An Implicit Regularization Approach to chiral models

NLO QCD corrections to Z_0 and (pseudo)scalar decays

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[1]

[1] Based on the paper "Rosado, R. J., Cherchiglia, A., Sampaio, M., Hiller, B. (2023). Infrared Subtleties and Chiral Vertices at NLO: An Implicit Regularization Analysis. EPJC 83 (2023) 9 879, arXiv:2305.07129."

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Motivation for Alternative Schemes

(Desired) Properties of Regularization Schemes[2] [3]:

- Mathematical Consistency
- Unitarity and Causality
- Symmetry preservation
- Quantum Action Principle
- Computational Efficiency

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[2] C Gnendiger, A Signer, D Stöckinger, A Broggio, AL Cherchiglia, F Driencourt-Mangin, AR Fazio, B Hiller, P Mastrolia, T Peraro, et al.To d, or not to d: recent developments and comparisons of regularization schemes.The European Physical Journal C, 77(7):1–39, 2017.

[3] WJ Torres Bobadilla, GFR Sborlini, P Banerjee, S Catani, AL Cherchiglia, L Cieri, PK Dhani, F Driencourt-Mangin, T Engel, G Ferrera, et al. May the four be with you: Novel ir-subtraction methods to tackle NNLO calculations. The European Physical Journal C, 81:1–61, 2021.

What is Implicit Regularization?

- Non-Dimensional framework
- it deals with UV divergences through a minimal-subtraction equivalent scheme.
- It is compatible with the BPHZ theorem, assuring locality, causality and Lorentz invariance. [4] [5]
- It complies with Abelian gauge symmetry at arbitrary loop level. [6] [7]

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[4] N. N. Bogoliubov and O. S. Parasiuk. On the Multiplication of the causal function in the quantum theory of fields. Acta Math., 97:227–266, 1957.

[5] A. Cherchiglia, M. Sampaio and M. Nemes, Systematic Implementation of Implicit Regularization for Multi-Loop Feynman Diagrams, Int. J. Mod. Phys. A 26 (2011) 2591–2635, [1008.1377].

[6] A. R. Vieira, A. L. Cherchiglia, and Marcos Sampaio. Momentum Routing Invariance in Extended QED: Assuring Gauge Invariance Beyond Tree Level. Phys. Rev. D, 93(2):025029, 2016

[7] Luellerson C. Ferreira, A. L. Cherchiglia, Brigitte Hiller, Marcos Sampaio, and M. C. Nemes. Momentum routing invariance in Feynman diagrams and quantum symmetry breakings. Phys. Rev. D, 86:025016, 2012.

Rules for IReg

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- Perform the Dirac algebra in the physical dimension

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Use the numerator-denominator consistency to remove squared internal momenta terms

$$\int_k \frac{k^2}{k^2(k-p)^2} = \int_k \frac{1}{(k-p)^2} \neq g_{\mu\nu} \int_k \frac{k^\mu k^\nu}{k^2(k-p)^2}$$

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Add a regulator mass to massless propagators

$$\frac{1}{(k \pm p)^2} \to \frac{1}{(k \pm p)^2 - \mu^2}$$

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Separation of UV Divergences; Apply as needed.

$$\frac{1}{(k \pm p)^2 - \mu^2} \to \frac{1}{k^2 - \mu^2} - \frac{p^2 \pm 2p \cdot k}{(k^2 - \mu^2)[(k \pm p)^2 - \mu^2]}$$

Rules for IReg: Basic Divergent integrals (BDIs)

Define a Set of BDIs

$$I_{log}^{\alpha_1\alpha_2\ldots\alpha_{2x}} = \int \frac{k^{\alpha_1}k^{\alpha_2}\ldots k^{\alpha_{2x}}}{(k^2-\mu^2)^{2+x}}\frac{d^4k}{(2\pi)^2}$$

$$I_{quad}^{\alpha_1\alpha_2\ldots\alpha_{2x}} = \int \frac{k^{\alpha_1}k^{\alpha_2}\ldots k^{\alpha_{2x}}}{(k^2-\mu^2)^{1+x}}\frac{d^4k}{(2\pi)^2}$$

