USE OF DISPERSIVE MESON-MESON ANALYSES IN GIANT CP VIOLATION IN B TO THREE LIGHT MESONS AT LHC BEYOND LEADING ORDER

J.R. Peláez and A. Reyes-Torrecilla, in progress

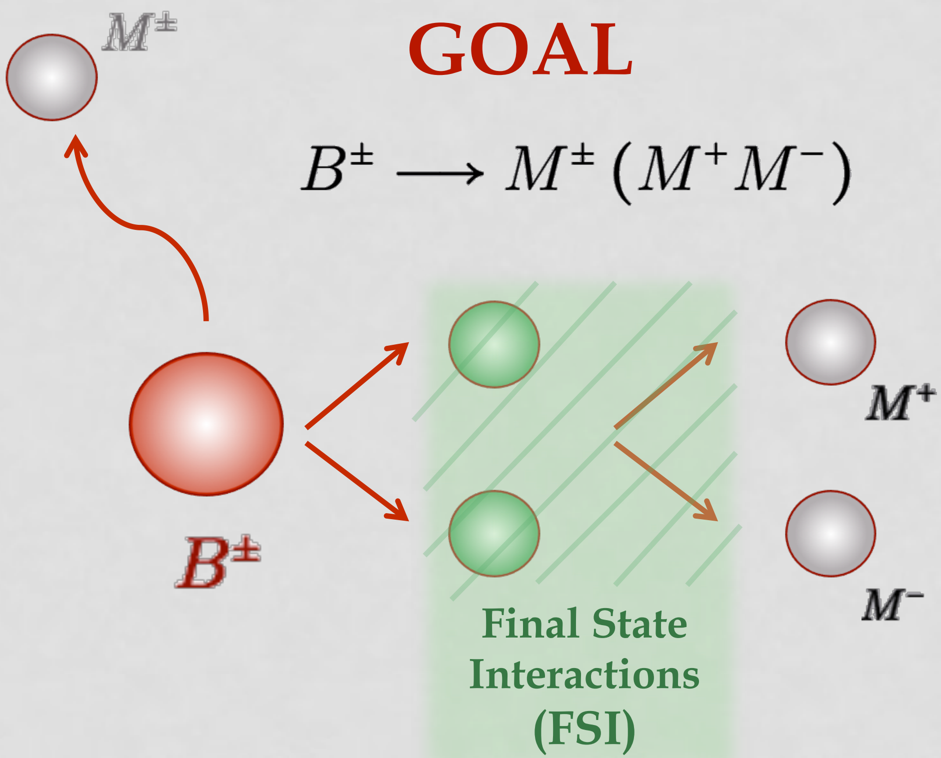
Supported by: Institute of Particle & Cosmos Physics (IPARCOS)

EXCITED QCD 2024

Departamento de Física Teórica
Universidad Complutense de Madrid

GOAL

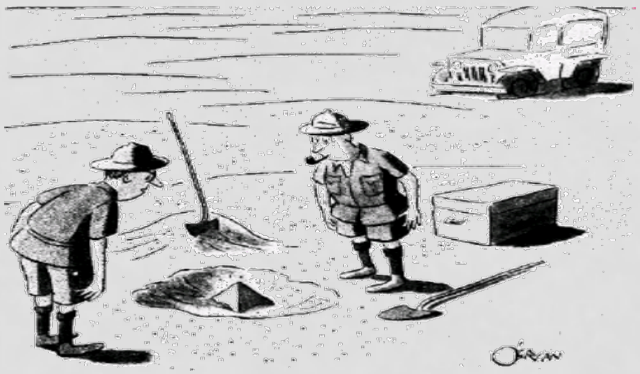
$$B^{\pm} \longrightarrow M^{\pm}\,(M^{+}M^{-})$$



Describe B meson decays into three light mesons

1. **2-body decays** vs **3-body decays**
2. **How can we approach these 3-body decays? Can we simplify them?**
3. **Into the simplest FSI model at LHCb**
   - Two unnecessary and crude estimates
   - Using dispersive $\pi\pi \longrightarrow KK$ parameterizations
4. **How can we improve this?**

Let's start with a brief introduction…



- CP violation (CPV) relevant for bariogenesis and other open questions within the SM

- CPV observed through
  - CP forbidden processes
  - Asymmetries between CP-conjugated decays
    $$\mathcal{A}_{CP} \equiv \Delta\Gamma_f = \Gamma(M \to f) - \Gamma(\bar{M} \to \bar{f})$$

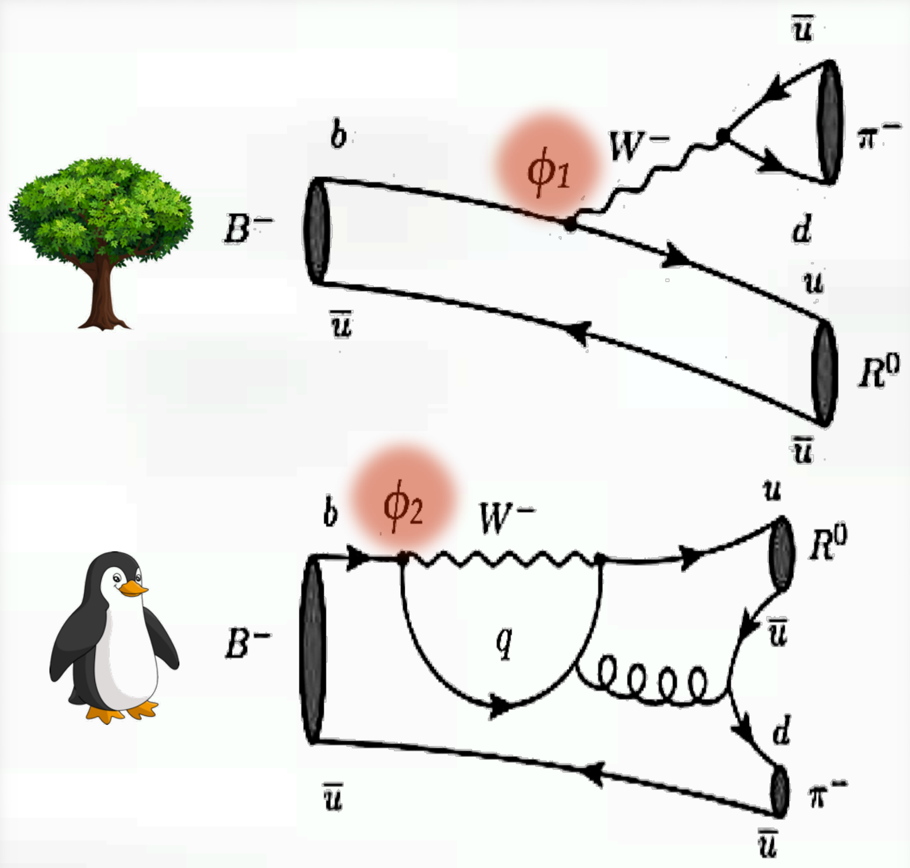
- In the SM quark/hadron CPV due to "weak" phase in the CKM matrix

To observe such CP phase in an asymmetry we need interference between at least two mechanisms/diagrams with different weak and strong phases

Standard CPV framework

[1] Bander Silverman & Soni PRL 43 (1979) 242

- “Tree+ Penguin”, BSS model



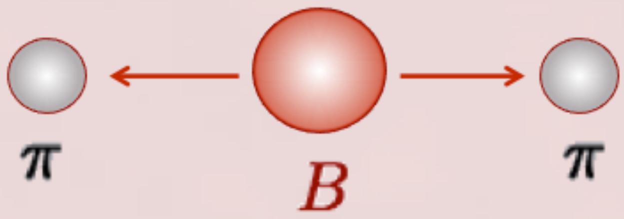
- CPV (weak) phase appears in the quark CKM matrix

- Strong imaginary part is due to the loop, which are not physical states

However, strong phases can be generated at the “hadron level” via Hadronic FSI

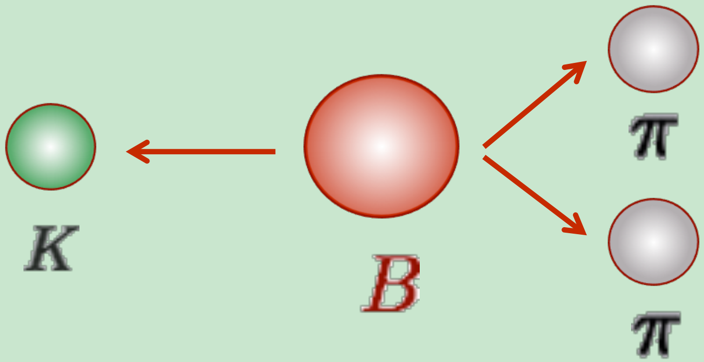
[2] L. Wolfenstein, Phys. Rev. D 43, (1991) 151
[3] M. Suzuki and L. Wolfenstein, Phys. Rev. D 60, (1999) 074019
[4] M. Suzuki, Phys. Rev. D 77, (2008) 054021

2-body decays



- Decaying particle mass fixes the resulting products' momenta
- Hadronization/FSI is just a number after angular integration
- Simpler to estimate

3-body decays



- Available energy is distributed between the three final particles
- Strong FSI phase is a **FUNCTION** of two energy variables. NOT FIXED
- No so simple to reproduce…

LHCb intense research program involving large CPV asymmetries in $B \longrightarrow 3M$ …

PRL **111,** 101801 (2013) PHYSICAL REVIEW LETTERS

Measurement of *CP* Violation in the Phase Space of $B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}$ and $B^{\pm} \to K^{\pm}K^{+}K^{-}$ Decays

R. Aaij *et al.**

(LHCb Collaboration)

(Received 5 June 2013; published 3 September 2013)

The charmless decays $B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}$ and $B^{\pm} \to K^{\pm}K^{+}K^{-}$ are reconstructed using data, corresponding to an integrated luminosity of 1.0 fb^{-1}, collected by LHCb in 2011. The inclusive charge asymmetries of these modes are measured as $A_{CP}(B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}) = 0.032 \pm 0.008(\mathrm{stat}) \pm 0.004(\mathrm{syst}) \pm 0.007(J/\psi K^{\pm})$ and $A_{CP}(B^{\pm} \to K^{\pm}K^{+}K^{-}) = -0.043 \pm 0.009\,(\mathrm{stat}) \pm 0.003\,(\mathrm{syst}) \pm 0.007(J/\psi K^{\pm})$, where the third uncertainty is due to the *CP* asymmetry of the $B^{\pm} \to J/\psi K^{\pm}$ reference mode. The significance of $A_{CP}(B^{\pm} \to K^{\pm}K^{+}K^{-})$ exceeds three standard deviations and is the first evidence of an inclusive *CP* asymmetry in charmless three-body *B* decays. In addition to the inclusive *CP* asymmetries, larger asymmetries are observed in localized regions of phase space.

LHCb intense research program involving large CPV asymmetries in $B \longrightarrow 3M$ …

PRL **111**, 101801 (2013) PHYSICAL REVIEW LETTERS

M

PRL **112**, 011801 (2014) PHYSICAL REVIEW LETTERS

Measurement of *CP* Violation in the Phase Space of $B^{\pm} \to K^+K^-\pi^{\pm}$ and $B^{\pm} \to \pi^+\pi^-\pi^{\pm}$ Decays

(Received 18 October 2013; published 7 January 2014)

The charmless decays $B^{\pm} \to K^+K^-\pi^{\pm}$ and $B^{\pm} \to \pi^+\pi^-\pi^{\pm}$ are reconstructed in a data set of pp collisions with an integrated luminosity of 1.0 fb^{-1} and center-of-mass energy of 7 TeV, collected by LHCb in 2011. The inclusive charge asymmetries of these modes are measured to be $A_{CP}(B^{\pm} \to K^+K^-\pi^{\pm}) = -0.141 \pm 0.040\,(\text{stat}) \pm 0.018\,(\text{syst}) \pm 0.007(J/\psi K^{\pm})$ and $A_{CP}(B^{\pm} \to \pi^+\pi^-\pi^{\pm}) = 0.117 \pm 0.021\,(\text{stat}) \pm 0.009\,(\text{syst}) \pm 0.007(J/\psi K^{\pm})$, where the third uncertainty is due to the *CP* asymmetry of the $B^{\pm} \to J/\psi K^{\pm}$ reference mode. In addition to the inclusive *CP* asymmetries, larger asymmetries are observed in localized regions of phase space.

LHCb intense research program involving large CPV asymmetries in $B \longrightarrow 3M$ ...

PHYSICAL REVIEW D **90**, 112004 (2014)

Measurements of *CP* violation in the three-body phase space of charmless $B^{\pm}$ decays

R. Aaij *et al.*[*]
(LHCb Collaboration)

(Received 25 August 2014; published 11 December 2014)

The charmless three-body decay modes $B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}$, $B^{\pm} \to K^{\pm}K^{+}K^{-}$, $B^{\pm} \to \pi^{\pm}K^{+}K^{-}$ and $B^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}$ are reconstructed using data, corresponding to an integrated luminosity of 3.0 fb^{-1}, collected by the LHCb detector. The inclusive *CP* asymmetries of these modes are measured to be

$$A_{CP}(B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}) = +0.025 \pm 0.004 \pm 0.004 \pm 0.007,$$
$$A_{CP}(B^{\pm} \to K^{\pm}K^{+}K^{-}) = -0.036 \pm 0.004 \pm 0.002 \pm 0.007,$$
$$A_{CP}(B^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}) = +0.058 \pm 0.008 \pm 0.009 \pm 0.007,$$
$$A_{CP}(B^{\pm} \to \pi^{\pm}K^{+}K^{-}) = -0.123 \pm 0.017 \pm 0.012 \pm 0.007,$$

where the first uncertainty is statistical, the second systematic, and the third is due to the *CP* asymmetry of the $B^{\pm} \to J/\psi K^{\pm}$ reference mode. The distributions of these asymmetries are also studied as functions of position in the Dalitz plot and suggest contributions from rescattering and resonance interference processes.

FSI experimental signs

THE LARGES CP ASYMMETRY REPORTED TO DATE

PHYS... ERS **123,** 231802 (2019)

Amplitude Analysis of $B^{\pm} \to \pi^{\pm} K^{+} K^{-}$ Decays

R. Aaij *et al.*[*]
(LHCb Collaboration)

(Received 12 June 2019; revised manuscript received 15 October 2019; published 6 December 2019)

The first amplitude analysis of the $B^{\pm} \to \pi^{\pm} K^{+} K^{-}$ decay is reported based on a data sample corresponding to an integrated luminosity of 3.0 fb^{-1} of pp collisions recorded in 2011 and 2012 with the LHCb detector. The data are found to be best described by a coherent sum of five resonant structures plus a nonresonant component and a contribution from $\pi\pi \leftrightarrow KK$ S-wave rescattering. The dominant contributions in the $\pi^{\pm} K^{\mp}$ and $K^{+} K^{-}$ systems are the nonresonant and the $B^{\pm} \to \rho(1450)^{0} \pi^{\pm}$ amplitudes, respectively, with fit fractions around 30%. For the rescattering contribution, a sizable fit fraction is observed. This component has the largest CP asymmetry reported to date for a single amplitude of $(-66 \pm 4 \pm 2)\%$, where the first uncertainty is statistical and the second systematic. No significant CP violation is observed in the other contributions.

of the $B^{\pm} \to J/\psi K^{\pm}$ reference mode. The distributions of these asymmetries are also studied as functions of position in the Dalitz plot and suggest contributions from rescattering and resonance interference processes.

