

*Branching ratios of  
the pseudoscalar glueball and  
its first excited state*

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# The QCD Lagrangian

- Contains ‘colored’ quarks and gluons but no ‘colored’ states have been appeared
- if all quark massless then we have chiral symmetry

$$U(N_f)_r \times U(N_f)_l = SU(N_f)_r \times SU(N_f)_l \times U(1)_V \times U(1)_A$$

- Spontaneous breaking of chiral symmetry by quark condensates.
  - Explicit breaking of global chiral symmetry by quark masses and chiral anomaly.
- Effective chiral models of (QCD).
    - Linear Sigma Model
    - Non-Linear Sigma Model

*Decay of the  
pseudoscalar glueball*

# A globally chirally invariant for three flavours

Interaction Lagrangian for the pseudoscalar glueball with vector, axial-vector, scalar and pseudoscalar mesons

$$\mathcal{L}_{eLSM, \tilde{G}}^{int} = i c \tilde{G} \text{Tr} [L_\mu (\partial^\mu \Phi \cdot \Phi^\dagger + \Phi \cdot \partial^\mu \Phi^\dagger) - R_\mu (\partial^\mu \Phi^\dagger \cdot \Phi + \Phi^\dagger \cdot \partial^\mu \Phi)] ,$$

where  $c$  is a dimensionless coupling constant and  $\Phi$  reads for three flavours,  $N_f = 3$  :

$$\Phi = (S^a + iP^a) t^a = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{(\sigma_N + a_0^0) + i(\eta_N + \pi^0)}{\sqrt{2}} & a_0^+ + i\pi^+ & K_S^+ + iK^+ \\ a_0^- + i\pi^- & \frac{(\sigma_N - a_0^0) + i(\eta_N - \pi^0)}{\sqrt{2}} & K_S^0 + iK^0 \\ K_S^- + iK^- & \bar{K}_S^0 + i\bar{K}^0 & \sigma_S + i\eta_S \end{pmatrix}$$

The vector  $V^a$  and axial-vector  $A^a$ , degree of freedom, are presented in the following left- and right handed matrices,  $L_\mu$  and  $R_\mu$

$$L_\mu = (V^a + i A^a)_\mu t^a = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N + \rho^0}{\sqrt{2}} + \frac{f_{1N} + a_1^0}{\sqrt{2}} & \rho^+ + a_1^+ & K^{*+} + K_1^+ \\ \rho^- + a_1^- & \frac{\omega_N - \rho^0}{\sqrt{2}} + \frac{f_{1N} - a_1^0}{\sqrt{2}} & K^{*0} + K_1^0 \\ K^{*-} + K_1^- & \bar{K}^{*0} + \bar{K}_1^0 & \omega_S + f_{1S} \end{pmatrix}_\mu ,$$