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Surface Terms

$$\Upsilon^{\mu\alpha_1\alpha_2\ldots\alpha_{2x-1}} = \int \frac{d}{dk_\mu} \frac{k^{\alpha_1} k^{\alpha_2} \ldots k^{\alpha_{2x-1}}}{(k^2-\mu^2)^{2+x}} \frac{d^4k}{(2\pi)^2}$$

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Gauge Invariance $\rightarrow$Momentum Routing Invariance $\rightarrow$

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Gauge Invariance $\rightarrow$Momentum Routing Invariance $\rightarrow$Surface Terms $=0$

Rules for IReg: Basic Divergent integrals (BDIs)

Use Surface Relations: Tensor Reduction (Examples)

$$0 = \int \frac{d}{dk_\nu} \frac{k^\mu}{(k^2-\mu^2)^2} \frac{d^4k}{(2\pi)^2} =$$

$$= \int \frac{g^{\mu\nu}}{(k^2-\mu^2)^2} \frac{d^4k}{(2\pi)^2} - 4 \int \frac{k^\mu k^\nu}{(k^2-\mu^2)^3} \frac{d^4k}{(2\pi)^2} \rightarrow$$

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$$\rightarrow g^{\mu\nu} I_{log} = 4 I^{\mu\nu}_{log}$$

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$$\rightarrow g^{\mu\nu} I_{log} = 4 I^{\mu\nu}_{log}$$

$$g^{\mu\nu} I_{quad} = 2 I^{\mu\nu}_{quad}$$

$$g^{\mu\nu} I^{\rho\sigma}_{log} + g^{\mu\rho} I^{\nu\sigma}_{log} + g^{\mu\sigma} I^{\nu\rho}_{log} = 6 I^{\mu\nu\rho\sigma}_{log} \rightarrow$$

$$(g^{\mu\nu} g^{\rho\sigma} + g^{\mu\rho} g^{\nu\sigma} + g^{\mu\sigma} g^{\nu\rho}) I_{log} = 24 I^{\mu\nu\rho\sigma}_{log}$$

Rules for IReg: Some Remarks

Renormalization scale

A renormalization group scale can be introduced by disentangling the UV/IR behavior of BDI's under the limit $\mu \to 0$. This is achieved by employing the identity

$$I_{log}(\mu^2) = I_{log}(\lambda^2) + b \cdot ln\left(\frac{\lambda^2}{\mu^2}\right), \quad b = \frac{i}{(4\pi)^2}$$

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It is possible to absorb the BDI's in the renormalization constants (without explicit evaluation), and renormalization functions can be readily computed using

$$\lambda^2 \frac{\partial I_{log}(\lambda^2)}{\partial \lambda^2} = -b$$

Rules for IReg: γ^5

[8] Rosado, R. J., Cherchiglia, A., Sampaio, M., Hiller, B. (2023). Infrared Subtleties and Chiral Vertices at NLO: An Implicit Regularization Analysis. arXiv preprint arXiv:2305.07129.

Rules for IReg: γ^5

Chiral models require a few remarks

- The γ^5 is a strictly 4-dimensional object.

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γ^5 inside traces are re-defined

$$\gamma^5 = i\frac{\epsilon_{\alpha\beta\delta\sigma}}{4!}\gamma^\alpha\gamma^\beta\gamma^\delta\gamma^\sigma$$

[8] Rosado, R. J., Cherchiglia, A., Sampaio, M., Hiller, B. (2023). Infrared Subtleties and Chiral Vertices at NLO: An Implicit Regularization Analysis. arXiv preprint arXiv:2305.07129.

Rules for IReg:γ^5

The Right(left)-most-position

similar to the T'Hooft-Veltman scheme, we also employ the right(left)-most-position method [a] [b]. The appearing γ^5 are moved all the way to the right(left)-most-position before performing the Dirac algebra.

