Spin density matrix elements from the photoproduction of the $\Delta^{++}(1232)$

Vanamali Shastry

JPAC

15 Jan, 2024

Funded by US DoE

Contents

Introduction

$$\vec{\gamma} + p \to \{1 + 2 + 3 + \ldots\} + \text{“}(\pi + p)\text{”}$$

- $\{1 + 2 + 3 + \ldots\}$ could be mesonic resonances (b_1, π_1, ...)
- ‘many-pion’ (or η or combination of π and η) production.
- $\pi\Delta$ production is the simplest process involving Δ.
  - π exchange process is to be understood (gauge invariance).
  - Δ is a well-defined, well-understood resonance.
  - What is the physics behind the photoproduction of $\pi\Delta$?
  - How does it compare against pion photoproduction?
- Available observables: $\dfrac{d\sigma}{dt}$, $\Sigma_{4\pi}$, $\rho^{0,1,2}$, ...

Introduction

$$\vec{\gamma} + p \to \{1 + 2 + 3 + \ldots\} + \text{“}(\pi + p)\text{”}$$

- $\{1 + 2 + 3 + \ldots\}$ could be mesonic resonances (b_1, π_1, ...)
- 'many-pion' (or η or combination of π and η) production.
- $\pi\Delta$ production is the simplest process involving Δ.
  - π exchange process is to be understood (gauge invariance).
  - Δ is a well-defined, well-understood resonance.
  - What is the physics behind the photoproduction of $\pi\Delta$?
  - How does it compare against pion photoproduction?
- Available observables: $\frac{d\sigma}{dt}$, $\Sigma_{4\pi}$, $\underline{\rho^{0,1,2}}$, ...
- SDMEs are coefficients of angular distributions (of final proton) and hence can be extracted from the intensity distribution [1].
- Theoretically, SDMEs give insights into the production process.

SDMEs

Consider the **2** $\rightarrow$ **2** scattering process,

$$\vec{1} + 2 \rightarrow 3 + 4 \qquad (1)$$

where the arrow indicates a polarised state. Let λ_i represent the helicity of the i^{th} particle, and $T_{\lambda_1,\lambda_2,\lambda_3,\lambda_4}(s,t)$ be the amplitude for the process.

Definition

$$\rho_{\lambda_4,\lambda_4'} = \frac{1}{2N} \sum_{\lambda_1,\lambda_1',\lambda_2,\lambda_3} T_{\lambda_1,\lambda_2,\lambda_3,\lambda_4}(s,t)\ \hat{\rho}_{\lambda_1,\lambda_1'}\ T^*_{\lambda_1',\lambda_2,\lambda_3,\lambda_4'}(s,t) \qquad (2)$$

where $\hat{\rho}_{\lambda_1,\lambda_1'}$ gives the polarization of particle 1. ($\hat{\rho}_{\lambda_1,\lambda_1'} = \delta_{\lambda_1,\lambda_1'}$ implies unpolarised beam), and

$$2N = \sum_{\{\lambda\}} |T_{\{\lambda\}}|^2$$

SDMEs

The GlueX process:

$$\vec{\gamma} + p \to \pi + \Delta \tag{3}$$

has linearly polarised photon beam.

$$\hat{\rho}_{\lambda_\gamma,\lambda'_\gamma} = \frac{1}{2}\begin{pmatrix} 1 & -P_\gamma e^{-2i\Phi} \\ -P_\gamma e^{2i\Phi} & 1 \end{pmatrix}_{\lambda_\gamma,\lambda'_\gamma} \tag{4}$$

- Three SDME operators: $\hat{\rho}^{0,1,2}_{\lambda_\gamma,\lambda'_\gamma}$
- $\hat{\rho}^{0}_{\lambda_\gamma,\lambda'_\gamma} = \frac{1}{2}\delta_{\lambda_\gamma,\lambda'_\gamma}$ is the unpolarised SDME
- $\hat{\rho}^{1}_{\lambda_\gamma,\lambda'_\gamma} = \frac{1}{2}P_\gamma\delta_{\lambda_\gamma,-\lambda'_\gamma}$, and $\hat{\rho}^{2}_{\lambda_\gamma,\lambda'_\gamma} = \frac{i}{2}P_\gamma\lambda_\gamma\delta_{\lambda_\gamma,-\lambda'_\gamma}$ are the polarized SDMEs.

