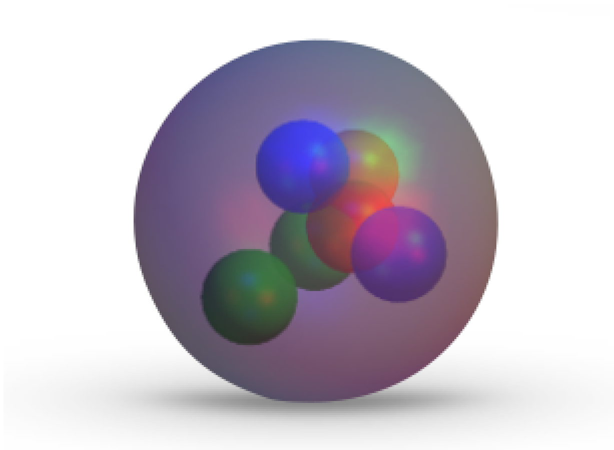
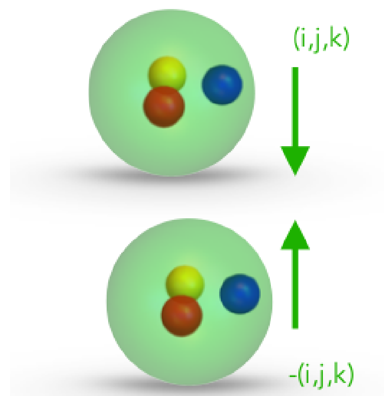
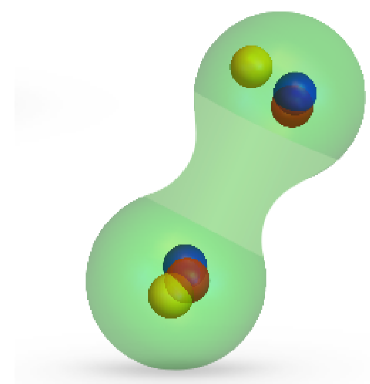
Two-Baryon Spectroscopy From Lattice QCD







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for NPLQCD

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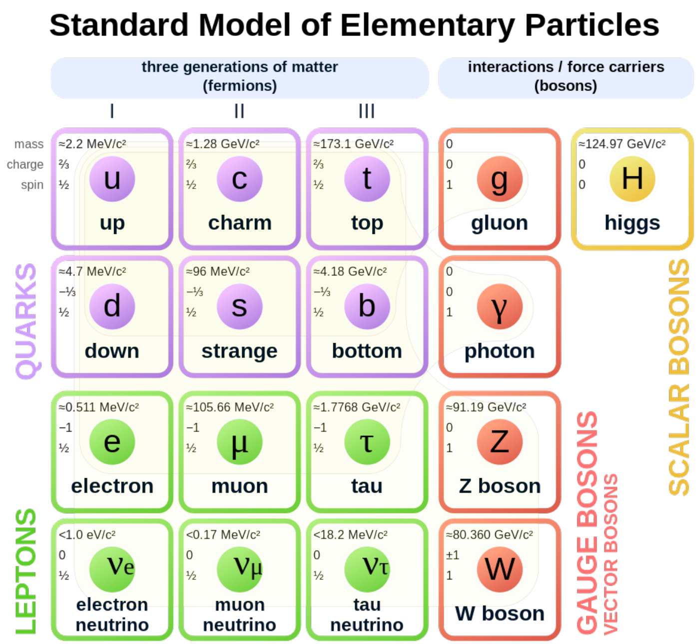
Outline

- Motivation
- Scattering Amplitudes from Lattice QCD
- Variational Analysis of NN
  - Observation of resonant-like state at de ~ 0.07.
- Conclusions

Motivation

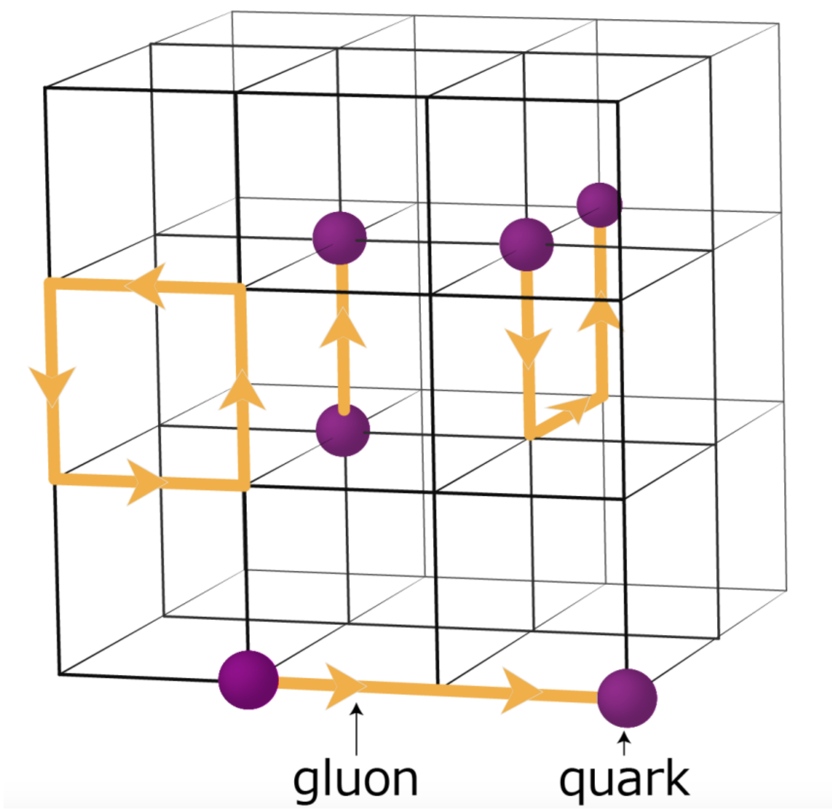


LQCD can provide important input for both understanding the SM and constraining BSM physics.

Nuclear matrix elements required for:

- double-beta decay,
- dark matter direct detection,
- neutrino-nucleus scattering
- Constrain nuclear EFTs

Introduction to LQCD



- Provides a non-perturbative definition of field theory
- Discretize space-time.
- Place in finite volume
- Regulates IR and UV divergences → suitable for numerical evaluation

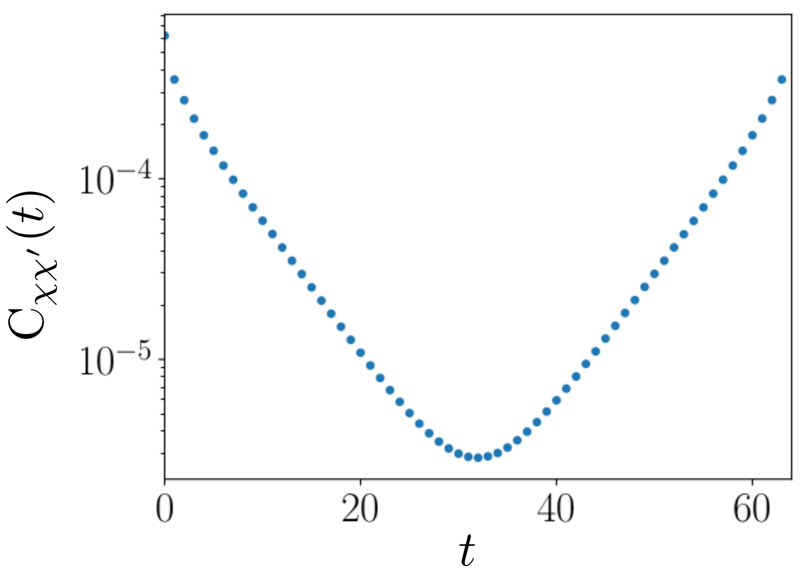
$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int D\phi \, \mathcal{O}[\phi] \, e^{-S[\phi]}$$

$$= \frac{1}{N} \sum_{i=1}^{N} \mathcal{O}[\phi_i], \qquad \phi \sim \frac{1}{Z} e^{-S[\phi]}$$

Hadron Spectroscopy in LQCD

$$C_{\chi\chi'}(t) = \langle 0|\mathcal{O}_\chi(t)\mathcal{O}^\dagger_{\chi'}(0)|0\rangle$$

- Study time dependence of two-point correlators
- $\mathcal{O}_\chi$ is operator with quantum numbers of state of interest
- Admits spectral decomposition: time dependence determined by energy eigenstates.



