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On the ultraviolet asymptotics of the glueball effective action in large-N Yang-Mills theory

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in collaboration with M. Bochicchio and M. Papinutto

Based on:

[1] M. Bochicchio, M. Papinutto and F. Scardino, JHEP 08 (2021) 142, [2104.13163].

[2] M. Bochicchio, Eur. Phys. J. C 81 (2021) 749, [2103.15527]

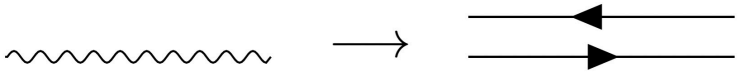
[3] M. Bochicchio, M. Papinutto and F. Scardino, Phys. Rev. D 108 (2023), [2208.14382].

[4] M. Bochicchio, M. Papinutto and F. Scardino, to appear.

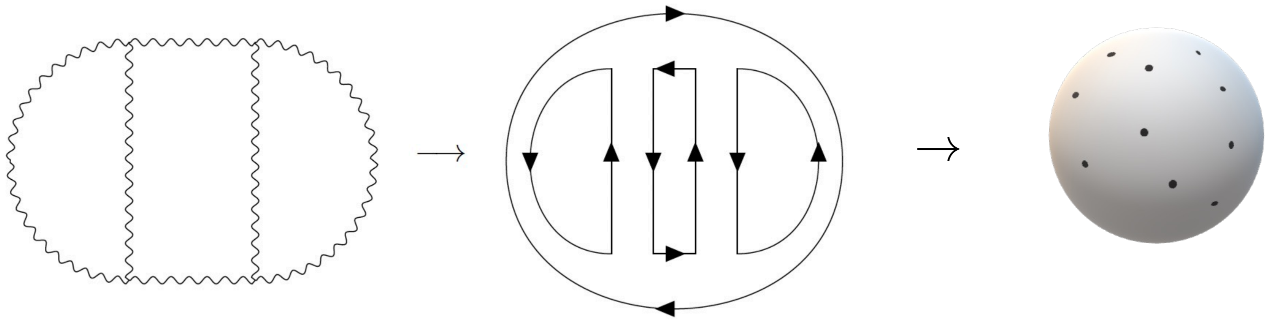
It has been known for more than forty years that SU(N) Yang-Mills (YM) theory admits the 't Hooft large-N topological expansion [5] for the n-point correlators of gauge-invariant single-trace operators.

To say it in a nutshell, the corresponding Feynman graphs in 't Hooft double-line representation, after a suitable gluing of reversely oriented lines, are topologically classified by the sum on the genus g of n-punctured closed Riemann surfaces, where each topology is weighted by a factor of N^{χ} with χ=2-2g-n the Euler characteristic of the Riemann surface:

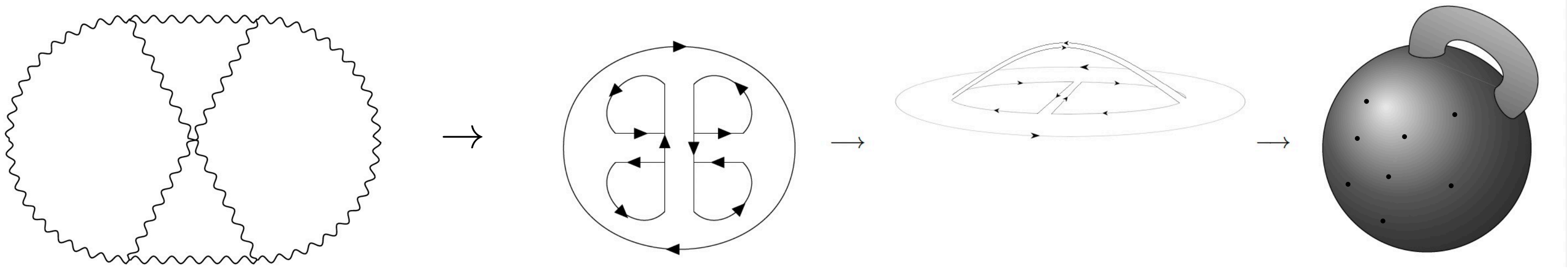
‘t Hooft double line prescription:



n-punctured spheres, g= 0, are triangulated by planar diagrams:



Higher genus, g, n-punctured surfaces are triangulated by nonplanar diagrams:



[5] G. 't Hooft, A Planar Diagram Theory for Strong Interactions, Nucl. Phys. B72 (1974) 461.

In the ‘t Hooft large-N expansion, Feynmann diagrams -in the double line representation- are topologically equivalent to closed Riemann surfaces (with punctures).

Then, the perturbative series can be rearranged so that the topologically equivalent contributions are resummed together.

Therefore, by assuming the ‘t Hooft expansion, the generating functional of single trace operators $\mathcal{O}$

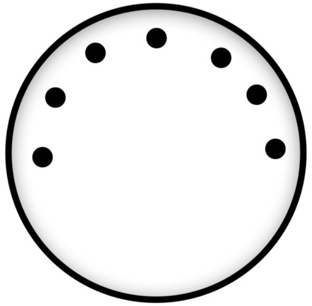
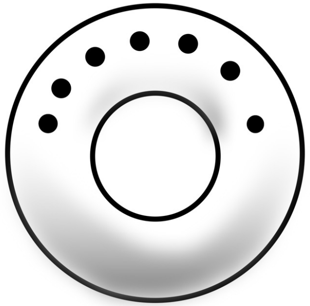
$$\mathcal{W}^E[J_{\mathcal{O}}] = \log \mathcal{Z}^E[J_{\mathcal{O}}] = \log \int \mathcal{D}A\, e^{-S_{YM}+\sum_s \int J_{\mathcal{O}_s}\mathcal{O}_s}$$

should be interpreted as a sum of topologically distinct terms

$$\mathcal{W} = \sum_n N^{2-n}\mathcal{W}_{\text{sphere}}(g,n) + N^{-n}\mathcal{W}_{\text{torus}}(g,n) + \dots$$

where n stands for the number of operator insertions and the ellipses for higher-genus contributions.

Graphically

$$\mathcal{W} = \sum_n N^{2-n}$$  $+ N^{-n}$  $+ \dots$

Nonperturbatively, $\mathcal{W}_{\rm sphere}[J_{\mathcal{O}}]$, which perturbatively resums the ('t Hooft) planar contributions, is a sum of tree diagrams involving glueball propagators and vertices,

while $\mathcal{W}_{\rm torus}[J_{\mathcal{O}}]$, which perturbatively resums the leading-non('t Hooft-)planar contributions, is a sum of glueball one-loop diagrams.