PHYSICAL REVIEW LETTERS **123**, 231802 (2

Amplitude Analysis of $B^{\pm} \to \pi^{\pm} K^{+} K^{-}$ Decays

Looking forward to the whole RUN2 data

PHYSICAL REVIEW LETTERS **124**, 031801 (2020)

Observation of Several Sources of CP Violation in $B^{+} \to \pi^{+}\pi^{+}\pi^{-}$ Decays

R. Aaij *et al.**
(LHCb Collaboration)

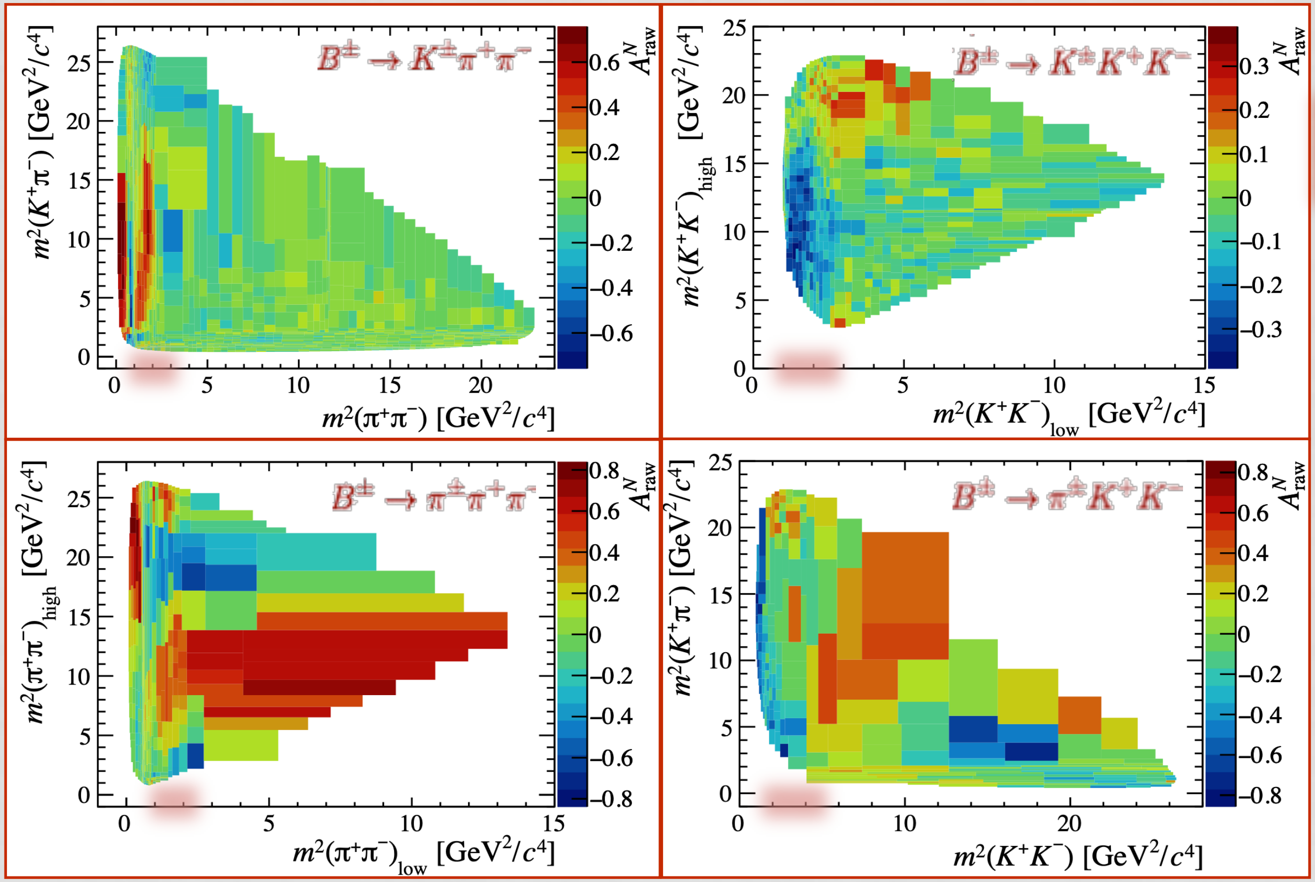
(Received 16 September 2019; published 21 January 2020)

Observations are reported of different sources of CP violation from an amplitude analysis of $B^{+} \to \pi^{+}\pi^{+}\pi^{-}$ decays, based on a data sample corresponding to an integrated luminosity of 3 fb^{-1} of pp collisions recorded with the LHCb detector. A large CP asymmetry is observed in the decay amplitude involving the tensor $f_2(1270)$ resonance, and in addition significant CP violation is found in the $\pi^{+}\pi^{-}$ S wave at low invariant mass. The presence of CP violation related to interference between the $\pi^{+}\pi^{-}$ S wave and the P wave $B^{+} \to \rho(770)^{0}\pi^{+}$ amplitude is also established; this causes large local asymmetries but cancels when integrated over the phase space of the decay. The results provide both qualitative and quantitative new insights into CP -violation effects in hadronic B decays.

CPV in $B^\pm \longrightarrow M^\pm (M^+ M^-)$

○ **Large integrated CPV asymmetries** $10\,\%$

$$A_{CP}(B^\pm \to K^\pm\pi^+\pi^-) = +0.011 \pm 0.002 \pm 0.003 \pm 0.003, \quad A_{CP}(B^\pm \to \pi^\pm\pi^+\pi^-) = +0.080 \pm 0.004 \pm 0.003 \pm 0.003,$$
$$A_{CP}(B^\pm \to K^\pm K^+K^-) = -0.037 \pm 0.002 \pm 0.002 \pm 0.003, \quad A_{CP}(B^\pm \to \pi^\pm K^+K^-) = -0.114 \pm 0.007 \pm 0.003 \pm 0.003,$$



RUN2 $5.9\ fb^{-1}$

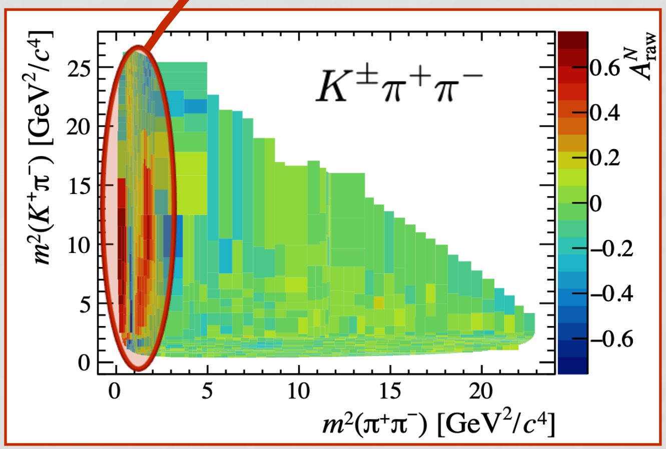
[6] R. Aaij, *et al* Phys. Rev. D 108.1 (2023) 012008

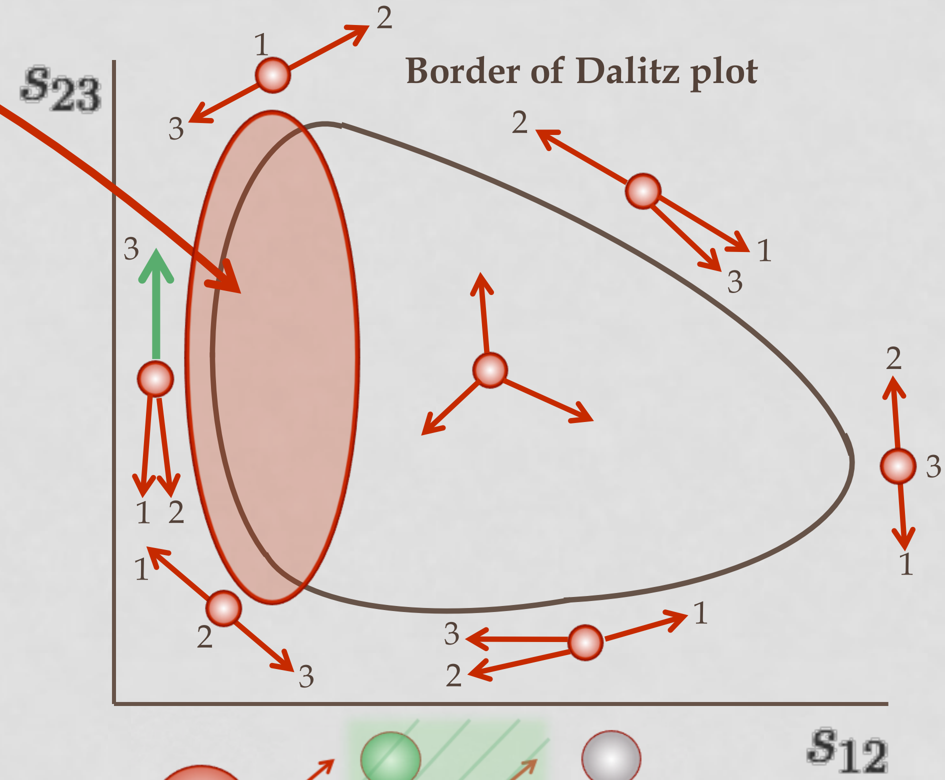
○ **GIANT local CPV asymmetries**

$60\,\% - 80\,\%$

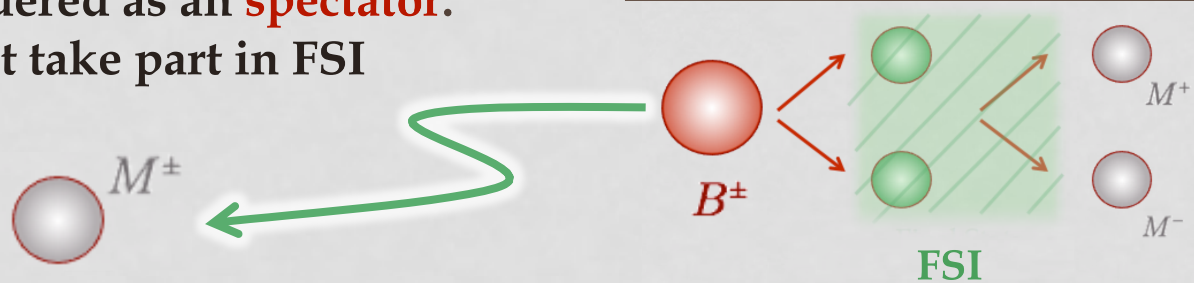
However, reproducing the full 3-body Dalitz plot needs many contributions and it is a difficult task. Analysis

LHCb follows this analysis [5]…





1. One of the three final particles is considered as an spectator. Does not take part in FSI



[5] Bediaga, Frederico, Lourenco, Phys. Rev. D 89 (2014) 094013

Simplest aproach [5]

1. One of the three final particles is considered as an spectator

2. Use CPT constrains at hadron level so that h and $\bar{h}$ must have the same TOTAL decay widths. CPV can only occur in PARTIAL decay widths

$$\Gamma_{Total} = \Gamma_1 + \Gamma_2 + \Gamma_3 + \Gamma_4 + \ldots$$
$$\bar{\Gamma}_{Total} = \underbrace{\bar{\Gamma}_1 + \bar{\Gamma}_2 + \bar{\Gamma}_3 + \bar{\Gamma}_4 + \ldots}$$

$$\Delta\Gamma_{Total} = \Gamma_{Total} - \bar{\Gamma}_{Total} = 0$$

[5] Bediaga, Frederico, Lourenco, Phys. Rev. D 89 (2014) 094013

Simplest aproach [5]

1. One of the three final particles is considered as an spectator

2. Use CPT constrains at hadron level so that h and $\bar{h}$ must have the same TOTAL decay widths. CPV can only occur in PARTIAL decay widths

$$\Gamma_{Total} = \Gamma_1 + \Gamma_2 + \Gamma_3 + \Gamma_4 + \ldots$$
$$\bar{\Gamma}_{Total} = \underbrace{\bar{\Gamma}_1 + \bar{\Gamma}_2 + \bar{\Gamma}_3 + \bar{\Gamma}_4 + \ldots}$$

$$\Delta\Gamma_{Total} = \Gamma_{Total} - \bar{\Gamma}_{Total} = 0$$

3. Golden mode → Just two coupled channels

$$\Gamma_{Total} = \Gamma_{\pi\pi} + \Gamma_{KK}$$
$$\bar{\Gamma}_{Total} = \bar{\Gamma}_{\pi\pi} + \bar{\Gamma}_{KK}$$

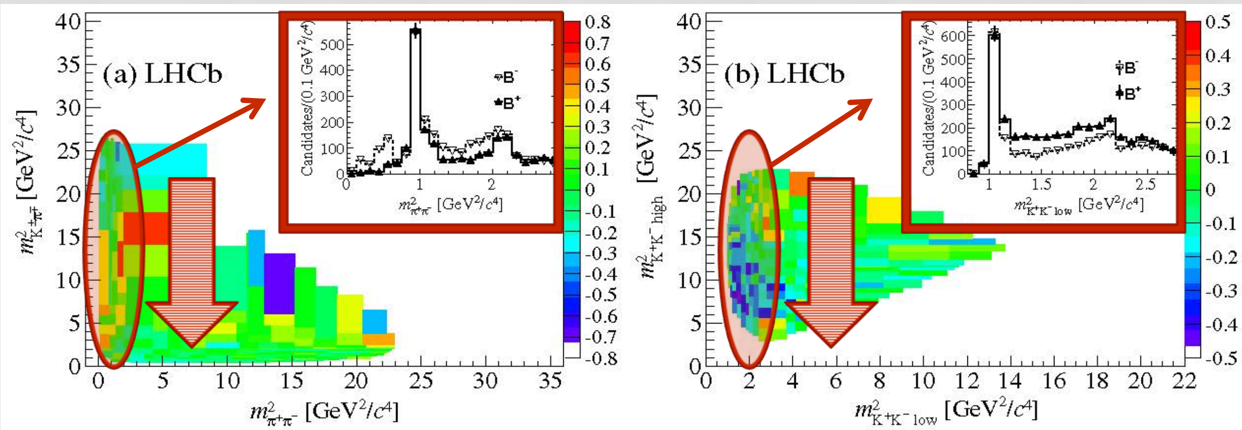
$$\Delta\Gamma_1 + \Delta\Gamma_2 = 0$$
$$\Delta\Gamma_2 \approx -\Delta\Gamma_1$$

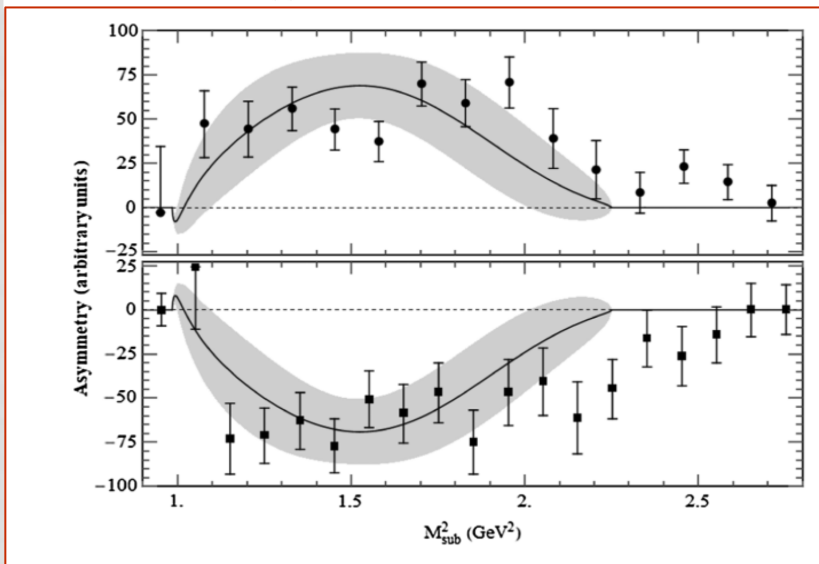
Is this found anywhere?