$$C_{\chi\chi'}(t) = \sum_{n=0}^{\infty} Z_{n\chi} Z^*_{n\chi'}\, e^{-tE_n}$$

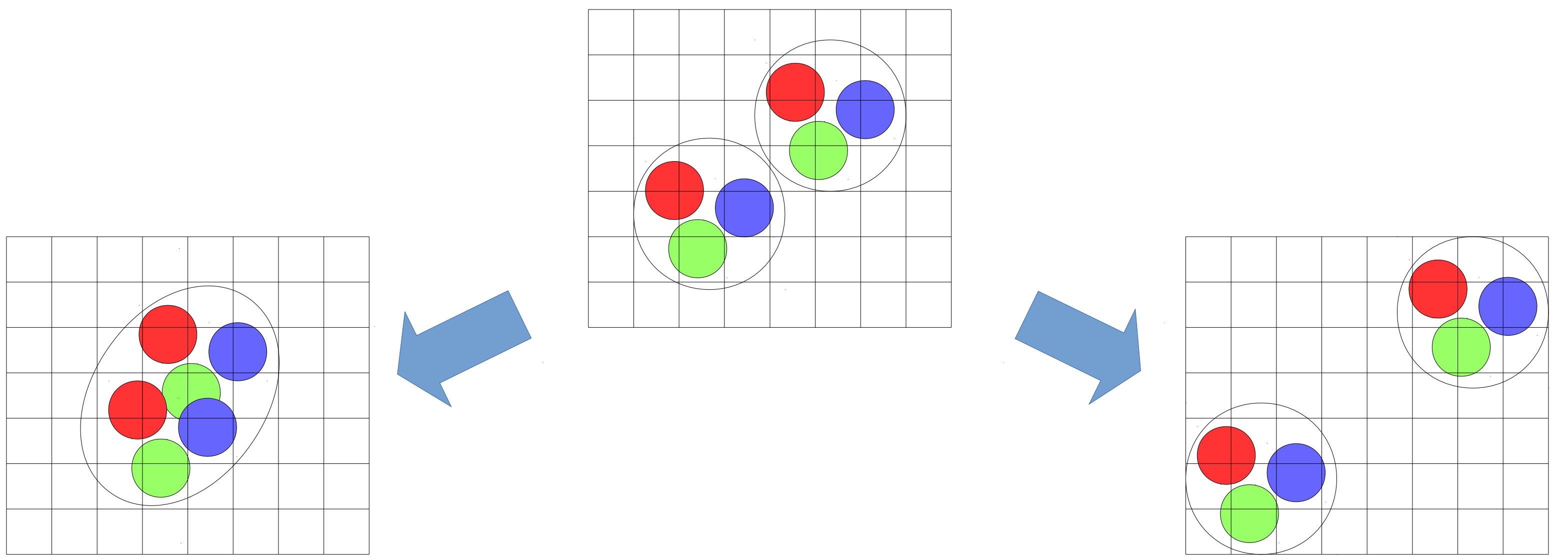
No-Go Theorem

- LSZ: Scattering amplitudes are residues of poles of n-point correlation function
- LQCD: Formulated with Euclidean time
- Maiani-Testa:

  S-matrix elements cannot be extracted from infinite-volume Euclidean-space Green functions except at kinematic thresholds
- Cannot isolate single particle states.

Volume Dependence

$$C_{\chi\chi'}(t) = \langle 0 | \mathcal{O}_{\chi}(t) \mathcal{O}^{\dagger}_{\chi'}(0) | 0 \rangle$$



Bound state

Scattering state

$$[E(L) - E(\infty)] \sim \frac{e^{-mL}}{L}$$

$$[E(L) - E(\infty)] \sim \frac{a}{ML^3}$$

Lüscher: Relate volume dependence to phase shift

The NN Channel

- Smallest multi-nucleon system
- I=0 Deuteron: bound pn state (E_b~2 MeV)
- I=0, d*(2380) resonance
- I=1 dineutron (pp, nn, pn equivalent in isospin limit)

To Bind or Not to Bind?

At m ~ 800 MeV,

- HALQCD: **unbound** deuteron and dineutron
  - Aoki et al, Comput. Sci. Dis. 1 (2008)
  - HAL QCD, Nucl. Phys. A881 (2012)
  - Iritani et al, JHEP 1610 (2016)
  - Iritani et al, PRD 96 (2017)
  - HAL QCD, PRD 99 (2019)
  - HAL QCD, JHEP 1903 (2019)
- PACS: **bound** deuteron and dineutron
  - PACS-CS, PRD 81 (2010)
- NPLQCD: **bound** deuteron and dineutron
  - NPLQCD, PRD 87 (2013)
  - NPLQCD, PRD 87 (2017)
  - NPLQCD PRD 107 (2023)
- CalLatt: **bound** deuteron and dineturon
  - Berkowitz et al, Phys. Lett. B 285 (2017)
- Francis et al: **unbound** dineutron
  - Francis, et al, PRD 99 (2019)

Why are Nuclear Systems Difficult?

- Common to all lattice calculations:
  - Light quarks are expensive
  - Continuum limit requires multiple lattice ensembles
- BB: 6 quark system → large cost computing quark contractions
- StN problem at large Euclidean time
- Small energy gap:

$$C_{\chi\chi}(t) = |Z_{0\chi}|^2 \, e^{-tE_0} + |Z_{1\chi}|^2 \, e^{-tE_1} + \ldots$$

$$\Delta E = E_1 - E_0 \sim \frac{1}{L^2}$$

Variational Method

- Solve the generalized eigenvalue problem (GEVP):

$$\sum_{\chi'} C_{\chi\chi'}(t) v_{n\chi'}(t,t_0) = \lambda_n(t,t_0) \sum_{\chi'} C_{\chi\chi'}(t_0) v_{n\chi'}(t,t_0)$$

- $\lambda_n(t,t_0) = e^{-(t-t_0)E_n(t,t_0)}$ are eigenvalues, $v_{n\chi}(t,t_0)$ are eigenvevtors
- Construct optimized interpolating operators

$$\mathcal{O}_n(t_{\mathrm{ref}}, t_0, t) = \sum_{\chi} v_{n\chi}(t_{\mathrm{ref}}, t_0) \mathcal{O}_\chi(t)$$

- Compute resulting two-point correlators

$$\hat{C}_n(t, t_0, t_{\mathrm{ref}}) = \sum_{n=0}^{\infty} |Z_n(t_0, t_{\mathrm{ref}})|^2 \, e^{-tE_n} > 0$$

INTERLACING THEOREM

Provides **rigorous bounds** on number of energy levels at or below effective mass*

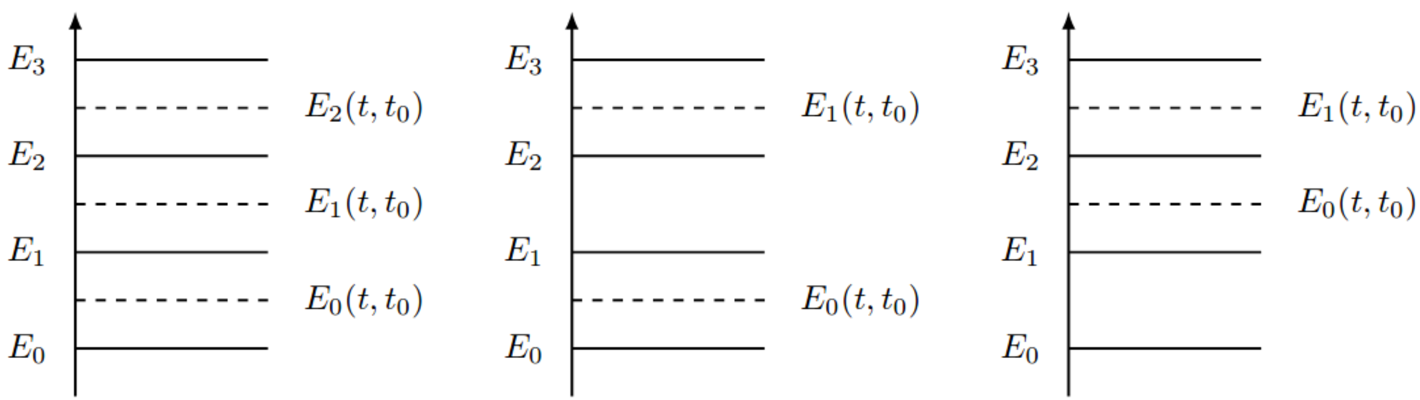
Theorem (Eigenvalue Interlacing Theorem) *Suppose $A \in \mathbb{R}^{n \times n}$ is symmetric. Let $B \in R^{m \times m}$ with $m < n$ be a principal submatrix (obtained by deleting both i-th row and i-th column for some values of i). Suppose A has eigenvalues $\lambda_1 \leq \cdots \leq \lambda_n$ and B has eigenvalues $\beta_1 \leq \cdots \leq \beta_m$. Then*

$$\lambda_k \leq \beta_k \leq \lambda_{k+n-m} \text{ for } k = 1, \ldots, m$$

And if $m = n - 1$,

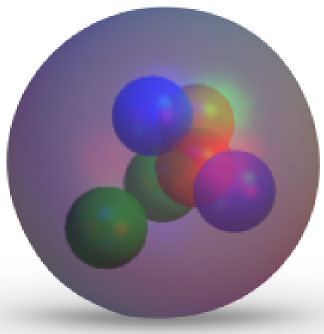
$$\lambda_1 \leq \beta_1 \leq \lambda_2 \leq \beta_2 \leq \cdots \leq \beta_{n-1} \leq \lambda_n$$

$$\lambda_n = e^{-E_n(t-t_0)} \qquad \beta_n = e^{-E_n(t,t_0)(t-t_0)}$$



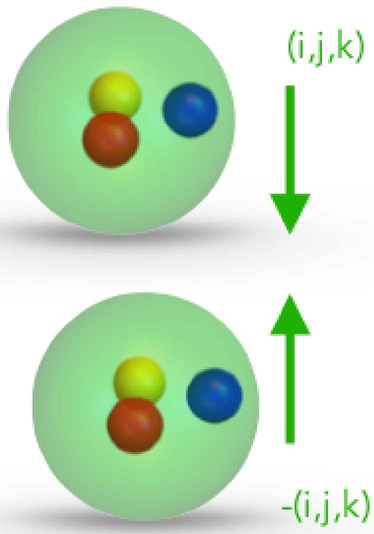
*Fleming, Lattice 2023 (2023)

