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σ(500) pole trajectories as a function of pion mass: analysis with a unitary coupled-channel model

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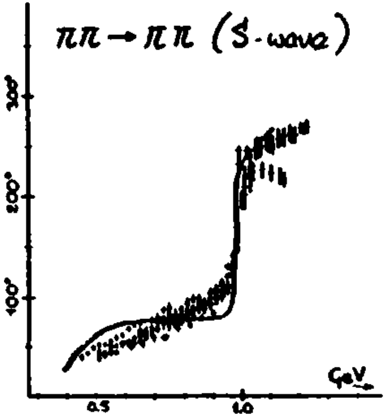
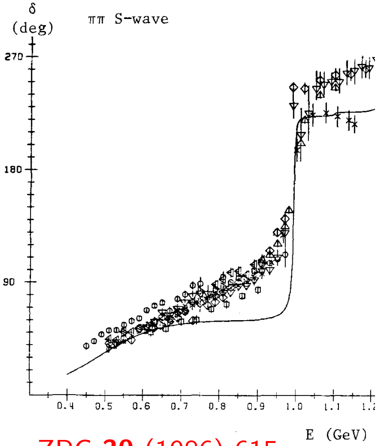
I. Introduction: the light scalar meson nonet as $q\bar{q}$ resonances

- The light scalar mesons have been haunting theorists and experimentalists for more than the past half-century; see e.g. minireview GR, E. van Beveren, Acta Phys. Polon. B Supp. **11** (2018) 455 [1806.00364] (eQCD2018).
- Their PDG experimental status stabilised in 2018, confirming the nonet $f_0(500)$, $f_0(980)$, $a_0(980)$, $K_0^\star(700)$ (see minireview).
- R. L. Jaffe, Phys. Rev. D **15** (1977) 267 proposed the $q^2\bar{q}^2$ assignments $\epsilon(650)$, $S^\star(1100)$, $\delta(1100)$, $\kappa(900)$ in the MIT-Bag model, due to a huge attractive colour-spin interaction.
- E. van Beveren *et al.*, Z. Phys. C **30** (1986) 615 [0710.4067] predicted the light scalar resonances as dynamical $q\bar{q}$ states in a unitary coupled-channel model, with pole positions (in MeV) $\epsilon(470 - i208)$, $S(994 - i20)$, $\delta(968 - i28)$, $\kappa(727 - i263)$, which are still compatible with present-day PDG limits.
- These were genuine predictions of the mentioned model, with its parameters already fitted in 1983 to ρ, K, $K^\star$, ψ, Υ spectra.

- The same model, with the same parameters, predicted S-wave $\pi\pi$ phase shifts with a roughly correct behaviour up to 1.2 **GeV** (right-hand plot). An earlier, simpler version of the model, C. Dullemond, T. A. Rijken, E. van Beveren, GR, THEF-NYM-83.09 (Proc. Kazimierz 1983), showed a similar trend (left plot).



