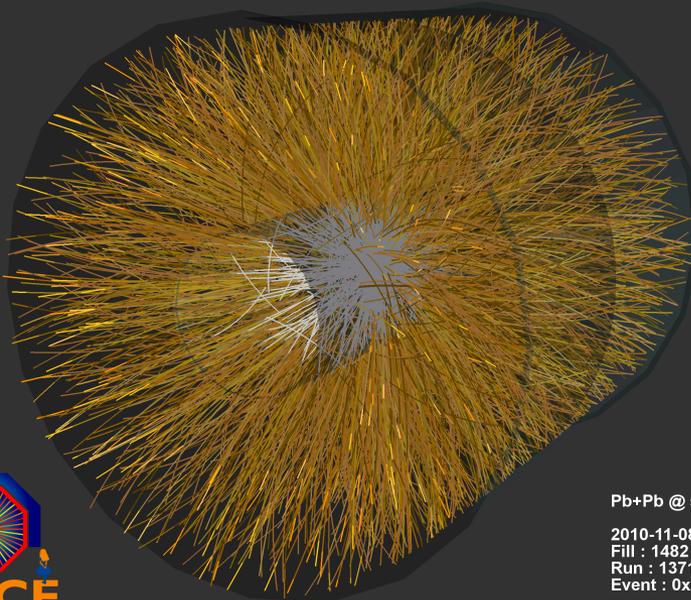


The Statistical Hadronization Model, the Hadron Resonance Gas and All That:

What do yields of the light nuclei and hypernuclei in heavy ion collisions tell us about QCD?

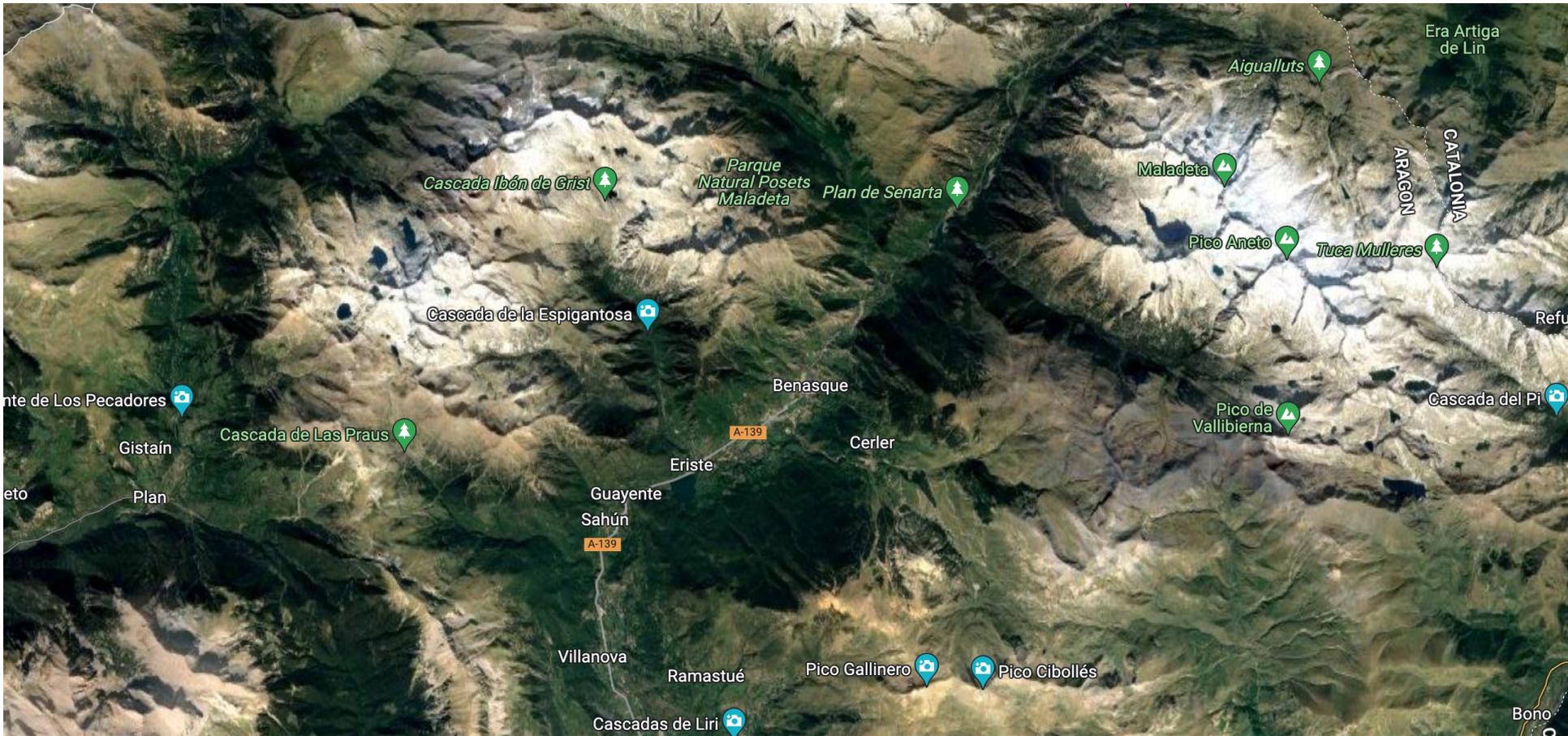
Manessha Praddeep & TDC



Pb+Pb @ \sqrt{s} = 2.76 ATeV
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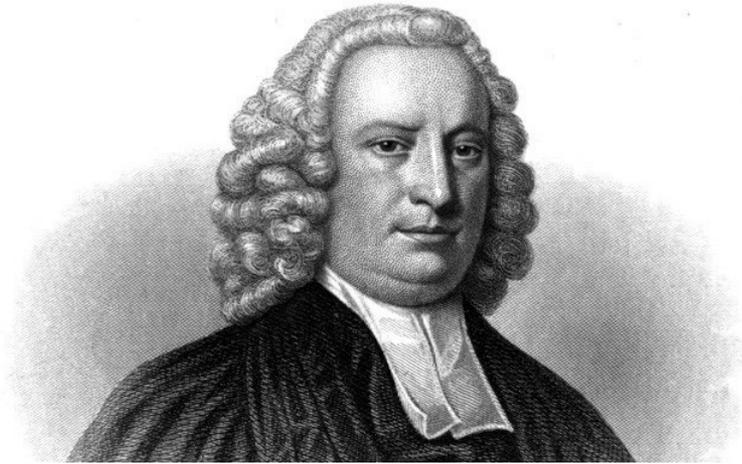


An Overview



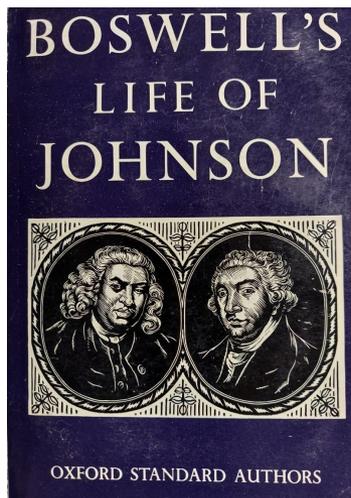
An Overview

- A very simple model, the Statistical Hadronization Model* (SHM) has been used to predict yields of hadrons and nuclei.
- The model is phenomenologically quite predictive given its simplicity.
- If one accepts the assumptions underlying the model, recent measurements at the LHC imply a remarkable picture of the dynamics
 - However these assumptions have been questioned.
- This talk focuses on the yield of light nuclei, which indicate the assumptions of the SHM cannot be justified.



Spirit of how this talk views the statistical hadronization model is inspired by a famous comment by the 18th century intellectual, Samuel Johnson

He described a person's activity he thought absurd and implausible as being "...like a dog's walking on his hinder legs. It's not done well; but you are surprised to find it done at all." July 31, 1763 (as recorded by Thomas Boswell)





- The SHM is so simple as to be cartoonlike.
- Yet despite this simplicity it efficiently describes a significant amount of data.
- Like a dog walking on its hind legs, it is not so much that it does it well, but you are surprised that it does it all.

Key question is what—if anything—one can learn from the phenomenological success, about the underlying dynamics of heavy ion physics.

Assumptions of SHM

1. The system equilibrates into a QGP
2. System expands and cools and becomes an equilibrated hadronic gas (including light nuclei) with the bulk of the system contained in a volume where:
 - a. System is sufficiently dilute enough so that hadrons (and nuclei) are sufficiently well-separated as to be discernible.
 - b. The system is sufficiently dilute so that the properties of the hadronic gas are well-approximated by a gas of noninteracting hadrons with a masses given by the vacuum value.
3. System sufficiently dense that interactions maintain both chemical and kinetic equilibrium. As system cools further it falls out of chemical equilibrium with the hadronic species freezing out chemically
 - a. All species freeze out at approximately the same temperature.
 - b. Yields given by the primordial yields given by the model for stable species at the freeze out temperature plus yields due from decay of unstable hadrons with branching ratios given by their free space values.

