Evolution of genuine states to molecular states with one channel/coupled channels: $T_{cc}(3875)/X(3872)$

Jing Song（宋 晶）

In Collaboration with: Prof. L-R.Dai, Prof. E.Oset

School of Physics, Beihang University
Departamento de Física Teórica and IFIC, Universidad de Valencia-CSIC

Outline

- **Introduction**
- **Theoretical formalism**
- **Results and discussion**
- **Summary**

Outline

- **Introduction**
- **Theoretical formalism**
- **Results and discussion**
- **Summary**

Introduction

- Motivation: What is the component of $T_{cc}(3875)/X(3872)$?

- $T_{cc}(3875)$:
  1. molecular state of DD^*
     - *A. Feijoo et al, PRD 104 (2021) 114015*
     - *M. Albaladejo, PLB 829 (2022) 137052, EPJC 82 (2022) 724, ...*
  2. compact tetraquark nature
     - *J. Carlson et al, PRD 37 (1988) 744, T. Guo et al, PRD 105 (2022) 014021*
     - *...*
  3. a mixture of both above components
     - *M.P. Valderrama,et al, PRD 105 (2022) 014007,*
     - *D. Janc et al, Few-Body Syst. 35 (2004) 175*
- $X(3872)$:
  1. molecular state of $\bar{D}^0 D^{*0}$& $D^- D^{*+}$
     - *D. Gamermann et al, PRD 81, 014029 (2010), E. Braaten, et al 103, 036014(2021)*
     - *M.Z. Liu , et al, EPJC 81, 179 (2021),L. Meng, et al, Sci. Bull. 66, 1288(2021), ,...*
  2. compact tetraquark state
     - *P.-P. Shi, et al, PRD 103, 094038 (2021), A. Esposito, et al, PRD 105, L031503 (2022)*
     - *C. Chen, et al, EPJC 83, 52 (2023), ...*
  3. mixture …

- Purpose: How to identify the compositeness of $T_{cc}(3875)/X(3872)$?

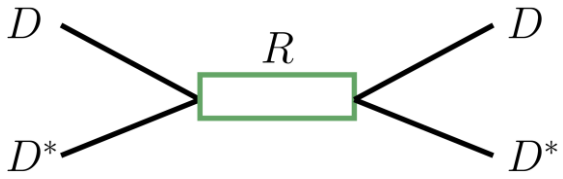
- Couplings and Probabilities: stressing the molecular nature of a state
- Scattering length and effective range: determining the compositeness of a state

Outline

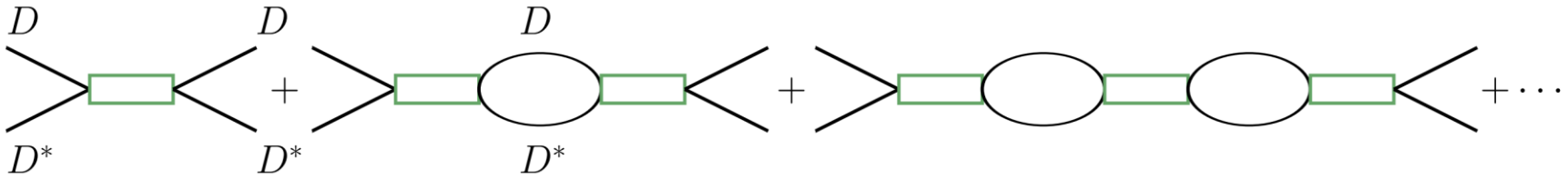
- Introduction
- **Theoretical formalism**
  1. One channel of DD^* **($T_{cc}(3875)$)**
  2. Couple channels of $\bar{D}^0 D^{*0}$ & $D^- D^{*+}$ **($X(3872)$)**

- *DD** amplitude

$$\tilde{t}_{DD^*,DD^*}(s) = \frac{\tilde{g}^2}{s - s_R}$$



- Iterated diagram of unitarity of the *DD** amplitude



- The *DD** amplitude insert the self-energy

$$t_{DD^*,DD^*}(s) = \frac{\tilde{g}^2}{s - s_R - \tilde{g}^2 G_{DD^*}(s)}$$

- Function *G* is regularized with a sharp cutoff,

$$G_{DD^*}(s) = \int_{|\boldsymbol{q}|<q_{\max}} \frac{d^3q}{(2\pi)^3} \frac{\omega_1 + \omega_2}{2\,\omega_1\,\omega_2} \frac{1}{s - (\omega_1 + \omega_2)^2 + i\epsilon}$$

- Function G is regularized with a sharp cutoff,

$$G_{DD^*}(s) = \int_{|\boldsymbol{q}|<q_{\max}} \frac{d^3q}{(2\pi)^3} \frac{\omega_1+\omega_2}{2\,\omega_1\,\omega_2} \frac{1}{s-(\omega_1+\omega_2)^2+i\epsilon}$$

- The molecular probability

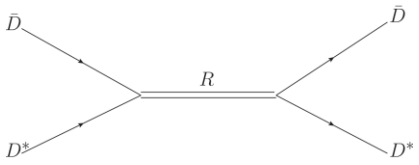
$$P = -\frac{\widetilde{g}^2 \frac{\partial G}{\partial s}}{1-\widetilde{g}^2 \frac{\partial G}{\partial s}}\Big|_{s=s_0}$$

◆ Several limits of molecular probability

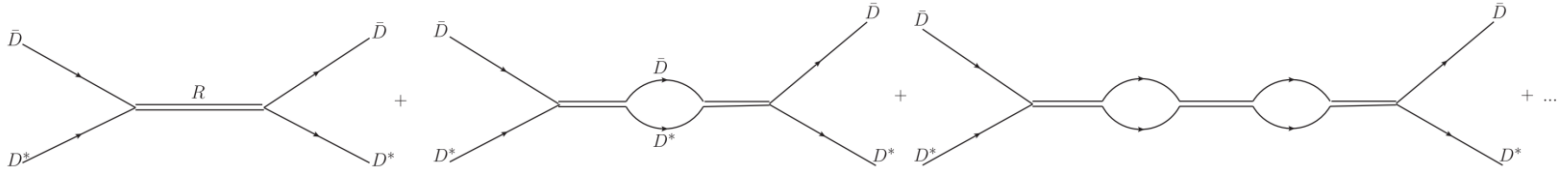
$$\begin{cases} \tilde{g}^2 \to 0, & P \to 0, & \text{the genuine state survives} \\ \tilde{g}^2 \to \infty, & P \to 1, & \text{the state becomes pure molecular} \\ s_0 \to s_{\text{th}}\,, & \frac{\partial G}{\partial s} \to -\infty, & P \to 1, \quad \text{the state becomes pure molecular} \end{cases}$$

- $\bar{D}D^*$ amplitude

$$t_{D^*\bar{D}}(I=0) = \frac{\tilde{g}^2}{s - s_R}$$



- Iterated diagram of unitarity of the $\bar{D}D^*$ amplitude



- The $\bar{D}D^*$ amplitude insert the self-energy

$$T = \tilde{V}_R + \tilde{V}_R G \tilde{V}_R + \tilde{V}_R G \tilde{V}_R G \tilde{V}_R + \cdots = \tilde{V}_R + \tilde{V}_R G T$$

$$\tilde{V}_R = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \frac{\tilde{g}^2}{s - s_R} \equiv \begin{pmatrix} \frac{1}{2}V_R & \frac{1}{2}V_R \\ \frac{1}{2}V_R & \frac{1}{2}V_R \end{pmatrix}, \qquad \text{with } V_R = \frac{\tilde{g}^2}{s - s_R}.$$

