Excited QCD 2024 Workshop, Benasque

Analysis of the recent LQCD data for the $\Lambda(1405)$

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Outline

- ❑ $\Lambda(1405)$
- ❑ Chiral approach
- ❑ $\Lambda(1405)$ in lattice QCD simulation
- ❑ Finite volume effect
- ❑ Results

Discovery of $\Lambda(1405)$

- The resonance first appeared in bubble chamber experiments

R. H. Dalitz and S. F. Tuan, PRL 2, 425(1959)
R. H. Dalitz and S. F. Tuan, Annals Phys, 10, 307

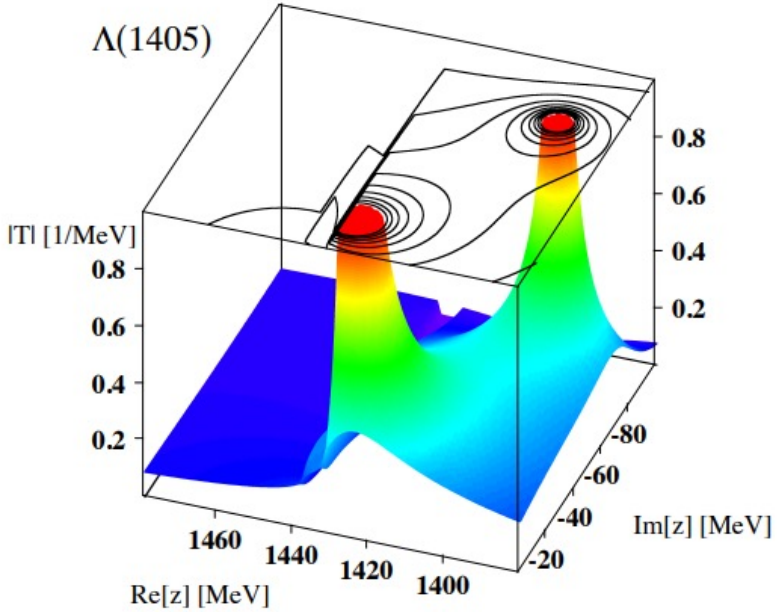
- $\Lambda(1405)$: $J^P = \frac{1}{2}^-, I = 0$

PDG, Prog. Theor. Exp. Phys. 2022, 083C01 (2022)

- mass: 1406.5 ± 4 MeV, width: 50 ± 2 MeV

- A good candidate of hadronic molecular state
  1. Coupled-channel framework
  2. Chiral approach

- Two pole structures of $\Lambda(1405)$
  - ✓ Two poles of the scattering amplitude found around nominal $\Lambda(1405)$ energy region.



T. Hyodo et al. Part. Nucl. Phys. 67, 1(2012)

Chiral unitary model(LO)

- Lagrangian at leading order

$$\mathcal{L}_{WT} = \frac{1}{4f^2}\mathrm{Tr}\left[\bar{B}i\gamma^{\mu}[\Phi\partial_{\mu}\Phi - \partial_{\mu}\Phi\Phi, B]\right]$$

$$\Phi = \begin{bmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\Lambda \end{bmatrix} \qquad B = \begin{bmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{bmatrix}$$

- Interaction kernel

$$V_{ij} = -\frac{1}{4f^2}C_{ij}\left(2\sqrt{s} - M_{Bi} - M_{Bj}\right)\left(\frac{M_{Bi}+E}{2M_{Bi}}\right)^{\frac{1}{2}}\left(\frac{M_{Bj}+E'}{2M_{Bj}}\right)^{\frac{1}{2}}$$

$$C_{ij} = \begin{array}{cc} \pi\Sigma & \bar{K}N \end{array}$$
$$\begin{pmatrix} 4 & -\sqrt{\frac{3}{2}} \\ -\sqrt{\frac{3}{2}} & 3 \end{pmatrix}$$

Chiral unitary approach up to NLO

$$\mathcal{L}_{\phi B}^{(2)} = b_D\langle \bar{B}\{\chi_+, B\}\rangle + b_F\langle \bar{B}[\chi_+, B]\rangle + b_0\langle \bar{B}B\rangle\langle\chi_+\rangle + d_1\langle \bar{B}\{u_\mu, [u^\mu, B]\}\rangle + d_2\left\langle \bar{B}\left[u_\mu, [u^\mu, B]\right]\right\rangle + d_3\langle \bar{B}u_\mu\rangle\langle u^\mu B\rangle + d_4\langle \bar{B}B\rangle\langle u^\mu u_\mu\rangle$$