The model has three parameters:

T_{cf} Temperature at chemical freeze out

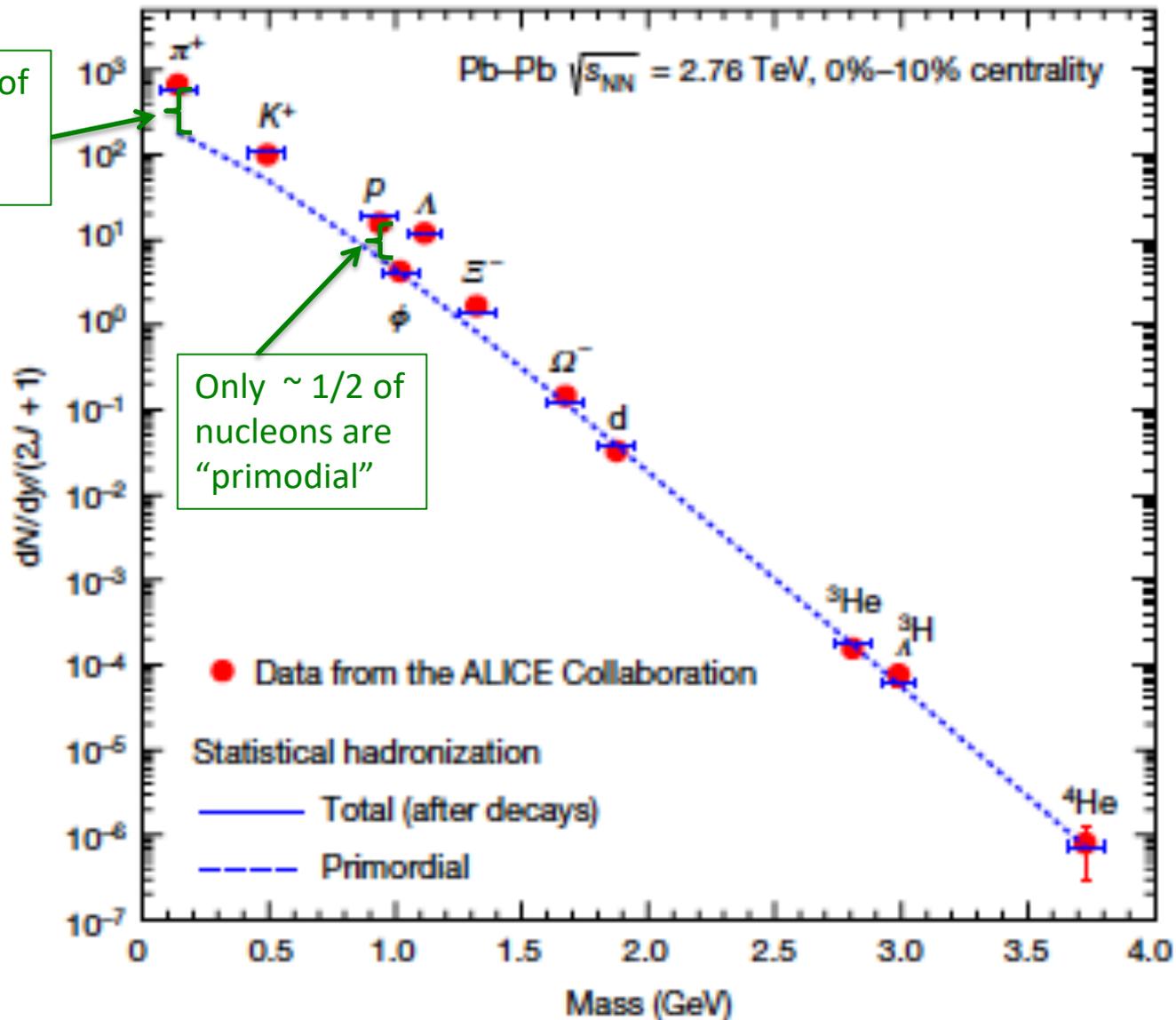
μ Baryon chemical potential at chemical freeze out

V Volume of hadronic gas at chemical freeze out

Predicts yields of hadrons (and light nuclei) for midrapidity in central collisions

Note that relative yields at high beam energy effectively only depend on T_{cf} : V does not affect relative yields (only absolute) and that $\mu \rightarrow 0$ as the beam energy gets high. (Only thing distinguishing baryons from antibaryons is the baryons in initial state which is a tiny fraction of baryons seen at midrapidity).

Only ~ 1/3 of pions are "primordial"



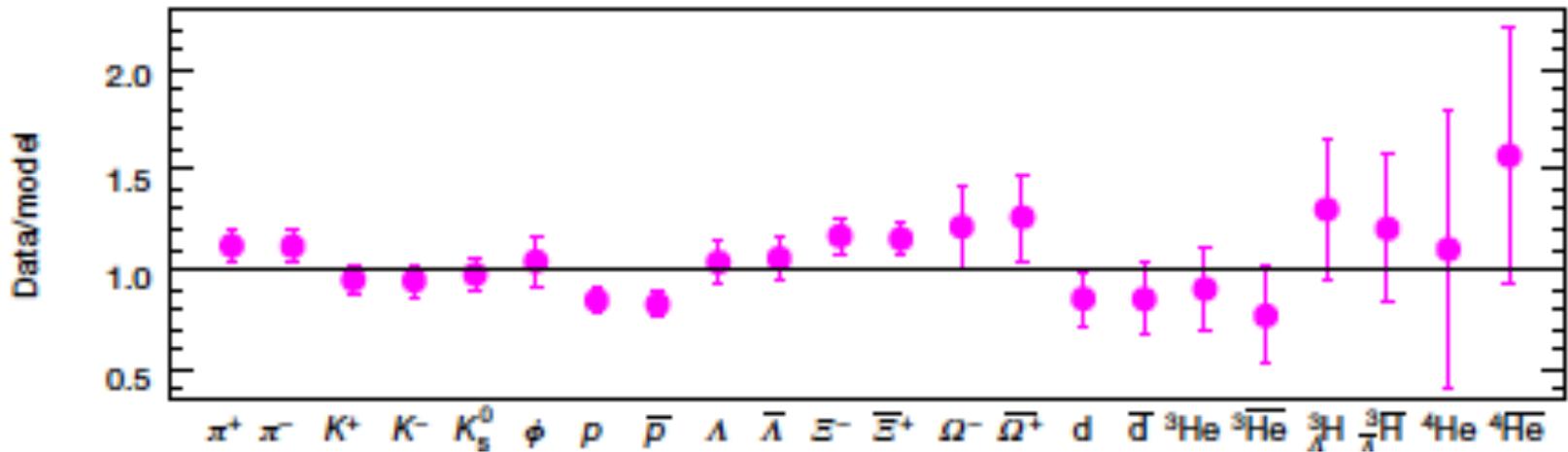
Only ~ 1/2 of nucleons are "primordial"