- Function G is regularized with a sharp cutoff,

$$G_i(s) = \int_{|\mathbf{q}|<q_{\max}} \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{w_1^{(i)} + w_2^{(i)}}{2w_1^{(i)} w_2^{(i)}} \times \frac{1}{s - (w_1^{(i)} + w_2^{(i)})^2 + i\epsilon}$$

- The diagonal G-loop matrix

$$G = \begin{pmatrix} G_{D^{*0}\bar{D}^0} & 0 \\ 0 & G_{D^{*+}D^-} \end{pmatrix}$$

- The molecular probability

$$P_1 = -g_1^2 \left.\frac{\partial G_1}{\partial s}\right|_{s_0} = -\left.\frac{\frac{1}{2}\tilde{g}^2 \frac{\partial G_1}{\partial s}}{1 - \frac{1}{2}\tilde{g}^2 \frac{\partial}{\partial s}(G_1 + G_2)}\right|_{s_0},$$

$$P_2 = -g_2^2 \left.\frac{\partial G_2}{\partial s}\right|_{s_0} = -\left.\frac{\frac{1}{2}\tilde{g}^2 \frac{\partial G_2}{\partial s}}{1 - \frac{1}{2}\tilde{g}^2 \frac{\partial}{\partial s}(G_1 + G_2)}\right|_{s_0}.$$

◆ Several limits of molecular probability at $s_0 \to s_{\rm th1}$

$$\begin{cases} \tilde{g}^2 \to 0, & P_1 \to 0,\ P_2 \to 0, & \text{genuine state} \\ \tilde{g}^2 \to \infty, & P_1 + P_2 = 1, & \text{completely molecular} \\ s_0 \to s_{\rm th1}, & \frac{\partial G_1}{\partial s} \to -\infty, \quad \frac{\partial G_2}{\partial s} \to \text{finite}, & P_1 \to 1, P_2 \to 0, \text{ completely molecular state} \end{cases}$$

$\bar{D}^0 D^{*0}$

- The relationship of *T*-matrix and scattering matrix in Quantum mechanics

$$T = (-8\pi\sqrt{s}) f^{QM} \simeq (-8\pi\sqrt{s}) \frac{1}{-\frac{1}{a} + \frac{1}{2} r_0 k^2 - ik}$$

- $\bar{D}D^*$ amplitude

$$T_{jj} = \frac{1}{\frac{s-s_R}{\frac{1}{2}[\tilde{g}^2 + \beta V(s-s_R)]} - G_1 - G_2}, \qquad j = 1, 2$$

- Scattering length and effective range of couple channel

$$-\frac{1}{a_1} = (-8\pi\sqrt{s}) \left[\frac{s - s_R}{\frac{1}{2}[\tilde{g}^2 + \beta V(s - s_R)]} - \mathrm{Re}G_1 - G_2 \right] \Bigg|_{s_{th1}},$$

$$r_{0,1} = 2\frac{\sqrt{s}}{\mu_1} \frac{\partial}{\partial s} \left\{ (-8\pi\sqrt{s}) \left[\frac{s - s_R}{\frac{1}{2}[\tilde{g}^2 + \beta V(s - s_R)]} - \mathrm{Re}G_1 - G_2 \right] \right\} \Bigg|_{s_{th1}},$$

$$-\frac{1}{a_2} = (-8\pi\sqrt{s}) \left[\frac{s - s_R}{\frac{1}{2}[\tilde{g}^2 + \beta V(s - s_R)]} - \mathrm{Re}G_2 - G_1 \right] \Bigg|_{s_{th2}},$$

$$r_{0,2} = 2\frac{\sqrt{s}}{\mu_2} \frac{\partial}{\partial s} \left\{ (-8\pi\sqrt{s}) \left[\frac{s - s_R}{\frac{1}{2}[\tilde{g}^2 + \beta V(s - s_R)]} - \mathrm{Re}G_2 - G_1 \right] \right\} \Bigg|_{s_{th2}},$$

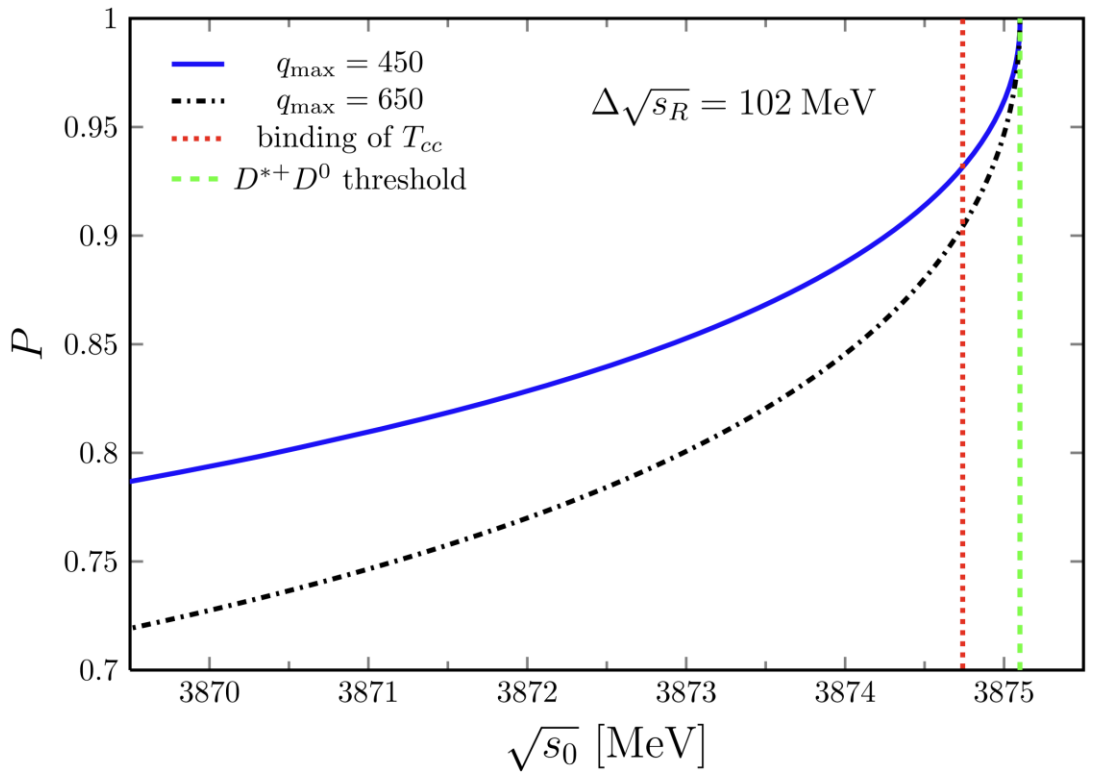
Outline

- Introduction
- Theoretical Formalism
- **Results and discussion**

1. Molecular probability
2. Scattering length and effective range
3. Mixture of compact and molecular components

Results| Molecular probability for different $s_R (= \sqrt{s}_{\text{th}} + \Delta\sqrt{s}_R)$