Types of Operators

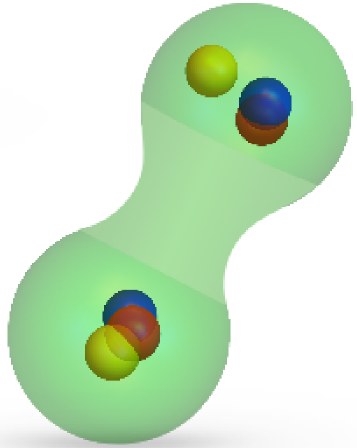


Local hexaquark operators
Six Gaussian smeared quarks at a point
Novel basis of hexaquark operators which project onto two nucleons
Contains operators which cannot be factored into 3-quark color singlets

Dibaryon Operators
Two spatially-separated plane-wave baryons with relative momenta
Operators built products of positive and negative partity nucleon operators
Relative momentum: up to four units → 5 operators



Quasi-local Operators
Two exponentially localized baryons
NN -EFT motivated deuteron-like structure



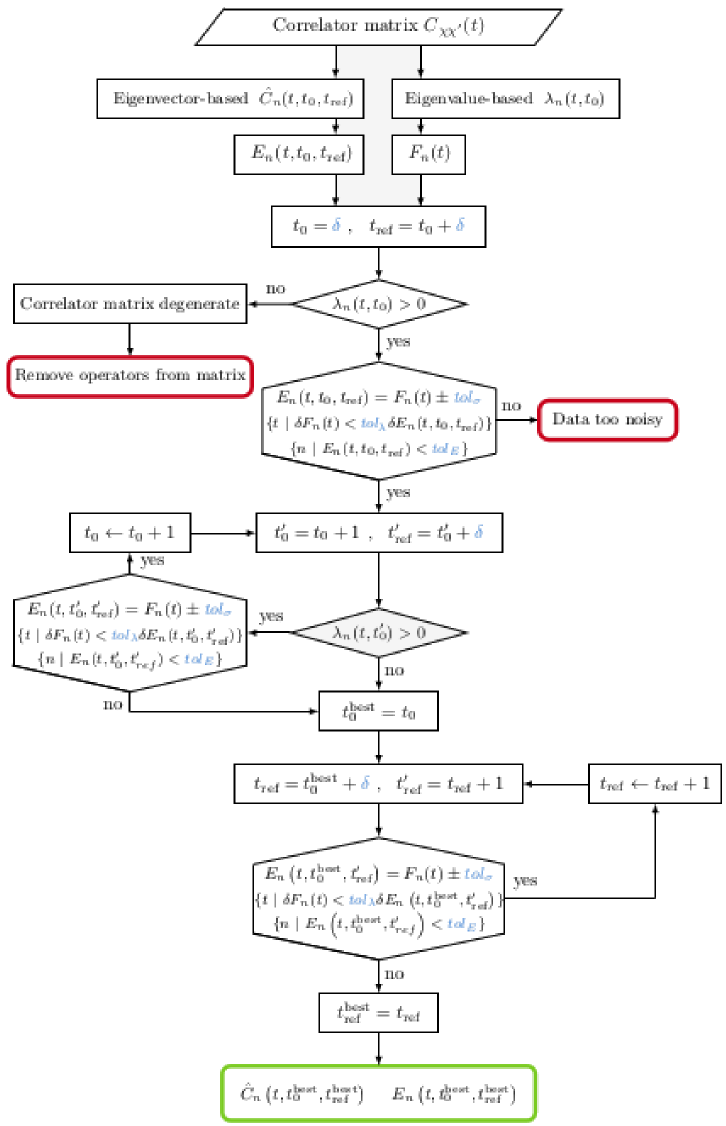
Operator Sets

Consider operator sets with

- Dibaryon only:
  - 5D: positive parity, 5 different momenta
  - 10D: positive & negative parity, 5 different momenta
  - 15D: positive & negative parity, lower spin components, 5 different momenta
- Hexaquark only:
  - 1H: Most important hexaquark
  - 2H: Most important hexaquarks
  - 16H: Full basis
- Union of these two sets:
  - 5D+1H, 5D+2H, 5D+16H, 15D+16H

