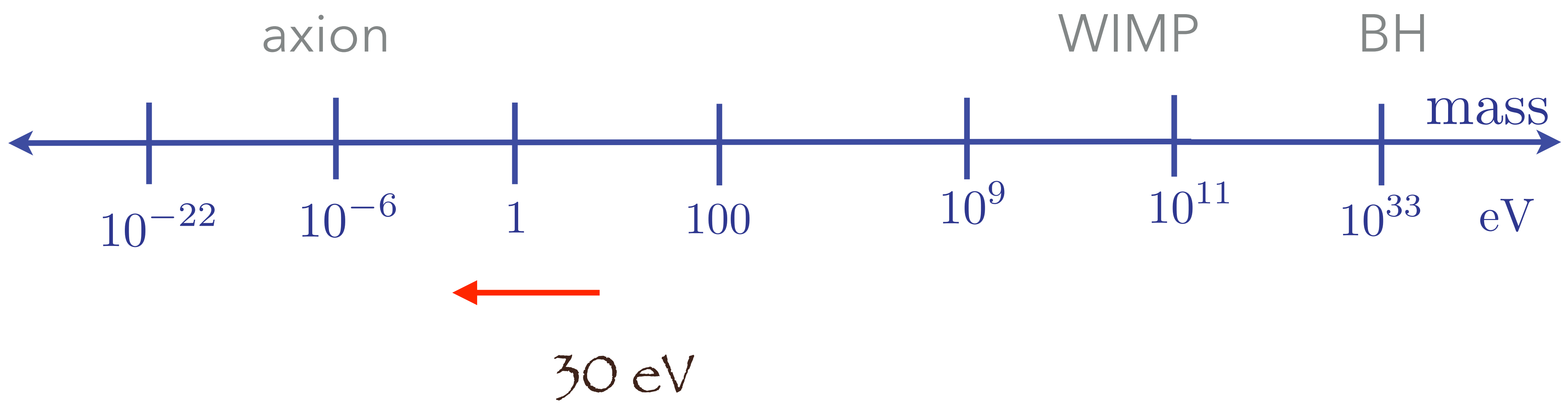

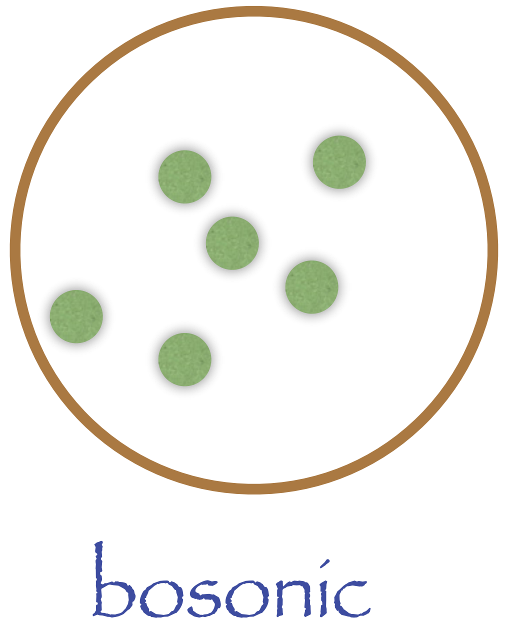

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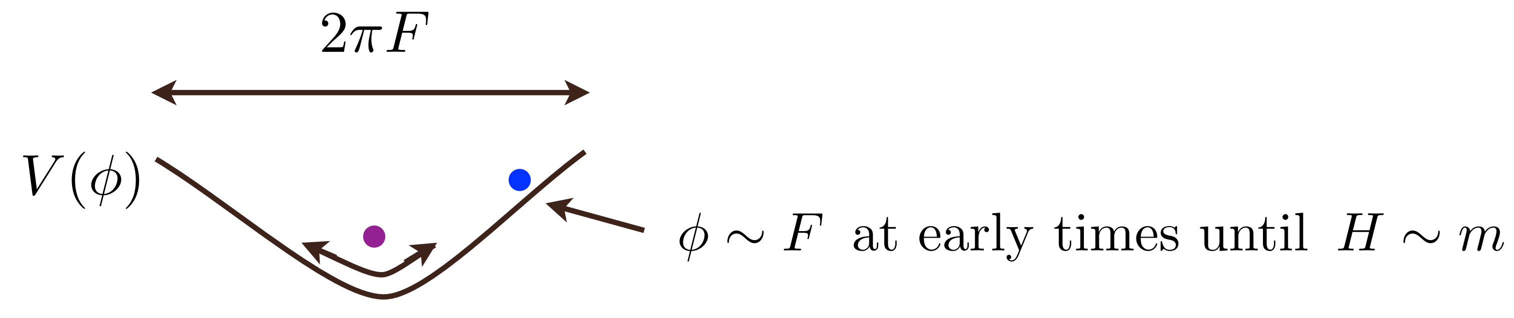
