Coherent Energy Loss: beyond leading-log accuracy

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Program

- Reminder of Fully Coherent Energy Loss (FCEL) at leading-log *(see talk at Excited QCD 2022)*
- Coherent radiation spectrum for $2 \to 2$ processes beyond leading-log
    - structure of the spectrum
    - matching with $2 \to 1$ processes
    - an unusual effect: *fully coherent energy gain*

Talk based on :

G. Jackson, S.P., K. Watanabe 2312.11650 [hep-ph]

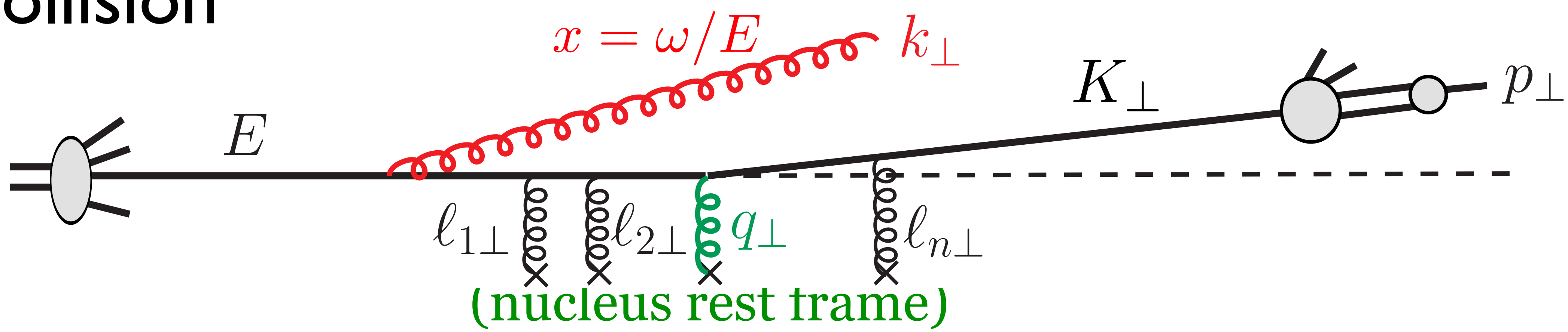
FCEL at leading-log

FCEL = induced radiative energy loss in parton small angle scattering

$2 \to 1$ processes

Arleo, S.P., Sami PRD 83 (2011)
S.P., Arleo, Kolevatov PRD 93 (2016)
Munier, S.P., Petreska PRD 95 (2017)

pA collision



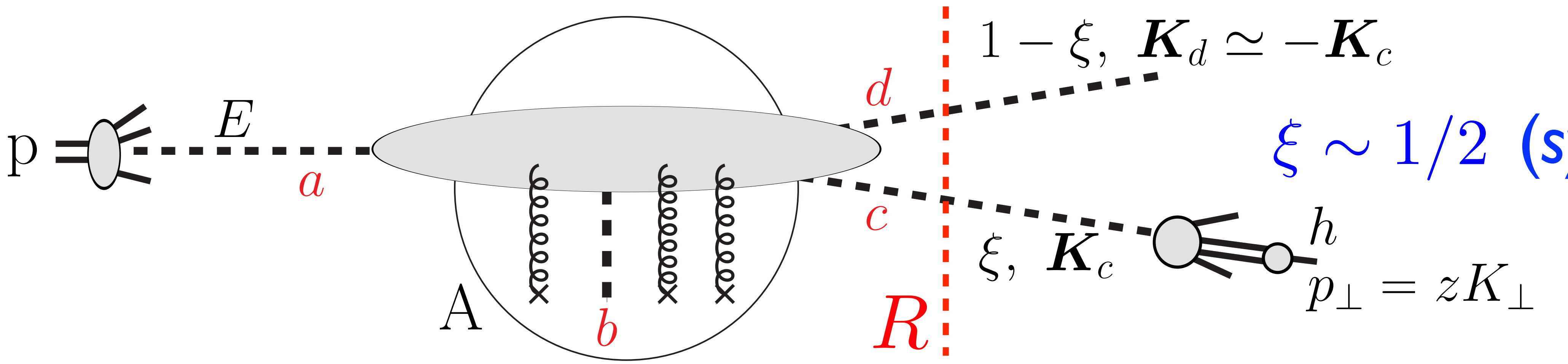
- *single* hard exchange $q_\perp \simeq K_\perp = p_\perp / z$
- soft rescatterings $\ell_\perp^2 = \left(\sum \vec{\ell}_{i\perp}\right)^2 \sim \hat{q}L \sim Q_s^2 \ll K_\perp^2$
- recoil parton is assumed to be soft

$$x \frac{\mathrm{d}I}{\mathrm{d}x}\bigg|_{1\to 1} = (C_1 + C_2 - C_t)\frac{\alpha_s}{\pi} \log\left(1 + \frac{\hat{q}L}{x^2 K_\perp^2}\right)$$

$$\Delta E = E \int_0^1 \mathrm{d}x\, x \frac{\mathrm{d}I}{\mathrm{d}x} \propto \alpha_s \frac{\sqrt{\hat{q}L}}{K_\perp} E$$

$2 \to 2$ processes

Liu, Mueller PRD 89 (2014)
S.P., Kolevatov JHEP 01 (2015)



$\xi \sim 1/2$ *(symmetric dijet)*

$K_\perp \equiv K_{c\perp} \simeq K_{d\perp} \gg \sqrt{\hat{q}L}$ and $|\vec{K}_c + \vec{K}_d| \lesssim \sqrt{\hat{q}L}$

to leading-log: radiated gluon does not probe the dijet

→ effectively equivalent to $2 \to 1$

$$x\frac{\mathrm{d}I}{\mathrm{d}x}\bigg|_{1\to 2}^{\mathrm{LL}} = \sum_R \rho_R\,(C_a + C_R - C_b)\,\frac{\alpha_s}{\pi}\log\left(1+\frac{\hat{q}L}{x^2 M^2_{\mathrm{dijet}}}\right)$$

