Disordering of exotic phases through bosonic quantum fluctuations in strongly-interacting matter

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in collaboration with Semeon Valgushev (Iowa State University)

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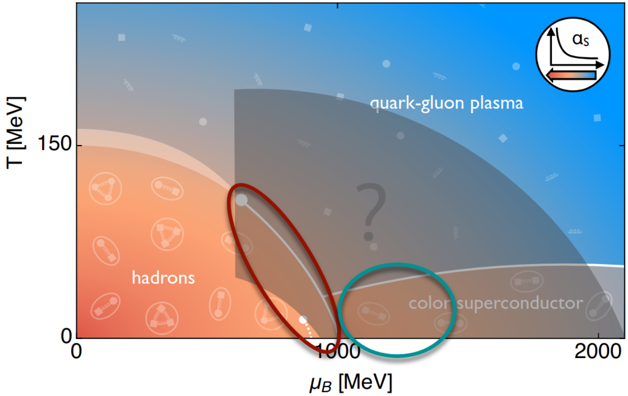
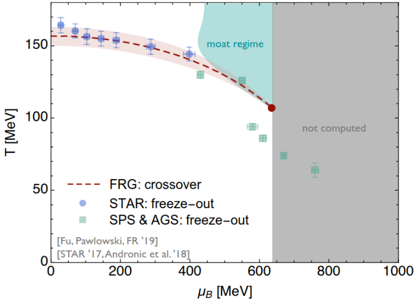
Introduction (I)

- QCD Phase diagram in $T - \mu_B$ plane is vastly unexplored, especially at intermediate densities
  - Location of CEP, quarkyonic matter, (crystalline) color superconductivity, mixed phases...
  - Moat regimes, inhomogeneous chiral phases: Spatial modulations of the order parameter?

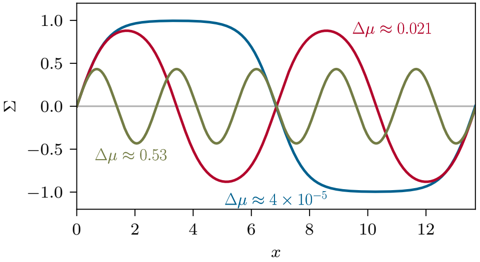


- Inhomogeneous, chiral phase (IP): $\langle\bar{\psi}\psi\rangle = f(\mathbf{x})$ (so far: mostly 1d modulations)
- Recently: Moat regime with $E^2 = p^4 + Zp^2 + m^2$ with $Z < 0$



FRG study

[Fu, Pawlowski, Rennecke, PRD **101**, 054032 (2020)]



1 + 1-dimensional Four-Fermion model

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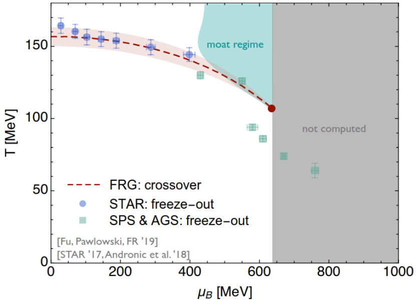

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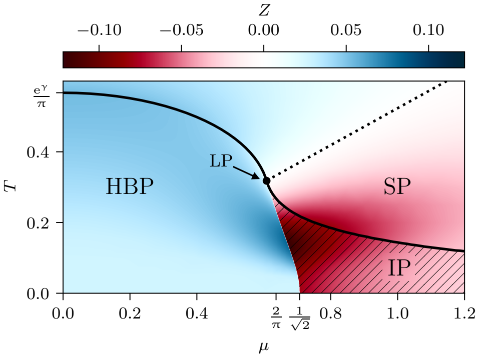

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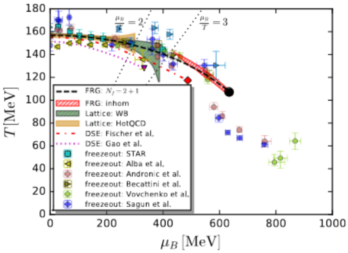
Chiral phase transition at finite density



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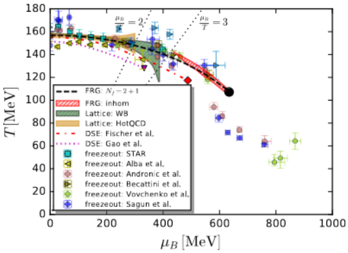

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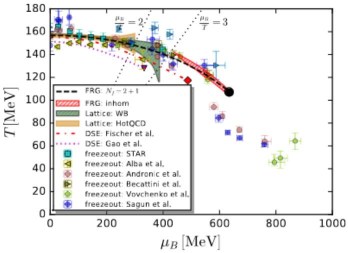

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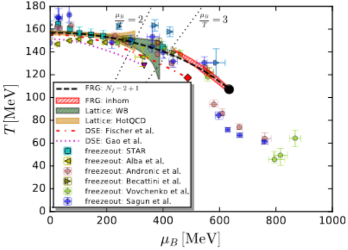

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A few studies beyond mean-field

[Mueller, Buballa, Wambach, Phy. Lett. B **727**, 240 (2013), arXiv: 1308.4303]