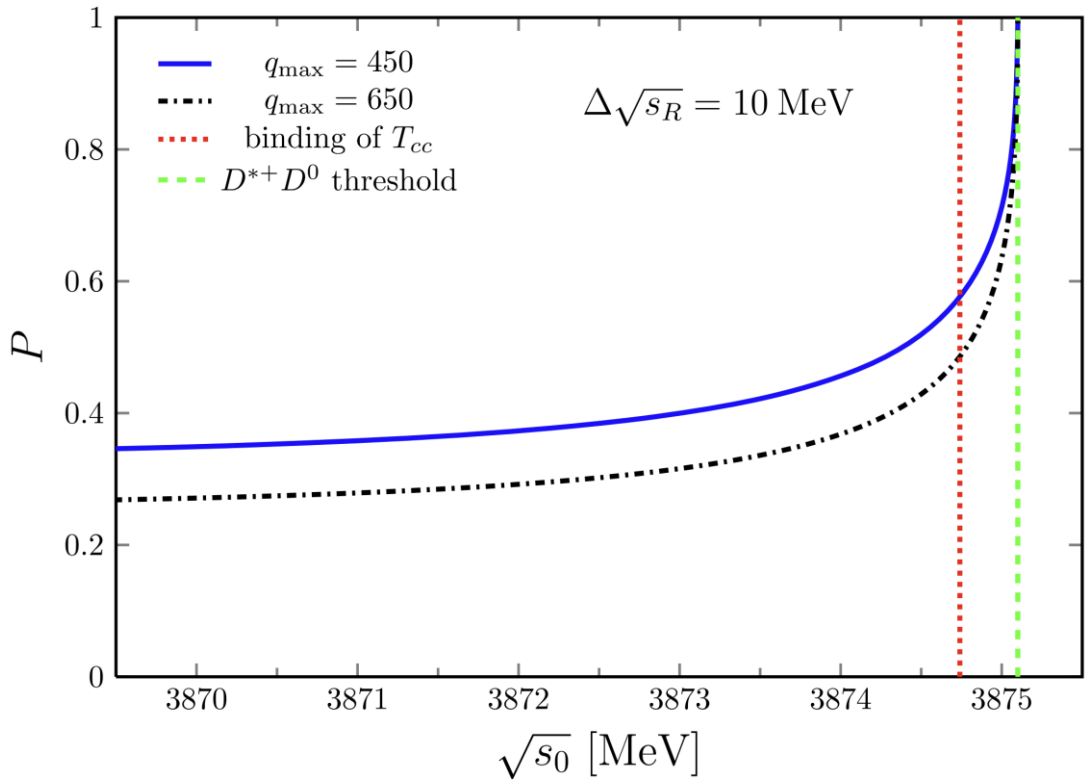
- P: molecular probability as a function of $\sqrt{s}_0$ at $\Delta\sqrt{s}_R = 102$ MeV



1. P goes to 1 at $s_0 \rightarrow s_{\text{th}}$;
2. P (~0.9) at T_{cc} depends a bit of $q_{\max}$: **genuine state → molecular state**.

Results| Molecular probability for different $s_R (= \sqrt{s}_{\text{th}} + \Delta\sqrt{s}_R)$

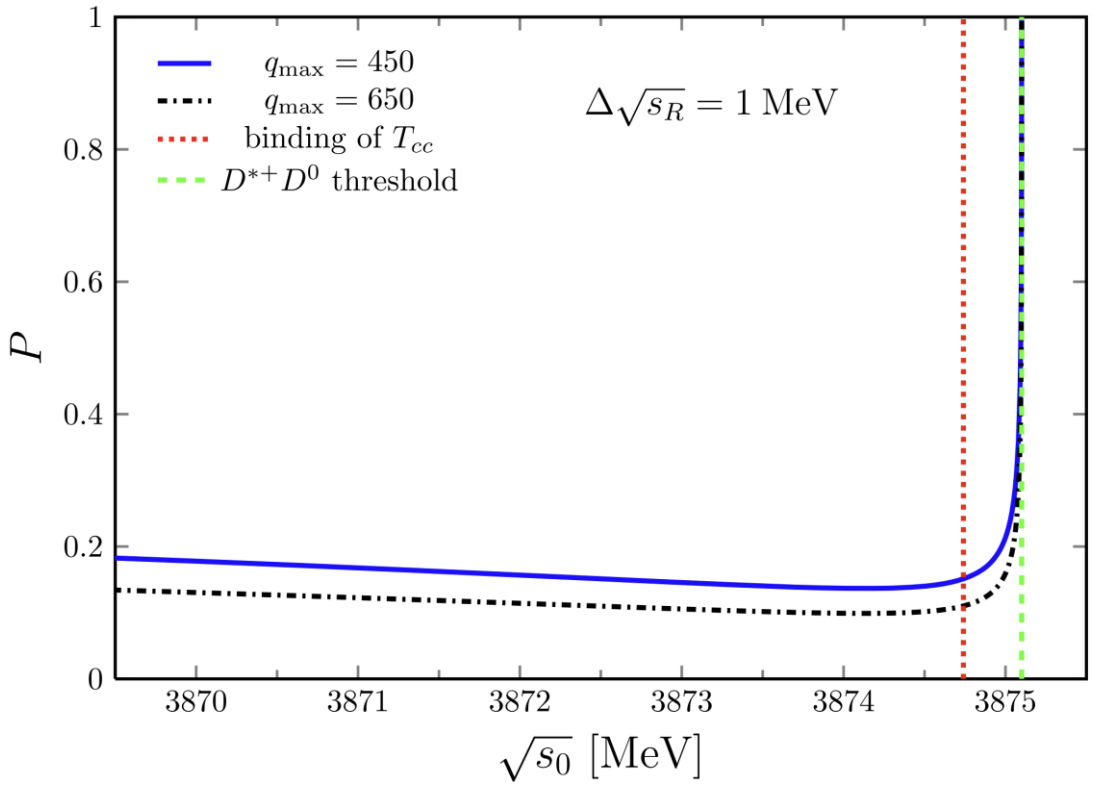
- P: molecular probability as a function of $\sqrt{s}_0$ at $\Delta\sqrt{s}_R = 10$ MeV



1. P goes to 1 at $s_0 \rightarrow s_{\text{th}}$;
2. The value P at T_{cc} smaller than before, of the order of 0.5.

Results| Molecular probability for different $s_R (= \sqrt{s}_{\text{th}} + \Delta\sqrt{s}_R)$