Analysis

$$\hat{C}_n(t, t_0, t_{\mathrm{ref}})$$



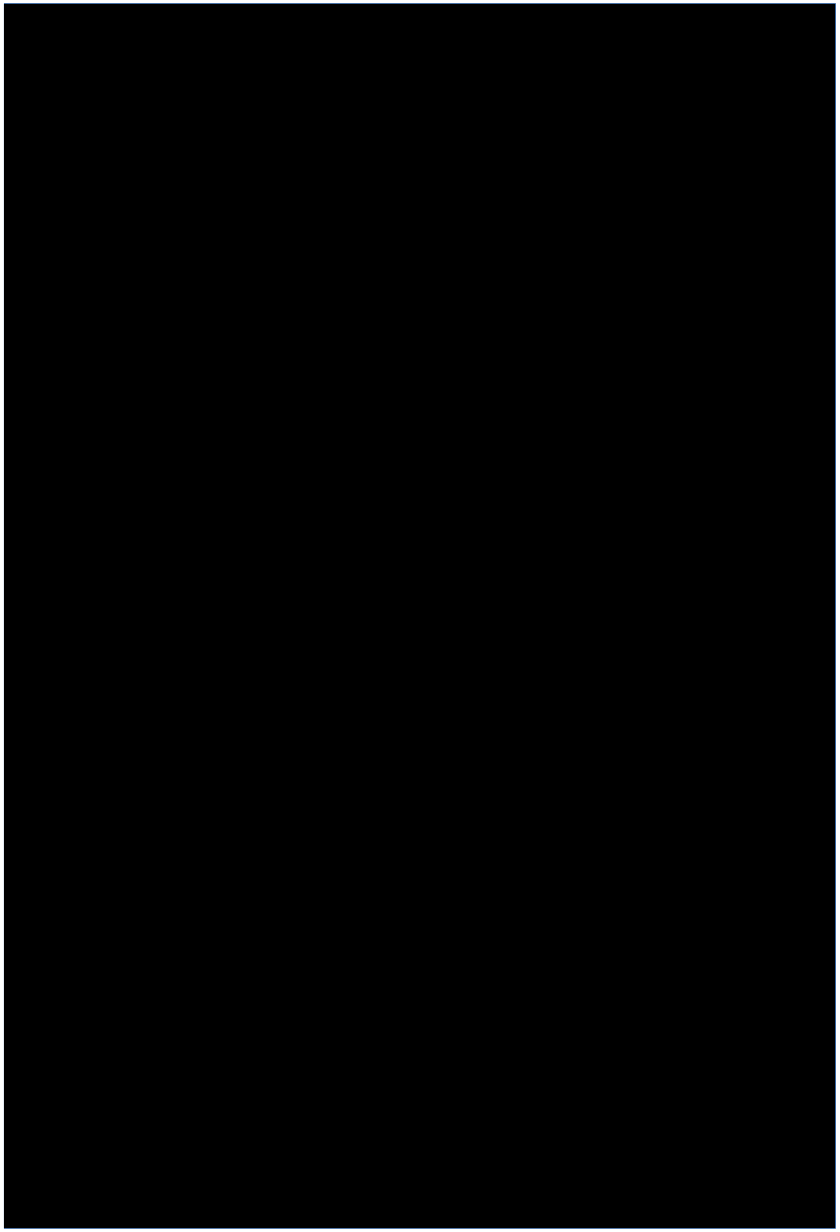
$$E_0 \leq e_0$$

$$E_1 \leq e_1$$

$$\vdots$$

ANALYSIS

$\hat{C}_n(t, t_0, t_{\text{ref}})$

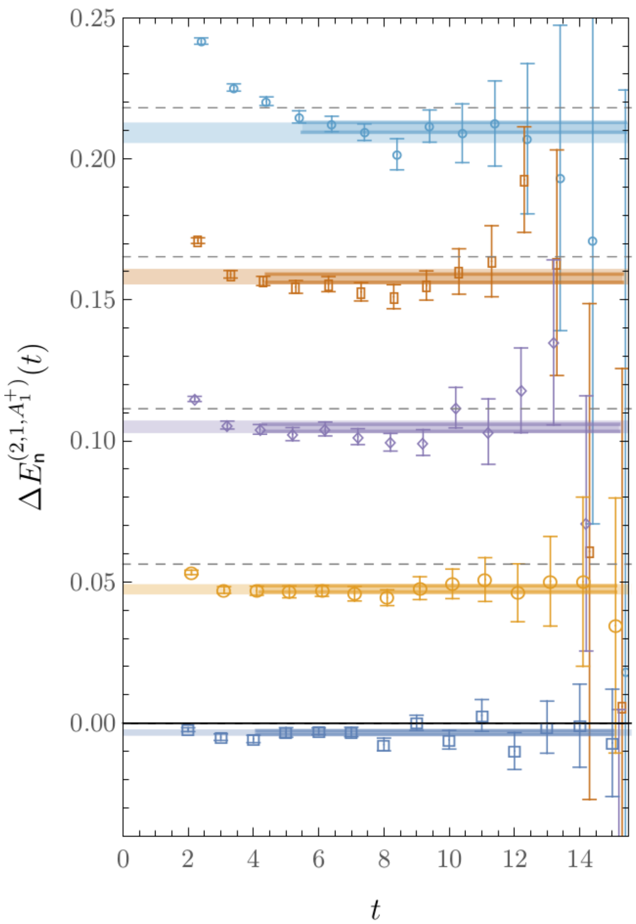


$E_0 \leq e_0$

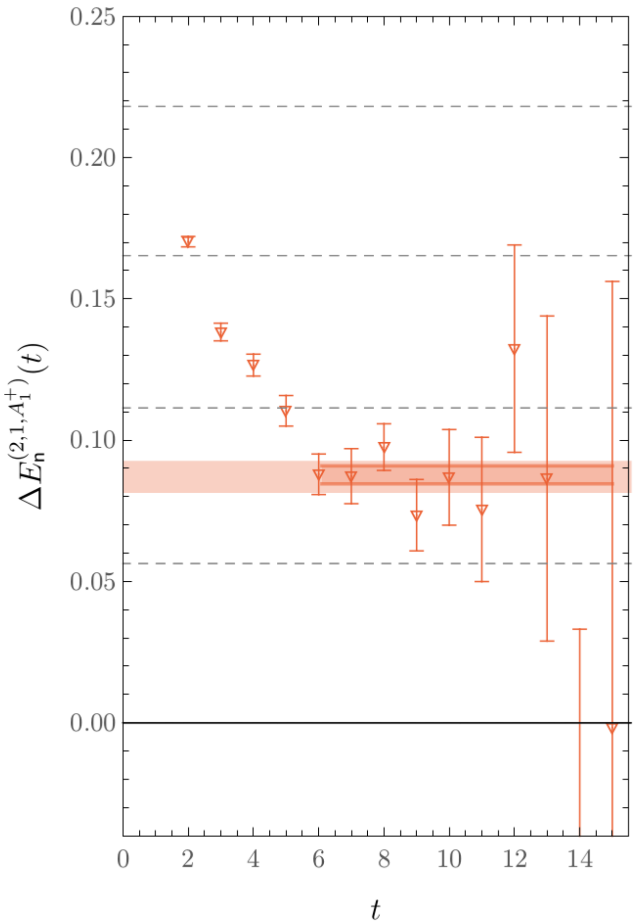
$E_1 \leq e_1$

$\vdots$

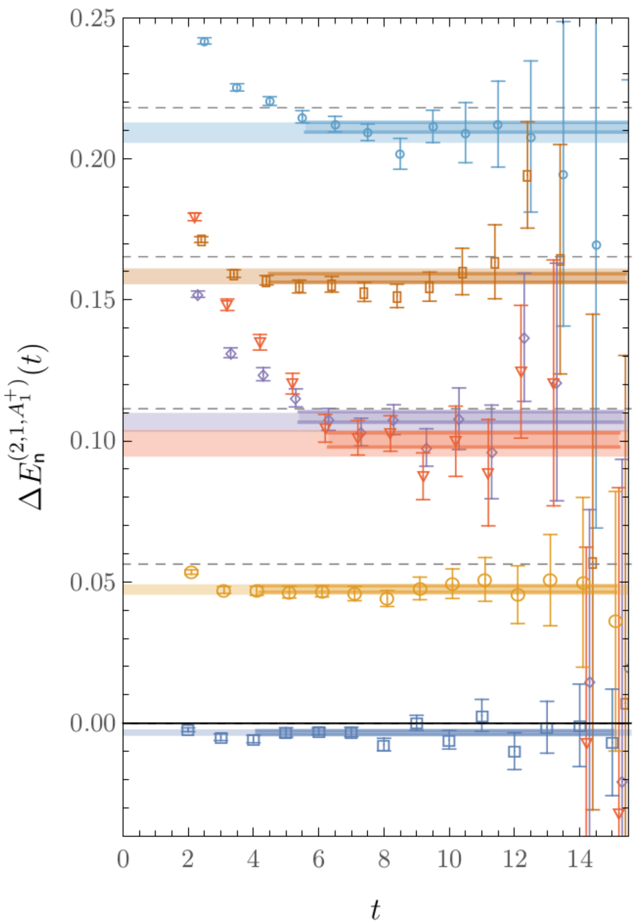
Dineutron (I=1)



5D

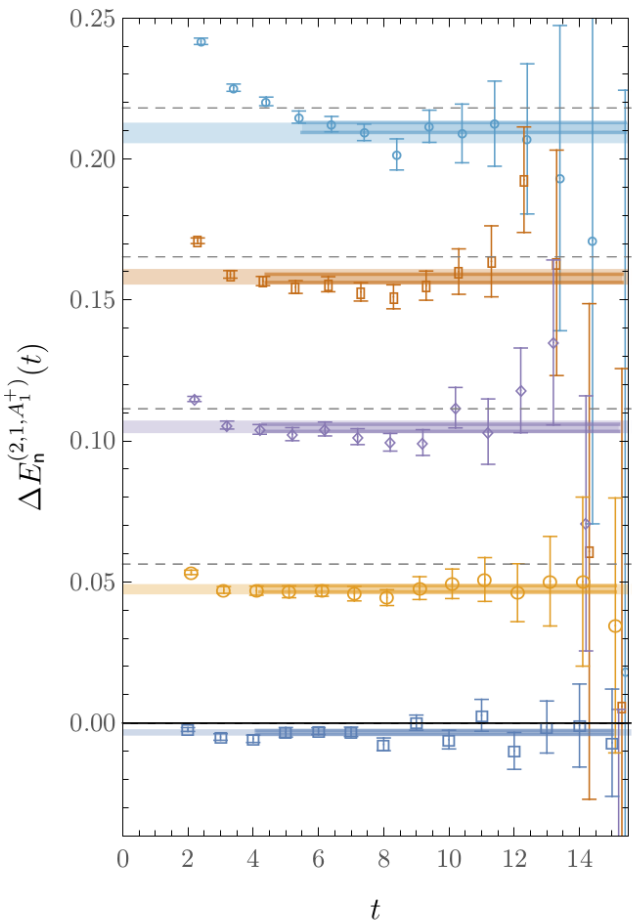


1H

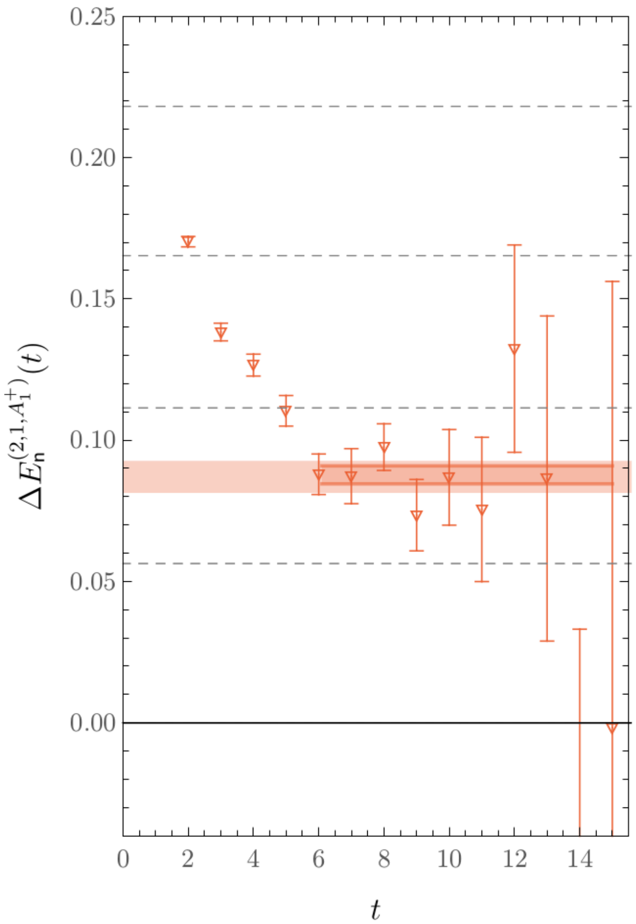


5D+1H

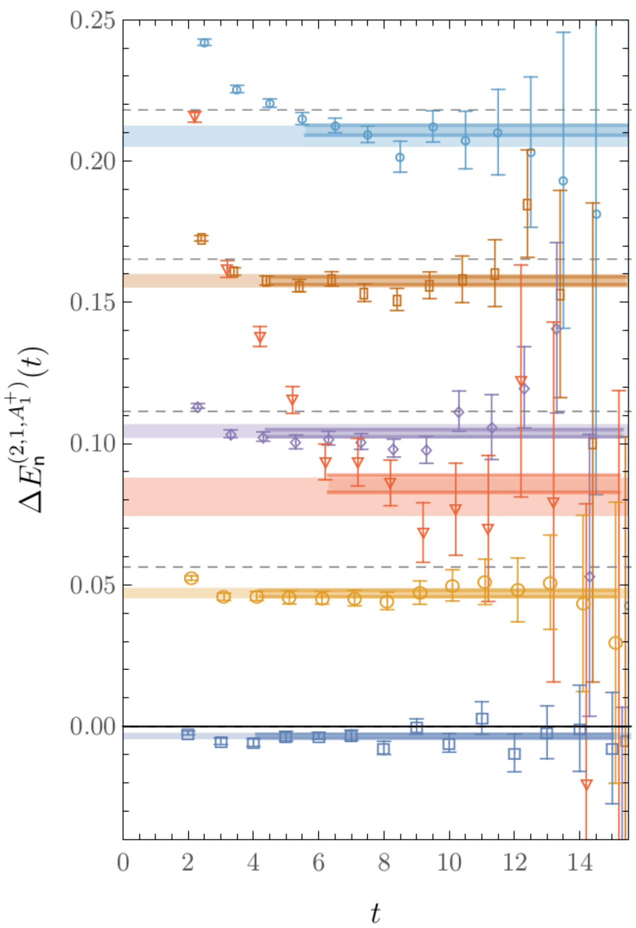
Dineutron (I=1)