$$T_{cf} = 156.5 \pm 1.5 \text{ MeV}$$

$$\mu_b = .7 \pm 3.8 \text{ MeV (Consistent with zero)}$$

$$V = 5280 \pm 410 \text{ fm}^3 = (17.4 \pm .4 \text{ fm})^3$$

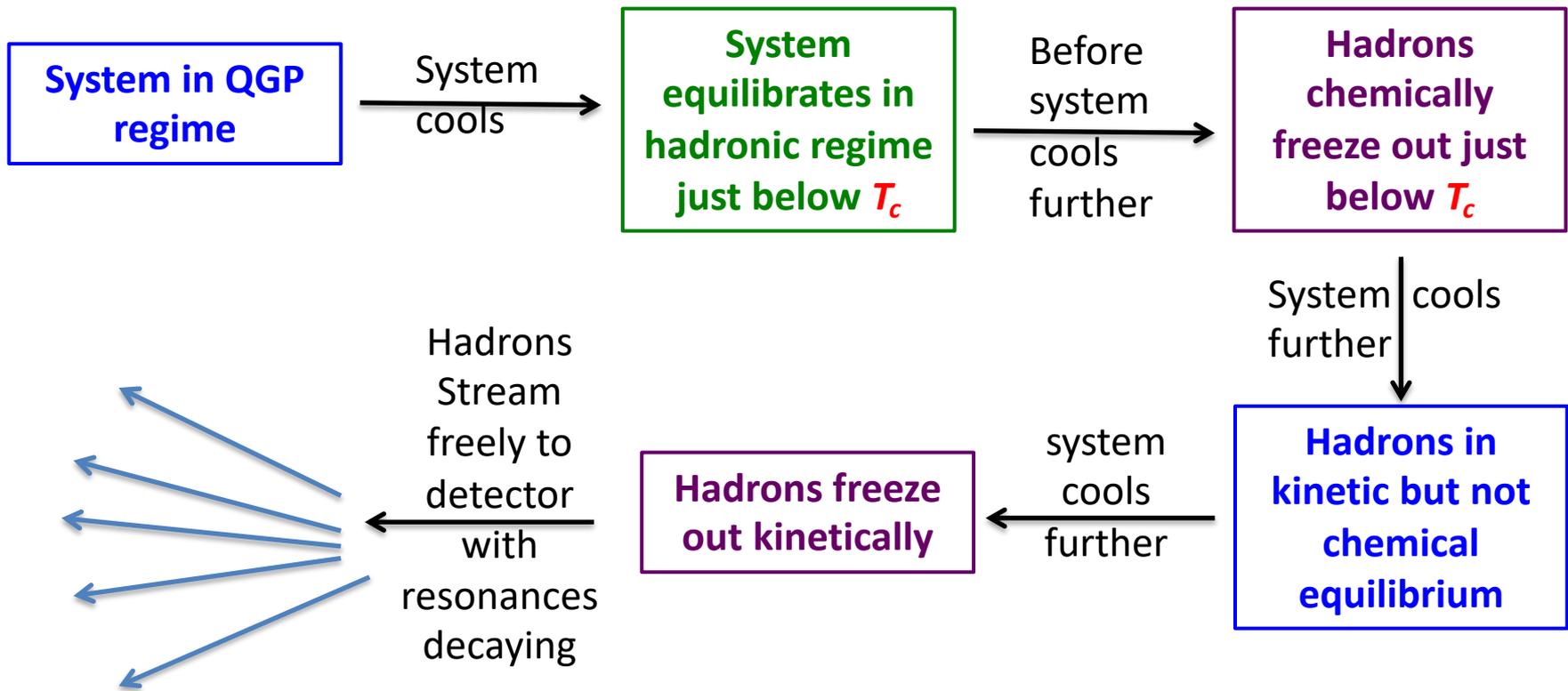
From A. Andronic, P. Braun-Muniziger, Krzysztof Redlich & J. Stachel, Nature 561, 312 (2018)



Not perfect, as seen above.

But relative yields of 11 quantities (ignoring difference of isospin and particles vs antiparticles) covering 9 orders of magnitude are fit to better than .12 orders of magnitude with one parameter, T_{cf} .





Remarkable fact: value of T_{cf} at highest energy is consistent with the chiral “cross-over temperature”, or pseudocritical temperature T_c , as determined from lattice studies in which $T_c : T_{cf} = 154 \pm 9 \text{ MeV}$ compared with $T_{cf} = 156.5 \pm 1.5 \text{ MeV}$. Remarkable thing is that $T_c = T_{cf}$. Logically nothing relates the two in the SHM beyond requirement $T_c \geq T_{cf}$: T_c is a thermodynamic property of equilibrated matter, while T_{cf} depends on the dynamics of expansion and how things fall out of equilibrium.

Remarkable scenario depends on the assumptions of the model being trustworthy. **Are they?**

- The dynamical assumptions of the model have been questioned for some time*.
 - Typical concern involves the time scales of the dynamics.
 - Concern: chemical equilibrium depends on processes with very different time scales; one does not expect universal chemical freezeout temp.
 - Concern that all hadrons in the system do not have time to chemically equilibrate in hadronic phase.
 - A “born in equilibrium” dynamical scenario has been considered as a way around these problems.

This talk takes an agnostic view of the detailed dynamics and asks a more basic question:

Suppose one accepts the dynamical assumptions of the SHM, is the description of the hadronic matter consistent in light of the parameters extracted from experiment?

- To test this focus on properties of matter just prior to freeze out.

*See for example: U. Heinz and G. Kestin, PoS CPOD 2006 038 (2006); P. Castorina, D. Kharzeev and H. Satz, Eur. Phys. J. C52 187 (2007); J. Schukraft, Phys. Scr. T158 014003 (2013).

Probe the assumptions of model in more detail

Key assumption: *Just before freeze out , the system is dilute enough that hadrons (and light nuclei) are sufficiently well-separated as to be discernible. Seems so obvious as to not require stating.* However, this assumption fails for light nuclei.

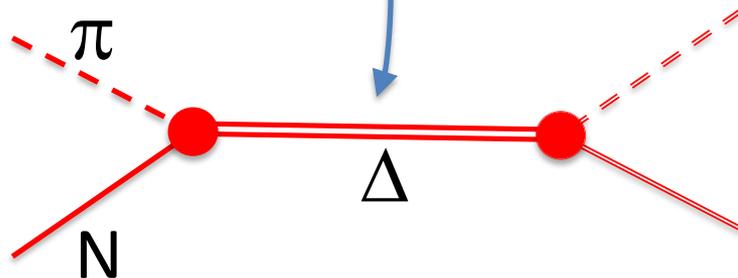
The physical picture:

- Almost all of the energy is in the mass and kinetic energy for discernible hadrons.
- Hadrons are freely propagating almost all of their time with their energies fixed via free space standard dispersion relation.
- The hadrons occasionally exchange energy enabling the establishment and maintenance of kinetic equilibrium.
- Chemical equilibrium established and maintained via rare inelastic interactions; “interactions” includes spontaneous decay of an unstable hadrons as well as inelastic collisions .

In support of this picture

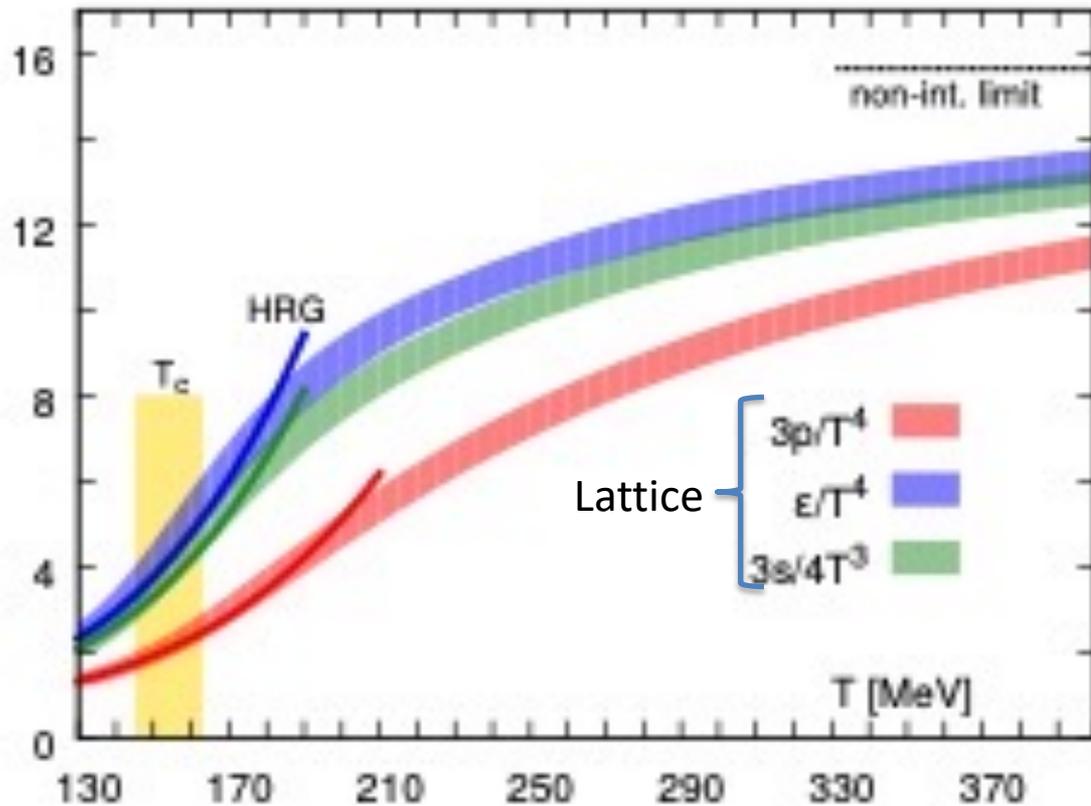
The success of the hadron resonance gas (HRG)

- The HRG model is implicitly assumed in SHM. It treats the thermodynamics of QCD (at low temperatures) as a gas of hadrons including resonances that are assumed to be long-lived and at their physical masses (as given by PDG)
- Justified if system dilute enough to exploit virial expansion and scattering amplitudes are negligible except near resonances; resonances are narrow (Dashen, Ma & Bernstein 1969)



Δ pole dominates scattering in this channel and is near enough to real axis to approximate by a mass.