[a] Er-Cheng Tsai. Gauge invariant treatment of 5 in the scheme of't hooft and veltman. Physical Review D, 83(2):025020, 2011.

[b] Er-Cheng Tsai. Maintaining gauge symmetry in renormalizing chiral gauge theories. Physical Review D, 83(6):065011, 2011.

The Z_0 Calculations

- The Z_0 decay is a prototype for Chiral Theories
- For a more complete model we also study the decay of a general Scalar and Pseudo-Scalar
- At the Energy Scale of Z_0 the strong coupling becomes weak [9]

[9] Albert M Sirunyan, Armen Tumasyan, Wolfgang Adam, Federico Ambrogi, Thomas Bergauer, Marko Dragicevic, Janos Erö, A Escalante Del Valle, Martin Flechl, Rudolf Fruehwirth, et al. Determination of the strong coupling constant $\alpha_s\ (m_Z)$ from measurements of inclusive $w^{\pm}$ and z boson production cross sections in proton-proton collisions at $\sqrt{s}$= 7 and 8 Tev. Journal of High Energy Physics, 2020(6):1–50, 2020.

KLN Theorem

We use massless quarks in order to verify the KLN theorem [10] [11]

[10] Toichiro Kinoshita. Mass singularities of feynman amplitudes. Journal of Mathematical Physics, 3(4):650–677, 1962.

[11] Tsung-Dao Lee and Michael Nauenberg. Degenerate systems and mass singularities.Physical Review, 133(6B):B1549, 1964.

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Kinoshita–Lee–Nauenberg (KLN) Theorem

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no $ln(\mu_0)$ with $\mu_0 \rightarrow 0$ in the end of the calculations of all terms up to any order. With $\mu_0 = \frac{\mu^2}{m_\Omega^2}$.

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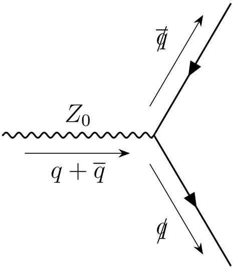
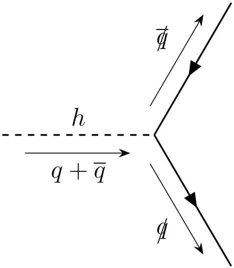
[11] Tsung-Dao Lee and Michael Nauenberg. Degenerate systems and mass singularities.Physical Review, 133(6B):B1549, 1964.

Z_0 Tree Level

Ricardo J. C. Rosado

$M_0 =$ 

$M_{ts/5} =$ 

$$= \overline{u}(q) \cdot \frac{-ie\gamma^{\mu} Z_{-}}{sin(2\omega)} \cdot v(\overline{q})\epsilon_{\mu}(z)$$

$$M_{ts} = \overline{u}(q) \cdot \frac{-iem_q}{2sin(\omega)w} \cdot v(\overline{q})$$

$$M_{t5} = \overline{u}(q) \cdot \frac{-iel_3\gamma^5 m_q}{sin(\omega)w} \cdot v(\overline{q})$$

Some more information [12]

- $$Z_{\pm} = g_V \pm \gamma^5 g_A$$
- $$g_V = I_3 - Q' sin^2(\omega)$$ $$g_A = I_3$$
- $$\xi_s = \frac{em_q}{2sin(\omega)m_W}$$ $$\xi_5 = \frac{eI_3 m_q}{sin(\omega)m_W}$$

[12] Romao, J. C., Silva, J. P. (2012). A resource for signs and Feynman diagrams of the Standard Model. International Journal of Modern Physics A, 27(26), 1230025.

