Tetraquark Tcc(3875): interpretación teórica. Funciones de correlación. Molécula o tetraquark compacto?

Tetraquark Tcc(3875): How to find out its origin and properties

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Experimental information and theoretical interpretation

General method to find out the nature of a state. Application to the Tcc(3875)

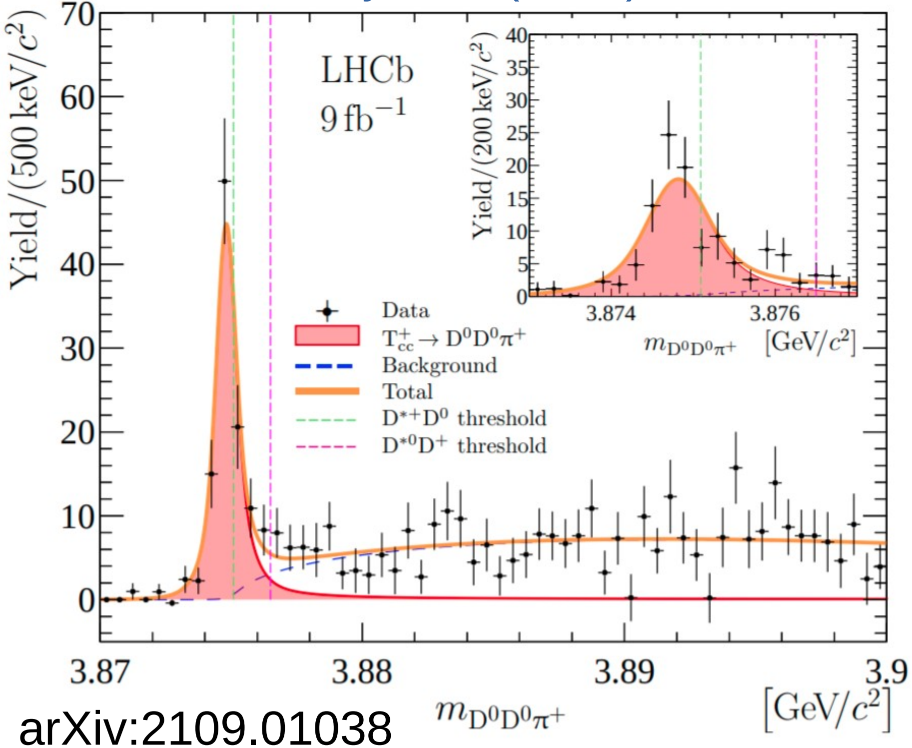
Correlation functions applied to the Tcc

Inverse problem of finding the properties of the Tcc from the correlation functions

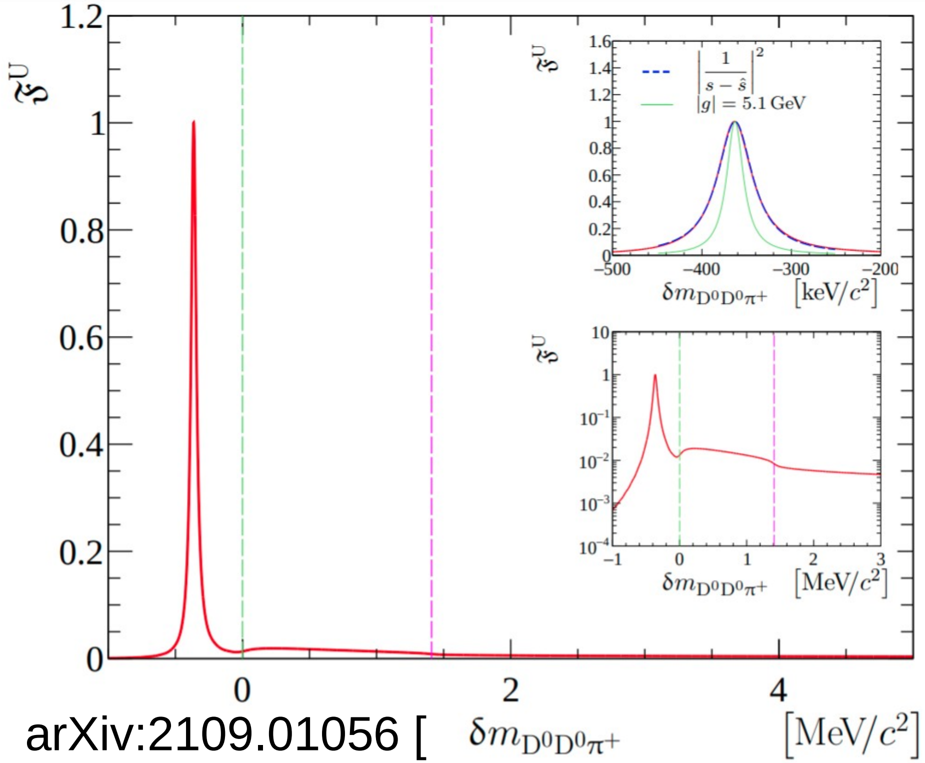
The Tcc discovery by the LHCb collaboration

Nature Phys. 18 (2022) 7, 751

Nature Commun. 13 (2022) 1, 3351



arXiv:2109.01038



arXiv:2109.01056 [

Spectra without correction by experimental resolution

$$m_{\rm exp} = 3875.09\ \text{MeV} + \delta m_{\rm exp},$$

$$\delta m_{\rm exp} = -273 \pm 61 \pm 5^{+11}_{-14}\ \text{keV}. \quad \Gamma = 410 \pm 165 \pm 43^{+18}_{-38}\ \text{keV}$$

Spectra corrected by resolution and analyzed with a unitary amplitude

$$\delta m_{\rm exp} = -360 \pm 40^{+4}_{-0}\ \text{keV} \qquad \Gamma = 48 \pm 2^{+0}_{-14}\ \text{keV}.$$

Effective theories for the interaction of hadrons.

Weinberg had the wisdom to propose an effective theory to describe the interaction at low energies between hadrons, eliminating the quarks and considering only the hadrons as elementary fields: Chiral Lagrangians

$$\mathcal{L}_2 = \frac{1}{12 f^2}\langle(\partial_\mu \Phi\Phi - \Phi\partial_\mu\Phi)^2 + M\Phi^4\rangle$$

Meson-Meson

$$\Phi \equiv \frac{\boldsymbol{\lambda}}{\sqrt{2}}\boldsymbol{\phi} = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix}$$

$$M = \begin{pmatrix} m_\pi^2 & 0 & 0 \\ 0 & m_\pi^2 & 0 \\ 0 & 0 & 2m_K^2 - m_\pi^2 \end{pmatrix},$$

With these Lagrangians one can do perturbation theory → chiral perturbation theory