- Lagrangian

- $\langle\dots\rangle$: the trace in flavor space
- $\chi_+ = 2B_0(u^+Mu^+ + uMu)$

  breaks chiral symmetry
- $M = \mathrm{Diag}(m_u, m_d, m_s)$
- $B_0 = -\frac{\langle 0|\bar{q}q|0\rangle}{f^2}$
- b_i and d_j, the low energy constants
- D_{ij} and L_{ij} depend on b_i and d_j

- Potential

1. $V^{NLO} = \frac{D_{ij} - 2k_\mu k'^\mu L_{ij}}{f^2} N_i N_j$
2. S-wave: $V^s = \frac{N_i N_j(D_{ij} - 2L_{ij}\omega_i\omega_j)}{f^2}$

↑

Partial wave decomposition $V_l = \frac{1}{2}\int_{-1}^{1} P_l(\cos\theta)V(\theta)d\cos\theta$

$$D_{ij} = \begin{pmatrix} 4(b_0 + b_D)m_\pi^2 & -\sqrt{\frac{3}{2}}(b_D - b_F)\mu_1^2 \\ -\sqrt{\frac{3}{2}}(b_D - b_F)\mu_1^2 & 2(2b_0 + 3b_D + b_F)m_K^2 \end{pmatrix} \quad L_{ij} = \begin{pmatrix} -4d_2 + 4d_3 + 4d_4 & \sqrt{\frac{3}{2}}(d_1 + d_2 - 2d_3) \\ \sqrt{\frac{3}{2}}(d_1 + d_2 - 2d_3) & d_1 + 3d_2 + 2(d_3 + d_4) \end{pmatrix}$$

$\Lambda(1405)$ in coupled-channel scattering(LO)

- Bethe-Salpether equation

$$T = \frac{1}{1 - VG} V$$

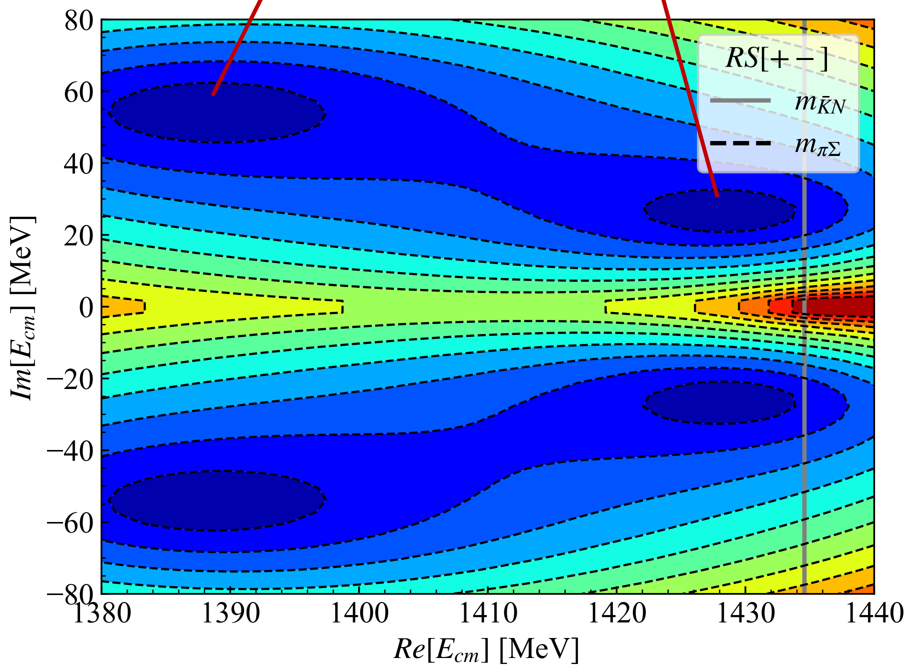
$$G_l = \frac{2M_l}{(4\pi)^2}\left\{a_l + \ln\frac{M_l^2}{\mu^2} + \frac{m_l^2 - M_l^2 + s}{2s}\ln\frac{m_l^2}{M_l^2} + \frac{q_{\rm cm}}{\sqrt{s}}\ln\left[\frac{(s+2\sqrt{s}q_{\rm cm})^2 - \left(M_l^2 - m_l^2\right)^2}{(s-2\sqrt{s}q_{\rm cm})^2 - \left(M_l^2 - m_l^2\right)^2}\right]\right\}$$