Tree Level

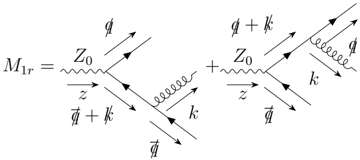
Z_0 tree level decay

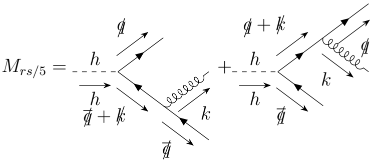
$$\Gamma_t = \frac{e^2(g_V^2 + g_A^2)z}{4\pi sin^2(2\omega)}$$

Scalar tree level decay

$$\Gamma_{ts/5} = \xi_{s/5}^2 \frac{h}{8\pi}$$

NLO Real Contributions Diagrams





$$M_{1r} = \epsilon_\mu(z)\overline{u}(q)\left[(-ig\gamma^\alpha t^a)\cdot\frac{-i}{\not q+\not k}\cdot\frac{-ie\gamma^\mu Z_-}{sin(2\omega)}+\right.$$
$$\left.+\frac{-ie\gamma^\mu Z_-}{sin(2\omega)}\cdot\frac{i}{\not{\overline{q}}+\not k}\cdot(-ig\gamma^\alpha t^a)\right]v(\overline{q})\epsilon^*_\alpha(k)$$

$$\begin{cases} M_{rs} = \overline{u}(q)\begin{pmatrix}(-ig\gamma^\alpha t^a)\cdot\frac{-i}{\not q+\not k}\cdot -i\xi_s \\ -i\xi_s\cdot\frac{i}{\not{\overline{q}}+\not k}\cdot(-ig\gamma^\alpha t^a)\end{pmatrix}v(\overline{q})\epsilon^*_\alpha(k) \\ M_{r5} = \overline{u}(q)\begin{pmatrix}(-ig\gamma^\alpha t^a)\cdot\frac{-i}{\not q+\not k}\cdot -i\xi_5\gamma^5 \\ -i\xi_5\gamma^5\cdot\frac{i}{\not{\overline{q}}+\not k}\cdot(-ig\gamma^\alpha t^a)\end{pmatrix}v(\overline{q})\epsilon^*_\alpha(k)\end{cases}$$

NLO Real Contributions

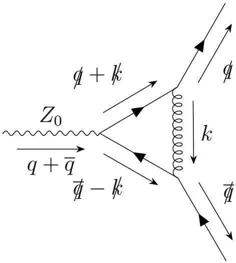
Z_0 NLO Real Contributions to the decay rate

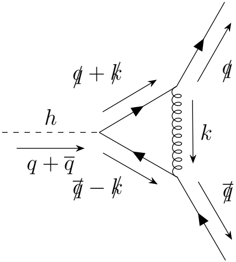
$$\Gamma_{1r} = \Gamma_t \frac{(t^a)^2 g^2}{(4\pi)^2}[2 ln^2(\mu_0) - 2\pi^2 + 6 ln(\mu_0) + 17]$$

Scalar NLO Real Contributions to the decay rate

$$\Gamma_{rs/5} = \Gamma_{ts/5} \frac{(t^a)^2 g^2}{(4\pi)^2}[2 ln^2(\mu_0) - 2\pi^2 + 6 ln(\mu_0) + 19]$$

NLO Virtual Contributions Diagrams

$M_{1v} =$ 

$M_{vs/5} =$ 

$$M_v^\mu = \int \overline{u}(q) \cdot (-ig\gamma^\alpha t^a) \cdot \frac{-i}{\not{q}+\not{k}} \cdot \frac{-ie}{sin(2\omega)} \gamma^\mu Z_- \cdot \\ \cdot \frac{i}{\not{\bar{q}}-\not{k}} \cdot (-ig\gamma^\beta t^b) \cdot \frac{-ig_{\alpha\beta}\delta_{ab}}{k^2} \cdot v(\overline{q}) \frac{d^4k}{(2\pi)^4}$$

