Federal University of Santa Maria
Physics Departament
Excited QCD 2024 Workshop

Influence of chiral chemical potential on the QCD phase diagram.

Francisco X. Azeredo

Benasque Science Center, Jan 14 - Jan 20, 2024

Outline

1. Motivation

2. QCD Phase diagram

3. Results and perspectives

In collaboration with:

- D.C.Duarte - UFSM - Brazil
- R.L.S. Farias - UFSM - Brazil
- G. Krein - IFT - Unesp - Brazil
- R.O. Ramos - UERJ - Brazil

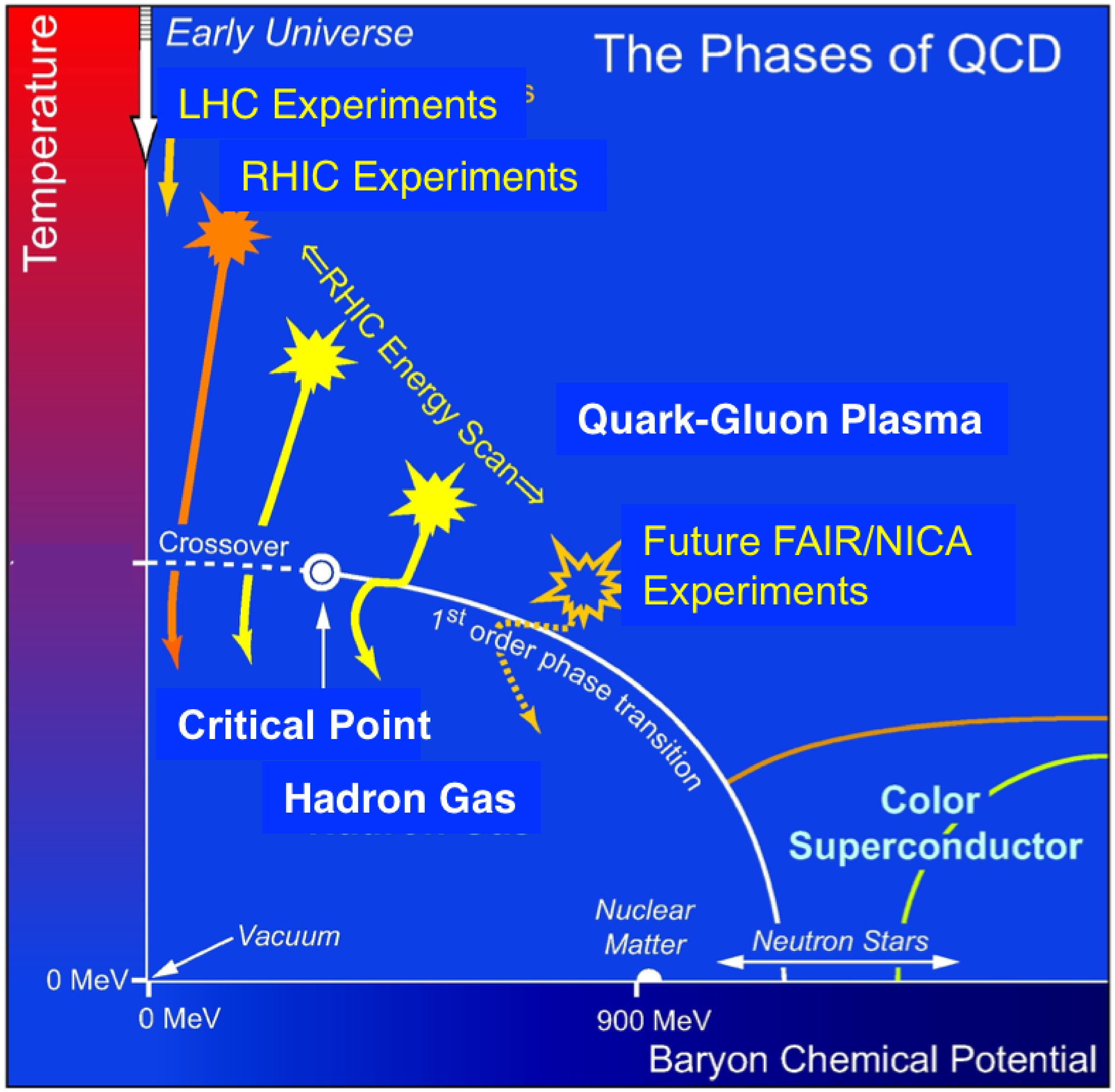


Figure: A schematic QCD phase diagram in the temperature (T) and baryon chemical potential (μ_B) plane. Font :https://cds.cern.ch/record/2729160/plots

- Early universe
- CP
- Heavy Ion collisions
- Compact objects
- Lattice QCD has a signal problem with finite μ

What is the effect of chiral imbalance on this phase diagram?

Chiral imbalance

Chirality: an intrinsic quantum number related to parity transformation - mirror image. For massless particles chirality can be identified with helicity

How does it happen? Axial anomaly: imbalance of chirality

Topological invariant :

$$Q_w = \frac{g^2}{32\pi^2} \int d^4x F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a. \tag{1}$$

In this context symmetries have a central role in theoretical physics, by Noether theorem

$$\partial_\mu J^\mu = 0. \tag{2}$$

When we quantize theory, something happens!
Non-conservation of axial current:

$$\partial_\mu J^5_\mu = 2\sum_f m_f \langle \bar{\psi}_f i\gamma_5 \psi_f \rangle_A - \frac{N_f g^2}{16\pi^2} F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a. \tag{3}$$

The anomaly will affect the ward identities of a quark interacting with a gauge field, and it is possible to relate the winding number with eigenvalues of the equation of motion. Summing over the eigenvalues of the chiral operator:

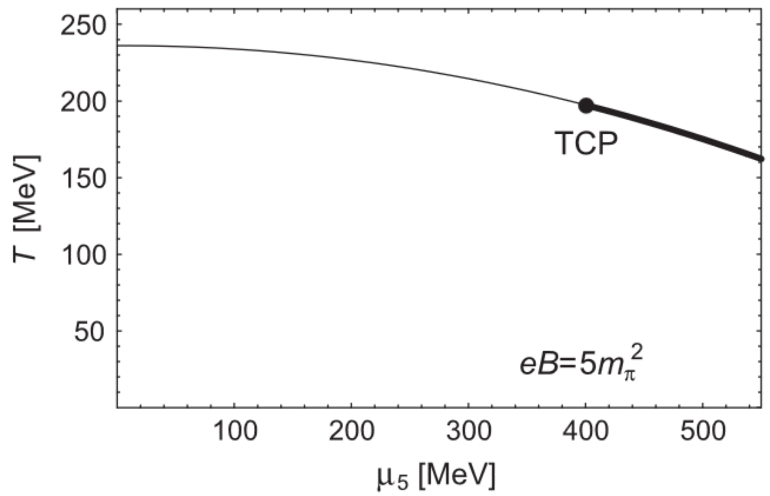
$$(N_R - N_L) = 2N_f Q_w$$

This is a specific form of the Atiyah-Singer theorem!