Nonperturbatively, $\mathcal{W}_{\rm torus}[J_{\mathcal{O}}]$ should have the structure of the logarithm of a functional determinant. Indeed, in the yet-to-come nonperturbative solution of large-N YM theory, the very same correlators should be computed by the correlators of a glueball field Φ with an infinite number of components, the corresponding generating functional being schematically to one loop accuracy:

$$\begin{aligned}\mathcal{W}_{\rm glueball}[j] = \log \mathcal{Z}_{\rm glueball}[j] = &-\frac{1}{2}\int \Phi_j *_2 (-\Delta + M^2)\Phi_j - \frac{1}{N}\frac{1}{3!}\int \Phi_j *_3 \Phi_j *_3 \Phi_j + \int j *_1 \Phi_j \\ &-\frac{1}{2}\log \mathrm{Det}\left(*_2(-\Delta+M^2) + \frac{1}{N} *_3 \Phi_j *_3 + \ldots\right) + \ldots\end{aligned}$$

with $\frac{\delta S}{\delta \Phi}\Big|_{\Phi_j} = *_1 j$ and $\Phi_j = (*_2(-\Delta+M^2))^{-1} *_1 j - \frac{1}{2N}(*_2(-\Delta+M^2))^{-1} *_3 \Phi_j *_3 \Phi_j + \ldots$

where $*_1$ and $*_2$ are fixed by matching the corresponding spectral representations as a sum of free propagators for 2-point correlators of $\mathcal{O}$ at $N=\infty$, while $*_3$ stands for a presently unkown algebraic structure on the glueball fields.

The minus sign in front of the logDet in $\mathcal{W}_{\text{glueball}}[j]$ arises from the spin-statistics theorem, since all the gauge-invariant glueball interpolating fields have integer spin, and thus the glueballs should be bosons

$$-\frac{1}{2}\log \mathrm{Det}\left(*_2(-\Delta + M^2) + \frac{1}{N} *_3 \Phi_j *_3 + \dots \right)$$

Until recently, nothing has been known quantitatively on

$$\mathcal{W}^E[J_{\mathcal{O}}] = \log \mathcal{Z}^E[J_{\mathcal{O}}] = \log \int \mathcal{D}A\, e^{-S_{YM} + \sum_s \int J_{\mathcal{O}_s} \mathcal{O}_s} \qquad \text{and} \qquad \mathcal{W}_{\text{glueball}}[j]$$

We briefly explain our recent computation of the renormalization-group (RG) improved generating functional of Euclidean twist-2 operators [3].

We start with twist-2 operators in Minkowskian space-time with maximal-spin projection on the p_+ light-cone direction in pure SU(N) Yang-Mills theory [3].

For simplicity in this talk we consider only the even-spin nonchiral operators, which are exactly bilinear in the light-cone gauge $A_+ = 0$:

$$\mathbb{O}_s = \frac{1}{2}\bar{A}^a(x)\mathcal{Y}^{\frac{5}{2}}_{s-2}(\overrightarrow{\partial}_+, \overleftarrow{\partial}_+)A^a(x)$$

$$\begin{aligned}\mathcal{Y}^{\frac{5}{2}}_{s-2}(\overrightarrow{\partial}_+, \overleftarrow{\partial}_+) &= \overleftarrow{\partial}_+(i\overrightarrow{\partial}_+ + i\overleftarrow{\partial}_+)^{s-2} C^{\frac{5}{2}}_{s-2}\left(\frac{\overrightarrow{\partial}_+ - \overleftarrow{\partial}_+}{\overrightarrow{\partial}_+ + \overleftarrow{\partial}_+}\right)\overrightarrow{\partial}_+ \\ &= \frac{\Gamma(3)\Gamma(s+3)}{\Gamma(5)\Gamma(s+1)} i^{s-2} \sum_{k=0}^{s-2} \binom{s}{k}\binom{s}{k+2}(-1)^{s-k}\overleftarrow{\partial}_+^{s-k-1}\overrightarrow{\partial}_+^{k+1}\end{aligned}$$

where **s** is the collinear spin, i.e. the eigenvalue of the spin operator along the light-cone direction and $C^{\frac{5}{2}}_{s-2}$ are Gegenbauer polynomials, which are a special case of the Jacobi polynomials

We have computed the conformal generating functional as a Gaussian functional integral in the light-cone gauge $A_+ = 0$ to the lowest order [3]

$$\mathcal{Z}_{conf}[J_{\mathbb{O}}] = \frac{1}{Z}\int \mathcal{D}A\mathcal{D}\bar{A}\, e^{-i\int d^4x\, \bar{A}^a \Box A^a} \exp\left(\int d^4x \sum_s J_{\mathbb{O}_s}\mathbb{O}_s\right)$$

The connected conformal generating functional reads

$$\mathcal{W}_{conf}[J_{\mathbb{O}}] = \log \mathcal{Z}_{conf}[J_{\mathbb{O}}] = -(N^2-1)\log \mathrm{Det}\left(\mathbb{I} + \frac{1}{2} i \Box^{-1} J_{\mathbb{O}_s} \otimes \mathcal{Y}^{\frac{5}{2}}_{s-2}\right)$$

From analytic continuation to Euclidean space-time we obtain

$$\mathcal{W}^{E}_{\text{conf}}[J_{\mathbb{O}^E}] = -(N^2-1)\log \text{Det}\left(\mathbb{I} + \frac{1}{2}\frac{1}{N}\Delta^{-1} J_{\mathbb{O}^E_s} \otimes \mathcal{Y}^{\frac{5}{2}}_{s-2}\right)$$

By means of a careful choice of the renormalization scheme that reduces the mixing of the above operators to the multiplicatively renormalizable case to all orders of perturbation theory [2], we have lifted $\mathcal{W}^{E}_{\text{conf}}[J_{\mathbb{O}^E}]$ to the generating functional of the RG-improved correlators $\mathcal{W}^{E}_{asym}$ -as all the coordinates are uniformly rescaled by a factor $\lambda \to 0$ -
that inherits the very same structure of the logarithm of a functional determinant [3]

$$\mathcal{W}^{E}_{\text{asym}}[J_{\mathbb{O}^E}] = -(N^2-1)\log \text{Det}\left(\mathbb{I} + \frac{1}{2}\frac{1}{N}\frac{Z_{\mathbb{O}_s}(\lambda)}{\lambda^{s+2}}\Delta^{-1} J_{\mathbb{O}^E_s} \otimes \mathcal{Y}^{\frac{5}{2}}_{s-2}\right)$$

In [2] it demonstrated that in the above scheme $Z(\lambda)$ is diagonalizable and one-loop exact to all orders of perturbation theory with eigenvalues

$$Z_{\mathcal{O}_i}(\lambda) = \left(\frac{g(\mu)}{g(\frac{\mu}{\lambda})}\right)^{\frac{\gamma_{0\mathcal{O}_i}}{\beta_0}} \quad \text{and} \quad g^2(\frac{\mu}{\lambda}) \sim \frac{1}{\beta_0 \log(\frac{1}{\lambda^2})}\left(1 - \frac{\beta_1}{\beta_0^2}\frac{\log\log(\frac{1}{\lambda^2})}{\log(\frac{1}{\lambda^2})}\right)$$

Explictly, we separate the sphere and torus asymptotic contributions:

$$\mathcal{W}^{E}_{asym}[J_{\mathbb{O}^E}] = -N^2 \log \mathrm{Det}\left(\mathbb{I} + \frac{1}{2}\frac{1}{N}\frac{Z_{\mathbb{O}_s}(\lambda)}{\lambda^{s+2}}\Delta^{-1} J_{\mathbb{O}^E_s} \otimes \mathcal{Y}^{\frac{5}{2}}_{s-2}\right)$$
$$+ \log \mathrm{Det}\left(\mathbb{I} + \frac{1}{2}\frac{1}{N}\frac{Z_{\mathbb{O}_s}(\lambda)}{\lambda^{s+2}}\Delta^{-1} J_{\mathbb{O}^E_s} \otimes \mathcal{Y}^{\frac{5}{2}}_{s-2}\right)$$

