Are the finite volume effects on the phase diagram fully understood?

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Excited QCD
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What are the typical sizes?

- Typical size of the fireball in heavy ion collisions is a few fm.
- Neutron stars and compact stars built up from strongly interacting matter (with extra structure) with a size ~ 10 km.
- Several models with (different) finite size.
- In field theoretical calculations (LSM, NJL, DS, etc) usually the size is infinite.

Why does it matter?

- It can be seen that the properties of the system can change significantly.
- Example: in the phase diagram of strong interaction the CEP (and the first-order region) might disappear.

Size of the physical system

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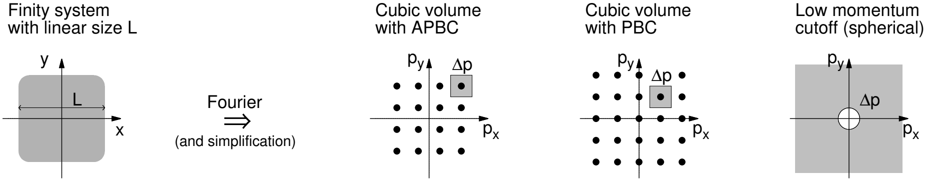
- Example: in the phase diagram of strong interaction the CEP (and the first-order region) might disappear.

The vicinity of the CEP is accessible with models that are in the thermodynamic limit.

How to consider the finite size effects without losing the advantages of these models?

It is usual to modify the momentum space.

- Discretization: $\int dp \to \sum_n$
- Low momentum cutoff: $\int_0^\infty dp \to \int_\lambda^\infty dp$



Finite size effects in different models

Various results in (P)NJL, (P)LSM, DS, etc. calculations.
With discretization and low momentum cutoff as well.

	Discretization	Low cutoff
LSM	J. Phys. G **38**, 085101 (2011) PoS **FACESQCD**, 017 (2010)	J. Phys. G **44**, no.2, 025101 (2017) Universe **5**, no.4, 94 (2019)
NJL	arXiv:1802.00258 [hep-ph] Mod. Phys. Lett. A **33**, no.39, 1850232 (2018) Universe **8**, no.5, 264 (2022)	Phys. Rev. D **87**, no.5, 054009 (2013) Phys. Rev. D **91**, no.5, 051501 (2015) Int. J. Mod. Phys. A **32**, no.13, 1750067 (2017)
FRG	Phys. Rev. D **73**, 074010 (2006) Phys. Rev. D **90**, no.5, 054012 (2014) Phys. Rev. D **95**, no.5, 056015 (2017) Phys. Rept. **707-708**, 1-51 (2017)	
D-S	Phys. Rev. D **81**, 094005 (2010) Phys. Rev. D **102**, 114011 (2020) Phys. Rev. D **104**, no.7, 074035 (2021) Phys. Lett. B **841**, 137908 (2023)	Nucl. Phys. B **938**, 298-306 (2019)

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Various results in (P)NJL, (P)LSM, DS, etc. calculations.
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Vacuum part		Discretization	Low cutoff
NO	**LSM**	J. Phys. G **38**, 085101 (2011) PoS **FACESQCD**, 017 (2010)	J. Phys. G **44**, no.2, 025101 (2017) Universe **5**, no.4, 94 (2019)
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Separated Renorm.	**FRG**	Phys. Rev. D **73**, 074010 (2006) Phys. Rev. D **90**, no.5, 054012 (2014) Phys. Rev. D **95**, no.5, 056015 (2017) Phys. Rept. **707-708**, 1-51 (2017)	
Renorm.	**D-S**	Phys. Rev. D **81**, 094005 (2010) Phys. Rev. D **102**, 114011 (2020) Phys. Rev. D **104**, no.7, 074035 (2021) Phys. Lett. B **841**, 137908 (2023)	Nucl. Phys. B **938**, 298-306 (2019)

Few examples for the phase diagram:

LSM

Palhares, Fraga and Kodama,
J. Phys. G **38**, 085101 (2011)