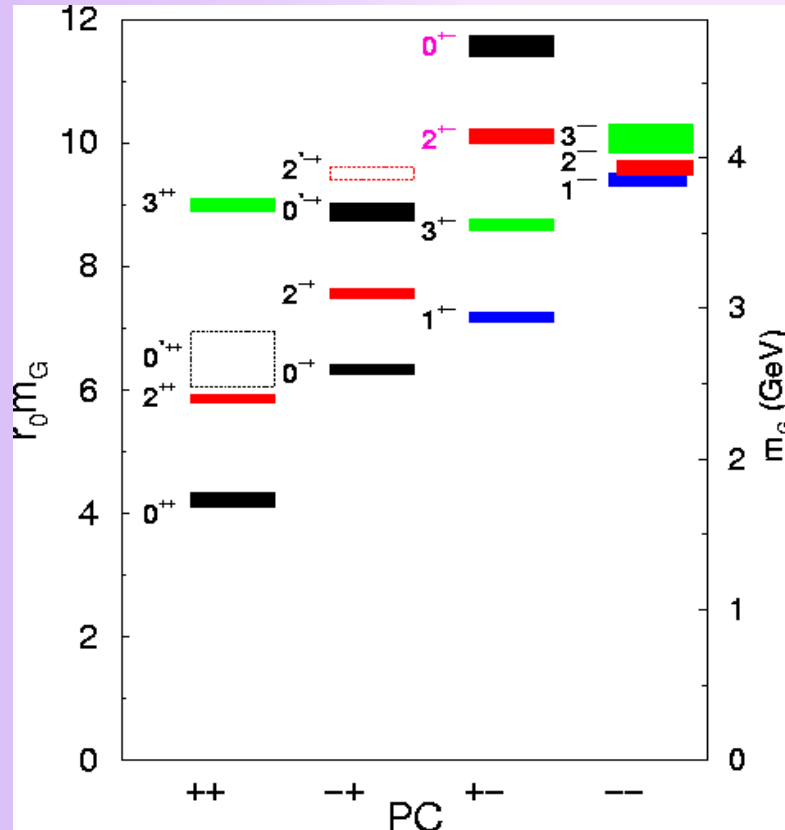
and

$$R_\mu = (V^a - i A^a)_\mu t^a = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N + \rho^0}{\sqrt{2}} - \frac{f_{1N} + a_1^0}{\sqrt{2}} & \rho^+ - a_1^+ & K^{*+} - K_1^+ \\ \rho^- - a_1^- & \frac{\omega_N - \rho^0}{\sqrt{2}} - \frac{f_{1N} - a_1^0}{\sqrt{2}} & K^{*0} - K_1^0 \\ K^{*-} - K_1^- & \bar{K}^{*0} - \bar{K}_1^0 & \omega_S - f_{1S} \end{pmatrix}_\mu .$$

W. I. Eshraim, *Eur.Phys.J.C* 83 (2023) 3, 262 [arXiv: [2005.11321](https://arxiv.org/abs/2005.11321) [hep-ph]]

# Glueballs

## Lattice QCD calculation



The pseudoscalar glueball  $\tilde{G} \equiv |gg\rangle$  at the border within light and heavy

$$M_{\tilde{G}} = 2.6 \quad , \quad J^{PC} = 0^{-+}, \quad I = 0.$$

The first excited pseudoscalar Glueball

$$M_{\tilde{G}'} = 3.7 \quad , \quad J^{PC} = 0^{*-+}.$$

[C. Morningstar and M. J. Peardon, AIP Conf. Proc. 688, 220 (2004)  
[arXiv:nucl-th/0309068]];

## Two experiments related to our work:

1. PANDA experiment at FAIR facility.

It will be capable to scan the mass region above 2.5 GeV.

2. BESIII experiment.

The resonance  $X(2370)$  could be a pseudoscalar glueball with a mass 2.37 GeV.

## Branching ratios for the two-body decays of the pseudoscalar glueball

Quantity	Case (i): $M_{\tilde{G}} = 2.6$ GeV	Case (ii): $M_{\tilde{G}} = 2.37$ GeV
$\Gamma_{\tilde{G} \rightarrow KK^*} / \Gamma_{\tilde{G} \rightarrow \pi\pi\eta}$	0.00026	0.00031
$\Gamma_{\tilde{G} \rightarrow a_0\pi} / \Gamma_{\tilde{G} \rightarrow \pi\pi\eta}$	0.1913	0.1858
$\Gamma_{\tilde{G} \rightarrow KK_S} / \Gamma_{\tilde{G} \rightarrow \pi\pi\eta}$	0.1745	0.1595
$\Gamma_{\tilde{G} \rightarrow f_0(1370)\eta} / \Gamma_{\tilde{G} \rightarrow \pi\pi\eta}$	0.0374	0.0349
$\Gamma_{\tilde{G} \rightarrow f_0(1500)\eta} / \Gamma_{\tilde{G} \rightarrow \pi\pi\eta}$	0.00399	0.00325
$\Gamma_{\tilde{G} \rightarrow f_0(1700)\eta} / \Gamma_{\tilde{G} \rightarrow \pi\pi\eta}$	0.00265	0.00134
$\Gamma_{\tilde{G} \rightarrow f_0(1370)\eta'} / \Gamma_{\tilde{G} \rightarrow \pi\pi\eta}$	0.00837	0.00343
$\Gamma_{\tilde{G} \rightarrow f_0(1500)\eta'} / \Gamma_{\tilde{G} \rightarrow \pi\pi\eta}$	0.00999	0