- $\mu = 630$ MeV
- $a_{\pi\Sigma} = -2$
- $a_{\bar{K}N} = -1.84$

E. Oset et al, PLB, 527(2002)

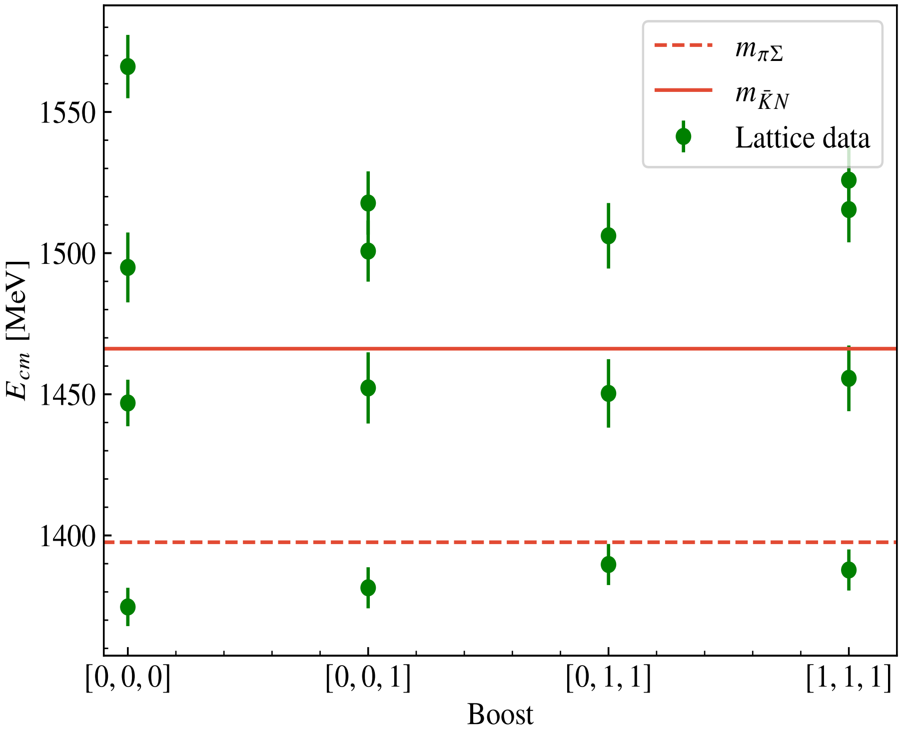
- $+/-$ stands for unphysical/physical sheet

$\Lambda(1380)$	$\Lambda(1405)$
$m_{\pi\Sigma} = 1331$ MeV	$m_{\bar{K}N} = 1433$ MeV
Lower pole	**Higher pole**



✓ Both of two poles are resonaces with physical hadronic masses.