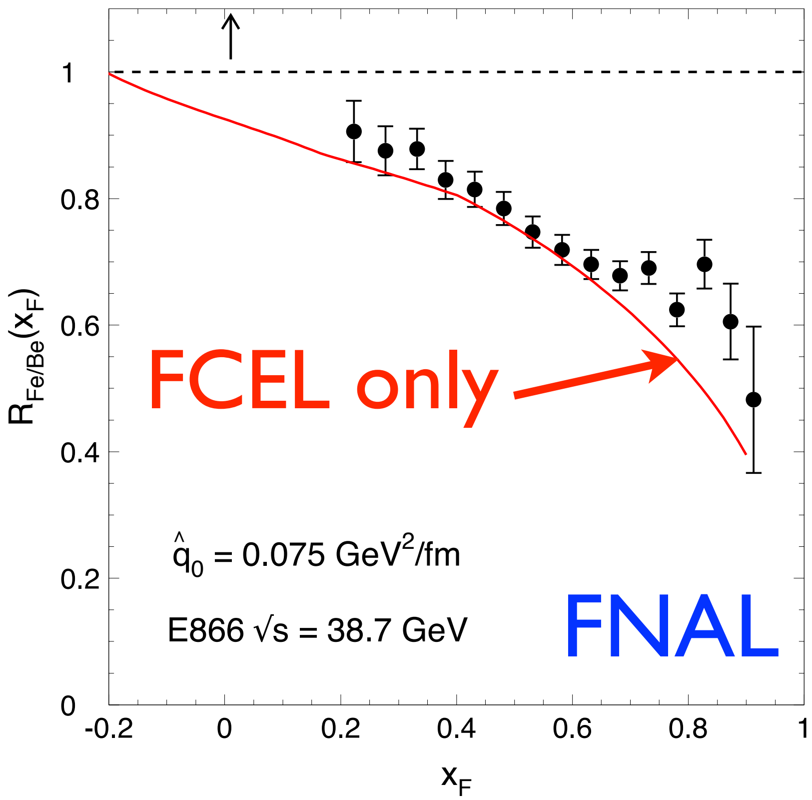
C_R global dijet color charge (Casimir) in state *R*

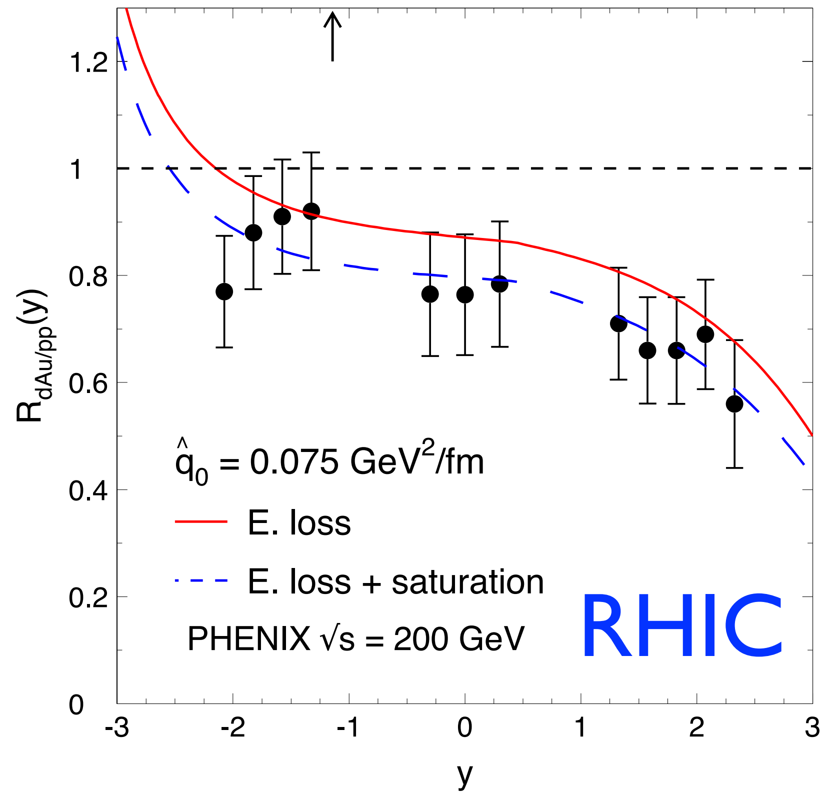
ρ_R proba for dijet to be produced in color state *R*

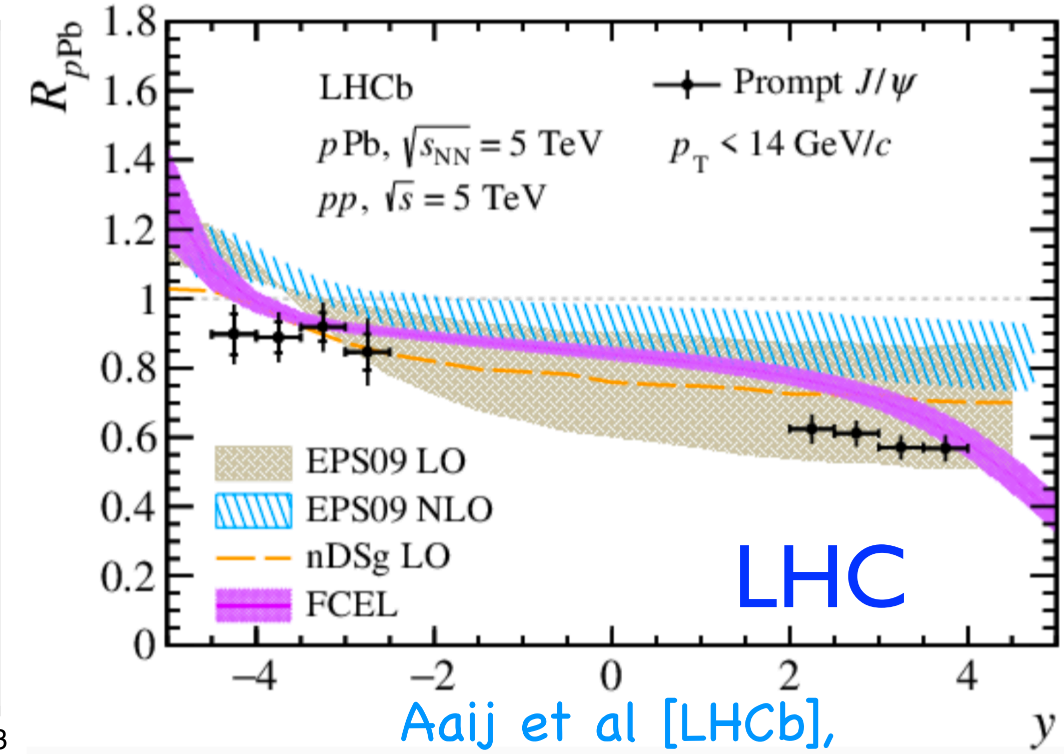
hadron suppression in pA collisions

FCEL in quarkonium production $(2 \to 1)$

Arleo and Peigne, PRL109, 122301 (2012), JHEP03, 122 (2013)

