Hybrid static potentials from Laplacian eigenmodes

arXiv:2401.09453

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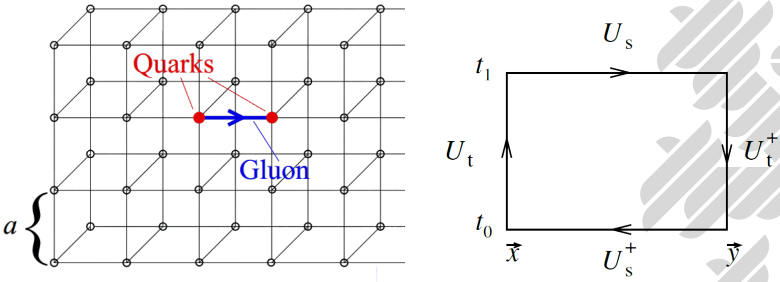
Lattice QCD



- link variables $U_\mu(x) = \exp(i \int_x^{x+a\hat{\mu}} A_\mu dx^\mu)$
- Wilson line $U_s(\vec{x},\vec{y}) = \exp(i \int_{\vec{x}}^{\vec{y}} A_\mu dx^\mu) = \prod U_\mu$
- path-ordered product of link variables, on-/off-axis
- plaquette, Wilson loop $W(R,T)$, static $\bar{Q}Q$ pair

Motivation

- hybrid mesons (valence glue, exotics, XYZ), (hybrid) static-light
- identify bound states using B-O approximation ⇒ Joan Soto, Braaten et al. (2014) or EFT Brambilla et al. (2005)
- observation of (hybrid) string breaking in QCD, *e.g.*, Bali et al. (2008), Bulava et al. (2019)
- calculate the (hybrid) static potential with high resolution

⇒ on the lattice we have to work with off-axis separated quarks

- the spatial part of the Wilson loop has to go over stair-like paths through the lattice → not unique, computationally expensive

⇒ alternative operator which ensures gauge invariance of the quark-anti-quark $\bar{Q}(\vec{x})U_s(\vec{x},\vec{y})Q(\vec{y})$ trial state

- required gauge transformation behavior:

$$U_s'(\vec{x},\vec{y}) = G(\vec{x})U_s(\vec{x},\vec{y})G^\dagger(\vec{y})$$

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Laplace trial states

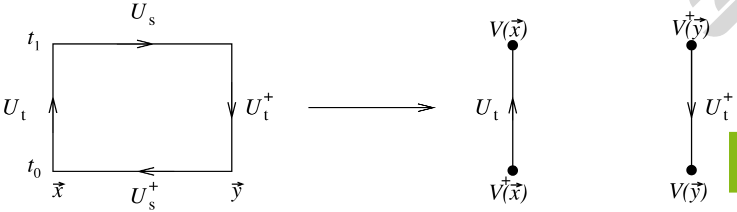
- adaptation of Neitzel et al. (2016) using SU(2)
- eigenvectors $v(\vec{x})$ of the 3D covariant lattice Laplace operator
- spatial Wilson line: $U'_s(\vec{x},\vec{y}) = G(\vec{x})U_s(\vec{x},\vec{y})G^\dagger(\vec{y})$

$$v'(\vec{x})v'^\dagger(\vec{y}) = G(\vec{x})v(\vec{x})v^\dagger(\vec{y})G^\dagger(\vec{y})$$

- Wilson loop of size $(R = |\vec{r}| = |\vec{x}-\vec{y}|) \times (T = |t_1 - t_0|)$

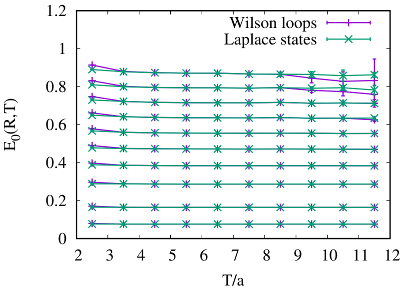
$$W(R,T) = \langle \mathrm{tr}\,[U_t(\vec{x};t_0,t_1)U_s(\vec{x},\vec{y};t_1)U_t^\dagger(\vec{y};t_0,t_1)U_s^\dagger(\vec{x},\vec{y};t_0)]\rangle$$

