Quarkyonic effective field theory

Dyana C. Duarte

In collaboration with K.S. Jeong, S.Hernandez-Ortiz and L. McLerran
Based on PRD 104, L091901, PRC 107, 065201

Excited QCD 2024 Workshop
Benasque Science Center, Jan 14-20, 2024

FAPERGS

serrapilheira

Universidade Federal de Santa Maria
SEDES SAPIENTIÆ
1960

Outline

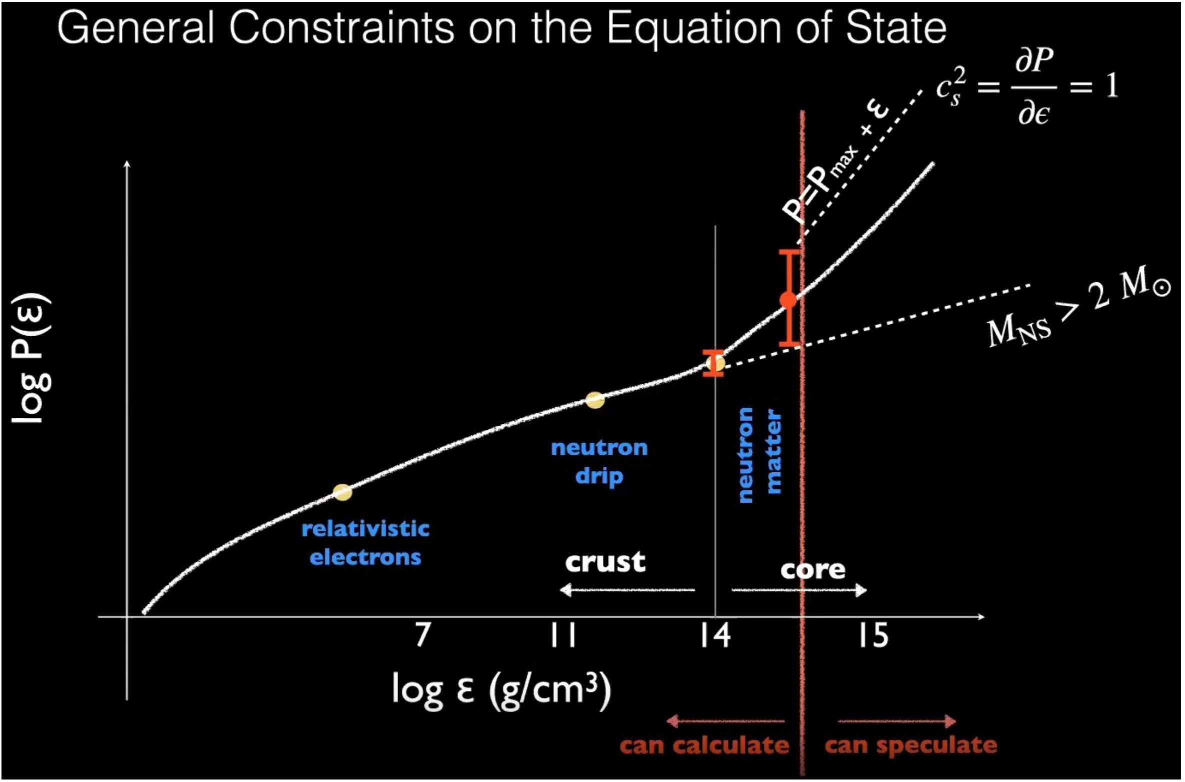
- Brief motivation;

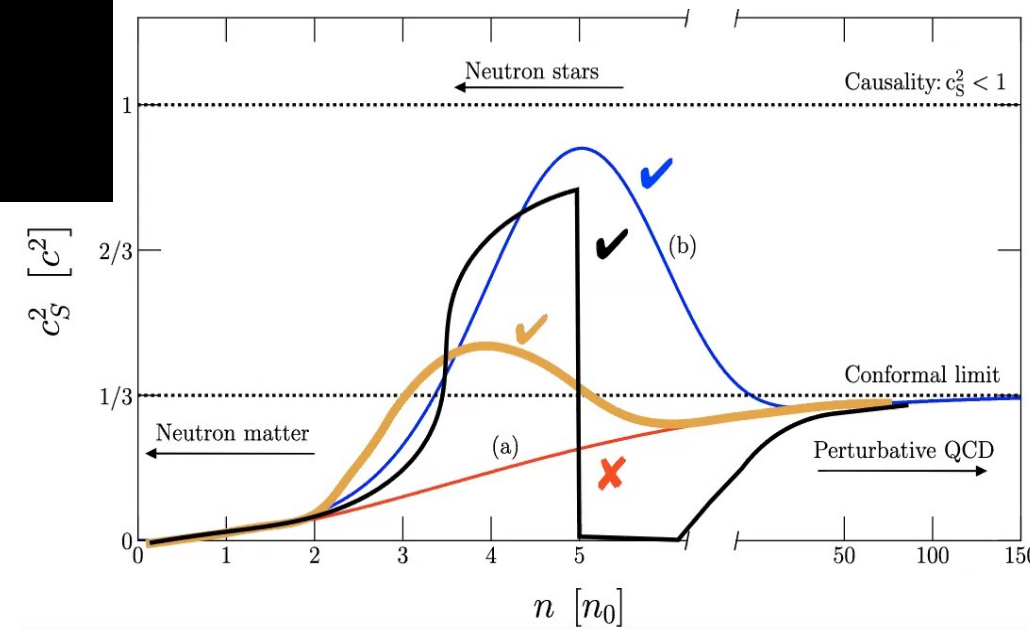
- Quarkyonic Matter for the EoS of neutron stars;

- Quarkyonic EFT;

- Final Remarks.

Constraints to the EoS of dense matter





GW170817

- $R_{M=1.4M_\odot} \leq 13.5$ km and $M_{max} > 2M_\odot$

GW190425

- $R_{M=1.4M_\odot} \leq 15$ km and $M_{max}\ (2-3)M_\odot$

Breakdown of the models!

Figs. from S. Reddy presentation @BNL (2021)

Many different approaches and possibilities!

- Extension of low-energy nuclear physics to higher densities: RMF and many body calculations with dependence of couplings and masses with n_B [Oertel et al., Rev.Mod.Phys. 89, 015007 (2017), Kaiser et al., Nucl. Phys. A. 697, 255 (2002), Drischler et al., Phys. Rev. Lett.122, 042501 (2019), Lonardoni et al., Phys. Rev. Res. 2, 022033 (2020)];

- Other interactions and degrees of freedom [Glendenning, Astrophys. J. 293, 470 (1985), Knorren, et al., Phys. Rev. C 52, 3470 (1995), Cai et Al., Phys. Rev. C 92, 015802 (2015)];

- Phase transition from nuclear to quark matter. [Annala et al. Nat. Phys. 16, 907 (2020), Nature Communications 14, 8451 (2023)].

Many different approaches and possibilities!

- Extension of low-energy nuclear physics to higher densities: RMF and many body calculations with dependence of couplings and masses with n_B [Oertel et al., Rev.Mod.Phys. 89, 015007 (2017), Kaiser et al., Nucl. Phys. A. 697, 255 (2002), Drischler et al., Phys. Rev. Lett.122, 042501 (2019), Lonardoni et al., Phys. Rev. Res. 2, 022033 (2020)];

- Other interactions and degrees of freedom [Glendenning, Astrophys. J. 293, 470 (1985), Knorren, et al., Phys. Rev. C 52, 3470 (1995), Cai et Al., Phys. Rev. C 92, 015802 (2015)];

- Phase transition from nuclear to quark matter. [Annala et al. Nat. Phys. 16, 907 (2020), Nature Communications 14, 8451 (2023)].