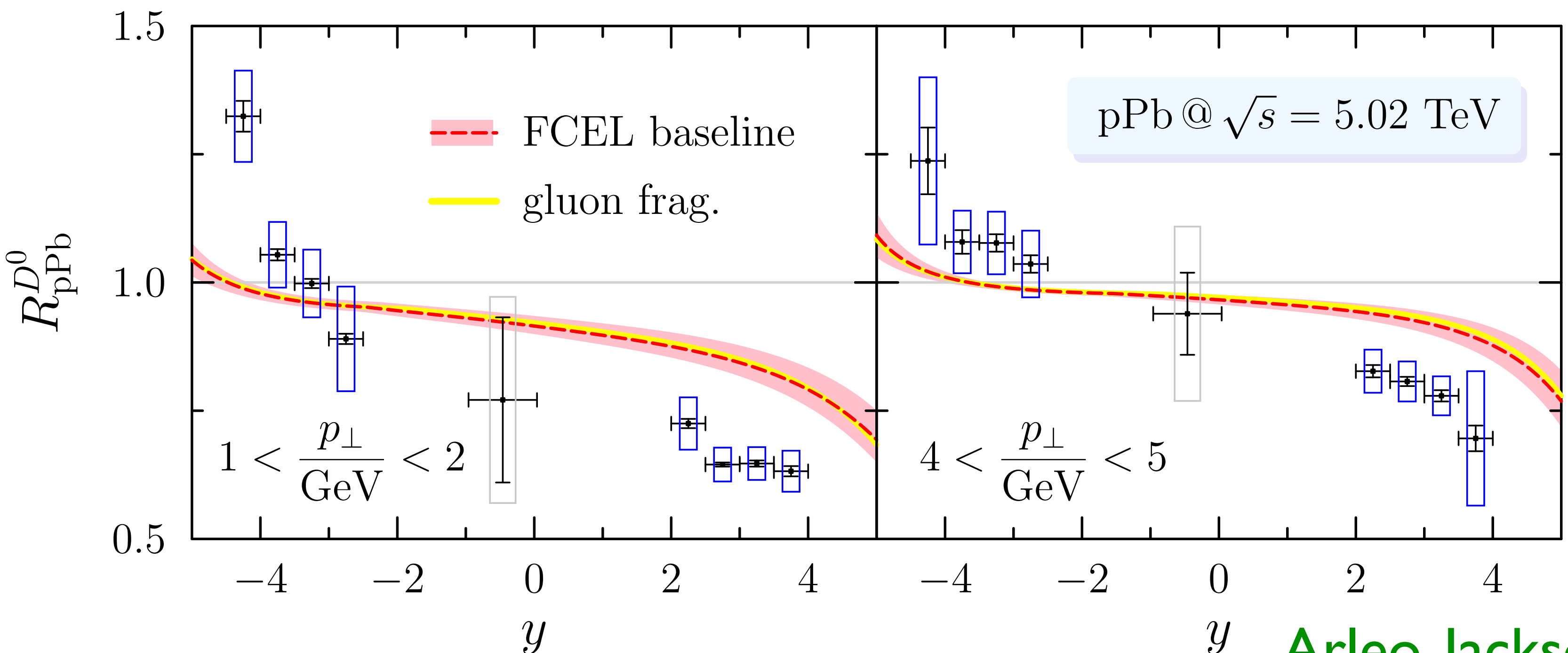






Aaij et al [LHCb], JHEP11, 181 (2021)

FCEL in heavy flavour production $(2 \to 2 \text{ and } \xi = 1/2)$



Aaij et al [**LHCb**], JHEP 10 (2017) 090

Abelev et al [**ALICE**], PRL 113 (2014) 232301

Arleo, Jackson, S.P. JHEP 01 (2022) 164

- FCEL contributes to *substantial* hadron suppression in pA

  at least as much as nuclear PDF effects

- FCEL is a (finite) pQCD prediction, with small uncertainty

→ implement FCEL in pQCD hadron cross sections *before* extracting nPDF sets

$\Longrightarrow$ needs coherent radiation spectrum in full $\xi-\mathrm{range}:$

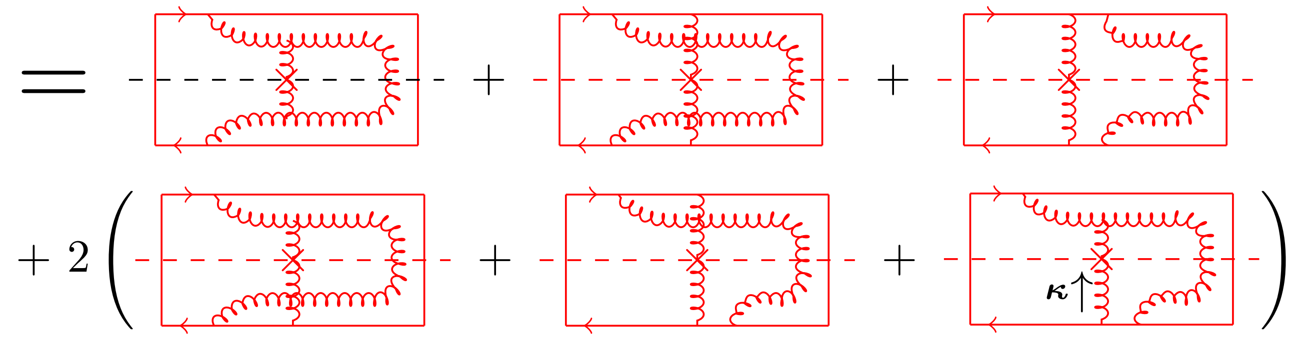
$$0 \leq \xi \leq 1$$

Coherent radiation spectrum beyond leading-log

G. Jackson, S.P., K. Watanabe 2312.11650 [hep-ph]

- structure of the spectrum

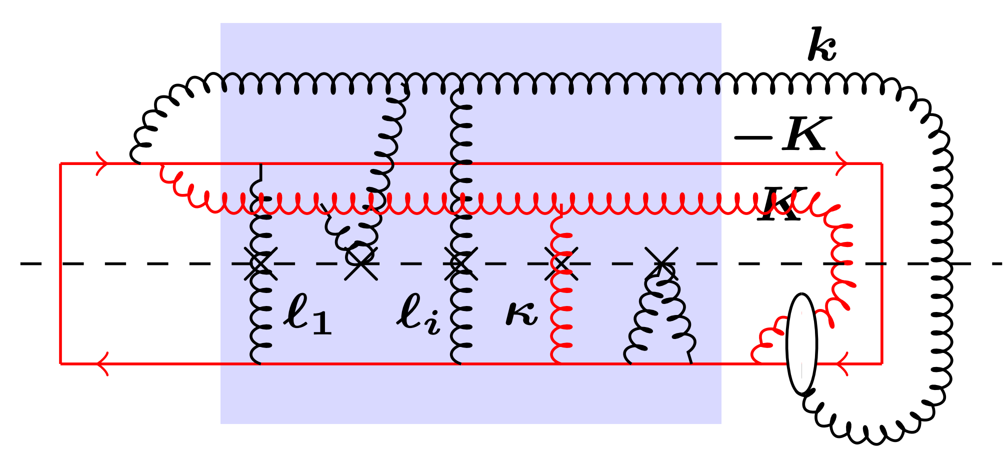
hard process $|\mathcal{M}|^2 =$



opacity expansion

$$x\frac{\mathrm{d}I^{(\tilde{n})}}{\mathrm{d}x} = \frac{\alpha_s}{\pi}\int\frac{\mathrm{d}^2\boldsymbol{k}}{\pi}\left[\prod_{i=1}^{\tilde{n}}\int\frac{\mathrm{d}z_i}{N\lambda_g}\int\mathrm{d}^2\boldsymbol{\ell}_i\,V(\boldsymbol{\ell}_i)\right]C_{\tilde{n}}(\boldsymbol{k},\boldsymbol{K})$$