W. I. Eshraim, *Eur.Phys.J.C* 83 (2023) 3, 262 [arXiv: [2005.11321](https://arxiv.org/abs/2005.11321) [hep-ph]]



## Branching ratios for the three-body decays of the pseudoscalar glueball

Quantity	Case (i): $M_{\bar{G}} = 2.6$ GeV	Case (ii): $M_{\bar{G}} = 2.37$ GeV
$\Gamma_{\bar{G} \rightarrow \pi\pi\eta'} / \Gamma_{\bar{G} \rightarrow \pi\pi\eta}$	0.4654	0.3986
$\Gamma_{\bar{G} \rightarrow KK\pi} / \Gamma_{\bar{G} \rightarrow \pi\pi\eta}$	0.9126	0.8553
$\Gamma_{\bar{G} \rightarrow KK\eta} / \Gamma_{\bar{G} \rightarrow \pi\pi\eta}$	0.0038	0.0031
$\Gamma_{\bar{G} \rightarrow KK\eta'} / \Gamma_{\bar{G} \rightarrow \pi\pi\eta}$	0.13799	0.07157
$\Gamma_{\bar{G} \rightarrow \eta\eta\eta} / \Gamma_{\bar{G} \rightarrow \pi\pi\eta}$	0.00012	0.000087
$\Gamma_{\bar{G} \rightarrow \eta\eta\eta'} / \Gamma_{\bar{G} \rightarrow \pi\pi\eta}$	0.0253	0.0102
$\Gamma_{\bar{G} \rightarrow \eta\eta'\eta'} / \Gamma_{\bar{G} \rightarrow \pi\pi\eta}$	0.0000012	0

W. I. Eshraim, *Eur.Phys.J.C* 83 (2023) 3, 262 [arXiv: [2005.11321](https://arxiv.org/abs/2005.11321) [hep-ph]]

*Decay modes of the  
excited pseudoscalar glueball*

# Interaction Lagrangian for the excited pseudoscalar glueball

with a pseudoscalar glueball and the ordinary scalar and pseudoscalar mesons

$$\mathcal{L}_{\tilde{G}\tilde{G}'}^{int} = c_{\tilde{G}\tilde{G}'} \tilde{G}\tilde{G}' Tr(\Phi^\dagger\Phi)$$

with a scalar glueball and the pseudo(scalar) mesons

$$\mathcal{L}_{\tilde{G}G}^{int} = ic_{\tilde{G}G\Phi} \tilde{G}G (\det\Phi - \det\Phi^\dagger)$$

with scalar and pseudoscalar mesons

$$\mathcal{L}_{\tilde{G}\Phi}^{int} = ic_{\tilde{G}\Phi} \tilde{G} (\det\Phi - \det\Phi^\dagger)$$

where  $c_{\tilde{G}\Phi}$  is a dimensionless coupling constant and  $\Phi$  for three flavours,  $N_f = 3$

Walaa I. Eshraim, Stefan Schramm, Phys.Rev. D95 (2017) 014028 [arXiv:1606.02207 [hep-ph]].

## Results

Branching ratios for the decay of the excited pseudoscalar glueball into the pseudoscalar glueball

Quantity	The theoretical result
$\Gamma_{\tilde{G} \rightarrow \tilde{G}' KK} / \Gamma_{\tilde{G}}^{tot}$	0.0277
$\Gamma_{\tilde{G} \rightarrow \tilde{G}' \pi\pi} / \Gamma_{\tilde{G}}^{tot}$	0.9697
$\Gamma_{\tilde{G} \rightarrow \tilde{G}' \eta\eta'} / \Gamma_{\tilde{G}}^{tot}$	0.0026
$\Gamma_{\tilde{G} \rightarrow \tilde{G}' \eta\eta} / \Gamma_{\tilde{G}}^{tot}$	0.000012

The branching ratio for the decay of the excited pseudoscalar glueball into charmonium state

$$\Gamma_{\tilde{G} \rightarrow \eta_C \pi\pi} / \Gamma_{\tilde{G}_3}^{tot} = 0.001$$

Walia I. Eshraim, Stefan Schramm, Phys.Rev. D95 (2017) 014028 [arXiv:1606.02207 [hep-ph]].