Resultant model/theory must:

$\surd$ Satisfy the well-known nuclear matter properties at saturation density; $n_B \sim (1-2)\rho_0$

$\surd$ Evolve to a phase of deconfined quarks at high densities; $n_B \gtrsim (2-5)\rho_0???$

$\surd$ Recover the pQCD results at asymptotically high densities and temperatures. $n_B \gtrsim 40\rho_0$

Bayesian Inference

(gaussian regression)

Neutron star EoS is characterized by:

- Nucleon mass scale in the hadronic part;
- Near-conformal properties (no mass scale).

nature communications

Article https://doi.org/10.1038/s41467-023-44051-y

Strongly interacting matter exhibits deconfined behavior in massive neutron stars

Received: 3 May 2023

Accepted: 28 November 2023

Published online: 19 December 2023

Eemeli Annala[1], Tyler Gorda[2,3], Joonas Hirvonen[1], Oleg Komoltsev[4], Aleksi Kurkela[4], Joonas Nättilä[5,6] & Aleksi Vuorinen[1]

Table 1 | Characteristic features of dense strongly interacting matter

	CEFT	Dense NM	Pert. QM	CFTs	FOPT	
c_s^2	$\ll 1$	[0.25, 0.6]	$\lesssim 1/3$	1/3	0	$= dp/d\varepsilon$
Δ	$\approx 1/3$	[0.05, 0.25]	[0, 0.15]	0	$1/3 - p_{PT}/\epsilon$	$= (\varepsilon - 3p)/(3\varepsilon)$
Δ'	≈ 0	[−0.4, −0.1]	[−0.15, 0]	0	$1/3 - \Delta$	$= d\Delta/d\ln(\varepsilon)$
d_c	$\approx 1/3$	[0.25, 0.4]	$\lesssim 0.2$	0	$\geq 1/(3\sqrt{2})$	$= \sqrt{\Delta^2 + (\Delta')^2}$
γ	≈ 2.5	[1.95, 3.0]	[1, 1.7]	1	0	$= d\ln(p)/d\ln(\varepsilon)$
p/p_{free}	$\ll 1$	[0.25, 0.35]	[0.5, 1]	—	p_{PT}/p_{free}	$= p/p_{free}$

Bayesian Inference
(gaussian regression)

Neutron star EoS is characterized by:

- Nucleon mass scale in the hadronic part;
- Near-conformal properties (no mass scale).

Table 1 | Characteristic features of dense strongly interacting matter

	CEFT	Dense NM	Pert. QM	CFTs	FOPT	
c_s^2	$\ll 1$	[0.25, 0.6]	$\lesssim 1/3$	1/3	0	$= dp/d\varepsilon$
Δ	$\approx 1/3$	[0.05, 0.25]	[0, 0.15]	0	$1/3 - p_{PT}/\epsilon$	$= (\varepsilon - 3p)/(3\varepsilon)$
Δ'	≈ 0	[−0.4, −0.1]	[−0.15, 0]	0	$1/3 - \Delta$	$= d\Delta/d\ln(\varepsilon)$
d_c	$\approx 1/3$	[0.25, 0.4]	$\lesssim 0.2$	0	$\geq 1/(3\sqrt{2})$	$= \sqrt{\Delta^2 + (\Delta')^2}$
γ	≈ 2.5	[1.95, 3.0]	[1, 1.7]	1	0	$= d\ln(p)/d\ln(\varepsilon)$
p/p_{free}	$\ll 1$	[0.25, 0.35]	[0.5, 1]	—	p_{PT}/p_{free}	$= p/p_{\mathrm{free}}$

nature communications

Article

https://doi.org/10.1038/s41467-023-44051-y

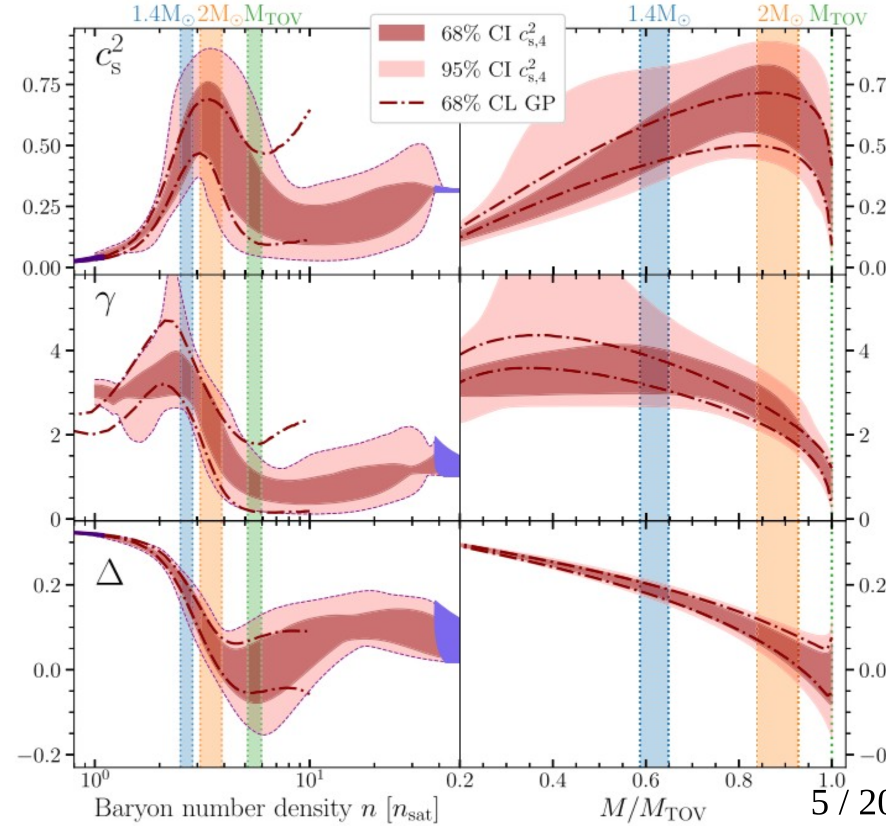
Strongly interacting matter exhibits deconfined behavior in massive neutron stars

