Search for glueballs in the presence of dynamical quarks

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Glueballs in Lattice QCD

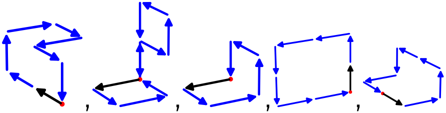
Bound states of only gluons arising from their self-interaction. Experimental detection is difficult due to decays and mixing with mesons. → "Experimentally (...) their status remains unclear and controversial" [F. Brünner and A. Rebhan, (2015)]
In the **lattice**:

- ✓ Spectrum is well-established in pure Yang-Mills theory. [Morningstar and Peardon (1999), Chen *et al.* (2005)]
- × Correlations from purely gluonic operators are heavily affected by a **signal-to-noise** problem. [Lepage (1989)]
- × **Large statistics** are needed to achieve precision similar to mesonic operators.
- × With **dynamical quarks**, glueballs either mix with or decay into mesonic states, making identification **difficult**.

Glueballs in Lattice QCD

Glueballs with defined quantum numbers J^{PC} in the continuum are often studied in the lattice with purely gluonic operators.

- Subduction relates $J^{PC} = 0^{\pm\pm}, 1^{\pm\pm}, 2^{\pm\pm}, \ldots$ with 20 lattice irreps $R = A_1^{\pm\pm}, A_2^{\pm\pm}, E^{\pm\pm}, T_1^{\pm\pm}, T_2^{\pm\pm}$.
- Simplest purely-gluonic operators are spatial Wilson loops:



, etc...

- Different shapes allow to access different irreps R as well as sample different spatial structure.
- Link smearing reduces UV-fluctuations and improves overlap with low-lying states.

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Glueballs in Lattice QCD

Including dynamical quark means that new iso-singlet states with quarks on the valence are possible, e.g $\bar{q}q$, and the spectrum becomes more dense.

- Lattice meson operators $\bar{q}\Gamma q$ can have the same lattice quantum numbers as the purely-gluonic operators.
- Energy eigenstates of each channel receive contributions from both types of operators $\rightarrow$ Both types must be included to search for glueballs.
- We need a tool to sort through operators of both types.

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Glueballs in Lattice QCD

The **G**eneralized **E**igen**v**alue **P**roblem (GEVP) formulation is particularly useful here. [Lüscher and Wolff (1999), Blossier *et al.* (2009)]
Start with N operators $\mathcal{O}_i$ in a fixed symmetry channel and calculate the correlation matrix

$$C_{ij}(t) = \left\langle \mathcal{O}_i(t)\bar{\mathcal{O}}_j(0) \right\rangle .$$

Solutions to $C(t)w_i(t_G, t) = \rho_i(t_G, t)C(t_G)w_i(t_G, t)$ give us:

- ✓ Energies of eigenstates $|i\rangle$ via $\rho_i(t, t_G) \overset{t\to\infty}{\propto} e^{-E_i t}$.
- ✓ Operators which best approximate $|i\rangle$ via*

$$\mathcal{O}_i = \sum_{k=1}^{N} w_i^{(k)}(t_G, t_1)\mathcal{O}_k$$

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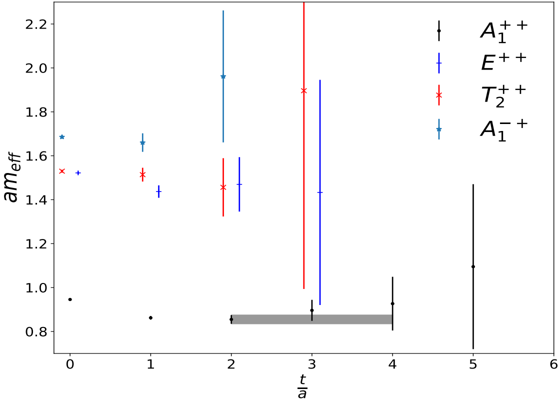
*Up to corrections.

Glueballs in a quenched setup

$24^3 \times 48$ lattice with $\beta = 5.85$ and periodic B.C.



- 35 shapes, 5 levels of APE smearing, 9000 cfgs.
- **Small** excited-state contamination at early times.
- Exponential signal-to-noise problem.

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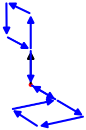
Behind the scenes: Glueball hunting

- Smearing link variables.
  - ✓ Improves overlap with low-lying states.
  - × Close degeneracies between small loop shapes requires reducing basis of operators. [Sakai and Sasaki (2023)]
- Large correlation matrix.
  - ✓ Big basis samples different degrees of freedom (Size, spatial distribution,...).
  - × GEVP becomes ill-defined so matrix must be pruned, i.e projected onto dominant singular vectors of a fixed time. [Balog *et al.* (1999), Niedermayer *et al.* (2001)]
- Choice of loop shapes.
  - ✓ Shapes with non-trivial geometry were useful.
  - ✓ "Flags" like  helped for the A_1^{-+}.