5D

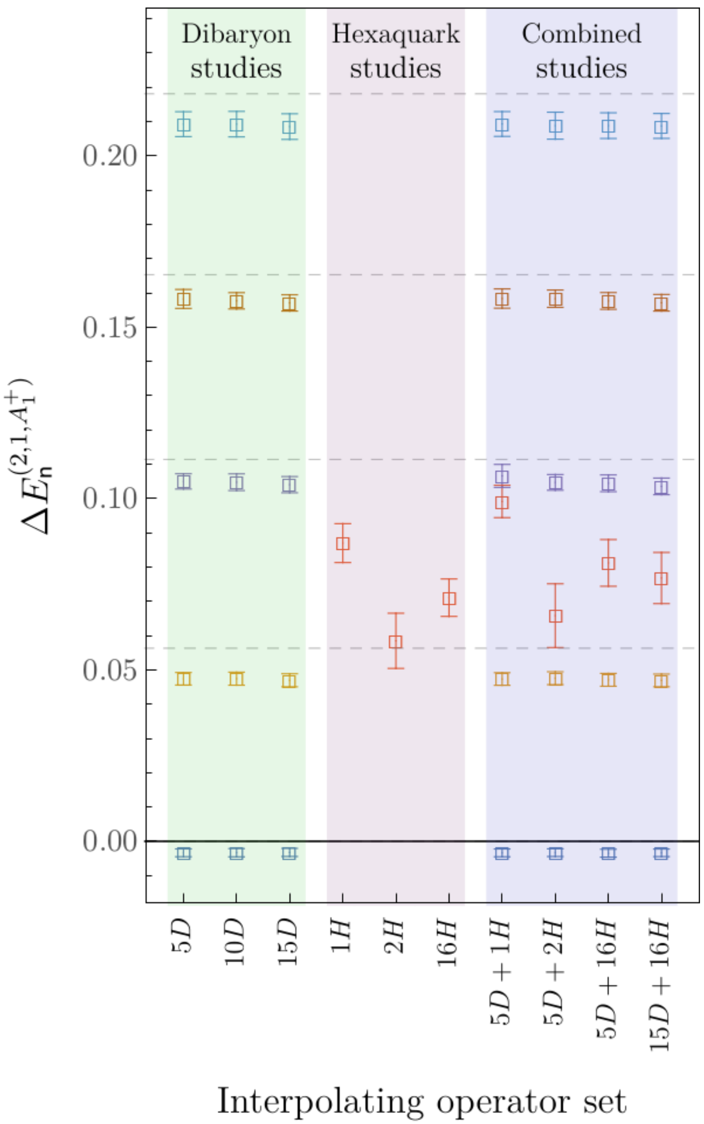


1H



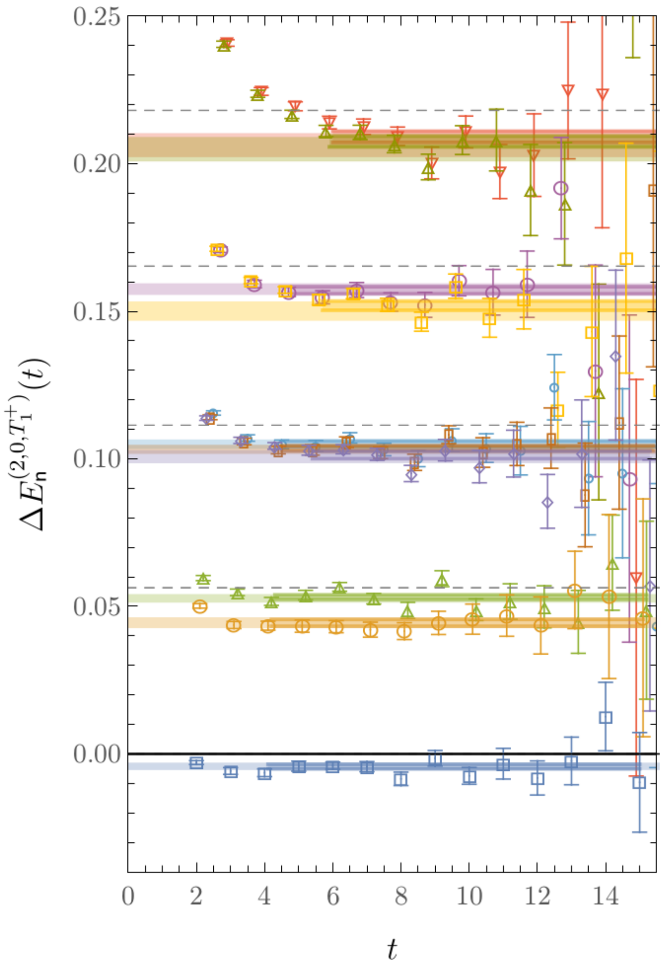
5D+1H

Dineutron (I=1)

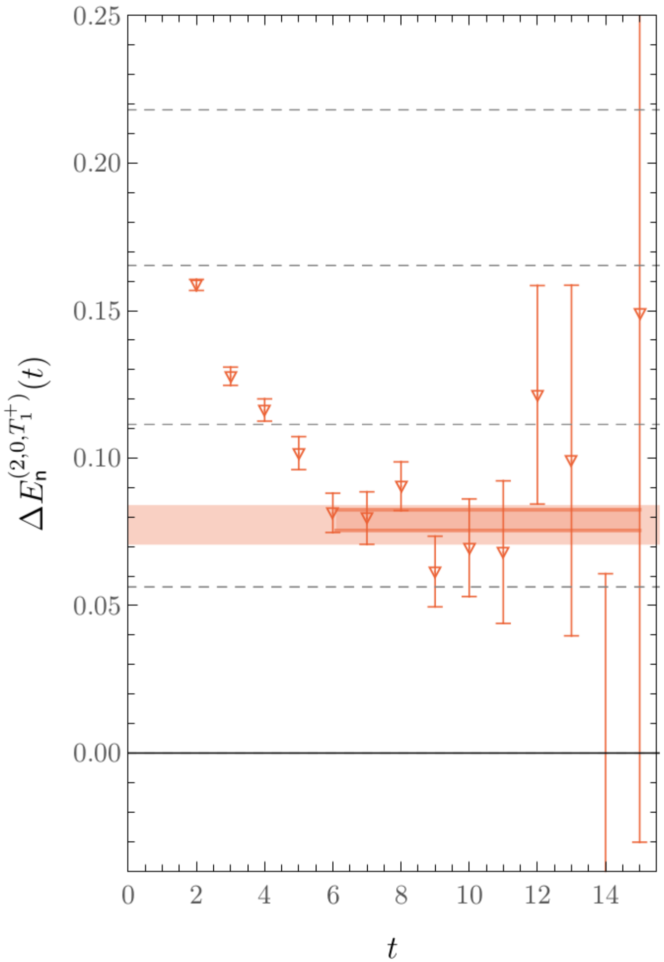


- 5D, 10D and 15D agree within 1 sigma
- 1H, 2H and 16H poor variational bounds
- 5D+1H etc lead to union of two sets of variational bounds!
- Additional state not present in non-interacting theory

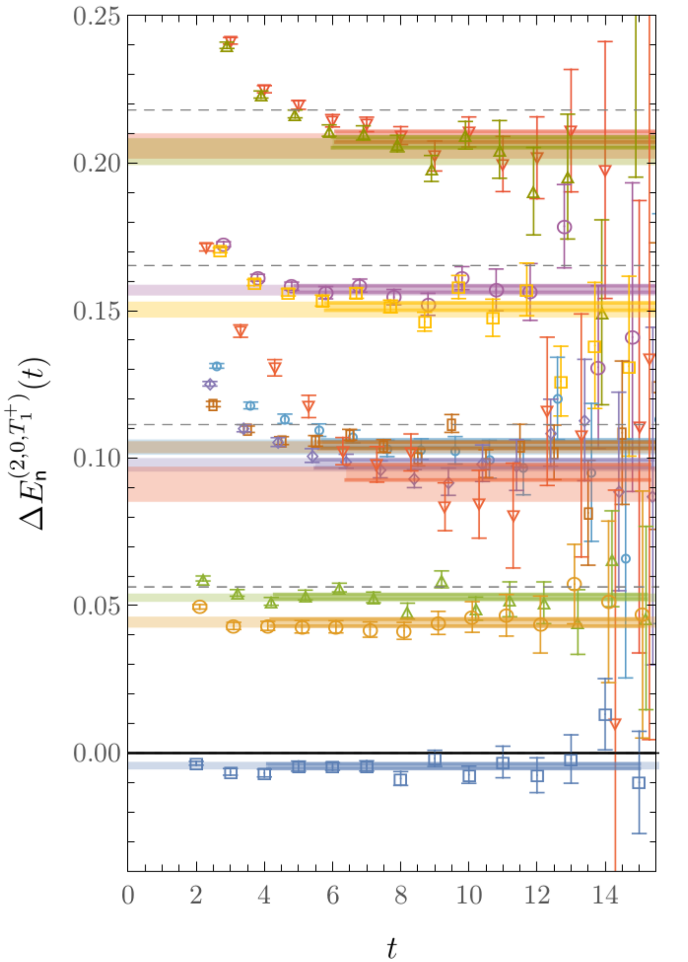
DEUTERON (I=0)