Received: 3 May 2023

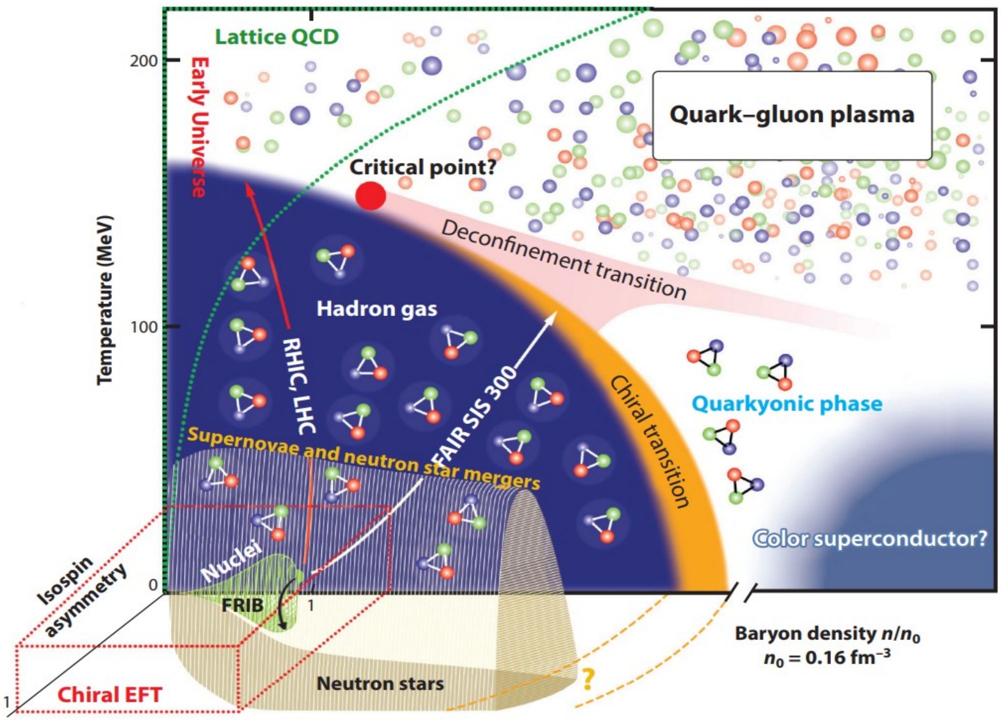
Accepted: 28 November 2023

Published online: 19 December 2023

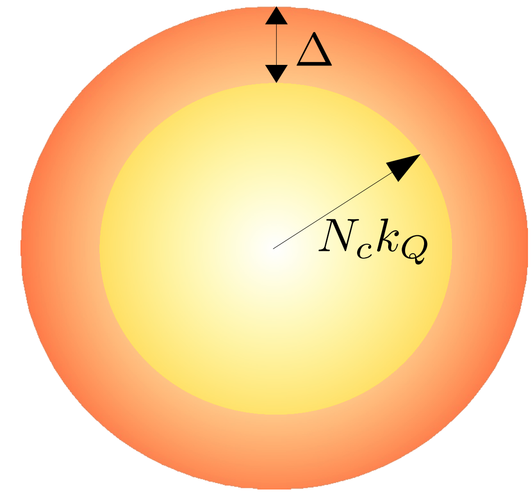
Eemeli Annala[1], Tyler Gorda[2,3], Joonas Hirvonen[1], Oleg Komoltsev[4], Aleksi Kurkela[4], Joonas Nättilä[5,6] & Aleksi Vuorinen[1]



Quarkyonic Matter



Drischler et al (2021).



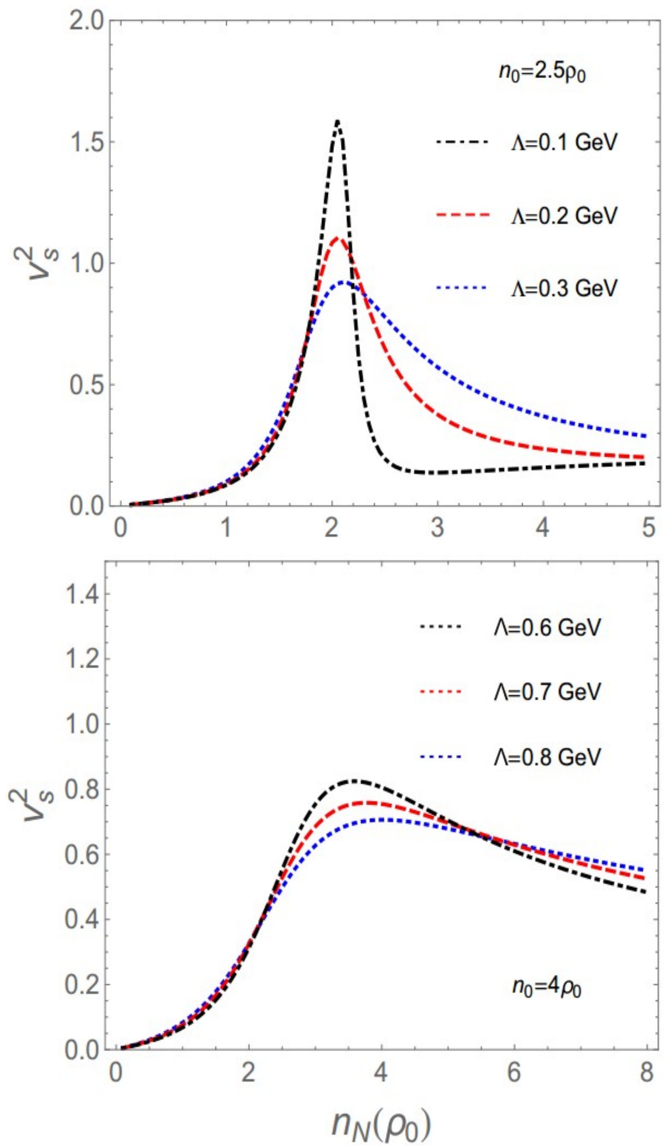
Pressure when $\mu_Q > M_q$ and $n_B \neq 0$:

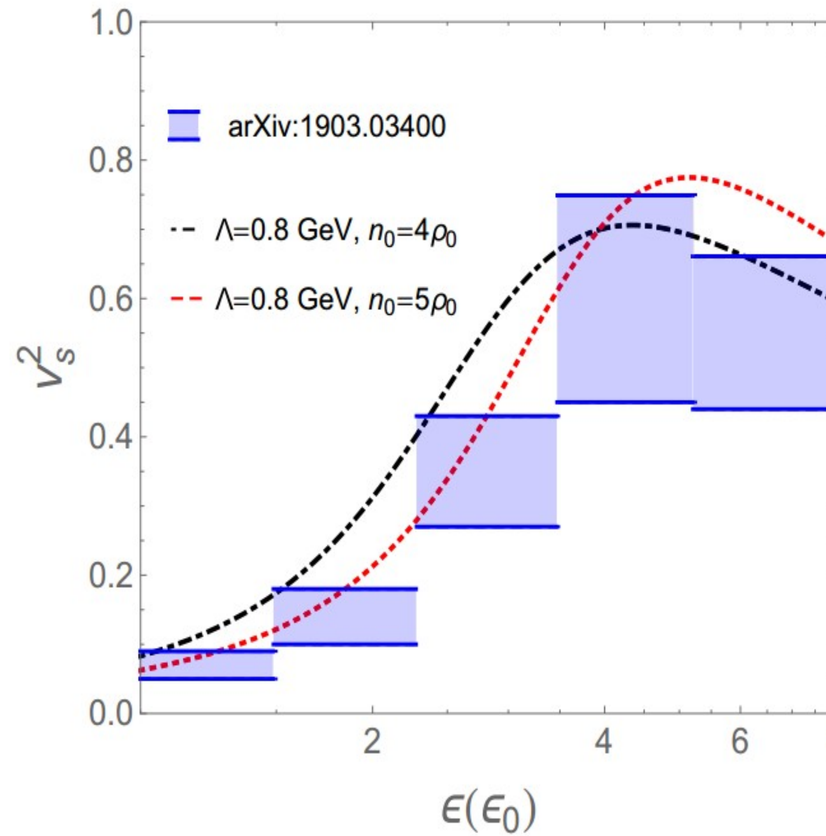
$$\mathcal{O}(N_c^0) \to \mathcal{O}(N_c)$$

(hadron) $\to$ (quarkyonic)

Weakly interacting quark system?
Baryonic system?