Remarkably, our asymptotic result reproduces the logDet structure of the glueball one-loop generating functional

$$-\frac{1}{2}\log \mathrm{Det}\left(*_2(-\Delta + M^2) + \frac{1}{N} *_3 \Phi_j *_3 + \dots \right)$$

Yet, surprisingly, the sign is the opposite of what would follow from the spin-statistics theorem

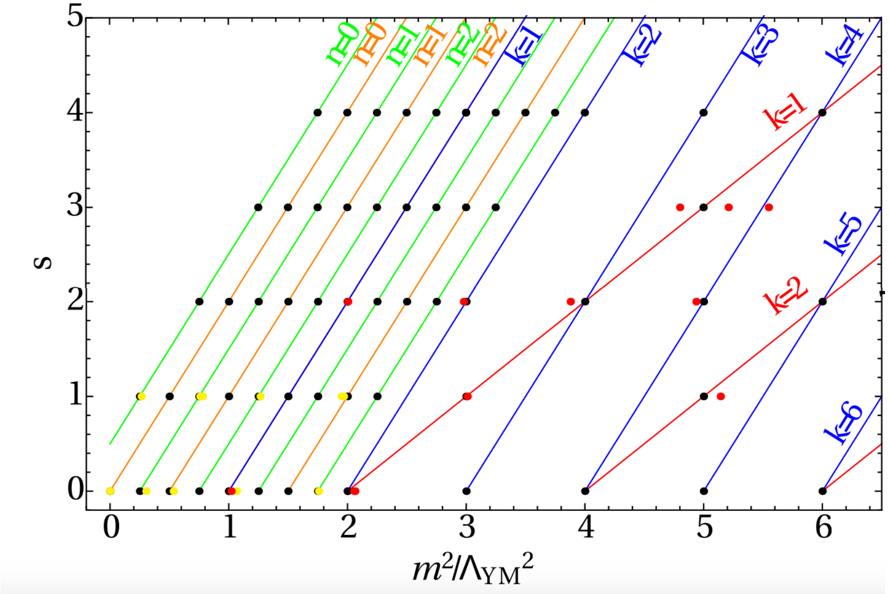
$$\mathcal{W}^{E}_{\text{Torus asym}}[J_{\mathbb{O}^E}] = +\log \mathrm{Det}\left(\mathbb{I} + \frac{1}{2}\frac{1}{N}\frac{Z_{\mathbb{O}_s}(\lambda)}{\lambda^{s+2}}\Delta^{-1} J_{\mathbb{O}^E_s} \otimes \mathcal{Y}^{\frac{5}{2}}_{s-2} \right)$$

The aim of this talk is to solve the above sign puzzle and to discuss the implications of the solution.

One of the hypotheses of the spin-statistics theorem is that the number of fields is finite. Indeed, in theories with an infinite number of fields, counterexamples to the theorem are known.
These counterexamples are based on massive infinite dimensional representations of the Lorentz group. For instance, in [10] examples are constructed of theories with an infinite number of integer-spin fields where fermionic statistics must be imposed in order to ensure positivity of the energy.

Conversely, there are examples of fields with half-integer spins that require bosonic quantization [10][11][12][13].

A significant issue with this hypothetical way out of the sign puzzle is that the aforementioned infinite-dimensional representations of the Lorentz group have infinite mass degeneracy. Namely, the fields constructed by means of these representations decompose –according to the Wigner theorem- into the sum of irreducible representations of the Poincaré group corresponding to an infinite number of particles of any spin, all having the same mass.
This would correspond to vertical Regge trajectories that is not acceptable in large-N Yang-Mills theory, as evidence from lattice calculations shows [14]



[10] Harish-Chandra. "Infinite Irreducible Representations of the Lorentz Group." Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences 189, no. 1018 (1947)

[11] Feldman, G. and Matthews, P. T., Unitarity, Causality, and Fermi Statistics, Phys. Rev. 151 , 4, (1966)

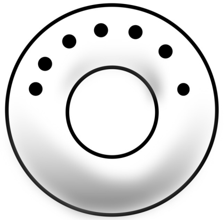
[12] Streater, R.F. Local fields with the wrong connection between spin and statistics. Commun.Math. Phys. **5**, 88–96 (1967).

[13] R. Casalbuoni, Majorana and the Infinite Component Wave Equations, PoS EMC2006 (2006), [hep-th/0610252].

[14] M. Bochicchio, Glueball and Meson Spectrum in Large-N QCD, Few Body Syst. 57 (2016) no.6, 455-459

We have found a different way out of the sign puzzle: New topologies actually occur in the large-N expansion of the generating functional that refine ‘t Hooft topological expansion

$$\mathcal{W}_{1-\mathrm{loop}} = \sum_n N^{-n}$$



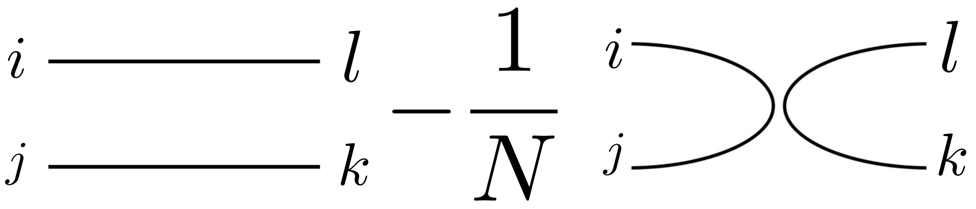
$$+\,\text{new topologies} \quad \sim + \log \mathrm{Det}$$

To understand where the extra topologies come from, we reconsider the proof of the ‘t Hooft topological expansion.

In SU(N) Yang-Mills theory the color dependence of the propagator has a leading and subleading contribution

$$\langle A_{ij} A_{lk} \rangle \propto \frac{1}{2}\left(\delta_{il}\delta_{jk} - \frac{1}{N}\delta_{ij}\delta_{lk}\right)$$

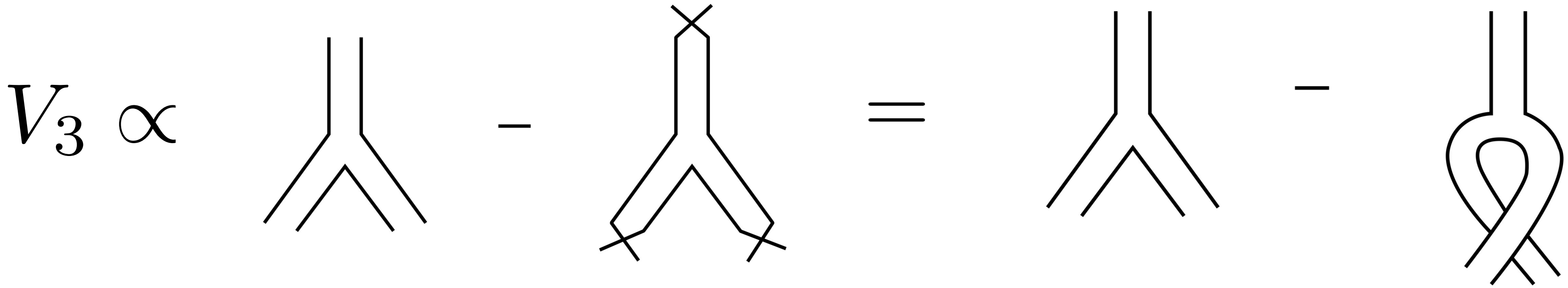
In the double line representation [15][16]