However, one can use the amplitudes obtained and consider them as the potential to be used I in the Shroedinger equations (Lippmann Schwinger equation, Bethe Salpeter equation) →
Chiral unitary theory

$$H\Psi = (H_0 + V)\Psi = E\Psi \Rightarrow (E - H_0)\Psi = V\Psi$$

$$(E - H_0)\Phi = 0$$

$$\Psi = \Phi + \frac{1}{E - H_0} V\Psi \Rightarrow \Psi = \Phi + \frac{1}{E - H_0} T\Phi$$

$$T\Phi \equiv V\Psi.$$

T= V+ VGT

$$T = V + V \frac{1}{E - H} V.$$

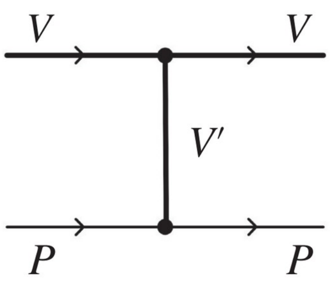
In coupled channels

$T=(1-VG)^{-1}$ V

T has a pole for eigenstates of H

VP INTERACTION IN THE LOCAL HIDDEN GAUGE APPROACH

Bando et al Phys Rep. 164



$$\mathcal{L}_{VVV} = ig\langle (V_\mu \partial_\nu V^\mu - \partial_\nu V_\mu V^\mu) V^\nu \rangle$$

$$g = M_V/2f \;\; (M_V \approx 800 \text{ MeV}, \; f = 93 \text{ MeV})$$

$$\mathcal{L}_{VPP} = -ig\langle V^\mu [P, \partial_\mu P] \rangle$$

Neglecting the k/M_V

$$\varepsilon_1(\mathbf{k}) = (0, 1, 0, 0)$$

$$\varepsilon_2(\mathbf{k}) = (0, 0, 1, 0)$$

$$\varepsilon_3(\mathbf{k}) = (|\mathbf{k}|, 0, 0, \omega_{\mathbf{k}})/m_W$$

$$-it = -g(V^\mu \partial_\nu V_\mu - \partial_\nu V_\mu V^\mu)_{ij} V^\nu_{ji} \frac{i}{q^2 - M_V^2} V^{\nu'}_{lm} [P, \partial_{\nu'} P]_{ml}$$

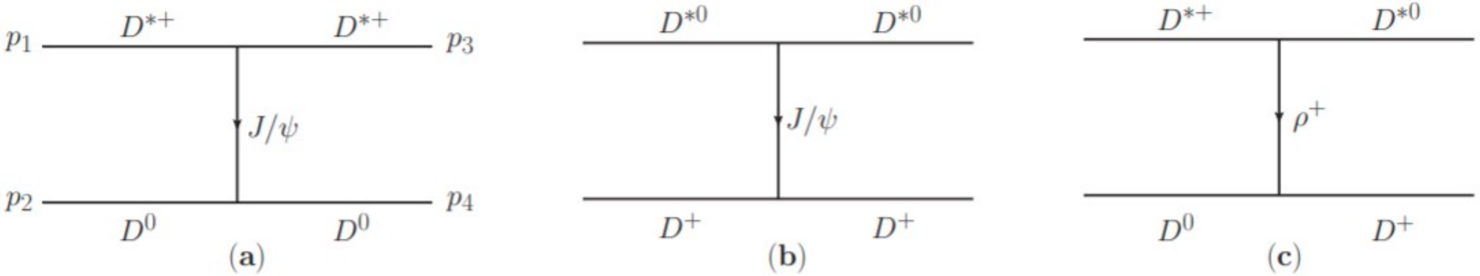
$$\sum_{pol} \epsilon^\nu_{ji} \epsilon^{\nu'}_{lm} = \left(-g^{\nu\nu'} + \frac{q^\nu q^{\nu'}}{M_V^2} \right) \delta_{jl} \delta_{im}$$

$$-it = -i \frac{g^2}{M_V^2} \langle (V^\mu \partial_\nu V_\mu - \partial_\nu V_\mu V^\mu)[P, \partial^\nu P] \rangle$$

$$\mathcal{L} = -\frac{1}{4f^2} \langle [V^\mu, \partial_\nu V^\mu][P, \partial^\nu P] \rangle$$

Chiral Lagrangian of M. C. Birse, Z. Phys. A 355, 231 (1996)

A. Feijoo, W.H. Liang, Eulogio Oset, Phys.Rev.D 104 (2021) 11, 114015



$$
\begin{aligned}
\mathcal{L}_{VPP} &= -ig\,\langle [P, \partial_\mu P] V^\mu \rangle, \\
\mathcal{L}_{VVV} &= ig\,\langle (V^\nu \partial_\mu V_\nu - \partial_\mu V^\nu V_\nu) V^\mu \rangle, \\
g &= \frac{M_V}{2\,f}, \quad (M_V = 800\ \text{MeV},\ f = 93\ \text{MeV}).
\end{aligned}
$$

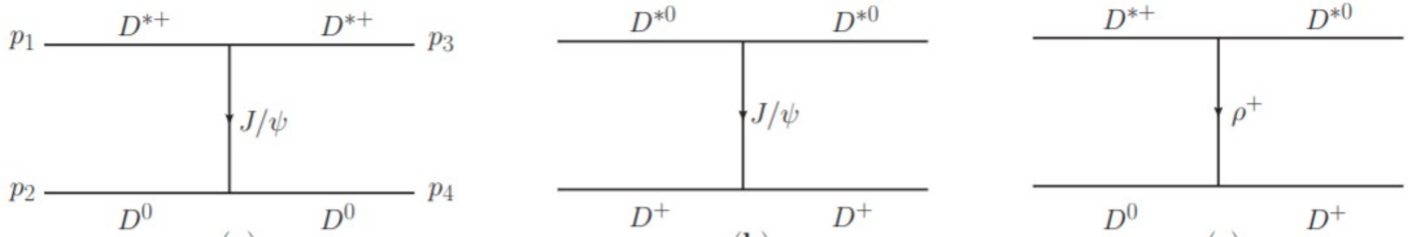
$$
P = \begin{pmatrix}
\frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} + \frac{\pi^0}{\sqrt{2}} & \pi^+ & K^+ & \bar{D}^0 \\
\pi^- & \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} - \frac{\pi^0}{\sqrt{2}} & K^0 & D^- \\
K^- & \bar{K}^0 & -\frac{\eta}{\sqrt{3}} + \sqrt{\frac{2}{3}}\eta' & D_s^- \\
D^0 & D^+ & D_s^+ & \eta_c
\end{pmatrix}
\qquad
V_\mu = \begin{pmatrix}
\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} & \bar{D}^{*0} \\
\rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} & D^{*-} \\
K^{*-} & \bar{K}^{*0} & \phi & D_s^{*-} \\
D^{*0} & D^{*+} & D_s^{*+} & J/\psi
\end{pmatrix}_\mu
$$

$D^{*+}D^0, D^{*0}D^+$ the 1, 2 channels, the interaction that we obtain is

$$
\begin{aligned}
V_{ij} &= C_{ij}\, g^2\, (p_1 + p_3)\cdot(p_2 + p_4)\, \vec{\epsilon}\cdot\vec{\epsilon}\,' \\
&\to C_{ij}\, g^2\, \frac{1}{2}[3s - (M^2 + m^2 + M'^2 + m'^2) \\
&\quad - \frac{1}{s}(M^2 - m^2)(M'^2 - m'^2)]\, \vec{\epsilon}\cdot\vec{\epsilon}\,',
\end{aligned}
$$

$$
C_{ij} = \begin{pmatrix} \frac{1}{M^2_{J/\psi}} & \frac{1}{m^2_\rho} \\ \frac{1}{m^2_\rho} & \frac{1}{M^2_{J/\psi}} \end{pmatrix}
$$

$$
T = [1 - VG]^{-1}\, V,
$$



(a) (b) (c)

$$
|D^*D, I=0\rangle = -\frac{1}{\sqrt{2}}(D^{*+}D^0 - D^{*0}D^+),
$$

$$
|D^*D, I=1, I_3=0\rangle = -\frac{1}{\sqrt{2}}(D^{*+}D^0 + D^{*0}D^+),
$$

$$
C_{00} = \frac{1}{M^2_{J/\psi}} - \frac{1}{m^2_\rho};\quad C_{11} = \frac{1}{M^2_{J/\psi}} + \frac{1}{m^2_\rho};\quad C_{01} = 0;
$$

There is attraction in I=0, repulsion in I=1, but due to different masses there is a bit of isospin breaking

Convolution of the G function:
Origin of the width.