SDMEs in the helicity basis

The SDMEs are defined as,

$$\rho^0_{\lambda_\Delta \lambda'_\Delta} = \frac{1}{2N} \sum_{\lambda_1} \left[T_{1\lambda_1\lambda_\Delta} T^*_{1\lambda_1\lambda'_\Delta} + T_{-1\lambda_1\lambda_\Delta} T^*_{-1\lambda_1\lambda'_\Delta} \right] \quad (5)$$

$$\rho^1_{\lambda_\Delta \lambda'_\Delta} = \frac{1}{2N} \sum_{\lambda_1} \left[T_{1\lambda_1\lambda_\Delta} T^*_{-1\lambda_1\lambda'_\Delta} + T_{-1\lambda_1\lambda_\Delta} T^*_{1\lambda_1\lambda'_\Delta} \right] \quad (6)$$

$$\rho^2_{\lambda_\Delta \lambda'_\Delta} = \frac{i}{2N} \sum_{\lambda_1} \left[T_{1\lambda_1\lambda_\Delta} T^*_{-1\lambda_1\lambda'_\Delta} - T_{-1\lambda_1\lambda_\Delta} T^*_{1\lambda_1\lambda'_\Delta} \right] \quad (7)$$

$\rho^0_{\lambda_\Delta \lambda_\Delta}$ is positive.

SDMEs in the helicity basis

The SDMEs are defined as,

$$\rho^0_{\lambda_\Delta \lambda'_\Delta} = \frac{1}{2N} \sum_{\lambda_1} \left[T_{1\lambda_1\lambda_\Delta} T^*_{1\lambda_1\lambda'_\Delta} + T_{-1\lambda_1\lambda_\Delta} T^*_{-1\lambda_1\lambda'_\Delta} \right] \tag{5}$$

$$\rho^1_{\lambda_\Delta \lambda'_\Delta} = \frac{1}{2N} \sum_{\lambda_1} \left[T_{1\lambda_1\lambda_\Delta} T^*_{-1\lambda_1\lambda'_\Delta} + T_{-1\lambda_1\lambda_\Delta} T^*_{1\lambda_1\lambda'_\Delta} \right] \tag{6}$$

$$\rho^2_{\lambda_\Delta \lambda'_\Delta} = \frac{i}{2N} \sum_{\lambda_1} \left[T_{1\lambda_1\lambda_\Delta} T^*_{-1\lambda_1\lambda'_\Delta} - T_{-1\lambda_1\lambda_\Delta} T^*_{1\lambda_1\lambda'_\Delta} \right] \tag{7}$$

$\rho^0_{\lambda_\Delta \lambda_\Delta}$ is positive. $\rho^1_{\lambda_\Delta \lambda_\Delta}$ is related to the beam spin asymmetry

Reflectivity basis

- Reflectivity operation involves 180° rotation about the "y-axis" + parity inversion $\Rightarrow$ inversion of the "y-axis" [2, 3].
- The amplitude in the reflectivity basis can be defined as (valid for $\gamma p \to \pi\Delta$):

$$T^{(\epsilon)}_{\lambda_1,\lambda_\Delta}(s,t) = \frac{1}{2}\left(T_{1,\lambda_1,\lambda_\Delta}(s,t) + \epsilon\, T_{-1,\lambda_1,\lambda_\Delta}(s,t)\right) \quad (8)$$

- The SDMEs take the form,

$$\rho^0_{\lambda_\Delta\lambda'_\Delta} = \frac{1}{N}\sum_{\lambda_1}\left[T^{(+)}_{\lambda_1,\lambda_\Delta} T^{(+)^*}_{\lambda_1,\lambda'_\Delta} + T^{(-)}_{\lambda_1,\lambda_\Delta} T^{(-)*}_{\lambda_1,\lambda'_\Delta}\right] \quad (9)$$

$$\rho^1_{\lambda_\Delta\lambda'_\Delta} = \frac{1}{N}\sum_{\lambda_1}\left[T^{(+)}_{\lambda_1,\lambda_\Delta} T^{(+)^*}_{\lambda_1,\lambda'_\Delta} - T^{(-)}_{\lambda_1,\lambda_\Delta} T^{(-)*}_{\lambda_1,\lambda'_\Delta}\right] \quad (10)$$

- The $\epsilon = (-)+$ amplitudes are dominated by (un)natural parity meson exchange.