5D

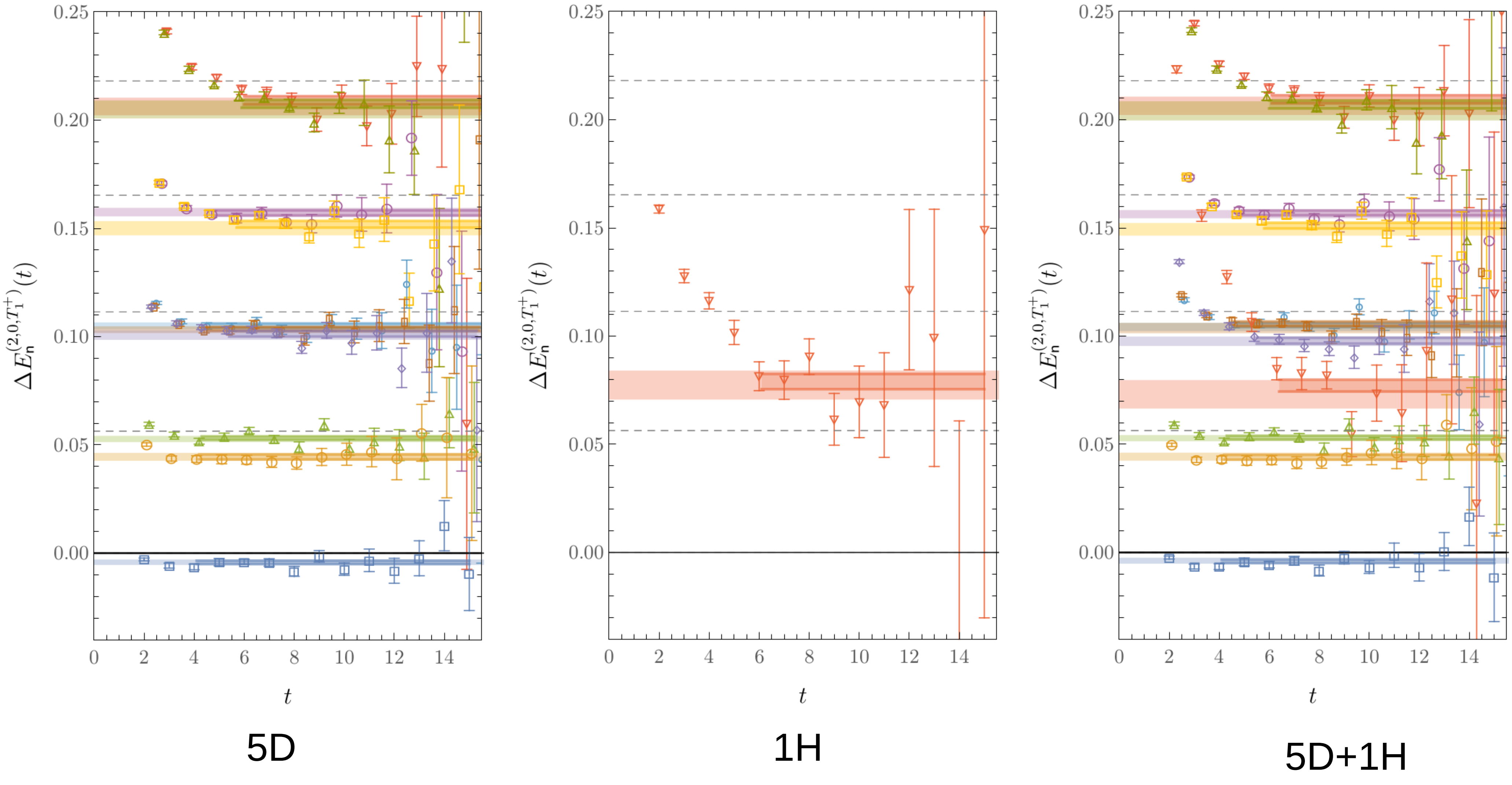


1H



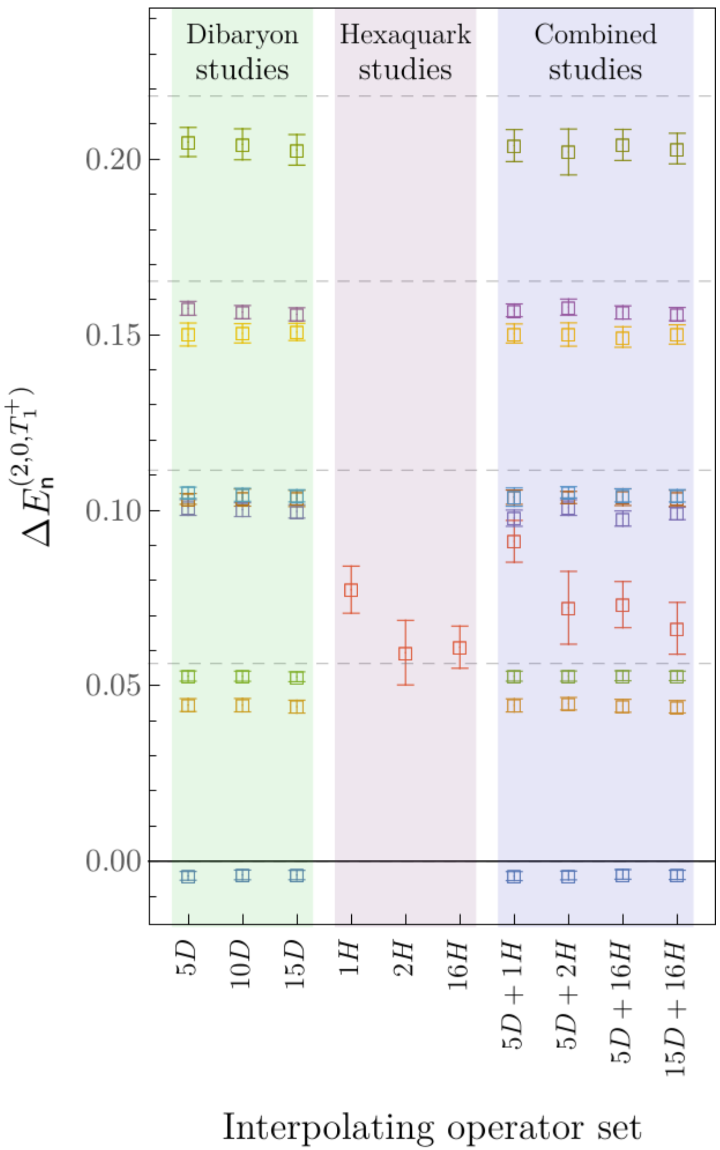
5D+1H

Deuteron (I=0)



0.25
0.20
0.15
0.10
0.05
0.00
$\Delta E_n^{(2,0,T_1^+)}(t)$
0 2 4 6 8 10 12 14
t
5D
1H
5D+1H

Deuteron (I=0)