[Tripolt et al., PRD **97**, 034022 (2018), arXiv: 1709.05991]

[Fu, Pawlowski, Rennecke, PRD **101**, 054032 (2020), arXiv: 1909.02991]

[Stoll et al., arXiv: 2108.10616]

[Lenz et al., PRD **101**, 094512 (2020), arXiv: 2004.00295]

[Ciccione, Di Petro, Serone, PRL **129**, 071603 (2022), arXiv: 2203.07451]

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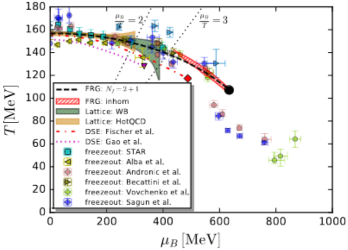

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Influence of bosonic quantum fluctuations on IPs

Fluctuations are expected to weaken ordering. Some evidence from model studies:

- In $1+1$-dimensional models:
  - Mermin-Wagner-Coleman theorem: Continuous symmetry cannot be spontaneously broken at $T>0$
  - Landau's argument: Even discrete symmetry should be restored, supported by [Stoll et al., arXiv: 2108.10616]
  - Inhomogeneous correlation function observed in LFT studies of GN model [Lenz et al., PRD $\mathbf{101}$, 094512]
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  - Regulator dependence: IP even on mean-field level unclear, moat regime seems to be robust [Nickel, PRD **80**, 074025 (2009)] [Pannullo, Wagner, MW, PoS LATTICE2022 156 (2023) & upcoming paper]
  - Landau-Peierls instability of CDW / 1d-modulations prohibits long-range order
  - Magnetic dual chiral density wave stable at large B-field [Ferrer, de la Incera, PRD **102**, 014010 (2020)]

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$\Rightarrow$ All of mentioned studies are either computationally expensive, some have contradictory results, some involve strong approximations / do not incorporate full quantum effects

Motivation and main idea

Motivation: Study the influence of bosonic fluctuation on an IP in a simple and effective setup

- Assuming IPs exist in some approximation, how do the bosonic fluctuations impact the inhomogeneous condensate ?

Main idea

- Quark determinant, bosonic loops, etc., can be expanded in powers of mesonic fields $\vec{\phi}, \partial\vec{\phi}$ in an effective theory for 'chiral physics' (see, e.g., Quark-Meson or Nambu-Jona-Lasinio model)
- At intermediate density and temperature CSB dominated by light-mesons $\vec{\phi} = (\sigma, \vec{\pi}) \Rightarrow \mathrm{O}(4)$ model for two-flavor QCD

$$\langle\sigma\rangle \sim \langle\bar{\psi}\psi\rangle \quad \langle\vec{\pi}\rangle \sim \langle\bar{\psi}\mathrm{i}\vec{\tau}\gamma_5\psi\rangle$$

$\Rightarrow$ Study $\mathrm{O}(N)$ model as effective theory

Studying IPs in an $\mathrm{O}(N)$ model

$$\mathcal{L} = \frac{1}{2}\left(\partial_0\vec{\phi}\right)^2 + \frac{1}{2}\left(\partial_j\vec{\phi}\right)^2 + \frac{m^2}{2}\vec{\phi}^2 + \frac{\lambda N}{4}(\vec{\phi}^2)^2$$

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- Approach path integral with standard LFT techniques (hybrid Monte Carlo, Metropolis)

$\mathrm{O}(N)$ model in the large-N limit

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- As expected: $k_0 \neq 0$ is solution for classical EoM

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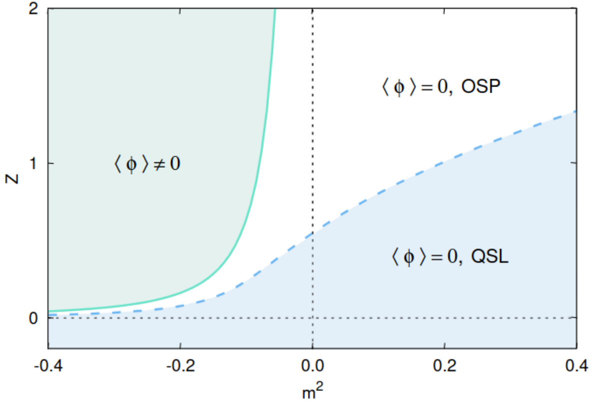

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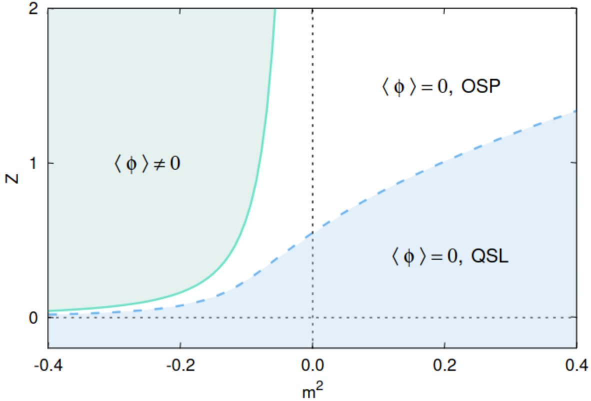