Hadron resonance gas model in fairly good agreement with lattice QCD at low temperature



HotQCD 2014

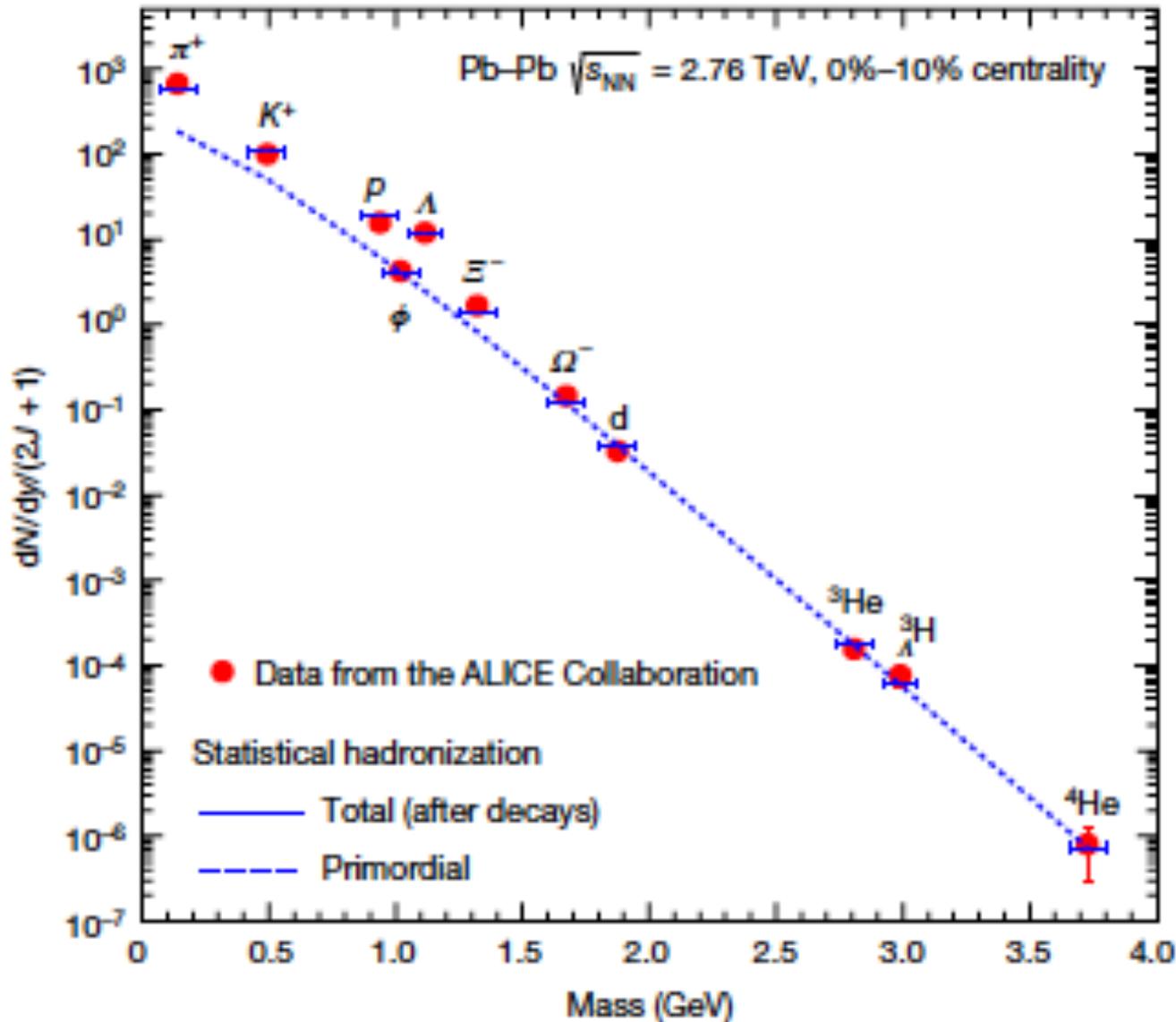
- **Qualitative/semi-quantitative agreement at low T.**
- **Suggests HRG qualitatively captures thermodynamics.**

Caveats:

- **Information is bulk thermodynamics and need not imply that each hadronic state contributes as in HRG.**
- **The curvature of the fit is noticeably imperfect (but model is crude).**
- **“Works” well past T_c.**

- Focus on yield of light nuclei.
 - Binding energies of light nuclei are much smaller than typical hadronic scales and the temperature. This causes tensions with assumptions.
 - Actually, more than tensions—outright inconsistencies.
 - Much of the phenomenological success of the SHM come from the light nuclei:
 - Of 9 orders of magnitudes in yields, 5 of them come from light nuclei.
 - The yields of light nuclei come from primordial densities (rather than feed down from decaying resonances); yields directly probe the putative equilibrated matter in HRG.

Note that the SHM describes the light nuclei rather well.



A fit to just the light nuclei rather than the whole set yields $T_{cf}=159\pm 5$ MeV
Consistent with full fit of $T_{cf}=156.5\pm 5$ MeV

The hypertriton yields are particularly sensitive test as it is extremely weakly bound state of D and Λ despite “success” the assumptions of the HRG are badly violated for hypertritons nuclei at $T \approx 155$ MeV

To kill off the model assumptions for the light nuclei I will adopt the Rasputin strategy and kill it off in multiple ways.



Russian nobles lead by Prince Yusupov concluded that Rasputin was a threat to Russia and decided to kill him.

The murder on Dec. 29-30, 1916 involved

- A poisoned cake (cyanide).
- Poisoned Madeira wine(cyanide).
- A pistol shot to chest believe by the conspirators to be fatal.
- Two subsequent pistol shots when Rasputin attempt to flee hours later.
- Ultimate cause of death drowning in the Neva river where his body was thrown.



The large size of the ${}^3_{\Lambda}\text{H}$ is incompatible with size of fireball



- Implicit assumption of the SHM: **The variation of temperatures over the size of a hadron or nucleus is small.**
 - Hadrons and nuclei are implicitly treated as points (presumably at their center of mass)
 - Yields proportional to HRG model density at T_{cf} .
 - But given the spatio-temporal evolution of the fireball, the temperature varies over the hadron or nucleus and with it all of the mechanisms to yield equilibrium.
 - Not an issue if predicted yields associated with the varying temp over the size of the hadron is small; it is just a modest source of theoretical uncertainty. If these uncertainties are large the model is not predictive

- Problem becomes severe as the hadron or nucleus gets heavy and spatially large: ${}^3_{\Lambda}\text{H}$ is both.
- Mass effect: yields proportional to density: for

$$M \gg T, \quad \rho \propto e^{-\frac{m}{T}} (mT)^{\frac{3}{2}} \quad \text{so}$$

$$\frac{\rho(T_1)}{\rho(T_2)} = \left(\frac{T_1}{T_2}\right)^{3/2} e^{-\frac{m(T_1-T_2)}{T_1 T_2}}$$

- Consider the difference in predicted yields between $T=155 \text{ MeV}$ and $T=165 \text{ MeV}$ for ${}^3_{\Lambda}\text{H}$:

$$\frac{\rho(165 \text{ MeV})}{\rho(155 \text{ MeV})} = 3.5$$

- If the ${}^3_{\Lambda}\text{H}$ extends over region with the nominal freeze out temp to one just 10 MeV higher the model cannot predict a yield.

- Fireball size is comparatively modest and region where temperature is between 155 MeV and 165 MeV is quite small.