[5] Bediaga, Frederico, Lourenco, Phys. Rev. D 89 (2014) 094013

$B^{\pm} \longrightarrow K^{\pm}(K^{+}K^{-})$ and $B^{\pm} \longrightarrow K^{\pm}(\pi^{+}\pi^{-})$ in the $1-1.5$ GeV region

Large asymmetries found in LHCb data when projected in the $1-1.5$ GeV region, where only $\pi\pi$ and KK are relevant [5] $(2.25\ \mathrm{GeV}^2)$





In that projection [5]:

$$\Delta\Gamma_{\pi\pi} + \Delta\Gamma_{KK} = 0$$

$$\Delta\Gamma_{KK} \approx -\Delta\Gamma_{\pi\pi}$$

- **Clear indication of DOMINANT role of FSI between the two hadronic states!**

[5] Bediaga, Frederico, Lourenco, Phys. Rev. D 89 (2014) 094013

Standard FSI model in detail

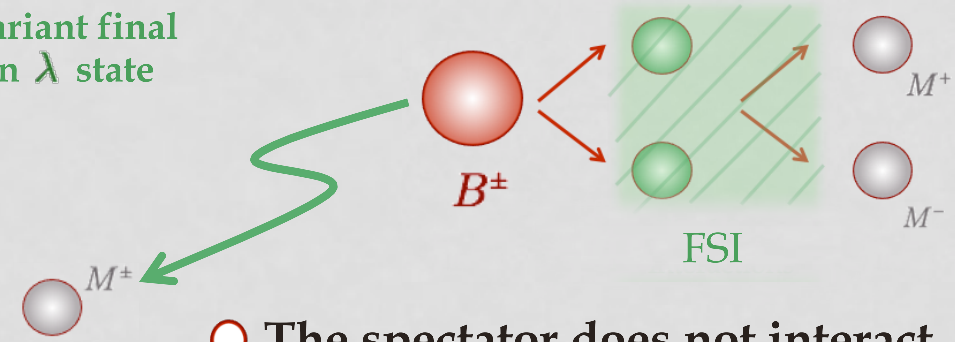
- **Decay amplitude as:**

CP invariant final hadron λ state

$$\mathcal{A}^- = \langle \lambda | \mathscr{H}_w | h \rangle \qquad h \longrightarrow \lambda$$

$$\mathcal{A}^+ = \langle \lambda | \mathscr{H}_w | \bar{h} \rangle \qquad \bar{h} \longrightarrow \lambda$$

CP conjugated hadron states



- **The spectator does not interact with the other two products, but can add some complex strong phases to the amplitude**

Complex coming from QCD interaction and therefore symmetric under CP

$$\boxed{\mathcal{A}^\pm = A_\lambda + B_\lambda e^{\pm i\gamma}}$$

γ **: weak phase changes sign under CP conjugation**

[5] Bediaga, Frederico, Lourenco, Phys. Rev. D 89 (2014) 094013

If λ can couple to other channels λ' (FSI):

[3] M. Suzuki and L. Wolfenstein, Phys. Rev. D 60, 074019 (1999)

$$\mathcal{A} = S^{1/2} A^0$$

Amplitude without FSI

(Over the right cut)

Hadronic FSI interaction

- S **to LO (LHCb) :**

$1 = \pi\pi$

$\lambda, \lambda' = 1, 2 \qquad 2 = KK$

$$S_{\lambda\lambda'} = \delta_{\lambda\lambda'} + 2i\hat{f}_{\lambda\lambda'}$$

$$\mathcal{A}^{\pm} = A_\lambda + B_\lambda e^{\pm i\gamma}$$

$$\mathcal{A}^{\pm}_{\mathrm{LO}} = A^0_\lambda + B^0_\lambda e^{\pm i\gamma} + i\sum_{\lambda'} \hat{f}_{\lambda\lambda'}\left(A^0_{\lambda'} + B^0_{\lambda'} e^{\pm i\gamma}\right)$$

- **Asymmetry in the simplest regime with only $\pi\pi \longrightarrow KK$ FSI in the S0 -wave :**

$$\Delta\Gamma_{\mathrm{KK}} \simeq \mathcal{C}\,|S_{\pi\pi\mathrm{KK}}|\cos\left(\delta_{\pi\pi\mathrm{KK}} + \Phi_{\mathrm{KK}}\right) F\left(M^2_{\mathrm{KK}}\right)$$

Two functions describing the dispersion

Two parameters

$$\mathcal{C} = 4|K|\sin\gamma$$

$$K = |K|\exp(i\Phi_{KK}) = B^*_{KK}A_{\pi\pi} - B^*_{\pi\pi}A_{KK}$$

$F(M^2_K) \longrightarrow$ **Dalitz plot kinematic factor**

[5] Bediaga, Frederico, Lourenco, Phys. Rev. D 89 (2014) 094013

S -matrix:

$$S_{\lambda\lambda'} = \begin{pmatrix} |S_{\pi\pi\pi\pi}|\, e^{2i\delta_{\pi\pi\pi\pi}} & i\,|S_{\pi\pi KK}|\, e^{i\delta_{\pi\pi KK}} & \cdots \\ i\,|S_{\pi\pi KK}|\, e^{i\delta_{\pi\pi KK}} & |S_{KKKK}|\, e^{2i\delta_{KKKK}} & \cdots \\ \vdots & \vdots & \vdots \end{pmatrix}$$

This is what we need

$$\Delta\Gamma_{KK} \simeq \mathcal{C}\,|S_{\pi\pi KK}| \cos\left(\delta_{\pi\pi KK} + \Phi_{KK}\right) F\left(M^2_{KK}\right)$$

- At the time this model was first implemented, only $\pi\pi \longrightarrow \pi\pi$ dispersive analysis existed
- In the two-coupled channels approximation $\longrightarrow$ $\delta_{\pi\pi KK} = \delta_{\pi\pi\pi\pi} + \delta_{KKKK}$

1. $KK \longrightarrow KK$ is unknown $\longrightarrow$ Very crude estimate $\delta_{KKKK} = \delta_{\pi\pi\pi\pi}$

$$\delta_{\pi\pi KK} = 2\delta_{\pi\pi\pi\pi}$$

2. Took η from old and imprecise $\pi\pi \longrightarrow \pi\pi$ parameterization

$$|S_{\pi\pi KK}| = \sqrt{1-\eta^2}$$

[5] Bediaga, Frederico, Lourenco, Phys. Rev. D 89 (2014) 094013

[7] Alvarenga, Bediaga, Frederico Phys. Rev. D 92 (2015) 0954010

S -matrix:

$$S_{\lambda\lambda'} = \begin{pmatrix} |S_{\pi\pi\pi\pi}|\, e^{2i\delta_{\pi\pi\pi\pi}} & i\,|S_{\pi\pi KK}|\, e^{i\delta_{\pi\pi KK}} & \cdots \\ i\,|S_{\pi\pi KK}|\, e^{i\delta_{\pi\pi KK}} & |S_{KKKK}|\, e^{2i\delta_{KKKK}} & \cdots \\ \vdots & \vdots & \vdots \end{pmatrix}$$

This is what we need

$$\Delta\Gamma_{KK} \simeq \mathcal{C}\,|S_{\pi\pi KK}| \cos\left(\delta_{\pi\pi KK} + \Phi_{KK}\right) F\left(M^2_{KK}\right)$$

- At the time this model was first implemented, only $\pi\pi \longrightarrow \pi\pi$ dispersive analysis existed
- In the two-coupled channels approximation $\longrightarrow \delta_{\pi\pi KK} = \delta_{\pi\pi\pi\pi} + \delta_{KKKK}$

1. $KK \longrightarrow KK$ is ... wn $\longrightarrow$ Very crude estimate $\delta_{KKKK} = \delta_{\pi\pi\pi\pi}$

$$\delta_{\pi\pi KK} = 2\delta_{\pi\pi\pi\pi}$$

2. Took η from old and imprec... eterization

$$|S_{\pi\pi KK}| = \sqrt{1-\eta\ldots}$$

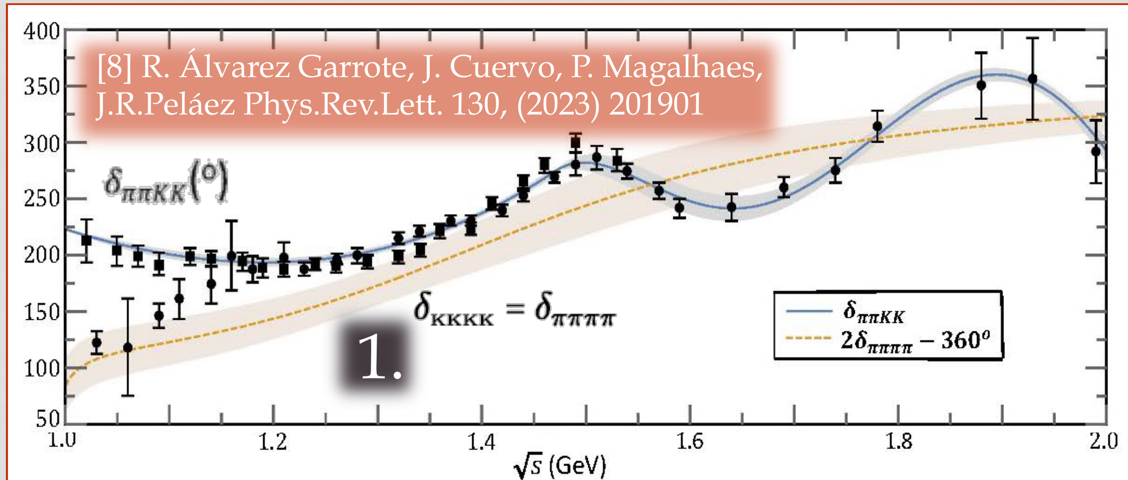
These estimates are what the LHCb is currently using!

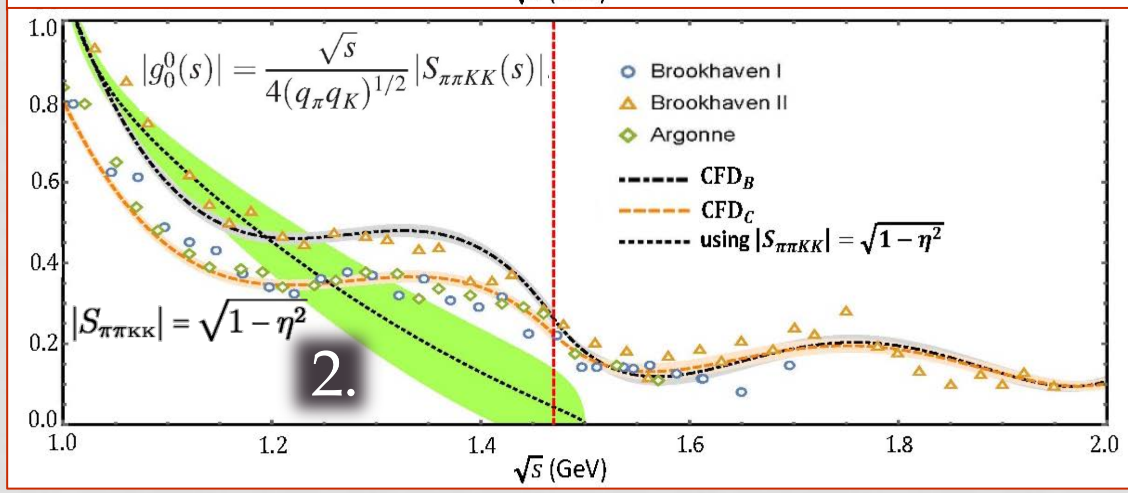
[5] Bediaga, Frederico, Lourenco, Phys. Rev. D 89 (2014) 094013

[7] Alvarenga, Bediaga, Frederico Phys. Rev. D 92 (2015) 0954010

Consequences of the crude estimates 1. and 2.

- The crude estimates do not describe de data
- They produce huge uncertainties
- They do not resolve resonance structures



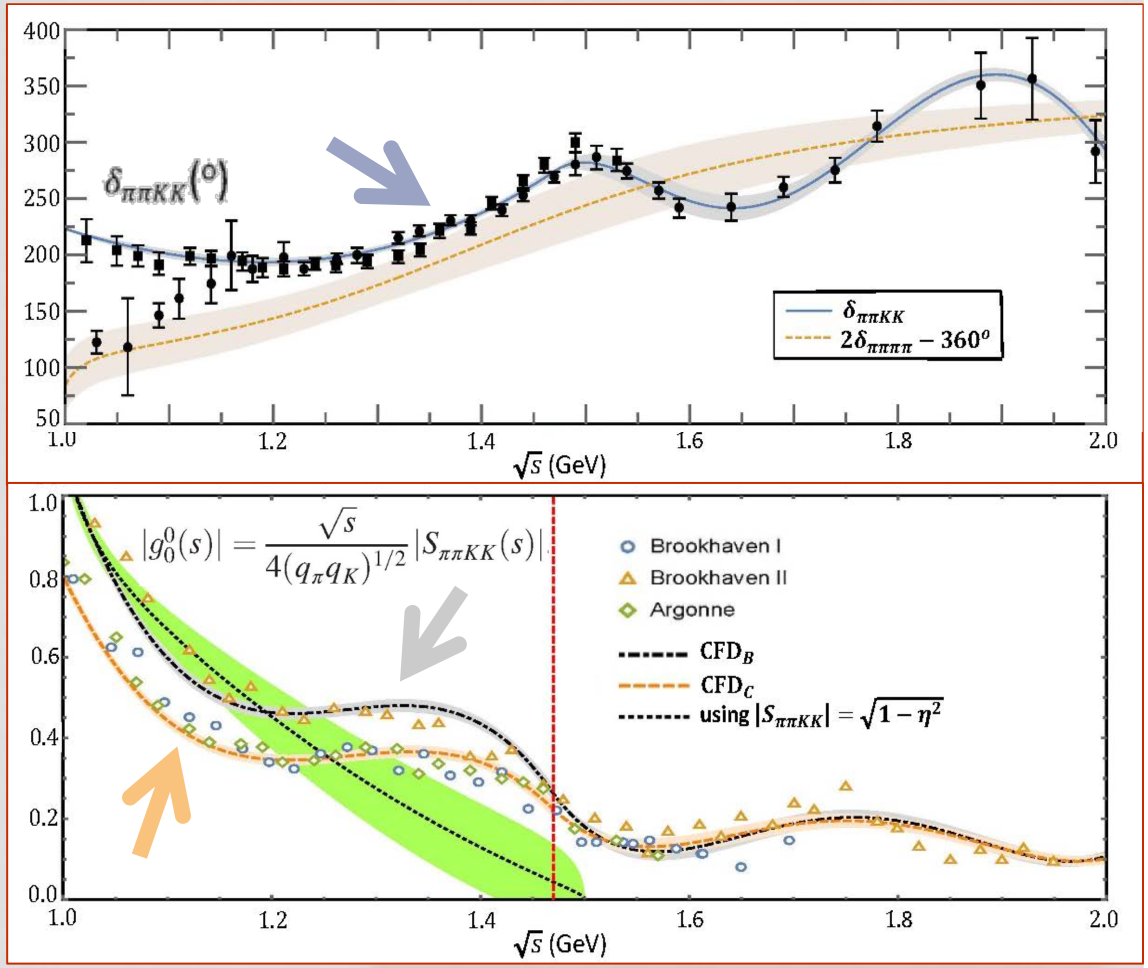


[5] Bediaga, Frederico, Lourenco, Phys. Rev. D 89 (2014) 094013

[7] Alvarenga, Bediaga, Frederico Phys. Rev. D 92 (2015) 0954010

But correct and precise $\pi\pi \longrightarrow KK$ parameterizations were implemented…