$$\begin{cases} M_{vs} = \int \overline{u}(q) \cdot (-ig\gamma^\alpha t^a) \cdot \frac{-i}{\not{q}+\not{k}} \cdot -i\xi_s \cdot \frac{i}{\not{\bar{q}}-\not{k}} \cdot \\ \cdot (-ig\gamma^\beta t^b) \cdot \frac{-ig_{\alpha\beta}\delta_{ab}}{k^2} \cdot v(\overline{q}) \frac{d^4k}{(2\pi)^4} \\ M_{v5} = \int \overline{u}(q) \cdot (-ig\gamma^\alpha t^a) \cdot \frac{-i}{\not{q}+\not{k}} \cdot -i\xi_5 \gamma^5 \cdot \frac{i}{\not{\bar{q}}-\not{k}} \cdot \\ \cdot (-ig\gamma^\beta t^b) \cdot \frac{-ig_{\alpha\beta}\delta_{ab}}{k^2} \cdot v(\overline{q}) \frac{d^4k}{(2\pi)^4} \end{cases}$$

NLO Virtual Contributions

Z_0 NLO Virtual Contributions to the decay rate

$$\Gamma_{1v} = -\Gamma_t \frac{(t^a)^2 g^2}{(4\pi)^2}[2ln^2(\mu_0) + 6ln(\mu_0) + 14 - 2\pi^2]$$

Scalar NLO Virtual Contributions to the decay rate

$$\Gamma_{vs/5} = -\Gamma_{vs/5} \frac{(t^a)^2 g^2}{(4\pi)^2}[2ln^2(\mu_0) - 2\pi^2]$$

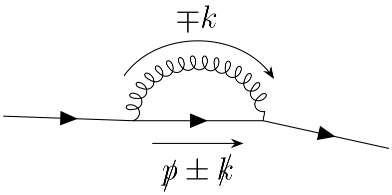
NLO Decay Rate (Preliminary)

$$\Gamma_1 = \Gamma_t + \Gamma_{1v} + \Gamma_{1r} =$$
$$= \Gamma_t \left(1 + 3\frac{(t^\alpha)g^2}{(4\pi)^2}\right) =$$
$$= \Gamma_t \left(1 + 3\frac{(t^\alpha)\alpha_s}{4\pi}\right) =$$
$$= \Gamma_t \left(1 + \frac{\alpha_s}{\pi}\right)$$

$$\Gamma_{1s/5} =$$
$$= \Gamma_{ts/5} + \Gamma_{vs/5} + \Gamma_{rs/5} =$$
$$= \Gamma_{ts/5}\left(1 + \frac{(t^\alpha)g^2}{(4\pi)^2}(6ln(\mu_0) + 19)\right)$$

Self-Energy Diagrams

$$-i\Sigma(p) =$$



$$-i\Sigma(\not{p})^{(1)} = \int (-ig\gamma^{\alpha} t^{a}) \frac{i}{\not{p} \pm \not{k} - m} (-ig\gamma_{\alpha} t^{a}) \frac{-i}{k^2} \frac{d^4 k}{(2\pi)^4} =$$

Self-Energy

$$i\Sigma^{(1)}(\not p) = g^2(t^\alpha)\{b(3\not p - m_q)+$$
$$+(\not p - 4m_q)\left[b \cdot ln\left(\frac{\mu^2}{m_q^2}\right) + I_{log}(\mu^2)\right]\}$$

Self-Energy

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$$+(\not{p} - 4m_q)\left[b \cdot ln\left(\frac{\mu^2}{m_q^2}\right) + I_{log}(\mu^2)\right]\}$$

$$m_0 = m\left(1 + \frac{g^2}{4\pi}\delta_m\right) = m\left(1 + \frac{\alpha_s}{(4\pi)^2}\delta_m\right) = m(1 - ibg^2\delta_m)$$

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$$\delta_m = -(t^\alpha)\left\{5 + 3\left[ln\left(\frac{\mu^2}{m_q^2}\right) + \frac{1}{b}I_{log}(\mu^2)\right]\right\}$$