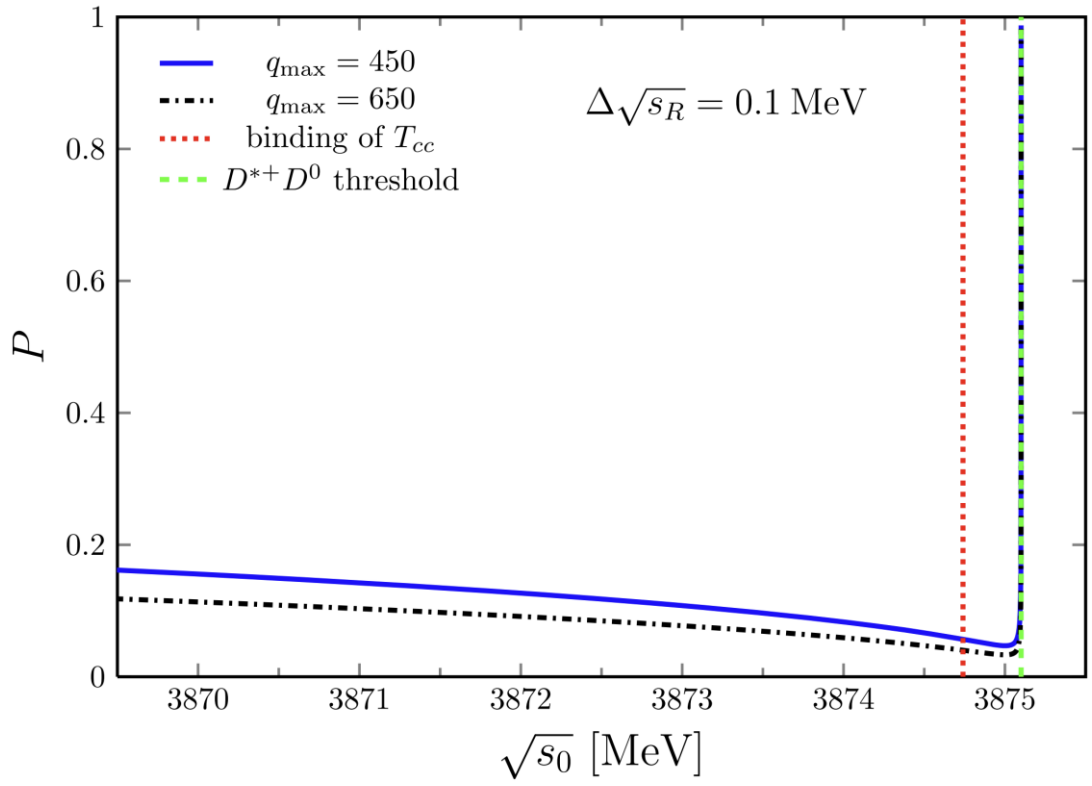
- P: molecular probability as a function of $\sqrt{s}_0$ at $\Delta\sqrt{s}_R = 1$ MeV



1. P shows the same trend at $s_0 \rightarrow s_{\text{th}}$;
2. The value P at T_{cc} smaller than 0.15 : **mostly nonmolecular.**

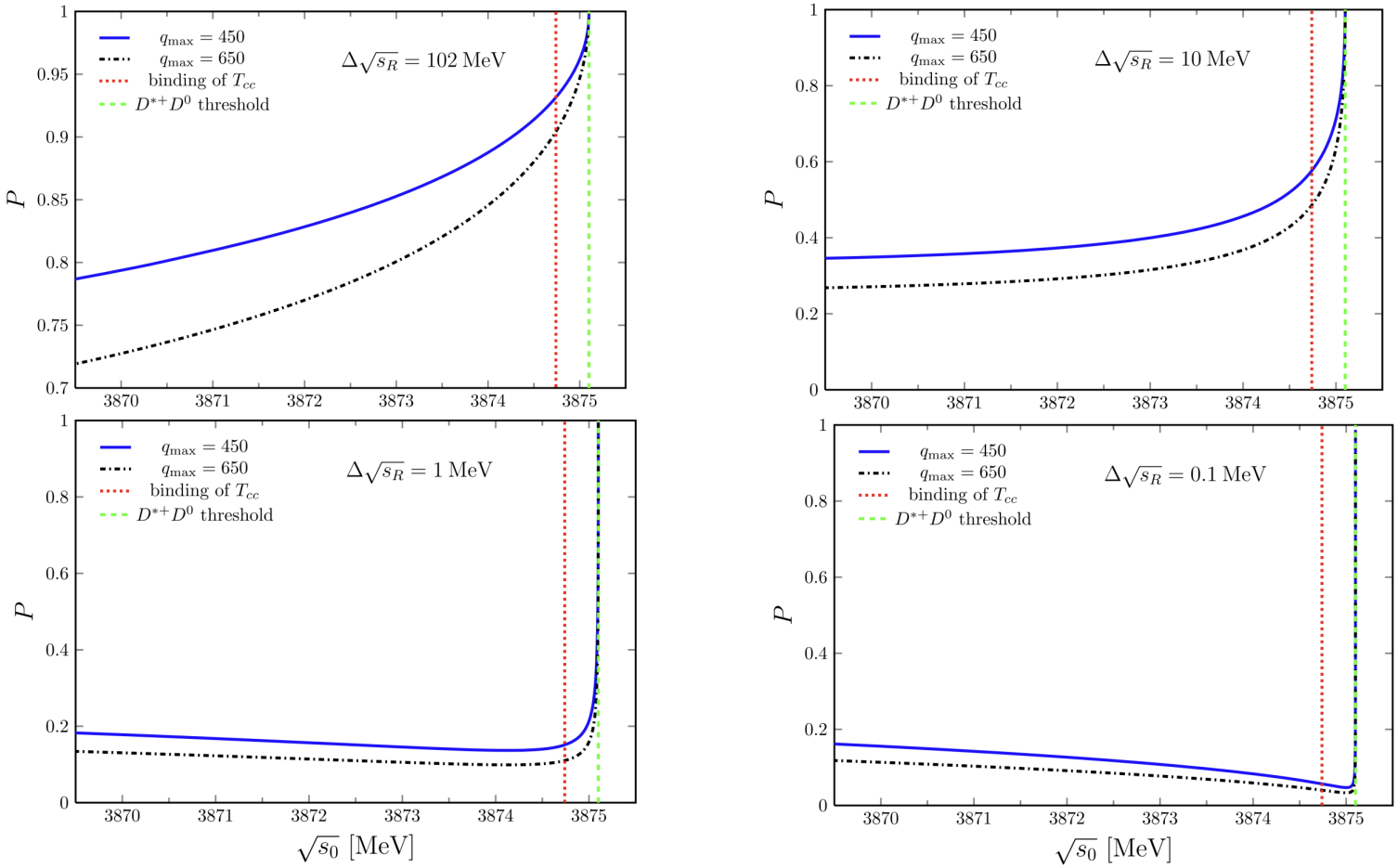
Results| Molecular probability for different $s_R (= \sqrt{s}_{\rm th} + \Delta\sqrt{s}_R)$

- P: molecular probability as a function of $\sqrt{s}_0$ at $\Delta\sqrt{s}_R = 0.1$ MeV