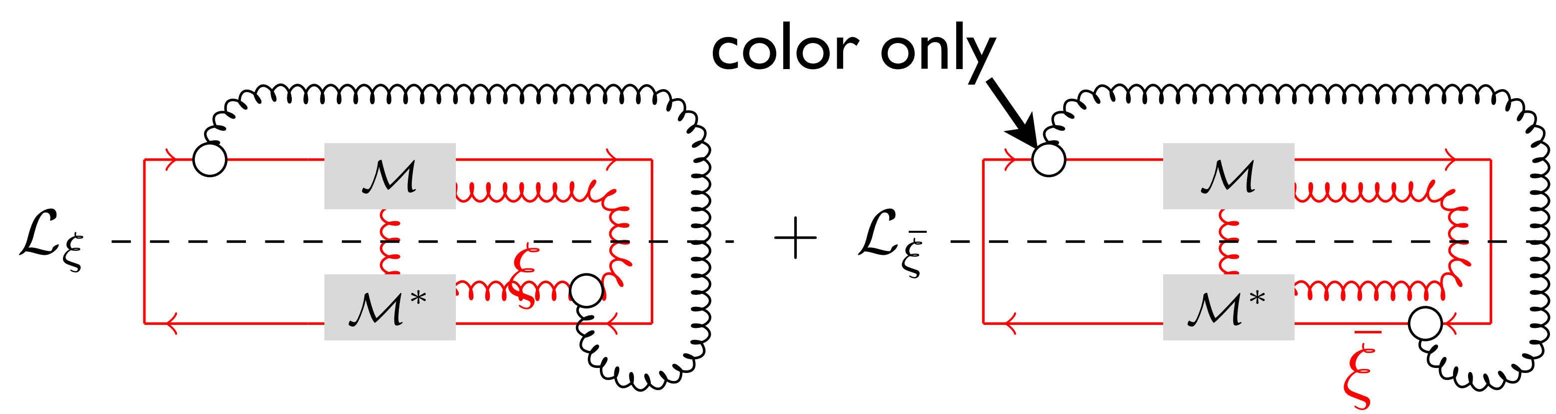
$$C_{\tilde{n}}(\boldsymbol{k},\boldsymbol{K}) = \frac{2}{|\mathcal{M}|^2}\left\{ \right.$$



$$\left. + \;\ldots \right\}$$

soft rescatterings factorize into $\mathcal{L}_\xi \simeq \log\left(1+\xi^2\frac{Q_s^2}{x^2m_\perp^2}\right) - \log\left(1+\xi^2\frac{Q_{s,\mathrm{p}}^2}{x^2m_\perp^2}\right)$

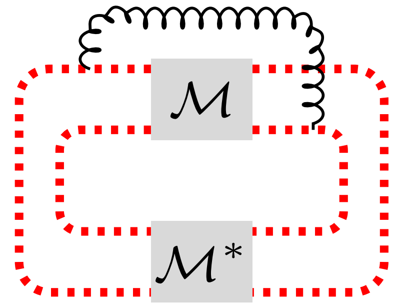
$\rightarrow$ $x\frac{\mathrm{d}I}{\mathrm{d}x} = \frac{\alpha_s}{\pi}\frac{2}{|\mathcal{M}|^2}\Big\{$



$\Big\}$

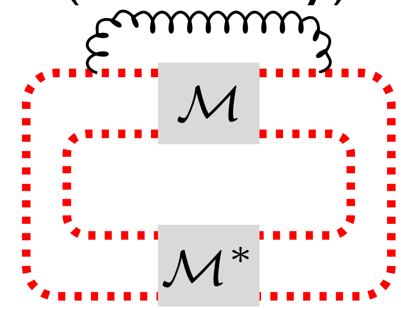
for any $2 \to 2$ process:

$$x\frac{\mathrm{d}I}{\mathrm{d}x} = \frac{\alpha_s}{\pi}\frac{2}{|\mathcal{M}|^2}\Bigg\{ \mathcal{L}_\xi$$



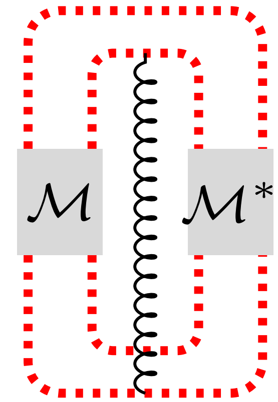
$$+ \ \mathcal{L}_{\bar{\xi}}$$

(color only)

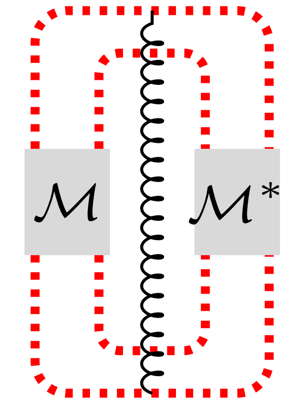


$$\Bigg\}$$

$$= \frac{\alpha_s}{\pi}\frac{2}{|\mathcal{M}|^2}\Bigg\{ \mathcal{L}_\xi$$



$$+ \ \mathcal{L}_{\bar{\xi}}$$



$$\Bigg\}$$

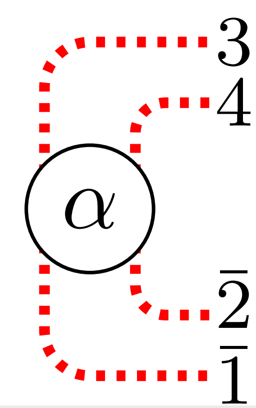
color decomposition of hard amplitude:

color indices only

$$\mathcal{M}_{12\to34} = \sum_\alpha \nu_\alpha \langle\alpha|$$

kinematics, spin, flavour

$$\langle\alpha| = \frac{1}{\sqrt{K_\alpha}}$$

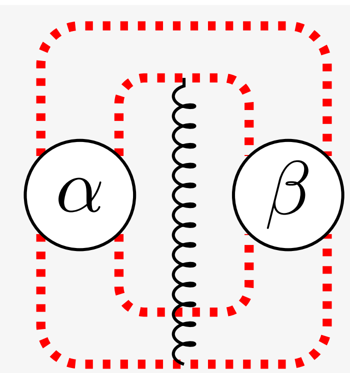


(*s*-channel basis)