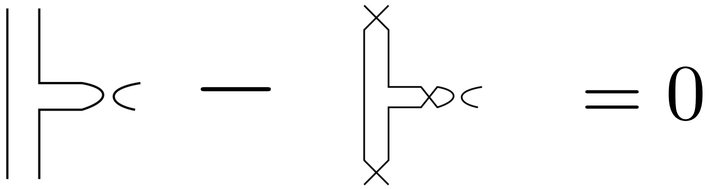
[15] M. Marino, Instantons and Large N: An Introduction to Non-Perturbative Methods in Quantum Field Theory,' Cambridge University Press, 2015.
[16] F. Maltoni, K. Paul, T. Stelzer and S. Willenbrock, Color Flow Decomposition of QCD Amplitudes, Phys. Rev. D 67 (2003), [hep-ph/0209271].

The vertices of the Lagrangian are $V_3 \propto \frac{g}{\sqrt{N}} f^{abc}$ and $V_4 \propto \frac{g^2}{N} f^{abl} f^{cdl} \propto V_3^2$

They are drawn in double line as [15]



The subleading propagator does not contribute when attached to the vertices of the Lagrangian. Indeed, the subleading propagator is a U(1) contribution, while the vertices are purely nonabelian. Graphically



Therefore, the topology of vacuum-vacuum diagrams matches the ‘t Hooft topological expansion [15]. The same holds for all the gluon composite operators whose local vertices are proportional to the matrix product of a string of f^{abc} [16].

Therefore, we conclude that in these cases we may drop the subleading contribution to the propagator and the ‘t Hooft expansion holds for SU(N) [15][16].

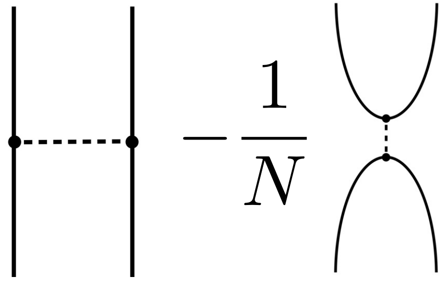
Yet, we point out that the above proof does not apply to 2-gluon operators – specifically, our twist-2 operators in the light-cone gauge. Indeed, even in the adjoint representation, their local vertex involves δ^{ab} as opposed to f^{abc}.

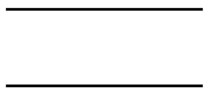
As a consequence, when the subleading part of the propagator is attached to the local vertex, a nonzero contribution is obtained.

Equivalently, in the SU(N) theory we may keep only the leading part of the propagator -in order to maintain the ‘t Hooft double line representation- provided that we project the above operators on their traceless part

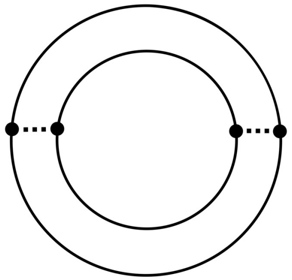
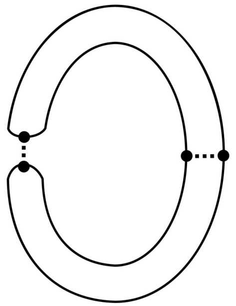
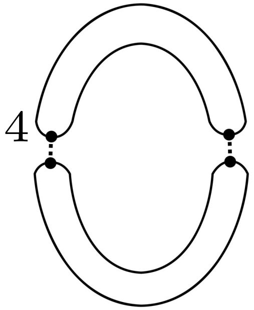
$$\mathrm{Tr}(\bar{A}A) \to \mathrm{Tr}(\bar{A}A) - \frac{1}{N}\mathrm{Tr}(\bar{A})\mathrm{Tr}(A)$$

Hence, the 2-gluon operators can be represented as a 2-point vertex with a subleading contribution

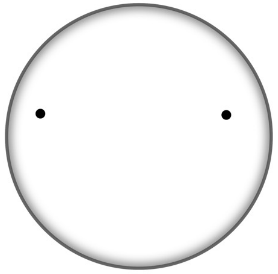
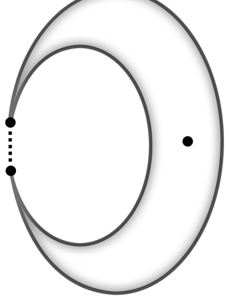
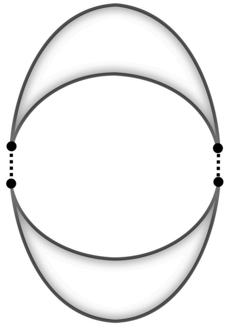


while keeping the propagator in the standard double line representation 

This subleading contribution to the operator vertex gives rise to new topologies. For example, we get the following color structure for the 2-point correlators of twist-2 operators to the leading perturbative order

$\langle \mathbb{O}_s(x)\mathbb{O}_s(0)\rangle = N^{-2}$  $-2N^{-3}$  $+N^{-4}$ 

Hence, new diagrams arise -in addition to the planar one above- that are possibly color-disconnected (but space-time connected) punctured disks. Following 't Hooft gluing prescription, the first one has the topology of a punctured sphere

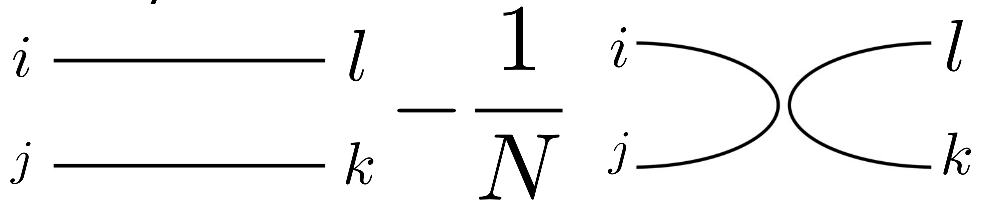
$\langle \mathbb{O}_s(x)\mathbb{O}_s(0)\rangle = N^{0}$  $+N^{-2}$  $+N^{-2}$ 

while the topology of the remaining ones arises by identifying the doubly punctured disk with an infinite strip and gluing together the opposite edges into an infinite cylinder that is a 2-punctured sphere, to get possibly disconnected punctured spheres with two punctures pairwise identified.

