Conventional and unconventional spin 2 mesons in the Linear Sigma Model

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Overview

QCD symmetries

- Color symmetry $\longrightarrow$ confinement + large N_c scaling limit
- CP symmetry $\longrightarrow$ restricts possible interactions
- Chiral symmetry: $U(N_f)_R \times U(N_f)_L$, exact in chiral limit ($m_i \to 0$)
- Breaking of chiral symmetry:
  1) spontaneously $\longrightarrow$ massive mesons + goldstone bosons (pions)
  2) explicitly by $m_i \neq 0 \longrightarrow$ pions are not massless
- Dilatation invariance $x^\mu \to \lambda^{-1} x^\mu$ classically in chiral limit
- Dilatation invariance is broken due to quantum effects (trace anomaly), leads to condensate
- $U(1)_A$, classical symmetry, broken by axial anomaly

The (extended) Linear Sigma Model is a hadronic model based on these symmetries and their breaking patterns

The conventional mesons

J^{PC}	I=1 $u\bar{d}$, $d\bar{u}$ $\frac{d\bar{d}-u\bar{u}}{\sqrt{2}}$	I=1/2 $u\bar{s}$, $d\bar{s}$ $s\bar{d}$, $s\bar{u}$	I=0 $\approx \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$	I=0 $\approx s\bar{s}$	Meson names
0^{-+}	π	K	$\eta(547)$	$\eta'(958)$	Pseudoscalar
0^{++}	$a_0(1450)$	$K_0^\star(1430)$	$f_0(1370)$	$f_0(1710)$	Scalar
1^{--}	$\rho(770)$	$K^\star(892)$	$\omega(782)$	$\phi(1020)$	Vector
1^{++}	$a_1(1260)$	K_{1A}	$f_1(1285)$	$f_1'(1420)$	Axial-vector
1^{+-}	$b_1(1235)$	K_{1B}	$h_1(1170)$	$h_1(1415)$	Pseudovector
1^{--}	$\rho(1700)$	$K^\star(1680)$	$\omega(1650)$	$\phi(?)$	Excited-vector
2^{++}	$a_2(1320)$	$K_2^\star(1430)$	$f_2(1270)$	$f_2'(1525)$	Tensor
2^{--}	$\rho_2(?)$	$K_2(1820)$	$\omega_2(?)$	$\phi_2(?)$	Axial-tensor
2^{-+}	$\pi_2(1670)$	$K_2(1770)$	$\eta_2(1645)$	$\eta_2(1870)$	Pseudotensor

Still missing resonances with quantum numbers 2^{--}. We can study this with a low energy effective model of QCD.

Linear Sigma Model

The LSM can be extended for tensor and axial tensor mesons and their decay products of vectors, axial vectors, etc.

$$\mathcal{L}_{\text{eLSM}} = \mathcal{L}_{\text{dil}} + \text{Tr}\left[\left(D_\mu \Phi\right)^\dagger \left(D_\mu \Phi\right)\right] - m_0^2 \left(\frac{G}{G_0}\right)^2 \text{Tr}\left[\Phi^\dagger \Phi\right]$$
$$- \frac{1}{4}\text{Tr}\left[\left(L_{\mu\nu}^2 + R_{\mu\nu}^2\right)\right] + \cdots ,$$

The objects we primarily work with are multiplets that transform in a simple way under chiral symmetry e.g. for spin 2:

$$\mathbf{R}^{\mu\nu} := T^{\mu\nu} - A_2^{\mu\nu}, \;\; \mathbf{L}^{\mu\nu} := T^{\mu\nu} + A_2^{\mu\nu}$$

$$T^{\mu\nu} = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{f_{2,N}^{\mu\nu} + a_2^{0\mu\nu}}{\sqrt{2}} & a_2^{+\mu\nu} & K_2^{*+\mu\nu} \\ a_2^{-\mu\nu} & \frac{f_{2,N}^{\mu\nu} - a_2^{0\mu\nu}}{\sqrt{2}} & K_2^{*0\mu\nu} \\ K_2^{*-\mu\nu} & \bar{K}_2^{*0\mu\nu} & f_{2,S}^{\mu\nu} \end{pmatrix}$$