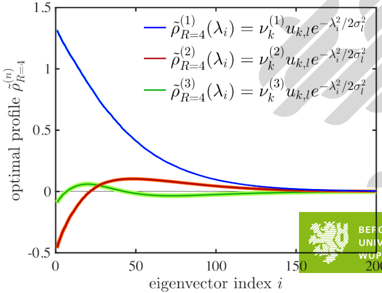
$$\to \langle \sum_{i,j}^{N_v} \mathrm{tr}\,[U_t(\vec{x})\rho_j(t_1)v_j(\vec{x},t_1)v_j^\dagger(\vec{y},t_1)U_t^\dagger(\vec{y})\rho_i(t_0)v_i(\vec{y},t_0)v_i^\dagger(\vec{x},t_0)]\rangle$$



Optimal trial state results

- Gaussian profile functions: $\rho_i^{(k)}(\lambda_i) = \exp(-\lambda_i^2/2\sigma_k^2)$
- define correlation matrix W_{kl} using 7 different $\sigma_{k,l}$, SVD ($u_{k,l}$)
- GEVP: $W(t)\nu^{(n)} = \mu^{(n)}W(t_0)\nu^{(n)}$, $\mu^{(n)}$ give effective energies
- optimal profiles $\tilde{\rho}_R^{(n)}(\lambda_i) = \nu_k^{(n)} u_{k,l} \exp(-\lambda_i^2/2\sigma_l^2)$
- $24^3 \times 48$, $\beta = 5.3$, $N_f = 2$, $\kappa = 0.13270$, $a = 0.0658$ fm





Static-hybrid potentials

Hybrid static potentials are characterized by the following quantum numbers Λ_η^ϵ, Bali et al. (2005), Bicudo et al. (2015):

- $\Lambda = 0, 1, 2, 3, \ldots \equiv \Sigma, \Pi, \Delta, \Phi, \ldots$, the absolute value of the total angular momentum with respect to the axis of separation of the static quark-antiquark pair
- $\eta = +, - \equiv g, u$, the eigenvalue corresponding to the operator $\mathcal{P} \circ \mathcal{C}$, i.e. the combination of parity and charge conjugation
- $\epsilon = +, -$, the eigenvalue corresponding to the operator $\mathcal{P}_x$, which denotes the spatial reflection with respect to a plane including the axis of separation ($\Lambda > 0$ degenerate)

$\Rightarrow$ derived from the continuous group $D_{\infty h}$, which leaves a cylinder along a chosen axis invariant

$\Rightarrow$ we can realize gluonic excitations via covariant derivatives

$$\nabla_{\vec{k}} v(\vec{x}) = \frac{1}{2}[U_k(\vec{x})v(\vec{x}+\hat{k}) - U_k^\dagger(\vec{x}-\hat{k})v(\vec{x}-\hat{k})]$$

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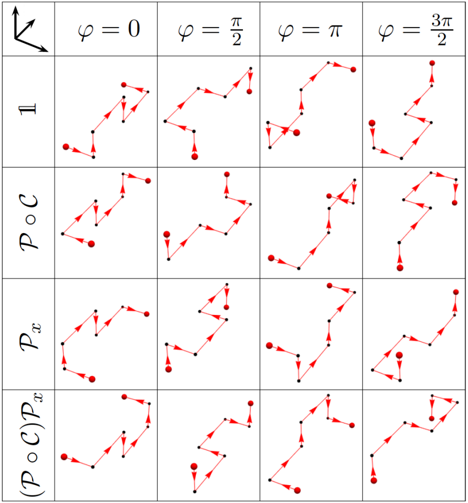
Wilson loop static-hybrid states

standard construction with handles, e.g. Σ_g^-, Capitani et al. (2019)



Laplacian static-hybrid states

- $\Sigma_g^+(R,T) = \sum_{\vec{x},t_0,i,j}$

  $\left\langle \mathrm{tr}\left[U_t(\vec{x};t_0,t_1)\rho(\lambda_j)v_j(\vec{x},t_1)v_j^\dagger(\vec{y},t_1)U_t^\dagger(\vec{y};t_0,t_1)\rho(\lambda_i)v_i(\vec{y},t_0)v_i^\dagger(\vec{x},t_0)\right]\right\rangle$

- $\Sigma_{u/g}^+(R,T) = \sum_{\vec{x},t_0,i,j,\vec{k}||\vec{r}=\vec{y}-\vec{x}}$