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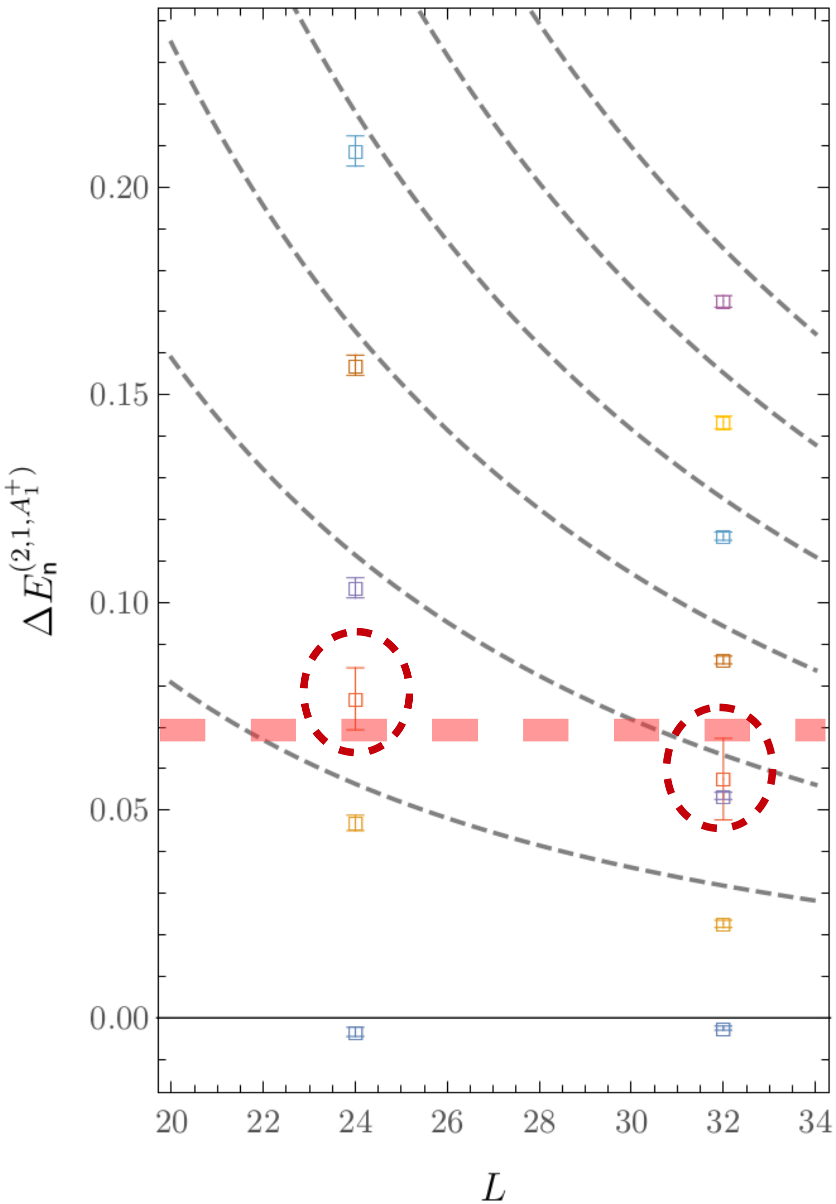
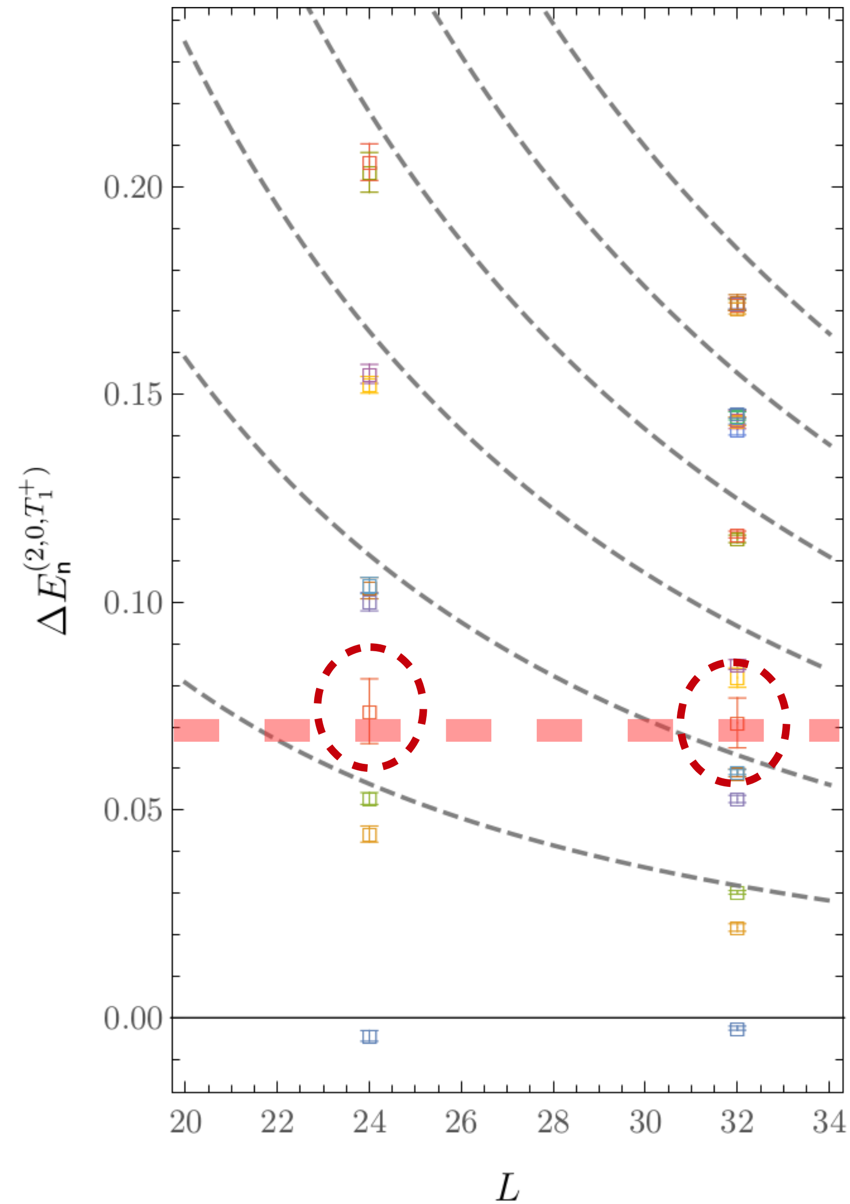
Volume Dependence



Additional level exhibits approximate volume independence! Suggestive of resonance.



L=32 data from NPLQCD: Phys.Rev.D 107 (2023) 9, 094508

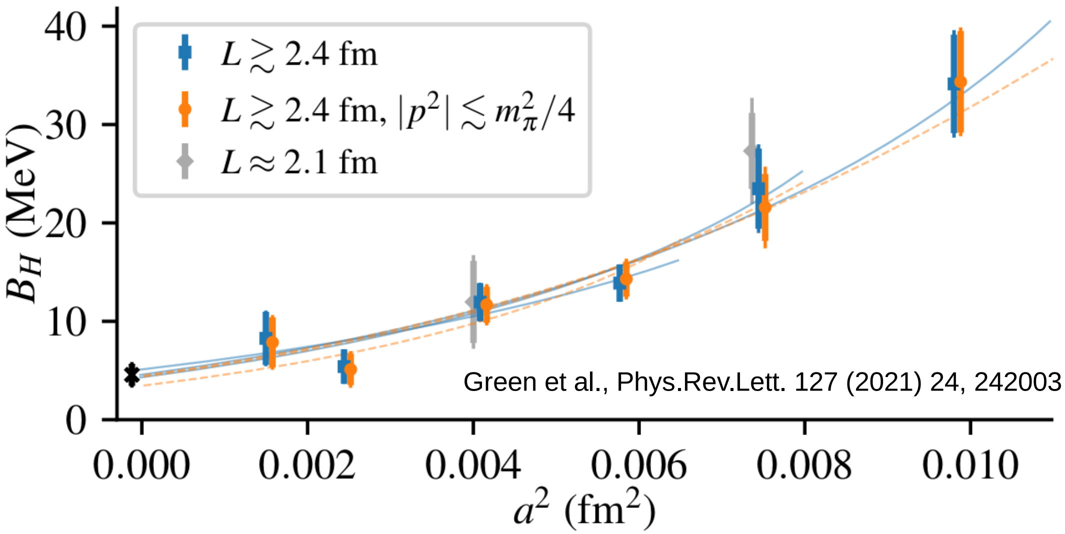
Additional State

- Lattice artifact?
- Symmetric spectra for dineutron and deuteron predicted in heavy quark limit. Natural to see approx. degenerate spectra at these quark masses.
- Lack of volume dependence consistent with resonance
- Difficult to interpret at heavy pion mass.

Conclusions

- LQCD can connect QCD to nuclear physics
- Challenge to control systematic uncertainties
- Variational Analysis of NN
  - Resonant-like state at dE~0.07.
  - Need to move towards physical point to interpret
  - Currently underway!

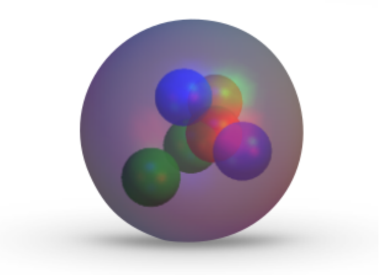
H-Dibaryon



$L \gtrsim 2.4$ fm
$L \gtrsim 2.4$ fm, $|p^2| \lesssim m_\pi^2/4$
$L \approx 2.1$ fm
B_H (MeV)
a^2 (fm^2)
0
10
20
30
40
0.000
0.002
0.004
0.006
0.008
0.010
Green et al., Phys.Rev.Lett. 127 (2021) 24, 242003

Two Volumes

Hexaquark Operators



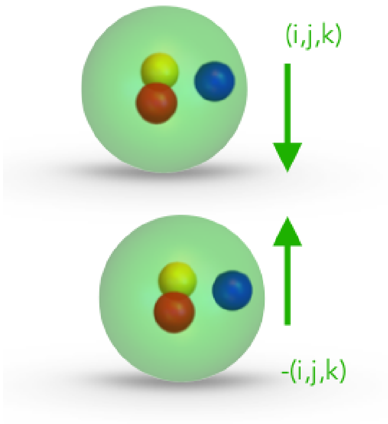
$$\mathcal{H}^K(x) = \mathcal{H}^{C_1 C_2 C_3}_{\Gamma_1, F_1; \Gamma_2, F_2; \Gamma_3, F_3}(x) = T^{C_1 C_2 C_3}_{abcdef} \mathcal{D}^{ab}_{\Gamma_1, F_1}(x) \mathcal{D}^{cd}_{\Gamma_2, F_2}(x) \mathcal{D}^{ef}_{\Gamma_3, F_3}(x)$$