We construct chirally invariant terms in the lagrangian, such as

$$g_i \text{Tr}\left[\Phi^\dagger \mathbf{L}^{\mu\nu} \mathbf{L}_{\mu\nu} \Phi + \Phi \mathbf{R}^{\mu\nu} \mathbf{R}_{\mu\nu} \Phi^\dagger\right]$$

Spin 2 masses

Mass terms appear by a shift in scalars due to spontaneous symmetry breaking

$$\mathcal{L} = h\mathrm{Tr}\left[\Phi^\dagger \mathbf{L}^{\mu\nu}\mathbf{L}_{\mu\nu}\Phi\right] + \ldots, \qquad \Phi \to \Phi + \Phi_0$$

Resonances	Masses (in MeV)	Resonances	Masses (in MeV)
$a_2(1320)$	**1317**	$\rho_2(?)$	1663
$K_2^*(1430)$	**1427**	$K_2(1820)$	**1819**
$f_2(1270)$	1297	$\omega_{2,N}(?)$	1663
$f_2'(1525)$	1538	$\omega_{2,S}(?)$	1971

PDG inputs, consistency checks and predictions

$$\text{e.g.} \qquad m_{\rho_2}^2 = m_{a_2}^2 + |h_3|\phi_N^2$$

Masses of chiral partners become equal as chiral condensate vanishes

Tensor to 2 pseudoscalar decays

$$\mathcal{L} = \frac{g_2^{\text{ten}}}{2}\left(\text{Tr}\left[\mathbf{L}_{\mu\nu}\{L^\mu, L^\nu\}\right] + \text{Tr}\left[\mathbf{R}_{\mu\nu}\{R^\mu, R^\nu\}\right]\right) + \ldots$$

$$A_{1\,\mu} \to A_{1\,\mu} + \partial_\mu \mathcal{P}$$

in total: 2 couplings and 1 mixing angle which are fit to data

Decay process (in model)	eLSM (MeV)	PDG (MeV)
$a_2(1320) \longrightarrow \bar{K}\,K$	4.06 ± 0.14	$7.0^{+2.0}_{-1.5}$
$a_2(1320) \longrightarrow \pi\,\eta$	25.37 ± 0.87	18.5 ± 3.0
$a_2(1320) \longrightarrow \pi\,\eta'(958)$	1.01 ± 0.03	0.58 ± 0.10
$K_2^*(1430) \longrightarrow \pi\,\bar{K}$	44.82 ± 1.54	49.9 ± 1.9
$f_2(1270) \longrightarrow \bar{K}\,K$	3.54 ± 0.29	8.5 ± 0.8
$f_2(1270) \longrightarrow \pi\,\pi$	168.82 ± 3.89	$157.2^{+4.0}_{-1.1}$
$f_2(1270) \longrightarrow \eta\,\eta$	0.67 ± 0.03	0.75 ± 0.14
$f_2'(1525) \longrightarrow \bar{K}\,K$	23.72 ± 0.60	75 ± 4
$f_2'(1525) \longrightarrow \pi\,\pi$	0.67 ± 0.14	0.71 ± 0.14
$f_2'(1525) \longrightarrow \eta\,\eta$	1.81 ± 0.05	9.9 ± 1.9

Qualitative agreement overall, some entries at odds with data

Tensor to vector + pseudoscalar decays

$$\mathcal{L}_{c^{\text{ten}}} = c^{\text{ten}}\,\text{Tr}\Big[\partial^{\mu}\mathbf{L}^{\nu\alpha}\tilde{L}_{\mu\nu}\,\partial_{\alpha}\Phi\,\Phi^{\dagger}\Big] + \ldots\,, \qquad \tilde{L}_{\mu\nu} := \frac{\varepsilon_{\mu\nu\rho\sigma}}{2}(\partial^{\rho}L^{\sigma} - \partial^{\sigma}L^{\rho})$$

Upon shifts contains

$$\mathcal{L}_{tvp} = c^{\text{ten}}\,\varepsilon_{\mu\nu\rho\sigma}\,\text{Tr}\Big[\,\partial^{\mu}T^{\nu\alpha}\Big(\partial^{\rho}V^{\sigma}(\partial_{\alpha}P)\Phi_0 - \Phi_0(\partial_{\alpha}P\partial^{\rho}V^{\sigma})\Big)\Big]$$