Analytically, the Euclidean conformal generating functional can be written without performing the color trace as

$$\mathcal{W}^E_{\text{conf}}[J_{\mathbb{O}^E}] = -\log \text{Det}\left(\mathcal{I} + \frac{1}{2}(I-P)\Delta^{-1}\frac{J_{\mathbb{O}^E_s}}{N} \otimes \mathcal{Y}^{\frac{5}{2}}_{s-2}\right)$$

where the term containing I-P involves the SU(N) propagator:



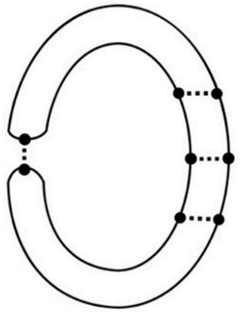
As we said earlier, in order to keep the 't Hooft double-line representation that only involves the leading propagator also beyond the planar limit of the SU(N) theory, we transfer the 1/N dependence from the propagator to the vertex by the following identity

$$\mathcal{W}^E_{\text{conf}}[J_{\mathbb{O}^E}] = -\log \text{Det}\left(\mathcal{I} + \frac{1}{2}\Delta^{-1} I \frac{J_{\mathbb{O}^E_s}}{N} \otimes \mathcal{Y}^{\frac{5}{2}}_{s-2}\right)$$
$$-\log \text{Det}\left(\mathcal{I} - \frac{1}{2}\left(\mathcal{I} + \frac{1}{2}\Delta^{-1} I \frac{J_{\mathbb{O}^E_s}}{N} \otimes \mathcal{Y}^{\frac{5}{2}}_{s-2}\right)^{-1}\Delta^{-1} P \frac{J_{\mathbb{O}^E_s}}{N} \otimes \mathcal{Y}^{\frac{5}{2}}_{s-2}\right)$$

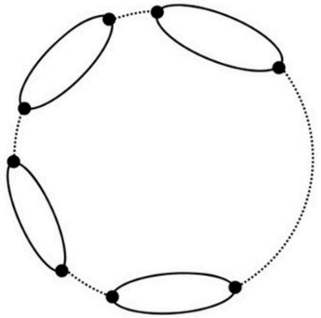
The first logDet above is the planar contribution -- involving the leading vertex -- and the second logDet the leading-nonplanar one -- involving at least one subleading vertex for 2-gluon operators carrying the factor of P.

Graphically, the leading-nonplanar conformal generating functional reads

$$\mathcal{W}_{\text{conf leading-nonplanar}} = \sum_{n} N^{-n} \sum_{p=1}^{n} N^{-1}$$



$$+ \dots + N^{-n}$$

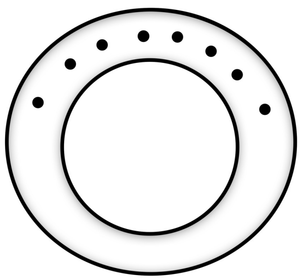
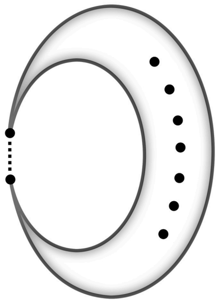
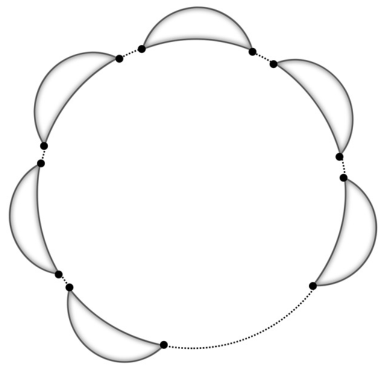


After the RG improvement, the leading-nonplanar asymptotic generating functional reads:

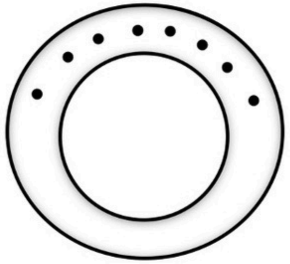
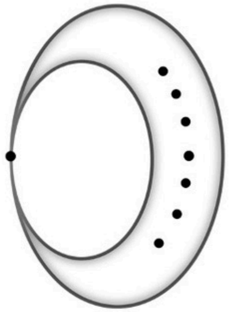
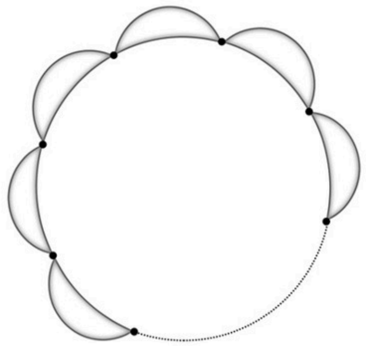
$$\begin{aligned}\mathcal{W}^{E}_{\text{asym nonplanar}}[J_{\mathbb{O}^E}, \lambda] &= -\log \operatorname{Det}\left(\mathcal{I} - \frac{1}{2}\left(\mathcal{I} + \frac{1}{2}\frac{Z_{\mathbb{O}_s}(\lambda)}{\lambda^{s+2}} \Delta^{-1} I \frac{J_{\mathbb{O}^E_s}}{N} \otimes \mathcal{Y}^{\frac{5}{2}}_{s-2}\right)^{-1} \frac{Z_{\mathbb{O}_s}(\lambda)}{\lambda^{s+2}} \Delta^{-1} P \frac{J_{\mathbb{O}^E_s}}{N} \otimes \mathcal{Y}^{\frac{5}{2}}_{s-2}\right) \\ &= + \log \operatorname{Det}\left(\mathbb{I} + \frac{1}{2}\frac{Z_{\mathbb{O}_s}(\lambda)}{\lambda^{s+2}} \Delta^{-1} \frac{J_{\mathbb{O}^E_s}}{N} \otimes \mathcal{Y}^{\frac{5}{2}}_{s-2}\right)\end{aligned}$$

Remarkably, now the overall sign of the first logDet is consistent with the bosonic statistics for the glueballs, but at the price of introducing a refined topological expansion.

Hence, our refined topological expansion of the one-loop generating functional of twist-2 operators reads:

$$\mathcal{W}_{1-\text{loop}} = \sum_n N^{-n}$$  $$+\, N^{-n}$$  $$+ \ldots +\, N^{-n}$$ 

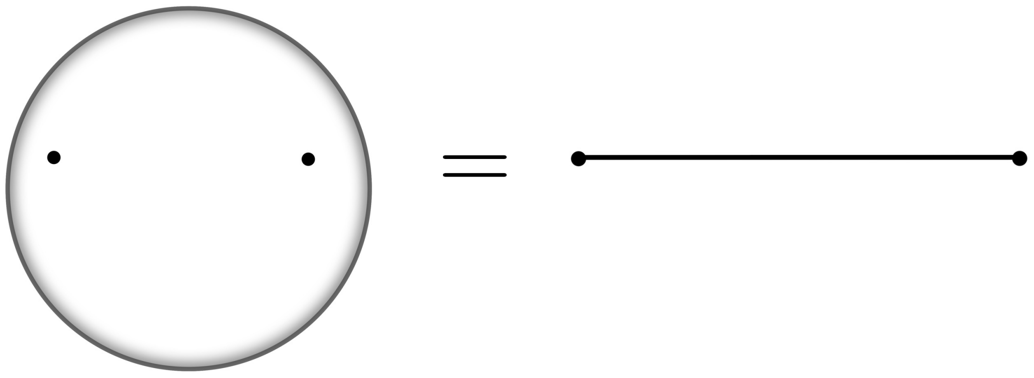
Remarkably, the new diagrams are the normalization of pinched tori, where the pinches are cut in possibly disconnected components that are punctured spheres with two punctures pairwise identified:

$$\mathcal{W}_{1-\text{loop}} = \sum_n N^{-n}$$  $$+\, N^{-n}$$  $$+ \ldots +\, N^{-n}$$ 