  $\left\langle \mathrm{tr}\left[U_t(\vec{x};t_0,t_1)\rho(\lambda_j)\{[\nabla_{\vec{k}}v_j](\vec{x},t_1)v_j^\dagger(\vec{y},t_1)\pm v_j(\vec{x},t_1)[\nabla_{\vec{k}}v_j]^\dagger(\vec{y},t_1)\}\right.\right.$

  $\left.\left.U_t^\dagger(\vec{y};t_0,t_1)\rho(\lambda_i)\{[\nabla_{\vec{k}}v_i](\vec{y},t_0)v_i^\dagger(\vec{x},t_0)\pm v_i(\vec{y},t_0)[\nabla_{\vec{k}}v_i]^\dagger(\vec{x},t_0)\}\right]\right\rangle$

- $\Pi_{u/g}(R,T) = \Pi_\mp(R,T) = \sum_{\vec{x},t_0,i,j,\vec{k}\perp\vec{r}=\vec{y}-\vec{x}}$

  $\left\langle \mathrm{tr}\left[U_t(\vec{x};t_0,t_1)\rho(\lambda_j)\{[\nabla_{\vec{k}}v_j](\vec{x},t_1)v_j^\dagger(\vec{y},t_1)\pm v_j(\vec{x},t_1)[\nabla_{\vec{k}}v_j]^\dagger(\vec{y},t_1)\}\right.\right.$

  $\left.\left.U_t^\dagger(\vec{y};t_0,t_1)\rho(\lambda_i)\{[\nabla_{\vec{k}}v_i](\vec{y},t_0)v_i^\dagger(\vec{x},t_0)\pm v_i(\vec{y},t_0)[\nabla_{\vec{k}}v_i]^\dagger(\vec{x},t_0)\}\right]\right\rangle$

- ...

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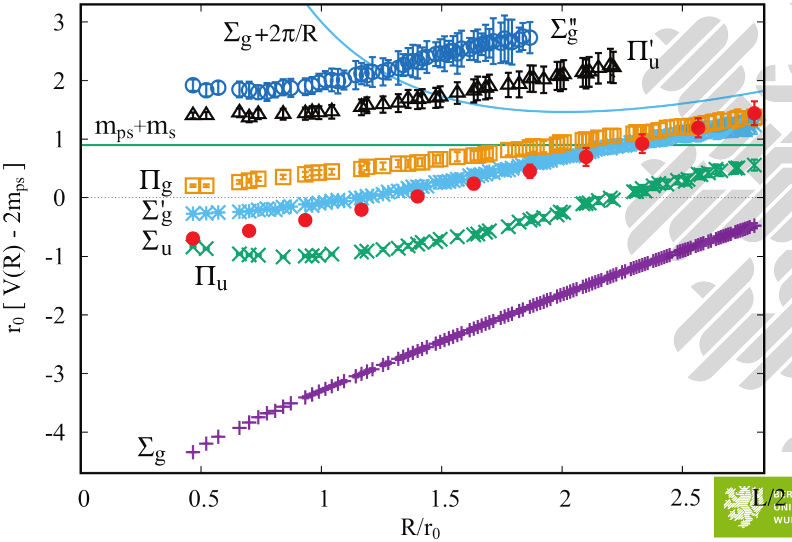
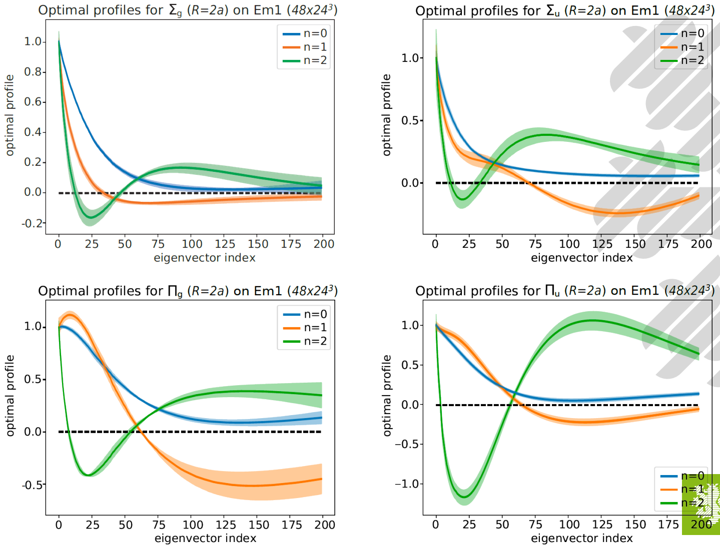
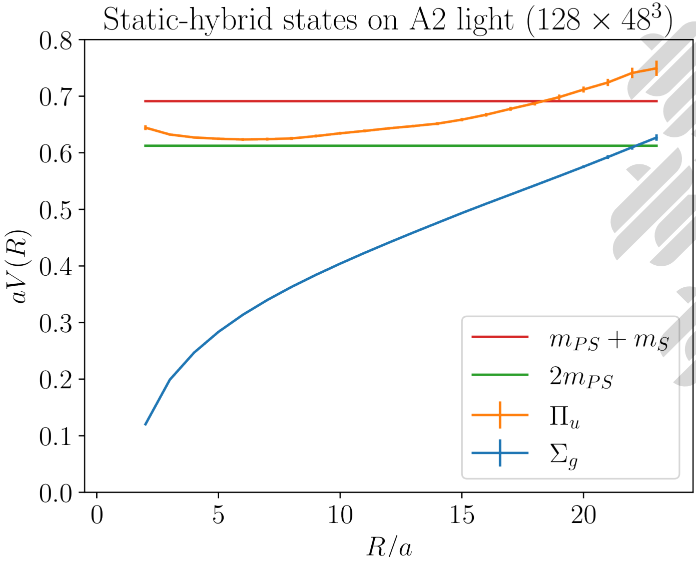
Laplacian static-hybrid states