Crude estimates 1. and 2. FIXED



- **Describe the data much better**
- **Significantly reduced uncertainties**
- **Visible resonance structures**

Using the current $\pi\pi \longrightarrow KK$ parameterization

[9] J. R. Peláez, A. Rodas, Eur. Phys. J. C 78 (2018) 11, 897 & Phys. Rept. 960 (2022) 1-126

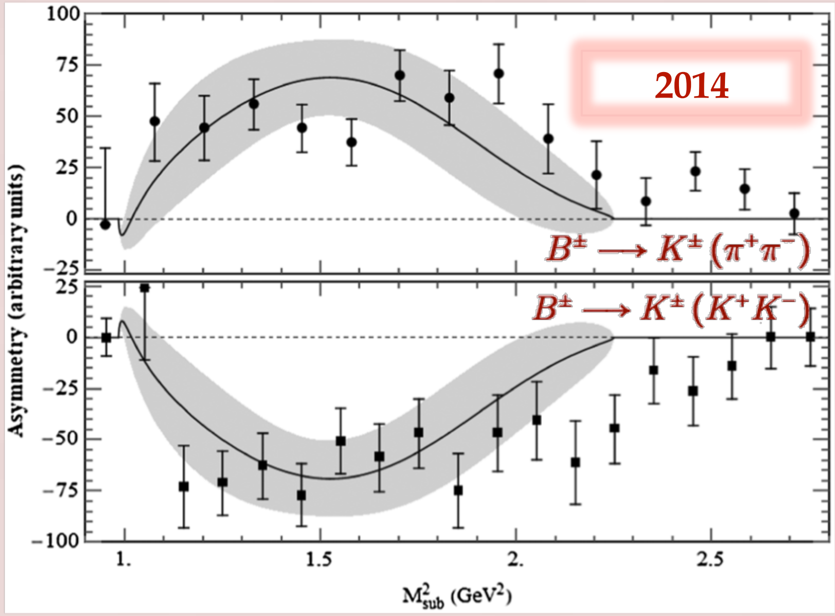
Let's see the improvements

[8] R. Álvarez Garrote, J. Cuervo, P. Magalhaes, J.R.Peláez Phys.Rev.Lett. 130, (2023) 201901

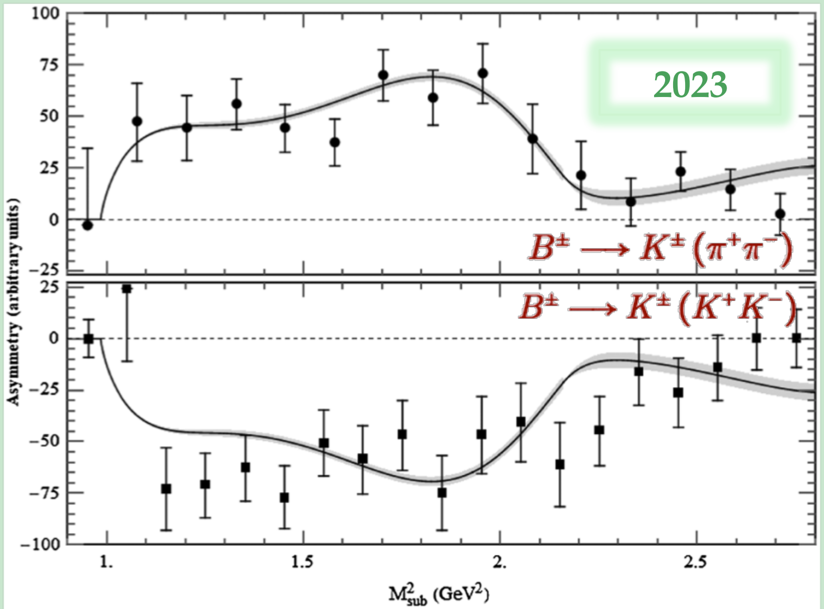
Using the dispersive $\pi\pi \longrightarrow KK$ parameterization

[8] R. Álvarez Garrote, J. Cuervo, P. Magalhaes, J.R.Peláez Phys.Rev.Lett. 130, (2023) 201901

With crude estimates 1. 2.



With dispersive $\pi\pi \longrightarrow KK$



- **Dramatic reduction in uncertainties**
- **Resonant features resolved**
- **Even beyond 1.5 GeV, still fairly reasonable**

It makes sense to add subdominant contributions

Slightly modified model including a mimicked source term with a mild energy dependence apart from the rescattering [7]

[7] Alvarenga, Bediaga, Frederico Phys. Rev. D 92 (2015) 0954010

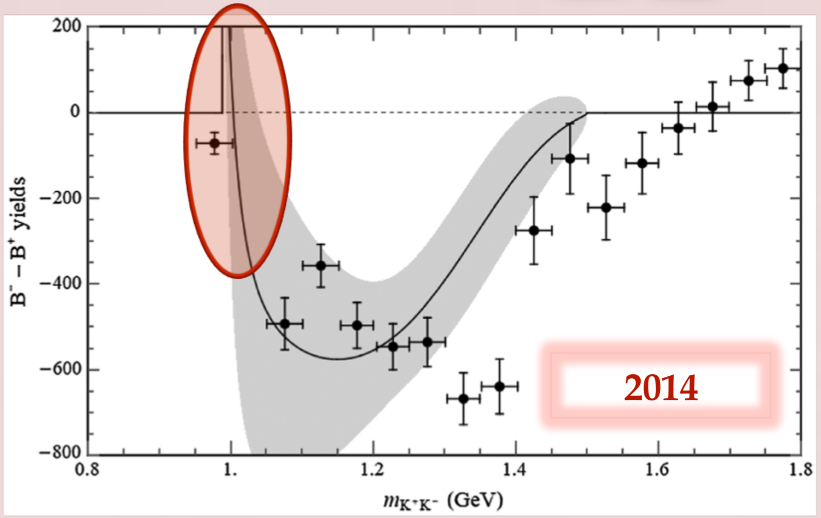
$$\Delta\Gamma_{KK} \simeq \frac{C\,|S_{\pi\pi KK}|\cos(\delta_{\pi\pi KK}+\Phi_{KK})\,F(M^2_{KK})}{(1+s/\Lambda^2_{\pi\pi})(1+s/\Lambda^2_{KK})}$$

$\Lambda_{\pi\pi} = 3$ GeV

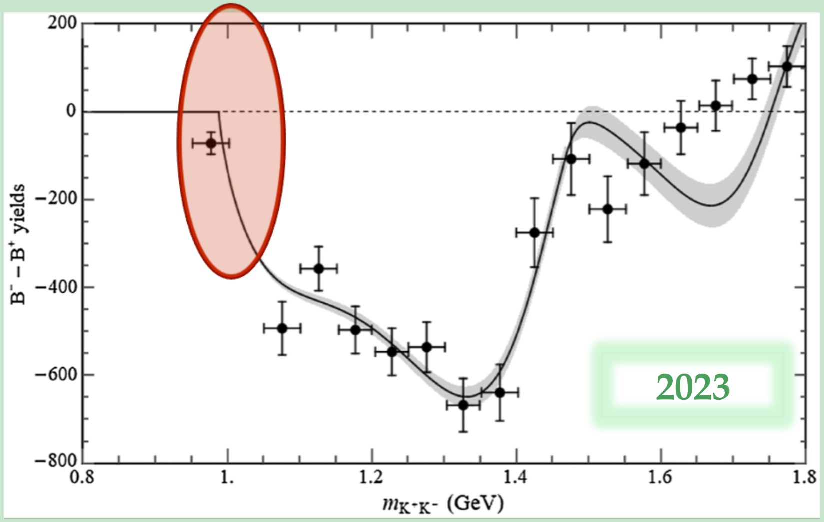
$\Lambda_{KK} = 4$ GeV

[8] R. Álvarez Garrote, J. Cuervo, P. Magalhaes, J.R.Peláez Phys.Rev.Lett. 130, (2023) 201901

With crude estimates 1. 2.



With dispersive $\pi\pi \longrightarrow KK$



Kinematic factor correction at threshold so that the strange non-physical peak disappears

$B \longrightarrow KKK$ data from the whole RUN1

- [5] showed the predominant role of FSI in giant CPV of charmless $B \longrightarrow 3M$ decays in the $1-1.5$ GeV range

[5] Bediaga, Frederico, Lourenco, Phys. Rev. D 89 (2014) 094013

- Very crude estimates where widely used in the literature, particularly by LHCb

- Recent proposal to amend these estimates in [8]

[8] R. Álvarez Garrote, J. Cuervo, P. Magalhaes, J.R.Peláez Phys.Rev.Lett. 130, 201901 (2023)

Confirmed that FSI providing the strong phase in CPV requires realistic and accurate $\pi\pi \longrightarrow KK$ amplitudes

Dramatic increase in precision and the unveiling of distinct meson-scale dynamics (resonant shapes) in the giant CPV observation

- The new precision paves the way for more rigorous studies of FSI

HOW CAN WE IMPROVE THIS?

Recalling…

If λ can couple to other channels λ' (FSI):

[3] M. Suzuki and L. Wolfenstein, Phys. Rev. D 60, 074019 (1999)

$$\mathcal{A} = S^{1/2} \mathcal{A}^0$$

Hadronic FSI interaction

Amplitude without FSI

(Over the right cut)

- **Expanding S to LO (LHCb) :**

$$S_{\lambda\lambda'} = \delta_{\lambda\lambda'} + 2i\hat{f}_{\lambda\lambda'}$$

$$\mathcal{A}^{\pm} = A_\lambda + B_\lambda e^{\pm i\gamma}$$

$$\mathcal{A}^{\pm}_{\text{LO}} = A^0_\lambda + B^0_\lambda e^{\pm i\gamma} + i\sum_{\lambda'} \hat{f}_{\lambda\lambda'}\left(A^0_{\lambda'} + B^0_{\lambda'} e^{\pm i\gamma}\right)$$

What we are doing :

If λ can couple to other channels λ' (FSI):

[3] M. Suzuki and L. Wolfenstein, Phys. Rev. D 60, 074019 (1999)

$$\mathcal{A} = \underbrace{S^{1/2}}_{\text{Hadronic FSI interaction}} \mathcal{A}^0$$

Amplitude without FSI

(Over the right cut)

~~**Expanding S to LO**~~ **(LHCb) :**

$$\cancel{S_{\lambda\lambda'} = \delta_{\lambda\lambda'} + 2i\hat{f}_{\lambda\lambda'}}$$

$$\mathcal{A}^{\pm} = A_\lambda + B_\lambda e^{\pm i\gamma}$$

$$\mathcal{A}^{\pm}_{\cancel{\mathrm{LO}}} = A^0_\lambda + B^0_\lambda e^{\pm i\gamma} + i\sum_{\lambda'} \hat{f}_{\lambda\lambda'}\left(A^0_{\lambda'} + B^0_{\lambda'} e^{\pm i\gamma}\right)$$

- Work with the complete S matrix instead of expanding it to leading order
- Explicit calculation of the square root of the matrix without Taylor series approximation
- Avoid unnecessary simplifications involving the parameters in the QCD amplitude A^0_λ , B^0_λ

We start with the general S matrix

$$S_{\lambda\lambda'} = \begin{pmatrix} |S_{\pi\pi\pi\pi}|\, e^{2i\delta_{\pi\pi\pi\pi}} & i\,|S_{\pi\pi KK}|\, e^{i\delta_{\pi\pi KK}} & \cdots \\ i\,|S_{\pi\pi KK}|\, e^{i\delta_{\pi\pi KK}} & |S_{KKKK}|\, e^{2i\delta_{KKKK}} & \cdots \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

- **We need to diagonalize the matrix $\longrightarrow$**
  $\longrightarrow$ Two coupled channels
- **In terms of $|S_{\pi\pi KK}|$, $\delta_{\pi\pi\pi\pi}$ and $\delta_{\pi\pi KK}$**
- **Skip 1. and 2. crude estimates**

$$S_{\lambda\lambda'} = \begin{pmatrix} \sqrt{1-|S_{\pi\pi KK}|^2}\, e^{2i\delta_{\pi\pi\pi\pi}} & i\,|S_{\pi\pi KK}|\, e^{i\delta_{\pi\pi KK}} \\ i\,|S_{\pi\pi KK}|\, e^{i\delta_{\pi\pi KK}} & \sqrt{1-|S_{\pi\pi KK}|^2}\, e^{2i(\delta_{\pi\pi KK}-\delta_{\pi\pi\pi\pi})} \end{pmatrix}$$

- **We compute the eigenvalues of the matrix without any unnecessary approximations $\longrightarrow$ We obtain the following two eigenvalues**

$$a_{\frac{1}{2}} = \frac{e^{2i(\delta_{\pi\pi KK}-\delta_{\pi\pi\pi\pi})}+e^{2i\delta_{\pi\pi\pi\pi}}}{2}\cos\alpha \mp \sqrt{\left(\frac{e^{2i(\delta_{\pi\pi KK}-\delta_{\pi\pi\pi\pi})}+e^{2i\delta_{\pi\pi\pi\pi}}}{2}\cos\alpha\right)^2 - e^{2i\delta_{\pi\pi KK}}}$$

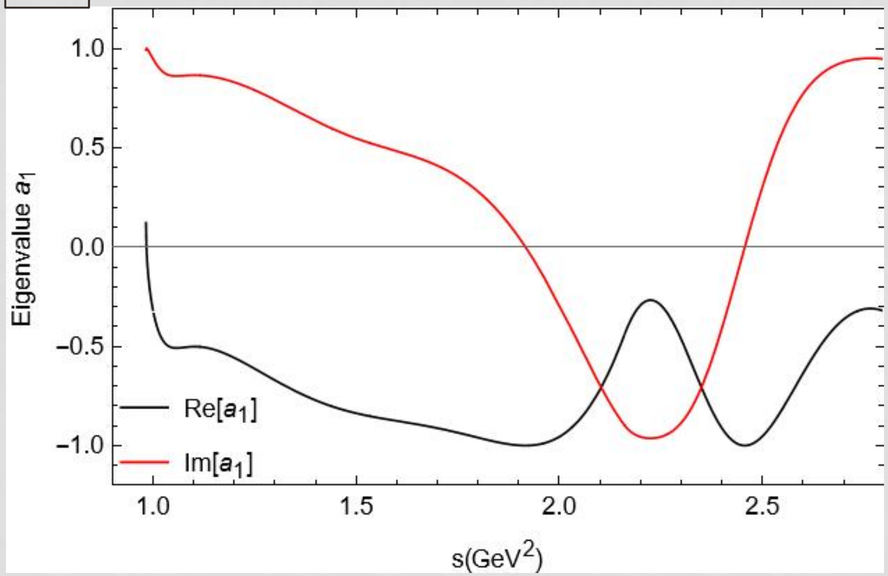
$\longrightarrow$ $|S_{\pi\pi KK}| \equiv \sin\alpha$, $\sqrt{1-|S_{\pi\pi KK}|^2} \equiv \cos\alpha$