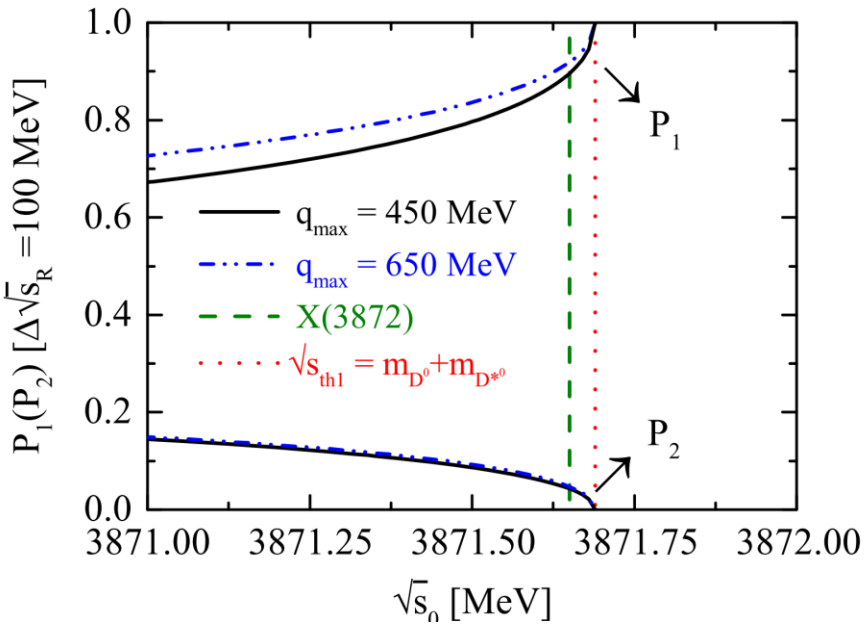
1. P goes to 1 at $s_0 \rightarrow s_{\rm th}$;
2. The value P at T_{cc} smaller than 0.05 : **basically nonmolecular.**

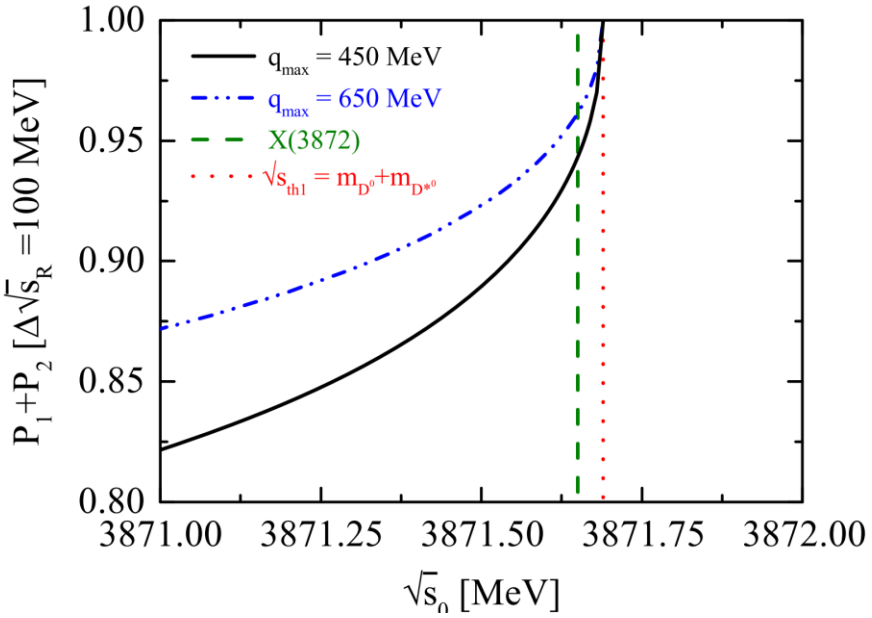
Conclusion1.



➢ The value of the binding energy alone cannot prove the nature of the state.

◆ P: molecular probability as a function of $\sqrt{s}_0$ at $\Delta\sqrt{s}_R = 100$ MeV

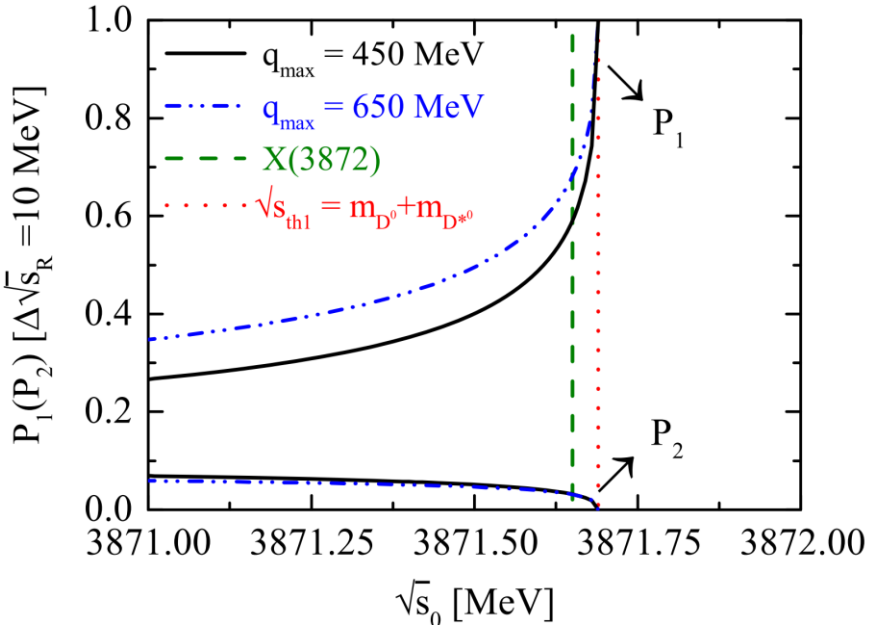


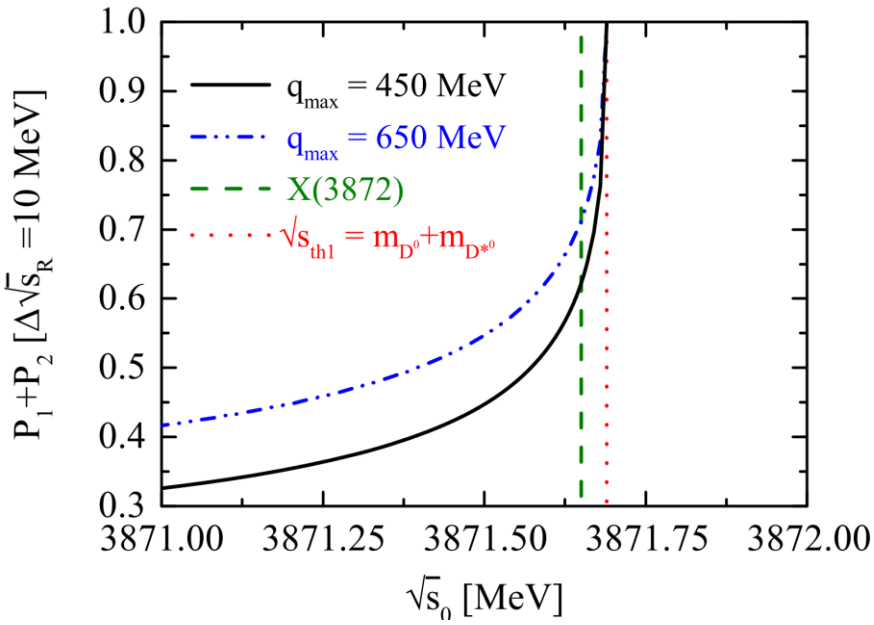


1. $P_1 \to 1$, $P_2 \to 0$ at $s_0 \to s_{\mathrm{th1}}$; $P_1 + P_2 \to 1$.
2. The value P_1 ~ 0.9, P_2 ~0.05 at $X(3872)$ depend a bit on q_{max}.
3. The total molecular probability is around 0.95 at $X(3872)$ energy.