- The phase diagram of QCD with the effects of chiral imbalance can be studied in the grand canonical ensemble by introducing a chiral chemical potential

$$\mu_5 \bar{\psi}\gamma_0\gamma_5\psi \longrightarrow \text{In the QCD Lagrangian density}$$

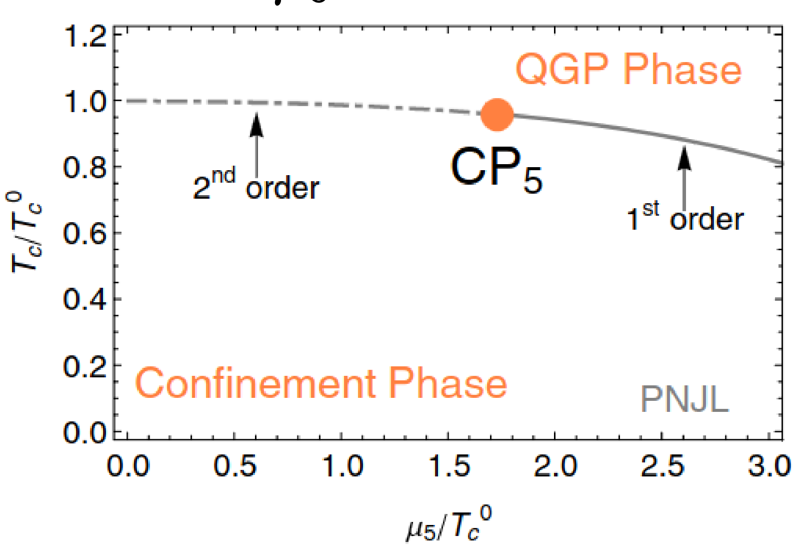
- Chiral magnetic effect in the Polyakov–Nambu–Jona-Lasinio model by Kenji Fukushima, Marco Ruggieri, and Raoul Gatto Phys. Rev. D 81, 114031 – Published 21 June 2010



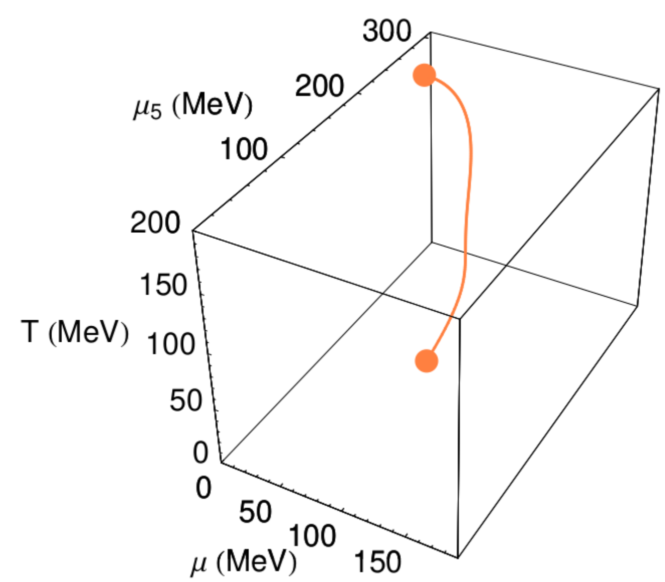
- Critical temperature decrease with μ_5
- Appear a Tricritical point TCP

- Critical end point of quantum chromodynamics detected by chirally imbalanced quark matter by Marco Ruggieri Phys. Rev. D 84, 014011 – Published 7 July 2011

- T_c decrease with μ_5

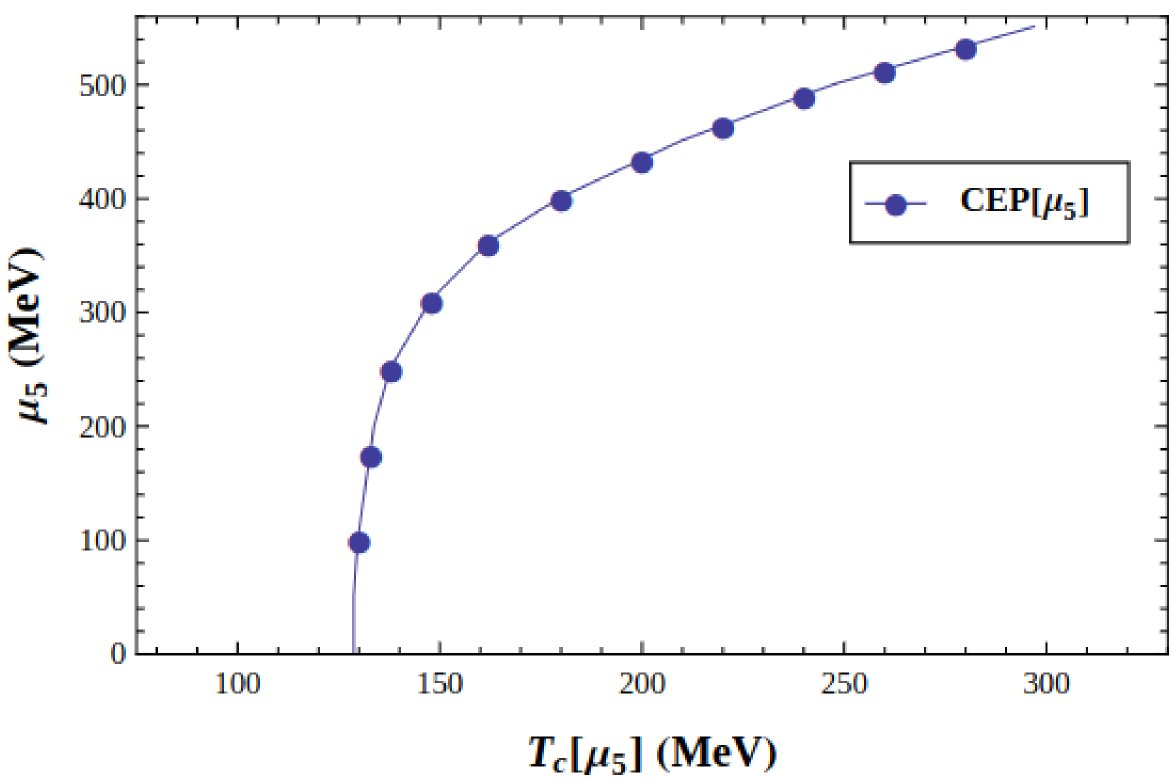


- Finding/mapping of CP with CP5



The chiral phase transition with a chiral chemical potential in the framework of Dyson-Schwinger equations

Shu-Sheng Xu[1,7], Zhu-Fang Cui[2,7], Bin Wang[3], Yuan-Mei Shi[4], You-Chang Yang[2,5], and Hong-Shi Zong[2,6,7,*]
[1] *Key Laboratory of Modern Acoustics, MOE, Institute of Acoustics, and Department of Physics, Nanjing University, Nanjing 210093, China*
[2] *Department of Physics, Nanjing University, Nanjing 210093, China*
[3] *Department of Physics, Huazhong University of Science and Technology, Wuhan 430074, China*
[4] *Department of Physics and Electron ic Engineering, Nanjing Xiaozhuang University, Nanjing 211171, China*
[5] *School of Physics and Mechanical-Electrical Engineering, Zunyi Normal College, Zunyi 563002, China*
[6] *Joint Center for Particle, Nuclear Physics and Cosmology, Nanjing 210093, China and*
[7] *State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, CAS, Beijing, 100190, China*



Lattice Results in 2015/2016

PHYSICAL REVIEW D **93,** 034509 (2016)