- **They are functions of $|S_{\pi\pi KK}|$, $\delta_{\pi\pi\pi\pi}$ and $\delta_{\pi\pi KK}$**

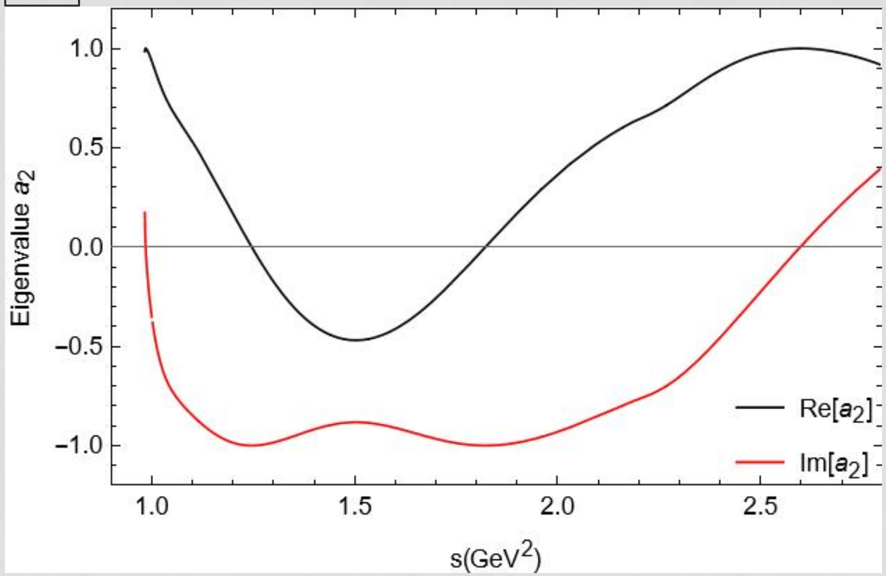
The eigenvalues we obtain:

$$a_{\substack{1\\2}} = \frac{e^{2i(\delta_{\pi\pi\mathrm{KK}}-\delta_{\pi\pi\pi\pi})}+e^{2i\delta_{\pi\pi\pi\pi}}}{2}\cos\alpha \mp \sqrt{\left(\frac{e^{2i(\delta_{\pi\pi\mathrm{KK}}-\delta_{\pi\pi\pi\pi})}+e^{2i\delta_{\pi\pi\pi\pi}}}{2}\cos\alpha\right)^2 - e^{2i\delta_{\pi\pi\mathrm{KK}}}}$$

a_1



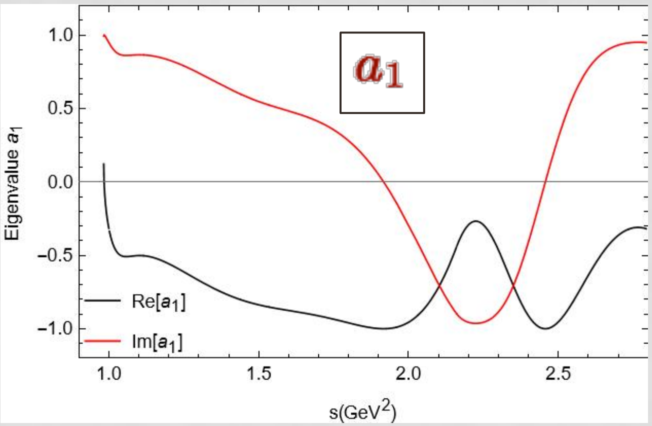
a_2

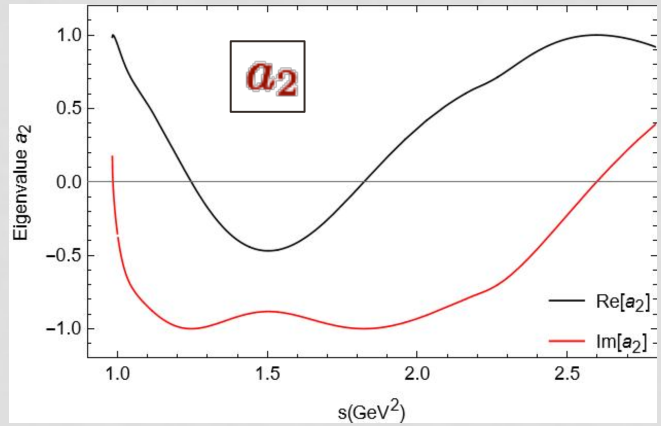


Computing the square root of the S matrix

The eigenvalues we obtain:

$$a_{\frac{1}{2}} = \frac{e^{2i(\delta_{\pi\pi KK}-\delta_{\pi\pi\pi\pi})}+e^{2i\delta_{\pi\pi\pi\pi}}}{2}\cos\alpha \mp \sqrt{\left(\frac{e^{2i(\delta_{\pi\pi KK}-\delta_{\pi\pi\pi\pi})}+e^{2i\delta_{\pi\pi\pi\pi}}}{2}\cos\alpha\right)^2 - e^{2i\delta_{\pi\pi KK}}}$$

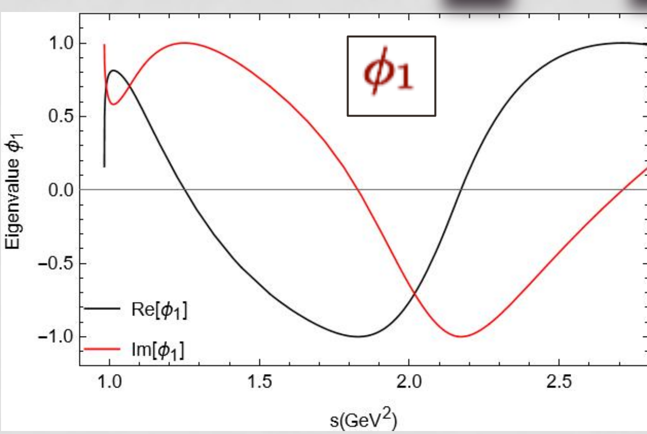


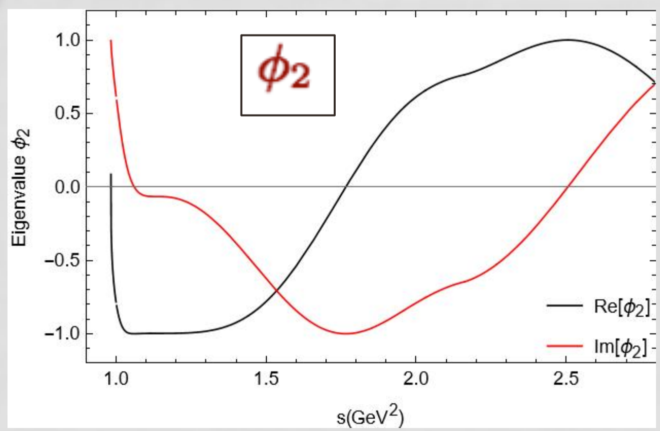


Quite different

The first attempt [10] to compute this square root also inherited 1. and 2. crude estimates

$$\phi_{\frac{1}{2}} = e^{i(2\delta_{\pi\pi\pi\pi}\mp\alpha)}$$





Crude estimates head to huge simplifications and unrealistic eigenvalues

[10] H. Y. Cheng, C.K. Chua, Z.Q. Zhang Phys. Rev. D 94.9 (2016) 094015

"Complete-S matrix FSI model"

$$S^{1/2}_{\lambda\lambda'} = \underbrace{\frac{\sqrt{a_1}-\sqrt{a_2}}{2\sqrt{\left(\frac{a_1+a_2}{2}\right)^2 - e^{i\delta_{\pi\pi KK}}}}}_{\equiv\, s_1}\left(\begin{pmatrix} e^{2i(\delta_{\pi\pi KK}-\delta_{\pi\pi\pi\pi})}\cos\alpha & -ie^{i\delta_{\pi\pi KK}}\sin\alpha \\ -ie^{i\delta_{\pi\pi KK}}\sin\alpha & e^{2i\delta_{\pi\pi\pi\pi}}\cos\alpha \end{pmatrix} - \underbrace{\frac{\sqrt{a_1^3}-\sqrt{a_2^3}}{\sqrt{a_1}-\sqrt{a_2}}}_{\equiv\, s_2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right)$$

$$\mathcal{A} = S^{1/2}\mathcal{A}^0$$

Amplitude without FSI

- **Amplitude beyond LO :**

$$\boxed{\begin{aligned}\mathcal{A}^{\pm}_{KK} = S_1\Big(\left(A^0_2 + B^0_2 e^{\pm i\gamma}\right)&\left(e^{2i(\delta_{\pi\pi KK}-\delta_{\pi\pi\pi\pi})}\cos\alpha - S_2\right) - \\ &- \left(A^0_1 + B^0_1 e^{\pm i\gamma}\right) ie^{i\delta_{\pi\pi KK}}\sin\alpha\Big)\end{aligned}}$$

"Complete-S matrix FSI model"

$$S^{1/2}_{\lambda\lambda'} = \underbrace{\frac{\sqrt{a_1}-\sqrt{a_2}}{2\sqrt{\left(\frac{a_1+a_2}{2}\right)^2-e^{i\delta_{\pi\pi KK}}}}}_{\equiv s_1}\left(\begin{pmatrix} e^{2i(\delta_{\pi\pi KK}-\delta_{\pi\pi\pi\pi})}\cos\alpha & -ie^{i\delta_{\pi\pi KK}}\sin\alpha \\ -ie^{i\delta_{\pi\pi KK}}\sin\alpha & e^{2i\delta_{\pi\pi\pi\pi}}\cos\alpha \end{pmatrix} - \underbrace{\frac{\sqrt{a_1^3}-\sqrt{a_2^3}}{\sqrt{a_1}-\sqrt{a_2}}}_{\equiv s_2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right)$$

$$\mathcal{A} = S^{1/2}\mathcal{A}^0$$

Amplitude without FSI

- Amplitude beyond LO
- Decay width asymmetry:

$$\Delta\Gamma_{KK} = |S_1|^2\Big(\mathcal{C}\sin\alpha\left(\mathrm{Re}\left[S_2 e^{-i(\delta_{\pi\pi KK}-\Phi_{KK})}\right] - \cos\left(2\delta_{\pi\pi\pi\pi}-\delta_{\pi\pi KK}-\Phi_{KK}\right)\cos\alpha\right) + \mathcal{K}\left(\cos^2\alpha-\sin^2\alpha+2\mathrm{Re}\left[S_2 e^{-2i(\delta_{\pi\pi KK}-\delta_{\pi\pi\pi\pi})}\right]+|S_2|^2\right)\Big)F(M_K^2)$$

→ Three functions describing the dispersion

→ Three parameters

$$\mathcal{C} = 4|K|\sin\gamma$$

$$K = |K|e^{i\Phi_{KK}} = A_1^{0*}B_2^0 - A_2^0 B_1^{0*}$$

$F(M_K^2) \longrightarrow$ Dalitz plot kinematic factor

$$\mathrm{Im}\left[A_1^{0*}B_1^0\right] = \mathrm{Im}\left[A_2^{0*}B_2^0\right] = 0$$

[5] Bediaga, Frederico, Lourenco, Phys. Rev. D 89 (2014) 094013

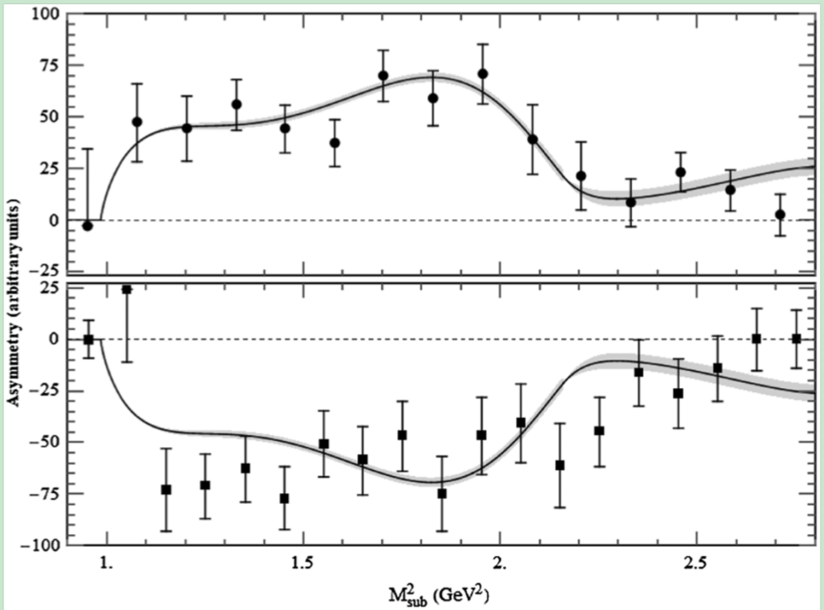
CPT constraint

$$\sum_{\lambda,J}\mathrm{Im}\left[A_{\lambda,J}^{0*}B_{\lambda,J}^0\right] = 0$$

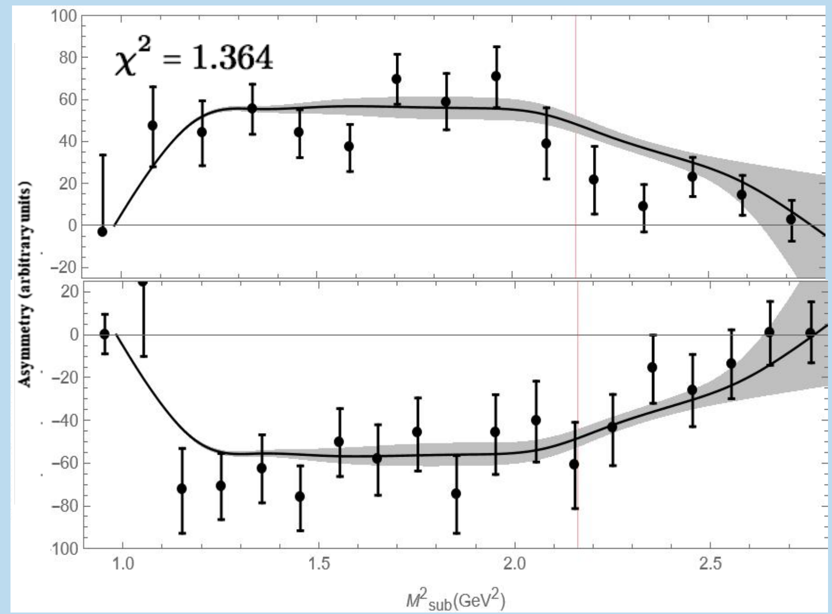
$$\mathcal{K} = 4\sin\gamma\ \mathrm{Im}\left[A_1^{0*}B_1^0\right] = -4\sin\gamma\ \mathrm{Im}\left[A_2^{0*}B_2^0\right]$$