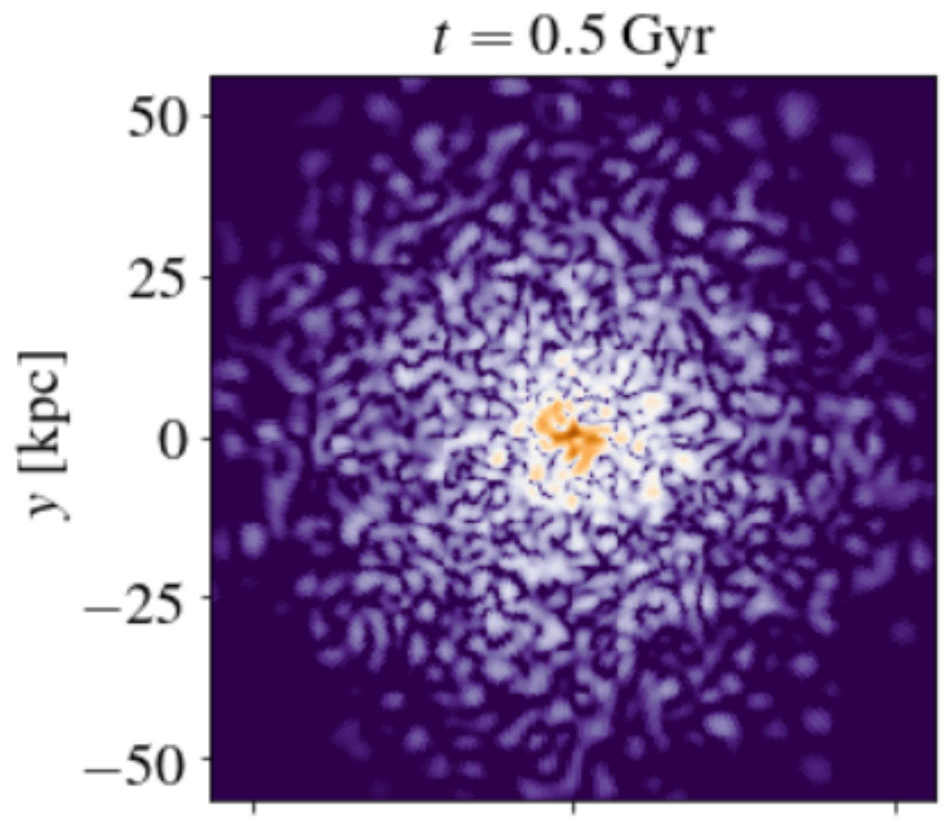


- Don't obsess about 10^{-22} eV ! Wave dark matter need not be fuzzy to be interesting e.g. inevitable time varying wave interference substructure on de Broglie scale:

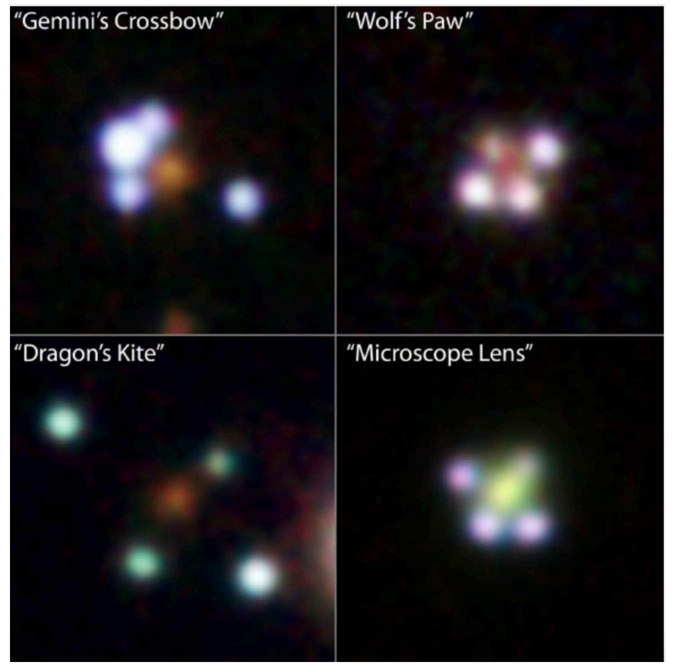


Wave dark matter: a few observations on existing constraints

- The existing constraints should be taken seriously. This means one should understand the assumptions behind them.

e.g. from lensing (flux anomaly):

Schutz 2020 rules out $m < 2.1 \times 10^{-21}$ eV for predicting too little substructure.
Laroche et al. 2022 rules out $m < 3 \times 10^{-21}$ eV for predicting too much substructure.
Correctly accounting for wave interference substructure? Effect of baryons?



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Effect of tidal stripping?

e.g. from Lyman alpha forest, etc.

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  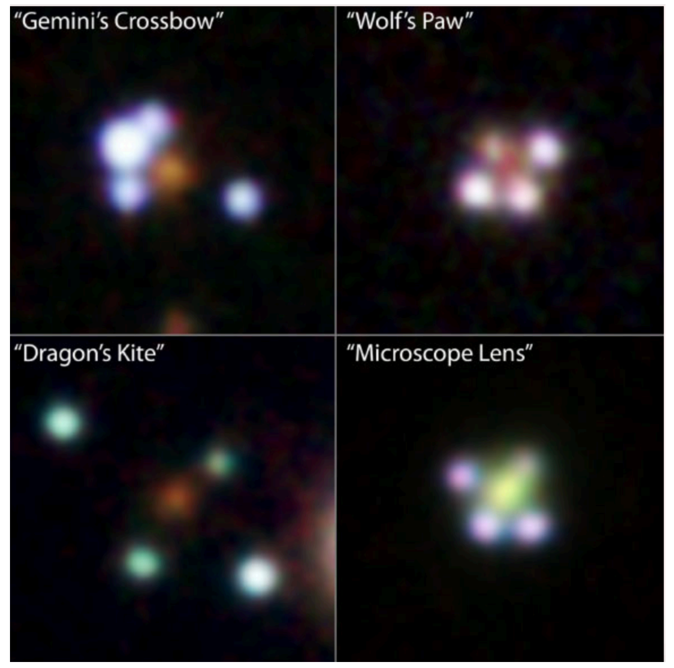

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- Strategies:

1. Understand what wave dark matter really predicts.
2. Wave dark matter need not be fuzzy to be interesting. Thus keep pushing the limits on m. Can they be further improved?
3. Because each astrophysical constraint comes with its own limitations, more variety of constraints is useful (e.g. Kim's talk).
4. From a string theory point of view, where axion-like particles arise generically as zero-modes of higher form fields, there could well be multiple axion/axion-like particles at play (e.g. Mateja 's talk).

   If interested, come to discussion organized by Mateja, Luis. Also, see ARAA review on wave dark matter.

a robust way to use highly nonlinear modes

use tidal deformation & nonlinear ringdown to test gravity / probe BH environment

Symmetries, broken or otherwise: soft theorems, black hole perturbation theory & wave dark matter

fuzzy or wavy?

Lam Hui

Columbia University