Study of the QCD phase diagram with a nonzero chiral chemical potential

V. V. Braguta[*]

Institute for High Energy Physics NRC "Kurchatov Institute", Protvino 142281, Russia; Institute of Theoretical and Experimental Physics, 117259 Moscow, Russia; Far Eastern Federal University, School of Natural Sciences, 690950 Vladivostok, Russia; and Moscow Institute of Physics and Technology, Dolgoprudny 141700, Russia

E.-M. Ilgenfritz[†]

Joint Institute for Nuclear Research, BLTP, 141980 Dubna, Russia

A. Yu. Kotov[‡]

Institute of Theoretical and Experimental Physics, 117259 Moscow, Russia; and National Research Nuclear University MEPhI (Moscow Engineering Physics Institute), Moscow 115409, Russia

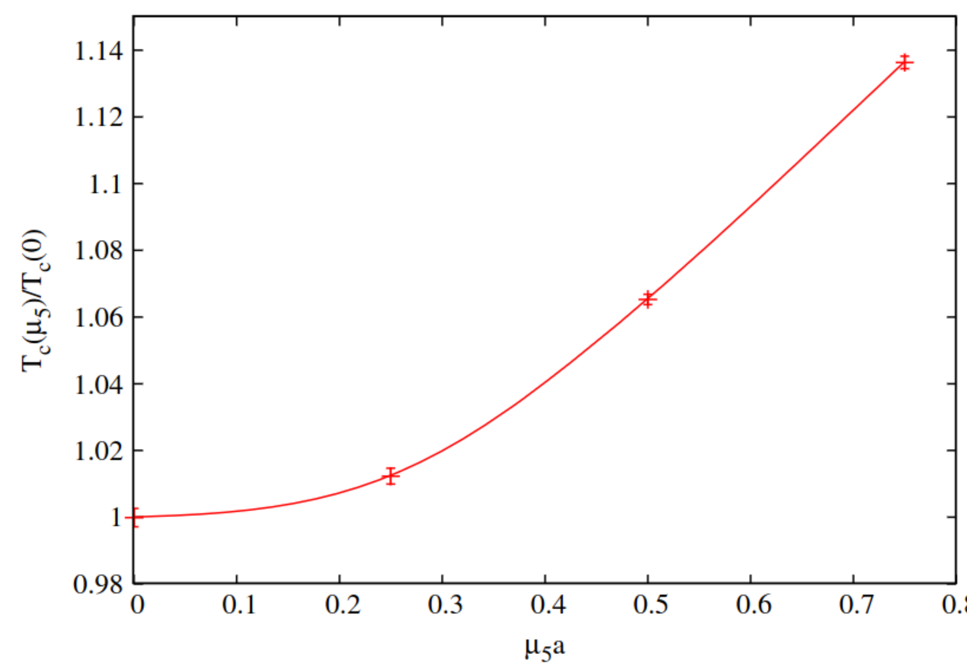
B. Petersson[§]

Humboldt-Universität zu Berlin, Institut für Physik, 12489 Berlin, Germany

S. A. Skinderev[‖]

Institute of Theoretical and Experimental Physics, 117259 Moscow, Russia

(Received 15 January 2016; published 25 February 2016)



- Contrary to the case of QCD in the presence of a baryon chemical potential, which has a sign problem.
- QCD in the presence of a chiral chemical potential is free from the sign problem and, therefore, amenable to Monte Carlo sampling in lattice simulations.
- There is hope that lattice simulations of QCD with μ_5 can be used as a benchmark platform for comparing different effective models used in the literature.
- The temperatures for chiral and deconfinement phase transition are the same.
- This transition is always a crossover.

The Lagrangian of the PNJL model for the SU(2) symmetry version is given by[1]

$$\mathscr{L}_{PNJL} = \bar{\psi}(i\gamma_\mu D^\mu - m_c)\psi + G[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2] - \mathscr{U}(\Phi[A], \Phi^\dagger[A], T). \quad (4)$$

In order to include the effects of baryonic density and chiral imbalance, we will write an effective Lagrangian given by,

$$\begin{aligned}\mathscr{L}_{eff} &= \bar{\psi}(i\gamma_\mu D^\mu - m_c + \mu_5\gamma^0\gamma^5 + \mu\gamma^0)\psi \\ &+ G[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2] - \mathscr{U}(\Phi[A], \Phi^\dagger[A], T),\end{aligned} \quad (5)$$

with μ being the potential for the quark number density and μ_5 is the chiral chemical potential for the chiral imbalance.

The quantity $\mathscr{U}(\Phi, \Phi^\dagger, T)$ is the effective potential expressed in terms of the traced Polyakov loop and its (charge) conjugate,

$$\Phi = (\mathrm{Tr}_c\, L)/N_c,\ \Phi^\dagger = (\mathrm{Tr}_c\, L^\dagger)/N_c, \quad (6)$$

$$L(x) = \mathscr{P}\left\{\exp\left[\int_0^\beta d\tau A_4(\vec{x}, t)\right]\right\}. \quad (7)$$

[1] Claudia Ratti, Michael A. Thaler, and Wolfram Weise. "Phases of QCD: Lattice thermodynamics and a field theoretical model". In: *Phys. Rev. D* 73 (1 Jan. 2006), p. 014019. DOI: 10.1103/PhysRevD.73.014019. URL: https://link.aps.org/doi/10.1103/PhysRevD.73.014019.

In order to reproduce the gluonic data of the Lattice QCD a potential $\mathscr{U}(\Phi, \Phi^\dagger, T)$ given in polynomial parametrization by[2],

$$\mathscr{U}(\Phi, \Phi^\dagger, T) \;=\; T^4\left[-\frac{b_2(T)}{2}\Phi\Phi^\dagger - \frac{b_3}{6}(\Phi^3 + \Phi^{\dagger 3}) + \frac{b_4}{4}(\Phi\Phi^\dagger)^2\right], \tag{8}$$

with

$$b_2(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3. \tag{9}$$

Table: Parameters for PNJL using polynomial parametrization.

a_0	a_1	a_2	a_3	b_3	b_4
6.75	-1.95	2.625	-7.44	0.75	7.5

The full thermodynamic potential in mean field aproximation is given by,

$$\begin{aligned}\Omega(M, \Phi, \Phi^\dagger, T, \mu, \mu_5) \;&=\; \mathscr{U}(\Phi, \Phi^\dagger, T) + \frac{(M - m_c)^2}{4G} + \Omega_V \\ &-\; N_f T \sum_{s=\pm 1}\int \frac{d^3p}{(2\pi)^3}[\log(F_+^s F_-^s)],\end{aligned} \tag{10}$$

$$F_-^s = 1 + 3\Phi e^{-\beta(\omega_s - \mu)} + 3\Phi^\dagger e^{-2\beta(\omega_s - \mu)} + e^{-3\beta(\omega_s - \mu)}, \tag{11}$$