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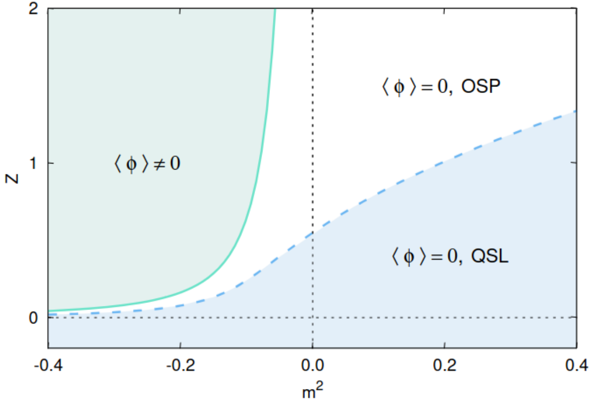

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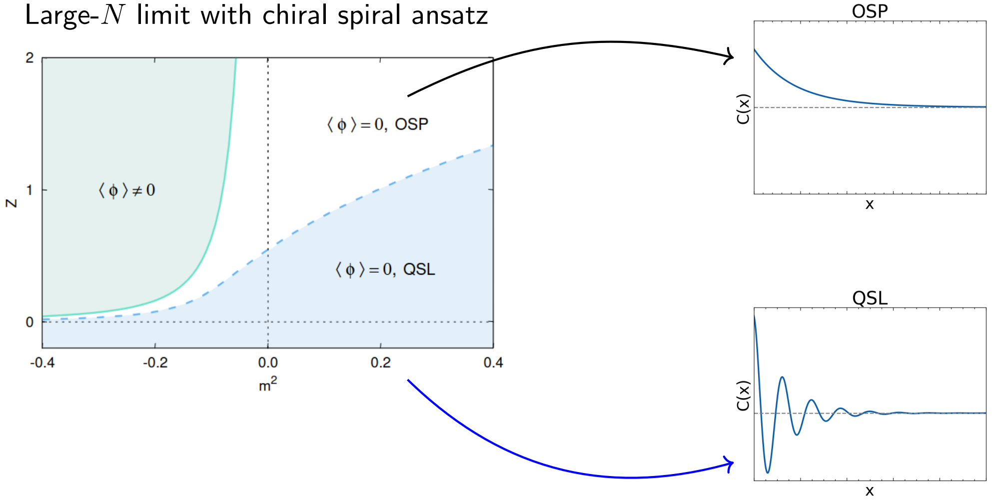

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  - "Phase diagram" in (Z, m^2) plane and dependence on N

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CRC-TR 211

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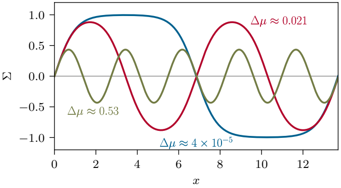
- Jackknife error: Typically smaller than dots

Different scenarios for spatial modulations at finite μ

IP

- Long-range order
- Spatial mesonic correlators $C(x) = \langle \phi(x)\phi(y=0)\rangle$ are oscillatory functions C_{osc} at large x

$\langle \phi_j \rangle \sim \langle \bar{\psi}\Gamma_j \psi \rangle = f(\mathbf{x})$



Liquid crystal

- Quasi-long range order
- Disordering through Goldstone modes of translational symmetry breaking (phonons)

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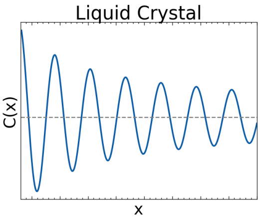

Quantum spin / π liquid

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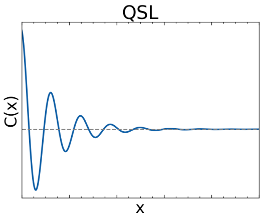

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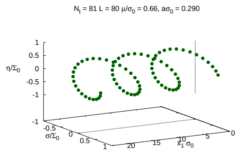

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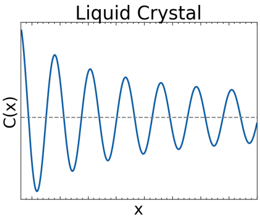

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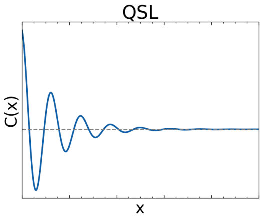