NJL

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QM model FRG

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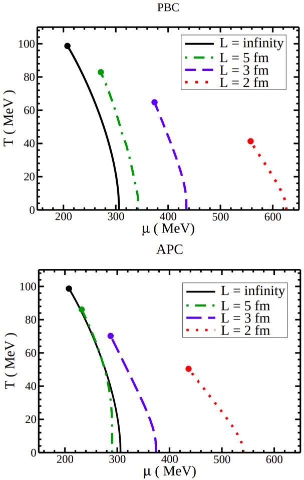

Only zero mode of vacuum part

No vacuum part

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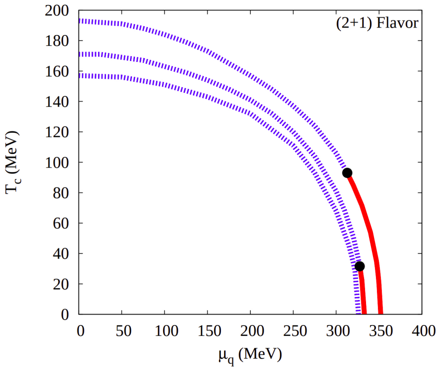

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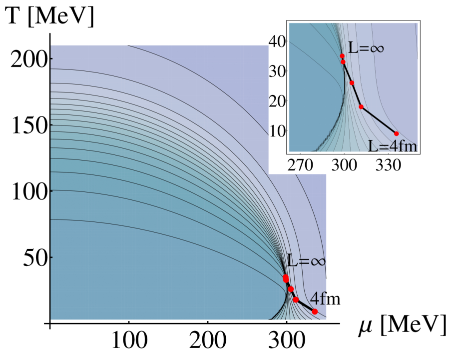

Finite size effects in different models

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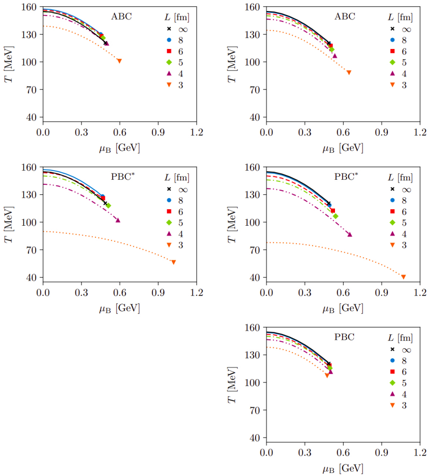

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Vector and axial vector meson **E**xtended **P**olyakov **Q**uark-**M**eson model. (ePQM or ELSM)
Effective model to study the phase diagram of strongly interacting matter at finite T and μ.

Phys. Rev. D 93, no. 11, 114014 (2016)

- **Q**uark-**M**eson model: "simple" linear sigma model with quarks and mesons
- **E**xtended: Vector and Axial vector nonets (besides to Scalar and Pseudoscalar)
Isospin symmetric case: 16 mesonic degrees of freedom.
- **P**olyakov: Polyakov loop variables give 2 order parameters Φ, $\bar{\Phi}$.
- The mesonic Lagrangian $\mathcal{L}_m$ with chiral symmetry

$$SU(3)_L \times SU(3)_R \times U(1)_V \times U(1)_A \rightarrow SU(2)_I \times U(1)_V$$

broken explicitly (and spontaneously) and with the axial anomaly taken into account.

- fermion-meson interaction in a Yukawa type Lagrangian.

Thermodynamics: **Mean field level** effective potential:

- Classical potential.
- Fermionic one-loop correction with vanishing fluctuating mesonic fields.

$$\bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - \mathrm{diag}(m_u, m_d, m_s)\right)\psi$$

  Functional integration over the fermionic fields.
  The momentum integrals are renormalized.
- Polyakov loop potential.

$$\Omega(T,\mu_q) = U_{Cl} + \mathrm{tr}\int_K \log\left(iS_0^{-1}\right) + U(\Phi,\bar{\Phi}) \tag{1}$$

Field equations (FE):

$$\frac{\partial\Omega}{\partial\bar{\Phi}} = \frac{\partial\Omega}{\partial\Phi} = \frac{\partial\Omega}{\partial\phi_N} = \frac{\partial\Omega}{\partial\phi_S} = 0 \tag{2}$$

Curvature meson masses:

$$M_{ab}^2 = \left.\frac{\partial^2\Omega}{\partial\varphi_a\partial\varphi_b}\right|_{\{\varphi_i\}=0} \tag{3}$$

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Functional integration over the fermionic fields.
The momentum integrals are renormalized.