Coupling c^{ten} is fit to data

Decay process (in model)	eLSM (MeV)	PDG (MeV)
$a_2(1320) \longrightarrow \rho(770)\,\pi$	71.0 ± 2.6	73.61 ± 3.35
$K_2^*(1430) \longrightarrow \bar{K}^*(892)\,\pi$	27.9 ± 1.0	26.92 ± 2.14
$K_2^*(1430) \longrightarrow \rho(770)\,K$	10.3 ± 0.4	9.48 ± 0.97
$K_2^*(1430) \longrightarrow \omega(782)\,\bar{K}$	3.5 ± 0.1	3.16 ± 0.88
$f_2'(1525) \longrightarrow \bar{K}^*(892)\,K + \text{c.c.}$	19.89 ± 0.73	

Good agreement with experiment and one new prediction

Radiative decays of tensor mesons I

Vector meson dominance induces the shift

$$V_{\mu\nu} \to V_{\mu\nu} + \frac{e}{g_\rho} F_{\mu\nu} Q$$

Decay process (in model)	eLSM	PDG
$K_2^{\pm}(1430) \longrightarrow \gamma\, K^{\pm}$	1.12 ± 0.47 MeV	0.24 ± 0.05 MeV
$K_2^{0}(1430) \longrightarrow \gamma\, K^{0}$	5.1 ± 2.2 keV	< 5.4 keV
$a_2^{\pm}(1320) \longrightarrow \gamma\, \pi^{\pm}$	1.01 ± 0.43 MeV	0.31 ± 0.03 MeV

Decay rates for $T \to \gamma P$, 0 parameter fit

Decay process (in model)	eLSM (keV)	PDG (keV)
$a_2(1320) \longrightarrow \gamma\gamma$	1.01 ± 0.06	1.00 ± 0.06
$f_2(1270) \longrightarrow \gamma\gamma$	1.95 ± 0.10	2.6 ± 0.5
$f_2'(1525) \longrightarrow \gamma\gamma$	0.083 ± 0.009	0.082 ± 0.009

Decay rates for $T \to \gamma\gamma$, 2 parameter fit

Radiative decays of tensor mesons II

Decay process (in model)	Decay Width (MeV)
$a_2(1320) \longrightarrow \rho(770)\,\gamma$	0.22 ± 0.04
$a_2^0(1320) \longrightarrow \omega(782)\,\gamma$	1.94 ± 0.04
$a_2(1320) \longrightarrow \phi(1020)\,\gamma$	0.0024 ± 0.0005
$K_2^{*\pm}(1430) \longrightarrow \bar{K}^{*\pm}(892)\,\gamma$	0.23 ± 0.04
$K_2^{*\,0}(1430) \longrightarrow \bar{K}^{*\,0}(892)\,\gamma$	0.94 ± 0.18
$f_2(1270) \longrightarrow \rho(770)\,\gamma$	0.70 ± 0.17
$f_2(1270) \longrightarrow \omega(782)\,\gamma$	0.068 ± 0.017
$f_2(1270) \longrightarrow \phi(1020)\,\gamma$	0.007 ± 0.002
$f_2'(1525) \longrightarrow \rho(770)\,\gamma$	0.32 ± 0.08
$f_2'(1525) \longrightarrow \omega(782)\,\gamma$	0.012 ± 0.005
$f_2'(1525) \longrightarrow \phi(1020)\,\gamma$	0.61 ± 0.12