Self-Energy

$$i\Sigma^{(1)}(\not{p}) = g^2(t^\alpha)\{b(3\not{p} - m_q)+ \\ +(\not{p} - 4m_q)\left[b \cdot ln\left(\frac{\mu^2}{m_q^2}\right) + I_{log}(\mu^2)\right]\}$$

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$$M_t \to M_t(m_h)\left[1 - ig^2(t^\alpha)^2 b\left(3ln\left(\frac{m_h^2}{m_q^2}\right) + 4\right) + O(g^4)\right](1 - ibg^2\delta_m)$$

Self-Energy

$$i\Sigma^{(1)}(\not p) = g^2(t^\alpha)\{b(3\not p - m_q) + \\ +(\not p - 4m_q)\left[b \cdot ln\left(\frac{\mu^2}{m_q^2}\right) + I_{log}(\mu^2)\right]\}$$

$$m_0 = m\left(1 + \frac{g^2}{4\pi}\delta_m\right) = m\left(1 + \frac{\alpha_s}{(4\pi)^2}\delta_m\right) = m(1 - ibg^2\delta_m)$$

$$\delta_m = -(t^\alpha)\left\{5 + 3\left[ln\left(\frac{\mu^2}{m_q^2}\right) + \frac{1}{b}I_{log}(\mu^2)\right]\right\}$$

$$M_t \to M_t(m_h)\left[1 - ig^2(t^\alpha)^2 b\left(3ln\left(\frac{m_h^2}{m_q^2}\right) + 4\right) + O(g^4)\right](1 - ibg^2\delta_m)$$

$$\Gamma_{vs/5} = -\Gamma_{ts/5}\frac{(t^\alpha)^2 g^2}{(4\pi)^2}(2ln^2(\mu_0) - 2\pi^2 + 2 + 6ln(\mu_0))$$

NLO Decay Rate

$$\Gamma_1 = \Gamma_t + \Gamma_{1v} + \Gamma_{1r} =$$
$$= \Gamma_t \left(1 + 3\frac{(t^\alpha)g^2}{(4\pi)^2}\right) =$$
$$= \Gamma_t \left(1 + 3\frac{(t^\alpha)\alpha_s}{4\pi}\right) =$$

$$= \Gamma_t \left(1 + \frac{\alpha_s}{\pi}\right)$$

$$\Gamma_{1s/5} =$$
$$= \Gamma_{ts/5} + \Gamma_{ms/5} + \Gamma_{vs/5} + \Gamma_{rs/5} =$$
$$= \Gamma_{ts/5} \left(1 + 17\frac{(t^\alpha)g^2}{(4\pi)^2}\right) =$$

$$= \Gamma_{ts/5} \left(1 + \frac{17\alpha_s}{3\pi}\right)$$

Examples of Dimensional Schemes

Dimensional Regularization Schemes

(HV) 't Hooft-Veltman: Internal Vectors are treated in D-dimension, External Vectors are strictly 4-dimensional.
(CDR) Conventional Dimensional Regularization: Both internal and External Vectors are treated in D-dimension.

Dimensional Reduction Schemes

(DREG) Dimensional Reduction: Both internal and External Vectors are quasi-4-dimensional.
(FDH) Four Dimensional Helicity: Only The Internal Vectors are Treated in the quasi-4-dimensional space, External Vectors are strictly 4-dimensional.

Considerations of DS: Evanescent Fields in Dimentional Reductions schemes [13]

$$d_s = d + n_\epsilon = 4 - 2\epsilon + n_\epsilon$$

QED

$$D^\mu_{[d_s]}\psi_i = \partial^\mu_{[d]}\psi_i + i\left(eA^\mu_{[d]} + e_e A^\mu_{[n_\epsilon]}\right) Q\psi_i$$

QCD

$$D^\mu_{[d_s]}\psi_i = \partial^\mu_{[d]}\psi_i + i\left(g_s A^\mu_{k[d]} + g_e A^\mu_{k[n_\epsilon]}\right) T^{ijk}\psi_j$$

[13] C Gnendiger, A Signer, D Stöckinger, A Broggio, AL Cherchiglia, F Driencourt-Mangin, AR Fazio, B Hiller, P Mastrolia, T Peraro, et al.To d, or not to d: recent developments and comparisons of regularization schemes.The European Physical Journal C, 77(7):1–39, 2017.