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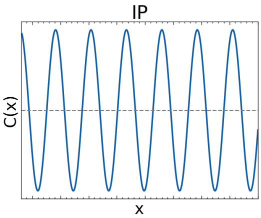
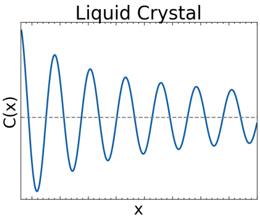
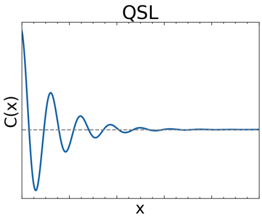
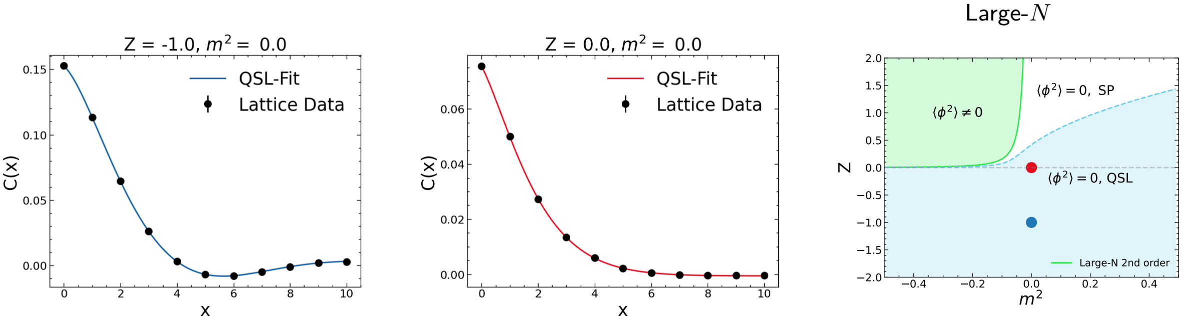






Correlators are (discrete) rotationally symmetric, study $C(\mathbf{x}) = C((x, 0, 0))$



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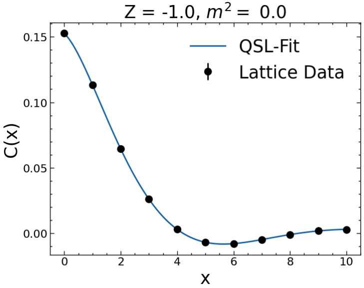
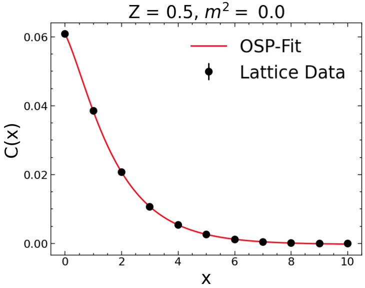
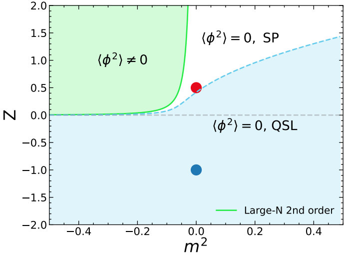




Large-N



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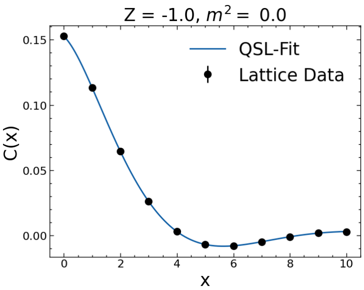

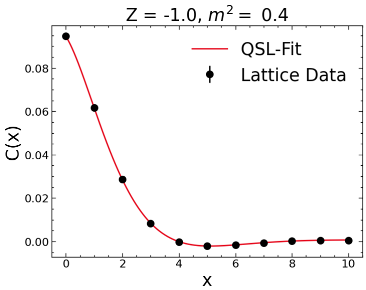

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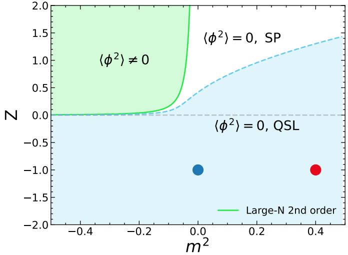

Correlators $N = 1$

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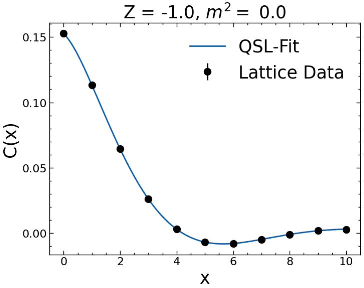

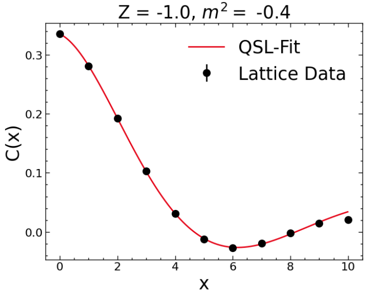

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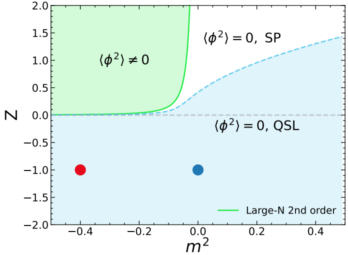