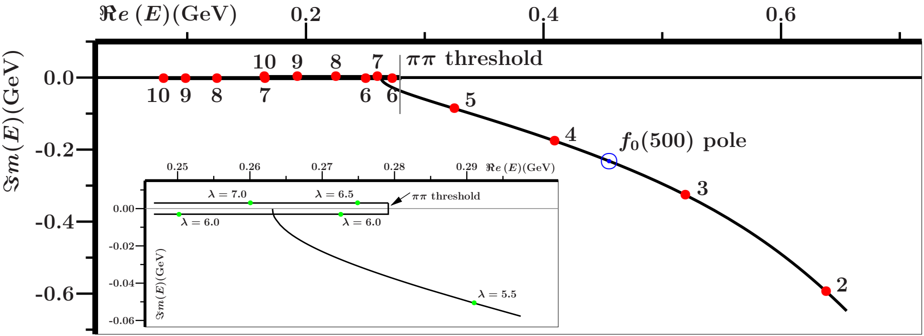
THEF-NYM-83.09



ZPC **30** (1986) 615

- Recent description of $f_0(500)$, $f_0(980)$, $f_0(1370)$, $a_0(980)$, $a_0(1450)$, using the momentum-space $\mathcal{R}SE$ model while fitting S-wave $\pi\pi$ phases up to 1.6 **GeV** and the $a_0(980)$ line shape: E. van Beveren, GR, World Scientific, Gribov-90 Memorial Volume, pp. 201–216 (2021) [2012.04994].
- Included decay channels with pairs of the lightest pseudoscalar, vector, and scalar mesons: 17 for the f_0s and 9 for the a_0s. Four fit parameters in each case, with similar optimum values. Constituent quark masses and radial splittings are taken at the values used in Z. Phys. C **30** (1986) 615 and prior work.
- Scalar resonance poles: $f_0(455 - i232)$, $f_0(1007 - i17.4)$, $f_0(1290 - i131)$, $a_0(1017 - i39.6)$, $a_0(1341 - i285)$.
- In the $I = 1/2$ case there are stability problems in the fit with the (modified) LASS data, which still violate unitarity at higher energies. An old fit with only pseudoscalar-pseudoscalar channels produced the poles $K_0^\star(722 - i266)$, $K_0^\star(1400 - i96)$; see GR, S Coito, E. van Beveren, Acta Phys. Polon. B Supp. **2** (2009) 437 [0905.3308] (eQCD2009).

- The above model for $f_0(500)$, $f_0(980)$, $f_0(1370)$, $a_0(980)$, $a_0(1450)$ was employed to study the trajectory of the $f_0(500)$ pole as a function of the overall decay coupling $\boldsymbol{\lambda}$, confirming the typical behaviours of an S-wave pole:

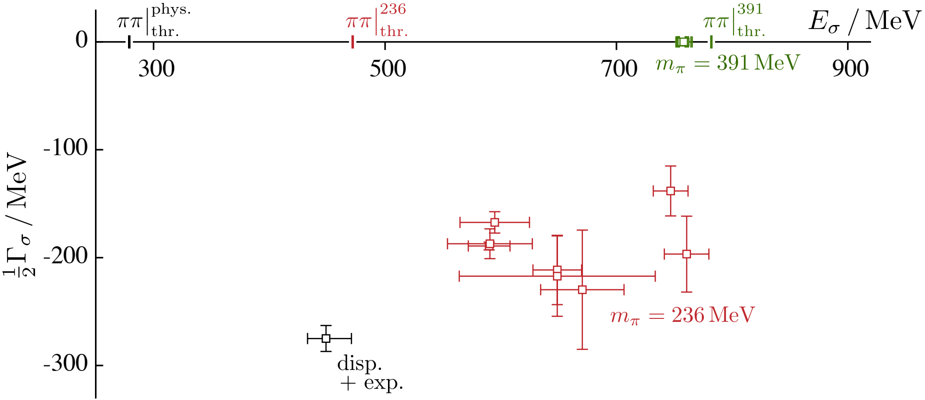


E. van Beveren, GR, World Scientific, Gribov-90, pp. 201–216 (2021) [2012.04994]; Phys. Rev. D **107** (2023) 058501 [2202.08809].

II. The light $\mathbf{q\bar{q}}$ scalars on the lattice

- From 2015 to 2018, the lattice *"Hadron Spectrum Collaboration"* (HSC) has published a series of papers on the light scalar mesons: PRD **91** (2015) 054008, **93** (2016) 094596, **97** (2018) 054513 ("HSC-2") and PRL **118** (2017) 022002 ("HSC-1").
- In all these works, only $q\bar{q}$ and meson-meson interpolators were employed, finding direct ($f_0(500)$, $f_0(980)$, $a_0(980)$) or indirect ($K_0^\star(700)$) indications of these resonances.
- In HSC-1 and HSC-2, $u\bar{u}+d\bar{d}$ and $s\bar{s}$ single-meson interpolating fields were included, as well as $\pi\pi$ and $K\bar{K}$ two-meson interpolators. In HSC-2 also $\eta\eta$ was added.
- In HSC-1, π masses of 391 **MeV** and 236 **MeV** were used, resulting in an $f_0(500)$ ("σ") bound state at $758\,(4)$ **MeV** resp. resonance pole positions with central real and imaginary parts in the ranges $590 – 760$ **MeV** resp. $-140 – -230$ **MeV** depending on parametrisation and also with large to very large error bars.
- In HSC-2, the σ bound state was found at $745\,(5)$ **MeV** for a π mass of 391 **MeV**.