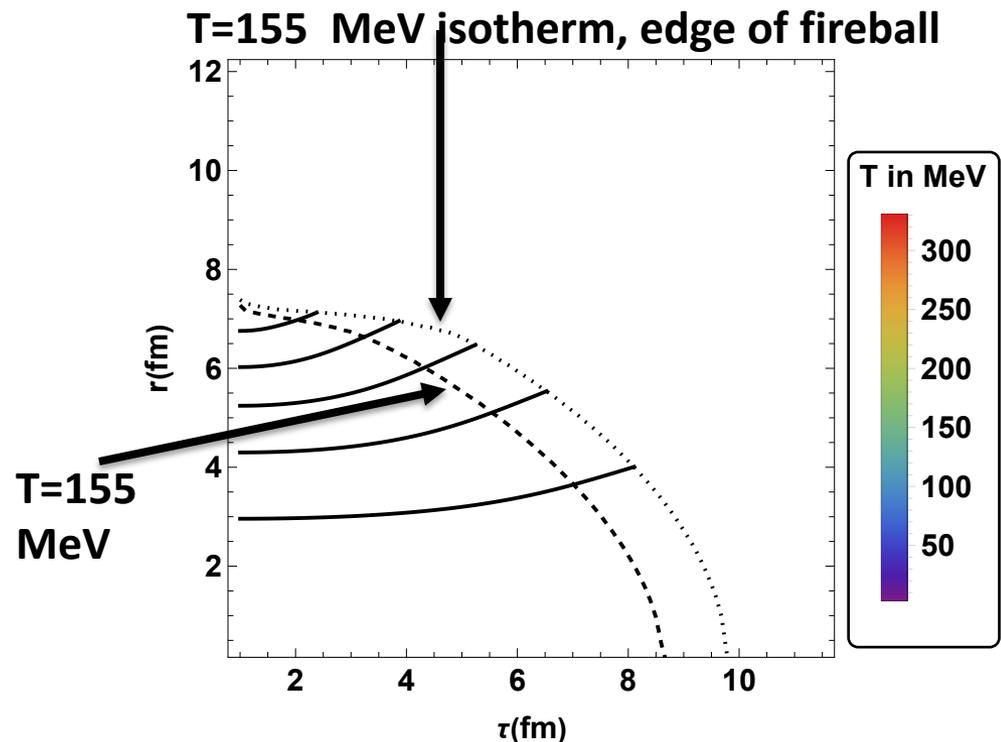
– Note: fireball size is a function of time.

- Somewhat counterintuitively it basically shrinks with time As the outer edge freezes out the remaining material is at shorter distances.

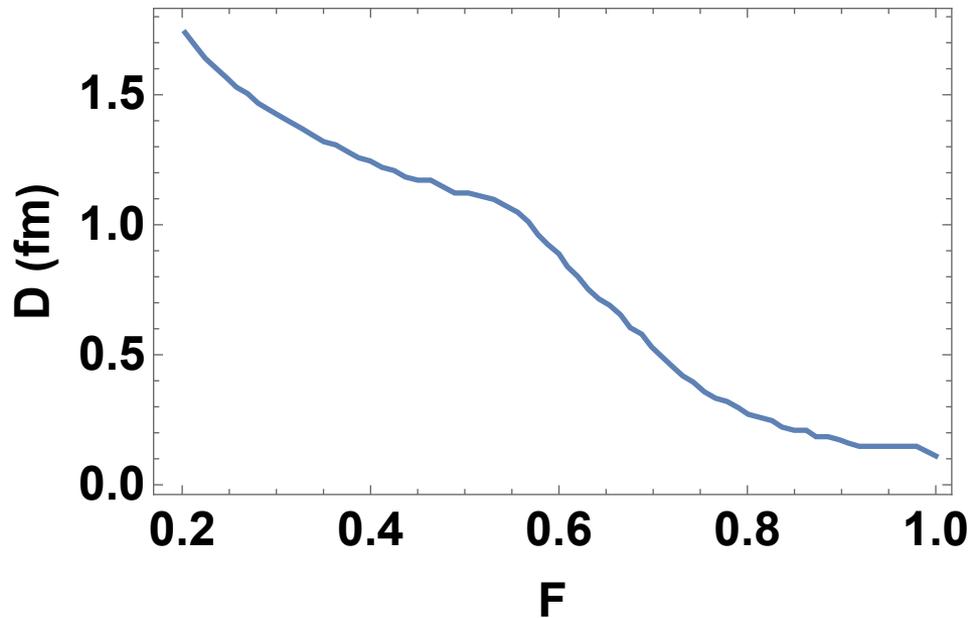
Fireball radius as function of time using standard hydro code:

200 GeV Au-Au central collisions. Longitudinally invariant

Solid lines correspond to equal fractions of eventual yields



Distance
between
isotherms with
 $T=155$ MeV &
 $T=165$ MeV



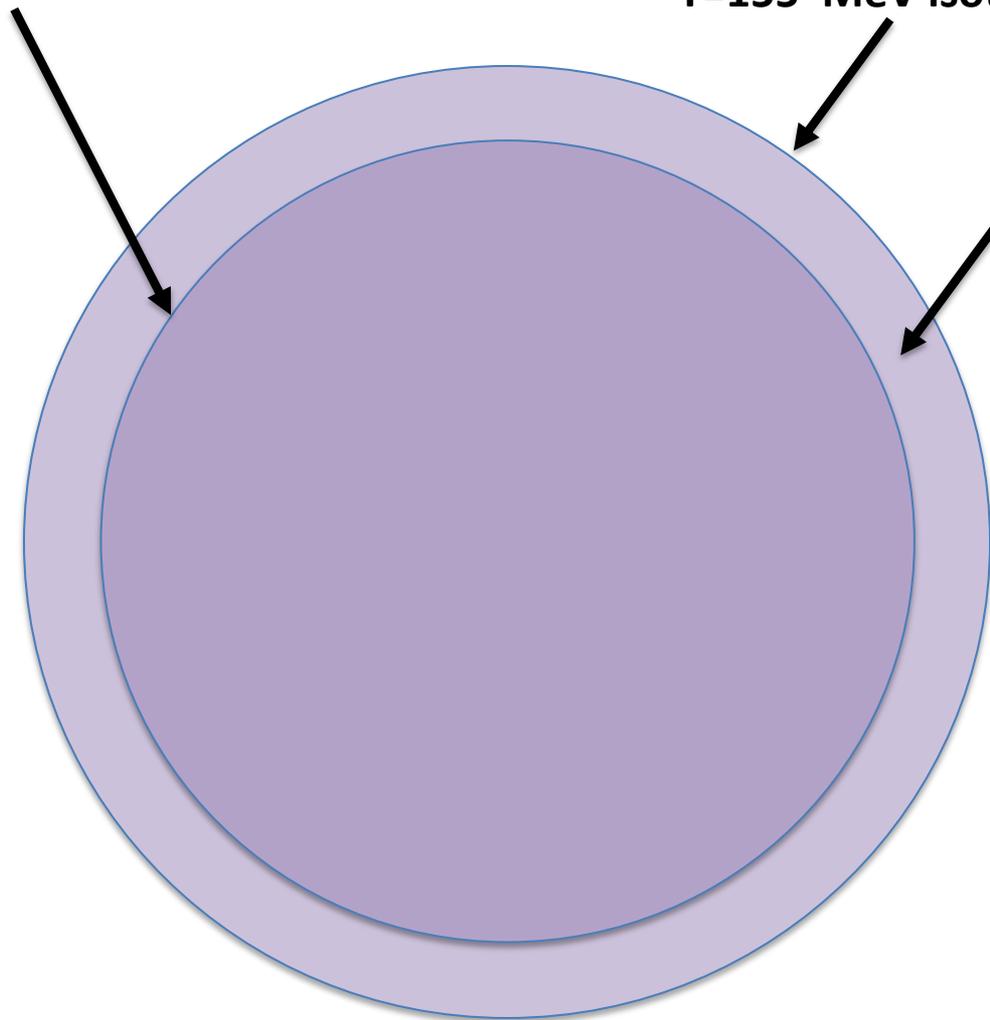
Fraction of initial fireball that has not yet
frozen

For vast majority of evolution of fireball the
distance is less than 1.5 fm.

Recall: ${}^3_{\Lambda}\text{H}$ needs to fit in that distance to have any
hope of predicting yields

T=165 MeV isotherm

T=155 MeV isotherm, edge of fireball



${}^3_{\Lambda}\text{H}$ needs to fit in shell for model predictions to make sense:

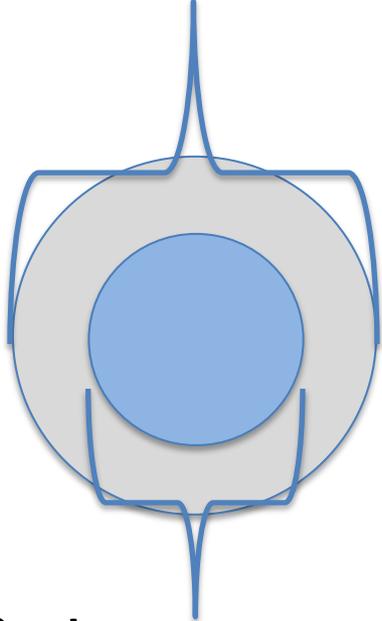
It doesn't!!