Optimal trial state profiles



Preliminary!!! $N_f = 3 + 1$, $m_\pi = 420$ MeV



Born-Oppenheimer Approximation

- solve the radial Schrödinger equation: $E_{\Lambda_\eta^\epsilon;L,n}\Psi_{\Lambda_\eta^\epsilon;L,n}(r) =$

$$\left(-\frac{1}{2\mu}\frac{d^2}{dr^2} + \frac{L(L+1) - 2\Lambda^2 + J_{\Lambda_\eta^\epsilon}(J_{\Lambda_\eta^\epsilon}+1)}{2\mu r^2} + V_{\Lambda_\eta^\epsilon}(r)\right)\Psi_{\Lambda_\eta^\epsilon;L,n}(r)$$

  $\mu = m_Q m_{\bar{Q}}/(m_Q + m_{\bar{Q}})$, for $m_c = 1628$ MeV or $m_b = 4977$ MeV
- parametrization of potentials $V_{\Lambda_\eta^\epsilon}(r)$ from Capitani et al. (2019)
  $V_{\Sigma_g^+}(r) = V_0 - \alpha/r + \sigma r$
  $V_{\Pi_u}(r) = A_1/r + A_2 + A_3 r^2$
- $E_{\Lambda_\eta^\epsilon;L,n}$ contain the self-energies of the static quarks

$$\Rightarrow m_{\Lambda_\eta^\epsilon;L,n} \quad = \quad E_{\Lambda_\eta^\epsilon;L,n} - E_{\Sigma_g^+;L=0,n=1} + \overline{m}$$

  with $\overline{m}$ the spin averaged mass Workman et al. [PDG] (2022)
  $\overline{m}_c = (m_{\eta_c(1S),\text{exp}} + 3m_{J/\Psi(1S),\text{exp}})/4 = 3069(1)$ MeV
  $\overline{m}_b = (m_{\eta_b(1S),\text{exp}} + 3m_{\Upsilon(1S),\text{exp}})/4 = 9445(1)$ MeV

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Born-Oppenheimer Approximation

$L = 0,\ m_Q = m_c = 1628$ MeV:

Braaten et al. (2014)

state	n=1	n=2
Σ_g^+	$\bar{m}_c$	4347(24)
$m_{J/\Psi}(nS)$	3096.900(6)	3674(1)
Π_u	4490(23)	

$L = 1,\ m_Q = m_b = 4977$ MeV:

state	n=1	n=2	n=3	n=4
Σ_g^+	$\bar{m}_b$	10401(18)	11032(32)	11741(42)
$\Upsilon(nS)$	9460.4(1)	10023.4(5)	10355.1(5)	10579.4(1.2)
Π_u	11023(22	11563(35))		

$m_{ps}^* - m_{ps} = 932(20) \Leftrightarrow B_c^{\pm}(2S) - B_c^{+}(1S) = 2397(1)$ MeV