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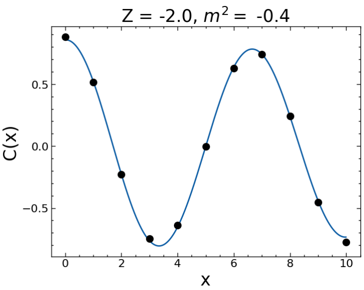
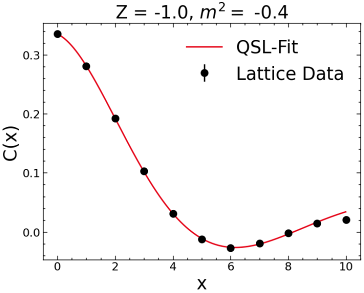
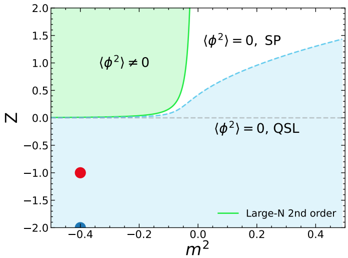
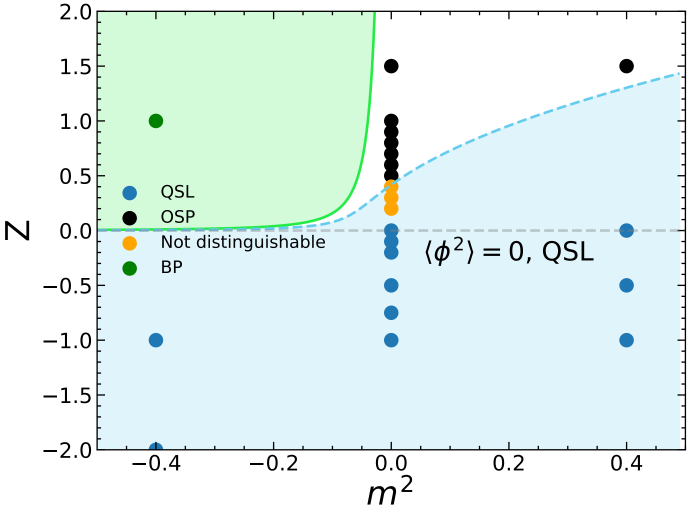


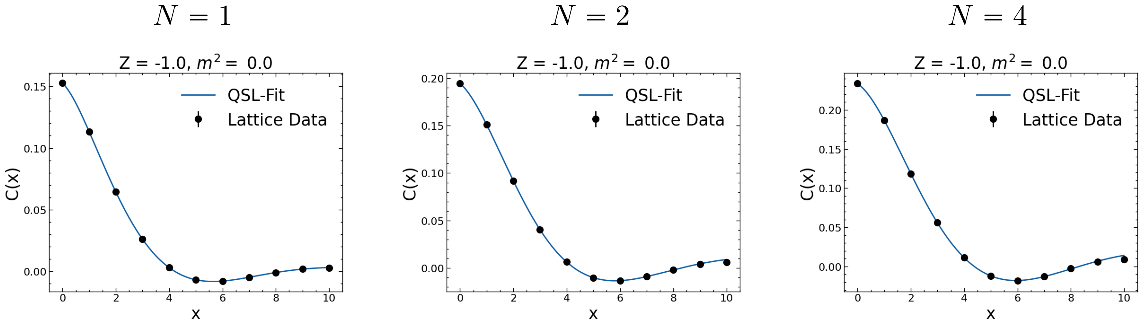


Large-N




2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
Z
−0.4
−0.2
0.0
0.2
0.4
m^2
QSL
OSP
Not distinguishable
BP
$\langle\phi^2\rangle = 0$, QSL

Independence of N



- $m^2 = 0.0, Z \in [-1.0, 1.0]$ Resulting regimes (QSL /OSP) almost independent of $N (= 1, 2, 4, 8, 10)$
- Mechanism through disordering with transverse modes cannot be the full picture
- Lacking a (local) order parameter to fully map out regime in (Z, m^2) plane

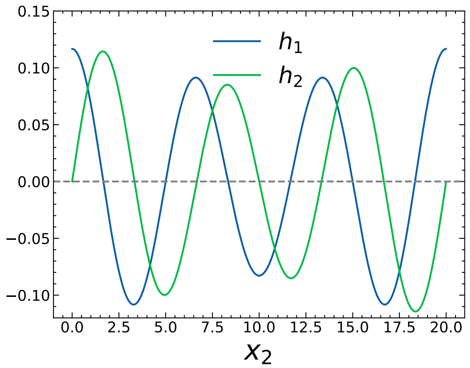
External breaking of translational & rotational symmetry

$$L_{\mathrm{br}} = L_{\mathrm{eff}} - \vec{h}(x)\vec{\phi}(x)$$

$$h(x) = \frac{h_0}{\sqrt{2\pi} L\,\sigma} \sum_{n=0}^{L-1} \mathrm{e}^{-\frac{1}{2\sigma}(p_n - k_0)^2} \begin{pmatrix} \cos(p_n x_2) \\ \sin(p_x x_2) \\ \vec{0} \end{pmatrix}$$