Fireball after approximately 5 fm; approx. half of the fireball has frozen out

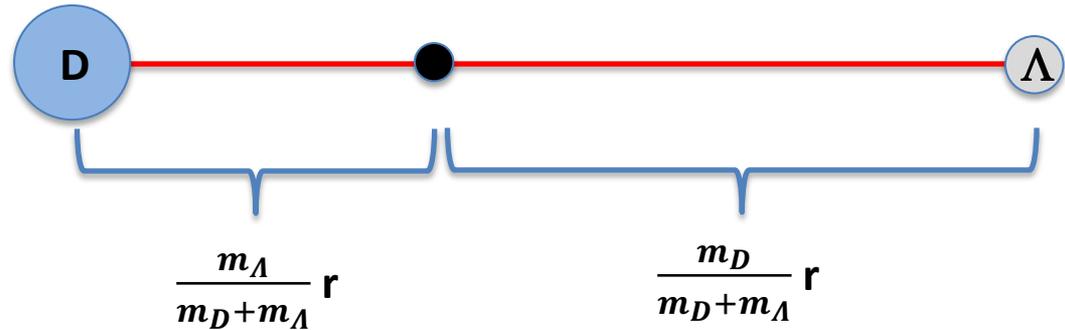
- ${}^3_{\Lambda}\text{H}$ is large as it is extremely weakly bound:
 $B \sim 148 \pm 40 \text{ KeV}$ (although it is quite uncertain) This creates contradictions:
- Very weakly bound states are necessarily large: Beyond the range of the effective potential between D & μ the probability drops off as $\exp(-r/r_0)$; with $r_0 = (8 \mu B)^{-1/2}$ as $B \rightarrow 0$; more state is beyond range of potential and the size gets big where r is distance between D & μ .
- $r_{rms} = \sqrt{\langle r^2 \rangle}$ for 2 simple models fit to $B = .148 \text{ MeV}$:
 - Model I: Zero range potential : $r_{rms} = 9.7 \text{ fm}$
 - Model II: Constant short-range potential to 2.5 fm: $r_{rms} = 10.6 \text{ fm}$
 - For simplicity take $r_{rms} = 10 \text{ fm}$ (exact size irrelevant)

- Size relative to center of mass:

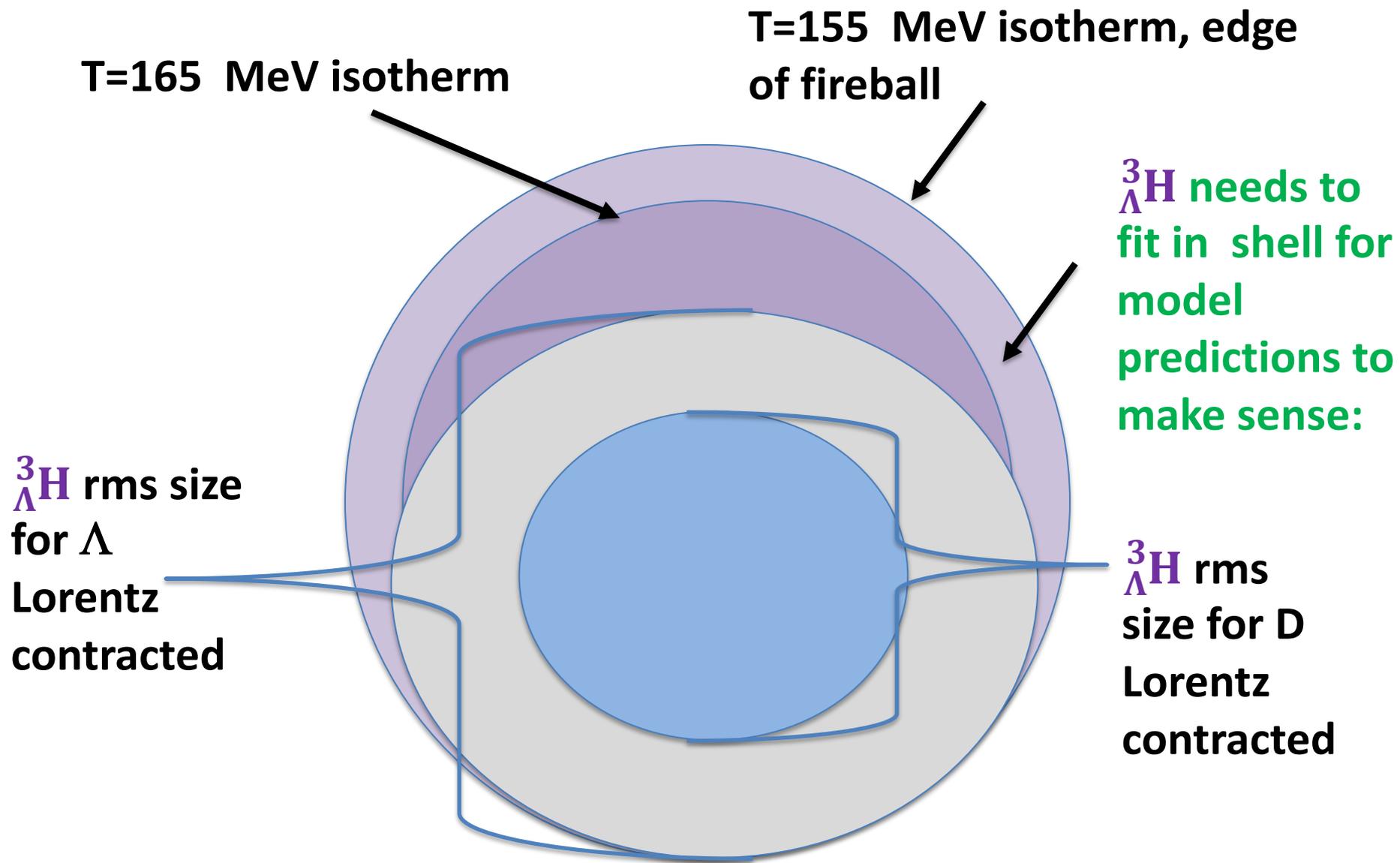
Lambda rms
diameter: **12.5 fm**



Deuteron rms
diameter: **7.5 fm**



The Lambda sticks out well
beyond the deuteron, but both
stick out a very long way



Neither D or Λ comes close to fitting in shell!

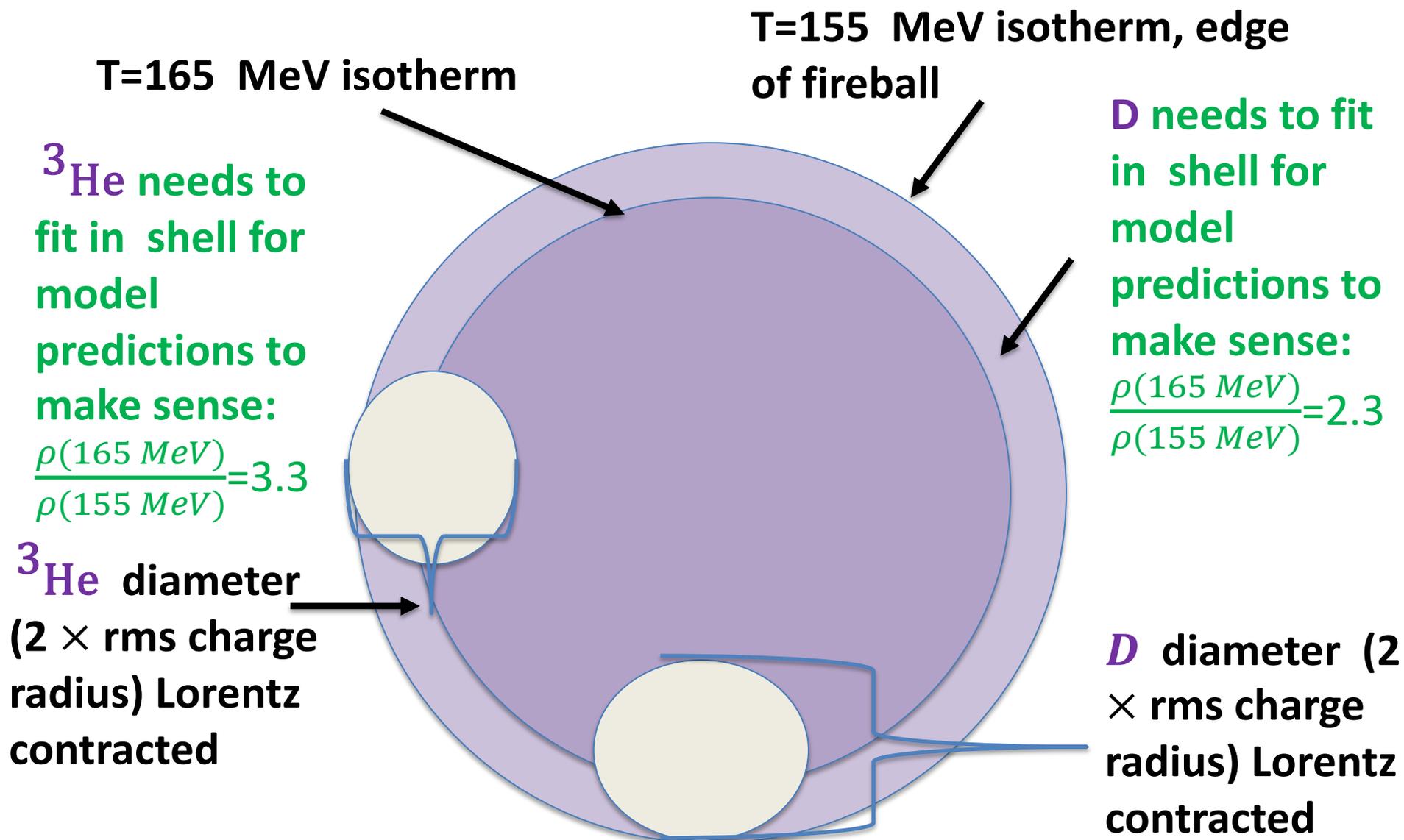
- **Caveats/Concerns:**

1. Figure was only for one time.
2. Calculations for AGS energies, target could be different at LHC
3. RMS radius exaggerates long distance tail.
4. Uncertainties in radius as B is unknown as form of short distance potential