PRELIMINARY First results using the "Complete-S matrix FSI model"

With dispersive $\pi\pi \longrightarrow KK$



Beyond Leading Order



- **We have fit the model only up to** 1.47 GeV, 2.16 GeV^2
- **Increase in the model uncertainty due to the inclusion of** $\pi\pi \longrightarrow \pi\pi$, **through** $\delta_{\pi\pi\pi\pi}$ **function. Still fairly reasonable**
- **Keeps describing the experimental data**

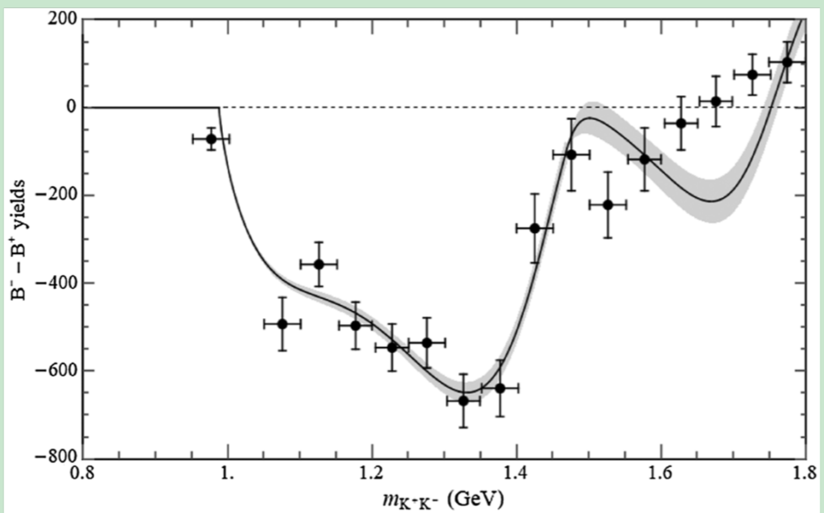
PRELIMINARY First results using the "Complete-S matrix FSI model"

Including the mimicked source in [7]:

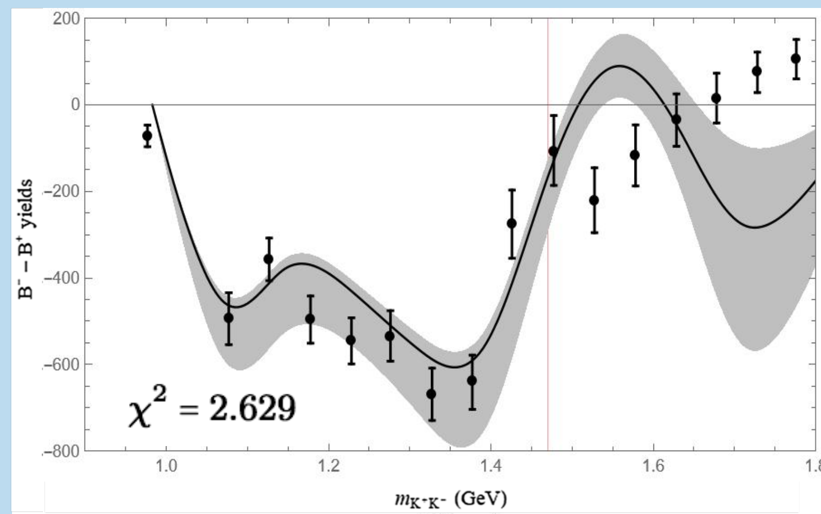
[7] Alvarenga, Bediaga, Frederico Phys. Rev. D 92 (2015) 0954010

$$\Delta\Gamma_{KK} = |S_1|^2 \left(\mathcal{C} \sin\alpha \frac{\mathrm{Re}\left[S_2 e^{-i(\delta_{\pi\pi KK} - \Phi_{KK})}\right] - \cos(2\delta_{\pi\pi\pi\pi} - \delta_{\pi\pi KK} - \Phi_{KK}) \cos\alpha}{(1+s/\Gamma_{\pi\pi}^2)(1+s/\Gamma_{KK})} + \mathcal{K}\left(-\frac{\sin^2\alpha}{(1+s/\Gamma_{\pi\pi}^2)^2} + \frac{\cos^2\alpha + 2\mathrm{Re}\left[S_2 e^{-2i(\delta_{\pi\pi KK} - \delta_{\pi\pi\pi\pi})}\right] + |S_2|^2}{(1+s/\Gamma_{KK})^2} \right) \right) F(M_K^2)$$

With dispersive $\pi\pi \longrightarrow KK$



Beyond Leading Order



- Resonant features well resolved

PRELIMINARY Discussion: If strong phase only comes from FSI

REAL coming from QCD interaction and therefore symmetric under CP

$$\mathcal{A}^{\pm} = A_\lambda + B_\lambda e^{\pm i\gamma}$$

γ **: weak phase changes sign under CP conjugation**

- **Decay width asymmetry:**

$$\Delta\Gamma_{KK} = |S_1|^2 \Big(\mathcal{C}\sin\alpha\Big(\mathrm{Re}\big[S_2 e^{-i(\delta_{\pi\pi KK}-\Phi_{KK})}\big] - \cos\left(2\delta_{\pi\pi\pi\pi} - \delta_{\pi\pi KK} - \Phi_{KK}\right)\cos\alpha\Big) + \mathcal{K}\Big(\cos^2\alpha - \sin^2\alpha + 2\mathrm{Re}\big[S_2 e^{-2i(\delta_{\pi\pi KK}-\delta_{\pi\pi\pi\pi})}\big] + |S_2|^2\Big)\Big) F(M_K^2)$$

→ **Three functions describing the dispersion**

→ **Three parameters**

$$\mathcal{C} = 4|K|\sin\gamma$$

$$K = |K|e^{i\Phi_{KK}} = A_1^{0*}B_2^0 - A_2^0 B_1^{0*}$$

$F(M_K^2) \longrightarrow$ **Dalitz plot kinematic factor**

PRELIMINARY Discussion: If strong phase only comes from FSI

REAL coming from QCD interaction and therefore symmetric under CP

$$\mathcal{A}^{\pm} = A_\lambda + B_\lambda e^{\pm i\gamma}$$

γ : **weak phase changes sign under CP conjugation**

- **Decay width asymmetry:**

$$\Delta\Gamma_{KK} = |S_1|^2 \Big(\mathcal{C} \sin\alpha \big(\mathrm{Re}\big[S_2 e^{-i(\delta_{\pi\pi KK} \qquad)} \big] - \cos\left(2\delta_{\pi\pi\pi\pi} - \delta_{\pi\pi KK} - \qquad \right) \cos\alpha \big) + \qquad \Big) F(M_K^2)$$

Three functions describing the dispersion

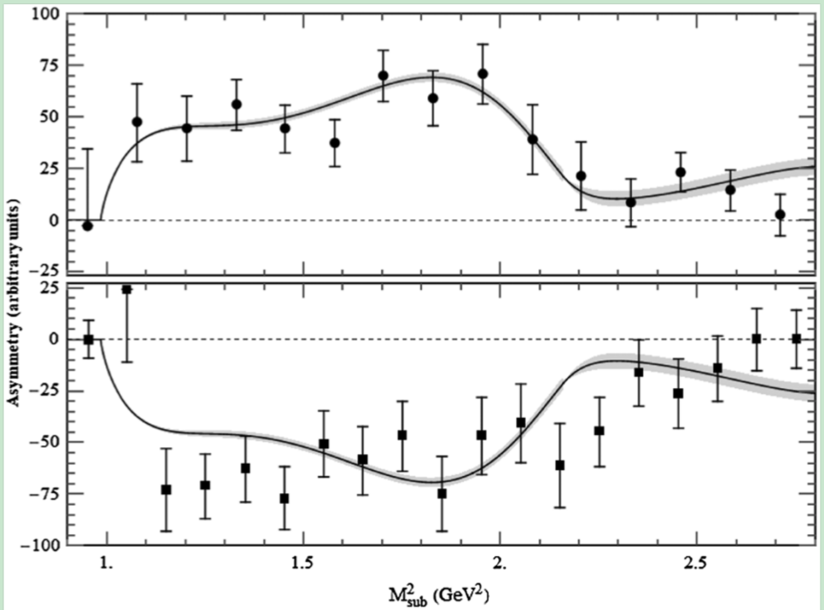
ONE **parameter**

$$\mathcal{C} = 4|K| \sin\gamma$$

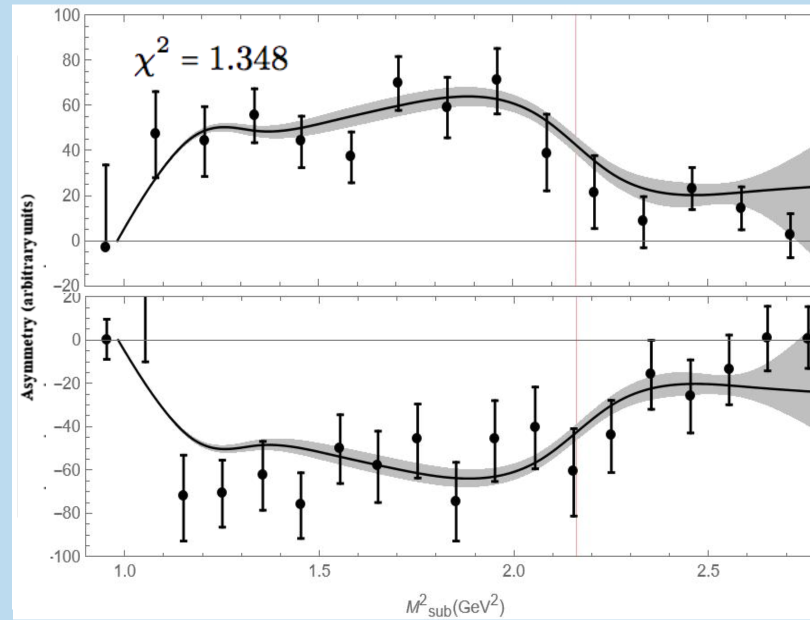
$F(M_K^2) \longrightarrow$ **Dalitz plot kinematic factor**

PRELIMINARY Discussion: If strong phase only comes from FSI

With dispersive $\pi\pi \longrightarrow KK$



Beyond LO Strong phase only from FSI



- **The fitting is comparable since $\chi^2 = 1.348$ in this limit and $\chi^2 = 1.364$ with the complete model**
- **This hypothesis is still a good description of the data**

PRELIMINARY Discussion: If strong phase only comes from FSI

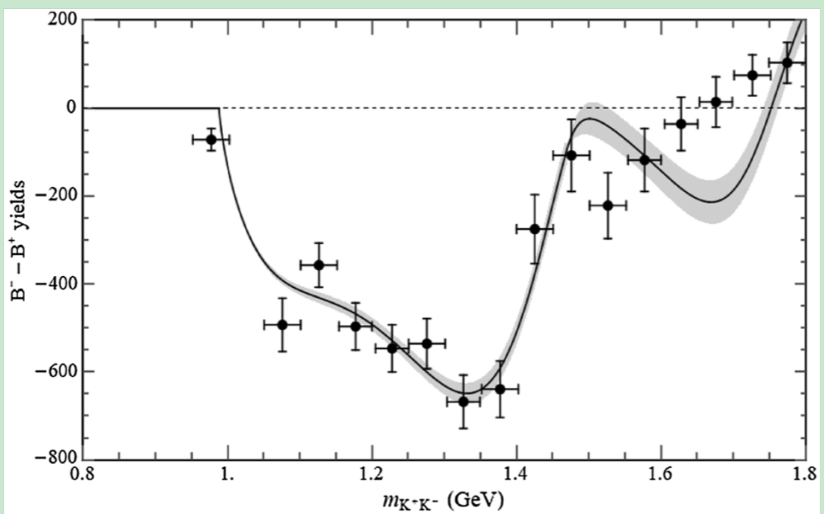
Including the mimicked source in [7]:

[7] Alvarenga, Bediaga, Frederico Phys. Rev. D 92 (2015) 0954010

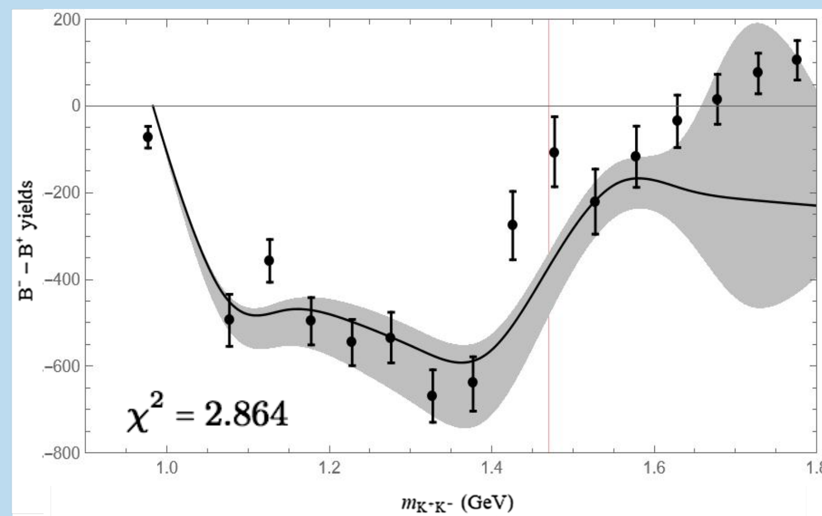
$$\Delta\Gamma_{KK} = |S_1|^2\, \mathcal{C} \sin\alpha \frac{\mathrm{Re}\left[S_2 e^{-i\delta_{\pi\pi KK}}\right] - \cos(2\delta_{\pi\pi\pi\pi} - \delta_{\pi\pi KK})\cos\alpha}{(1+s/\Gamma^2_{\pi\pi})(1+s/\Gamma_{KK})} F(M_K^2)$$

$B \longrightarrow KKK$ data from the whole RUN1

With dispersive



Beyond LO **Strong phase only from FSI**



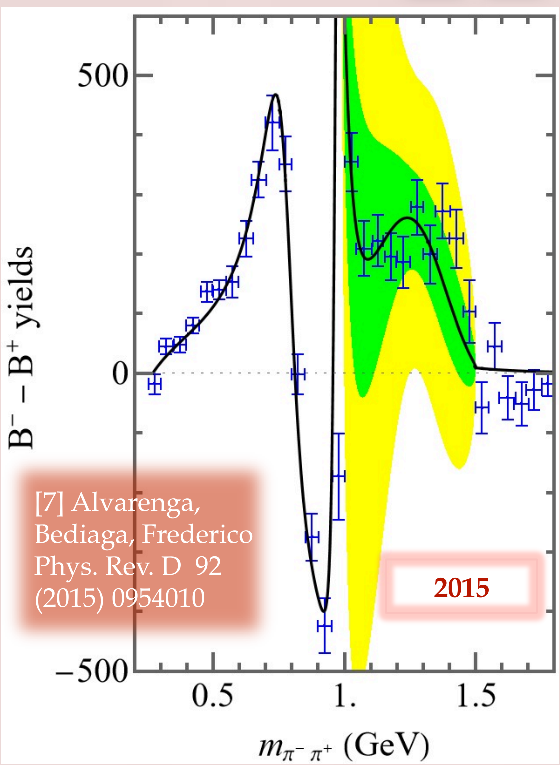
- The fitting is still comparable; $\chi^2 = 2.864$ now and $\chi^2 = 2.629$ before
- Describes the data