Only this term is modified

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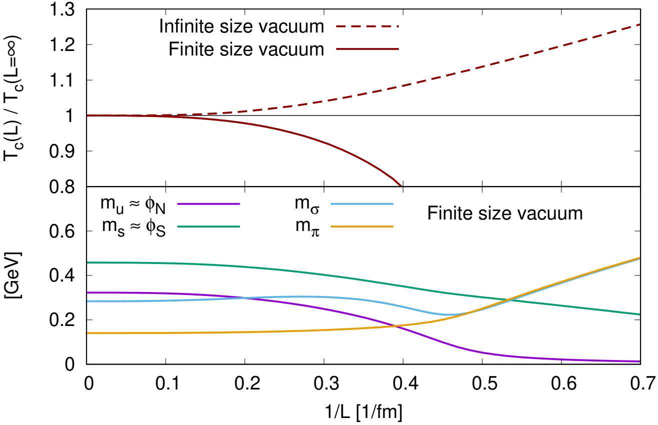
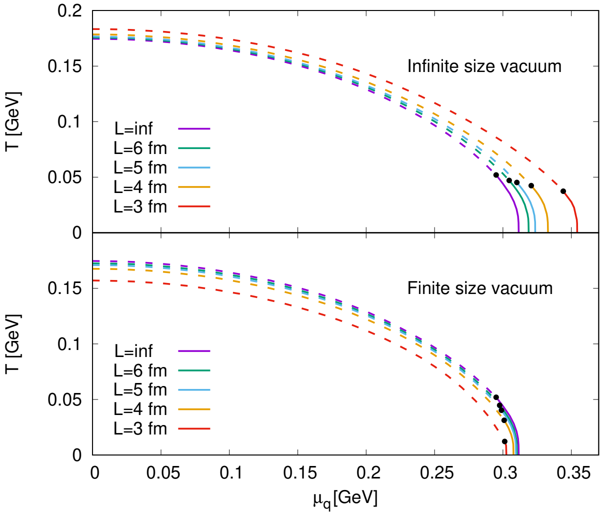
- Fermionic vacuum and matter contribution:

$$\Omega_{\bar{q}q}(T,\mu_q) = \Omega_{\bar{q}q}^{\mathrm{vac}} + \Omega_{\bar{q}q}^{\mathrm{mat}}(T,\mu_q)$$

- The size dependence of $\Omega_{\bar{q}q}^{\mathrm{vac}}$ pushes the system towards chiral restoration

- At $T = 0$ and $\mu_q = 0$ the physical quantities also show the restoration



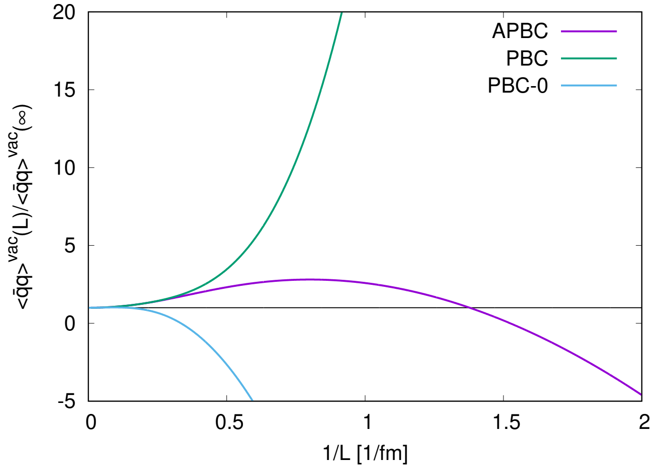


- With $L = \infty$ vacuum part the chirally broken phase expands and the CEP is present even for very small sizes
- With L dependent vacuum part the chirally broken phase will be reduced.

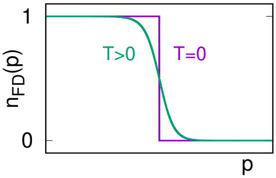
  The CEP disappears at $L = 2.5$ fm, as well as the broken phase at $L = 2$ fm.