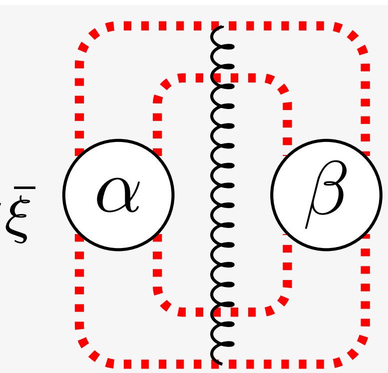
color density matrix $\Phi_{\alpha\beta} = \dfrac{\mathrm{tr}_\mathrm{d}(\nu_\alpha \nu^*_\beta)}{\mathrm{tr}_\mathrm{d}\,\mathrm{tr}_\mathrm{c}|\mathcal{M}|^2}$ $\qquad \rho_\alpha \equiv \Phi_{\alpha\alpha}$ *s*-channel probability

soft radiation matrix

$$S(x)_{\alpha\beta} \equiv \frac{\alpha_s}{\pi\, x}\frac{2}{\sqrt{K_\alpha K_\beta}}\Bigg(\mathcal{L}_\xi$$



$$+ \ \mathcal{L}_{\bar{\xi}}$$



$$\Bigg)$$

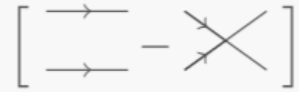
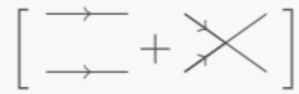
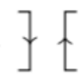
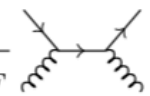
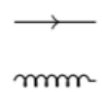
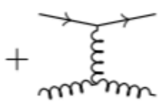
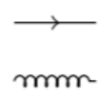
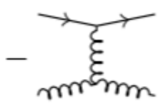
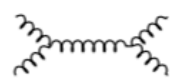
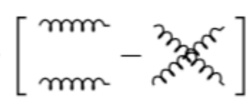
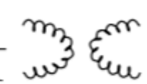
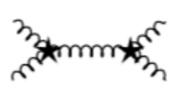
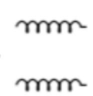
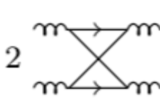
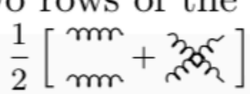
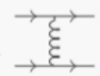
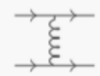
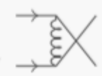
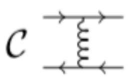
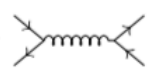
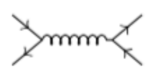
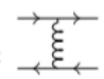
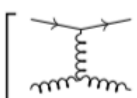
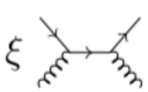
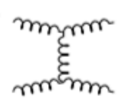
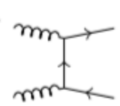
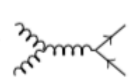
system	irrep α	projector $\mathbb{P}_\alpha$	dimension K_α	Casimir C_α
$\mathbf{3}\otimes\mathbf{3}$	$\bar{\mathbf{3}}$	$\frac{1}{2}$ 	$\frac{1}{2}N(N-1)$	$2C_F-\frac{N+1}{N}$
	$\mathbf{6}$	$\frac{1}{2}$ 	$\frac{1}{2}N(N+1)$	$2C_F+\frac{N-1}{N}$
$\mathbf{3}\otimes\bar{\mathbf{3}}$	$\mathbf{1}$	$\frac{1}{N}$ 	1	0
	$\mathbf{8}$	2 	N^2-1	N
$\mathbf{3}\otimes\mathbf{8}$	$\mathbf{3}$	$\frac{1}{C_F}$ 	N	C_F
	$\bar{\mathbf{6}}$	$\frac{1}{2}$  $-\frac{N+1}{2}\mathbb{P}_{\mathbf{3}}+$ 	$\frac{1}{2}N(N+1)(N-2)$	C_F+N-1
	$\mathbf{15}$	$\frac{1}{2}$  $+\frac{N-1}{2}\mathbb{P}_{\mathbf{3}}-$ 	$\frac{1}{2}N(N-1)(N+2)$	C_F+N+1
$\mathbf{8}\otimes\mathbf{8}$	$\mathbf{8}_{\mathrm{a}}$	$\frac{1}{N}$ 	N^2-1	N
	$\mathbf{10}\oplus\overline{\mathbf{10}}$	$\frac{1}{2}$  $-\mathbb{P}_{\mathbf{8}_{\mathrm{a}}}$	$\frac{1}{2}(N^2-1)(N^2-4)$	$2N$
	$\mathbf{1}$	$\frac{1}{N^2-1}$ 	1	0
	$\mathbf{8}_{\mathrm{s}}$	$\frac{N}{N^2-4}$ 	N^2-1	N
	$\mathbf{27}$	$\left(\frac{1}{2}\right.$  $+\,2$  $\left.\right)\mathbb{Q}$	$\frac{1}{4}N^2(N-1)(N+3)$	$2(N+1)$
	$\mathbf{0}$	$\left(\frac{1}{2}\right.$  $-\,2$  $\left.\right)\mathbb{Q}$	$\frac{1}{4}N^2(N+1)(N-3)$	$2(N-1)$

Table 1. Projectors, dimensions and quadratic Casimirs of the SU(N) irreps associated to the qq, $q\bar{q}$, qg and gg systems. An SU(N) irrep is labelled according to its dimension for $N=3$. We express the arising color factors via N and $C_F=(N^2-1)/(2N)$. In the last two rows of the table, for the projectors $\mathbb{P}_{\mathbf{27}}$ and $\mathbb{P}_{\mathbf{0}}$ of a gg pair we use the shorthand notation $\mathbb{Q}\equiv\frac{1}{2}$  $-\mathbb{P}_{\mathbf{8}_{\mathrm{s}}}-\mathbb{P}_{\mathbf{1}}$.

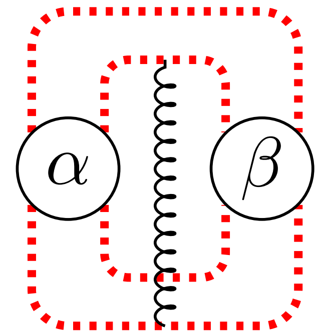
G. Jackson, S.P., K. Watanabe 2312.11650 [hep-ph]