- HRC lattice results of $\pi\pi$ bound state (green) and resonance pole positions (red), for $m_\pi = 391$ **MeV** and 236 **MeV**, respectively:

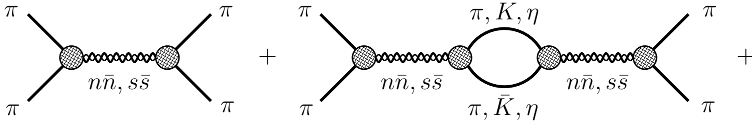


R. A. Briceno, J. J. Dudek, R. G. Edwards, D. J. Wilson, Phys. Rev. Lett. **118** (2017) 022002 [1607.05900] (HSC-1).

III. Unitary RSE model for scalar $q\bar{q}$, $s\bar{s} \leftrightarrow \pi\pi$, $K\bar{K}$, $\eta\eta$ system

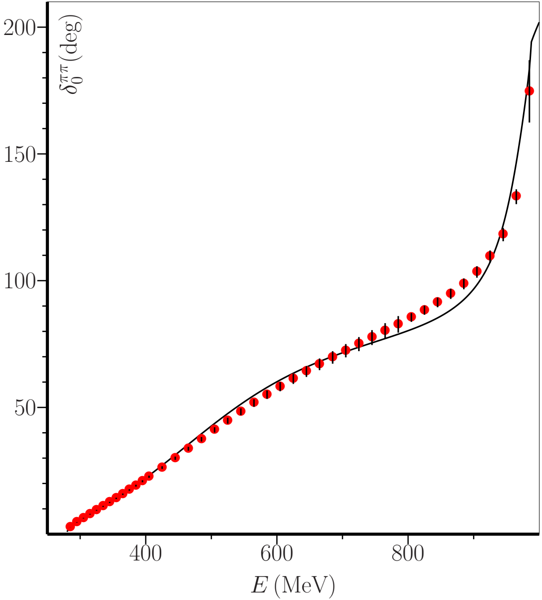
- A simplified version of the above $\mathcal{RSE}$ model, limited to the $\pi\pi$, $K\bar{K}$, and $\eta\eta$ two-meson channels, is used to fit $\pi\pi$ phase shifts up to **1 GeV**. For the quality of the fit, see figure on next slide. Graphical representation of the $\mathcal{RSE}$ $T_{\pi\pi}$ amplitude:

$T_{\pi\pi\to\pi\pi}$ =

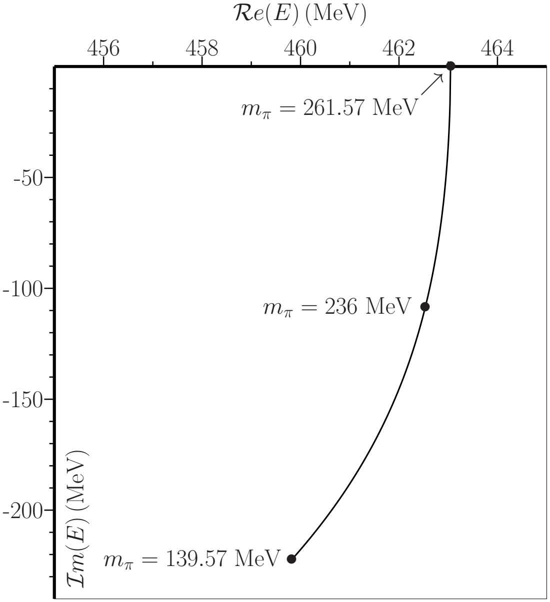


- The three free parameters fitted to the data are the overall decay coupling λ, the sharp decay radius a, and the intrinsic scalar mixing angle Θ_S between $u\bar{u}+d\bar{d}$ and $s\bar{s}$. While Θ_S comes out smaller than in [2012.04994], λ and a change very little.
- The resulting isoscalar scalar pole positions are (in MeV): $\sigma(460 - i222)$ and $f_0(978 - i37.2)$.
- The S-wave $\pi\pi$ scattering length resulting from the fit is $a_0^0 = 0.211\, m_\pi^{-1}$.

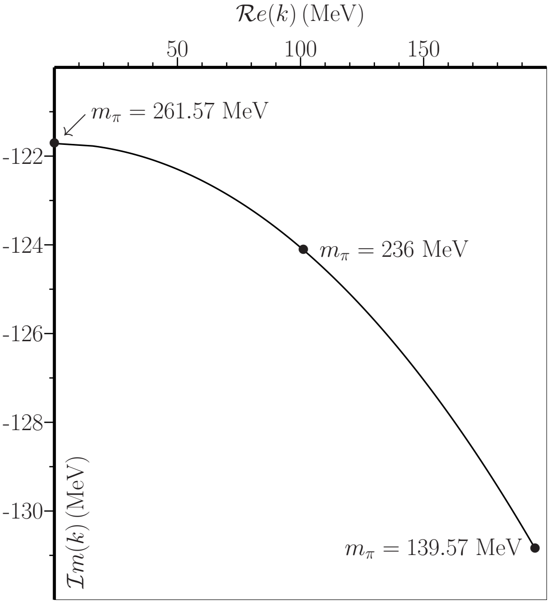
Model fit to *S*-wave $\pi\pi$ phase shifts from D. V. Bugg and H. Leutwyler:



IV. E- and k-plane $\sigma(500)$ pole trajectories as a function of m_π

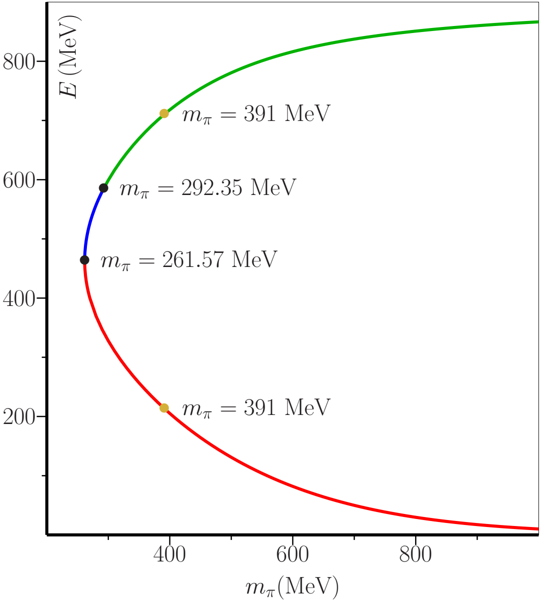


Resonance pole trajectory of $\sigma(500)$ on the second Riemann sheet of the complex energy plane, as a function of m_π.



Resonance pole trajectory of $\boldsymbol{\sigma(500)}$ in the complex momentum plane, as a function of $\boldsymbol{m_\pi}$.

Note that analyticity imposes an additional mirrored trajectory with $\mathcal{R}e(\boldsymbol{k}) < \mathbf{0}$.