G.-J. Wang, Z. Yang, J.-J. Wu, M. Oka, and S.-L. Zhu. arXiv:2306.12406

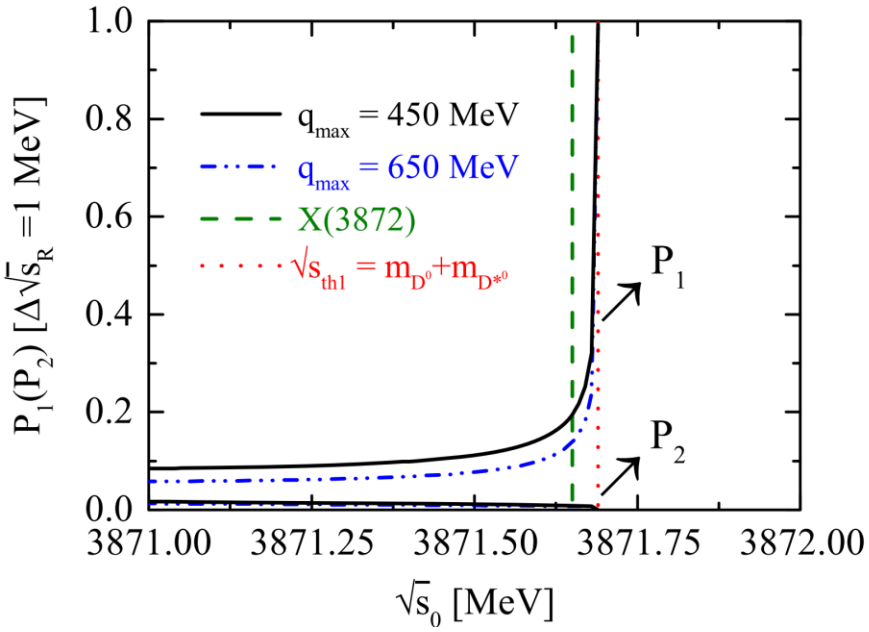
◆ P: molecular probability as a function of $\sqrt{s}_0$ at $\Delta\sqrt{s}_R = 10$ MeV

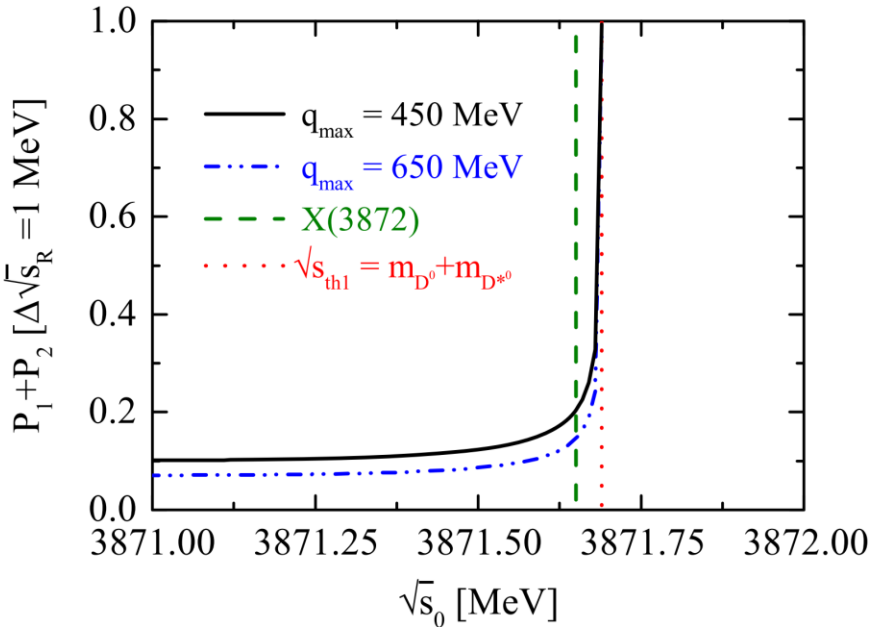




1. $P_1 \to 1$, $P_2 \to 0$ at $s_0 \to s_{\mathrm{th1}}$; $P_1 + P_2 \to 1$.
2. The value P_1 ~ 0.6-0.7, P_2 ~ 0.03 at $X(3872)$.
3. $P_1 + P_2$~ 0.6-07 at $X(3872)$ energy: **molecular component is smaller**.

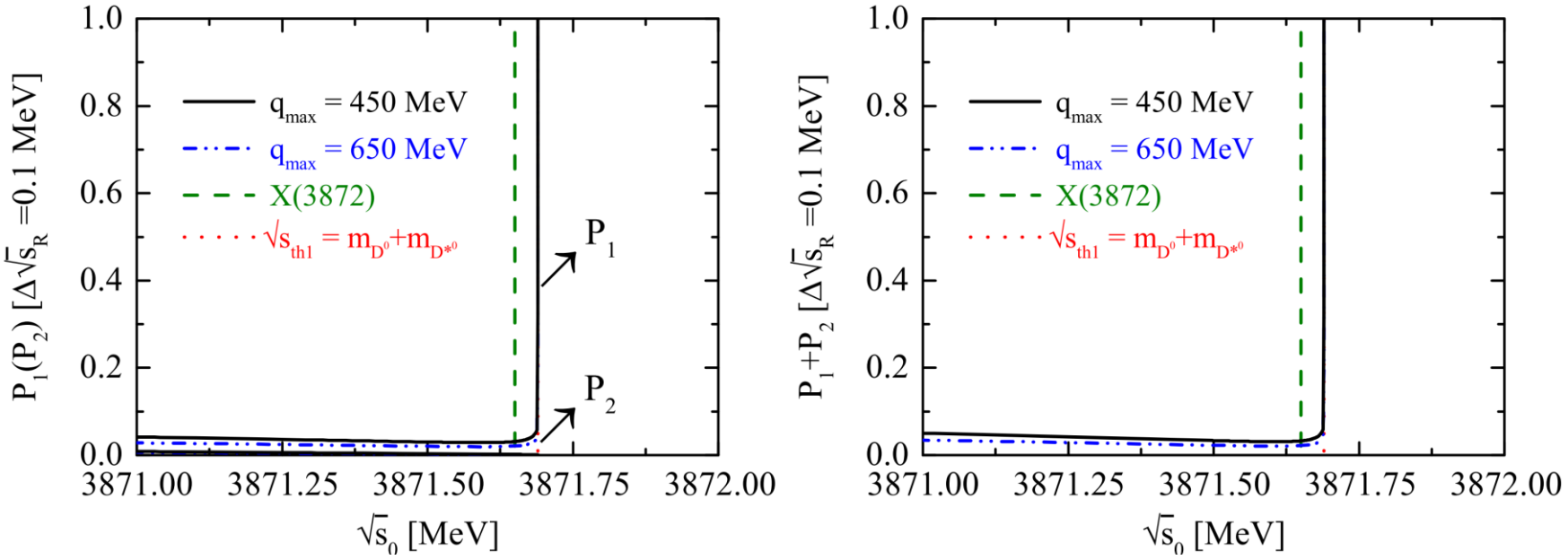
◆ P: molecular probability as a function of $\sqrt{s}_0$ at $\Delta\sqrt{s}_R = 1$ MeV





1. $P_1 \rightarrow$0.15-0.2, $P_2 \rightarrow$0.01 at $s_0 \rightarrow s_{th1}$; $P_1 + P_2 \rightarrow$ 0.15-0.2.
2. $P_1 + P_2 \sim$ 0.15-0.2 at the $X(3872)$ energy: **very small**.