Spectral function
Mass distribution

$$\mathrm{Im}[D(s_V)] = \mathrm{Im}\Big(\frac{1}{s_V - M_V^2 + iM_V\Gamma_V}\Big)$$

$$G(\sqrt{s}, M_k, m_k) = \frac{\int_{(M_V-2\Gamma_V)^2}^{(M_V+2\Gamma_V)^2} ds_V\, G(\sqrt{s}, \sqrt{s_V}, m_k) \times \mathrm{Im}[D(s_V)]}{\int_{(M_V-2\Gamma_V)^2}^{(M_V+2\Gamma_V)^2} ds_V\, \mathrm{Im}[D(s_V)]}$$

$$G_l = i\int \frac{d^4q}{(2\pi)^4}\frac{M_l}{E_l(\mathbf{q})}\frac{1}{k^0+p^0-q^0-E_l(\mathbf{q})+i\epsilon}$$

$$\Gamma_{D^{*+}}(M_{\rm inv}) = \Gamma(D^{*+})\left(\frac{m_{D^{*+}}}{M_{\rm inv}}\right)^2 \cdot \left[\frac{2}{3}\left(\frac{p_\pi}{p_{\pi,{\rm on}}}\right)^3 + \frac{1}{3}\left(\frac{p'_\pi}{p'_{\pi,{\rm on}}}\right)^3\right]$$

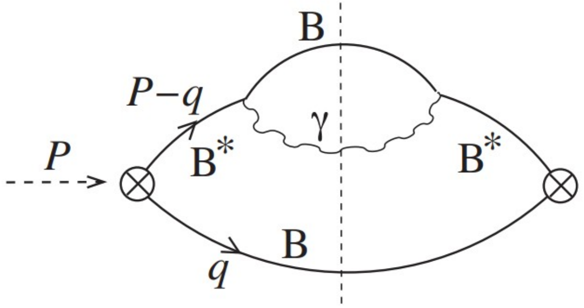
where p_π is the π^+ momentum in $D^{*+} \to D^0\pi^+$ decay

$p'_\pi, p'_{\pi,{\rm on}}$ are the same magnitudes for $D^{*+} \to D^+\pi^0$.

$$\Gamma_{D^{*0}}(M_{\rm inv}) = \Gamma(D^{*0})\left(\frac{m_{D^{*0}}}{M_{\rm inv}}\right)^2 \cdot \left[0.647\left(\frac{p_\pi}{p_{\pi,{\rm on}}}\right)^3 + 0.353\right]$$

$D^{*0} \to D^0\pi^0 \qquad D^{*0} \to D^0\gamma$

Alternative method including vector selfenergy

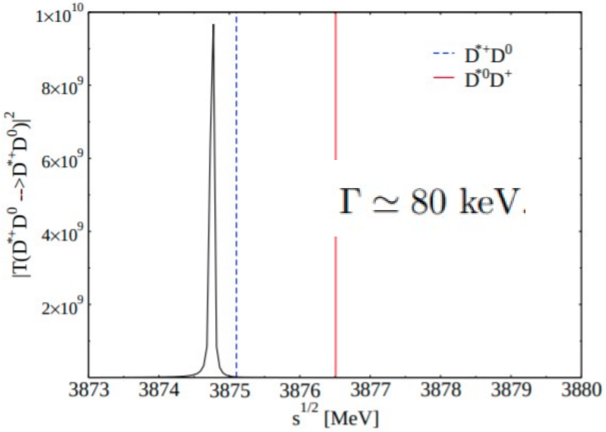


$$G(s) = i\int \frac{d^4q}{(2\pi)^4}\frac{1}{q^2-m_B^2+i\epsilon}\frac{1}{(P-q)^2-m_{B^*}^2+i\sqrt{(P-q)^2}\Gamma_{B^*}((P-q)^2)}$$

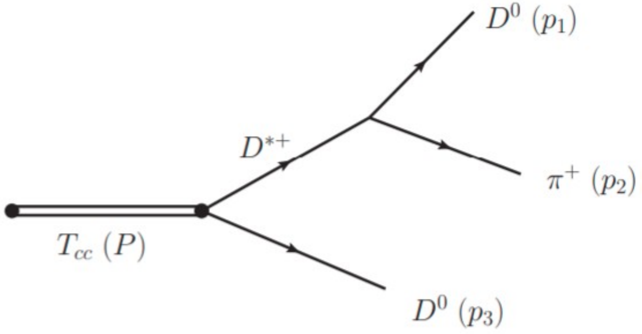
$$\Gamma_{B^*}(s') = \Gamma_{B^*}(m_{B^*}^2)\frac{m_{B^*}^2}{s'}\left(\frac{p_\gamma(s')}{p_\gamma(m_{B^*}^2)}\right)^3\Theta(\sqrt{s'}-m_B)$$

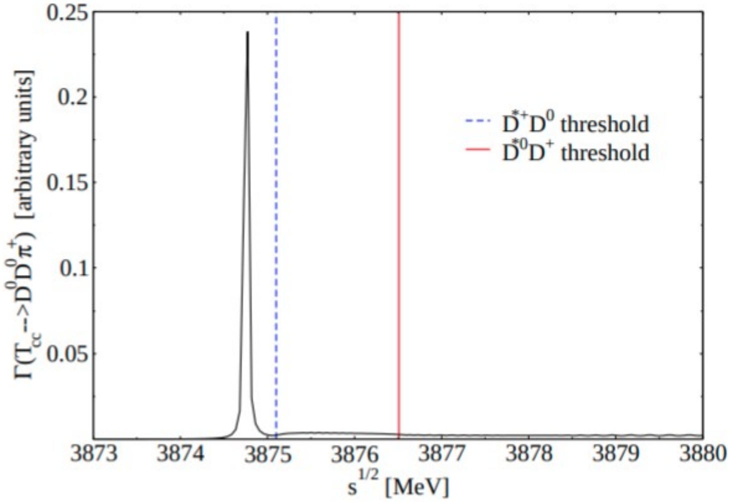
$$G(s) \simeq \int_0^{q_{\max}} dq\,\frac{q^2}{4\pi^2}\frac{\omega_B+\omega_{B^*}}{\omega_B\omega_{B^*}}\frac{1}{\sqrt{s}+\omega_B+\omega_{B^*}}$$
$$\times\frac{1}{\sqrt{s}-\omega_B-\omega_{B^*}+i\frac{\sqrt{s'}}{2\omega_{B^*}}\Gamma_{B^*}(s')},$$

$\omega_{B(B^*)} = \sqrt{\vec{q}^2+m_{B(B^*)}^2}$ and $s' = (\sqrt{s}-\omega_B)^2-\vec{q}^2$.

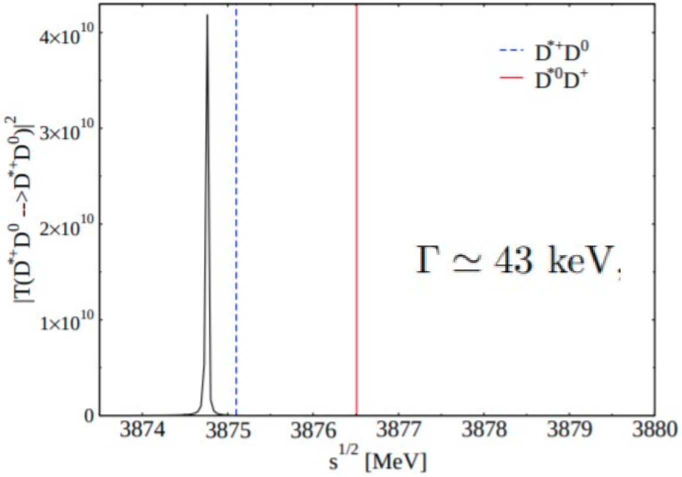


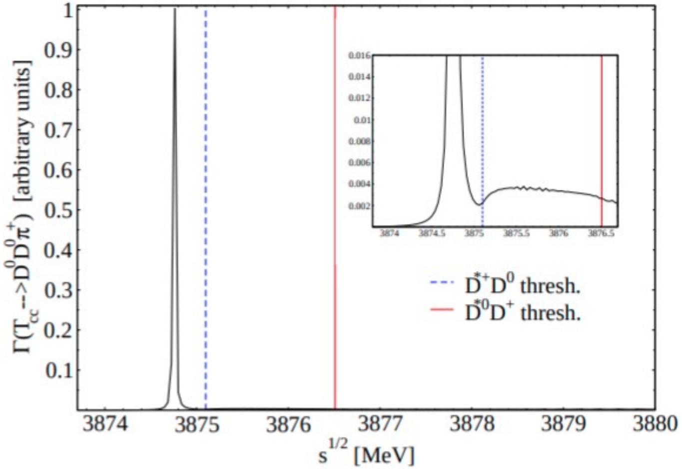
With mass of experimental raw data





With mass from unitary reanalysis of LHCb data , Mikhasenko





Compositeness of a state. Derivation of the sum rule.