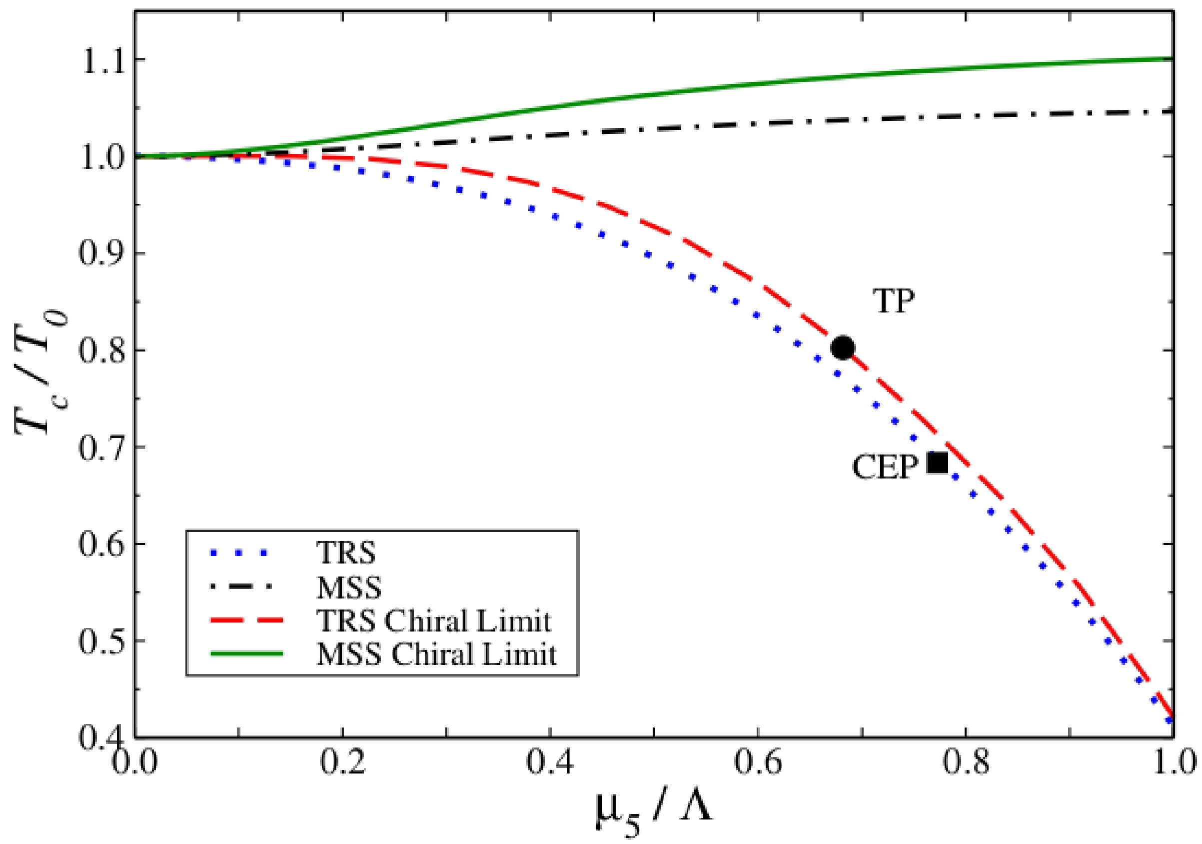
$$F_+^s = 1 + 3\Phi^\dagger e^{-\beta(\omega_s + \mu)} + 3\Phi e^{-2\beta(\omega_s + \mu)} + e^{-3\beta(\omega_s + \mu)}, \tag{12}$$

and

$$\omega_s = \sqrt{(|\vec{p}| + s\mu_5)^2 + M^2}. \tag{13}$$

[2] S. Rößner, C. Ratti, and W. Weise. "Polyakov loop, diquarks, and the two-flavor phase diagram". In: *Phys. Rev. D* 75 (3 Feb. 2007), p. 034007. DOI: 10.1103/PhysRevD.75.034007. URL: https://link.aps.org/doi/10.1103/PhysRevD.75.034007.

• Thermodynamics of quark matter with a chiral imbalance by Ricardo L.S. Farias, Dyana C. Duarte, Gastão Krein, and Rudnei O. Ramos Phys. Rev. D 94, 074011 – Published 6 October 2016



• Purely vacuum contributions are separated from medium-dependent regularized momentum integrals in such a way that one is left with ultraviolet divergent momentum integrals that depend on vacuum quantities only.

Vaccum regularization schemes

The Traditional Regularization Scheme (TRS), we regularize the moment integrals with a 3D cutoff, Λ.

$$\Omega_V^{TRS} = -N_f N_c \sum_{s=\pm 1} \int_0^\Lambda \frac{p^2}{(2\pi^2)} \omega_s dp. \tag{14}$$

The Medium Separation Scheme (MSS) is based on the idea that all divergences in the model are in a vacuum. as seen in the work of Farias et al. (2016)[3]. Thus, the contribution of the vacuum in this regularization scheme is

$$\begin{aligned} \Omega_V^{MSS} \; = \; & -2N_c N_f \left\{ \frac{M^2}{2} I_{quad} + \left[\mu_5^2 M^2 - \frac{M^4}{4} + \frac{M^2 M_0^2}{2} \right] \frac{I_{log}}{2} \right. \\ & \left. - \; \frac{3M^4}{64\pi^2} + \frac{M^2 M_0^2}{16\pi^2} + \frac{M^2}{8\pi^2} \left[\frac{M^2}{4} - \mu_5^2 \right] \ln\left(\frac{M^2}{M_0^2} \right) \right\}. \end{aligned} \tag{15}$$

Where M_0 is the mass of quarks in the regime of $T = \mu = \mu_5 = 0$. The remaining definitions are:

$$I_{quad}(\Lambda, M_0) = \int_0^\Lambda \frac{p^2}{2\pi^2} \frac{dp}{\sqrt{p^2 + M_0^2}}, \tag{16}$$

$$I_{log}(\Lambda, M_0) = \int_0^\Lambda \frac{p^2}{2\pi^2} \frac{dp}{(p^2 + M_0^2)^{3/2}}. \tag{17}$$

[3] Ricardo L. S. Farias et al. "Thermodynamics of quark matter with a chiral imbalance". In: *Phys. Rev. D* 94 (7 Oct. 2016), p. 074011. DOI: 10.1103/PhysRevD.94.074011. URL: https://link.aps.org/doi/10.1103/PhysRevD.94.074011.

Gap equations

Minimizing the thermodynamic potential with respect to to $M, \Phi,$ and $\Phi^\dagger$, the three gap equations required for this work will be obtained this work

$$\frac{\partial\Omega(M,\Phi,\Phi^\dagger,T,\mu,\mu_5)}{\partial M} = \frac{\partial\Omega(M,\Phi,\Phi^\dagger,T,\mu,\mu_5)}{\partial \Phi} = \frac{\partial\Omega(M,\Phi,\Phi^\dagger,T,\mu,\mu_5)}{\partial \Phi^\dagger} = 0. \tag{18}$$

Parametrization of the model

The parameters used[4] are such that they reproduce experimental results for the quark condensate, $|\langle\bar{\psi}_u\psi_u\rangle|^{1/3} = 251$ MeV, Pion decay constant, $f_\pi = 92.3$ MeV and Pion mass, $m_\pi = 139.3$ MeV.

Λ [MeV]	m_c [MeV]	G [GeV^{-2}]
651	5.5	5.04

[4] Claudia Ratti, Michael A. Thaler, and Wolfram Weise. "Phases of QCD: Lattice thermodynamics and a field theoretical model". In: *Phys. Rev. D* 73 (1 Jan. 2006), p. 014019. DOI: 10.1103/PhysRevD.73.014019. URL: https://link.aps.org/doi/10.1103/PhysRevD.73.014019.