Quarkyonic Matter can satisfy constraints to the EoS of NS

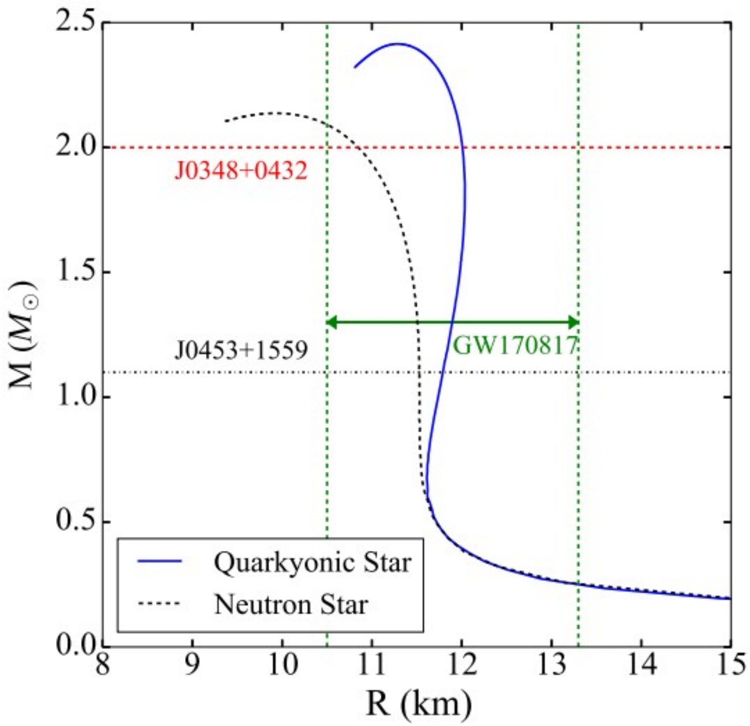




Good agreement with sound velocity obtained from an equation of state extracted from neutron stars properties using deep neural network.

K.S. Jeong, L. McLerran, S. Sen, Phys. Rev. C 101 035201 (2020)

Quarkyonic Matter can satisfy constraints to the EoS of NS



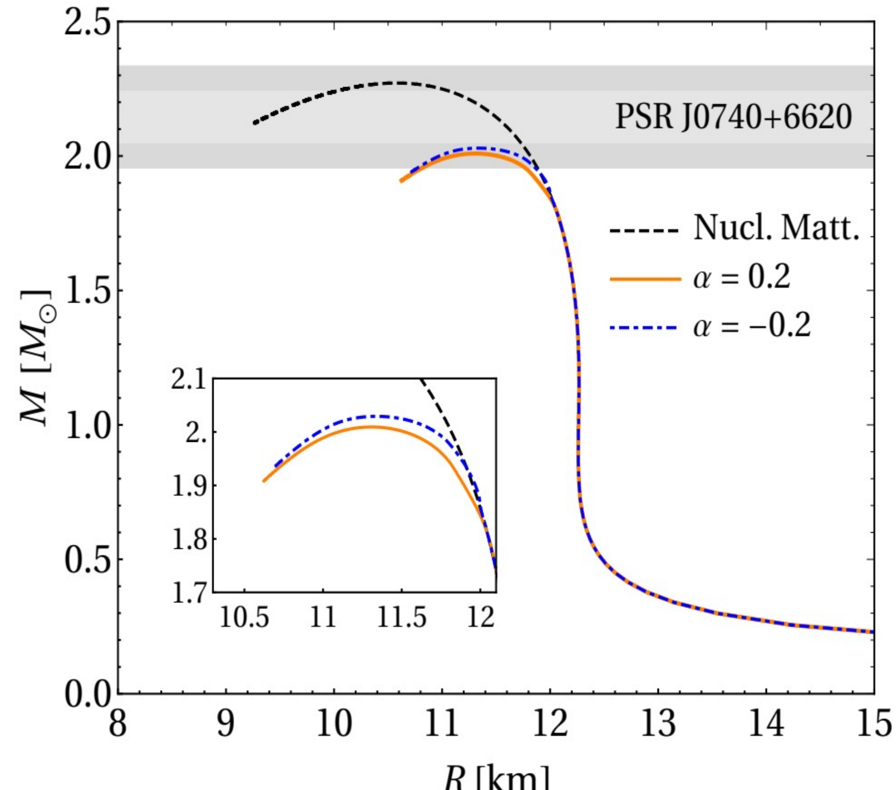
Model for the dependence of Δ with the baryon density:

$$\Delta = \frac{\Lambda^3}{k_{FB}^2} + \kappa \frac{\Lambda}{N_c^2}$$

McLerran and Reddy, PRL 122,122701 (2019)

Three flavor excluded volume model, shell appearing dynamically with the increase of baryon density.

DD, Jeong, Hernandez-Ortiz, PRC 102 065202 (2020)



Quarkyonic EFT

Field theory of quarkyonic matter: nucleonic and quark degrees of freedom in the same description: How to deal with??

Quarkyonic EFT

Field theory of quarkyonic matter: nucleonic and quark degrees of freedom in the same description: How to deal with??

- Nucleons are composed by quarks: since inside the Fermi sea the lower momentum states are occupied by quarks, the nucleons cannot propagate in the region of momentum $k_N < N_c k_Q$.

Quarkyonic EFT

Field theory of quarkyonic matter: nucleonic and quark degrees of freedom in the same description: How to deal with??

- Nucleons are composed by quarks: since inside the Fermi sea the lower momentum states are occupied by quarks, the nucleons cannot propagate in the region of momentum $k_N < N_c k_Q$.
- In addition to quarkyonic matter, such EFT should reduce to a theory of nucleons and mesons at low densities, and evolve to quarks at high density and temperature.

Quarkyonic EFT

Field theory of quarkyonic matter: nucleonic and quark degrees of freedom in the same description: How to deal with??

- Nucleons are composed by quarks: since inside the Fermi sea the lower momentum states are occupied by quarks, the nucleons cannot propagate in the region of momentum $k_N < N_c k_Q$.
- In addition to quarkyonic matter, such EFT should reduce to a theory of nucleons and mesons at low densities, and evolve to quarks at high density and temperature.

Our purpose: Inclusion of a negative metric ghost field, that fill precisely the same state as the quarks, and cancel away the degrees of freedom of unconstrained nucleons.

Quarkyonic EFT

Nucleons can be thought of as an ensemble of quarks: If there is a quark Fermi sea with a chemical potential μ_Q in the low momentum states, the quarks composing the nucleon cannot occupy the same states.

$$\rho^G = \frac{1}{1+e^{\beta(N_C E_Q - \mu_G)}} - \frac{1}{1+e^{\beta(N_C E_Q + \mu_G)}}$$

Density of the states in which nucleons cannot exist: Region of quark Fermi sea up to $\mu_G = N_c \mu_Q$.

Quarkyonic EFT

Nucleons can be thought of as an ensemble of quarks: If there is a quark Fermi sea with a chemical potential μ_Q in the low momentum states, the quarks composing the nucleon cannot occupy the same states.

$$\rho^G = \frac{1}{1 + e^{\beta(N_C E_Q - \mu_G)}} - \frac{1}{1 + e^{\beta(N_C E_Q + \mu_G)}}$$

Density of the states in which nucleons cannot exist: Region of quark Fermi sea up to $\mu_G = N_c \mu_Q$.

If this state is a nucleon: Energy of N_c quarks thought of as nucleon energy:

$$\rho^G = \frac{1}{1 + e^{\beta(E_N - \mu_G)}} - \frac{1}{1 + e^{\beta(E_N + \mu_G)}}$$

Quarkyonic EFT

Nucleons can be thought of as an ensemble of quarks: If there is a quark Fermi sea with a chemical potential μ_Q in the low momentum states, the quarks composing the nucleon cannot occupy the same states.

$$\rho^G = \frac{1}{1+e^{\beta(N_C E_Q - \mu_G)}} - \frac{1}{1+e^{\beta(N_C E_Q + \mu_G)}}$$

Density of the states in which nucleons cannot exist: Region of quark Fermi sea up to $\mu_G = N_c \mu_Q$.