$$\langle \vec{p}'|V|\vec{p}\rangle = V(\vec{p}', \vec{p}) = v\Theta(\Lambda - p)\Theta(\Lambda - p')$$

$\Lambda \equiv q_{max}$ Gives the range of the interaction In momentum space

$$T = V + V\frac{1}{E - H_0}T$$

$$\langle \vec{p}|T|\vec{p}'\rangle = \langle \vec{p}|V|\vec{p}'\rangle + \int_{k<\Lambda} d^3k \frac{\langle \vec{p}|V|\vec{k}\rangle}{E - m_1 - m_2 - \frac{\vec{k}^2}{2\mu}} \times \langle \vec{k}|T|\vec{p}'\rangle,$$

Gamermann, Nieves
E. O, Ruiz Arriola
Phys.Rev.D 81 (2010) 014029

$$\langle \vec{p}|T|\vec{p}'\rangle = \Theta(\Lambda - p)\Theta(\Lambda - p')t$$

$$t = v + vGt, \qquad t = \frac{v}{1 - vG}$$

$$G = \int_{p<\Lambda} d^3p \frac{1}{E - m_1 - m_2 - \frac{\vec{p}^2}{2\mu}}$$

Wave functions

$$(H_0 + V)|\psi\rangle = E|\psi\rangle$$

$$\langle\vec{p}|\psi\rangle = \int d^3k \int d^3k' \langle\vec{p}|\frac{1}{E-H_0}|\vec{k}'\rangle\langle\vec{k}'|V|\vec{k}\rangle\langle\vec{k}|\psi\rangle$$

$$|\psi\rangle = \frac{1}{E-H_0}V|\psi\rangle,$$

$$= v\frac{\Theta(\Lambda - p)}{E - m_1 - m_2 - \frac{\vec{p}^2}{2\mu}}\int_{k<\Lambda} d^3k\langle\vec{k}|\psi\rangle,$$

Generalization to coupled channels

$$\langle\vec{p}|\psi_i\rangle = \Theta(\Lambda - p)\frac{1}{E - M_i - \frac{\vec{p}^2}{2\mu_i}}\sum_j v_{ij}\int_{k<\Lambda} d^3k\langle\vec{k}|\psi_j\rangle,$$

$$g_i = \sum_j v_{ij}\int_{k<\Lambda} d^3k\langle\vec{k}|\psi_j\rangle$$

$$T = V + V\frac{1}{E-H}V.$$

$$\sum_i\langle\psi_i|\psi_i\rangle = \int d^3p\sum_i|\langle\vec{p}|\psi_i\rangle|^2$$

$$T_{ij} = v_{ij} + \sum_{mn} v_{im}\int_{k<\Lambda} d^3k\langle\vec{k}|\psi_m\rangle\frac{1}{E-E_\alpha} \times \int_{k'<\Lambda} d^3k'\langle\vec{k}'|\psi_n\rangle v_{nj},$$

$$\sum_i g_i^2\frac{dG_{ii}}{dE}\bigg|_{E=E_\alpha} = -1$$

$T_{ij} \approx g_i\, g_j / (E-E_\alpha)$

Around the pole

Each term with minus sign gives the probability of the corresponding channel

$$\langle \vec{p}|\psi_i\rangle = \Theta(\Lambda - p)\frac{1}{E - M_i - \frac{\vec{p}^2}{2\mu_i}}\sum_j v_{ij}\int_{k<\Lambda} d^3k\langle\vec{k}|\psi_j\rangle$$

$$\int_{p<\Lambda} d^3p\langle\vec{p}|\psi_i\rangle = G_{ii}\sum_j v_{ij}\int_{k<\Lambda} d^3k\langle\vec{k}|\psi_j\rangle$$

$G_i\, g_i = \psi_i\,(r=0)$

IMPORTANT: isospin is a symmetry of the strong interaction

Strong interaction is of short range

What matters for isospin is the wave function at the origin of a channel

Not the probability (tied to the binding of the channel)

Since G_i are similar for isospin partners, g_i is what matters for isospin

The derivation implicitly assumed that the potential is energy independent

How to account for missing channels?

Aceti, Dai, Geng, Zhang
Eur.Phys.J.A 50 (2014) 57

T. Hyodo
Int.J.Mod.Phys.A 28 (2013)1330045

$$v = \begin{pmatrix} v_{11} & v_{12} \\ v_{12} & 0 \end{pmatrix} \qquad T_{11} = \frac{v_{11} + v_{12}^2 G_2}{1 - (v_{11} + v_{12}^2 G_2) G_1}$$

$$V_{\rm eff} = v_{11} + v_{12}^2 G_2 \qquad T_{\rm eff} = \frac{V_{\rm eff}}{1 - V_{\rm eff} G_1}$$

$$g_{\rm eff}^2 = \lim \frac{(E - E_0) V_{\rm eff}}{1 - V_{\rm eff} G_1} = \frac{V_{\rm eff}}{-\frac{\partial V_{\rm eff}}{\partial E} G_1 - V_{\rm eff} \frac{\partial G_1}{\partial E}}, \tag{69}$$

which, using the pole condition $1 - V_{\rm eff} G_1 = 0$, can be rewritten as

$$g_{\rm eff}^2 = \frac{1}{-G_1^2 \frac{\partial V_{\rm eff}}{\partial E} - \frac{\partial G_1}{\partial E}}. \tag{70}$$

Probability of Channel 1

$$-g_{\rm eff}^2 G_1^2 \frac{\partial V_{\rm eff}}{\partial E} - g_{\rm eff}^2 \frac{\partial G_1}{\partial E} = 1$$

Probability of channel 2

Comes from $d\, G_2 / d\, E$

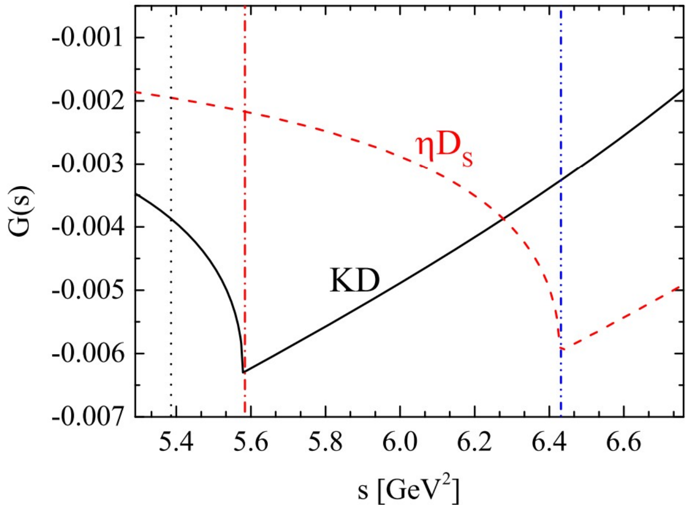
What if we have a genuine state (compact tetraquark …..) ?

