# Parton cascades at DLA: the role of the evolution variable 

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## Heavy Ion Collisions



The spacetime evolution of QCD matter covers a wide range of time/energy scales

Heavy Ion collisions are valuable as a laboratory to study the QCD phase diagram


## Parton Showers in a Coloured Medium



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Is jet quenching sensitive to the ordering of vacuum-like splittings?

## First, a look at vacuum (proton-proton) showers

## QCD Vacuum Splittings

## Estimate some scales:

- Formation time: $t_{\text {form }} \propto \frac{1}{m} \frac{E}{m} \sim \frac{E}{p^{2}} \sim \frac{1}{\omega \theta^{2}}$




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For an antenna: $\quad \lambda_{\perp}<\Delta b_{\perp} \Longleftrightarrow \theta<\Theta$

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* Larger $\lambda_{T}$
$\rightarrow$ Gluon cannot resolve the antenna legs
$\rightarrow$ Emission by the antenna as a whole
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This is the angular ordering property of vacuum splittings $\rightarrow$ Showers are collimated
*Can be generalised to non-singlets \& mutiple emissions


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## How to build a parton shower



Splittings with decreasing scale $\mu$

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Building blocks: QCD splittings


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Splitting probability given by PQCD:


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Building blocks: QCD splittings


Splitting probability given by PQCD :


Probability of not emitting until some scale $S$ :

$$
\Delta\left(s_{\text {prev }}, s\right)=\exp \left\{-\frac{\alpha C_{R}}{\pi} \int_{s}^{s_{\text {prev }}} \frac{\mathrm{d} \mu}{\mu} \int_{z_{\text {cut }}(\mu)}^{1} \frac{\mathrm{~d} z}{z}\right\}
$$

Yields the next emission scale $s$, given the previous scale $S_{\text {prev }}$

## Building differently ordered cascades

No-emission probability:

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\Delta\left(s_{\text {prev }}, s\right)=\exp \left\{-\frac{\alpha C_{R}}{\pi} \int_{s}^{s_{\text {prev }}} \frac{\mathrm{d} \mu}{\mu} \int_{z_{\text {cut }}(\mu)}^{1} \frac{\mathrm{~d} z}{z}\right\}
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## Interpretations for the scale:

$$
\begin{aligned}
& s \rightarrow p^{2}=\frac{\left|\boldsymbol{p}_{\text {rel }}\right|^{2}}{z(1-z)} \\
& \underset{\text { (Virtuality) }}{s \rightarrow t_{\text {form }}^{-1}}=\frac{p^{2}}{E}=\frac{\left|\boldsymbol{p}_{\text {rel }}\right|^{2}}{E z(1-z)}
\end{aligned}
$$

Splitting variables:


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\underset{\text { (Angle) }}{\rightarrow \rightarrow}=\frac{p^{2}}{E^{2} z(1-z)}=\left(\frac{\left|\boldsymbol{p}_{\text {rel }}\right|}{E z(1-z)}\right)^{2}
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To generate a splitting:


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$$

1. Sample a scale from $\Delta\left(s_{\text {prev }}, s\right)$
2. Sample a fraction from $\hat{P}(z) \propto 1 / z$ Ensure that $\left|\boldsymbol{p}_{\text {rel }}\right|^{2}>\Lambda^{2}$

## Building differently ordered cascades

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\text { (Formation time) }
\end{array} n^{2}
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$$




$$
\underset{\text { (Angle) }}{s \rightarrow} \zeta=\frac{p^{2}}{E^{2} z(1-z)}
$$

## Parton Shower Details



No-emission probability:

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- Splittings must happen above an hadronisation scale: $\left|\boldsymbol{p}_{\text {rel }}\right|^{2}>\Lambda^{2}$
- This provides a soft cutoff: $\quad z>z_{\text {cut }}(s)$
e.g.: Formation time ordering $\left|\boldsymbol{p}_{\text {rel }}\right|^{2}>\Lambda^{2} \Longleftrightarrow z(1-z)>\frac{\Lambda^{2}}{t_{\text {form }}^{-1} E}$
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- Initialisation condition for the shower: $t_{\text {form }}^{-1}<E$
- For consistency between orderings:

$$
\zeta<4 \Longrightarrow\left|p_{\text {rel }}\right|<\frac{E}{2}
$$

(Enforced via retrials)

Results (Work in Progress)

## Differences in Ordering Choices

Splittings along the quark branch



The strictly decreasing scale is different for the three algorithms

Different orderings $\rightarrow$ Different phase-space for allowed splittings

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Relative transverse momentum (1 ${ }^{\text {st }}$ splitting)

Transverse momentum distributions follow $\frac{\mathrm{d} p_{\text {rel }}^{2}}{p_{\text {rel }}^{2}}$

## Lund Plane Densities



Consider the shower evolution along the quark branch:
*Exaggerated scale



## Lund Plane Densities



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Shower evolution: Both transverse momentum and angle decrease.

## Lund Plane Densities



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## Lund Plane Trajectories



# Differences between phase-space trajectories <br> $\rightarrow$ Uncertainty at DLA Accuracy 

## Inversions in Kinematic Variables



## Formation Time Inversions:

Splittings with a formation time shorter that their immediate predecessor.

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Angular inversions

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## Angular inversions

Can this discrepancy translate into differences in quenching magnitude?

Now, a simple jet quenching model!

## Choosing a quenching condition

## Medium parameters (for a simple model):

- Medium length: L
- Transport coefficient: $\quad \hat{q} \sim \frac{\left\langle k_{\perp}^{2}\right\rangle}{\lambda}$


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Medium resolves splittings on
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$\rightarrow$ Daughters lose energy
individually (cf. antenna)


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What role do time-inversions play in these quenching differences?

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For angular ordered showers: $\Rightarrow \zeta$ strictly decreasing
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Increasing quenching effects

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quenching effects
The implementation details of the jet interface with a time-evolving medium are crucial!

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## Acknowledgements



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## Backup Slides

## Without the consistency condition



If the condition $\zeta<4$ is used simply to initialise the angular shower, the time and angle distributions do not behave consistently across algorithms

## With the consistency condition




When the condition $\zeta<4$ is used as a veto for all emissions, the distributions become consistent.

## Excluding time inversions - 1D Distributions




## Inclusive Sample - 1D Distributions




## Vetoing time inversions - 1D Distributions




## Excluding time inversions - Lund Planes

*Ordered in angle


## Inclusive Sample - Lund Planes

*Ordered in angle


## Vetoing time inversions - Lund Planes

*Ordered in angle


## Quenching Weights

Very Preliminary!
$\mathrm{E}_{\text {jet }}=1000 \mathrm{GeV}, \Lambda=1 \mathrm{GeV}$

$\mathrm{E}_{\text {jet }}=\mathbf{5 0 0} \mathbf{~ G e V}$


$$
\Lambda=0.1 \mathrm{GeV}
$$



An apparent dependence on the hadronisation cutoff and initial jet energy

## Quenching Weights - Radius Cut

Very Preliminary!

$$
\mathrm{E}_{\mathrm{jet}}=1000 \mathrm{GeV}, \Lambda=1 \mathrm{GeV}
$$

$$
\mathrm{E}_{\mathrm{jet}}=500 \mathrm{GeV}
$$

$\Lambda=0.1 \mathrm{GeV}$


Cut all events whose quark branch has a splitting wider than $\mathbf{R}_{\max }=0.2$

- This defines the new vacuum sample, and the quenching model is applied on top of this cut

An aggressive cut, but it returns independence of $\mathrm{E}_{\mathrm{jet}}$ and $\Lambda$.