Chiral and deconfinement transitions

In the PNJL model at $\mu = 0$, we have two different (pseudo)critical temperatures, T^c_{pc} and T^d_{pc}, for the chiral and deconfinement transitions respectively.

$$\left.\frac{\partial^2 M}{\partial T^2}\right|_{T=T^c_{pc}} = \left.\frac{\partial^2 \Phi}{\partial T^2}\right|_{T=T^d_{pc}} = 0 \tag{19}$$

Table: Values of pseudocritical temperatures for chiral (T^c_{pc}) and deconfinement (T^d_{pc}) phase transitions.

	T^c_{pc} (GeV)	T^d_{pc} (GeV)
TRS	0.229	0.225
MSS	0.233	0.227

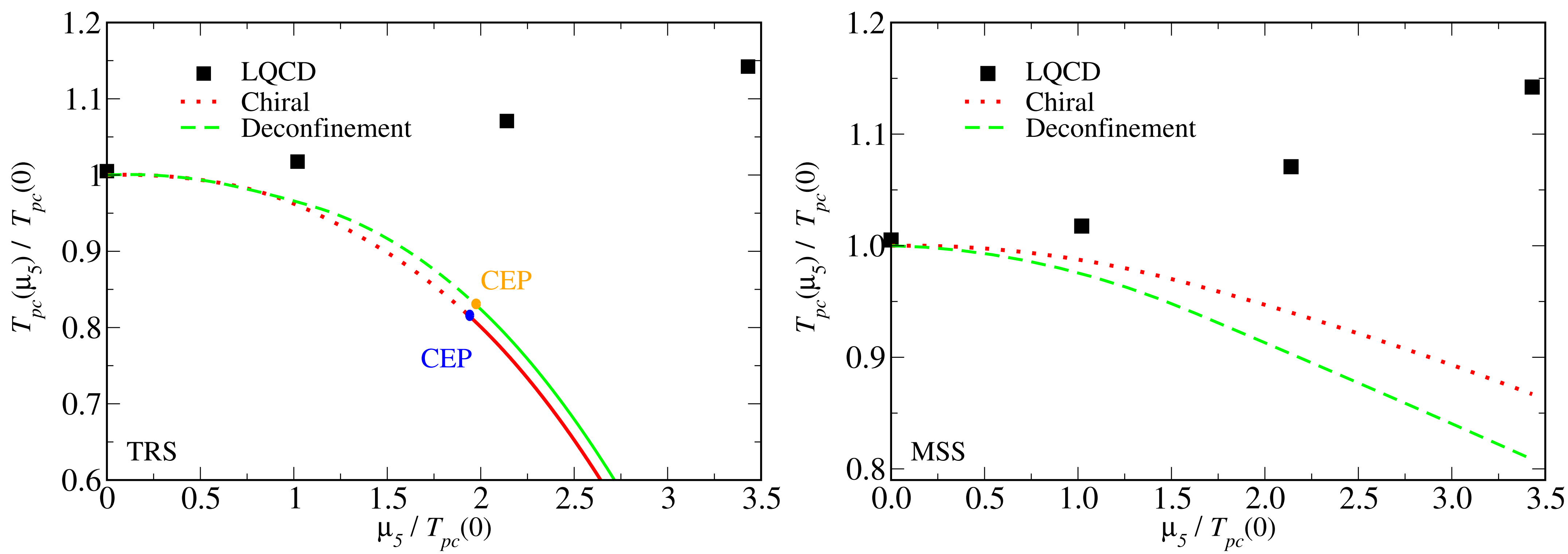


Figure: Phase diagrams in the plane $T \times \mu_5$ for chiral restoration and deconfinement, comparing lattice results from Ref.[5], TRS and MSS.

[5] V. V. Braguta et al. "Study of the QCD phase diagram with a nonzero chiral chemical potential". In: *Phys. Rev. D* 93 (3 Feb. 2016), p. 034509. DOI: 10.1103/PhysRevD.93.034509. URL: https://link.aps.org/doi/10.1103/PhysRevD.93.034509.

In the usual PNJL model, we obtain the pseudocritical temperatures as decreasing functions of μ_5. Our attempt to solve this problem and obtain behavior similar to that of lattice QCD simulations consists of including a dependence of the Polyakov loop potential on the chiral chemical potential using the following recipe

$$\mathscr{U}(\Phi, T) \to \mathscr{U}(\Phi, T, \mu_5) \; = \; T^4 \left[-\frac{\bar{b}_2(T, \mu_5)}{2} \Phi \Phi^\dagger - \frac{b_3}{6}(\Phi^3 + \Phi^{\dagger 3}) + \frac{b_4}{4}(\Phi \Phi^\dagger)^2 \right], \qquad (20)$$

in which

$$\bar{b}_2(T, \mu_5) = a_0 + a_1 \left(\frac{T_0}{T} \right) + a_2 \left(\frac{T_0}{T} \right)^2 + a_3 \left(\frac{T_0}{T} \right)^3 + k_1 \left(\frac{\mu_5}{T} \right) + k_2 \left(\frac{\mu_5}{T} \right)^2 + k_3 \left(\frac{\mu_5}{T} \right)^3 . \qquad (21)$$

Table: Parameters for Eq. (20).