Branching ratios for the decays of the excited pseudoscalar glueball into  $PS$  and scalar-isoscalar states as well as  $\eta$  and  $\eta'$

Case (i): $\mathcal{L}_{\tilde{G}\tilde{G}}^{int}$	The theoretical result	Case (ii): $\mathcal{L}_{\tilde{G}\Phi}^{int}$	The theoretical result
$\Gamma_{\tilde{G} \rightarrow a_0 \pi} / \Gamma_{\tilde{G}_2}^{tot}$	0.0325	$\Gamma_{\tilde{G} \rightarrow a_0 \pi} / \Gamma_{\tilde{G}_3}^{tot}$	0.0313
$\Gamma_{\tilde{G} \rightarrow KK_S} / \Gamma_{\tilde{G}_2}^{tot}$	0.032	$\Gamma_{\tilde{G} \rightarrow KK_S} / \Gamma_{\tilde{G}_3}^{tot}$	0.001
$\Gamma_{\tilde{G} \rightarrow \eta f_0(1370)} / \Gamma_{\tilde{G}_2}^{tot}$	0.00004	$\Gamma_{\tilde{G} \rightarrow \eta f_0(1370)} / \Gamma_{\tilde{G}_3}^{tot}$	0.0014
$\Gamma_{\tilde{G} \rightarrow \eta' f_0(1370)} / \Gamma_{\tilde{G}_2}^{tot}$	0.048	$\Gamma_{\tilde{G} \rightarrow \eta' f_0(1370)} / \Gamma_{\tilde{G}_3}^{tot}$	0.031
$\Gamma_{\tilde{G} \rightarrow \eta f_0(1500)} / \Gamma_{\tilde{G}_2}^{tot}$	0.0068	$\Gamma_{\tilde{G} \rightarrow \eta f_0(1500)} / \Gamma_{\tilde{G}_3}^{tot}$	0.0067
$\Gamma_{\tilde{G} \rightarrow \eta' f_0(1500)} / \Gamma_{\tilde{G}_2}^{tot}$	0.0219	$\Gamma_{\tilde{G} \rightarrow \eta' f_0(1500)} / \Gamma_{\tilde{G}_3}^{tot}$	0.0214
$\Gamma_{\tilde{G} \rightarrow \eta f_0(1710)} / \Gamma_{\tilde{G}_2}^{tot}$	0.0008	$\Gamma_{\tilde{G} \rightarrow \eta f_0(1710)} / \Gamma_{\tilde{G}_3}^{tot}$	0.0007
$\Gamma_{\tilde{G} \rightarrow \eta' f_0(1710)} / \Gamma_{\tilde{G}_2}^{tot}$	0.001	$\Gamma_{\tilde{G} \rightarrow \eta' f_0(1710)} / \Gamma_{\tilde{G}_3}^{tot}$	0.001

Walaa I. Eshraim, Stefan Schramm, Phys.Rev. D95 (2017) 014028 [arXiv:1606.02207 [hep-ph]].