If this state is a nucleon: Energy of N_c quarks thought of as nucleon energy:

$$\rho^G = \frac{1}{1+e^{\beta(E_N - \mu_G)}} - \frac{1}{1+e^{\beta(E_N + \mu_G)}}$$

$$\rho^Q = \frac{1}{1+e^{\beta(E_Q - \mu_Q)}} - \frac{1}{1+e^{\beta(E_Q + \mu_Q)}}$$

Density of quarks in a noninteracting gas of quarks and nucleons;

$$\rho^N_{\text{const.}} = \rho^n = \frac{1}{1+e^{\beta(E_N - \mu_N)}} - \frac{1}{1+e^{\beta(E_N - \mu_G)}}$$

Density of nucleons constrained to not propagate in the quark Fermi sea:

Quarkyonic EFT

Nucleons can be thought of as an ensemble of quarks: If there is a quark Fermi sea with a chemical potential μ_Q in the low momentum states, the quarks composing the nucleon cannot occupy the same states.

$$\rho^G = \frac{1}{1+e^{\beta(N_C E_Q - \mu_G)}} - \frac{1}{1+e^{\beta(N_C E_Q + \mu_G)}}$$

Density of the states in which nucleons cannot exist: Region of quark Fermi sea up to $\mu_G = N_c \mu_Q$.

If this state is a nucleon: Energy of N_c quarks thought of as nucleon energy:

$$\rho^G = \frac{1}{1+e^{\beta(E_N - \mu_G)}} - \frac{1}{1+e^{\beta(E_N + \mu_G)}}$$

$$\rho^Q = \frac{1}{1+e^{\beta(E_Q - \mu_Q)}} - \frac{1}{1+e^{\beta(E_Q + \mu_Q)}}$$

Density of quarks in a noninteracting gas of quarks and nucleons;

$$\rho^N_{\text{const.}} = \rho^n = \frac{1}{1+e^{\beta(E_N - \mu_N)}} - \frac{1}{1+e^{\beta(E_N - \mu_G)}}$$

Density of nucleons constrained to not propagate in the quark Fermi sea:

Nucleons are placed in a shell of momenta above the momenta of quark Fermi sea, where they are not Pauli blocked!

Quarkyonic EFT

$$\mathcal{L}_{\text{eff}} = \overline{N}\left(i\partial - M_N + \gamma^0 \mu_N\right) N + \overline{G}\left(i\partial - M_N + \gamma^0 \mu_G\right) G$$
$$+ \overline{Q}\left(i\partial - M_Q + \gamma^0 \mu_Q\right) Q$$

Ghosts: Nucleon fields with M_N and $\mu_G = N_c \mu_Q$, with the same Lorentz structure of nucleonic field, satisfying anti-periodic boundary conditions in immaginary time. It is, however, a commuting field!

But what does it mean? Let $S(\mu,M)$ be the propagator for a given field.

$$Z_{\text{Grassman}} = \det{}^{-1} S(\mu, M)$$
$$Z_{\text{c-number}} = \det S(\mu, M)$$

Quarkyonic EFT

$$\mathcal{L}_{\text{eff}} = \overline{N}\left(i\partial - M_N + \gamma^0 \mu_N\right) N + \overline{G}\left(i\partial - M_N + \gamma^0 \mu_G\right) G$$
$$+ \overline{Q}\left(i\partial - M_Q + \gamma^0 \mu_Q\right) Q$$

Ghosts: Nucleon fields with M_N and $\mu_G = N_c \mu_Q$, with the same Lorentz structure of nucleonic field, satisfying anti-periodic boundary conditions in immaginary time. It is, however, a commuting field!

But what does it mean? Let $S(\mu,M)$ be the propagator for a given field.

$$Z_{\text{Grassman}} = \det{}^{-1} S(\mu, M)$$
$$Z_{\text{c-number}} = \det S(\mu, M)$$

$$\Omega = -gT \int \frac{d^3k}{(2\pi)^3} \left\{ \ln\left[1 + e^{-\beta(E_N - \mu_N)}\right] - \ln\left[1 + e^{-\beta(E_N - \mu_G)}\right] + N_c \ln\left[1 + e^{-\beta(E_Q - \mu_Q)}\right] \right\}$$

$$\mathcal{E}_{\text{quarkyonic}} = \underbrace{\mathcal{E}_N - \mathcal{E}_G}_{\text{Energy density of nucleons}} + \mathcal{E}_Q$$

DD, K.S. Jeong, S. Hernandez-Ortiz, L.McLerran, PRD 104, L091901(2021)

Quarkyonic EFT

- Independent parameters: nucleon density $n_{_N}$ and quark baryon density $n_Q^B = n_Q/N_c$
- At fixed total baryon density...

$$n_B = n_N - n_G(n_Q^B) + n_Q^B$$

$$dn_B = dn_N + \left(1 - \frac{dn_G(n_Q^B)}{dn_Q^B}\right) dn_Q^B = 0$$

... the configuration that minimized the energy density is....

$$d\varepsilon = \frac{\partial\varepsilon}{\partial n_N} dn_N - \frac{\partial\varepsilon}{\partial n_G(n_Q^B)} dn_G(n_Q^B) + \frac{\partial\varepsilon}{\partial n_Q^B} dn_Q^B = 0$$

... and this leads to the relation

$$\mu_B = \frac{d\varepsilon}{dn_B} = \mu_N$$

Quarkyonic EFT

- Independent parameters: nucleon density n_N and quark baryon density $n_Q^B = n_Q/N_c$
- At fixed total baryon density...

$$n_B = n_N - n_G(n_Q^B) + n_Q^B$$

$$dn_B = dn_N + \left(1 - \frac{dn_G(n_Q^B)}{dn_Q^B}\right) dn_Q^B = 0$$

$\mu_Q^B \simeq N_c \mu_Q$

and

$\frac{dn_G(n_Q^B)}{dn_Q^B} \simeq N_c^3$

... the configuration that minimized the energy density is....

$$d\varepsilon = \frac{\partial \varepsilon}{\partial n_N} dn_N - \frac{\partial \varepsilon}{\partial n_G(n_Q^B)} dn_G(n_Q^B) + \frac{\partial \varepsilon}{\partial n_Q^B} dn_Q^B = 0$$

... and this leads to the relation

$$\mu_B = \frac{d\varepsilon}{dn_B} = \mu_N$$

Quarkyonic EFT: MF theory

$$n_v = \int_0^{k_N} dk \frac{k^2}{2\pi^2}$$

$$\begin{aligned} S &= \int d^4x \Big\{ \overline{\psi}\left(\frac{1}{i}\partial - g_v V - \gamma^0 \mu_N^* + M_N - g_\sigma \sigma\right)\psi \\ &+ \overline{G}\left(\frac{1}{i}\partial - g_v V - \gamma^0 \mu_G^* + M_N - g_\sigma \sigma\right) G \\ &+ \frac{M_\sigma^2}{2}\sigma^2 + \frac{M_v^2}{2} V^2 \Big\} \end{aligned}$$

$$\Downarrow$$

$$\begin{aligned} \varepsilon &= \varepsilon_{\text{kin}}^N(\mu_N) - \left(1 - \frac{1}{N_c^3}\right)\varepsilon_{\text{kin}}^N(N_c\mu_Q) \\ &+ \frac{g_v^2}{2M_v^2}\Big[n_v(\mu_N) - n_v(N_c\mu_Q)\Big]^2 \\ &- \frac{g_\sigma^2}{2M_\sigma^2}\left[n_s(\mu_N) - \left(1 - \frac{1}{N_c^3}\right) n_s(N_c\mu_Q)\right]^2 \end{aligned}$$

DD, K.S. Jeong, S. Hernandez-Ortiz, L.McLerran, PRC 107, 065201.