channel	$\mathcal{M}$	$\dfrac{\mathrm{tr_d\,tr_c}\,\lvert\mathcal{M}\rvert^2}{4g^4(N^2-1)}$	α	$\dfrac{\nu_\alpha}{\sqrt{K_\alpha}}$
$qq' \to qq'$	$\mathcal{A}$ 	$\dfrac{1+\bar\xi^2}{2\xi^2}$	$\bar{\mathbf{3}}$ $\mathbf{6}$	$\mathcal{A}\,\frac{N+1}{2N}$ $-\mathcal{A}\,\frac{N-1}{2N}$
$qq \to qq$	$\mathcal{B}_t$  $+\,\mathcal{B}_u$ 	$\dfrac{1+\xi^2}{2\bar\xi^2}+\dfrac{1+\bar\xi^2}{2\xi^2}-\dfrac{1}{N\xi\bar\xi}$	$\bar{\mathbf{3}}$ $\mathbf{6}$	$\frac{N+1}{4N}\,(\mathcal{B}_t-\mathcal{B}_u)$ $-\frac{N-1}{4N}\,(\mathcal{B}_t+\mathcal{B}_u)$
$q\bar q' \to q\bar q'$	$\mathcal{C}$ 	$\dfrac{1+\bar\xi^2}{2\xi^2}$	$\mathbf{1}$ $\mathbf{8}$	$C_F\,\mathcal{C}$ $-\frac{1}{2N}\,\mathcal{C}$
$q\bar q \to q'\bar q'$	$\mathcal{D}$ 	$\dfrac{\xi^2+\bar\xi^2}{2}$	$\mathbf{1}$ $\mathbf{8}$	0 $\frac{1}{2}\,\mathcal{D}$
$q\bar q \to q\bar q$	$\mathcal{E}_s$  $+\,\mathcal{E}_t$ 	$\dfrac{\xi^2+\bar\xi^2}{2}+\dfrac{1+\bar\xi^2}{2\xi^2}+\dfrac{\bar\xi^2}{N\xi}$	$\mathbf{1}$ $\mathbf{8}$	$C_F\,\mathcal{E}_t$ $\frac{1}{2}(\mathcal{E}_s-\frac{1}{N}\mathcal{E}_t)$
$qg \to qg$	$\mathcal{F}$ [ $-\,\xi$ ]	$(1+\bar\xi^2)\left(\dfrac{N}{\xi^2}+\dfrac{C_F}{\bar\xi}\right)$	$\mathbf{3}$ $\bar{\mathbf{6}}$ $\mathbf{15}$	$\left(\frac{1}{2N}+\bar\xi C_F\right)\mathcal{F}$ $\frac{1}{2}\,\mathcal{F}$ $-\frac{1}{2}\,\mathcal{F}$
$gg \to gg$	$\mathcal{G}$ [ $-\,\xi$ ]	$4N^2\dfrac{(1-\xi\bar\xi)^3}{\xi^2\bar\xi^2}$	$\mathbf{8}_{\mathrm{a}}$ $\mathbf{10}\oplus\overline{\mathbf{10}}$ $\mathbf{1}$ $\mathbf{8}_{\mathrm{s}}$ $\mathbf{27}$ $\mathbf{0}$	$\frac{N}{2}(\bar\xi-\xi)\,\mathcal{G}$ 0 $N\,\mathcal{G}$ $\frac{N}{2}\,\mathcal{G}$ $-\mathcal{G}$ $\mathcal{G}$
$gg \to q\bar q$	$\mathcal{H}$ [ $-\,\xi$ ]	$(\xi^2+\bar\xi^2)\left(\dfrac{C_F}{\xi\bar\xi}-N\right)$	$\mathbf{1}$ $\mathbf{8}_{\mathrm{a}}$ $\mathbf{8}_{\mathrm{s}}$	$\frac{\sqrt{N^2-1}}{2\sqrt{N}}\,\mathcal{H}$ $\frac{1}{2}\,(\bar\xi-\xi)\,\frac{\sqrt{N}}{\sqrt{2}}\,\mathcal{H}$ $\frac{\sqrt{N^2-4}}{2\sqrt{2N}}\,\mathcal{H}$

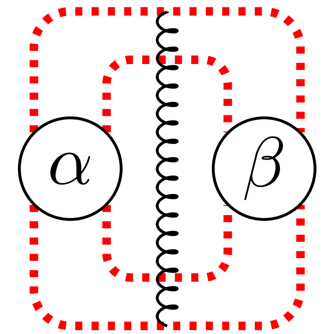
G. Jackson, S.P., K. Watanabe 2312.11650 [hep-ph]

$$\frac{\mathrm{d}I}{\mathrm{d}x} = \Phi_{\alpha\beta}\, S(x)_{\alpha\beta} = \mathrm{Tr}\left\{\Phi \cdot S(x)\right\}$$

$$S(x)_{\alpha\beta} \equiv \frac{\alpha_s}{\pi\, x} \frac{2}{\sqrt{K_\alpha K_\beta}} \left(\mathcal{L}_\xi \right.$$



$$+ \mathcal{L}_{\bar{\xi}}$$



$$\left. \right)$$

$$\xi = \bar{\xi} = 1/2 \text{ or } \mathcal{L}_\xi \simeq \mathcal{L}_{\bar{\xi}} \gg 1 \qquad T_3 + T_4 = T_\alpha = T_1 + T_2$$

final parton pair is pointlike $\Longrightarrow$ soft radiation conserves pair irrep

$$\langle \alpha | 2\, T_1\, T_\alpha | \beta \rangle = \langle \alpha | T_1^2 + T_\alpha^2 - T_2^2 | \beta \rangle = (C_1 + C_\alpha - C_2)\, \delta_{\alpha\beta}$$

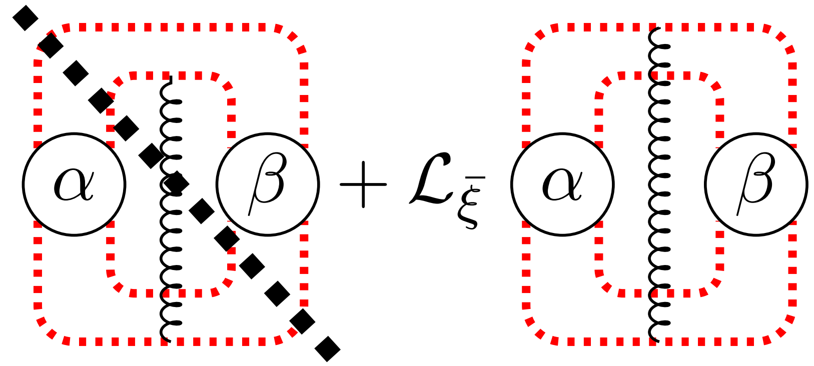
$$\left.\frac{\mathrm{d}I}{\mathrm{d}x}\right|_{\xi=\frac{1}{2}} = \mathrm{Tr}\left\{\Phi \cdot S(x)\right\} = \sum_\alpha \rho_\alpha\, (C_1 + C_\alpha - C_2)\, \frac{\alpha_s}{\pi\, x}\, \mathcal{L}_{1/2}$$

$\xi \neq 1/2$ and beyond leading-log

- soft gluon can change parton pair irrep: « color transitions »
- soft gluon can probe color structure of parton pair without probing its spatial size *(non-abelian feature)*

• limit $\xi \to 0$: matching with $2 \to 1$ processes

$$S(x)_{\alpha\beta} \equiv \frac{\alpha_s}{\pi\, x}\frac{2}{\sqrt{K_\alpha K_\beta}} \left(\mathcal{L}_\xi \,[\ldots]\, + \mathcal{L}_{\bar\xi}\,[\ldots] \right)$$



$$2T_1T_3 = T_1^2 + T_3^2 - (T_1 - T_3)^2 = C_1 + C_3 - T_t^2$$

color exchange in *t*-channel

$$\mathcal{M} = \sum_\alpha \nu_\alpha \langle\alpha| = \sum_{\alpha^t} \nu_{\alpha^t} \langle\alpha^t|$$ independent of color-basis

$$\Rightarrow \frac{\mathrm{d}I}{\mathrm{d}x} = \mathrm{Tr}\left\{\Phi \cdot S(x)\right\} = \Phi_{\alpha\beta}\, S(x)_{\alpha\beta} = \Phi^t_{\alpha^t\beta^t}\, S(x)^t_{\alpha^t\beta^t}$$