Properties

- Under rotations, $\rho \to \mathscr{D}^{J\,\dagger} \rho \mathscr{D}^{J}$; $\mathscr{D} \to$ Wigner-D function.
- $\mathrm{Tr}\rho^0 = 1$. $\mathrm{Tr}\rho^1 = \Sigma$.
- $0 < \rho^0_{\lambda_\Delta \lambda_\Delta} < \frac{1}{2}$ (valid for $\gamma p \to 0^{\pm}\Delta$).
- Possibility of a "zero-crossing" for $\rho^1_{\lambda_\Delta \lambda_\Delta}$ indicating a switch from natural exchange dominance to unnatural exchange dominance (more later).

Parity relations (valid for $\gamma p \to \pi\Delta$):

$$\rho^{0,1}_{-\lambda_\Delta -\lambda'_\Delta} = -(-1)^{\lambda_\Delta + \lambda'_\Delta} \rho^{0,1}_{\lambda_\Delta \lambda'_\Delta} \tag{11}$$

$$\rho^{2}_{-\lambda_\Delta -\lambda'_\Delta} = (-1)^{\lambda_\Delta + \lambda'_\Delta} \rho^{2}_{\lambda_\Delta \lambda'_\Delta}. \tag{12}$$

Regge model

Goal: To model the photoproduction of $\pi\Delta$.

- GlueX photon: $E_{\text{lab}} \approx 9$ GeV
- SDMEs to be measured at low-t
- Large value of s $\Rightarrow$ Regge exchange
- Simple model; exchange of π, ρ, b_1, a_2

  Lagrangians given in Refs. [4, 5, 6, 7, 8]
- Upper and lower vertices factorize
- Poor man's absorption (PMA) model for π-exchange [9].

Model reported in [10].

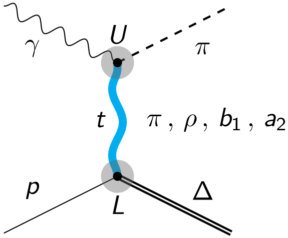


Figure: The t-channel photoproduction process of $\pi\Delta$. U and L are the upper and lower vertices.

Regge model

General form of the helicity amplitude is [10]

$$T_{\lambda_\gamma,\lambda_1,\lambda_\Delta}(s,t) = \sum_{\times}\left[\xi_{\lambda_\gamma\lambda_1\lambda_\Delta}\, T^{\times}_{\lambda_\gamma,\lambda_1,\lambda_\Delta}(s,t)\right] \quad ; \qquad \times \in \{\pi, \rho, b_1, a_2\} \tag{13}$$

$$T^{\times}_{\lambda_\gamma,\lambda_1,\lambda_\Delta}(s,t) = \sqrt{-t}^{\;|\lambda_\gamma|}\sqrt{-t}^{\;|\lambda_1-\lambda_\Delta|}\;\hat{\beta}^{\times,\gamma\pi}_{\lambda_\gamma}(t)\;\hat{\beta}^{\times,p\Delta}_{\lambda_1,\lambda_\Delta}(t)\;\mathcal{P}^{\times}_{R}(s,t) \tag{14}$$

- $\xi_{\lambda_\gamma\lambda_1\lambda_\Delta}$ is the half-angle factor.
- At large s, $T_{-\lambda_\gamma,-\lambda_1,-\lambda_\Delta}(s,t) \approx -(-1)^{\lambda_1-\lambda_\Delta}\, T_{\lambda_\gamma,\lambda_1,\lambda_\Delta}(s,t)$
- Like exchanges take the same form of the vertex (exchange degeneracy):

$$\hat{\beta}^{\pi,p\Delta}_{\lambda_1,\lambda_\Delta}(t) = \hat{\beta}^{b_1,p\Delta}_{\lambda_1,\lambda_\Delta}(t); \qquad \hat{\beta}^{\rho,p\Delta}_{\lambda_1,\lambda_\Delta}(t) = \hat{\beta}^{a_2,p\Delta}_{\lambda_1,\lambda_\Delta}(t)$$

- Overall (exponential + polynomial) suppression factors for each exchange.