# Branching ratios for the decays of the excited pseudoscalar glueball

into scalar-isoscalar states  
and (pseudo)scalar mesons

Case (i): $\mathcal{L}_{\tilde{G}_2}^{int}$	The theoretical result	Case (ii): $\mathcal{L}_{\tilde{G}_3}^{int}$	The theoretical result
$\Gamma_{\tilde{G} \rightarrow \eta\pi\pi} / \Gamma_{\tilde{G}_2}^{tot}$	0.095	$\Gamma_{\tilde{G} \rightarrow \eta\pi\pi} / \Gamma_{\tilde{G}_3}^{tot}$	0.1376
$\Gamma_{\tilde{G} \rightarrow \eta'\pi\pi} / \Gamma_{\tilde{G}_2}^{tot}$	0.111	$\Gamma_{\tilde{G} \rightarrow \eta'\pi\pi} / \Gamma_{\tilde{G}_3}^{tot}$	0.1069
$\Gamma_{\tilde{G} \rightarrow a_0 K K_S} / \Gamma_{\tilde{G}_2}^{tot}$	0.0026	$\Gamma_{\tilde{G} \rightarrow a_0 K K_S} / \Gamma_{\tilde{G}_3}^{tot}$	0.0025
$\Gamma_{\tilde{G} \rightarrow \eta a_0 a_0} / \Gamma_{\tilde{G}_2}^{tot}$	0.0001	$\Gamma_{\tilde{G} \rightarrow \eta a_0 a_0} / \Gamma_{\tilde{G}_3}^{tot}$	0.0001
$\Gamma_{\tilde{G} \rightarrow a_0 \pi f_0(1370)} / \Gamma_{\tilde{G}_2}^{tot}$	0.0003	$\Gamma_{\tilde{G} \rightarrow a_0 \pi f_0(1370)} / \Gamma_{\tilde{G}_3}^{tot}$	0.0003
$\Gamma_{\tilde{G} \rightarrow a_0 \pi f_0(1500)} / \Gamma_{\tilde{G}_2}^{tot}$	0.0034	$\Gamma_{\tilde{G} \rightarrow a_0 \pi f_0(1500)} / \Gamma_{\tilde{G}_3}^{tot}$	0.0032
$\Gamma_{\tilde{G} \rightarrow a_0 \pi f_0(1710)} / \Gamma_{\tilde{G}_2}^{tot}$	0.0001	$\Gamma_{\tilde{G} \rightarrow a_0 \pi f_0(1710)} / \Gamma_{\tilde{G}_3}^{tot}$	0.0001
$\Gamma_{\tilde{G} \rightarrow \eta f_0^2(1370)} / \Gamma_{\tilde{G}_2}^{tot}$	0.0003	$\Gamma_{\tilde{G} \rightarrow \eta f_0^2(1370)} / \Gamma_{\tilde{G}_3}^{tot}$	0.001
$\Gamma_{\tilde{G} \rightarrow \eta' f_0^2(1370)} / \Gamma_{\tilde{G}_2}^{tot}$	$0.03 \times 10^{-6}$	$\Gamma_{\tilde{G} \rightarrow \eta' f_0^2(1370)} / \Gamma_{\tilde{G}_3}^{tot}$	$0.006 \times 10^{-6}$
$\Gamma_{\tilde{G} \rightarrow \eta f_0^2(1500)} / \Gamma_{\tilde{G}_2}^{tot}$	0.00004	$\Gamma_{\tilde{G} \rightarrow \eta f_0^2(1500)} / \Gamma_{\tilde{G}_3}^{tot}$	0.00001
$\Gamma_{\tilde{G} \rightarrow \eta f_0(1370) f_0(1500)} / \Gamma_{\tilde{G}_2}^{tot}$	0.00003	$\Gamma_{\tilde{G} \rightarrow \eta f_0(1370) f_0(1500)} / \Gamma_{\tilde{G}_3}^{tot}$	0.0001
$\Gamma_{\tilde{G} \rightarrow \eta f_0(1370) f_0(1710)} / \Gamma_{\tilde{G}_2}^{tot}$	$3.798 \times 10^{-6}$	$\Gamma_{\tilde{G} \rightarrow \eta f_0(1370) f_0(1710)} / \Gamma_{\tilde{G}_3}^{tot}$	$7.25 \times 10^{-6}$