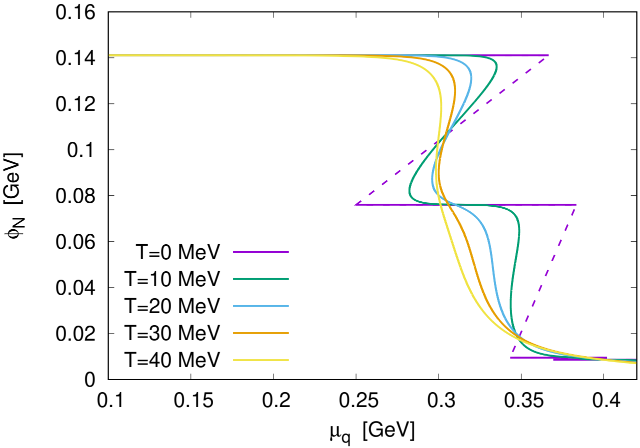
- APBC and PBC is considered
- With size-dependent vacuum contribution the condensates will increase for APBC (if $L > 1$ fm) and for PBC
- The common solution of the field equation is lost around 4.5 fm.

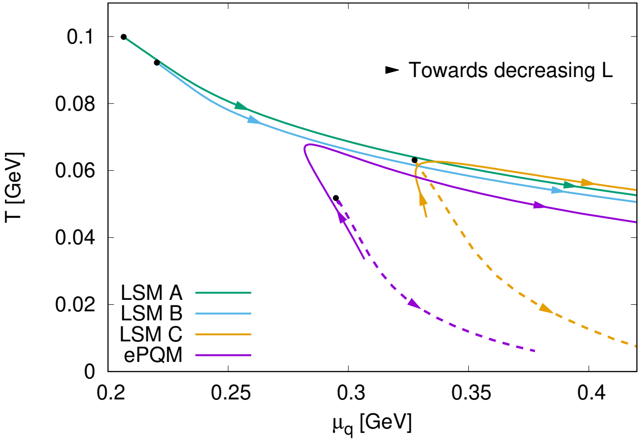
FEs: $\mathcal{P}[\phi_N, \phi_S] + \alpha\langle\bar{q}q\rangle = 0$



- APBC and PBC is considered
- Fermi–Dirac distribution in the matter part $\rightarrow$ step function for $T \rightarrow 0$
- The modes dropping below the Fermi surface cause drops in $\phi_{N/S}$.





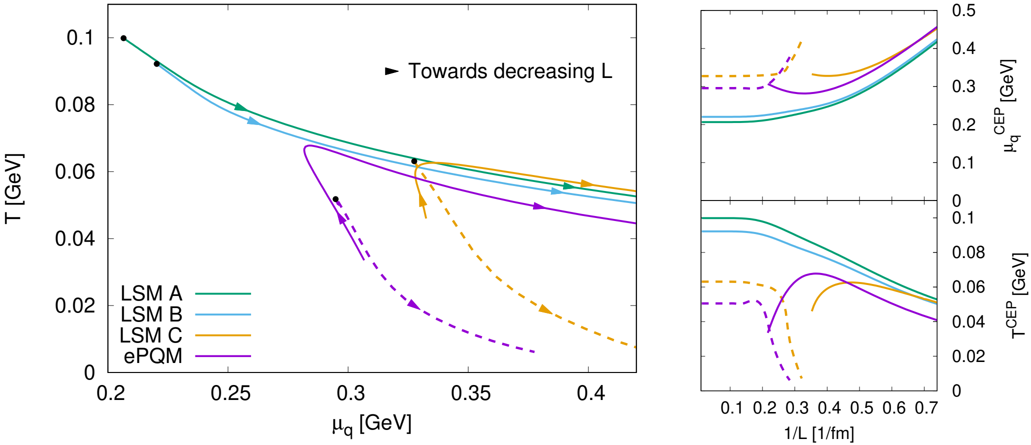


LSM A: J. Phys. G **38**, 085101 (2011)