Quarkyonic EFT: MF theory

$$n_v = \int_0^{k_N} dk \frac{k^2}{2\pi^2}$$

$$\begin{aligned} S \;=\; & \int d^4x \Big\{ \overline{\psi}\left(\frac{1}{i}\partial - g_v V - \gamma^0 \mu_N^* + M_N - g_\sigma \sigma\right)\psi \\ & + \overline{G}\left(\frac{1}{i}\partial - g_v V - \gamma^0 \mu_G^* + M_N - g_\sigma \sigma\right) G \\ & + \frac{M_\sigma^2}{2}\sigma^2 + \frac{M_v^2}{2} V^2 \Big\} \end{aligned}$$

$$\begin{aligned} \varepsilon \;=\; & \varepsilon_{\text{kin}}^N(\mu_N) - \left(1 - \frac{1}{N_c^3}\right)\varepsilon_{\text{kin}}^N(N_c\mu_Q) \\ & + \frac{g_v^2}{2M_v^2}\Big[n_v(\mu_N) - n_v(N_c\mu_Q)\Big]^2 \\ & \cancel{- \frac{g_\sigma^2}{2M_\sigma^2}\left[n_s(\mu_N) - \left(1 - \frac{1}{N_c^3}\right) n_s(N_c\mu_Q)\right]^2} \end{aligned}$$

DD, K.S. Jeong, S. Hernandez-Ortiz,L.McLerran, PRC 107, 065201.

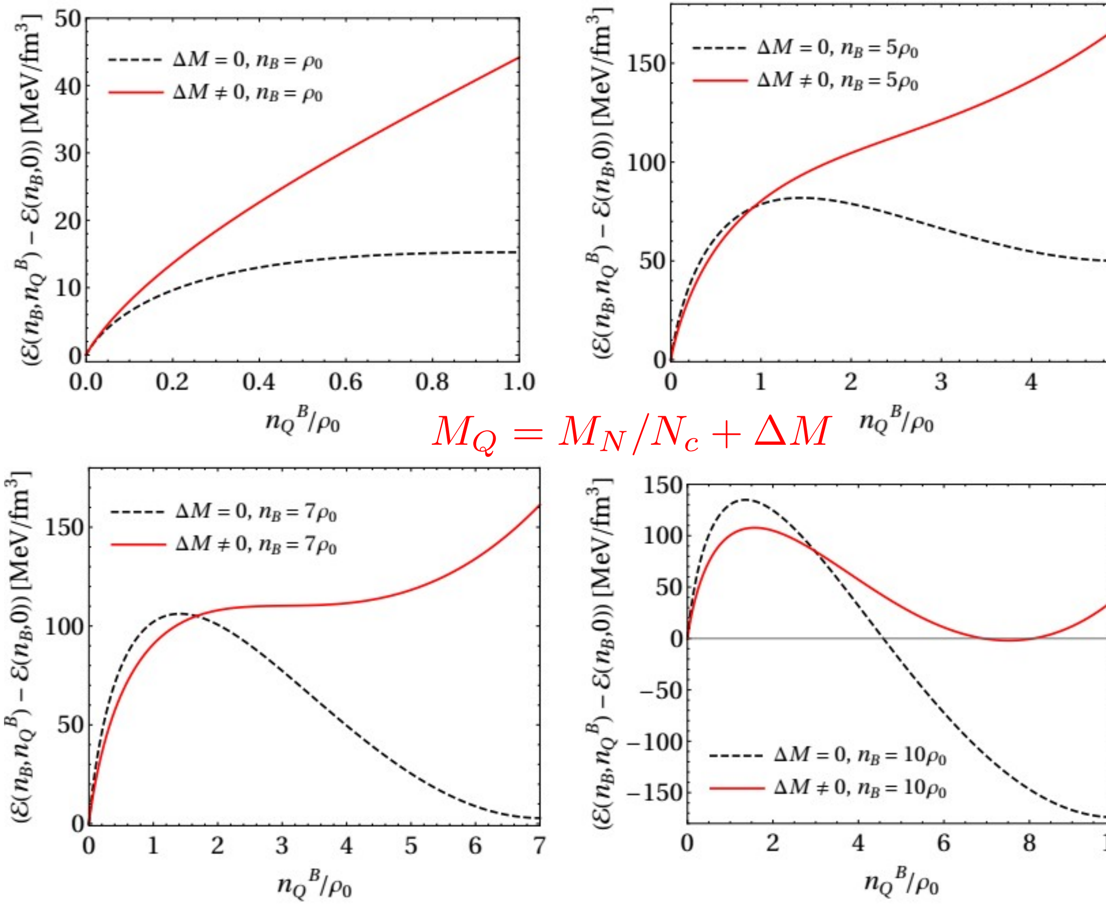
Quarkyonic EFT: MF theory

$$n_v = \int_0^{k_N} dk \frac{k^2}{2\pi^2}$$

$$\begin{aligned} S &= \int d^4x \Big\{ \overline{\psi} \left(\frac{1}{i}\partial - g_v V - \gamma^0 \mu_N^* + M_N - g_\sigma \sigma \right) \psi \\ &\quad + \overline{G} \left(\frac{1}{i}\partial - g_v V - \gamma^0 \mu_G^* + M_N - g_\sigma \sigma \right) G \\ &\quad + \frac{M_\sigma^2}{2}\sigma^2 + \frac{M_v^2}{2} V^2 \Big\} \end{aligned}$$

$$\mu_G = N_c(\mu_Q - \Delta M) + g_v V_0$$

$$\begin{aligned} \varepsilon &= \varepsilon_{\rm kin}^N(\mu_N) - \left(1 - \frac{1}{N_c^3}\right) \varepsilon_{\rm kin}^N(N_c \mu_Q) \\ &\quad + \frac{g_v^2}{2M_v^2} \Big[n_v(\mu_N) - n_v(N_c\mu_Q) \Big]^2 \\ &\quad \cancel{- \frac{g_\sigma^2}{2M_\sigma^2} \left[n_s(\mu_N) - \left(1 - \frac{1}{N_c^3}\right) n_s(N_c\mu_Q) \right]^2} \end{aligned}$$