Predictions for $T \rightarrow \gamma V$

Axial tensor to vector+pseudoscalar decays I

Decay process (in model)	eLSM (MeV)
$\rho_2(?) \longrightarrow \rho(770)\,\eta$	$\approx 99 \pm 50$
$\rho_2(?) \longrightarrow \bar{K}^*(892)\,K + \text{c.c.}$	$\approx 85 \pm 43$
$\rho_2(?) \longrightarrow \omega(782)\,\pi$	$\approx 419 \pm 210$
$\rho_2(?) \longrightarrow \phi(1020)\,\pi$	≈ 0.8
$K_{2,A} \longrightarrow \rho(770)\,K$	$\approx 195 \pm 98$
$K_{2,A} \longrightarrow \bar{K}^*(892)\,\pi$	$\approx 316 \pm 158$
$K_{2,A} \longrightarrow \bar{K}^*(892)\,\eta$	≈ 0.01
$K_{2,A} \longrightarrow \omega(782)\,\bar{K}$	$\approx 51 \pm 26$
$K_{2,A} \longrightarrow \phi(1020)\,\bar{K}$	$\approx 50 \pm 25$
$\omega_{2,N} \longrightarrow \rho(770)\,\pi$	$\approx 1314 \pm 657$
$\omega_{2,N} \longrightarrow \bar{K}^*(892)\,K + \text{c.c.}$	$\approx 85 \pm 43$
$\omega_{2,N} \longrightarrow \omega(782)\,\eta$	$\approx 93 \pm 47$
$\omega_{2,N} \longrightarrow \phi(1020)\,\eta$	≈ 0.06
$\omega_{2,S} \longrightarrow \bar{K}^*(892)\,K + \text{c.c.}$	$\approx 510 \pm 255$
$\omega_{2,S} \longrightarrow \omega(782)\,\eta$	$\approx 1.0 \pm 0.5$
$\omega_{2,S} \longrightarrow \omega(782)\,\eta'(958)$	≈ 0.3
$\omega_{2,S} \longrightarrow \phi(1020)\,\eta$	$\approx 101 \pm 51$

Predictions based on coupling in tensor data. Qualitative prediction, but clearly axial tensor states are broad.

Axial tensor to vector+pseudoscalar decays II

Decay process (in model)	eLSM (MeV)	LQCD (MeV)
$\rho_2(?) \longrightarrow \rho(770)\,\eta$	≈ 30	
$\rho_2(?) \longrightarrow \bar{K}^*(892)\,K + \mathrm{c.c.}$	≈ 27	36
$\rho_2(?) \longrightarrow \omega(782)\,\pi$	≈ 122	125
$\rho_2(?) \longrightarrow \phi(1020)\,\pi$	≈ 0.3	
$K_{2,A} \longrightarrow \rho(770)\,K$	≈ 53	
$K_{2,A} \longrightarrow \bar{K}^*(892)\,\pi$	≈ 87	
$K_{2,A} \longrightarrow \bar{K}^*(892)\,\eta$	≈ 0.004	
$K_{2,A} \longrightarrow \omega(782)\,\bar{K}$	≈ 13.8	
$K_{2,A} \longrightarrow \phi(1020)\,\bar{K}$	≈ 13.7	
$\omega_{2,N} \longrightarrow \rho(770)\,\pi$	≈ 363	365
$\omega_{2,N} \longrightarrow \bar{K}^*(892)\,K + \mathrm{c.c.}$	≈ 25	36
$\omega_{2,N} \longrightarrow \omega(782)\,\eta$	≈ 27	17
$\omega_{2,N} \longrightarrow \phi(1020)\,\eta$	≈ 0.02	
$\omega_{2,S} \longrightarrow \bar{K}^*(892)\,K + \mathrm{c.c.}$	≈ 100	148
$\omega_{2,S} \longrightarrow \omega(782)\,\eta$	≈ 0.2	
$\omega_{2,S} \longrightarrow \omega(782)\,\eta'(958)$	≈ 0.02	
$\omega_{2,S} \longrightarrow \phi(1020)\,\eta$	≈ 17	44

1 Parameter fit based on LQCD. Ratios are consistent with previous prediction, coupling constant approx halved, states still quite broad.

Axial tensor to tensor + pseudoscalar decays

Decay process (in model)	eLSM (MeV)
$\rho_2(?) \longrightarrow a_2(1320)\,\pi$	≈ 88
$K_{2,A} \longrightarrow K_2^\star(1430)\,\pi$	≈ 49
$K_{2,A} \longrightarrow a_2(1320)\,K$	≈ 84
$K_{2,A} \longrightarrow f_2(1270)\,K$	≈ 4
$\omega_{2,S} \longrightarrow K_2^\star(1430)\,K + \mathrm{c.c.}$	≈ 15

in summary for the axial tensor states:

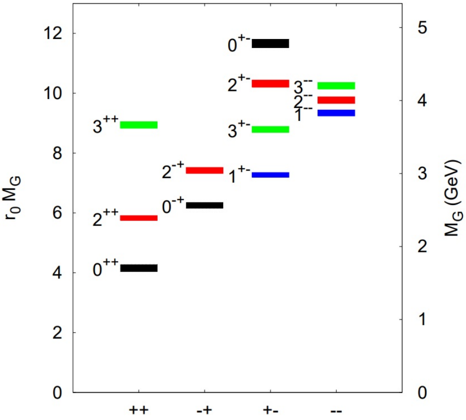
- Masses are found, $\sim$ 300-400 MeV larger than their chiral partners
- Quite sizeable decays in some channels
- Promising for future investigation of axial tensor states
- Sum of various decay channels for axial tensor is quite large

Tensor glueball

Lattice calculations have found a large spectrum of pure gluon states.