k_1	k_2	k_3
-0.53	-0.54	-0.55

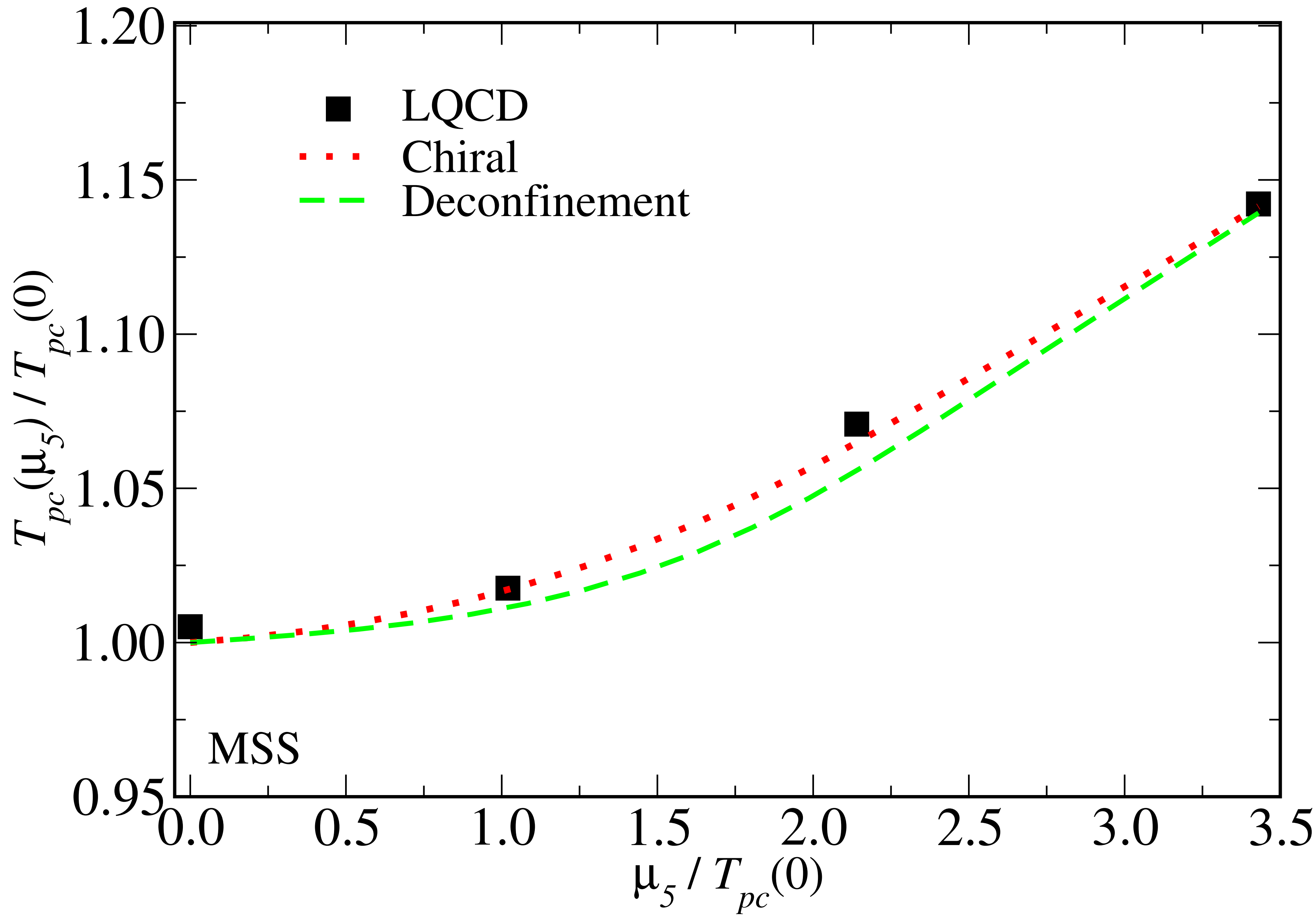


Figure: Phase diagram to MSS scheme with the ansatz.

Final Remarks

- Tpcs increase with μ_5.
- We purpose a way to conciliate results for the chiral and deconfinement transition obtained with lattice results.
- Lattice QCD simulations predict that for the phases transitions (chiral and deconfinement), the critical temperature increases with the chiral chemical potential, and this result is not observed in the simple version of PNJL.
- The next step would be to calculate PNJL$+\mu_5+\mu$.

Thank you for your attention!

Acknowledgements: UFSM, CNPq, Capes, Fapergs and Serrapilheira!

Backup!

Lattice results : signal problem

The Dirac operator $\not{D}$ obeys the relation $\gamma^5 \not{D} \gamma^5 = \not{D}^\dagger$, and therefore

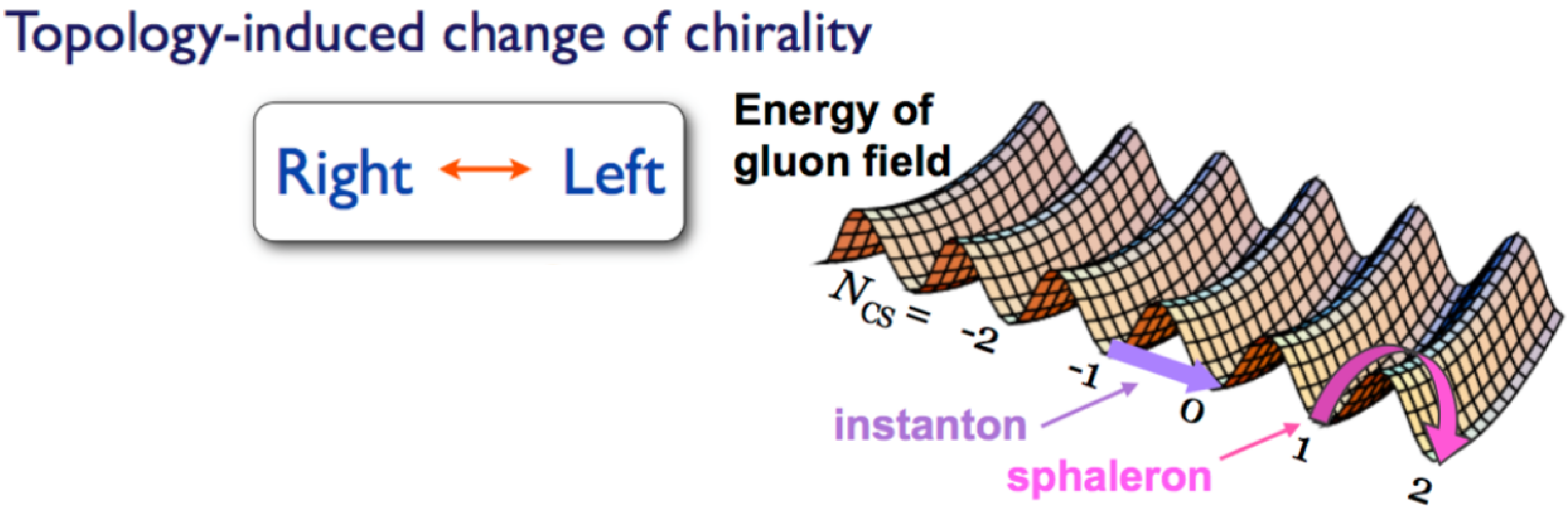
$$\gamma^5(\not{D} + m + \mu\gamma^0)\gamma^5 = (\not{D} + m - \mu^*\gamma^0)^\dagger. \tag{22}$$

Taking the determinant on both sides of the equation 22

$$\det(\not{D} + m + \mu\gamma^0) = [\det(\not{D} + m - \mu^*\gamma^0)]^*. \tag{23}$$

- Monte-Carlo ensemble has an expected value of the number of baryons is also zero or purely imaginary
- Standard lattice methods cannot be carried out in the finite density regime!

Vaccum of QCD



- The QCD vacuum has a non-trivial topological structure characterized by an integer-valued Chern-Simons number Ncs.
- At zero and low temperatures, the different Chern-Simons sectors are connected by quantum tunneling transitions, i.e. the instantons.
- At finite temperature, the gauge configurations that change the Chern-Simons number can also be activated thermally through sphaleron transitions.

- The phase diagram of QCD with the effects of chiral imbalance can be studied in the grand canonical ensemble by introducing a chiral chemical potential

$$\mu_5 \bar{\psi}\gamma_0\gamma_5\psi \longrightarrow \text{In the QCD Lagrangian density}$$

PHYSICAL REVIEW D **78,** 074033 (2008)

Chiral magnetic effect

Kenji Fukushima,[1,*] Dmitri E. Kharzeev,[2,+] and Harmen J. Warringa[2,‡]
[1] *Yukawa Institute, Kyoto University, Kyoto, Japan*
[2] *Department of Physics, Brookhaven National Laboratory, Upton New York 11973, USA*
(Received 2 September 2008; published 31 October 2008)

Topological charge changing transitions can induce chirality in the quark-gluon plasma by the axial anomaly. We study the equilibrium response of the quark-gluon plasma in such a situation to an external magnetic field. To mimic the effect of the topological charge changing transitions we will introduce a chiral chemical potential. We will show that an electromagnetic current is generated along the magnetic field. This is the chiral magnetic effect. We compute the magnitude of this current as a function of magnetic field, chirality, temperature, and baryon chemical potential.