In this case we should add an explicit state adding to the potential

$C_{ij} / (E- E_R)$

the energy dependence of the potential will make the sum rule smaller than 1
to accommodate the probability of having this genuine state in the wave function.

Realistic example: $D_{s0}^*(2317)$ is made from K D and η D_s channels. One can eliminate η D_s



$$V_{\text{eff}} = V_0 + \beta(s - s_0)$$

s_0 is the value of s for the bound state

To account for the possible genuine state we also allow a possible potential linear in s.

Back to the Tcc

L.~R.~Dai, L.~M.~Abreu, A.~Feijoo and E.~Oset, Eur.Phys.J.C 83 (2023) 983

$$V = \begin{pmatrix} V_{11} & V_{12} \\ V_{12} & V_{22} \end{pmatrix} \qquad T = [1 - VG]^{-1}\, V, \qquad G = \int_{|\boldsymbol{q}|<q_{\max}} \frac{d^3 q}{(2\pi)^3} \frac{\omega_1 + \omega_2}{2\,\omega_1\omega_2} \frac{1}{s - (\omega_1 + \omega_2)^2 + i\epsilon}$$

$$|D^*D, I=0\rangle \;=\; -\frac{1}{\sqrt{2}}(D^{*+}D^0 - D^{*0}D^+)\,,$$

$$|D^*D, I=1\rangle \;=\; -\frac{1}{\sqrt{2}}(D^{*+}D^0 + D^{*0}D^+)\,.$$

Assuming isospin symmetry in the potential $\longrightarrow$ $\langle I=0|V|I=1\rangle = 0$ $\longrightarrow$ $V_{11} = V_{22}$

$$\langle I=0|V|I=0\rangle \;=\; V_{11} - V_{12}\,,$$

$$\langle I=1|V|I=1\rangle \;=\; V_{11} + V_{12}\,.$$

We assume a general potential. The parameters will be fitted to data

$$V_{11} \;=\; V'_{11} + \frac{\alpha}{m_V^2}(s - s_0)\,,$$

$$V_{12} \;=\; V'_{12} + \frac{\beta}{m_V^2}(s - s_0)\,,$$

Fit to scattering lengths and effective range

two channels, $D^0 D^{*+}$ and $D^+ D^{*0}$,

$$f(s) = \frac{1}{-\frac{1}{a} + \frac{1}{2} r_0 k^2 - ik}$$

$$T(s) = -8\pi\sqrt{s} f(s)$$

$$\begin{aligned}
a_1 &= 6.134 \pm 0.51 \text{ fm}\,, \\
r_{0,1} &= -3.516 \pm 0.50 \text{ fm}\,, \\
a_2 &= (1.707 \pm 0.30) - i\,(1.07 \pm 0.30) \text{ fm}\,, \\
r_{0,2} &= (0.259 \pm 0.30) - i\,(3.769 \pm 0.30) \text{ fm}\,.
\end{aligned}$$

LHCb data in the limit of D* width zero

Courtesy of M. Mikhasenko

$$T = \frac{1}{\text{DET}} \begin{pmatrix} V_{11} + (V_{12}^2 - V_{11}^2) G_2 & V_{12} \\ V_{12} & V_{11} + (V_{12}^2 - V_{11}^2) G_1 \end{pmatrix} \; ; \; \text{DET} = 1 - V_{11}(G_1 + G_2) - (V_{12}^2 - V_{11}^2)\, G_1 G_2$$

The bound state appears when $\text{DET} = 0$ at s_0, hence

$$\text{DET}\,(s = s_0) = 0$$

Allows to write V_{12} in terms of V_{11}

We have 4 parameters and 6 data. We get a good fit

$$P_1 = -g_1^2 \frac{\partial G_1}{\partial s}\Big|_{s=s_0}, \quad P_2 = -g_2^2 \frac{\partial G_2}{\partial s}\Big|_{s=s_0} \qquad Z = 1 - P_1 - P_2$$

TABLE I: The obtained scattering lengths and effective ranges.

a_1 [fm]	$r_{0,1}$ [fm]	a_2 [fm]	$r_{0,2}$ [fm]
6.110 ± 0.065	-3.455 ± 0.194	$(1.761 \pm 0.031) - i\,(1.063 \pm 0.024)$	$(0.265 \pm 0.148) - i\,(3.760 \pm 0.142)$

TABLE II: The obtained coupling constants and probabilities.

g_1 [MeV]	g_2 [MeV]	P_1	P_2	Z
3759.88 ± 32.95	-3820.52 ± 43.61	0.685 ± 0.011	0.285 ± 0.005	0.030 ± 0.016

Recall LHCb data

$$\begin{aligned} a_1 &= 6.134 \pm 0.51 \text{ fm}, \\ r_{0,1} &= -3.516 \pm 0.50 \text{ fm}, \\ a_2 &= (1.707 \pm 0.30) - i\,(1.07 \pm 0.30) \text{ fm}, \\ r_{0,2} &= (0.259 \pm 0.30) - i\,(3.769 \pm 0.30) \text{ fm}. \end{aligned}$$

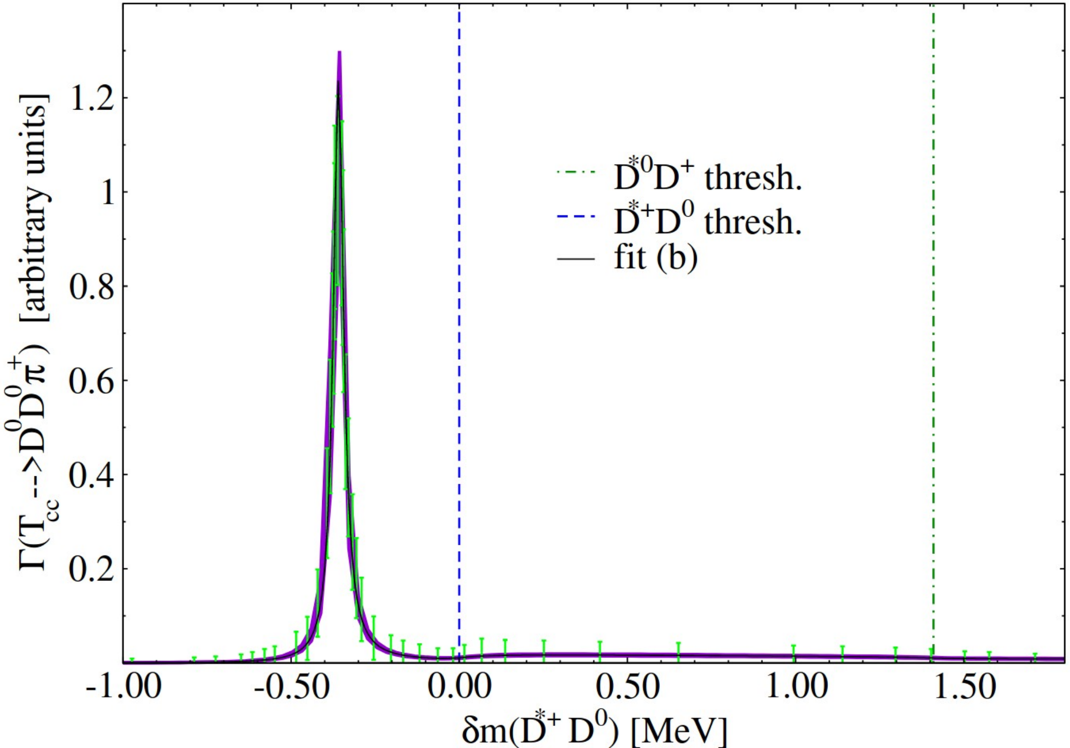
Direct fit to the $D^0D^0\pi^+$ mass distribution

There are correlations
In the parameters

make fits with $q_{\rm max} = 400, 550$ and 700 MeV

to evaluate systematic errors

$$V'_{11} = -66 \pm 67, \quad V'_{12} = 367 \pm 67, \quad \alpha = 0 \pm 68, \quad \beta = 0 \pm 55.$$



The precise values of the parameters do not matter, because of existing correlations.