The tensor ($J^{PC} = 2^{++}$) is the second lightest glueball and so one of the best candidates for experimental verification.

Lattice calculations have some difficulties computing decay rates, so there is room for us to find new information using our chiral model.



Chen et al, 2005

Glueball chiral interactions

Compared to the work on tensor mesons, we need to replace the tensors to realize flavour blindness:

$$T_{\mu\nu} \longrightarrow G_{\mu\nu} \cdot \mathbf{1}$$

The lagrangian leading to tensor glueball decays involves solely left- and right-handed chiral fields:

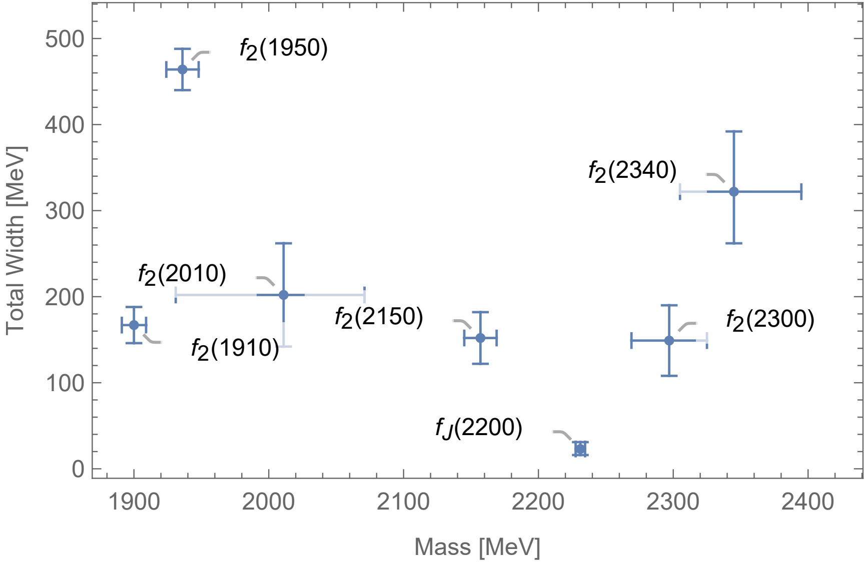
$$\mathcal{L} = \lambda G_{\mu\nu} \Big(\mathrm{Tr}\Big[\{ L^{\mu}, L^{\nu} \} \Big] + \mathrm{Tr}\Big[\{ R^{\mu}, R^{\nu} \} \Big] \Big)$$

The Lagrangian leads to three kinematically allowed decay channels

- Into two pseudoscalar mesons: $G_2 \longrightarrow P^{(1)} P^{(2)}$
- Into two vector mesons: $G_2 \rightarrow V^{(1)} V^{(2)}$
- Into an axial-vector and pseudoscalar meson: $G_2 \longrightarrow A_1 P$

We have no experimental data to fit λ to, so we calculate decay ratios.