$\Gamma_{\tilde{G} \rightarrow K K_S f_0(1370)} / \Gamma_{\tilde{G}_2}^{tot}$	0.0025	$\Gamma_{\tilde{G} \rightarrow K K_S f_0(1370)} / \Gamma_{\tilde{G}_3}^{tot}$	0.0025
$\Gamma_{\tilde{G} \rightarrow K K_S f_0(1500)} / \Gamma_{\tilde{G}_2}^{tot}$	0.00013	$\Gamma_{\tilde{G} \rightarrow K K_S f_0(1500)} / \Gamma_{\tilde{G}_3}^{tot}$	0.00013
$\Gamma_{\tilde{G} \rightarrow K K_S f_0(1710)} / \Gamma_{\tilde{G}_2}^{tot}$	$6.2 \times 10^{-6}$	$\Gamma_{\tilde{G} \rightarrow K K_S f_0(1710)} / \Gamma_{\tilde{G}_3}^{tot}$	$4.75 \times 10^{-6}$
$\Gamma_{\tilde{G} \rightarrow K K \eta} / \Gamma_{\tilde{G}_2}^{tot}$	0.0668	$\Gamma_{\tilde{G} \rightarrow K K \eta} / \Gamma_{\tilde{G}_3}^{tot}$	0.0643
$\Gamma_{\tilde{G} \rightarrow K K \eta'} / \Gamma_{\tilde{G}_2}^{tot}$	0.045	$\Gamma_{\tilde{G} \rightarrow K K \eta'} / \Gamma_{\tilde{G}_3}^{tot}$	0.044
$\Gamma_{\tilde{G} \rightarrow K_S K_S \eta} / \Gamma_{\tilde{G}_2}^{tot}$	0.0002	$\Gamma_{\tilde{G} \rightarrow K_S K_S \eta} / \Gamma_{\tilde{G}_3}^{tot}$	0.0002
$\Gamma_{\tilde{G} \rightarrow \eta^3} / \Gamma_{\tilde{G}_2}^{tot}$	0.024	$\Gamma_{\tilde{G} \rightarrow \eta^3} / \Gamma_{\tilde{G}_3}^{tot}$	0.0233
$\Gamma_{\tilde{G} \rightarrow \eta'^3} / \Gamma_{\tilde{G}_2}^{tot}$	0.0048	$\Gamma_{\tilde{G} \rightarrow \eta'^3} / \Gamma_{\tilde{G}_3}^{tot}$	0.0046
$\Gamma_{\tilde{G} \rightarrow \eta' \eta^2} / \Gamma_{\tilde{G}_2}^{tot}$	0.005	$\Gamma_{\tilde{G} \rightarrow \eta' \eta^2} / \Gamma_{\tilde{G}_3}^{tot}$	0.0048
$\Gamma_{\tilde{G} \rightarrow \eta'^2 \eta} / \Gamma_{\tilde{G}_2}^{tot}$	0.0035	$\Gamma_{\tilde{G} \rightarrow \eta'^2 \eta} / \Gamma_{\tilde{G}_3}^{tot}$	0.0034
$\Gamma_{\tilde{G} \rightarrow K K \pi} / \Gamma_{\tilde{G}_2}^{tot}$	0.489	$\Gamma_{\tilde{G} \rightarrow K K \pi} / \Gamma_{\tilde{G}_3}^{tot}$	0.471
$\Gamma_{\tilde{G} \rightarrow K_S K_S \pi} / \Gamma_{\tilde{G}_2}^{tot}$	0.002	$\Gamma_{\tilde{G} \rightarrow K_S K_S \pi} / \Gamma_{\tilde{G}_3}^{tot}$	0.0057

Walaa I. Eshraim, Stefan Schramm, Phys.Rev. D95 (2017) 014028 [arXiv:1606.02207 [hep-ph]].

## Conclusions

1. Construction of the interaction Lagrangian for the pseudoscalar Glueball with light mesons.
2. Decay of the pseudoscalar glueball.
3. Construction of several interaction Lagrangians for the excited pseudoscalar glueball with a scalar glueball and (pseudo)scalar mesons
4. Decay of the first excited pseudoscalar state.

شكراً جزيلاً

**THANKS A LOT**