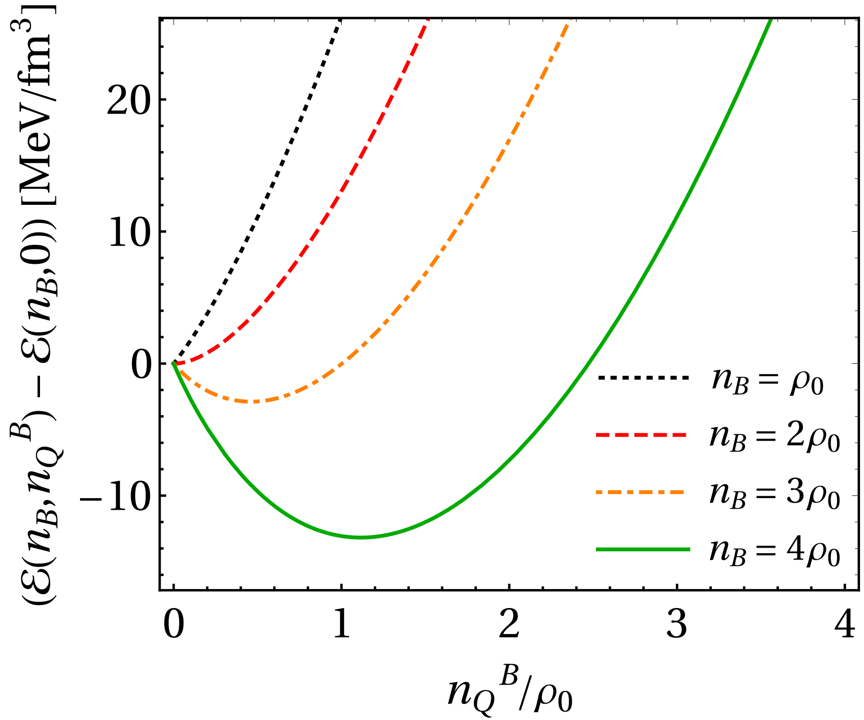


$$M_Q = M_N/N_c + \Delta M$$

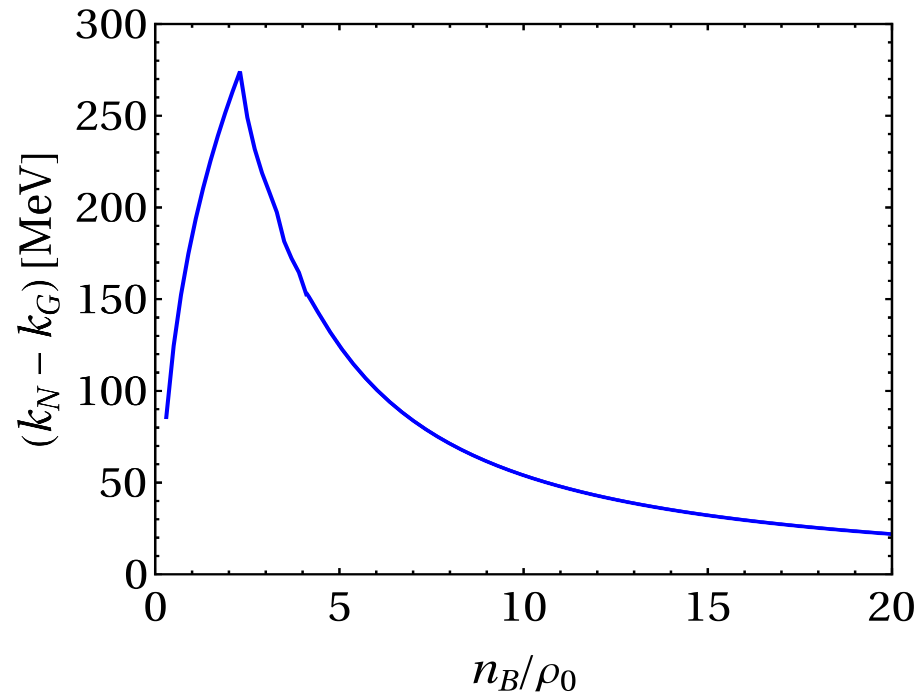
DD, K.S. Jeong, S. Hernandez-Ortiz, L.McLerran, PRC 107, 065201.

Quarkyonic EFT: MF theory

Interactions are important!! Phenomenological relation: $\mu_Q = M_Q + \frac{n_B^Q}{\Lambda^2}$



Continuous transition at $T = 0$.



Shell thickness

To work at finite temperature: Extremize free energy density with the same constraints.

$$f = \varepsilon - Ts = p + \mu_B n_B \qquad \mu_G = N_c(\mu_Q - \Delta M) + g_v V_0$$

DD, K.S. Jeong, S. Hernandez-Ortiz, L.McLerran, PRC 107, 065201 (2023).

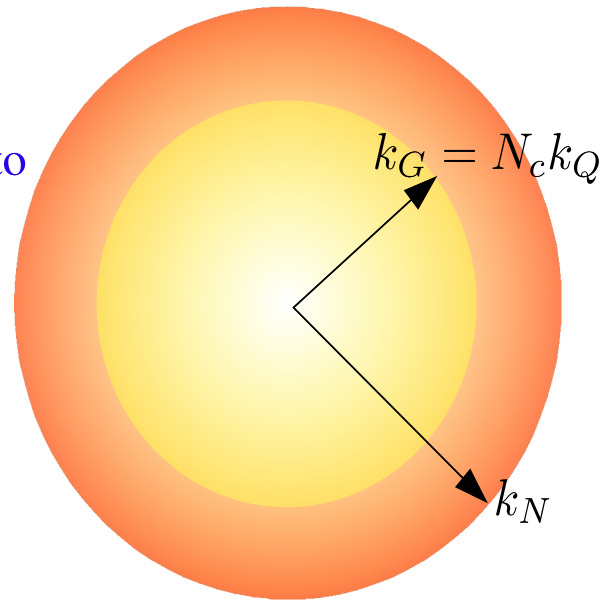
EFT with ghosts: Possibilities/Questions

- Inclusion of different nucleon and quark potentials directly in the Lagrangian density;
- Isospin asymmetry?
- Dynamic generation of the shell with different nucleonic interactions: Is it possible to construct a model that respect nuclear properties at saturation density and, at the same time respect the contraints from observational data?

$$\bar{N}\Gamma_i N + \bar{G}\Gamma_i G$$

- Quarkyonic sigma model: How chiral symmetry restoration is related to quarkyonic matter?

$$M_N = g_N(\phi + i\pi\gamma^5)$$
$$M_Q = g_Q(\phi + i\pi\gamma^5)$$



- Future FAIR/NICA simulations: Possible clues of quarkyonic matter: How it would appear in the QCD phase diagram?
- ....

Thanks to Sanjay Reddy, Toru Kojo, Rob Pisarski and Jinfeng Liao for all the useful discussions during the development of this work.

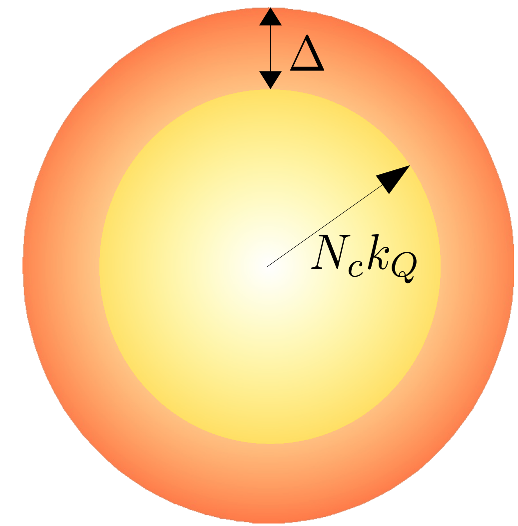
Thanks for your attention!

Dynamically generated shell of nucleons: Excluded Volume Model

$$n_{ex} = \frac{n_N}{1 - n_N/n_0} = \frac{N_f}{\pi^2}\int_{k_F}^{k_F+\Delta} dk\, k^2$$

$$k_Q = k_F/N_c \qquad m = M/N_c$$

$$\varepsilon = \frac{N_f}{\pi^2}\left(1 - \frac{n_N}{n_0}\right)\int_{k_F}^{k_F+\Delta} dk\, k^2\sqrt{k^2+M^2} + \varepsilon_Q$$



Free gas of quarks contribution

$$\begin{cases} n_Q = \dfrac{N_f}{\pi^2}\displaystyle\int_0^{k_Q} dk\, k^2 = \dfrac{N_f}{3\pi^2}k_Q^3 \\ \varepsilon_Q = \dfrac{N_c N_f}{\pi^2}\displaystyle\int_0^{k_Q} dk\, k^2\sqrt{k^2+m^2} \end{cases}$$

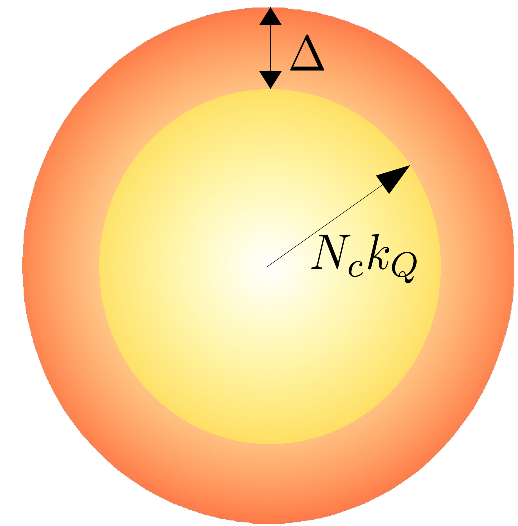
K.S. Jeong, L. McLerran, S. Sen, Phys. Rev. C 101 035201 (2020)