Regge model - absorption and gauge invariance

The amplitude has the factor [10, 9]:

$$\sqrt{-t}^{\;|\lambda_\gamma-\lambda_1+\lambda_\Delta|}\sqrt{-t}^{\;|\lambda_1-\lambda_\Delta|+|\lambda_\gamma|-|\lambda_\gamma-\lambda_1+\lambda_\Delta|} \quad (15)$$

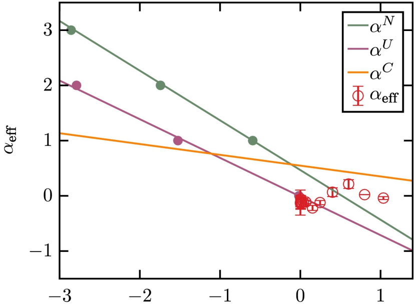
PMA: evaluate the second factor of $\sqrt{-t}$ at $t = m_\pi^2$, the pion pole.

- The vertex factor is of the form $\sqrt{-t}^{\;n+x}$, where $n\ (= |\lambda_\gamma - \lambda_1 + \lambda_\Delta|)$ arises due to angular momentum considerations, and x due to the model.
- In the PMA, $\sqrt{-t}^{\;x}$ corresponding to the unnatural exchange is evaluated at the pion pole.
- The residual term has an additional exponential factor to take care of the large-t fall-off.
- The π-exchange parameter in the lower vertex fixed from $\Delta \to \pi N$ decay width
- The parameters in the upper vertex fixed from the corresponding radiative decay widths $(\rho/b_1/a_2 \to \pi\gamma)$

"Pole" and "Cut" models

- Pole model: All exchange mesons are Regge poles.
- Cut model: Natural parity exchanges are modified by the Pomeron-Reggeon rescattering $\implies$ 1. Altered trajectory, 2. Logarithmic s-dependence.

$\rightarrow$ $\alpha^C(t) = \alpha_0^N + \alpha_0^{\mathbb{P}} - 1 + t\frac{\alpha_1^N \alpha_1^{\mathbb{P}}}{\alpha_1^N + \alpha_1^{\mathbb{P}}}$; ($\alpha_0$ and α_1 are the intercept and slopes of the respective trajectories)



$$\frac{d\sigma}{dt} \simeq f(t) s^{2\alpha_{\text{eff}}(t)-2}$$

- Small-t behavior dominated by pion exchange.
- 'Flat' behavior for large-t $\rightarrow$ Reggeon-Pomeron rescattering [10].

Regge model

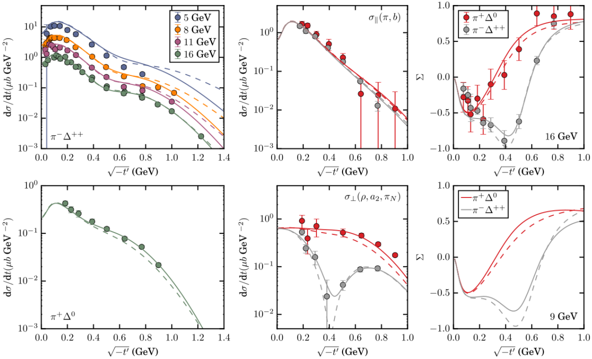
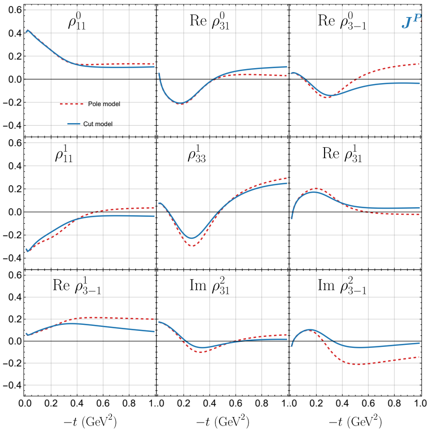


Figure: The pole and cut models fitted to the data (snapshot from [10]).

The $p\rho\Delta$ parameters and the phenomenological suppression factors fitted to the cross section.