$\Lambda(1405)$ in lattice QCD simulation



- Total momentum: $\vec{P} = \frac{2\pi}{L}\vec{N}$

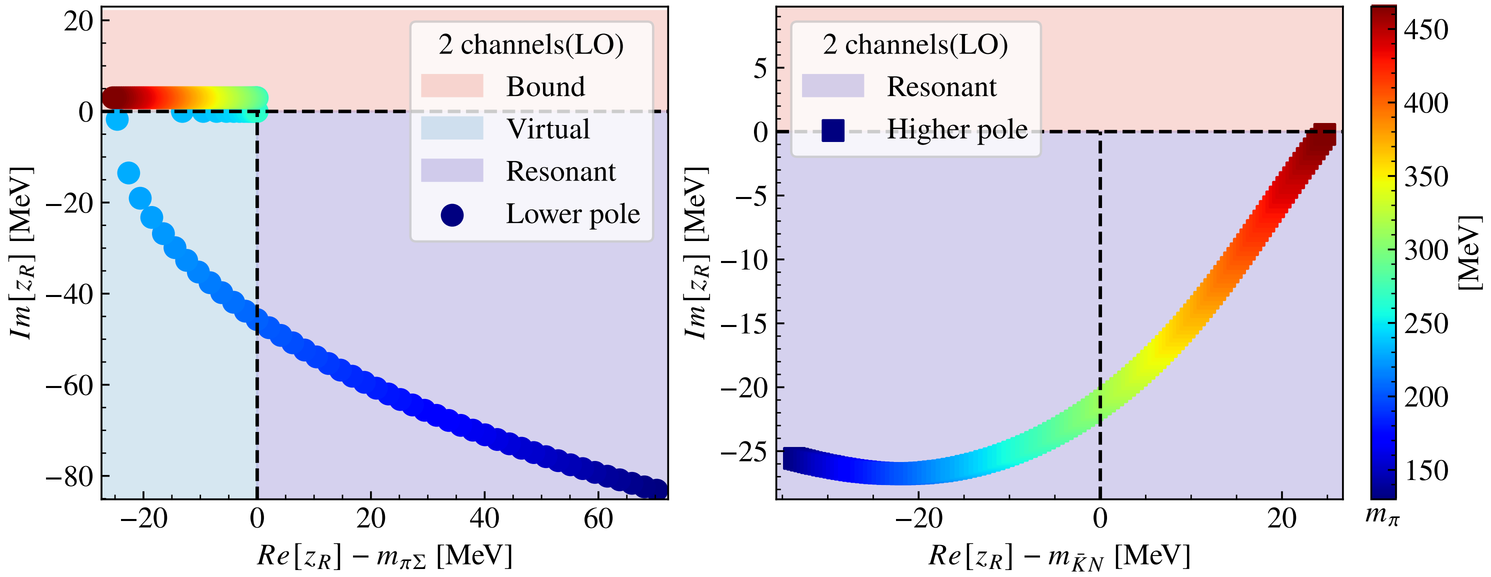
BaSc, arXiv: 2307.13471

➢ Hadron masses and decay constants(units in MeV)

m_π	m_K	m_N
203	486	979
m_Σ	f_π	f_K
1193	93	108

- Virtual state: 1392 ± 27 MeV
- Resonance: $(1455 \pm 32, 11.5 \pm 8)$ MeV

The trajectories of two poles of $\Lambda(1405)$



$$M_B(m_\pi) = M_0 + \sum_{\phi=\pi,K} \xi_{B,\phi} m_\phi^2$$

- $a_{\pi\Sigma} = -1.7$, $a_{\bar{K}N} = -2$
- Pion masses are denoted by colors

1. $m_\pi > 207$ MeV, lower pole become a virtual state
2. $m_\pi > 250$ MeV, lower pole become a bound state
3. Higher pole is resonance
4. $m_\pi > 250$ MeV, $\mathrm{Re}[z_R] > m_{\bar{K}N}$

✓ **The lower pole of $\Lambda(1405)$ becomes a virtual state in *unphys.***

Coupled-channel approach in a finite volume

➢ Loop function in a finite volume

$$\tilde{G} = \int \frac{d^3q}{(2\pi)^3} I(|\vec{q}|) = \frac{1}{L^3} \sum_{\vec{n}}^{|\vec{q}|<q_{max}} I(|\vec{q}|)$$

$$I(|\vec{q}|) = \frac{2m_B}{2\omega_1\omega_2} \frac{\omega_1 + \omega_2}{E^2 - (\omega_1 + \omega_2)^2}$$