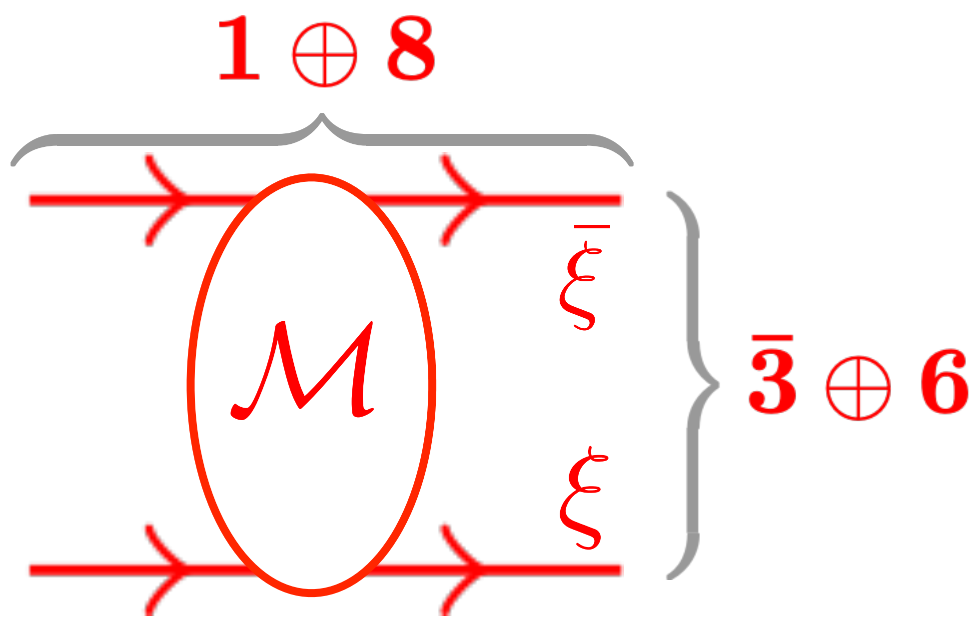
choose *t*-basis where *S(x)* is diagonal:

proba for *t*-channel pair to be in irrep α^t

$$\left.\frac{\mathrm{d}I}{\mathrm{d}x}\right|_{\xi\to 0} = \underbrace{\sum_{\alpha^t} \rho^t_{\alpha^t}\,(C_1 + C_3 - C_{\alpha^t})}\ \frac{\alpha_s}{\pi\, x}\,\mathcal{L}_1$$

matches with $C_1 + C_3 - N$ for $2 \to 1$ processes studied previously (with purely octet *t*-channel exchange)

illustration: $qq' \to qq'$ and $qq \to qq$ processes

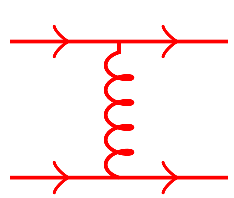
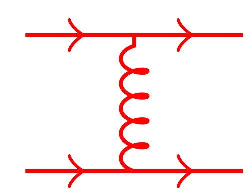
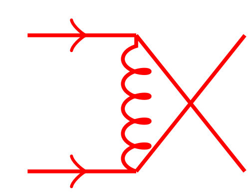


$$\left.\frac{\mathrm{d}I}{\mathrm{d}x}\right|_{\xi=\frac{1}{2}} = \frac{\alpha_s}{\pi\, x}\,\mathcal{L}_{1/2}\left[\,\rho_{\bar{\mathbf{3}}}C_{\bar{\mathbf{3}}} + \rho_{\mathbf{6}}C_{\mathbf{6}}\,\right]$$

$$\left.\frac{\mathrm{d}I}{\mathrm{d}x}\right|_{\xi\to 0} = \frac{\alpha_s}{\pi\, x}\,\mathcal{L}_{1}\left[\,\rho^t_{\mathbf{1}}(2C_F) + \rho^t_{\mathbf{8}}(2C_F - N)\,\right]$$

$$\left.\frac{\mathrm{d}I}{\mathrm{d}x}\right|_{\xi\to 1} = \frac{\alpha_s}{\pi\, x}\,\mathcal{L}_{1}\left[\,\rho^u_{\mathbf{1}}(2C_F) + \rho^u_{\mathbf{8}}(2C_F - N)\,\right]$$

probas depend on specific process:

$\mathcal{M}_{qq'\to qq'} \propto$  $\quad \mathcal{M}_{qq\to qq} \propto \mathcal{B}_t$  $+ \mathcal{B}_u$ 

- an unusual effect: *fully coherent energy gain* (FCEG)