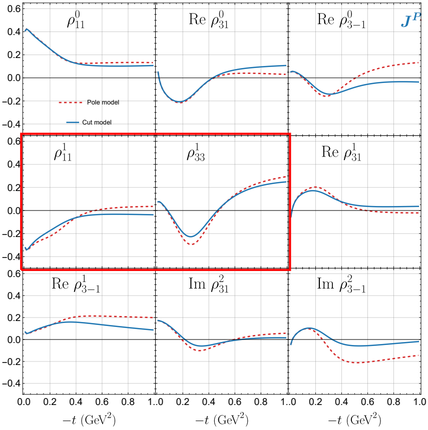
SDMEs



SDMEs of Δ in the Helicity frame.

- Marked change in the behavior of the SDMEs at $-t \sim M_\rho^2$
- Large-t behavior is well-explained by the model.
- Small-t behavior is poorly understood.

SDMEs



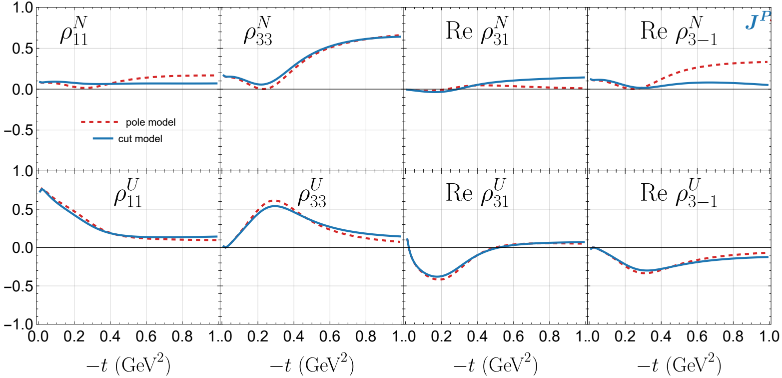
SDMEs of Δ in the Helicity frame.

$$\rho^0_{\lambda_\Delta \lambda'_\Delta} = \frac{1}{N} \sum_{\lambda_1} T^{(+)}_{\lambda_1,\lambda_\Delta} T^{(+)^*}_{\lambda_1,\lambda'_\Delta} + T^{(-)}_{\lambda_1,\lambda_\Delta} T^{(-)*}_{\lambda_1,\lambda'_\Delta} \quad (6)$$

$$\rho^1_{\lambda_\Delta \lambda'_\Delta} = \frac{1}{N} \sum_{\lambda_1} T^{(+)}_{\lambda_1,\lambda_\Delta} T^{(+)^*}_{\lambda_1,\lambda'_\Delta} - T^{(-)}_{\lambda_1,\lambda_\Delta} T^{(-)*}_{\lambda_1,\lambda'_\Delta} \quad (7)$$

- The zero in the ρ^1_{11} marks a clear separation in the $\pm$-reflectivity dominant regions.
- This separation is "system-dependent".
- ρ^1_{33} also has (two) zero crossings

(Un)Natural components



- Low-t region is dominated by unnatural exchange; large-t region by natural exchange.
- Unnatural component involves pion which has the PMA. Better models may be required.

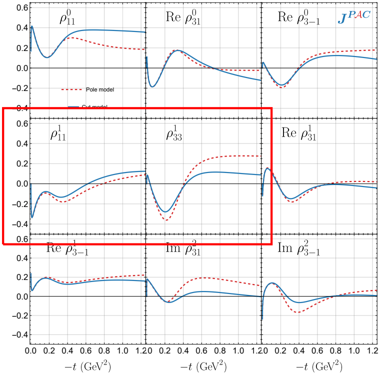
Frames of reference

The frame of reference used to extract the SDMEs can influence their behavior.

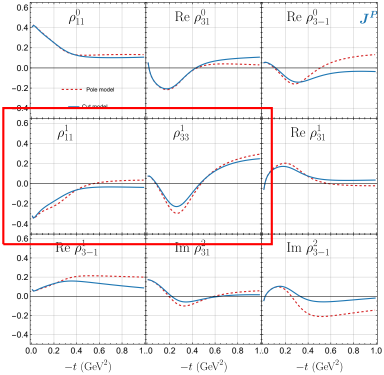
$$^{GJ}\rho_{\lambda_\Delta \lambda'_\Delta} = \sum_{\lambda\lambda'} d^{3/2}_{\lambda_\Delta \lambda}(\theta)\, ^{H}\rho_{\lambda\lambda'}\, d^{3/2}_{\lambda'_\Delta \lambda'}(\theta) \tag{16}$$

Different ρ's mix during the transformation $\implies$ features in one frame need nt be the same in the other.