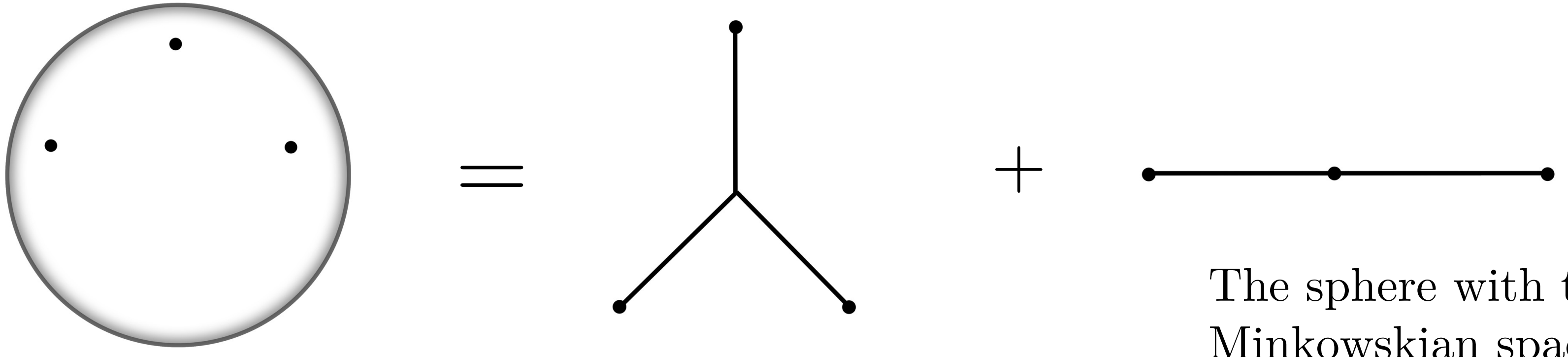
Interestingly, the smooth-torus contribution is suppressed in perturbation theory with respect to the remaining diagrams, since the smooth torus inevitably involves the Lagrangian vertices (that carry powers of the coupling).

Yet, in the RG-improved generating functional the smooth tori are essential to provide the renormalization factors due to the anomalous dimensions also for the new topologies, since they mix by renormalization because they have the same weights.

We now provide a nonperturbative interpretation of our refined topological expansion in terms of the effective theory of glueballs. It has been known for more than 40 years that punctured spheres correspond to glueball tree diagrams [16][17]



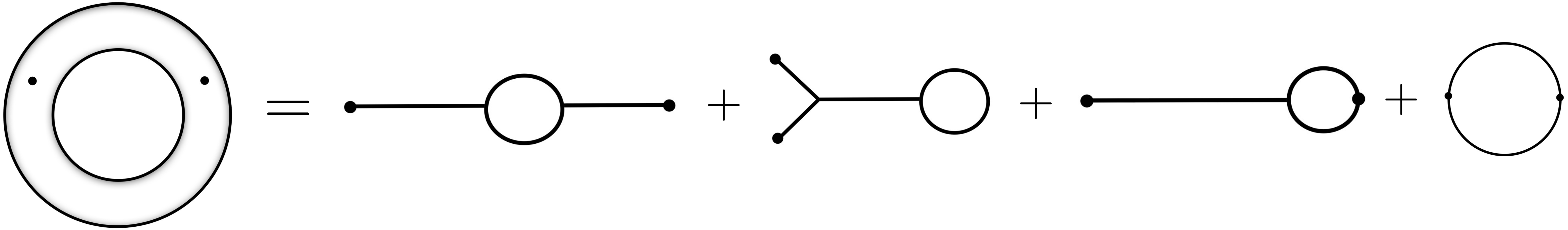
The sphere with two punctures corresponds to an infinite sum of glueball propagators [16].



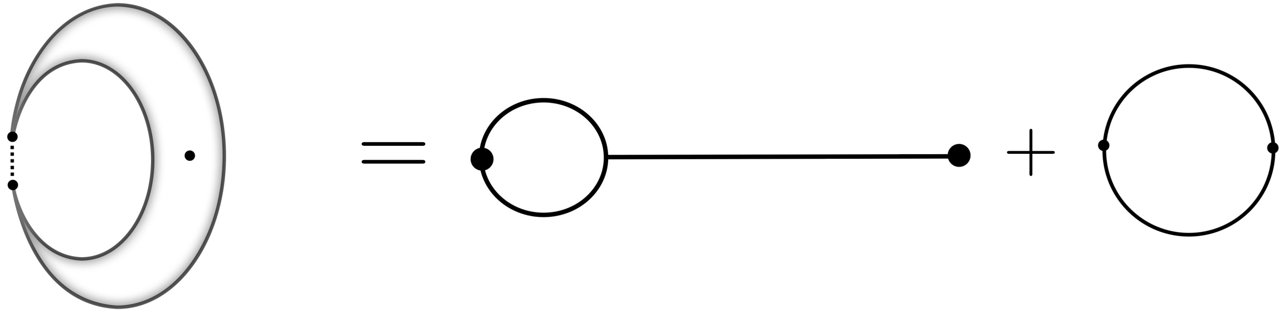
The sphere with three punctures corresponds in Minkowskian space-time to vertices that may carry sums of 3 or 2 poles [17]. We point out that the 2-pole graph contributes zero to the S matrix because of the missing glueball external leg [4].

[16] A. Migdal, Multicolor QCD as Dual Resonance Theory, Annals Phys. 109 (1977) 365.
[17] E. Witten Baryons in the 1/N expansion, Nucl. Phys. B160 (1979) 57-115.

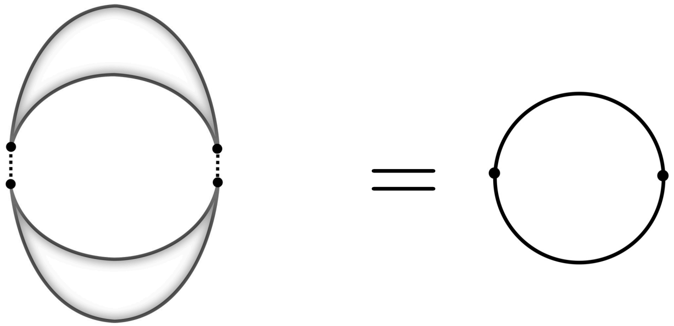
Glueball loops in the effective theory are represented by higher-genus surfaces. Specifically, 2-point glueball one-loop diagrams correspond to the punctured torus:



Interestingly, we observe [4] that in the effective theory on the right-hand side of the above picture also diagrams without external glueball legs occur. Moreover, for our new topologies we obtain the following interpretation



The punctures that arise from the normalization of pinches are identified in spacetime and cannot represent an external glueball leg. Consequently, both diagrams contribute zero to the S matrix as well.



The maximally pinched (color-disconnected) diagrams are the only ones that have a 1-to-1 correspondence with the effective theory, since they do not carry any external leg.