We use the resampling
or bootstrap method. Many fits with random centroids of data. In each of them the observables are evaluated, and averages and dispersions are calculated.

	a_i [fm]	$r_{0,i}$ [fm]
$i=1$ $(D^{*+}D^0)$	$(7.60 \pm 0.13(\pm 0.13)) - i\,(1.73 \pm 0.06(\pm 0.06))$	$-2.94 \pm 0.03(\pm 0.27)$
$i=2$ $(D^{*0}D^+)$	$(1.99 \pm 0.06(\pm 0.06)) - i\,(1.25 \pm 0.05(\pm 0.06))$	$(0.11 \pm 0.12(\pm 0.28)) - i\,(2.74 \pm 0.19(\pm 0.21))$

B [KeV]	Γ [KeV]	g_1 [MeV]	g_2 [MeV]	P_1	P_2
$360 \pm 2(\pm 1)$	$38 \pm 2(\pm 1)$	$3875 \pm 50(\pm 53)$	$-4077 \pm 63(\pm 59)$	$0.697 \pm 0.017(\pm 0.008)$	$0.301 \pm 0.009(\pm 0.007)$

$$a_1^{exp} = [(7.16 \pm 0.51) - i\,(1.85 \pm 0.28)]\ \text{fm}\,, \quad a_2^{exp} = (1.76 - i\,1.82)\ \text{fm}$$

$$P_1 + P_2 = 0.998 \pm 0.025(\pm 0.0004)$$

The Tcc is then a molecular state, consistent with 100% molecular probability with 3% uncertainty.

Isospin of the Tcc:

$$|D^*D, I=0\rangle = -\frac{1}{\sqrt{2}}(D^{*+}D^0 - D^{*0}D^+),$$
$$|D^*D, I=1\rangle = -\frac{1}{\sqrt{2}}(D^{*+}D^0 + D^{*0}D^+).$$

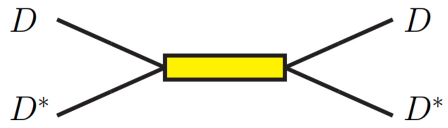
$$g_1 = 3854\ \mathrm{MeV}, \quad g_2 = -4119\ \mathrm{MeV}$$

This means that the state that we have found has I=0 very approximately

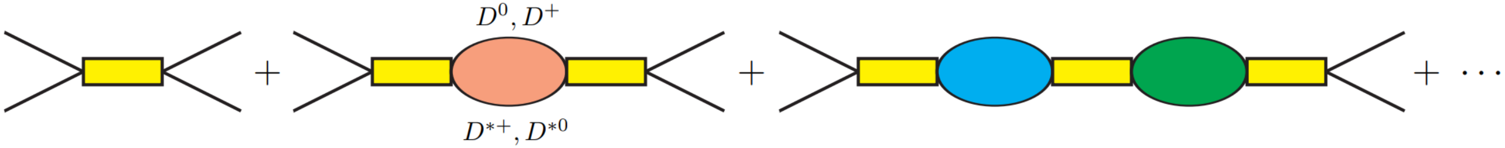
LIMITING CASE OF A NONMOLECULAR STATE

Assume I=0 single channel

$$\widetilde{T}_{D^*D,D^*D} = \frac{g^2}{s-s_0}$$



Unitarity demands



$$T_{D^0D^{*+},D^0D^{*+}} = \frac{1}{2}\,\frac{g^2}{s-\widetilde{s}_0-\frac{1}{2}g^2G_{D^0D^{*+}}-\frac{1}{2}g^2G_{D^+D^{*0}}}\,.$$

Genuine state in the limit of g → 0 $\qquad a\to 0;\quad a_1\to 0\,,\quad a_2\to 0$

$$\begin{aligned}
\frac{1}{2}\,r_{0,1} &= \lim_{g^2\to 0} -\frac{\sqrt{s_1}}{\mu_1}\frac{16\pi}{g^2}\frac{\partial}{\partial s}\left\{s^{1/2}(s-s_0)-O(g^2)\right\}\\
&= \lim_{g^2\to 0} -\frac{\sqrt{s_1}}{\mu_1}\frac{16\pi}{g^2}\frac{1}{2\sqrt{s}}(3s-s_0)\Big|_{s=s_1}\to -\infty\,,\\
\frac{1}{2}\,r_{0,2} &= \lim_{g^2\to 0} -\frac{\sqrt{s_2}}{\mu_2}\frac{16\pi}{g^2}\frac{1}{2\sqrt{s}}(3s-s_0)\Big|_{s=s_2}\to -\infty\,.
\end{aligned}$$

Striking discrepancy with the data

If g is not so small, make an expansion in s-s_0 and apply the former method

L.~R.~Dai, J.~Song and E.~Oset, Phys.Lett.B 843 (2023) 138200
%``Evolution of genuine states to molecular ones: The $T_{cc}(3875)$ case,''

Take one channel of I=0.

$$\tilde{t}_{DD^*,DD^*}(s) = \frac{\tilde{g}^2}{s - s_R}$$

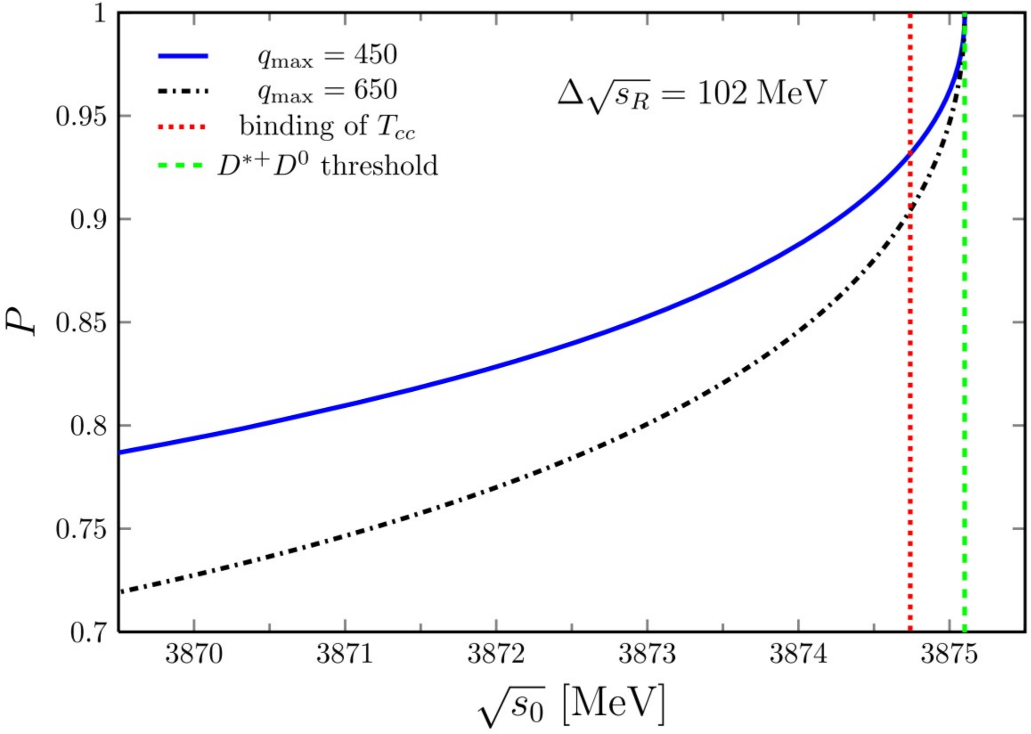
$$t_{DD^*,DD^*}(s) = \frac{\tilde{g}^2}{s - s_R - \tilde{g}^2 G_{DD^*}(s)}$$

$$g^2 = \lim_{s\to s_0} (s - s_0) \frac{\tilde{g}^2}{s - s_R - \tilde{g}^2 G_{DD^*}(s)} = \frac{\tilde{g}^2}{1 - \tilde{g}^2 \frac{\partial G}{\partial s}}\Big|_{s=s_0}$$