- Idea: External breaking through parameter h_0 and $h_0 \to 0$ to ensure that there is no symmetry breaking
- Enforce chiral spiral and avoid interference of multiple degenerate solutions
- Lattice momentum $p_n = 2\pi n / L_2$
- Extract momentum parameter k_0 through QSL fit at $h_0 = 0$ and then study $h_0 \in \{-0.1, \ldots, 0.1\}$
- $\langle \phi^i(x)\phi^j(y)\rangle$ loose translational invariance in x_2 direction and rotational symmetry

$$C(x,y) = C(x_0 - y_0, x_1 - y_1, x_2, y_2)$$



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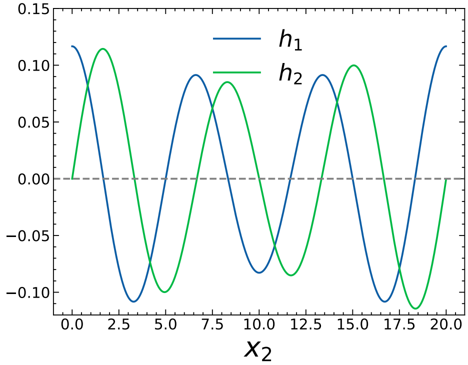

CRC-TR 211

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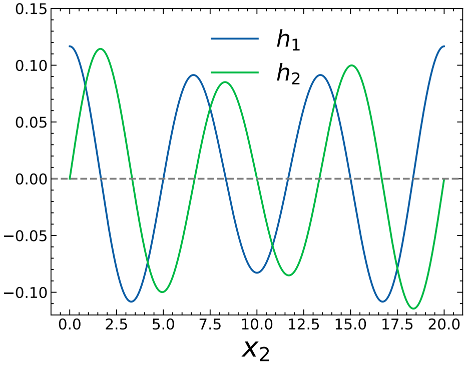
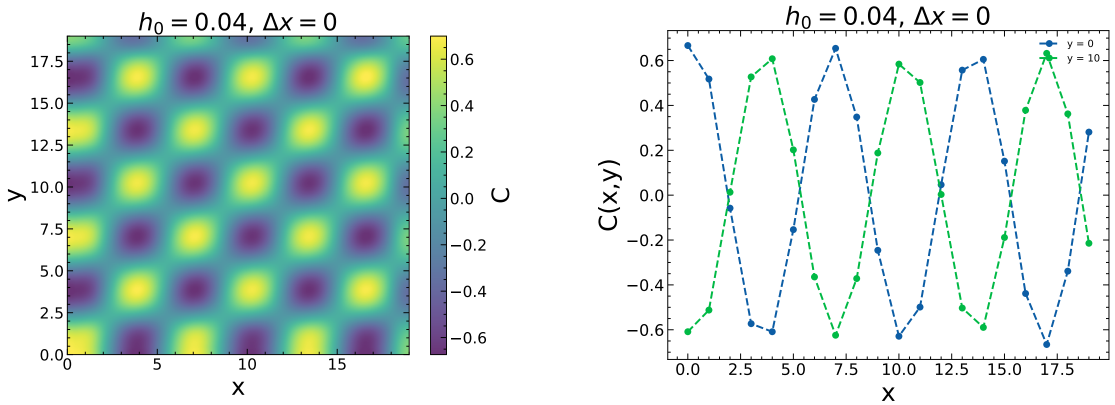


CRC-TR 211

$C(\Delta x_0, 0, x, y)$ for $Z = -2.0, m^2 = 0.0$



Translational symmetry clearly broken, C(x,y) qualitatively identical for $|h_0| > 0.01$

External breaking for $N = 1$

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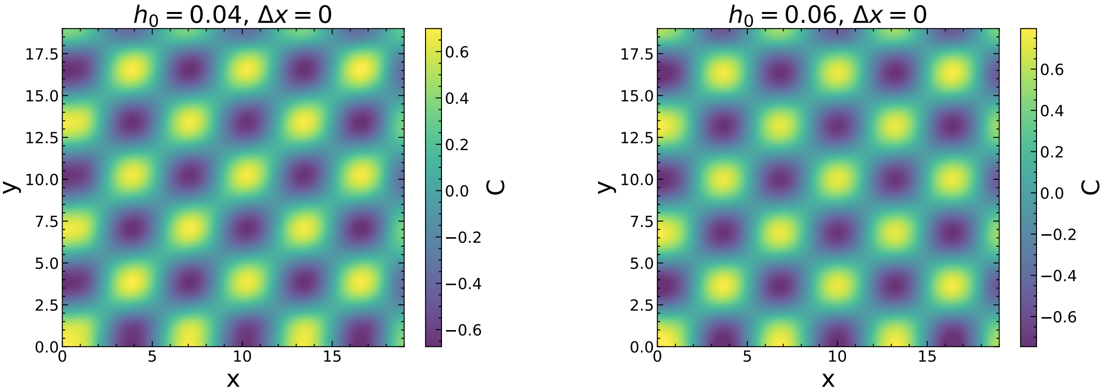