For the graphs that only contain bivalent vertices, i.e., the maximally (disconnected) pinched ones, we verify the spin-statistics theorem directly by the sign of the asymptotic result in the last line

$$\mathcal{W}^{E}_{\text{asym maximally pinched}}[J_{\mathbb{O}^E},\lambda] = -\log \mathrm{Det}\left(\mathcal{I} - \frac{1}{2}\frac{Z_{\mathbb{O}_s}(\lambda)}{\lambda^{s+2}}\Delta^{-1}P\frac{J_{\mathbb{O}^E_s}}{N}\otimes\mathcal{Y}^{\frac{5}{2}}_{s-2}\right)$$
$$= -\log \mathrm{Det}\left(\mathbb{I} - \frac{1}{2}\frac{Z_{\mathbb{O}_s}(\lambda)}{\lambda^{s+2}}\Delta^{-1}\frac{J_{\mathbb{O}^E_s}}{N}\otimes\mathcal{Y}^{\frac{5}{2}}_{s-2}\right)$$

Hence the new topologies solve the sign puzzle and specifically in the maximally pinched sector.

Moreover, the new topological sector dominates the UV asymptotics of the correlators of twist-2 operators, but contributes zero to the nonperturbative S matrix, since nonperturbatively it consists of tori with at least one pinch corresponding to glueball one-loop diagrams with at least one glueball external leg missing.

Conclusions

In the SU(N) YM theory it exists a new topological sector for twist-2 operators that refines the ‘t Hooft topological expansion, both perturbatively and nonperturbatively.

Instead, in the U(N) YM theory the new sector is absent perturbatively for twist-2 operators.

Yet, this does not solve the aforementioned sign problem nonperturbatively, since in the U(N) theory the asymptotics of the generating functional is the sum of the RG-improved SU(N) result and the free U(1) part

$$\mathcal{W}_{U(N)}[J_{\mathbb{O}}] \sim -(N^2-1)\log\operatorname{Det}\left(\mathbb{I}+\frac{1}{2N}\sum_s \frac{Z_{\mathbb{O}_s(\lambda)}}{\lambda^{s+2}}\Delta^{-1}J_{\mathbb{O}_s}\otimes\mathcal{Y}^{\frac{5}{2}}_{s-2}\right)$$

$$-\log\operatorname{Det}\left(\mathbb{I}+\frac{1}{2N}\sum_s \frac{1}{\lambda^{s+2}}\Delta^{-1}J_{\mathbb{O}_s}\otimes\mathcal{Y}^{\frac{5}{2}}_{s-2}\right)$$

with the U(1) contribution in the second line being actually exactly conformal and not only asymptotic, since the U(1) theory is free.
Hence, nonperturbatively -even in the U(N) theory- the solution of the sign puzzle is the one outlined above in the SU(N) theory.

The new topological sector dominates the UV of the correlators of twist-2 operators in the SU(N) theory, and its existence is necessary for consistency with the spin-statistics theorem for the glueballs.

Nonperturbatively the entire new sector does not contribute to the S matrix, since in the effective theory it consists of diagrams with at least one external glueball leg missing.

Hence, by limiting ourselves to the nonperturbative S matrix only, the original ‘t Hooft topological expansion is complete even for the twist-2 operators.

A crucial consequence is that a canonical string solution matching the topology of closed punctured Riemann surfaces cannot exist for the Yang-Mills correlators, but it may exist for the S matrix.

Yet, a noncanonical string solution may exist also for the correlators provided that it contains extra couplings to D-branes [8] ([M.B.] to appear).

Indeed, the new topological sector may be exactly solvable because of the vanishing S matrix ([M.B.] to appear).

In fact, the maximally color-disconnected diagrams have been conjectured [8] to be described by Chern-Simons theory on non-commutative space-time coupled to D-branes [8], which would provide the necessary logDet matching the UV asymptotics ([M.B.] to appear) in the present talk.

[8] M. Bochicchio, An asymptotic solution of large-N QCD, for the glueball and meson spectrum and the collinear S-matrix, HADRON 2015, AIP Conf. Proc. (2016).

EXTRA

Yang-Mills theory is conformal to order g^0 (leading order) and also g^2 (next-to-leading) of perturbation theory (as the beta function enters the solution of the Callan-Symanzik equation to order g^4).

Twist-2 operators transform to the leading order as primary operators with respect to the conformal group [6]

$$\mathbb{O}^{\mathcal{T}=2}_{\rho_1\ldots\rho_s} = \mathrm{Tr}\, F^{\mu}_{(\rho_1} \overleftrightarrow{D}_{\rho_2} \ldots \overleftrightarrow{D}_{\rho_{s-1}} F_{\rho_s)\mu} - \text{traces}$$ Generalization of the stress-energy tensor

$$\tilde{\mathbb{O}}^{\mathcal{T}=2}_{\rho_1\ldots\rho_s} = \mathrm{Tr}\, \tilde{F}^{\mu}_{(\rho_1} \overleftrightarrow{D}_{\rho_2} \ldots \overleftrightarrow{D}_{\rho_{s-1}} F_{\rho_s)\mu} - \text{traces}$$

$$\mathbb{S}^{\mathcal{T}=2}_{\mu\nu\rho_1\ldots\rho_{s-2}\lambda\sigma} = \mathrm{Tr}\,(F_{\mu(\nu} + i\tilde{F}_{\mu(\nu)}) \overleftrightarrow{D}_{\rho_1} \ldots \overleftrightarrow{D}_{\rho_{s-2}} (F_{\lambda)\sigma} + i\tilde{F}_{\lambda)\sigma}) - \text{traces}$$

$\mathbb{O}^{\mathcal{T}=2}_{\rho_1\ldots\rho_s}$, $\tilde{\mathbb{O}}^{\mathcal{T}=2}_{\rho_1\ldots\rho_s}$ are the "balanced" operators that appear as leading terms in the OPE of tensor currents in Minkowskian space-time near the light-cone.
They are balanced as the number of dotted and undotted indices in the spinor representation coincides.

$\mathbb{S}^{\mathcal{T}=2}_{\mu\nu\rho_1\ldots\rho_{s-2}\lambda\sigma}$ are the "unbalanced" operators as they are chiral, i.e. have a different number of dotted and undotted indices in the spinor representation

[6] N. Beisert, G. Ferretti, R. Heise and K. Zarembo, One-loop QCD spin chain and its spectrum, Nucl. Phys. B 717 (2005) 137–189, [hep-th/0412029].

Step 2

Defining

$$\langle \mathcal{O}_{k_1}(x_1)\dots\mathcal{O}_{k_n}(x_n)\rangle = G^{(n)}_{k_1\dots k_n}(x_1,\dots,x_n;\mu,g(\mu))$$

in general, the above operators mix with derivatives of lower spin operators of twist-2.
As a consequence we get from the Callan-Symanzik equation as $\lambda \to 0$

$$G^{(n)}_{k_1\dots k_n}(\lambda x_1,\dots,\lambda x_n;\mu,g(\mu)) = \sum_{j_1\dots j_n} Z_{k_1 j_1}(\lambda)\dots Z_{k_n j_n}(\lambda)\;\lambda^{-\sum_{i=1}^n D_{\mathcal{O}_i}} G^{(n)}_{j_1\dots j_n}(x_1,\dots,x_n;\mu,g\Big(\frac{\mu}{\lambda}\Big))$$

where $Z(\lambda) = P\exp\Big(\int_{g(\mu)}^{g(\frac{\mu}{\lambda})}\frac{\gamma(g')}{\beta(g')}dg'\Big)$ is the renormalized mixing matrix that involves the computation of a path-ordered exponential.