Static-light meson operators

$$C^{sl}_{\mathcal{O}}(t) = \frac{1}{N_f} \sum_{\vec{x},t_0,i} \left\langle \bar{Q}(\vec{x},t_0+t)\mathcal{O}q^i(\vec{x},t_0+t)\bar{q}^i(\vec{x},t_0)\gamma_4\mathcal{O}Q(\vec{x},t_0) \right\rangle$$

$$= -\sum_{\vec{x},t_0} \left\langle \mathrm{tr}_{\,c,d}\left(\mathcal{O}\,\underbrace{\mathcal{D}(\vec{x},t_0+t;\vec{x},t_0)\gamma_4}_{\text{light propagator}}\,\mathcal{O}\,\underbrace{U_t(\vec{x};t_0,t_0+t)P_-}_{\text{static propagator}} \right) \right\rangle$$

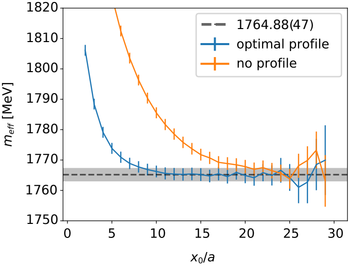
$\mathcal{O}$	$J^{\mathcal{P}}$	$j^{\mathcal{P}}$	$\mathrm{O_h}$	not.
$\gamma_5\ ,\ \gamma_5\gamma_j\nabla_j$	$0^-\ [1^-]$	$(1/2)^-$	A_1	S
$1\ ,\ \gamma_j\nabla_j$	$0^+\ [1^+]$	$(1/2)^+$		P_-
$\gamma_1\nabla_1 - \gamma_2\nabla_2$ (and cyclic)	$2^+\ [1^+]$	$(3/2)^+$	E	P_+
$\gamma_5(\gamma_1\nabla_1 - \gamma_2\nabla_2)$ (and cyclic)	$2^-\ [1^-]$	$(3/2)^-$		$D_\pm$
$\gamma_1\nabla_2\nabla_3 + \gamma_2\nabla_3\nabla_1 + \gamma_3\nabla_1\nabla_2$	$3^-\ [2^-]$	$(5/2)^-$	A_2	D_+
$\gamma_5(\gamma_1\nabla_2\nabla_3 + \gamma_2\nabla_3\nabla_1 + \gamma_3\nabla_1\nabla_2)$	$3^+\ [2^+]$	$(5/2)^+$		$F_\pm$

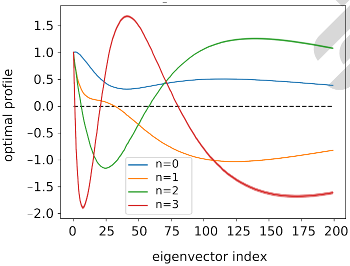
$$C^{sl}_{S/P_-}(t) = -\sum_{\vec{x},t_0} \left\langle \mathrm{tr}_{\,c,d}\left(\mathcal{D}(\vec{x},t_0+t;\vec{x},t_0)\gamma_4 P_\pm U_t(\vec{x};t_0,t_0+t) \right) \right\rangle$$

Static-light meson operators

$$C^{sl}_{S/P_-}(t) = -\sum_{t_0,i,j}\Big\langle \rho(\lambda_i)\rho(\lambda_j)\mathrm{Tr}_d\{[v_i^\dagger D^{-1}\gamma_4 v_j](t_0+t,t_0)P_\pm\} \sum_{\vec{x}} v_j^\dagger(\vec{x},t_0)U_t(\vec{x};t_0,t_0+t)v_i(\vec{x},t_0+t)\Big\rangle$$

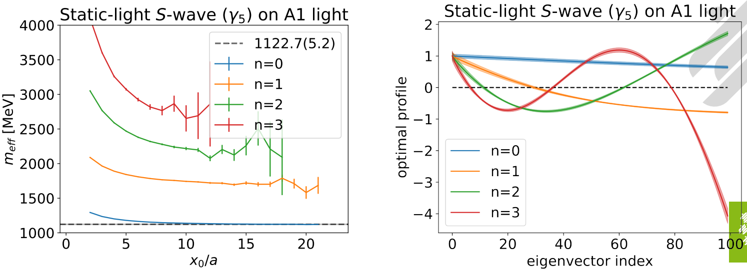
- light perambulators $v_i^\dagger(t_1)D^{-1}_{\alpha\beta}\gamma_4 v_j(t_0)$ from distillation framework Peardon et al. (2009), Knechtli et al (2022)
- static perambulators $v^\dagger(\vec{x},t_0)U_t(\vec{x};t_0,t_0+t)v(\vec{x},t_0+t)$
- Gaussian profiles ρ, SVD, GEVP $\Rightarrow$ optimal profiles...