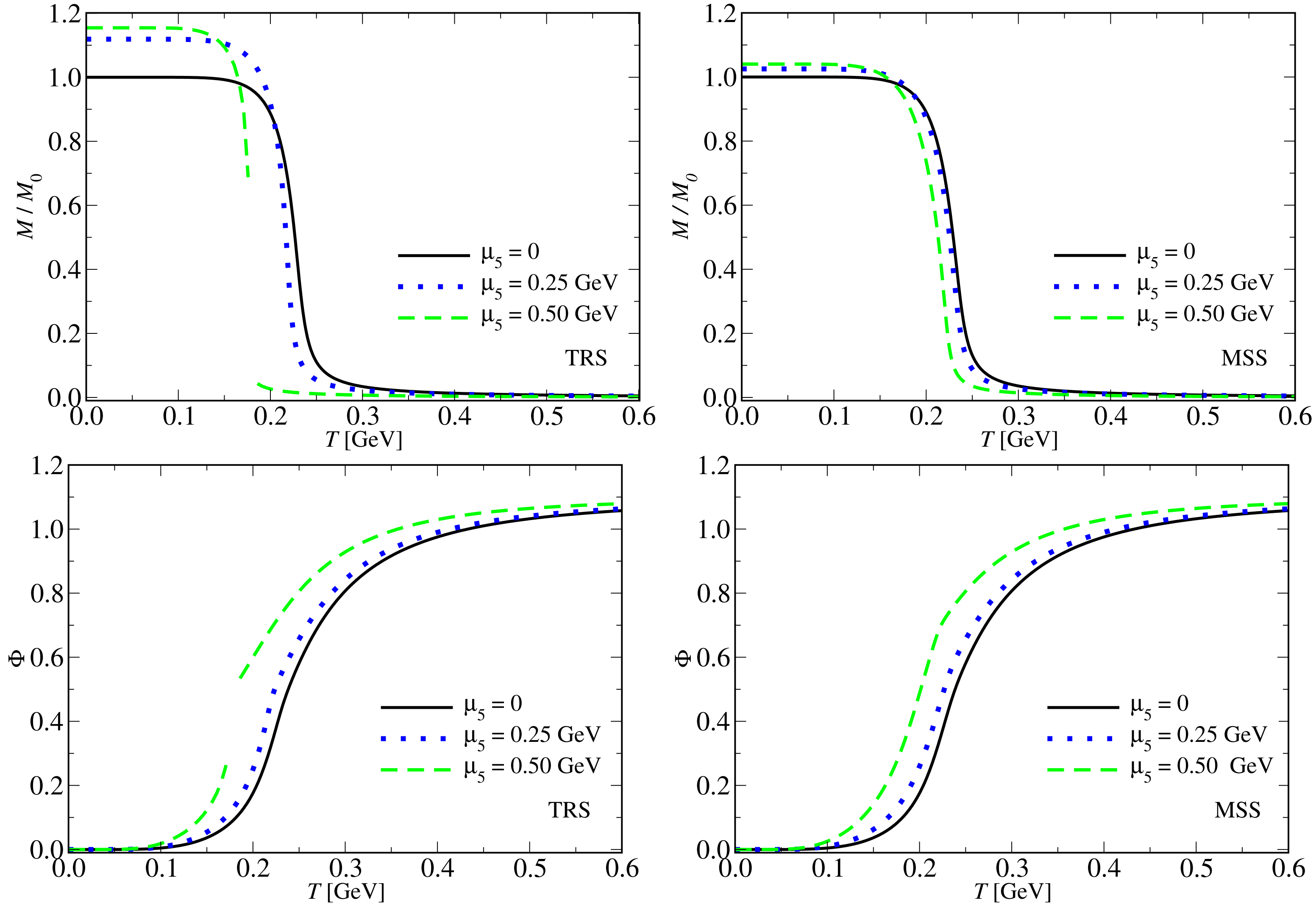
DOI: 10.1103/PhysRevD.78.074033 PACS numbers: 12.38.−t, 12.38.Mh

$$J_z \sim \sum_f q_f^2 B\mu_5 \longrightarrow \text{Independent of mass or temperature}$$

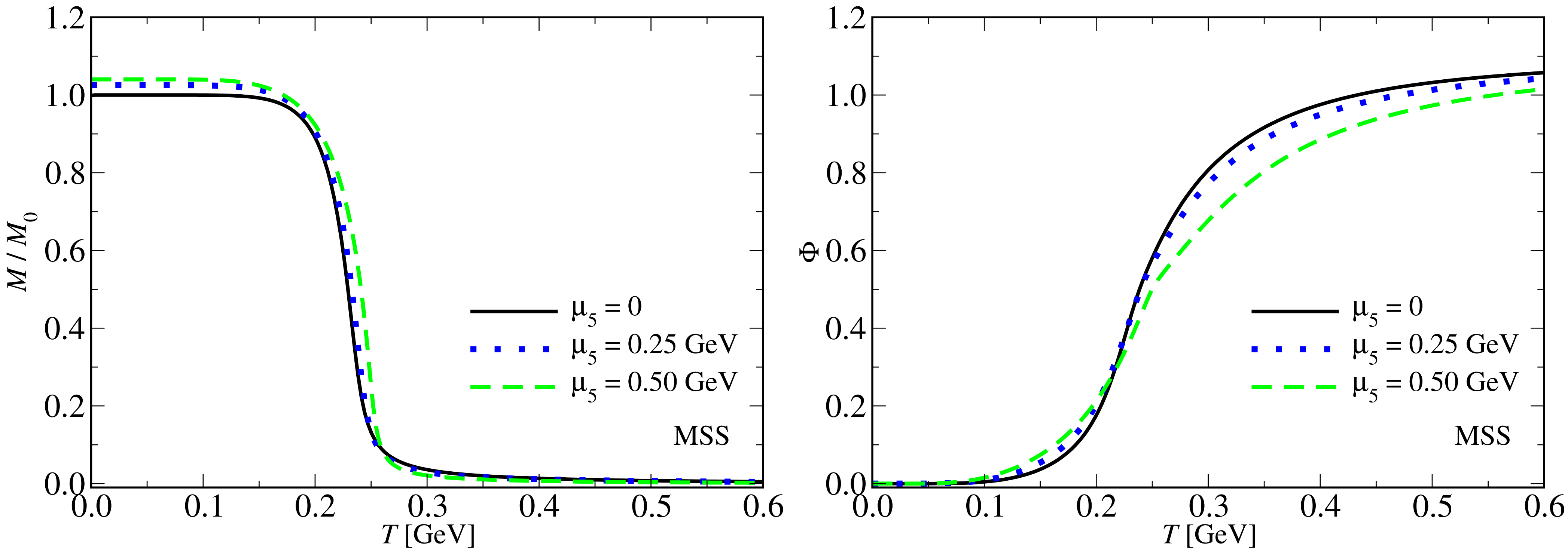
- Chiral separation effect: for $\mu \neq 0$, $J_5 \sim \sum_f q_f^2 B\mu$