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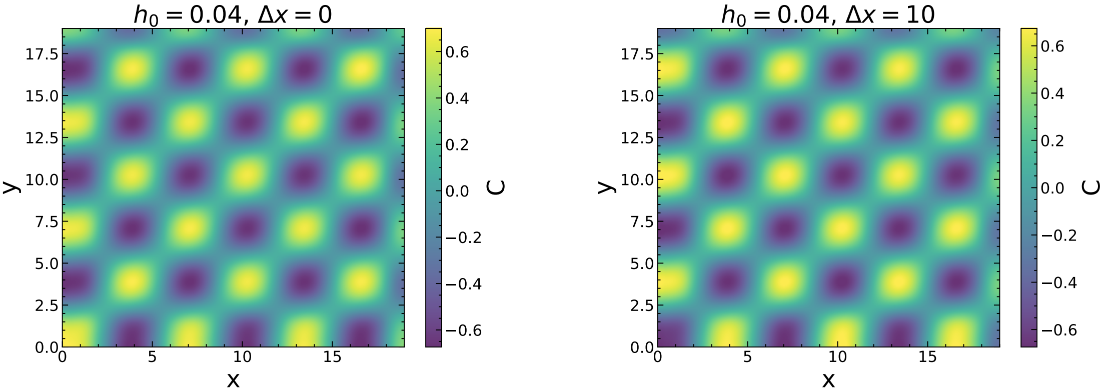

Δx does not change amplitude, only global sign, No rotational symmetry breaking? Further studies needed

External breaking for $N = 1$ – Small $|h_0|$

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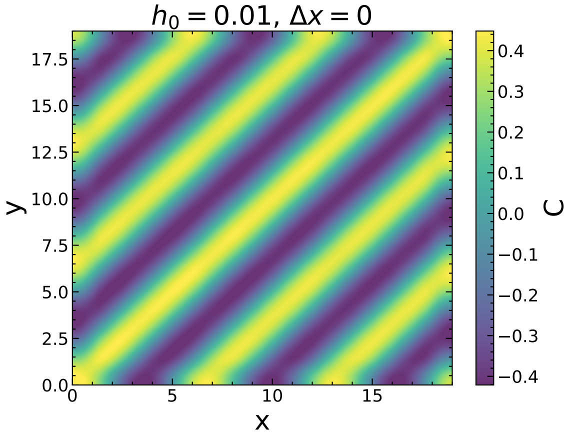


$h_0 = 0.01$ translational symmetry breaking already starts getting restored!

Summary & Open questions

- Bosonic quantum fluctuations lead to disordering of inhomogeneous condensate
- QSL regime with oscillating (damped) correlation functions
- Regime almost independent of N & also $N = 1, 2$

  $\Rightarrow$ Mechanism of disordering through $N - 2$ Goldstone bosons cannot be the full explanation

Open questions

- Suitable observables for classification of QSL vs OSP?
- Map out phase diagrams for different N
- Volume dependence of results?
- (Discrete), rotational symmetry might even be present with external field
- Is there really $\mathrm{O}(N)$ symmetry breaking with external field? Study $N = 2, 4$

Another important takeaway

Regimes with spatial oscillations can be highly relevant for QCD!

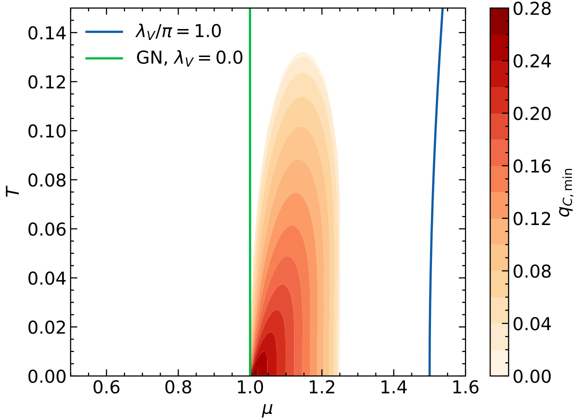
- $\mathcal{C}$-symmetry breaking at finite μ, but invariance under combined $\mathcal{CH}$ symmetry
- $\mathcal{PT}$ symmetric field theories can feature spatial oscillations in correlation functions!
  [Schindler, Schindler, Medina, Oglivie, PoS LATTICE2021 555 (2022), arXiv:2110.07761 & PRD **102**, arXiv:1906.07288]
- Also, simple four-fermion models with vector mesons show such oscillations [MW, upcoming paper]
  [Haensch, Rennecke, von Smekal, (2023) arXiv:2308.16244]

Methods / techniques to get better access to these exotic phases need to be developed

- Heavy-Ion-Collisions: Access physical consequences via moat regime dispersion relation

Appendix

$$\mathcal{S}_{\mathrm{FF}}[\bar{\psi},\psi] = \int \mathrm{d}^3x \left\{ \bar{\psi}\left(\partial\!\!\!/ + \gamma_3\mu\right)\psi - \left[\sum_{j=1}^{16} \left(\frac{\lambda_S}{2N}\left(\bar{\psi}\, c_j\, \psi\right)^2 + \frac{\lambda_V}{2N}\left(\left(\bar{\psi}\, \mathrm{i} c_j \gamma_\nu\, \psi\right)^2 \right) \right) \right] \right\}$$