➢ Scattering matrix

$$\tilde{T} = \frac{1}{1 - V\tilde{G}} V$$

➢ Energy levels

$$\mathrm{Det}\left(1 - V\tilde{G}\right) = 0$$

➢ With the effect of finite volume, $\vec{P} \neq \vec{0}$,

Lorentz invariance of the system will be broken

- Momentum in the moving frame

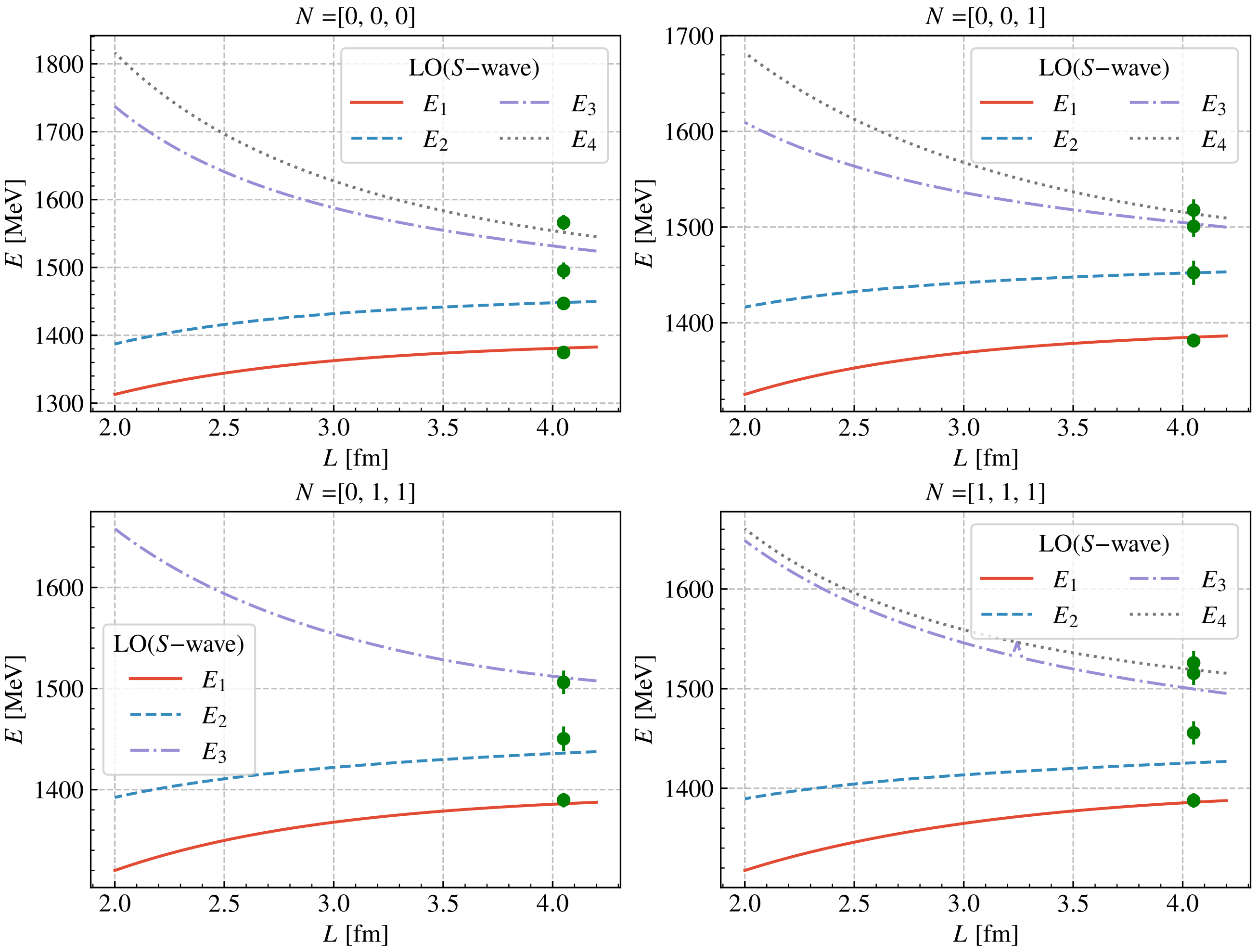
$$\vec{q}^* = \vec{q} + \left[\left(\frac{\sqrt{s}}{P^0} - 1\right) \frac{\vec{q} \cdot \vec{P}}{\left|\vec{P}\right|^2} - \frac{q^{*0}}{P^0}\right] \vec{P}$$

1. $(P^0, \vec{P})$: the total momentum of system
2. $\vec{q}^*$: the momentum in center of mass frame

➢ Moving frame

$$\tilde{G}(P) = \frac{1}{L^3} \frac{\sqrt{s}}{P^0} \sum_{n} I(|\vec{q}^*(\vec{q})|)$$