Isoscalar-tensor resonances



Decay ratios

Decay Ratio	theory	Decay Ratio	theory	Decay Ratio	theory
$\frac{G_2(2369)\longrightarrow \bar{K}\,K}{G_2(2369)\longrightarrow \pi\,\pi}$	0.4	$\frac{G_2(2369)\longrightarrow \rho(770)\,\rho(770)}{G_2(2369)\longrightarrow \pi\,\pi}$	51	$\frac{G_2(2369)\longrightarrow a_1(1260)\,\pi}{G_2(2369)\longrightarrow \pi\,\pi}$	0.26
$\frac{G_2(2369)\longrightarrow \eta\,\eta}{G_2(2369)\longrightarrow \pi\,\pi}$	0.1	$\frac{G_2(2369)\longrightarrow \bar{K}^*(892)\,\bar{K}^*(892)}{G_2(2369)\longrightarrow \pi\,\pi}$	44	$\frac{G_2(2369)\longrightarrow K_{1\,,A}\,K}{G_2(2369)\longrightarrow \pi\,\pi}$	0.12
$\frac{G_2(2369)\longrightarrow \eta\,\eta'}{G_2(2369)\longrightarrow \pi\,\pi}$	0.005	$\frac{G_2(2369)\longrightarrow \omega(782)\,\omega(782)}{G_2(2369)\longrightarrow \pi\,\pi}$	17	$\frac{G_2(2369)\longrightarrow f_1(1285)\,\eta}{G_2(2369)\longrightarrow \pi\,\pi}$	0.03
$\frac{G_2(2369)\longrightarrow \eta'\,\eta'}{G_2(2369)\longrightarrow \pi\,\pi}$	0.01	$\frac{G_2(2369)\longrightarrow \phi(1020)\,\phi(1020)}{G_2(2369)\longrightarrow \pi\,\pi}$	7	$\frac{G_2(2369)\longrightarrow f_1(1420)\,\eta}{G_2(2369)\longrightarrow \pi\,\pi}$	0.008

Decay ratios of G_2 w.r.t. $\pi\pi$ for a mass of 2369 MeV. The columns are sorted as PP on the left, VV in the middle, and A_1P on the right. Vector channels are dominant, in particular $\rho\rho$ and K^*K^*

Data Comparison

Resonance	Decay Ratio	PDG	Model Prediction
$f_2(1910)$	$\rho\rho/\omega\omega$	2.6 ± 0.4	3.1
$f_2(1910)$	$f_2(1270)\eta/a_2(1320)\pi$	0.09 ± 0.05	0.07
$f_2(1910)$	$\eta\eta/\eta\eta'$	< 0.05	~ 8
$f_2(1910)$	$\omega\omega/\eta\eta'$	2.6 ± 0.6	~ 200
$f_2(1950)$	**$\eta\eta/\pi\pi$**	**0.14 ± 0.05**	**0.081**
$f_2(1950)$	**$K\overline{K}/\pi\pi$**	**~ 0.8**	**0.32**
$f_2(1950)$	**$4\pi/\eta\eta$**	**> 200**	**> 700**
$f_2(2150)$	$f_2(1270)\eta/a_2(1320)\pi$	0.79 ± 0.11	0.1
$f_2(2150)$	$K\overline{K}/\eta\eta$	1.28 ± 0.23	~ 4
$f_2(2150)$	$\pi\pi/\eta\eta$	< 0.33	~ 10

Decay ratios for the decay channels with available data.

Glueball candidates

Resonances	Interpretation status
$f_2(1910)$	Agreement with some data, but large discrepancies in $\eta\eta\prime$ mode
$f_2(1950)$	**$\eta\eta/\pi\pi$ agrees with data, no contradictions to data** **but broad tensor glueball** **Best fit as predominantly glueball**
$f_2(2010)$	Likely primarily strange-antistrange content
$f_2(2150)$	All available data contradicts theoretical prediction
$f_J(2220)$	Data on $\pi\pi/K\bar{K}$ disagrees with theory Only smallest predicted decay channels are seen
$f_2(2300)$	Likely primarily strange-antistrange content
$f_2(2340)$	Likely primarily strange-antistrange content would also imply a broad glueball

Spin 2 resonances and their status as the tensor glueball.

Summary

- We have extended the Linear Sigma Model to the tensor and axial-tensor mesons, and the tensor glueball.
- With the data on tensor mesons we can make predictions on axial-tensor mesons which have limited experimental data.
- The axial-tensor states are very broad based on the predicted decay rates (explains why they are hard to find)
- For the tensor glueball decay we obtain decay ratios; vector channels are dominant, in particular $\rho\rho$ and K^*K^*
- The $f_2(1950)$ is clearly favored as a tensor glueball candidate by the eLSM, but data on resonances is limited, in particular for the states $f_J(2220)$, $f_2(2300)$, and $f_2(2340)$.

Thank you for your attention!