The corresponding UV asymptotics for $\lambda \to 0$ is

$$G^{(n)}_{k_1\dots k_n}(\lambda x_1,\dots,\lambda x_n;\mu,g(\mu)) \sim \sum_{j_1\dots j_n} Z_{k_1 j_1}(\lambda)\dots Z_{k_n j_n}(\lambda)\;\lambda^{-\sum_{i=1}^n D_{\mathcal{O}_i}} G^{(n)}_{conf\; j_1\dots j_n}(x_1,\dots,x_n)$$

provided that $G^{(n)}_{conf\; j_1\dots j_n}(x_1,\dots,x_n)$, which can be computed at the lowest order of perturbation theory, is not zero.

Hence, the evaluation of the asymptotics of correlators involves the computation of sums of products of $G^{(n)}_{conf\, j_1 \ldots j_n}(x_1, \ldots, x_n)$ and $Z_{ij}(\lambda)$

These computations are technically challenging, even in our case where the anomalous dimension matrix is triangular to all perturbative orders, so that the expansion of the path-ordered exponential terminates to a finite order [7].

Therefore, it is of the outmost importance to establish whether a renormalization scheme exists where Z is diagonalizable to all perturbative orders.

[7] M. Becchetti and M. Bochicchio, Operator mixing in massless QCD-like theories and Poincaré-Dulac theorem, Eur. Phys. J. C 82 (2022), [2103.16220].

The idea in [2] is to find a (formal) holomorphic gauge transformation depending on the coupling that defines a finite change of renormalization scheme

$$\mathcal{O}'(x) = S(g)\mathcal{O}(x)$$

Under the above gauge transformation $-\frac{\gamma(g)}{\beta(g)} = \frac{1}{g}\left(\frac{\gamma_0}{\beta_0} + \sum_{n=1}^{\infty} C_n g^{2n}\right)$ transforms as a gauge connection with a pole at g=0 [2].

In [2] it demonstrated, by means of the Poincaré-Dulac theorem, that under the "non-resonant" condition

$$\lambda_i - \lambda_j - 2k \neq 0$$

with i>j, k a positive integer and λ_i the eigenvalues of $\frac{\gamma_0}{\beta_0}$ in non-increasing order, an S(g) exists such that

to all orders of perturbation theory $-\frac{\gamma(g)}{\beta(g)} = \frac{1}{g}\frac{\gamma_0}{\beta_0}$ is one-loop exact, so that only the pole part survives.

Besides, if γ_0 is diagonalizable, $Z(\lambda)$ is diagonalizable as well with eigenvalues $Z_{\mathcal{O}_i}(\lambda) = \left(\frac{g(\mu)}{g(\frac{\mu}{\lambda})}\right)^{\frac{\gamma_{0\mathcal{O}_i}}{\beta_0}}$

Moreover, for twist-2 operators in SU(N) Yang-Mills theory the non-resonant condition is verified [8][3] and the non-resonant diagonal renormalization scheme restricts at the leading and next-to-leading order of perturbation theory to the standard conformal basis (defined above), since in the latter γ_0 is already diagonal.

Hence, to summarize, in the non-resonant diagonal renormalization scheme, the UV asymptotics is greatly simplified

$$G^{(n)}_{j_1\ldots j_n}(\lambda x_1,\ldots,\lambda x_n;\mu,g(\mu)) \sim \frac{Z_{\mathcal{O}_{j_1}}(\lambda)\ldots Z_{\mathcal{O}_{j_n}}(\lambda)}{\lambda^{D_{\mathcal{O}_1}+\cdots+D_{\mathcal{O}_n}}} G^{(n)}_{conf\; j_1\ldots j_n}(x_1,\ldots,x_n)$$

with $\quad Z_{\mathcal{O}_i}(\lambda) = \left(\frac{g(\mu)}{g(\frac{\mu}{\lambda})}\right)^{\frac{\gamma_{0\mathcal{O}_i}}{\beta_0}}$ $\quad$ and $\quad g^2(\frac{\mu}{\lambda}) \sim \frac{1}{\beta_0 \log(\frac{1}{\lambda^2})}\left(1-\frac{\beta_1}{\beta_0^2}\frac{\log\log(\frac{1}{\lambda^2})}{\log(\frac{1}{\lambda^2})}\right)$

[8] U. Aglietti, M. Becchetti, M. Bochicchio, M. Papinutto and F. Scardino, Operator mixing, UV asymptotics of nonplanar/planar 2-point correlators, and nonperturbative large-N expansion of QCD-like theories, [2105.11262].

Topological considerations

Finally, the occurrence of pinched tori with punctures resembles the Deligne-Mumford (DM) compactification of the moduli space of punctured Riemann surfaces that arises in canonical string theories due to their underlying conformal structure.

Yet, our pinched tori do not occur in the DM compactification of n-punctured tori, whose components only involve punctured Riemann surfaces with negative χ because in the DM compactification pinches and punctures never collide according to the underlying conformal structure of canonical string theories, while the components of our pinched tori may contain 2-punctured spheres with $\chi = 0$.

Indeed, an essential feature of the DM compactification is that the corresponding graphs dual to Riemann surfaces only contain trivalent vertices, while the dual graphs to our pinched tori contain at least one bivalent vertex.

As a consequence, no canonical closed-string theory may contain the new topological sector, even by including the DM compactification of the moduli space or, more generally, any compactification that involves stable surfaces.

$\mathcal{O}'(x) = S(g)\mathcal{O}(x)$ Formally analytic gauge transformation

Under the action of the aforementioned gauge transformation, the matrix: $A(g) = -\frac{\gamma(g)}{\beta(g)} = \frac{1}{g}\Big(\frac{\gamma_0}{\beta_0} + \cdots\Big)$

associated with the differential equation for $Z(\lambda)$ $\Big(\frac{\partial}{\partial g} - A(g)\Big)Z(\lambda) = 0$

defines a connection $A(g) = \frac{1}{g}\left(A_0 + \sum_{n=1}^{\infty} A_{2n} g^{2n}\right)$ with a regular singularity at g = 0 that transforms as:

$$A'(g) = S(g)A(g)S^{-1}(g) + \frac{\partial S(g)}{\partial g} S^{-1}(g)$$

It follows from the Poincarè-Dulac theorem that, if any two eigenvalues λ_i of the matrix $\frac{\gamma_0}{\beta_0}$ in nonincreasing order do not differ by a positive even integer: $\lambda_i - \lambda_j - 2k \neq 0$

then a formal holomorphic gauge transformation exists that sets A(g) in the canonical nonresonant form:

$$A'(g) = \frac{\gamma_0}{\beta_0}\frac{1}{g}$$

that is one-loop exact to all orders of perturbation theory.