Frames of reference



SDMEs of Δ in the GJ frame.



SDMEs of Δ in the Helicity frame.

Summary

- SDMEs determine the angular distribution.
- First step towards understanding the photoproduction of Δ with associated mesonic resonances $(\rho,\ b_1,\ \pi_1\ a_2, \dots)$.
- Possibility to study doubly polarised scattering process.
- Regge model involves exchange of $\pi,\ \rho,\ b_1,\ a_2$.
- Pion exchange amplitude involves PMA
- Natural exchange dominates the large-t behavior of the SDMEs
- Low-t behavior, dominated pion exchange, is poorly understood.
- System dependent "zeros" in ρ^1_{11} and ρ^1_{33}.

Thank you!

Bibliography I

K. Schilling, P. Seyboth, and Guenter E. Wolf.
On the Analysis of Vector Meson Production by Polarized Photons.
Nucl. Phys. B, 15:397–412, 1970.
[Erratum: Nucl.Phys.B 18, 332 (1970)].

S. U. Chung and T. L. Trueman.
Positivity Conditions on the Spin Density Matrix: A Simple Parametrization.
Phys. Rev. D, 11:633, 1975.

V. Mathieu, M. Albaladejo, C. Fernández-Ramírez, A. W. Jackura, M. Mikhasenko, A. Pilloni, and A. P. Szczepaniak.
Moments of angular distribution and beam asymmetries in $\eta\pi^0$ photoproduction at GlueX.
Phys. Rev. D, 100(5):054017, 2019.

Seung-il Nam and Byung-Geel Yu.
Charged pion photoproduction with the Delta(1232) baryon beyond the resonance region.
Phys. Rev. C, 84:025203, 2011.

Byung-Geel Yu, Tae Keun Choi, and Kook-Jin Kong.
Features of ω photoproduction off nucleon target at forward angles: dominance of π exchange with Regge cuts and scaling of differential cross-sections.
J. Phys. G, 46(7):075005, 2019.

Bibliography II

Byung Geel Yu, Tae Keun Choi, and W. Kim.
Regge phenomenology of pion photoproduction off the nucleon at forward angles.
Phys. Rev. C, 83:025208, 2011.

T. Sato and T-. S. H. Lee.
Meson exchange model for pi N scattering and gamma N —> pi N reaction.
Phys. Rev. C, 54:2660–2684, 1996.

S. Bellucci, J. Gasser, and M. E. Sainio.
Low-energy photon-photon collisions to two loop order.
Nucl. Phys. B, 423:80–122, 1994.
[Erratum: Nucl.Phys.B 431, 413–414 (1994)].

P. K. Williams.
Extrapolation model for pi pi scattering.
Phys. Rev. D, 1:1312–1318, 1970.

J. Nys, V. Mathieu, C. Fernández-Ramírez, A. Jackura, M. Mikhasenko, A. Pilloni, N. Sherrill, J. Ryckebusch, A. P. Szczepaniak, and G. Fox.
Features of $\pi\Delta$ Photoproduction at High Energies.
Phys. Lett. B, 779:77–81, 2018.

Intensity

The $\vec{\gamma} p \to \pi\Delta \to \pi(p\pi)$ amplitude is given by,

$$A_{\lambda_\gamma,\lambda_1,\lambda_2}(\Omega) = \sum_{\lambda_\Delta} T_{\lambda_\gamma,\lambda_1,\lambda_\Delta}(s,t) D^{3/2*}_{\lambda_\Delta,\lambda_2}(\Omega) \tag{17}$$

The intensity is given by,

$$I(\Omega,\Phi,P_\gamma) = \frac{\kappa}{2} \sum_{\lambda_\gamma^{(\prime)},\lambda_1,\lambda_2} A_{\lambda_\gamma,\lambda_1,\lambda_2}(\Omega)\ \hat{\rho}_{\lambda_\gamma,\lambda'_\gamma}\ A^*_{\lambda'_\gamma,\lambda_1\lambda_2}(\Omega) \tag{18}$$

where κ is the phase space factor. $\Omega = (\theta,\phi)$ are the Δ-decay angles.