Dynamically generated shell of nucleons: Excluded Volume Model

$$n_{ex} = \frac{n_N}{1 - n_N/n_0} = \frac{N_f}{\pi^2} \int_{k_F}^{k_F+\Delta} dk\ k^2$$

$$\varepsilon = \frac{N_f}{\pi^2} \left(1 - \frac{n_N}{n_0}\right) \int_{k_F}^{k_F+\Delta} dk\ k^2 \sqrt{k^2 + M^2} + \varepsilon_Q$$

$$k_Q = k_F/N_c \qquad m = M/N_c$$



Free gas of quarks contribution

$$\begin{cases} n_Q = \dfrac{N_f}{\pi^2} \displaystyle\int_0^{k_Q} dk\ k^2 = \dfrac{N_f}{3\pi^2} k_Q^3 \\ \varepsilon_Q = \dfrac{N_c N_f}{\pi^2} \displaystyle\int_0^{k_Q} dk\ k^2 \sqrt{k^2 + m^2} \end{cases}$$

At small k_Q quark density increase very fast to generate acceptable sound velocity ➞ Modification in the low density Fermi distribution in a way that does not affect its behavior for large Fermi momenta:

$$1 \to \frac{\sqrt{k_Q^2 + \Lambda^2}}{k_Q}$$

$$n_Q = \frac{N_f}{3\pi^2} \left[\left(k_Q^2 + \Lambda^2\right)^{3/2} - \Lambda^3 \right]$$

$$\varepsilon_Q = \frac{N_c N_f}{\pi^2} \int_0^{k_Q} dk\ k \sqrt{k^2 + \Lambda^2} \sqrt{k^2 + m^2}$$

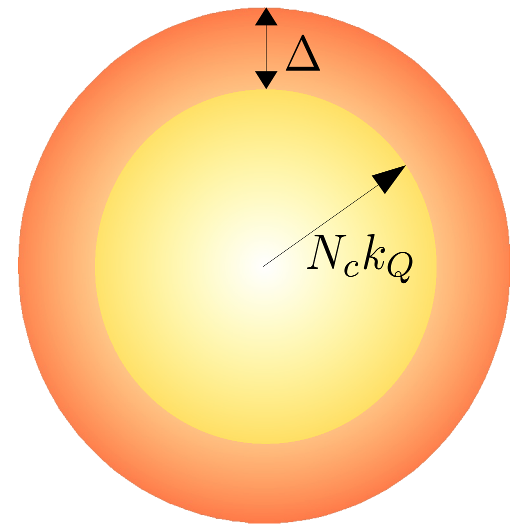
K.S. Jeong, L. McLerran, S. Sen, Phys. Rev. C 101 035201 (2020)

Dynamically generated shell of nucleons: Excluded Volume Model

$$n_{ex} = \frac{n_N}{1 - n_N/n_0} = \frac{N_f}{\pi^2} \int_{k_F}^{k_F+\Delta} dk\ k^2$$

$$\varepsilon = \frac{N_f}{\pi^2}\left(1 - \frac{n_N}{n_0}\right) \int_{k_F}^{k_F+\Delta} dk\ k^2 \sqrt{k^2 + M^2} + \varepsilon_Q$$

$k_Q = k_F/N_c \qquad m = M/N_c$



Free gas of quarks contribution

$$\begin{cases} n_Q = \dfrac{N_f}{\pi^2} \displaystyle\int_0^{k_Q} dk\ k^2 = \dfrac{N_f}{3\pi^2} k_Q^3 \\ \varepsilon_Q = \dfrac{N_c N_f}{\pi^2} \displaystyle\int_0^{k_Q} dk\ k^2 \sqrt{k^2 + m^2} \end{cases}$$

At small k_Q quark density increase very fast to generate acceptable sound velocity ➞ Modification in the low density Fermi distribution in a way that does not affect its behavior for large Fermi momenta:

$$1 \to \frac{\sqrt{k_Q^2 + \Lambda^2}}{k_Q}$$

$$n_Q = \frac{N_f}{3\pi^2}\left[\left(k_Q^2 + \Lambda^2\right)^{3/2} - \Lambda^3\right]$$

At the minimum of the energy density: $\mu_N = N_c \mu_Q$

$$\varepsilon_Q = \frac{N_c N_f}{\pi^2} \int_0^{k_Q} dk\ k \sqrt{k^2 + \Lambda^2} \sqrt{k^2 + m^2}$$

K.S. Jeong, L. McLerran, S. Sen, Phys. Rev. C 101 035201 (2020)

Quarkyonic EFT: Excluded volume model

- By assuming $\mu_G = N_c \mu_Q$ one may derive the excluded volume model and verify the shell-like distribution on nucleons in the ideal gas limit.

$$\Omega = -p = \varepsilon_n - \mu_N n_n - (\mu_n - \mu_G) n_G + \varepsilon_{\tilde{Q}} - \mu_{\tilde{Q}} n_{\tilde{Q}}$$

$$\begin{aligned} \varepsilon_n &= \varepsilon_N - \varepsilon_G \\ &= 4 \int_{k_G}^{k_N} dk \frac{k^2}{2\pi^2} \sqrt{k^2 + M_N^2} \end{aligned}$$

$$n_n = n_N - n_G = 4 \int_{k_G}^{k_N} dk \frac{k^2}{2\pi^2}$$

$$k_{N,G} = \sqrt{\mu_{N,G}^2 - M_N^2}$$

Quarkyonic EFT: Excluded volume model

- By assuming $\mu_G = N_c \mu_Q$ one may derive the excluded volume model and verify the shell-like distribution on nucleons in the ideal gas limit.

$$\Omega = -p = \varepsilon_n^{\rm ex} - \mu_N^{\rm ex} n_n^{\rm ex} - (\mu_n^{\rm ex} - \mu_G^{\rm ex}) n_G^{\rm ex} + \varepsilon_{\tilde{Q}} - \mu_{\tilde{Q}} n_{\tilde{Q}}$$

$$\begin{aligned}\varepsilon_n^{\rm ex} &= \varepsilon_N^{\rm ex} - \varepsilon_G^{\rm ex} \\ &= 4 \int_{k_G^{\rm ex}}^{k_N^{\rm ex}} dk \frac{k^2}{2\pi^2} \sqrt{k^2 + M_N^2}\end{aligned}$$

$$n_n^{\rm ex} = \frac{n_N - n_G}{1 - \frac{n_N - n_G}{n_0}} = 4 \int_{k_G^{\rm ex}}^{k_N^{\rm ex}} dk \frac{k^2}{2\pi^2}$$

$$k_{N,G}^{\rm ex} = \sqrt{(\mu_{N,G}^{\rm ex})^2 - M_N^2}$$

Quarkyonic EFT: Excluded volume model

- The other quantities in the reduced volume are related to the ones of the total system from:

$$\varepsilon^{\mathrm{ex}} = \frac{\varepsilon}{1 - \frac{n}{n_0}}$$

$$\mu^{\mathrm{ex}} = \frac{\partial \varepsilon^{\mathrm{ex}}}{\partial n^{\mathrm{ex}}}$$

- At fixed total baryon density (and zero temperature) the system configuration is obtained by extremizing the energy density, and the chemical potentials are related as $\mu_B = \mu_{N.}$.