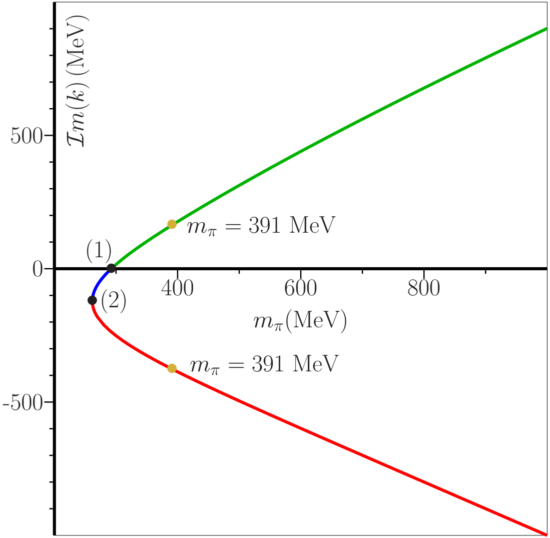


Bound-state and virtual-state energies of $\sigma(500)$, as a function of m_π.

Green: bound state

Blue: first virtual state

Red: second virtual state



Imaginary part of bound-state and virtual-state momenta of $\boldsymbol{\sigma(500)}$, as a function of $\boldsymbol{m_\pi}$.

Green: bound state

Blue: first virtual state

Red: second virtual state

(1): $m_\pi = 292.35$ MeV

(2): $m_\pi = 261.57$ MeV

V. Discussion and Conclusions

- The above unitary coupled-channel model achieves a remarkably good description of S-wave $\pi\pi$ phase shifts and scattering length $\mathbf{a_0^0}$, as well as the $f_0(500)$ and $f_0(980)$ resonance pole positions.
- For a hypothetical pion mass of 391 **MeV** and so a $\pi\pi$ threshold at 782 **MeV**, the model predicts a $\pi\pi$ bound state at 710.3 **MeV**, to be compared with $758\,(4)$ in HRC-1 and $745\,(5)$ in HRC-2.
- This difference may in part be due to the use of scale-adjusted stable π, K, and η masses of, respectively, 391 **MeV**, 549 **MeV**, and 587 **MeV** in HSC-2, including the same degrees of freedom as our model with the physical K and η masses.
- Doing a model calculation with exactly the same meson masses as in HSC-2 yields a $\pi\pi$ bound state at 718.0 **MeV**. If we also include a phenomenological subthreshold damping of kinematically closed two-meson channels as in the more general model of WS, Gribov-90 (2021) 201–216 [2012.04994] and PRD **107** (2023) 058501, the latter value increases to 752.0 **MeV**.

- The model produces resonance, bound-state, and virtual-state pole trajectories as a function of pion mass that are conform expectations from analyticity for S-wave scattering, as described in J. R. Taylor, *"Scattering Theory . . . "*, John Wiley & Sons (1972) ISBN 0-471-84900-6 (see pp. 232–247).
- In particular, the complex $\sigma(500)$ energy pole moves below the $\pi\pi$ threshold for m_π increasing beyond 231.2 **MeV** and only reaches the real axis at about 463 **MeV** for $m_\pi = 261.57$ **MeV**.
- Moreover, the real part of the $\sigma(500)$ resonance pole is remarkable stable over a wide range of pion masses, only increasing roughly from 460 **MeV** to 463 **MeV**, for $m_\pi = 139.57 \to 261.57$ **MeV**.
- Here, the subthreshold resonance pole splits into two virtual-state poles, with one moving to lower energies and the other upwards towards the $\pi\pi$ threshold, where it turns into a bound-state pole.
- Contrarily, the HSC-1 widely spread out $\sigma(500)$ resonance poles, for $m_\pi = 236$ **MeV**, all lie well above the corresponding $\pi\pi$ threshold at 472 **MeV**, including their large error bars.

So let me conclude by quoting the authors of HSC-1:

> *"The amplitude parameterizations we explored to describe the finite-volume spectrum determined with 236 MeV pions all feature a σ appearing as a broad resonance, but the pole position is not precisely determined, showing variation with parameterization choice. We believe that this comes about because our parameterizations, while maintaining elastic unitarity, do not necessarily respect the analytical constraints placed on them by causality and crossing symmetry. In the future we plan to adapt dispersive approaches so that they are applicable to describing the lattice data, and we expect this will allow us to pin down the σ pole position with precision directly from QCD."*

THANKS FOR YOUR ATTENTION!