$$qq' \to qq',\ qq \to qq: \qquad 2C_F - N = -\frac{1}{N} < 0\ !$$

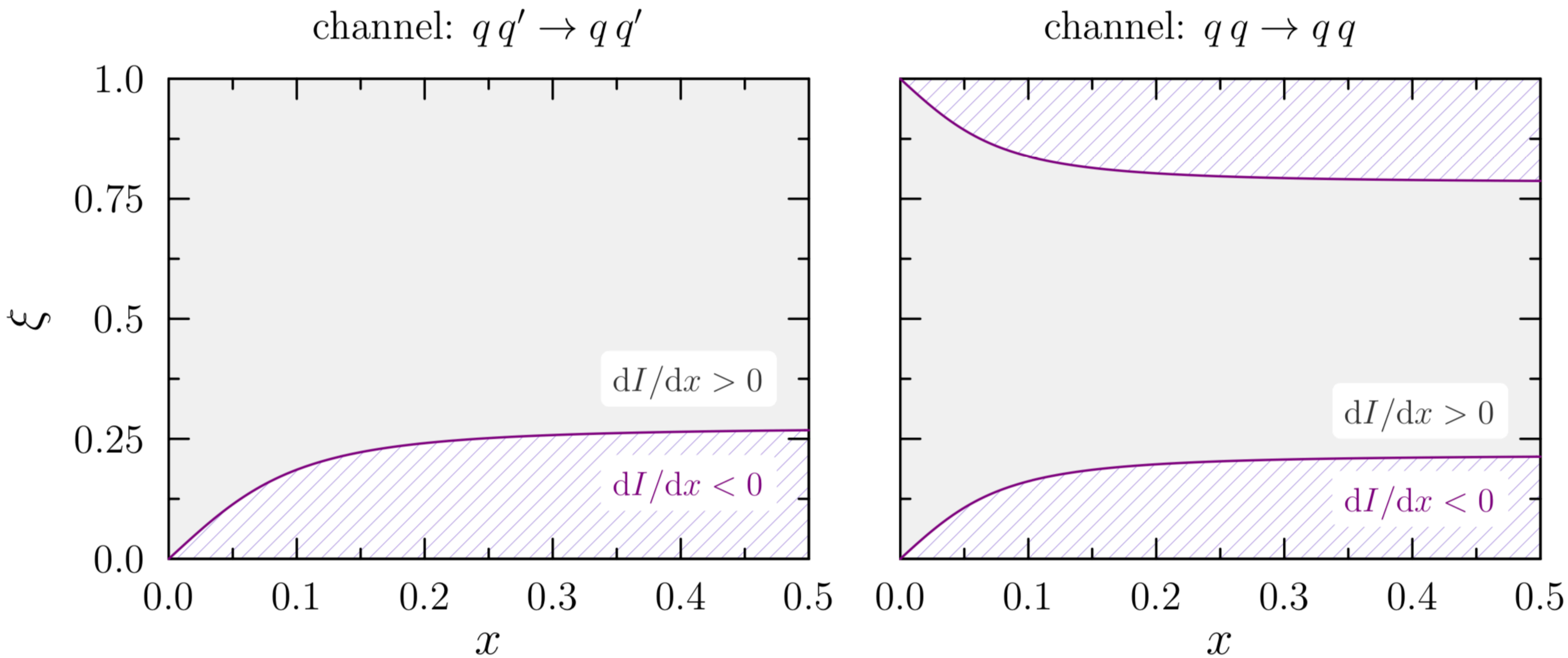


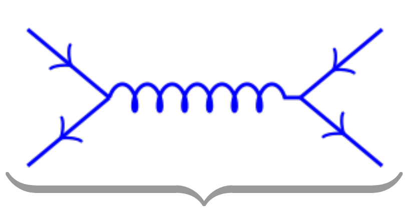
Figure 2. Regions in the (x, ξ)-plane, corresponding to energy-loss (solid gray) or energy-gain (hatched purple). In this figure, $N = 3$, $Q_{s,\mathrm{A}} = \frac{1}{4} m_\perp$ and $Q_{s,\mathrm{p}} = \frac{1}{10} m_\perp$.

G. Jackson, S.P., K. Watanabe 2312.11650 [hep-ph]

FCEG contributions appear in other channels:

e.g., $q\bar{q} \to q'\bar{q}'$

$\mathcal{M} \propto$

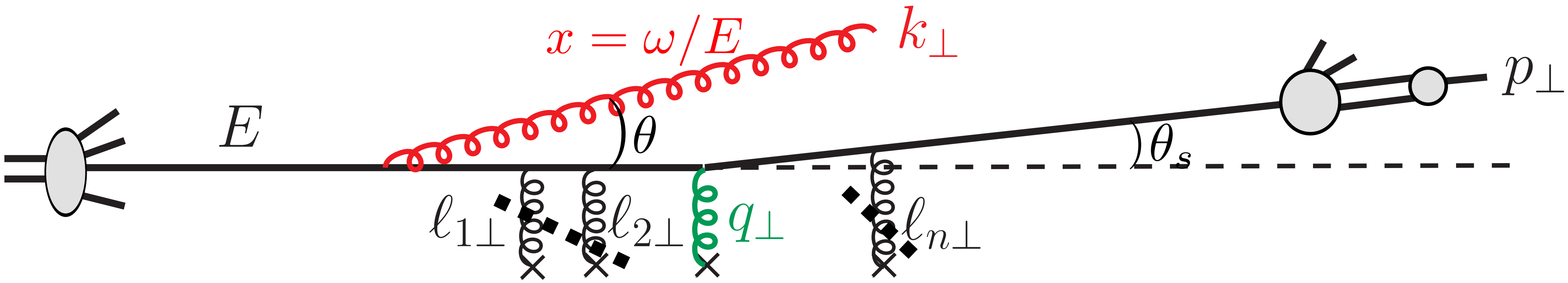


$\bar{\mathbf{3}} \oplus \mathbf{6}$

$$\left.\frac{\mathrm{d}I}{\mathrm{d}x}\right|_{\xi\to 0} = \frac{\alpha_s}{\pi\, x}\,\mathcal{L}_1\left[\rho^t_{\bar{\mathbf{3}}}(2C_F - C_{\bar{\mathbf{3}}}) + \rho^t_{\mathbf{6}}\underbrace{(2C_F - C_{\mathbf{6}})}_{<0}\right]$$

FCEG could have been inferred *heuristically* from features of *total* spectrum = spectrum at 0th order in opacity $(n = 0)$

S.P., Arleo, Kolevatov PRD 93 (2016)



$qg \to q$

$\theta < \theta_s$ (abelian) $\quad\quad$ $\theta > \theta_s$ (non-abelian)

$$x\frac{\mathrm{d}I^{(0)}}{\mathrm{d}x} \simeq \frac{\alpha_s}{\pi}\left[2C_R \log\left(\frac{x^2\boldsymbol{q}^2}{\Lambda_{\mathrm{IR}}^2}\right) + N_c \log\left(\frac{\Lambda_{\mathrm{S}}^2}{x^2\boldsymbol{q}^2}\right) + N_c \log\left(\frac{\boldsymbol{q}^2}{\Lambda_{\mathrm{S}}^2}\right) + N_c \log\left(\frac{\boldsymbol{q}^2}{\Lambda_{\mathrm{IR}}^2}\right)\right]$$

$k_\perp \lesssim xq_\perp$ $\quad\quad$ $xq_\perp < k_\perp < \Lambda_{\mathrm{S}}$ $\quad\quad$ $k_\perp > \Lambda_{\mathrm{S}}$

Λ_{S} arbitrary scale: $xq_\perp \ll \Lambda_{\mathrm{S}} \ll q_\perp$

additional radiation expected from rescattering $\ell_\perp \sim Q_s$?

- $k_\perp \lesssim Q_s \ll \Lambda_S \Rightarrow$ only $k_\perp < \Lambda_{\mathrm{S}}$ region is affected
- $\boldsymbol{q}^2 \to \boldsymbol{q}^2 + \mathcal{O}\left(Q_s^2\right) \Rightarrow \Delta\left(x\frac{\mathrm{d}I^{(0)}}{\mathrm{d}x}\right) \propto 2C_F - N < 0$

Remark: *total* spectrum *increases* when $q_\perp \nearrow$ with rate $\propto 2C_R + N_c$

Summary

- coherent radiation spectrum beyond leading-log and for any ξ now available for any $2 \to 2$ process
  - FCEL beyond LL depends on *color transitions*
  - some contributions to spectrum can be negative

Outlook

- implement FCEL in hadron pA cross sections to extract « better » and more precise nPDFs
- challenge: derive the quenching weight (valid for any ξ) associated with $\mathrm{d}I/\mathrm{d}x$

Thanks!