Intensity

Substituting and expanding,

$$\begin{aligned}I(\Omega,\Phi) = 2N\Big\{ & \rho^0_{33}\sin^2\theta + \rho^0_{11}\left(\frac{1}{3}+\cos^2\theta\right) \\ & -\frac{2}{\sqrt{3}}\,\mathrm{Re}\,\rho^0_{31}\sin 2\theta\cos\phi - \frac{2}{\sqrt{3}}\,\mathrm{Re}\,\rho^0_{3-1}\sin^2\theta\cos 2\phi \\ & - P_\gamma\cos 2\Phi\left[\rho^1_{33}\sin^2\theta + \rho^1_{11}\left(\frac{1}{3}+\cos^2\theta\right)\right. \\ & \left. -\frac{2}{\sqrt{3}}\,\mathrm{Re}\,\rho^1_{31}\sin 2\theta\cos\phi - \frac{2}{\sqrt{3}}\,\mathrm{Re}\,\rho^1_{3-1}\sin^2\theta\cos 2\phi\right] \\ & - P_\gamma\sin 2\Phi\left[\frac{2}{\sqrt{3}}\,\mathrm{Im}\,\rho^2_{31}\sin 2\theta\sin\phi\right. \\ & \left. +\frac{2}{\sqrt{3}}\,\mathrm{Im}\,\rho^2_{3-1}\sin^2\theta\sin 2\phi\right]\Big\}. \end{aligned} \tag{19}$$

SDME: properties

$$\frac{d\sigma}{dt} = \frac{K}{4} \sum_{\mu_\Delta, \mu_N, \mu_\gamma} |A_{\mu_\Delta, \mu_N \mu_\gamma}|^2, \qquad (7)$$

$$\Sigma \frac{d\sigma}{dt} = \frac{K}{4} \sum_{\mu_\Delta, \mu_N} 2\,\mathrm{Re}\, A_{\mu_\Delta, \mu_N \mu_\gamma = +1} A^*_{\mu_\Delta, \mu_N \mu_\gamma = -1}, \qquad (8)$$

$$\frac{d\sigma_{\perp/\parallel}}{dt} = \frac{K}{4} \sum_{\mu_\Delta, \mu_N} |A_{\mu_\Delta, \mu_N \mu_\gamma = +1} \pm A_{\mu_\Delta, \mu_N \mu_\gamma = -1}|^2, \qquad (9)$$

Picture from [10].

Reflectivity basis

The amplitude in the reflectivity basis is,

$$T^{(\epsilon)}_{\lambda_1,\lambda_\Delta}(s,t) = \frac{1}{2}\left(T_{1,\lambda_1,\lambda_\Delta}(s,t) + \epsilon\, T_{-1,\lambda_1,\lambda_\Delta}(s,t)\right) \tag{20}$$

where, $\epsilon = \pm$ represents the reflectivity quantum number. The reflectivity amplitudes obey the parity relation,

$$T^{(\epsilon)}_{-\lambda_1,-\lambda_\Delta}(s,t) = -\epsilon(-1)^{\lambda_1-\lambda_\Delta}\, T^{(\epsilon)}_{\lambda_1,\lambda_\Delta}(s,t). \tag{21}$$

In the reflectivity basis the SDMEs take the form,

$$\rho^0_{\lambda_\Delta\lambda'_\Delta} = \frac{1}{N}\sum_{\lambda_1} T^{(+)}_{\lambda_1\lambda_\Delta}\, T^{(+)^*}_{\lambda_1\lambda'_\Delta} + T^{(-)}_{\lambda_1\lambda_\Delta}\, T^{(-)*}_{\lambda_1\lambda'_\Delta} \tag{22}$$

$$\rho^1_{\lambda_\Delta\lambda'_\Delta} = \frac{1}{N}\sum_{\lambda_1} T^{(+)}_{\lambda_1\lambda_\Delta}\, T^{(+)^*}_{\lambda_1\lambda'_\Delta} - T^{(-)}_{\lambda_1\lambda_\Delta}\, T^{(-)*}_{\lambda_1\lambda'_\Delta} \tag{23}$$

$$\rho^2_{\lambda_\Delta\lambda'_\Delta} = \frac{i}{N}\sum_{\lambda_1} -T^{(+)}_{\lambda_1\lambda_\Delta}\, T^{(-)^*}_{\lambda_1\lambda'_\Delta} + T^{(-)}_{\lambda_1\lambda_\Delta}\, T^{(+)*}_{\lambda_1\lambda'_\Delta} \tag{24}$$