Large-N Quantum pion liquid – mechanism

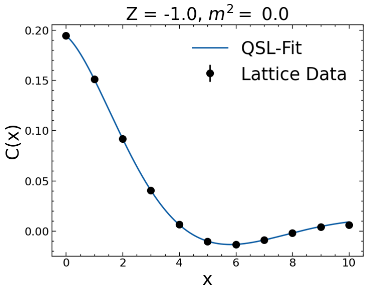
[Pisarski, Tsvelsik, Valgushev, PRD **102**, 016015 (2020)]

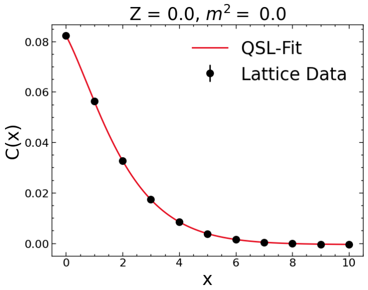
$$\text{QSL} : \langle \phi^i(x)\phi^j(0)\rangle|_{x\to\infty} \sim \delta^{ij} \mathrm{e}^{-m_r} c_1 \cos(m_i x)$$

- Ansatz: Chiral spiral $\vec{\phi} = \phi_0 \left(\cos(k_0 z), \sin(k_0 z), \phi_\perp = \vec{0}\right)$
- Propagator $S_{\phi_\perp}$ has double pole at $k_0 \neq 0$; IR divergences occur at $T > 0$
  - Fluctuations about transverse modes $\phi_\perp$ disorder IP
  - Complex poles $m_r + \mathrm{i}m_i$ arise instead of the usual purely imaginary poles $p^2 = -m_\pm^2$, leads to oscillatory behavior in QSL
  - For large $|Z|$ and $Z < 0$

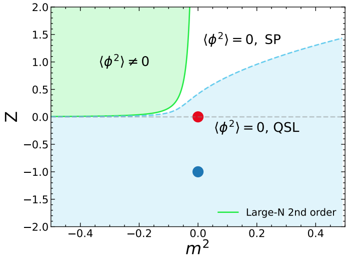
$$m_R \sim Z^{-2}, \quad m_I \sim \sqrt{-Z}$$

Increasing to $N = 2$ makes no major difference!

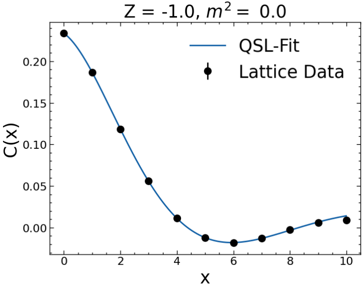


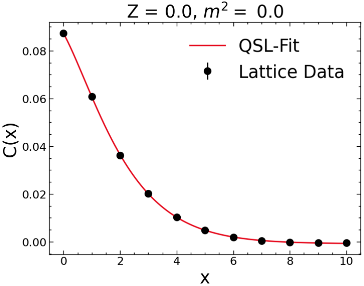


Large-N

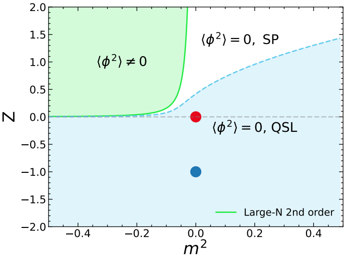


Increasing to $N = 4$ makes no major difference!



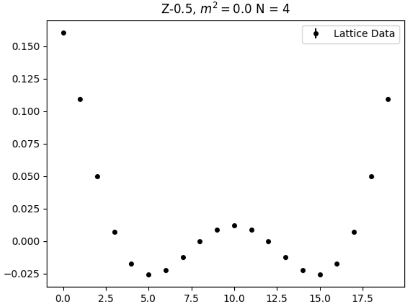


Large-N

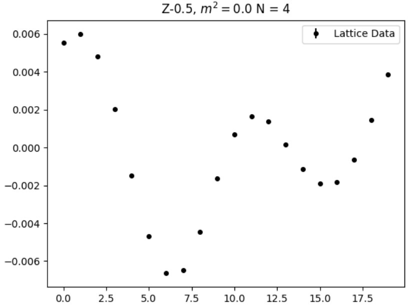


$$C^{ij}(x) = \langle c^{ij}(x) \rangle = \frac{1}{V} \sum_y \langle \phi^i(y+x) \phi^j(y) \rangle,$$

$i = 0, j = 0$



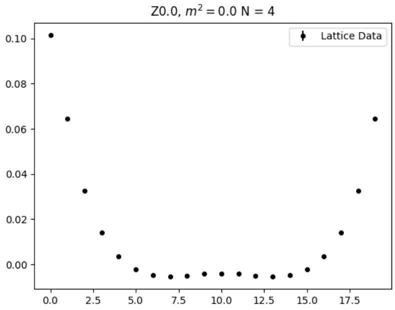
$i = 0, j = 1$



Off-diagonal correlators $N = 4$

$$C^{ij}(x) = \langle c^{ij}(x) \rangle = \frac{1}{V} \sum_y \langle \phi^i(y+x) \phi^j(y) \rangle,$$

$i = 0, j = 0$



$i = 0, j = 1$