The speed of sound peak of isospin-asymmetric QCD

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Excited QCD 2024 Workshop

Benasque
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FAPERGS

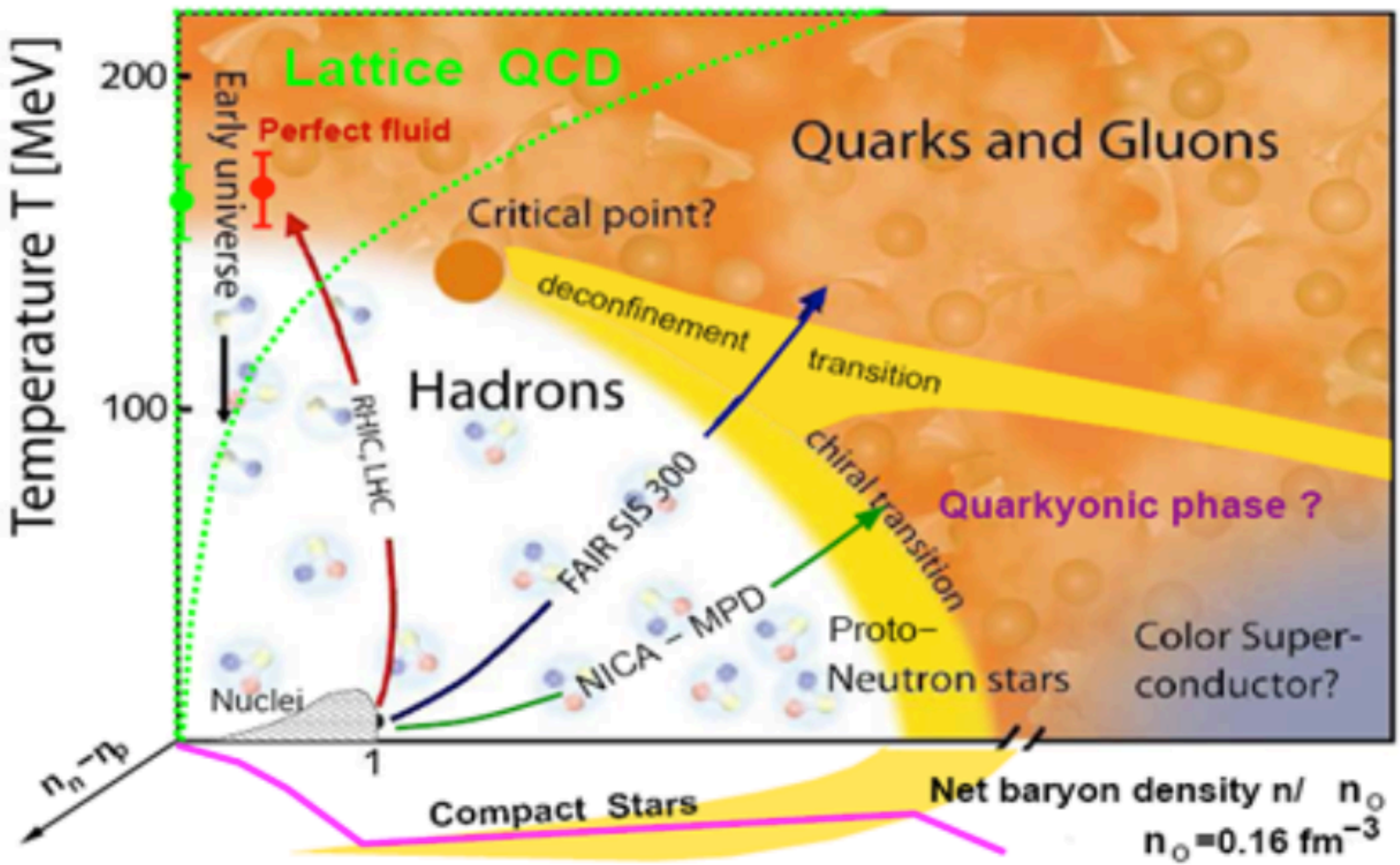
Outline

- Motivation
- i) How a isospin asymmetry can influence the phase diagram of QCD?
- ii) Effective models of QCD under extreme conditions X LQCD.
- iii) Role of the medium dependent coupling constants.
- Conclusions and perspectives

In Collaboration with:

- Alejandro Ayala - UNAM - Mexico
- Gastão Krein - IFT - Brazil
- Norberto Scoccola - CONICET - Argentina
- Rudnei O. Ramos, UERJ - Brazil
- Sidney Avancini, UFSC - Brazil
- Veronica Dexheimer - KSU - USA
- Aritra Bandyopadhyay - Heidelberg University - Germany
- William R. Tavares - UERJ - Brazil
- Dyana C. Duarte, UFSM - Brazil
- Marcus B. Pinto, UFSC - Brazil
- Our PostDocs and students…

QCD Phase Diagram



NICA - Nuclotron-based
Ion Collider fAcility

FAIR - Facility for Antiproton and
Ion Research

Lattice Results $\mu \neq 0$: Sign Problem

- fermion determinant is complex

$$[\det\, M(\mu)]^* = \det\, M(-\mu^*) \in \mathbb{C}$$

- no positive weight in path integral

$$Z = \int \mathcal{D}U\, e^{-S_{YM}}\, \det\, M(\mu)$$

- standard lattice methods base on importance sampling cannot be used!