- **Why they don't matter:**

1. Shorter times, problem worse—shell smaller; longer times still big problems. For $T > 6$ fms ${}^3_{\Lambda}\text{H}$ bigger than fireball.
2. SHM works at AGS energies targets as well as LHC. Expect similar results at LHC
3. Other definition of size still have major inconsistency.
4. Effect is so large none of the uncertainty matters

A similar but less dramatic problem with yields of D and ${}^3\text{He}$ but still enough to conclude the model is not valid.





Punch line

- Size of fireball and scale of temperature variations in it are **much** too small for the SHM to be valid for yields of ${}^3_{\Lambda}\text{H}$ given the very large size of the hypertriton
- This is more than enough to kill of the SHM for ${}^3_{\Lambda}\text{H}$.
- The same argument kills the model for **D**
- And ${}^3\text{He}$
- The argument can be extended to the yield of ${}^4\text{He}$. The size is a bit smaller than **D** but the shell is much thinner



- Another way to see incompatibility involves the short lifetime of the hypertriton in matter assuming HRG.

– Fully determining lifetime requires detailed dynamical model. However, there are reliable upper bounds:

- Upper bound of $\tau_{\Lambda^3\text{H}}$ obtained from (in medium) lifetime of nucleons or hyperons composing it: constituent decay \rightarrow vanishing $\tau_{\Lambda^3\text{H}}$ (and other mechanisms exist to destroy nucleus). $\tau_{nuc} < \left(\sum_{j=\text{constituents}} \frac{1}{\tau_j} \right)^{-1}$

- Upper bound on lifetime of nucleon from detailed balance: N is “destroyed” and replaced by a Δ when it absorbs a resonant π and “created” when a Δ decays. Detailed balance: same rate. Density of Δ s and N s in medium known in HRG. Other resonance decays also contribute, and the decay of Δ is faster than its spontaneous from collisions: we obtain an upper bound for nucleon lifetime

- Analogous argument for lifetime of the Λ based on decays of $\Sigma(1385)$

- Together this gives an extremely conservative upper bound: $\tau_{\Lambda^3\text{H}} < 1.0 \text{ fm}$

★ Y. Cai, T Cohen, B. Gelman & Y. Yamauchi *Phys.Rev.C* 100 (2019) 2, 024911

- On the other , the hypertriton is extremely weakly bound: This creates a contradiction due to causality

- Causality problem: Very weakly bound states implies large size for $\Lambda^3\text{H} \sim 10 \text{ fm}$. Causality: time to create state must $> 10 \text{ fm}$; otherwise the state does not “know” it has been made. For HRG to be valid, lifetimes must be \gg than creation times. But $\Lambda^3\text{H}$ lifetime is less than 1.0 fm . This is a serious contradiction which by itself should kill SHM for $\Lambda^3\text{H}$. (see Maryland mafia 2019 paper)

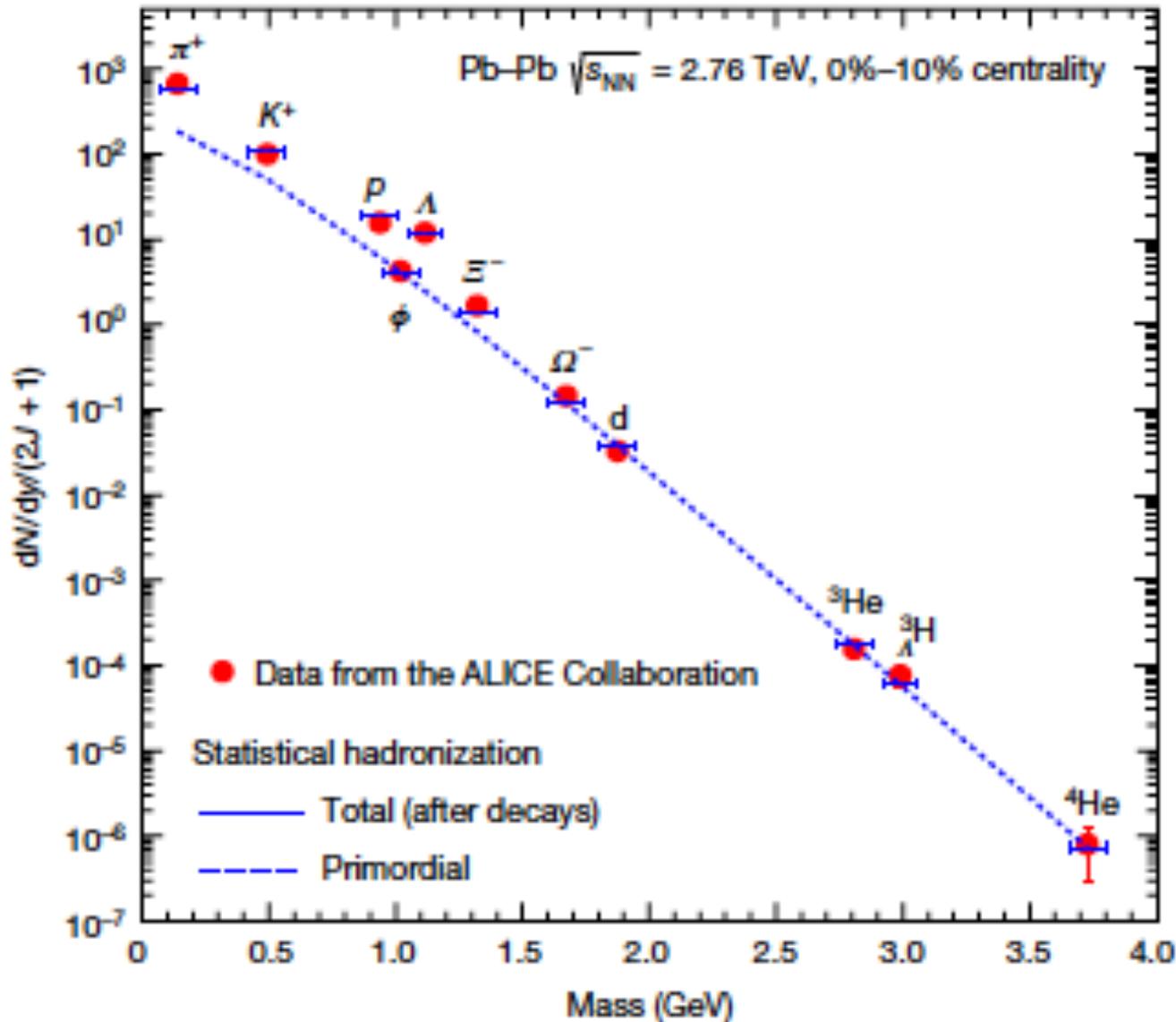


- A similar argument for the deuteron shows an incompatibility sufficient to kill the model for D

Conclusions and a key challenge

- The analysis here is sufficient to conclude the SHM is not valid for the yields of any of the light nuclei and hypernuclei—the arguments are compelling for all of these but particularly compelling for the ${}^3_{\Lambda}\text{H}$.
- This raises questions about the validity of the overall model
 - Recall of the phenomenological success of the SHM come from the light nuclei:
 - Of 9 orders of magnitudes in yields, 5 of them come from light nuclei

Note that the SHM describes the light nuclei rather well.



A fit to just the light nuclei rather than the whole set yields $T_{cf}=159\pm 5$ MeV
Consistent with full fit of $T_{cf}=156.5\pm 5$ MeV

Conclusions and a key challenge

- The model works in places where it is incompatible with its assumptions:
 - Clearly its successful phenomenologically is for reasons that are not understood. Whatever it is, it is not nuclei forming in the fireball and then freezing out.