Backup slides

$f_J(2220)$

$f_J(2220)$ is historically seen as a good candidate for the tensor glueball

$f_J(2220)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$\pi\pi$	not seen
Γ_2	$\pi^+\pi^-$	not seen
Γ_3	$K\overline{K}$	not seen
Γ_4	$p\overline{p}$	not seen
Γ_5	$\gamma\gamma$	not seen
Γ_6	$\eta\eta'(958)$	seen
Γ_7	$\phi\phi$	not seen
Γ_8	$\eta\eta$	not seen

- Only $\eta\eta\prime$ is seen, but we find it is $\sim 10^{-3}$ times $\pi\pi$ mode.
- PDG lists decay ratio $\pi\pi/\bar{K}K = 1.0 \pm 0.5$, we find $\pi\pi/\bar{K}K \sim 2.5$

Estimating glueball width

- A rough guess on the width of the tensor glueball can be made.
- Consider $f_2 \equiv f_2(1270) \simeq \sqrt{1/2}(\bar{u}u + \bar{d}d)$ and $f_2' \equiv f_2'(1525) \simeq \bar{s}s$, with $\Gamma_{f_2 \to \pi\pi} = 157.2$ MeV and $\Gamma_{f_2' \to \pi\pi} = 0.71$ MeV.
- The amplitude for $f_2 \to \pi\pi$ requires the creation of a single $\bar{q}q$ pair from the vacuum and scales as $1/\sqrt{N_c}$, where N_c is the number of colors. On the other hand, the amplitude for $f_2' \to \pi\pi$ scales as $1/N_c^{3/2}$ and goes schematically like

$$\bar{s}s \to gg \to \sqrt{1/2}(\bar{u}u + \bar{d}d)$$

Estimating glueball width

- Consider a transition Hamiltonian

$$H_{int} = \lambda \left(|\bar{u}u\rangle \langle gg| + \left|\bar{d}d\right\rangle \langle gg| + |\bar{s}s\rangle \langle gg| + h.c. \right), \quad \lambda \propto 1/\sqrt{N_c}.$$

  Then: $A_{f_2' \to \pi\pi} \simeq \sqrt{2}\lambda^2 A_{f_2 \to \pi\pi}$, hence $\Gamma_{f_2' \to \pi\pi} \simeq 2\lambda^4 \Gamma_{f_2 \to \pi\pi}$,
- Tensor glueball decay into $\pi\pi$ intuitively speaking, is at an 'intermediate stage', since it starts with a *gg* pair. One has:

$$A_{G_2 \to \pi\pi} \simeq \sqrt{2}\lambda A_{f_2 \to \pi\pi},$$
$$\Gamma_{G_2 \to \pi\pi} \simeq 2\lambda^2 \Gamma_{f_2 \to \pi\pi} \simeq \sqrt{2}\sqrt{\Gamma_{f_2 \to \pi\pi} \Gamma_{f_2' \to \pi\pi}} \simeq 15\text{MeV}.$$

- We emphasize that this is a **rough estimate**, based on large N_c scaling.
- Similar results to some holographic models: very large decay widths in vector modes.

Tensor glueball decays

The Lagrangian leads to three kinematically allowed decay channels

- Decaying of the tensor glueball to the two pseudoscalar mesons have the following decay rate formula

$$\Gamma_{G_2 \longrightarrow P^{(1)} P^{(2)}} = \frac{\kappa_{gpp,i}\, \lambda^2\, |\vec{k}_{p^{(1)},p^{(2)}}|^5}{60\, \pi\, m_{G_2}^2} \, ;$$

- while for two vector mesons

$$\Gamma_{G_2 \rightarrow V^{(1)} V^{(2)}} = \frac{\kappa_{gvv,i} \lambda^2 |\vec{k}_{v^{(1)},v^{(2)}}|}{120\, \pi\, m_{G_2}^2} \Big(15 + \frac{5|\vec{k}_{v^{(1)},v^{(2)}}|^2}{m_{v^{(1)}}^2} + \frac{5|\vec{k}_{v^{(1)},v^{(2)}}|^2}{m_{v^{(2)}}^2} + \frac{2|\vec{k}_{v^{(1)},v^{(2)}}|^4}{m_{v^{(1)}}^2\, m_{v^{(2)}}^2}\Big) \, ;$$

- and for the axial-vector and pseudoscalar mesons

$$\Gamma_{G_2 \longrightarrow A_1 P} = \frac{\kappa_{gap,i}\, \lambda^2\, |\vec{k}_{a_1,p}|^3}{120\, \pi\, m_{G_2}^2} (5 + \frac{2\, |\vec{k}_{a_1,p}|^2}{m_{a_1}^2}) \, .$$