Fitting result(Leading order)



$$\chi^2 / d.o.f. = 1.66$$

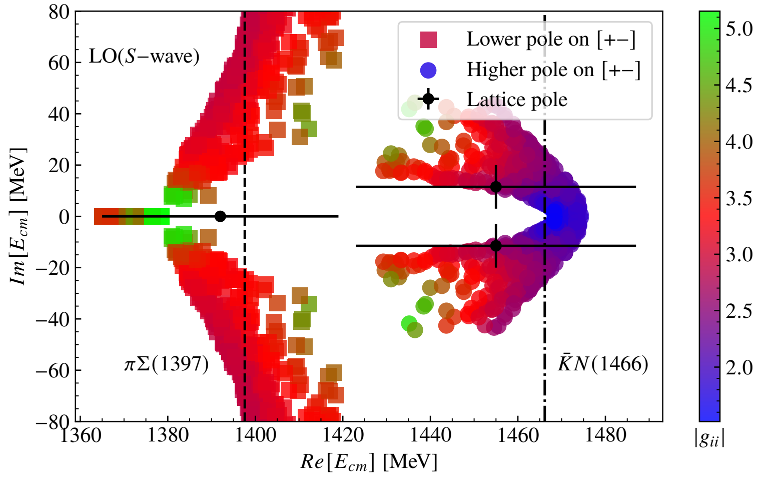
- **Green circle:** lattice energy levels
- E_i: the prediction of energy levels
- N_i: boost

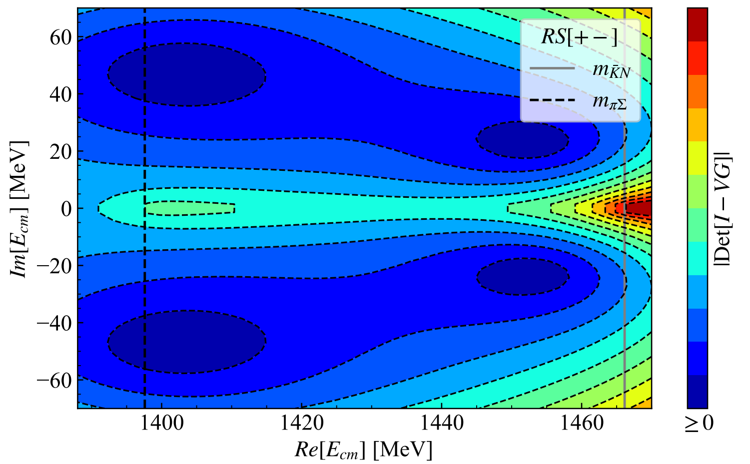
➢ **Subtraction constants**

$a_{\pi\Sigma}$	$a_{\bar{K}N}$