LHCb is actually using a more complete model where not only the is added, but also more waves

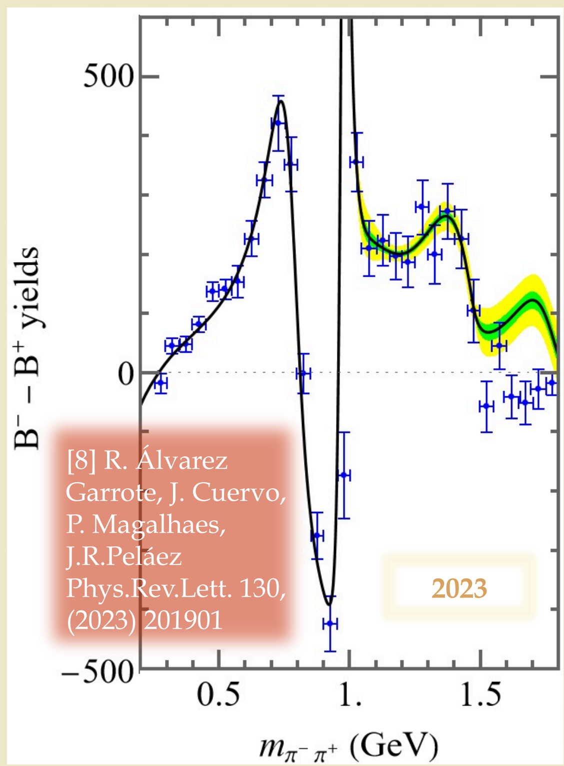
[7] Alvarenga, Bediaga, Frederico Phys. Rev. D 92 (2015) 0954010

$$\mathcal{A}^{\pm}_{\mathrm{LO}} = \underbrace{\sum_J (a^J_{\lambda\mathrm{NR}} + b^J_{\lambda\mathrm{NR}} e^{\pm i\gamma})/(1+s/\Lambda^2_\lambda)}_{\text{Non-resonant}} + \overbrace{\sum_{JR}(a^R_\lambda + b^R_\lambda e^{\pm i\gamma}) F^{\mathrm{BW}}_{R\lambda}}^{\text{Breit-Wigner resonances for } \rho(770) \text{ and } f_0(980)} P_J(\cos\theta) + i\underbrace{\sum_{\lambda',J} \hat{f}^J_{\lambda'\lambda}(a^J_{\lambda'\mathrm{NR}} + b^J_{\lambda'\mathrm{NR}} e^{\pm i\gamma})/(1+s/\Lambda^2_{\lambda'})}_{\text{FSI for } \pi\pi \longrightarrow KK}$$

With crude estimates 1. 2.



With dispersive $\pi\pi \longrightarrow KK$



$B \longrightarrow K\pi\pi$ data from the whole RUN1

- **Consistent FSI interactions**
- **Improved accuracy**
- **Clearer resonant structures**
- **Includes a crude $\pi\pi \longrightarrow \pi\pi$ analyses**

Summary

- [5] showed the **predominant role of FSI in Giant CPV** of charmless $B \longrightarrow 3M$ decays in the $1-1.5$ GeV range

[5] Bediaga, Frederico, Lourenco, Phys. Rev. D 89 (2014) 094013

- **Very crude estimates** in the literature, also at LHCb

[8] R. Álvarez Garrote, J. Cuervo, P. Magalhaes, J.R.Peláez Phys.Rev.Lett. 130, (2023) 201901

- Recent proposal to amend these estimates in [8] through **realistic and accurate** $\pi\pi \longrightarrow KK$ **amplitudes** that **dramatic increased the precision**

BUT all of these results arise from a **first-order expansion**

Summary

- [5] showed the predominant role of FSI in Giant CPV of charmless $B \longrightarrow 3M$ decays in the $1 - 1.5$ GeV range [5] Bediaga, Frederico, Lourenco, Phys. Rev. D 89 (2014) 094013
- Very crude estimates in the literature, also at LHCb [8] R. Álvarez Garrote, J. Cuervo, P. Magalhaes, J.R.Peláez Phys.Rev.Lett. 130, (2023) 201901
- Recent proposal to amend these estimates in [8] through realistic and accurate $\pi\pi \longrightarrow KK$ amplitudes that dramatic increased the precision

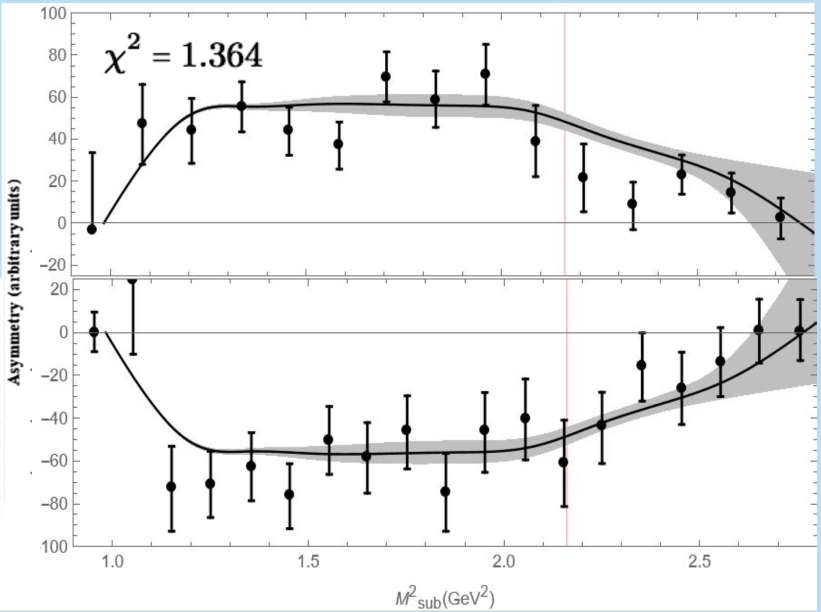
BUT all of these results arise from a first-order expansion

- Implementing a formalism beyond-Leading Order is much more complicated
- We are developing a complete formalism that contains the information in the whole S matrix based on [3] [3] M. Suzuki and L. Wolfenstein, Phys. Rev. D 60, (1999) 074019
- The eigenvalues we obtain differ from the ones in the literature [10] , since they inherit the crude estimates [10] H. Y. Cheng, C.K. Chua, Z.Q. Zhang Phys. Rev. D 94.9 (2016) 094015
- Our preliminary results still describe experimental data, confirming with a complete formalism the very relevant role of FSI in Giant CPV
- Similar good fits assuming the strong phase only comes from FSI

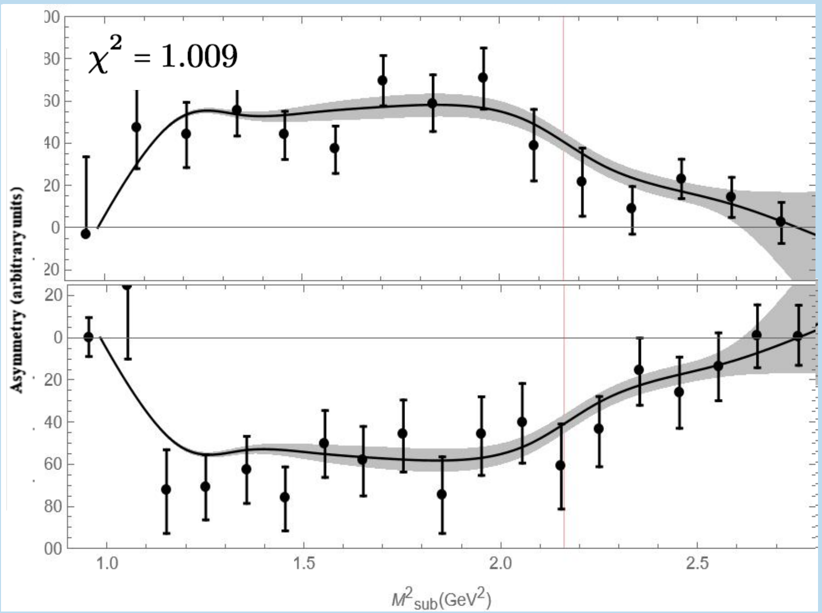
SPARE SLIDES

PRELIMINARY First results using the "Complete-S matrix FSI model"

Beyond LO 1.47 GeV



Beyond LO all data



- Comparing fitting only up to 1.47 GeV or fitting all the data

$$\chi^2 = 1.364 \longleftrightarrow \chi^2 = 1.009$$

- The second curve (with all the data) also provides a good fitting up to 1.47 GeV with $\chi^2 = 1.394$
- Beyond 1.5 GeV other decay modes become relevant

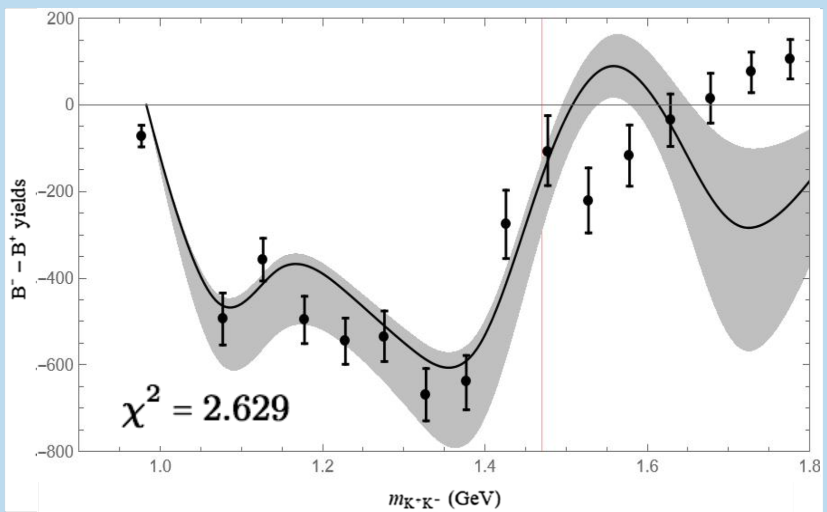
PRELIMINARY First results using the "Complete-S matrix FSI model"

Including the mimicked source in [7]:

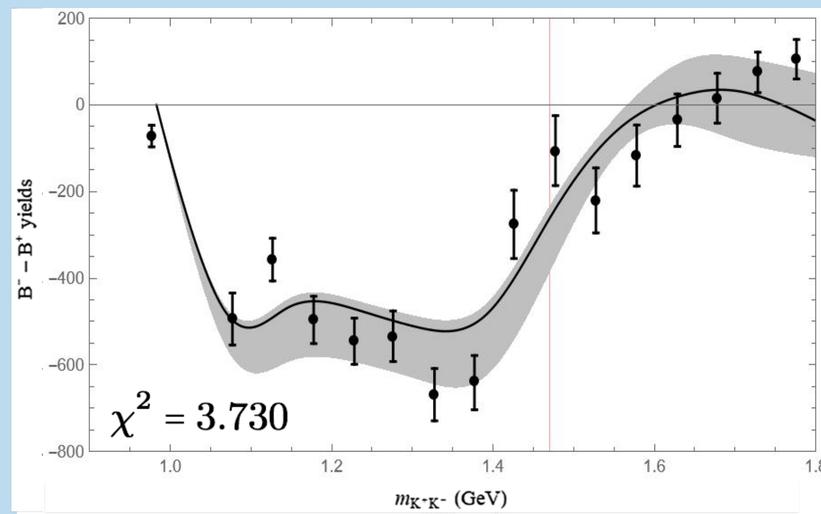
[7] Alvarenga, Bediaga, Frederico Phys. Rev. D 92 (2015) 0954010

$$\Delta\Gamma_{KK} = |S_1|^2 \left(\mathcal{C}\sin\alpha \frac{\mathrm{Re}\left[S_2 e^{-i(\delta_{\pi\pi KK}-\Phi_{KK})}\right] - \cos(2\delta_{\pi\pi\pi\pi}-\delta_{\pi\pi KK}-\Phi_{KK})\cos\alpha}{(1+s/\Gamma^2_{\pi\pi})(1+s/\Gamma_{KK})} + \mathcal{K}\left(-\frac{\sin^2\alpha}{(1+s/\Gamma^2_{\pi\pi})^2} + \frac{\cos^2\alpha + 2\mathrm{Re}\left[S_2 e^{-2i(\delta_{\pi\pi KK}-\delta_{\pi\pi\pi\pi})}\right] + |S_2|^2}{(1+s/\Gamma_{KK})^2}\right)\right) F(M_K^2)$$

Beyond LO 1.47 GeV

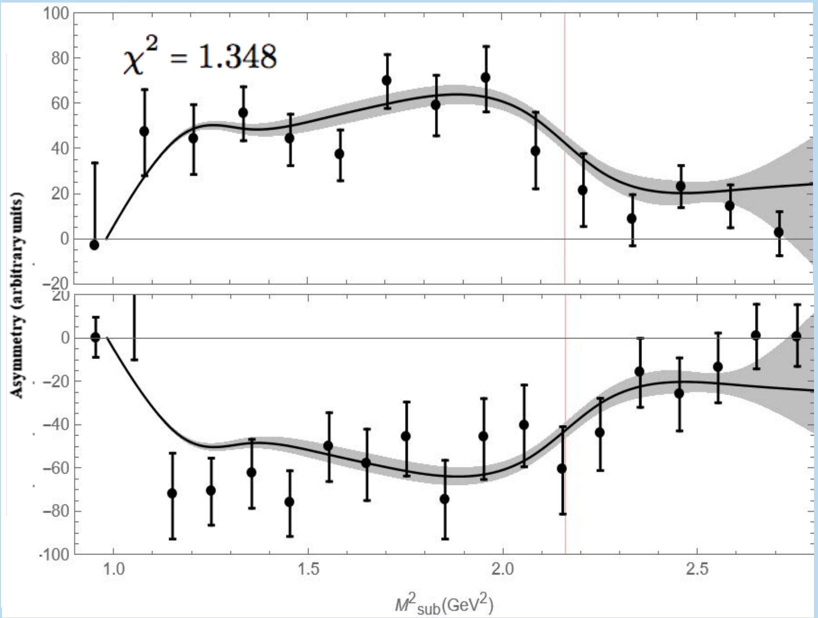


Beyond LO all data

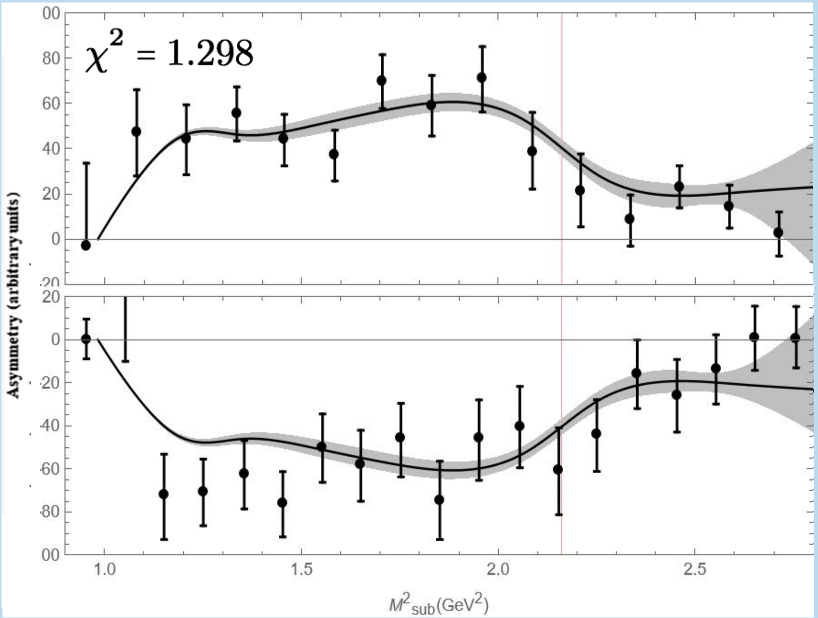


PRELIMINARY Discussion: If strong phase only comes from FSI

Strong phase only from FSI 1.47 GeV



Strong phase only from FSI all data



- Comparing fitting only up to 1.47 GeV or fitting all the data

$$\chi^2 = 1.348 \longleftrightarrow \chi^2 = 1.298$$

- The second curve (with all the data) also provides a good fitting up to 1.47 GeV with $\chi^2 = 1.382$
- Beyond 1.5 GeV other decay modes become relevant