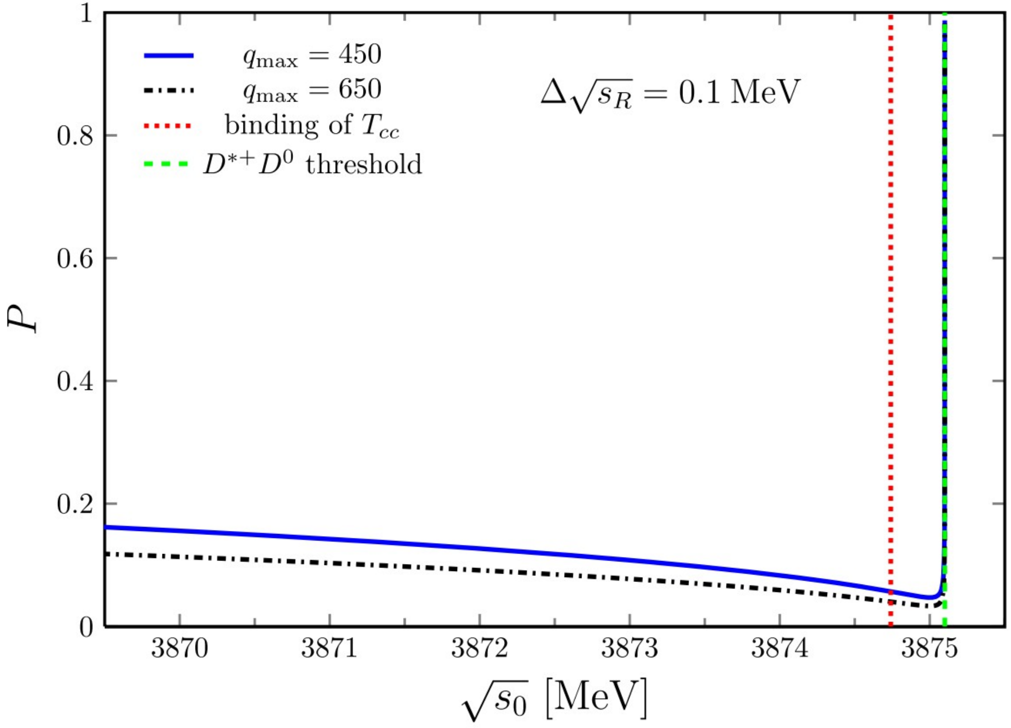
$$P = -g^2 \frac{\partial G}{\partial s}\Big|_{s=s_0}$$

$$P = -\frac{\tilde{g}^2 \frac{\partial G}{\partial s}}{1 - \tilde{g}^2 \frac{\partial G}{\partial s}}\Big|_{s=s_0}$$

$$\begin{cases} \tilde{g}^2 \to 0\,, & P \to 0\,, \quad \text{the genuine state survives} \\ \tilde{g}^2 \to \infty\,, & P \to 1\,, \quad \text{the state becomes pure molecular} \\ s_0 \to s_{\text{th}}\,, & P \to 1\,, \quad \text{the state becomes pure molecular} \end{cases}$$



$q_{\rm max} = 450$
$q_{\rm max} = 650$
binding of T_{cc}
$D^{*+}D^0$ threshold
$\Delta\sqrt{s_R} = 102$ MeV
P
$\sqrt{s_0}$ [MeV]



$q_{\rm max} = 450$
$q_{\rm max} = 650$
binding of T_{cc}
$D^{*+}D^0$ threshold
$\Delta\sqrt{s_R} = 0.1$ MeV
P
$\sqrt{s_0}$ [MeV]

Table 1
The obtained scattering length and effective range.

$\Delta\sqrt{s_R}$ [MeV]	$q_{max} = 450$ MeV		$q_{max} = 650$ MeV	
	a [fm]	r_0 [fm]	a [fm]	r_0 [fm]
0.1	0.87	-114.07	0.61	-168.39
0.3	1.19	-79.33	0.85	-117.23
1	2.10	-38.20	1.56	-56.68
2	3.04	-21.77	2.36	-32.49
5	4.62	-9.26	3.85	-14.07
10	5.74	-4.51	5.07	-7.08
30	6.94	-1.16	6.54	-2.14
50	7.25	-0.47	6.95	-1.13
70	7.39	-0.17	7.15	-0.69
102	7.51	0.06	7.31	-0.34

Experiment

$$a \sim 6 - 7 \text{ fm}, \; r_0 \sim -3.9 \text{ fm}$$

Striking discrepancy with experiment if we force the state to be nonmolecular

Femtoscopic correlation functions

In heavy ion collisions or p p collisions one observes pairs of particles and defines the correlation function as the ratio of probabilities to see the pair to the product of observing Individualy each particle. Under certain assumptions one finds

$$C(\vec{p}) = \int d^3\vec{r} S_{12}(\vec{r}) |\Psi(\vec{r},\vec{p})|^2$$

$$S_{12}(\vec{r}) = S_{12}(r) = \frac{1}{(\sqrt{4\pi})^3 R^3} \exp\left(-\frac{r^2}{4R^2}\right)$$

$$C(\vec{p}) = 1 + 4\pi \int_0^{+\infty} dr r^2 S_{12}(r) \times \left(\sum_j w_j |\widetilde{\Psi}_j(\vec{r},\vec{p})|^2 - j_0^2(pr) \right)$$

$$\Psi_j(\vec{r},\vec{p}) = \delta_{ij} j_0(pr) + T_{ji}(E)\theta(q_{\text{max}} - |\vec{p}\,|) \times \int_{|\vec{q}\,|<q_{\text{max}}} d^3\vec{q} \frac{j_0(qr)}{E - \omega_1^{(j)}(q) - \omega_2^{(j)}(q) + i\eta}$$

For the case of the observation of channel i

Particular case of the Tcc

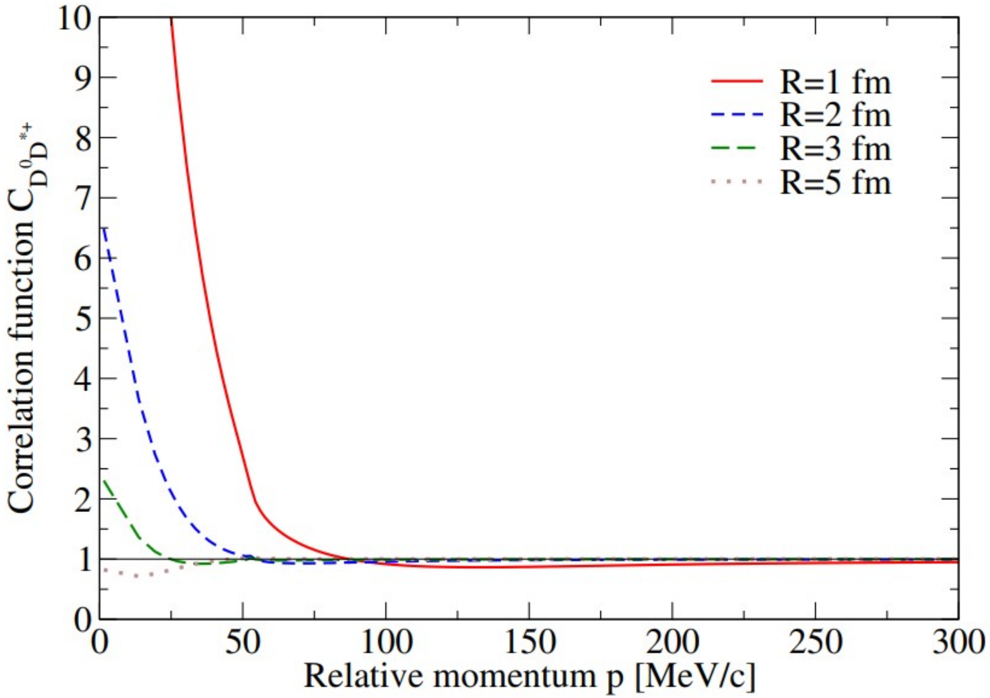
$$C_{D^0D^{*+}}(p_{D^0}) = 1 + 4\pi\theta(q_{\max} - p_{D^0})\times \int_0^{+\infty} dr r^2 S_{12}(r)\Big\{\big|j_0(p_{D^0}r) + T_{11}(E)\widetilde{G}^{(1)}(r;E)\big|^2 + \big|T_{21}(E)\widetilde{G}^{(2)}(r;E)\big|^2 - j_0^2(p_{D^0}r)\Big\}$$