◆ P: molecular probability as a function of $\sqrt{s}_0$ at $\Delta\sqrt{s}_R = 0.1$ MeV



- $P_1 + P_2 \sim 0.02$ at the $X(3872)$ energy: **molecular component is negligible.**

◆ *Conclusion2.*

➢ The binding energy itself does not give a molecular probability.

Results| Scattering length and effective range of *Tcc(3875)*

- The results of a and r_0 as a function of $\Delta\sqrt{s}_R$ at T_{cc}.

$\Delta\sqrt{s_R}$ [MeV]	$q_{max} = 450$ MeV		$q_{max} = 650$ MeV	
	a [fm]	r_0 [fm]	a [fm]	r_0 [fm]
0.1	0.87	-114.07	0.61	-168.39
0.3	1.19	-79.33	0.85	-117.23
1	2.10	-38.20	1.56	-56.68
2	3.04	-21.77	2.36	-32.49
5	4.62	-9.26	3.85	-14.07
10	5.74	-4.51	5.07	-7.08
30	6.94	-1.16	6.54	-2.14
50	7.25	-0.47	6.95	-1.13
70	7.39	-0.17	7.15	-0.69
102	7.51	0.06	7.31	-0.34

- Experimental values: $a \sim 6-7$ fm, $r_0 \sim -3.9$ fm
  - *R. Aaij, et al., LHCb Collaboration, Nat. Phys. 18 (2022) 751, Nat. Commun. 13 (2022) 3351*

1. $\Delta\sqrt{s}_R \searrow$, $a \searrow$ and $r_0 \to \infty$, decreasing the molecular probability;
2. a and r_0 are very useful to determine the molecular probability of the state.

◆ The results of a and r_0 as a function of $\Delta\sqrt{s}_R$ at $s_{\rm th1}(\bar{D}^0 D^{*0})$

	$q_{\rm max}=450$ MeV				$q_{\rm max}=650$ MeV			
$\Delta\sqrt{s}_R$ (MeV)	a_1 (fm)	$r_{0,1}$ (fm)	a_2 (fm)	$r_{0,2}$ (fm)	a_1 (fm)	$r_{0,1}$ (fm)	a_2 (fm)	$r_{0,2}$ (fm)
0.1	1.42	−663.61	$0.0073-i0.00003$	$-664.79-i1.56$	0.954	−1011.3	$0.0048-i0.00002$	$-1014.0-i1.56$
0.3	3.16	−273.51	$0.0176-i0.00020$	$-273.04-i1.56$	2.181	−416.86	$0.0116-i0.00009$	$-417.03-i1.56$
1	7.48	−89.71	$0.0530-i0.00180$	$-88.46-i1.56$	5.544	−136.78	$0.0350-i0.00078$	$-135.77-i1.56$
2	11.09	−45.95	$0.1014-i0.00660$	$-44.52-i1.56$	8.760	−70.098	$0.0674-i0.00292$	$-68.81-i1.56$
5	15.80	−18.86	$0.2305-i0.03475$	$-17.31-i1.56$	13.67	−28.816	$0.1571-i0.01597$	$-27.35-i1.56$
10	18.45	−9.68	$0.3957-i0.10756$	$-8.10-i1.56$	16.87	−14.837	$0.2827-i0.05290$	$-13.31-i1.56$
20	20.16	−5.07	$0.5902-i0.26910$	$-3.47-i1.56$	19.11	−7.8049	$0.4593-i0.14915$	$-6.25-i1.56$
50	21.35	−2.29	$0.7558-i0.58190$	$-0.68-i1.56$	20.79	−3.5725	$0.6801-i0.39616$	$-2.00-i1.56$
70	21.59	−1.76	$0.7761-i0.68790$	$-0.15-i1.56$	21.14	−2.7652	$0.7296-i0.50085$	$-1.19-i1.56$
100	21.78	−1.37	$0.7818-i0.78157$	$0.25-i1.56$	21.41	−2.1595	$0.7611-i0.60330$	$-0.58-i1.56$

1. $\Delta\sqrt{s}_R$ ↘, a ↘ and $r_0 \to \infty$, decreasing the molecular probability;
2. a and r_0 are very useful to determine the molecular probability of the state.

◆ Add local hidden gauge potential between *DD**

➢ Scattering amplitude

$$V' = V + \frac{\tilde{g}^2}{s - s_R}$$

$$T = \frac{V}{1 - VG}$$

$$V = \beta V_{\rm LHG} = \beta(-1)\frac{1}{2}g'^2\left[3s - \left(M^2 + m^2 + M'^2 + m'^2\right) - \frac{1}{s}\left(M^2 - m^2\right)\left(M'^2 - m'^2\right)\right]\frac{1}{M_\rho^2}$$