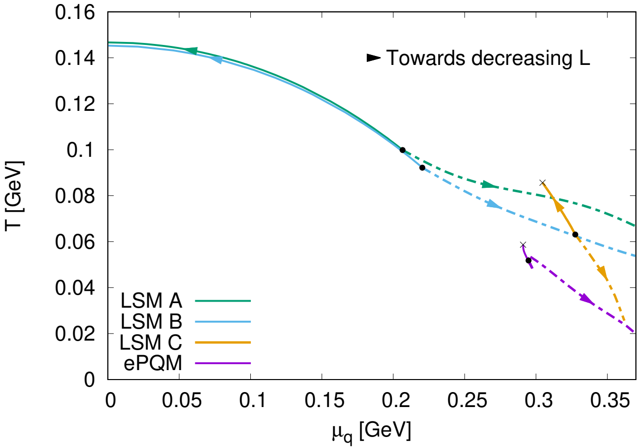
LSM B, C: Phys. Rev. D **79**, 014018 (2009)

- For the "low lying" models the $L \to \infty$ CEP and the $L \to 0$ CEP is not continuously connected
- For "higher lying" models there is a continuous path
- At $L < 2$ fm the CEP is governed by the first mode entering below the Fermi surface

CEP with APBC



CEP with PBC (and the vacuum zero mode)



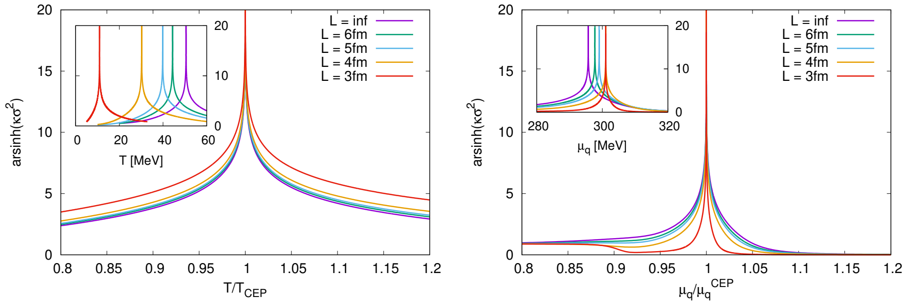
- With vacuum part of $L = \infty$ new unphysical first-order transition below $L \approx 5.5$ fm
- With (partially included) size-dependent vacuum part the trend is reversed.
- LSM A with zero mode added: J. Phys. G **38**, 085101 (2011)

Summary

- We investigated the finite size effects on the phase diagram with different momentum space constraints, boundary conditions, and treatment of the vacuum term.
- The CEP (probably) moves to lower T and higher μ_q with the decreasing size in most scenarios.
- **The details of the finite size effects depend on the used momentum space constraint, the boundary condition, and the treatment of the vacuum part.**
- With low momentum cutoff, the size-dependent vacuum term leads to the reduction of the chirally broken phase, and the disappearance of the CEP when the size decreases, contrary to the case with infinite size vacuum.
- With discretization the CEP will be determined by the modes entering below the Fermi surface. The location of the $L \to \infty$ CEP strongly affects its trajectory with the decreasing size. The vacuum part has an especially strong effect in the case of PBC.

Thank you!

Backup: Barion fluctuations – kurtosis through the CEP



- Moderate modification, only at very small sizes.
- The CEP is already very close to the μ_q axis.
- Note: divergence at the CEP.

- $\mathcal{L}_m$ contains the dynamical, the symmetry breaking, and the meson-meson interaction terms.
  - $U(1)_A$ anomaly and explicit breaking of the chiral symmetry.
  - Each meson-meson terms upto 4th order that are allowed by the chiral symmetry.
- Constituent quarks ($N_f = 2 + 1$) in Yukawa Lagrangian

$$\mathcal{L}_Y = \bar{\psi}\left(i\gamma^\mu \partial_\mu - g_F(S - i\gamma_5 P) - g_V \gamma^\mu (V_\mu + \gamma_5 A_\mu)\right)\psi \qquad (4)$$

In the 2016 version $g_V = 0$ was used. Phys. Rev. D 104, 056013 (2021)

- SSB with nonzero vev. for scalar-isoscalar sector ϕ_N, ϕ_S.
  $\Rightarrow m_{u,d} = \frac{g_F}{2}\phi_N$, $m_s = \frac{g_F}{\sqrt{2}}\phi_S$ fermion masses in $\mathcal{L}_Y$.
- Mean field level effective potential → the meson masses and the thermodynamics are calculated from this.