Static-light meson states

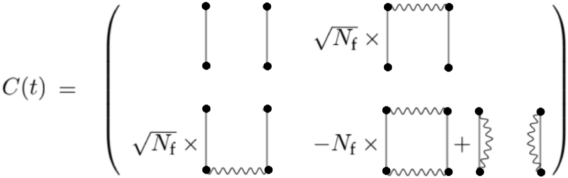
ensemble	$N_s^3 \times N_t$	N_f	m_π	m_s (P_-) - m_{ps} (S) [MeV]
Em1	$24^3 \times 48$	2	2.2 GeV	2394 - 1765 = 629(2)
A0 heavy	$24^3 \times 72$	3+1	800 MeV	1432 - 1062 = 370(5)
A1 heavy	$32^3 \times 96$	3+1	800 MeV	1427 - 1059 = 368(3)
A1 light	$32^3 \times 96$	3+1	420 MeV	1412 - 1123 = 289(5)



String breaking and tetra-quark operators

- combine static and light(charm)-quark perambulators
- building blocks for observation of string breaking

$\vert = v^\dagger(0) U_t v(t) \to \mathcal{P}$, $\bullet\!\sim\!\sim\!\bullet = v^\dagger(t) D^{-1}_{\alpha\beta} \gamma_4 v(0) \to \mathcal{D}$



$$
\begin{aligned}
C_{11}(t) &\to \mathcal{P}(\vec{x})\mathcal{P}^\dagger(\vec{y}) \qquad \hat{r} = |\vec{y} - \vec{x}|,\, P_\pm = (1 \pm \gamma_4)/2 \\
C_{12}(t) &\to \sqrt{N_f}\,\mathrm{Tr}_{c,d}\mathcal{P}(\vec{x})P_-\gamma\hat{r}\mathcal{D}(\vec{x},\vec{y},t)\mathcal{P}^\dagger(\vec{y}) \\
C_{21}(t) &\to -\sqrt{N_f}\,\mathrm{Tr}_{c,d}\mathcal{P}(\vec{x})\mathcal{P}^\dagger(\vec{y})P_+\gamma\hat{r}\mathcal{D}^\dagger(\vec{x},\vec{y},0) \\
C_{22}(t) &\to N_f\,\mathrm{Tr}_{c,d}\mathcal{P}(\vec{x})P_+\mathcal{D}(\vec{x},\vec{y},t)\mathcal{P}^\dagger(\vec{y})P_-\mathcal{D}^\dagger(\vec{x},\vec{y},0) \\
&\quad -\delta_{ij}\,\mathrm{Tr}_{c,d}[\mathcal{P}(\vec{x})P_+\mathcal{D}_i^\dagger(\vec{x},0,t)]\,\mathrm{Tr}_{c,d}[\mathcal{P}^\dagger(\vec{y})P_-\mathcal{D}_j(\vec{y},0,t)
\end{aligned}
$$

Conclusions & Outlook

- ✓ alternative operator for a static quark-anti-quark potentials based on Laplacian eigenmodes, replacing Wilson loop
- ✓ improved Laplace trial states (optimal profiles) give earlier effective mass plateaus and better signal
- ✓ computational advantage for high resolution of the potential energy as off-axis distances basically come ”for free”
- ✓ hybrid static potentials (hybrid meson masses), instead of ”gluonic handles”(excitations) use derivatives of v
- ✓ implementation of static-light (charm) correlator using ”perambulators”$v(t_1)D^{-1}v(t_2)$ from distillation framework
- 🔧 putting together building blocks for string breaking in QCD (mixing matrix of static and light quark propagators)
- 🔧 more (hybrid) static(-light) and multi-quark potentials

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Acknowledgements

THANK YOU

GCS Gauss Centre for Supercomputing

lrz Leibniz-Rechenzentrum der Bayerischen Akademie der Wissenschaften

NRW-FAIR

WWU Münster

Ruhr Universität Bochum RUB

Jülich Forschungszentrum

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