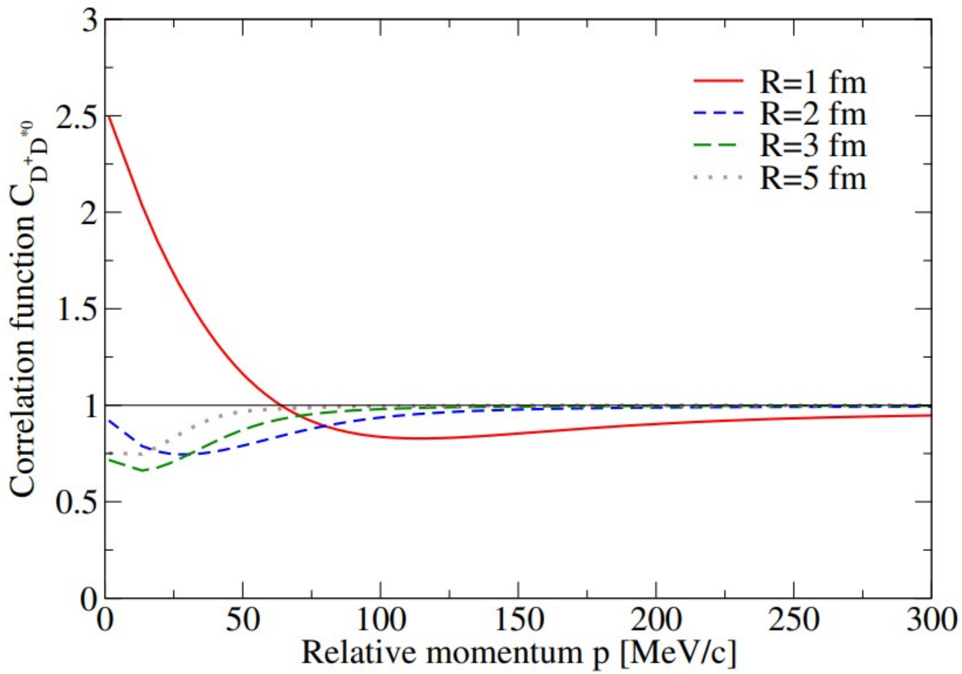
$$C_{D^+D^{*0}}(p_{D^+}) = 1 + 4\pi\theta(q_{\max} - p_{D^+})\times \int_0^{+\infty} dr r^2 S_{12}(r)\Big\{\big|j_0(p_{D^+}r) + T_{22}(E)\widetilde{G}^{(2)}(r;E)\big|^2 + \big|T_{12}(E)\widetilde{G}^{(1)}(r;E)\big|^2 - j_0^2(p_{D^+}r)\Big\}$$

$$\widetilde{G}^{(i)}(r;E) = \int_{|\vec{q}|<q_{\max}} \frac{d^3\vec{q}}{(2\pi)^3}\,\frac{\omega_1^{(i)}(q)+\omega_2^{(i)}(q)}{2\omega_1^{(i)}(q)\,\omega_2^{i}(q)} \times \frac{j_0(qr)}{s-\left[\omega_1^{(i)}(q)+\omega_2^{(i)}(q)\right]^2+i\eta}$$

I~Vidana, A.~Feijoo, M.~Albaladejo, J.~Nieves and E.~Oset 2303.06079

Y. Kamiya, T Hyodo and A. Ohnishi Eur.Phys.J.A 58 (2022) 7, 131





Note: the correlation functions start from threshold of each channel. Can we induce the Tcc and its properties from there, if these c. f. are measured?

Inverse problem

We assume we have data for the correlation functions and we carry a fit to the data with

V_{11}, V_{12}, α, β, q_{max}, R . Once we determine the parameters we can calculate everything

THE METHOD WORKS.   IT OPENS A NEW WINDOW TO INVESTIGATE HADRON HADRON INTERACTIONS FOR CASES WHERE THE FINAL STATE CANNOT BE PRODUCED OTHERWISE.

$\chi 2$ =0.0005, q_{max}=400.1 MeV,
V11=34.51 MeV, V12=490.00 MeV, α= 103.8, β=50.61, R=1.01 fm

a1,a2
(7.54 -i 1.98) fm   (2.07 -i 1.24)   fm
r01,r02
(-2.94043541,0.00000000) fm   (-6.091799214E-02,-2.41491890) fm
p1, p2
(0.699283779,-0.00000000)   (0.312470883,-0.00000000)

gcoup1,gcoup2
(3889.33765,0.00000000) MeV   (-4170.44043,0.00000000) MeV

The values of the couplings, very similar and of opposite sign, indicate that one has an I=0 state.

Error analysis of the magnitudes obtained is being done.
Errors of a few per cent. The most important: P_1=0.684+- 0.027, P_2=0.305+- 0.013
P_1+P_2= 0.989+-0.03, R=0.998+- 0.019 (the input was R=1)

It is remarkable that the fits also provide the value of R, not only the parameters related to the interaction.

Applications to KD scattering done in Z.~W.~Liu, J.~X.~Lu and L.~S.~Geng
%``Study of the DK interaction with femtoscopic correlation functions,''
Phys. Rev. D \textbf{107}, no.7, 074019 (2023)

M.~Albaladejo, J.~Nieves and E.~Ruiz-Arriola,
%``Femtoscopic signatures of the lightest S-wave scalar open-charm mesons,''
[arXiv:2304.03107 [hep-ph]].

The inverse problem is done in: N. Ikeno , G. Toledo , E. Oset From correlation functions one can induce that there is a bound state corresponding to the Ds*(2317). e-Print: 2305.16431

Conclusions:

We have studied the Tcc(3875) and found it to be well reproduced by the interaction of the $D^0 D^{*+}$, $D^+ D^{*0}$ channels, obtained from the local hidden gauge approach, forming a molecular state of two mesons.

We have studied the inverse problem of determining the interaction of the $D^0 D^{*+}$, $D^+ D^{*0}$ Channels from the experimental data. Conclusions from this analysis
The Tcc has I=0 and is of molecular nature, with $P_{D0\ D^{*+}} \approx 70$ %, $P_{D+\ D^{*0}} \approx 30\%$.

We study also the femtoscopic correlation functions for these two channels and exploit the Inverse problem of getting the Tcc and its nature from the correlation functions. It works !! One can even get the size parameter of the source.

The success of this study opens the door to use the correlation functions for systems that cannot be studied otherwise. From data above threshold of the channels on can induce the existence of bound states by 50 MeV or more, not just close the threshold, and the nature of these states.