-1.67	-2.0

Pole structures of $\Lambda(1405)$(LO)

- Pole structures of $\Lambda(1405)$ within 1-σ confidence level(CL)





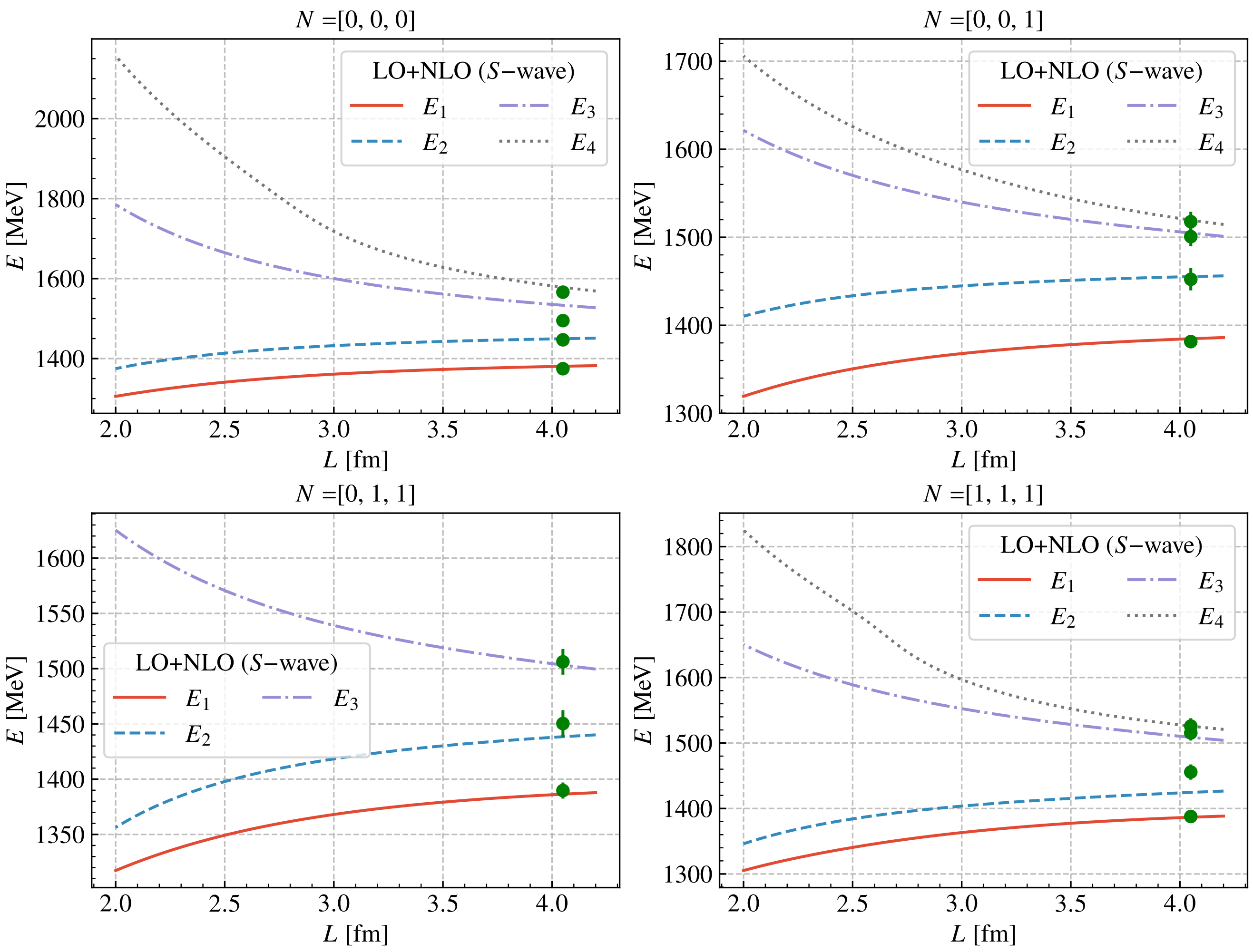
- Couple channels scattering(LO in S-wave)
- Poles coupling with $\pi\Sigma$ or $\bar{K}N$ are encoded by colors