Reflectivity basis - any J^P

- Reflectivity operation involves 180° rotation about the "y-axis" + parity inversion $\Rightarrow$ inversion of the "y-axis".
- The amplitude in the reflectivity basis can be defined as:

$$T^{(\epsilon)}_{\lambda_1,\Lambda,\lambda_\Delta}(s,t) = \frac{1}{2}\left(T_{1,\lambda_1,\Lambda,\lambda_\Delta}(s,t) + \epsilon P(-1)^{J+\Lambda} T_{-1,\lambda_1,\Lambda,\lambda_\Delta}(s,t)\right) \tag{25}$$

- The SDMEs take the form,

$$\rho^0_{\lambda_\Delta\lambda'_\Delta} = \frac{1}{N}\sum_{\lambda_1}\left[T^{(+)}_{\lambda_1\Lambda\lambda_\Delta} T^{(+)^*}_{\lambda_1,\Lambda,\lambda'_\Delta} + T^{(-)}_{\lambda_1,\Lambda,\lambda_\Delta} T^{(-)*}_{\lambda_1,\Lambda,\lambda'_\Delta}\right] \tag{26}$$

$$\rho^1_{\lambda_\Delta\lambda'_\Delta} = \frac{1}{N}\sum_{\lambda_1}(-1)^\Lambda\left[T^{(+)}_{\lambda_1,\Lambda,\lambda_\Delta} T^{(+)^*}_{\lambda_1,\Lambda,\lambda'_\Delta} - T^{(-)}_{\lambda_1,\Lambda,\lambda_\Delta} T^{(-)*}_{\lambda_1,\Lambda,\lambda'_\Delta}\right] \tag{27}$$

- The $\epsilon = (-)+$ amplitudes are dominated by (un)natural parity meson exchange.

Reflectivity basis

Consider the operator $\hat{\Pi}_y$ defined as,

$$\hat{\Pi}_y = \hat{P} e^{i\pi \hat{J}_y} \tag{28}$$

where $\hat{P}$ is the parity operator. This operator, called the reflectivity operator, represents a rotation by an angle π with respect to the y-axis and a subsequent parity operation. Equivalently, this is a reflection about the production plane. The eigen value is either positive or negative for a mesonic state. *i.e.*, for a state with spin-parity J^P and 3-momentum $\vec{p}$,

$$\hat{\Pi}_y \, |\vec{p}; \lambda\rangle = P(-1)^{J-\lambda} \, |\vec{p}; -\lambda\rangle. \tag{29}$$

Reflectivity basis and exchange

The amplitude can be written as,

$$T_{\lambda_\gamma \lambda_1 \lambda_\Delta} = \sum_J a^J_{\lambda_\gamma \lambda_1 \lambda_\Delta} d^J_{\lambda_\gamma \lambda_1 - \lambda_\Delta}(\theta) \tag{30}$$

a^J obeys the parity relation, $a^J_{-\lambda_\gamma \lambda_1 \lambda_\Delta} = P(-1)^J a^J_{\lambda_\gamma \lambda_1 \lambda_\Delta}$.
At large s, $\cos\theta \propto s$, and

$$d^J_{-\lambda_\gamma \lambda_1 \lambda_\Delta}(\theta) \simeq (1-)^{\lambda_\gamma} d^J_{\lambda_\gamma \lambda_1 \lambda_\Delta}(\theta)$$

$$\implies T_{\lambda_\gamma \lambda_1 \lambda_\Delta} \approx P(-1)^{J+\lambda_\gamma} T_{-\lambda_\gamma \lambda_1 \lambda_\Delta}$$

Frames

Helicity frame: Boost the center of mass frame to the rest frame of the required resonance. The z-axis is the direction of the 3-momentum of the resonance in the c.o.m frame.

GJ-frame: Rest frame of the resonance. The z-axis is the beam direction in the frame.