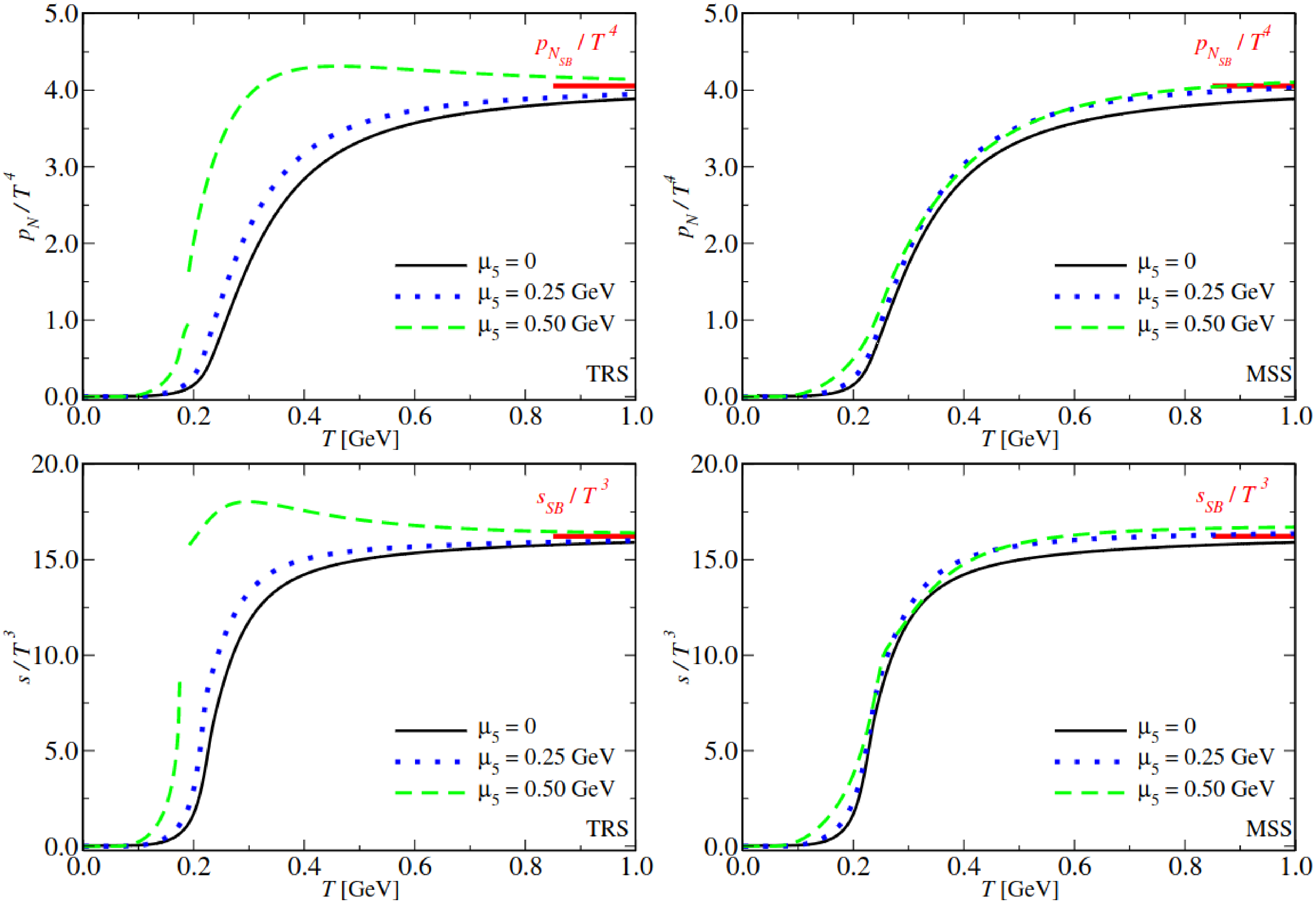
Solving a system of three equations by means of the thermodynamic potential minimization procedure with respect to M, Φ, $\Phi^\dagger$

$$\frac{\partial \Omega}{\partial M} = \frac{\partial \Omega}{\partial \Phi} = \frac{\partial \Omega}{\partial \Phi^\dagger} = 0. \tag{24}$$



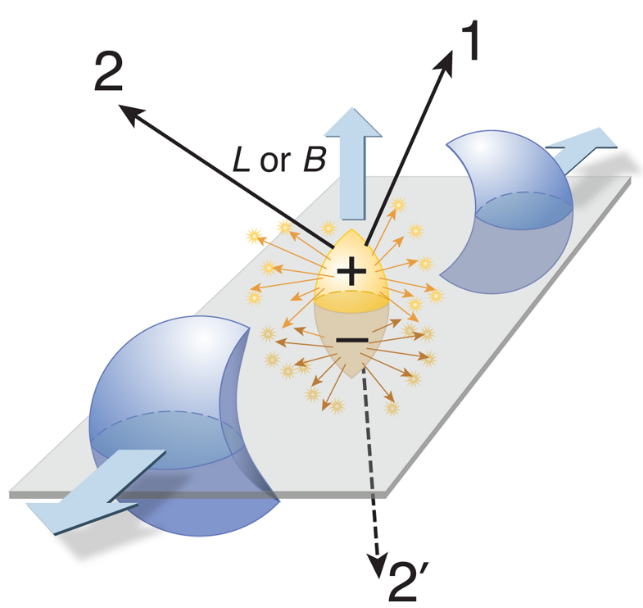
Order parameters





5.0
4.0
3.0
2.0
1.0
0.0
$p_{N_{SB}} / T^4$
p_N / T^4
$\mu_5 = 0$
$\mu_5 = 0.25$ GeV
$\mu_5 = 0.50$ GeV
TRS
MSS
0.0 0.2 0.4 0.6 0.8 1.0
T [GeV]
20.0
15.0
10.0
5.0
0.0
s_{SB} / T^3
s / T^3

- Such a chirality imbalance is expected to occur in event- by-event C and CP violating processes in heavy-ion collisions



To keep in mind: Chiral imbalance + strong magnetic field ⇒ Electric current in the direction of $\vec{B}$.

- In off-central collisions a magnetic field is created and the presence of a chiral imbalance gives rise to an electric current along the magnetic field, whose effect is to produce charge separation, an effect dubbed chiral magnetic effect (CME)

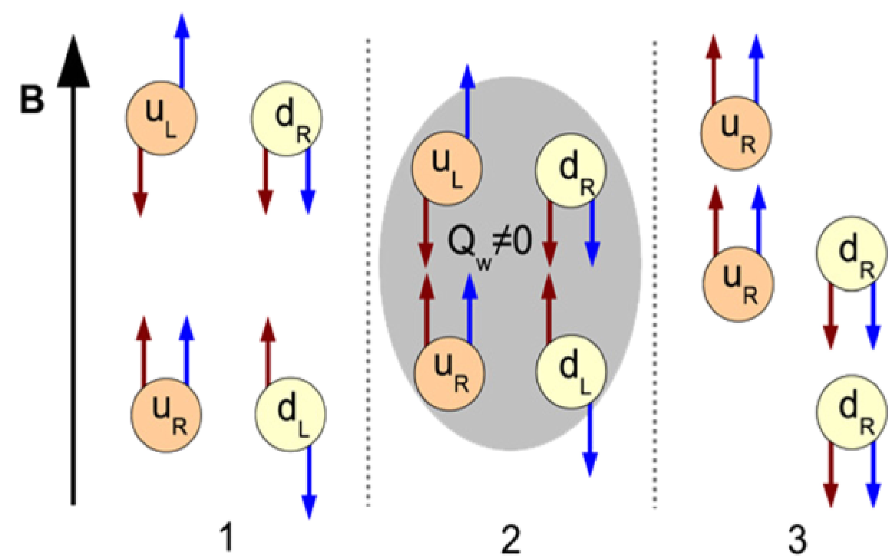


Figure: The illustration of the charge separation effect and chiral magnetic effect. The blue arrows and the red arrows respectively denote the spin and the momentum of quarks. Font :https://nica.jinr.ru/physics.php

• Effect of the chiral chemical potential on the chiral phase transition in the NJL model with different regularization schemes by Lang Yu, Hao Liu, and Mei Huang Phys. Rev. D 94, 014026 – Published 21 July 2016