Lower pole [MeV]	Higher pole [MeV]
$1402.59^{+15.87}_{-37.50} - i\,47.60^{+42.35}_{-47.60}$	$1452.71^{+21.99}_{-23.46} - i\,23.18^{21.14}_{-23.17}$

Coupling	Lower pole	Higher pole
$\lvert g_{\pi\Sigma}\rvert$	$3.41^{+1.74}_{-0.55}$	$2.05^{+2.10}_{-1.39}$
$\lvert g_{\bar{K}N}\rvert$	$2.89^{+2.30}_{-1.52}$	$3.18^{+1.73}_{-1.67}$

1. Both of $\Lambda(1405)$ states are resonances states
2. Within 1-σ CL, the lower pole becomes a virtual state partly

Fitting result (Up to NLO, S-wave)



$$\chi^2 / d.o.f. = 1.79$$

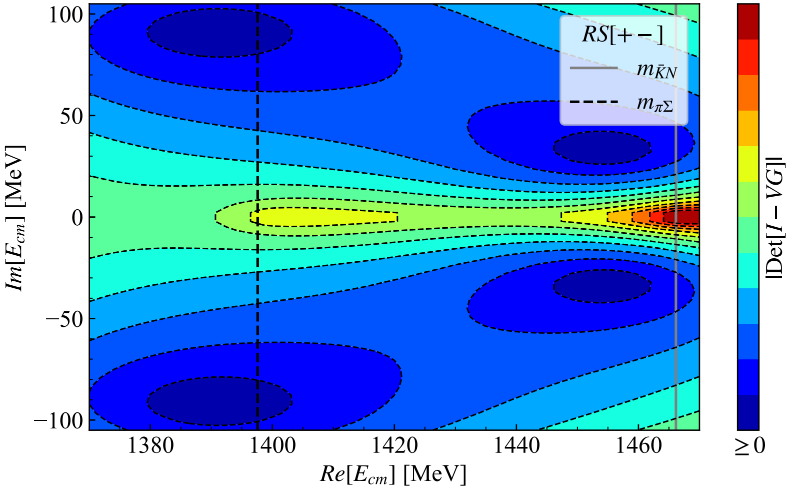
➢ **Lower energy constants d_i**

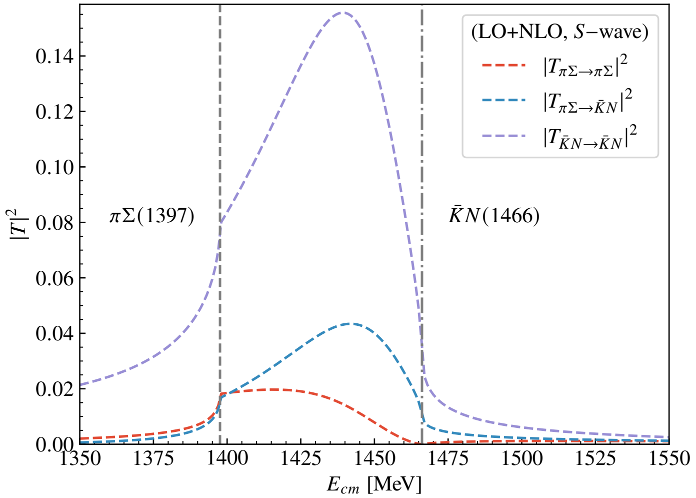
d_1	d_2	d_3	d_4
-0.26	0.0048	0.094	-0.77

- Units of d_i: GeV^{-1}
- $a_{\pi\Sigma} = -1.7, a_{\bar{K}N} = -2$
- b_i: fixed by baryon mass from LQCD

Up to next to leading order (S-wave)

- Pole positions of $\Lambda(1405)$ up to NLO





- A resonance
- A virtual state with non-zero imaginary part

	Lower pole	Higher pole
Mass[MeV]	$1390.37 - i91.86$	$1454.72 - i32.78$
$\lvert g_{\pi\Sigma}\rvert$	4.18	2.84
$\lvert g_{\bar{K}N}\rvert$	3.62	4.6

Effect of higher partial waves

- Partial wave mixing with boost

M. Doring et al, EPJA(2012)48,114

$$\tilde{G}_{lm,l'm'} = \frac{4\pi}{L^3}\sum_n \left(\frac{q}{q_{on}}\right)^{l+l'} Y^*_{lm}(\hat{q}) I(q)\, Y_{l'm'}(\hat{q})$$

- q_{on}: $E^2 - \left(\omega_1(q_{on}) + \omega_2(q_{on})\right)^2 = 0$, Energy level: $\mathrm{Det}\left[\delta_{ll'}\delta_{mm'} - V_l \tilde{G}_{lm,l'm'}\right] = 0$

1. The case of $\vec{P} = \frac{2\pi}{L}[0,0,0]$: $l = 0,\ \det\left[I - V_0\tilde{G}_{00,00}\right] = 0$
2. The case of $\vec{P} = \frac{2\pi}{L}[0,0,1]$: $l = 0,1,\ \det\left[\left(I - V_0\tilde{G}_{00,00}\right)\left(I - V_1\tilde{G}_{10,10}\right)\right] = 0$
3. The case of $\vec{P} = \frac{2\pi}{L}[0,1,1]$:

$$l = 0,1,\ \det\left[\left(I - V_0\tilde{G}_{00,00}\right)\left[I - V_1\left(\tilde{G}_{1-1,1-1} - i\tilde{G}_{11,1-1}\right) - V_0V_1\left(\tilde{G}_{00,11} - \tilde{G}_{00,1-1}\right)^2\right]\right] = 0$$

4. The case of $\vec{P} = \frac{2\pi}{L}[1,1,1]$:

$$l = 0,1,\ \det\left[\left(I - V_0\tilde{G}_{00,00}\right)\left(I - V_1\left(\tilde{G}_{1-1,1-1} + 2i\tilde{G}_{1-1,11}\right)\right) - 3V_0V_1\tilde{G}^2_{00,10}\right] = 0$$

Summary and Outlook

- **Summary:** study the lattice data of $\Lambda(1405)$ with chiral unitary approach

1. The lower pole in the first time becomes a virtual state from a resonance by varying the mass of pion
2. At LO(S-wave), the higher pole is a resonance, the lower pole could be a virtual state or a resonance
3. Up to NLO(S-wave), both of resonance and virtual state are present

- **Outlook**

1. Include the effect of P-wave in potential and fit to energy levels to obtain the parameters
2. Analyze the pole structures of $\Lambda(1405)$ in infinite volume to fix the poles position
3. Accomplish the paper

Thank you for your attention!

Thanks for Prof. Raquel and Dr. Jing for polishing slides