PRELIMINARY Discussion: If strong phase only comes from FSI

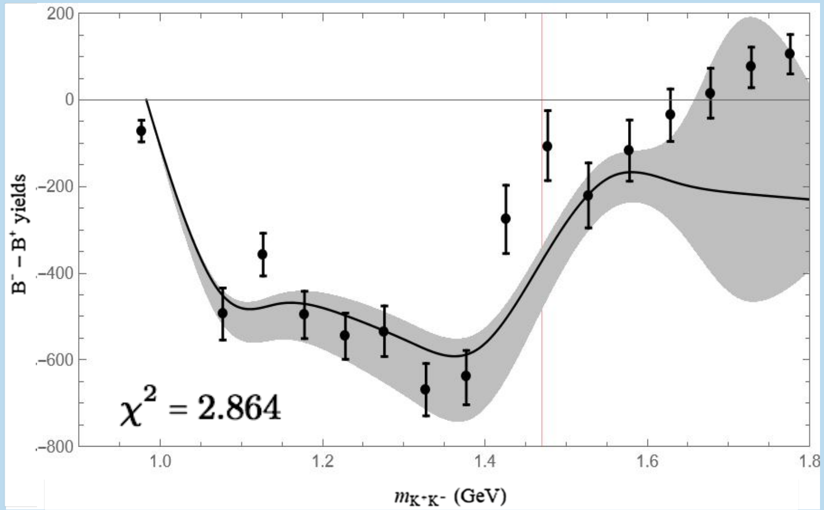
Including the mimicked source in [7]:

[7] Alvarenga, Bediaga, Frederico Phys. Rev. D 92 (2015) 0954010

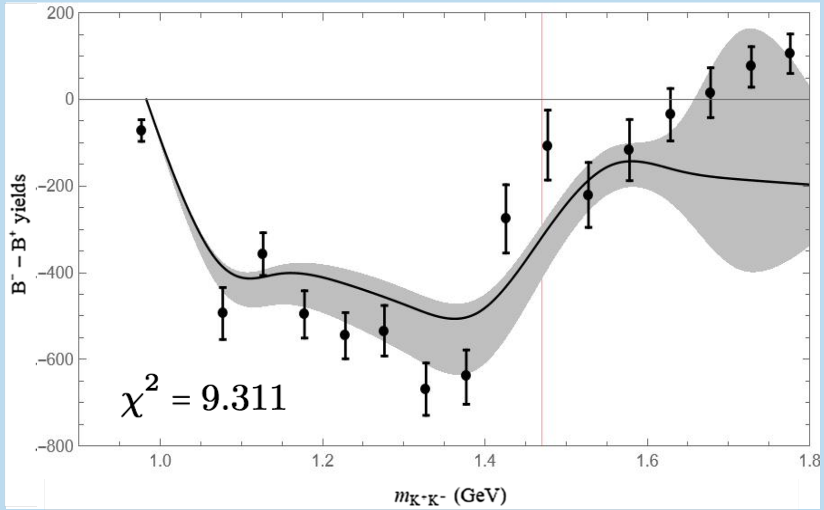
$$\Delta\Gamma_{KK} = |S_1|^2\, \mathcal{C} \sin\alpha \frac{\mathrm{Re}\left[S_2 e^{-i\delta_{\pi\pi KK}}\right] - \cos(2\delta_{\pi\pi\pi\pi} - \delta_{\pi\pi KK})\cos\alpha}{(1+s/\Gamma^2_{\pi\pi})(1+s/\Gamma_{KK})} F(M_K^2)$$

$B \longrightarrow KKK$ data from the whole RUN1

Strong phase only from FSI 1.47 GeV



Strong phase only from FSI all data



Using the dispersive $\pi\pi \longrightarrow KK$ parameterization

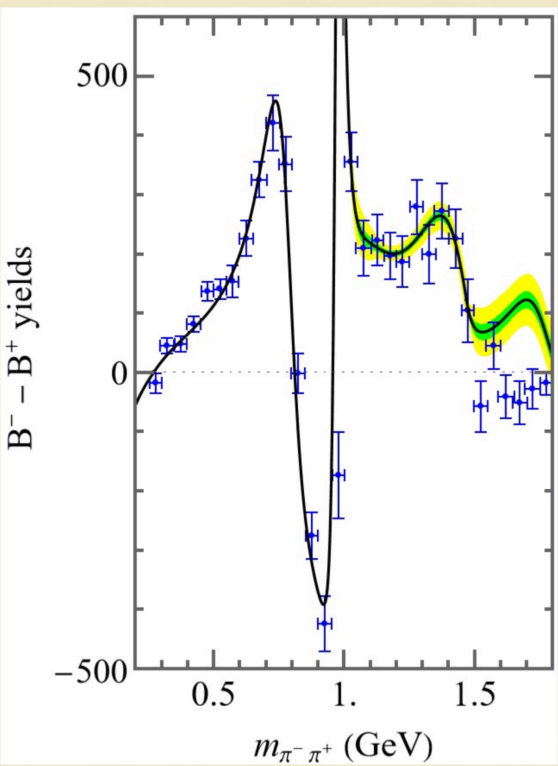
The simplest improvement to make the model more realistic:

[8] R. Álvarez Garrote, J. Cuervo, P. Magalhaes, J.R.Peláez Phys.Rev.Lett. 130, (2023) 201901

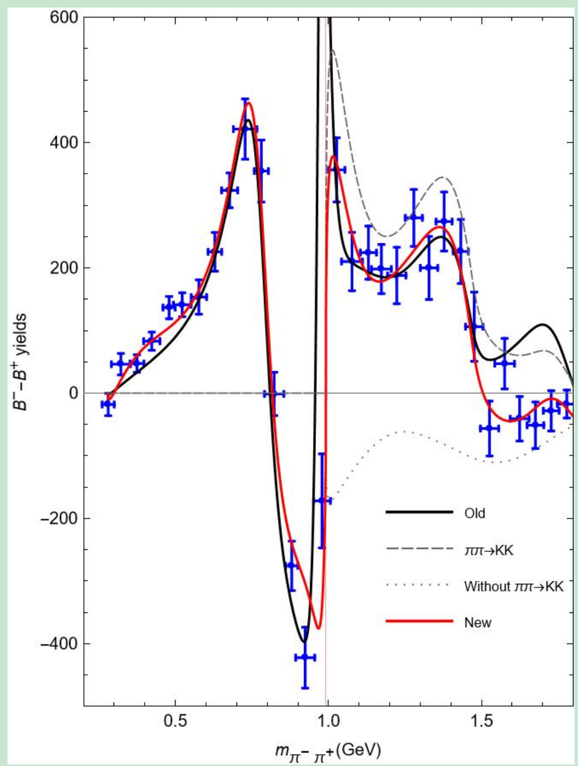
Replacing the naive $f_0(980)$ Breit-Wigner with realistic dispersive $\pi\pi \longrightarrow \pi\pi$ S-wave analysis

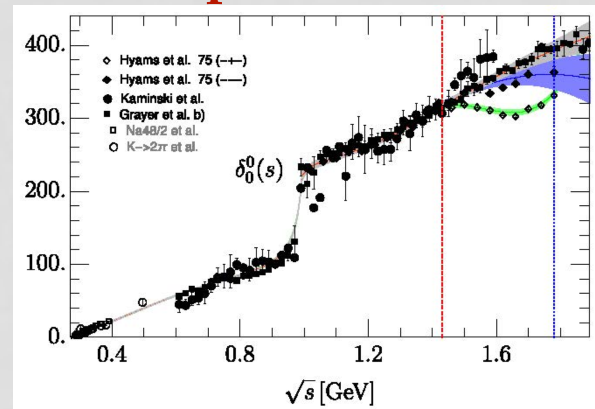
$B \longrightarrow K\pi\pi$ data from the whole RUN1

With $f_0(980)$ Breit-Wigner

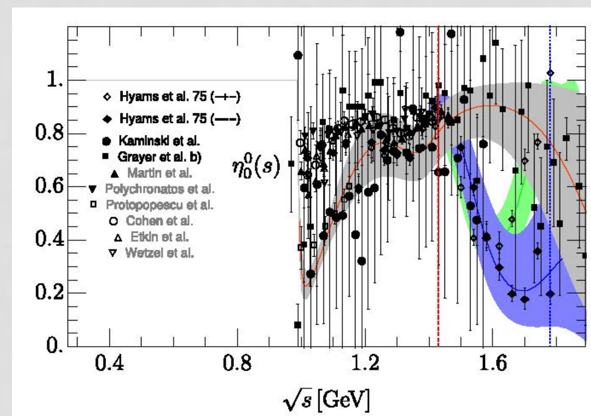


With realistic $\pi\pi \longrightarrow \pi\pi$





Phase shift



Elasticity

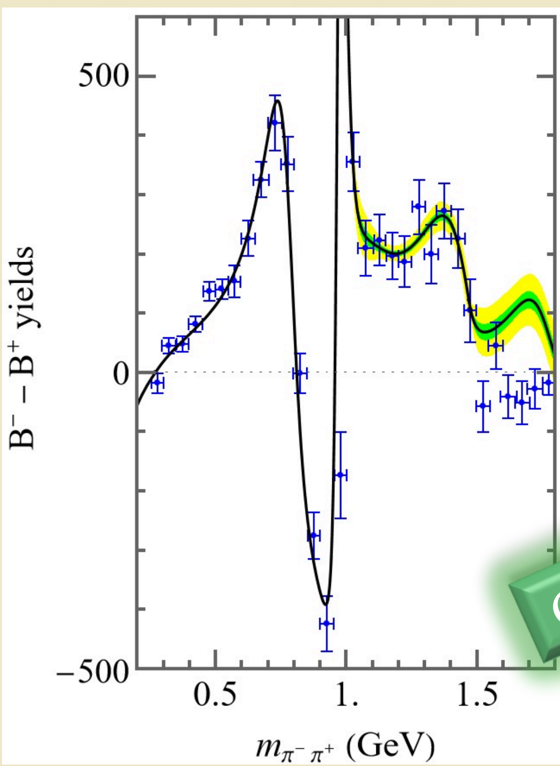
The simplest improvement to make the model more realistic:

[8] R. Álvarez Garrote, J. Cuervo, P. Magalhaes, J.R.Peláez Phys.Rev.Lett. 130, (2023) 201901

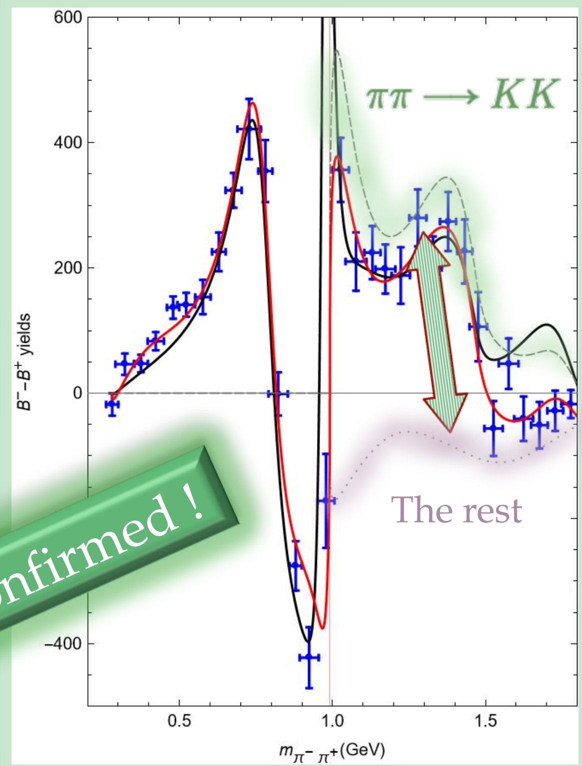
Replacing the naive $f_0(980)$ Breit-Wigner with realistic dispersive $\pi\pi \longrightarrow \pi\pi$ S-wave analysis

$B \longrightarrow K\pi\pi$ data from the whole RUN1

With $f_0(980)$ Breit-Wigner



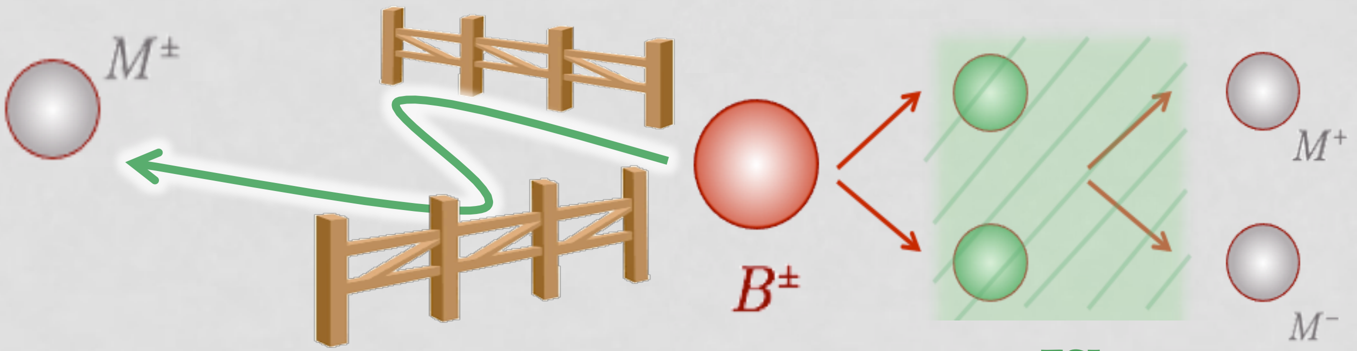
With realistic $\pi\pi \longrightarrow \pi\pi$



Confirmed !

- **$f_0(500)/\sigma$ resonance is present**
- **Spurious $f_0(980)$ peak is removed**
- **Not a bad behavior even beyond 1.5 GeV**
- **FSI $\pi\pi \longrightarrow KK$ dominates CPV between 1 to 1.5 GeV**

- The spectator does not interact at all, **any complex phase is added**



REAL coming from QCD interaction and therefore symmetric under CP

$$\mathcal{A}^{\pm} = A_{\lambda} + B_{\lambda} e^{\pm i\gamma}$$

γ : **weak phase changes sign under CP conjugation**