Scalar Contributions[14]

$$\Gamma^{(v)}_{s/5(FDH)} = \\ = \Gamma^{(t)}_{s/5(FDH)} C_f \left[\frac{\alpha_s}{4\pi}\left(-\frac{4}{\epsilon^2} - \frac{6}{\epsilon} - 4 + 2\pi^2 + O(\epsilon)\right) + \right. \\ \left. + \frac{\alpha_\epsilon}{4\pi}\left(\frac{n_\epsilon}{\epsilon} + O(\epsilon)\right)\right]$$

$$\Gamma^{(r)}_{s/5(FDH)} = \\ = \Gamma^{(t)}_{s/5(FDH)} C_f \left[\frac{\alpha_s}{4\pi}\left(\frac{4}{\epsilon^2} + \frac{6}{\epsilon} + 21 - 2\pi^2 + O(\epsilon)\right) + \right. \\ \left. + \frac{\alpha_\epsilon}{4\pi}\left(-\frac{n_\epsilon}{\epsilon} + O(\epsilon)\right)\right]$$

[14] C Gnendiger, A Signer, D Stöckinger, A Broggio, AL Cherchiglia, F Driencourt-Mangin, AR Fazio, B Hiller, P Mastrolia, T Peraro, et al. To d, or not to d: recent developments and comparisons of regularization schemes. The European Physical Journal C, 77(7):1–39, 2017.

$e^- e^+ \to Z_0 \to q\bar{q}$ Contributions[15]

$$\sigma^{(v)}_{\gamma(FDH)} = \sigma^{(0)} C_f \left[\frac{\alpha_s}{4\pi}\left(-\frac{4}{\epsilon^2} - \frac{6}{\epsilon} - 16 + 2\pi^2 + O(\epsilon)\right) + \right. \left. + \frac{\alpha_\epsilon}{4\pi}\left(\frac{n_\epsilon}{\epsilon} + O(\epsilon)\right)\right]$$

$$\sigma^{(r)}_{\gamma(FDH)} = \sigma^{(0)} C_f \left[\frac{\alpha_s}{4\pi}\left(\frac{4}{\epsilon^2} + \frac{6}{\epsilon} + 19 - 2\pi^2 + O(\epsilon)\right) + \right. \left. + \frac{\alpha_\epsilon}{4\pi}\left(-\frac{n_\epsilon}{\epsilon} + O(\epsilon)\right)\right]$$

[15] C Gnendiger, A Signer, D Stöckinger, A Broggio, AL Cherchiglia, F Driencourt-Mangin, AR Fazio, B Hiller, P Mastrolia, T Peraro, et al.To d, or not to d: recent developments and comparisons of regularization schemes.The European Physical Journal C, 77(7):1–39, 2017.

Conclusions

- In this work we have verified that the KLN theorem is satisfied in our framework.[16]
- It is not necessary to introduce evanescent particles, unlike in partially dimensional methods such as FDH and DRED.
- We also compared IReg with these methods, showing that, regarding IR divergences, there is a precise matching rule between IReg and dimensional results at NLO.
- γ_5 right-most-position approach is sufficient to render IReg a gauge invariant procedure in this case while reproducing the results obtained with more involved schemes literature.

[16] Rosado, R. J., Cherchiglia, A., Sampaio, M., Hiller, B. (2023). Infrared Subtleties and Chiral Vertices at NLO: An Implicit Regularization Analysis. arXiv preprint arXiv:2305.07129.

Thank You

For more information check out the main article "Infrared Subtleties and Chiral Vertices at NLO: An Implicit Regularization Analysis" https://arxiv.org/abs/2305.07129