Glueballs in $N_f = 2$ QCD

The setup:

- $N_f = 2$ degenerate dynamical Wilson quarks at half the physical charm quark mass.
- $24^3 \times 48$ lattice with $\beta = 5.3$, $\kappa = 0.13270$, periodic B.C. and $23,000$ gauge configurations.
- ✓ Simplified model: Absence of light quarks restricts mixing to only charmonium.

A first approach:

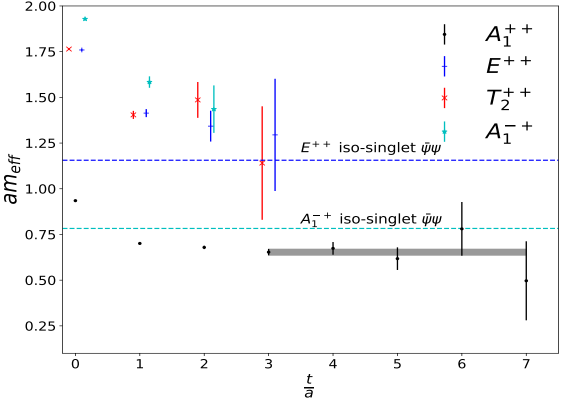
- Only gluonic operators: Same loop shapes as in quenched.

A more complete approach:

- Include both gluonic and mesonic operators.

Glueballs in $N_f = 2$ QCD: First approach

Ground state with only Wilson loops.



- **Larger** excited-state contamination at early times $\rightarrow$ Denser spectrum.
- Ordering of states must be done carefully.

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Glueballs in $N_f = 2$ QCD: Second approach

Include iso-scalar $\bar{\psi}\Gamma\psi$ with $\Gamma = \mathbb{I}$ for A_1^{++}:

$$C(t) = \begin{pmatrix} C_{MM}(t) & C_{MG}(t) \\ C_{GM}(t) & C_{GG}(t) \end{pmatrix},$$

with

$$C_{MM}(t) = \left\langle \bar{\psi}(t)\psi(t) \cdot \bar{\psi}(0)\psi(0) \right\rangle$$
$$C_{MG}(t) = \left\langle \bar{\psi}(t)\psi(t) \cdot W(0) \right\rangle \text{ (W(t): Wilson loop operator)}$$
$$C_{GG}(t) = \left\langle W(t)W(0) \right\rangle .$$

Mixing ($C_{MG}(t) \neq 0$) between gluonic and mesonic operators is enabled and the energy eigenstates are approximated as a **combination** of both types.
→ But we need **good** mesonic operators!

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Glueballs in $N_f = 2$ QCD: Second approach

Let $V[t]$ contain the N_v lowest eigenvectors of the 3D lattice covariant Laplacian operator

$$\nabla^2[t]_{\vec{x}\vec{y}} = -6\delta_{\vec{x}\vec{y}} + \sum_{i=1}^{3} U_i(\vec{x},t)\delta_{\vec{x}+\hat{i},\vec{y}} + U_i(\vec{x}-\hat{i},t)\delta_{\vec{x}-\hat{i},\vec{y}}$$

- Distillation: $\psi(t) \to V[t]V[t]^\dagger\psi(t)$ [M. Peardon *et al.* (2009)]
- Improved Distillation:

$J_k[t]_{ij} = g_k\left(\lambda_i[t]\right)\delta_{ij}$ Use basis of quark profiles...

$\psi_k(t) = V[t]J_k[t]V[t]^\dagger\psi(t)$ ..to distill quark fields...

$\mathcal{O}_i(t) = \bar{\psi}_k(t)\Gamma\psi_k(t)$ ...create basis of meson operators...

$\mathcal{O} = \sum_k c_k\bar{\psi}_k(t)\Gamma\psi_k(t)$ ...combine them optimally.

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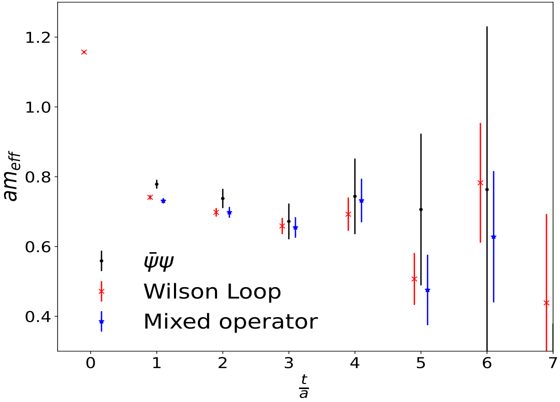
See Phys. Rev. D 106, 034501 (2022).