Many ways to construct color singlet operator.

$$\mathbf{3} \otimes \mathbf{3} = \mathbf{6} \oplus \overline{\mathbf{3}}$$

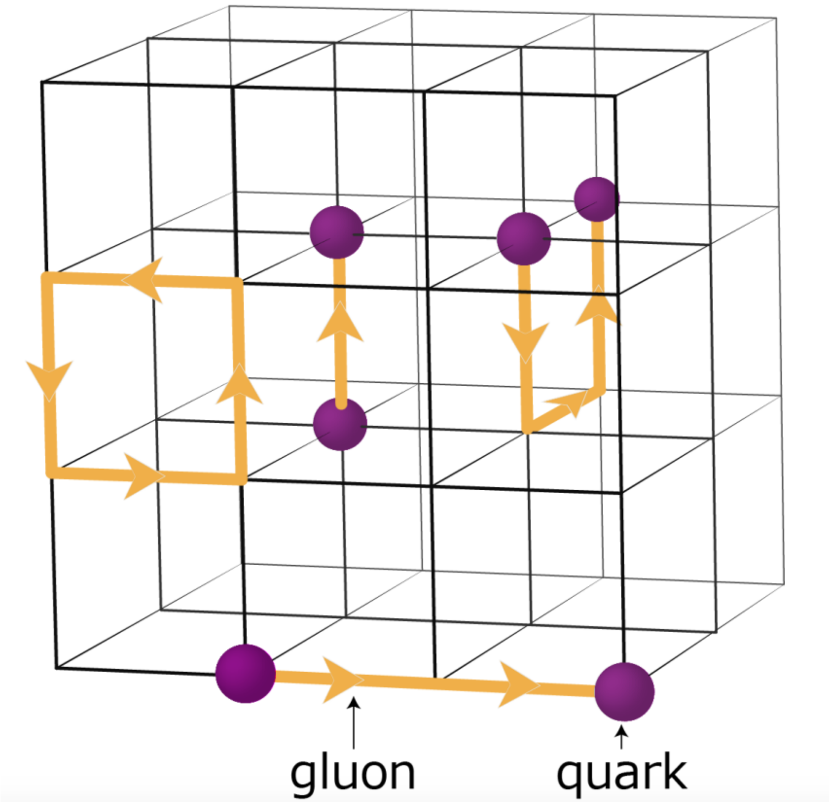
$$(\mathbf{3} \otimes \mathbf{3}) \otimes (\mathbf{3} \otimes \mathbf{3}) \otimes (\mathbf{3} \otimes \mathbf{3}) = (\mathbf{6} \oplus \overline{\mathbf{3}}) \otimes (\mathbf{6} \oplus \overline{\mathbf{3}}) \otimes (\mathbf{6} \oplus \overline{\mathbf{3}})$$

Dibaryon Operators

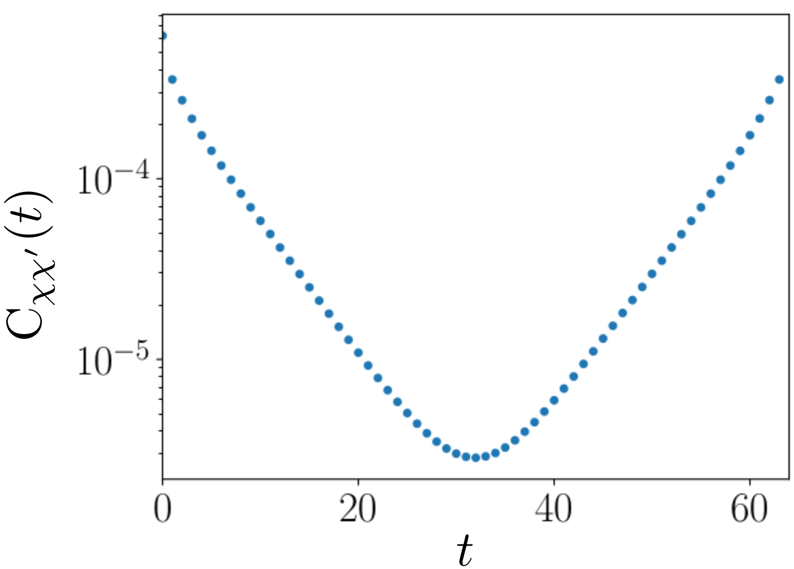


$L^3 \times T$	β	m_q	a [fm]	L [fm]	T [fm]	$m_\pi L$	$m_\pi T$	N_{cfg}	N_{src}
$24^3 \times 48$	6.1	-0.2450	0.1453(16)	3.4	6.7	14.3	28.5	469	216

Hadron Spectroscopy in LQCD



$$C_{\chi\chi'}(t) = \langle 0|\mathcal{O}_\chi(t)\mathcal{O}^\dagger_{\chi'}(0)|0\rangle$$

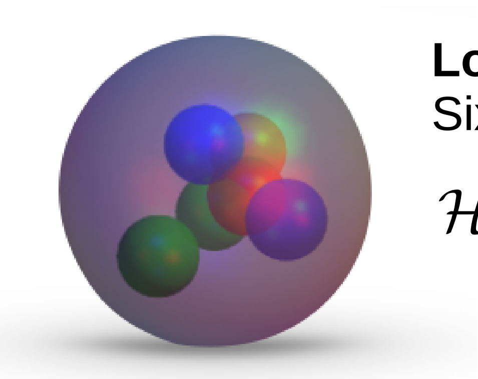


$$\langle\mathcal{O}\rangle = \frac{1}{Z}\int D\phi\,\mathcal{O}[\phi]\,e^{-S[\phi]}$$

$$= \frac{1}{N}\sum_{i=1}^{N}\mathcal{O}[\phi_i], \qquad \phi \sim \frac{1}{Z}e^{-S[\phi]}$$

$$C_{\chi\chi'}(t) = \sum_{n=0}^{\infty} Z_{n\chi}Z^*_{n\chi'}\,e^{-tE_n}$$

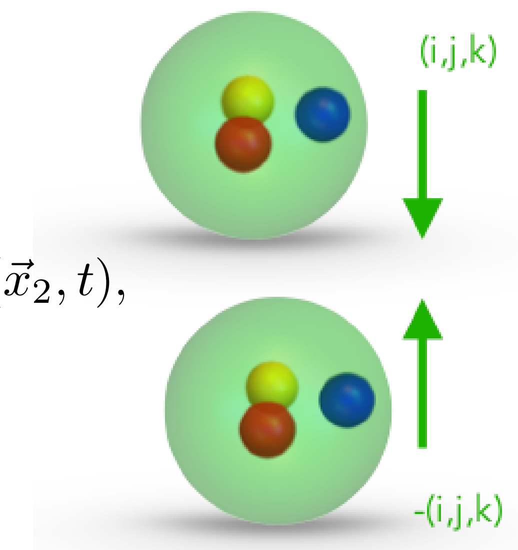
Types of Operators



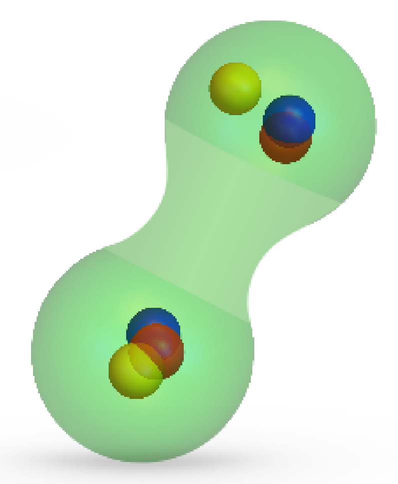
Local hexaquark operators
Six Gaussian smeared quarks at a point

$$\mathcal{H}^{K}(x) = T^{C_1 C_2 C_3}_{abcdef}\mathcal{D}^{ab}_{\Gamma_1,F_1}(x)\mathcal{D}^{cd}_{\Gamma_2,F_2}(x)\mathcal{D}^{ef}_{\Gamma_3,F_3}(x)$$

Dibaryon Operators
Two spatially-separated plane-wave baryons
with relative momenta



$$D^{\Gamma}_{\rho}(\vec{n},t) = \sum_{\vec{x}_1,\vec{x}_2} e^{i2\pi\vec{n}/L\cdot(\vec{x}_1-\vec{x}_2)} \sum_{\sigma,\sigma'} v^{\sigma\sigma'}_{\rho} N^{\Gamma}_{\sigma}(\vec{x}_1,t) N^{\Gamma}_{\sigma'}(\vec{x}_2,t),$$



Quasi-local Operators
Two exponentially localized baryons
NN -EFT motivated deuteron-like structure

$$Q^{\Gamma}_{\rho}(\kappa,t) = \sum_{\vec{R}} \sum_{\vec{x}_1,\vec{x}_2} e^{-\kappa|\vec{x}_1-\vec{R}|} e^{-\kappa|\vec{x}_2-\vec{R}|} \sum_{\sigma,\sigma'} v^{\sigma\sigma'}_{\rho} N^{\Gamma}_{\sigma}(\vec{x}_2,t) N^{\Gamma}_{\sigma'}(\vec{x}_1,t),$$