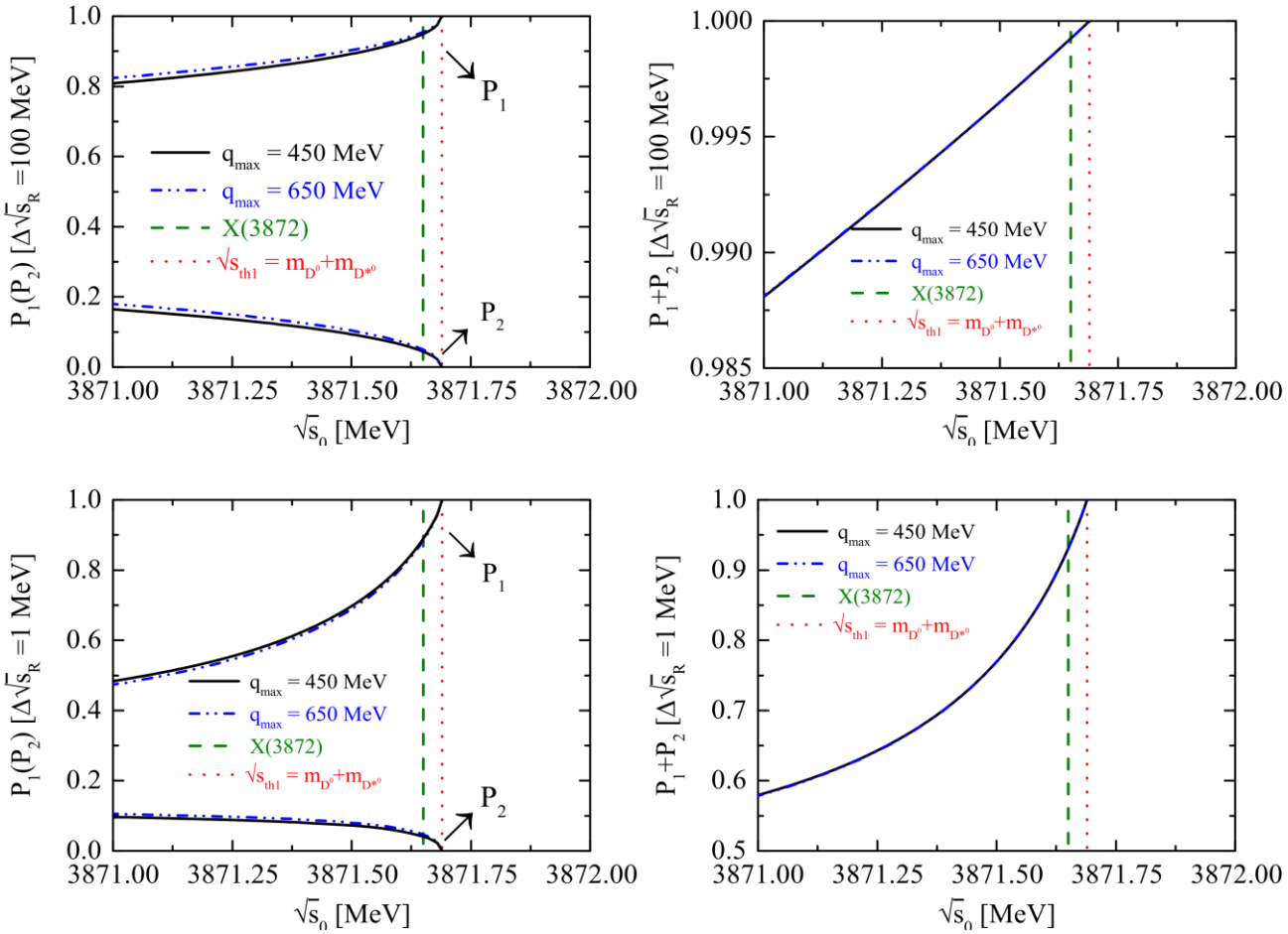
◆ *P*: molecular probability at threshold of *DD**

$$P = -\frac{\left[\tilde{g}^2 + (s - s_R)V\right]\frac{\partial G}{\partial s}}{1 - \left[\tilde{g}^2 + (s - s_R)V\right]\frac{\partial G}{\partial s} - VG}\Big|_{s=s_0}$$

$\Delta\sqrt{s_R}$ [MeV]	$q_{\max} = 450$ MeV		$q_{\max} = 650$ MeV	
	$\beta = 0$	$\beta = 0.74$	$\beta = 0$	$\beta = 0.52$
10	0.58	0.94	0.49	0.94
20	0.73	0.97	0.65	0.97
50	0.87	0.99	0.82	0.99

1. The potential is strong enough, but not enough to bind by itself.
2. The bigger $\sqrt{s}_R$, the relative increase in the compositeness is smaller.
3. Its effect is to increase the molecular probability bringing it close to unity.

◆ *P*: molecular probability as a function of $\sqrt{s}_0$ at $\Delta\sqrt{s}_R = 100$ &1 MeV



1. $P_1 + P_2 \rightarrow 1$.
2. Meson-meson interaction increases the molecular probability.

- The results of a and r_0 as a function of $\Delta\sqrt{s}_R$ at *X(3872)*

	$q_{\max} = 450$ MeV				$q_{\max} = 650$ MeV			
$\Delta\sqrt{s}_R$ (MeV)	a_1 (fm)	$r_{0,1}$ (fm)	a_2 (fm)	$r_{0,2}$ (fm)	a_1 (fm)	$r_{0,1}$ (fm)	a_2 (fm)	$r_{0,2}$ (fm)
0.1	15.60	−24.97	$0.7068 - i1.116$	$1.17 - i1.56$	15.84	−25.74	$0.7476 - i1.011$	$0.82 - i1.56$
0.3	19.65	−7.13	$0.7060 - i1.118$	$1.16 - i1.56$	19.50	−7.55	$0.7470 - i1.012$	$0.81 - i1.56$
1	21.38	−2.30	$0.7024 - i1.125$	$1.14 - i1.56$	21.23	−2.64	$0.7448 - i1.019$	$0.78 - i1.56$
2	21.79	−1.35	$0.6957 - i1.139$	$1.08 - i1.56$	21.64	−1.68	$0.7406 - i1.032$	$0.72 - i1.56$
5	22.05	−0.81	$0.6394 - i1.232$	$0.23 - i1.56$	21.90	−1.13	$0.7035 - i1.128$	$-0.11 - i1.56$
10	22.13	−0.63	$0.7818 - i0.780$	$-3.62 - i1.56$	21.98	−0.94	$0.7767 - i0.693$	$-4.32 - i1.56$
20	22.17	−0.54	$0.7514 - i0.998$	$0.92 - i1.56$	22.02	−0.85	$0.7731 - i0.898$	$0.56 - i1.56$
50	22.20	−0.48	$0.7410 - i1.031$	$1.12 - i1.56$	22.05	−0.80	$0.7677 - i0.930$	$0.77 - i1.56$
70	22.21	−0.47	$0.7396 - i1.035$	$1.14 - i1.56$	22.05	−0.79	$0.7669 - i0.934$	$0.79 - i1.56$
100	22.21	−0.47	$0.7385 - i1.038$	$1.15 - i1.56$	22.06	−0.78	$0.7663 - i0.937$	$0.80 - i1.56$

- Experimental values: $a_1 = 28$ fm, $r_{0,1} = -5.34$ fm, -2.78 fm $< r_{0,1} < 1$ fm
  - *R. Aaij et al. (LHCb), Phys. Rev. D 102, 092005 (2020), A. Esposito, et al, PRD 105, L031503 (2022)*
  - *V. Baru, et al., PLB 833, 137290 (2022, V. Baru, private communication (2023)*

1. Comparing the **experiments**: a_1 and r_{01} still unacceptable at $\Delta\sqrt{s}_R = 0.1$MeV, case of $\Delta\sqrt{s}_R = 1$ MeV they are agree within the uncertainty, and $P_1 + P_2 \sim 0.95$.
2. Comparing the **genuine state**: the direct meson-meson interaction effects the increase of a and the decrease of the size of r_0.

Outline

- Introduction
- Theoretical Formalism
- Results and discussion
- **Summary**

Summary

1. The binding energy of the state goes to the meson-meson threshold, the state becomes 100% molecular.
2. If the bare mass of the genuine state is very close to threshold, the raise of the molecular probability to unity occurs at even smaller distances to this threshold.
3. The value of the binding energy together with the measurement of scattering length and effective range can provide an answer to the compositeness of a state.
4. The mixture of a genuine state and an additional direct attractive meson-meson interaction, the state becomes clearly molecular.
5. With present experimental values of scattering length and effective range, both the $T_{cc}(3875)$ and $X(3872)$ are molecular states.
6. The results and conclusions are general, and the method employed in the analysis can be easily extrapolated to any other hadronic cases.

Thanks for your attention!