Glueballs in $N_f = 2$ QCD: Second approach

Simplest case: One $\bar{\psi}\psi$ and one Wilson loop.



Individual contributions of each type:

- $$\frac{\langle 0|\hat{\mathcal{O}}^{\dagger}_{\bar{\psi}\psi}|\Omega\rangle}{\langle \mathbf{1}|\hat{\mathcal{O}}^{\dagger}_{\bar{\psi}\psi}|\Omega\rangle} = -0.569(48) \ , \ \frac{\langle \mathbf{0}|\hat{\mathcal{O}}^{\dagger}_{W}|\Omega\rangle}{\langle 1|\hat{\mathcal{O}}^{\dagger}_{W}|\Omega\rangle} = -5.6(2.1)$$

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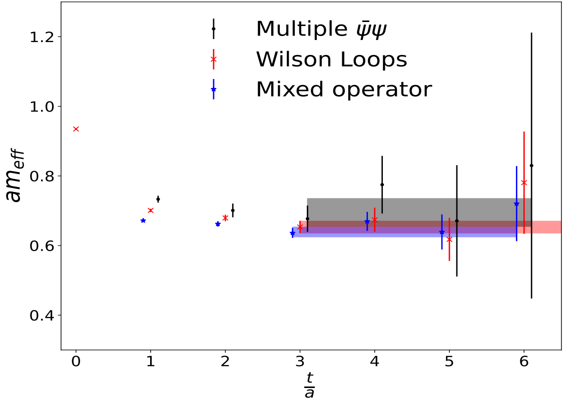
Glueballs in $N_f = 2$ QCD: Second approach

Multiple Wilson loops and profiles + pruning:



- Mixed operator is **better** than purely gluonic/mesonic.
- Energy eigenstate contains **both**.
- Interplay of profiles, shapes, smearing helps.

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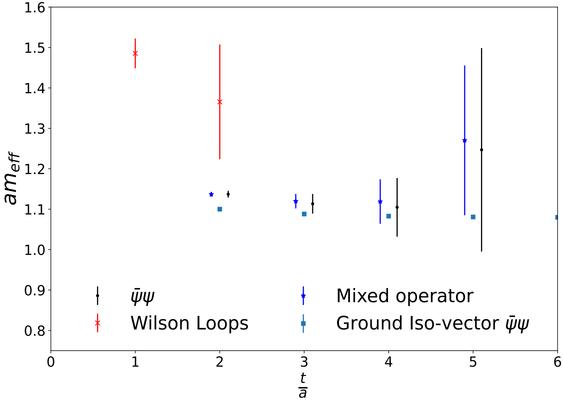
Glueballs in $N_f = 2$ QCD: Second approach

First excited state:



- State is mostly mesonic and very close to the ground-state iso-vector A_1^{++} (Possible χ_{c0}).
- Supports ground state being mostly gluonic.

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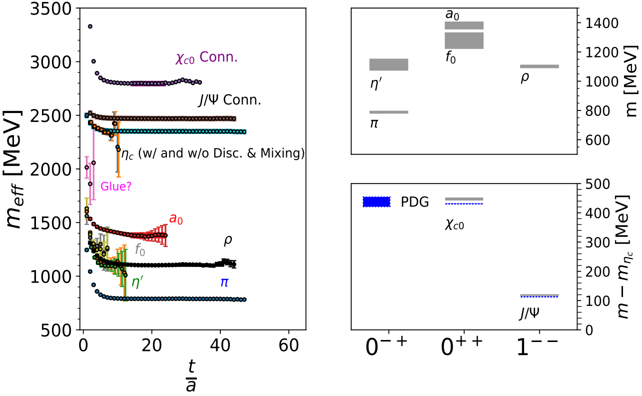
Glueballs in $N_f = 3 + 1$

Non-perturbatively improved Wilson fermions + Lüscher-Weisz gauge action + SU(3) flavor-symmetric point + heavy pions



2 (heavy) pion threshold for scalar glueball ≈ 1.6 GeV. Need 2-particle operators! (See 2312.16740 for other details.)

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Conclusions and Outlook

- Hunting for glueballs is **not easy** even in quenched QCD: SNR problem, noisy operators, cost of statistics.
- With dynamical quarks:
  - Denser spectrum $\rightarrow$ Degraded performance of loop operators.
  - States are not purely mesonic/gluonic $\rightarrow$ Need mixing.
  - What is a glueball here?

Work in progress within DFG 5269:

- Long list of loop shapes + other gluonic operators + well-tuned smearing.
- Multi-level updates for quenched (See 2312.11372) and with dynamical quarks (in progress).
- Hybrid operators (e.g $\bar{\psi}\epsilon_{ijk}F_{jk}\psi$) + Optimized distillation
- Multi-particle operators (e.g $\pi - \pi$) + Optimized distillation.

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Thank you for your attention!