Conclusions and a key challenge

- Since it works for unknown reasons for the light nuclei and hypernuclei, we do not know whether it's success for hadrons are for similarly unknown reasons or for the reasons the model assumes.
- **Why does this matter?**
 - It has been claimed that that the model enables the “**Decoding the phase structure of QCD via particle production at high energy**” from the heavy ion data (A. Andronic, P. Braun-Muniziger, Krzysztof Redlich & J. Stachel , Nature 561, 312 (2018))
 - Only if one can trust what the model tells us should this claim be taken seriously. But we know it cannot for light nuclei, and it is unclear whether we can trust it for hadrons.
 - The authors of that Nature paper were aware the weak binding and large size ${}^3_{\Lambda}\text{H}$, state that it is remarkable that the model works for these* , provides no viable explanation why it does, but still uses the model to draw conclusions about the phase structure of of QCD. This is troubling.

*But do not recognize that the spatio-temporal evolution

Conclusions and a key challenge

- The key theoretical challenge: find a viable explanation for why the model does such a good job in predicting yields for nuclei and hypernuclei despite the fact that the model's assumptions are not consistent with the results.

EXTRA SLIDES

Some useful quantities in probing HRG

Symbol	Quantity
T	Temperature
n_i	Density of species i
ε_i	Energy density of species i
$C_i = A_i$	Creation & annihilation rate per volume of species i
$\tau_i = \frac{n_i}{C_i} = \frac{n_i}{A_i}$	Equilibrium lifetime time for member of species i
τ_i^{interact}	Characteristic interaction time during for member of species i is created or annihilated

Condition for validity of SHM:

$$\tau_i^{\text{interact}} \ll \tau_i$$

Equilibrium phase-space density only yields ideal gas results independent of detailed mechanism and timing if this is satisfied.

Model predictions for equilibrium

For T=156.5 MeV

$$n_i(T) = g_i \int \frac{d^3p}{(2\pi)^3} \frac{1}{\exp((m_i^2+p^2)^{1/2}/T) \pm 1}$$

↑ degeneracy factor ↑ +fermions
↓ ↓ -bosons

$$\varepsilon_i(T) = g_i \int \frac{d^3p}{(2\pi)^3} \frac{(m_i^2+p^2)^{1/2}}{\exp((m_i^2+p^2)^{1/2}/T) \pm 1}$$

meson	n (fm ⁻³)	$\langle\gamma\rangle =$ $\varepsilon/(n m)$	v
pions	.143	3.62	.96
kaons	.052	1.61	.78
$f_0(500)$.013	1.60	.78
η	.010	1.55	.76
$K_0(700)$.010	1.41	.70
ρ	.032	1.37	.68
ω	.010	1.36	.68
K^*	.024	1.31	.65

	n (fm ⁻³)
All mesons with mass < 1250 MeV	.302

Baryons*	n (fm ⁻³)	$\langle\gamma\rangle =$ $\varepsilon/(n m)$	v
nucleon	.0124	1.29	.63
Λ	.0025	1.24	.59
Σ	.0051	1.23	.58
Δ	.0107	1.21	.57
All baryons with mass < 1250 MeV	.0254		
*includes antibaryons			

Unstable hadrons lifetime is shorter than in free space. Resonances can decay spontaneously as in free space and can also be destroyed in a collision. Yields an **upper bound** for τ_i for without full knowledge of interaction rates.

lifetime of the hadron in the medium.

$$\tau_i < \frac{\langle \gamma_i \rangle}{\Gamma_i} \quad C_i = A_i > \frac{n_i \Gamma_i}{\langle \gamma_i \rangle}$$

Time dilation factor

Width of resonance

Stable hadrons have finite lifetimes in HRG: When two pions resonate into a ρ meson, they cease to be pions.

Even without full knowledge of interaction rates one can deduce lower bounds for their lifetimes given the equilibrium assumption of the SHM.

$$\tau_{\text{nucleon}} < 2.38 \text{ fm}$$

Basic idea: nucleons created at certain rate by decay of Δ and in equilibrium must be destroyed at same rate by $\pi + N \rightarrow \Delta$

$$C_{\text{istable}} = C_i^{\text{resonance decays}} + C_i^{\text{collisions}} > C_i^{\text{resonance decays}}$$

$$= \sum_{j=\text{resonances}} \langle N_{i,j} \rangle A_j > \sum_{j=\text{resonances}} \langle N_{i,j} \rangle \frac{n_j \Gamma_j}{\langle \gamma_j \rangle} > \langle N_{i,k} \rangle \frac{n_k \Gamma_k}{\langle \gamma_k \rangle}$$

Average number of particles of type i produced in decay of resonance j

k is any of the resonances

Therefore

$$\tau_{\text{istable}} < \frac{n_i}{C_i} < \frac{n_i}{\sum_{j=\text{resonances}} \langle N_{i,j} \rangle \frac{n_j \Gamma_j}{\langle \gamma_j \rangle}} < \frac{n_i}{n_k} \frac{\langle \gamma_k \rangle}{\langle N_{i,k} \rangle \Gamma_k}$$

For the nucleon using the Δ for k with densities and $\langle \gamma_k \rangle$ from the tables above and using PDG, $\langle N_{\text{nucleon}, \Delta} \rangle \approx 1$, $\Gamma_{\Delta} \approx 117 \text{ MeV}$

Some light nuclei properties; velocity uses $T=156.5$ MeV

Focus on D and ${}^3_{\Lambda}H$ as they are extremely weakly bound

Claims:

- Lifetime of a nucleus in medium is same as lifetime of its constituents.
 - D and ${}^3_{\Lambda}H$ are so weakly bound that nucleons & Λ s interact with pions same as for unbound.
 - Only difference is that velocity of nucleons in nucleus smaller than unbound

Nucleus	Binding energy to nearest threshold	velocity
D	2.23 MeV	.48
${}^3_{\Lambda}H$	$\sim 148 \pm 40$ KeV*	.40

- A simple kinetic theory calculation^{*} shows that at $T=156.5$ MeV the $\pi + N \rightarrow \Delta$ reaction rate monotonically decreases with v of Δ implying lifetime of bound constituent shorter than unbound.

$$\tau_i^{\text{intinelas}} \ll \tau_i$$

Combined with $\frac{1}{B} \ll \tau_i^{\text{intinelas}}$ implies

$$1 \gg \frac{1}{B \tau_{\text{bound state}}}$$

for SHM to make sense for bound states. To predict the existence of bound states, they must hang around long enough to be bound.

Constituent of a loosely bound state such a D or hypertriton, interact with hadrons in the medium essentially as they do when unbound: the scales of the nuclear binding are much smaller than those of the hadrons in the gas. One ceases to have the bound state when a constituent vanishes (eg. there is no deuteron when a nucleon becomes a Δ)

Thus for SHM to make sense

$$\frac{1}{\tau_{\text{bound state}}} < \sum_{j=\text{constituents}} \frac{1}{\tau_j}$$

It is $<$ rather than $=$ as the bound state could dissociate the bound state leaving constituents intact.

- **Implication:** Both D and ${}^3H_{\Lambda}$ are very short-lived states in medium: $\tau < \frac{1}{\sum_i \frac{1}{\tau_i}}$ (τ_i are constituents lifetimes).
 $\tau_D < 1.2 \text{ fm}$ $\tau_{{}^3H_{\Lambda}} < 1.0 \text{ fm}$

- On the other , causality implies that weakly-bound states take a long time to create:

- Characteristic size of weakly bound 2-body state: $R \sim \frac{1}{\sqrt{4 \mu B}}$, μ is reduced mass, B is binding energy large for small B .
 Estimate of “size” assume a uniform sphere which matches rms charge radius $R^2 = \sqrt{\frac{5}{3}} \langle r^2 \rangle$

$$R_D \approx 2.74 \text{ fm}$$

(from D rms charge radius)

$$R_{{}^3H_{\Lambda}} > \sim 10 \text{ fm}$$

(est. from binding energy)

- Causality bound: with reasonable assumptions, time to create state must be larger than the size of the state ($c=1$); otherwise, the state does not “know” it has been formed and presumably much larger as system is nonrelativistic.

- **Implication: Neither D nor ${}^3H_{\Lambda}$ satisfy causality bound:**

D : 2.74 fm size is not much less than $\tau_D < 1.2$ fm

${}^3H_{\Lambda}$: ~15 fm size is not much less than $\tau_{{}^3H_{\Lambda}} < 1.0$ fm

Maryland Group *Phys.Rev.C* 100 (2019) 2, 024911