To make progress

QCD is nonperturbative at relevant scales!

- We use Quantum Field Theory (in medium)
- DSE (beyond RL truncation…)
- Lattice (limitations…)
- Large N
- Effective models (just a few degrees of freedom): NJL/PNJL, Linear σ model, MIT, Chiral perturbation theory…

QCD under extreme conditions

- Finite temperature T
- Finite magnetic field eB
- Chiral chemical potential μ_5
- Finite isospin chemical potential μ_I

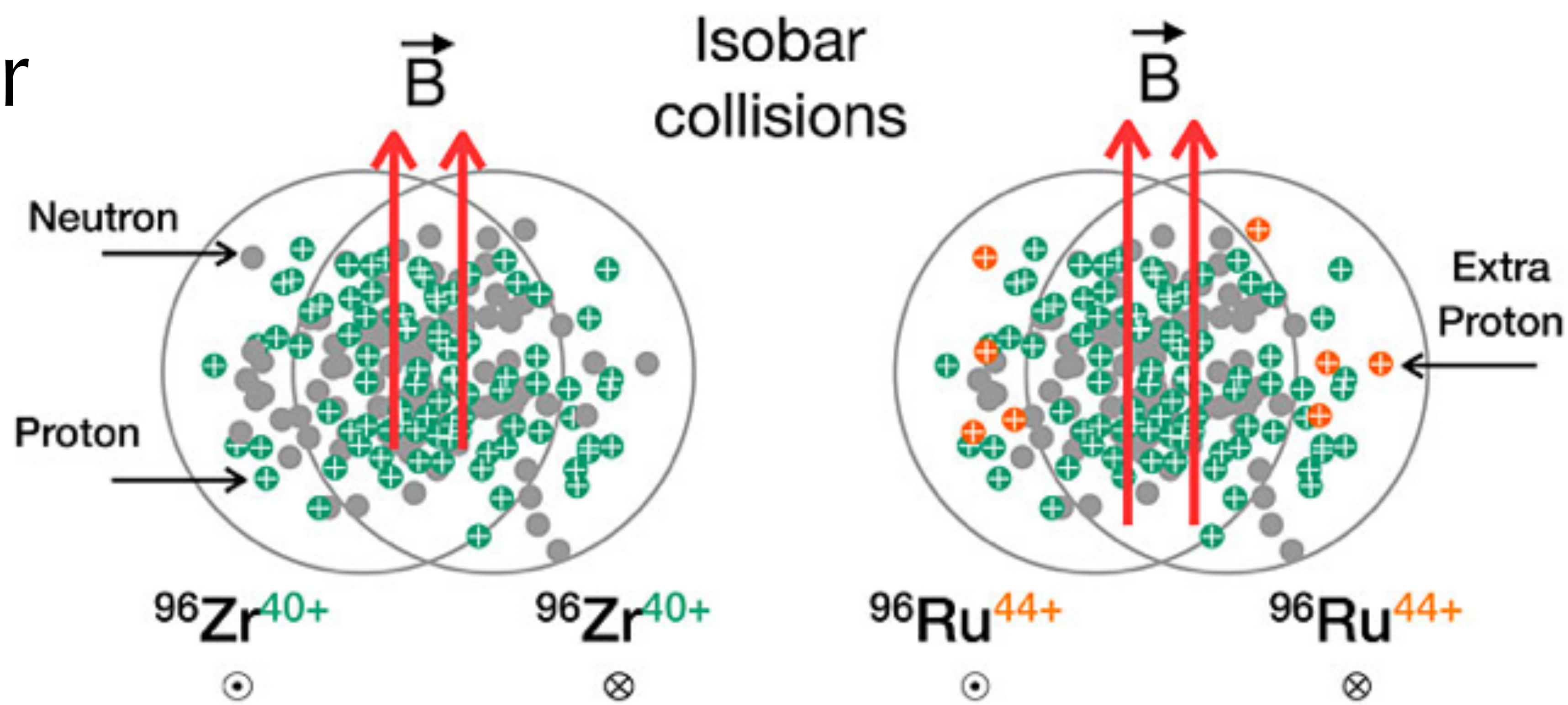
NO
Sign Problem!

LQCD can be used as a benchmark platform for comparing different effective models used in the literature.

Motivation

Why relevant?

- In RHIC isobar program



- Excess of neutrons over protons
- Excess of π^- over π^+
- Neutron star interiors

$$n_I < 0$$

$$n_I = n_u - n_d$$

Pion Condensation

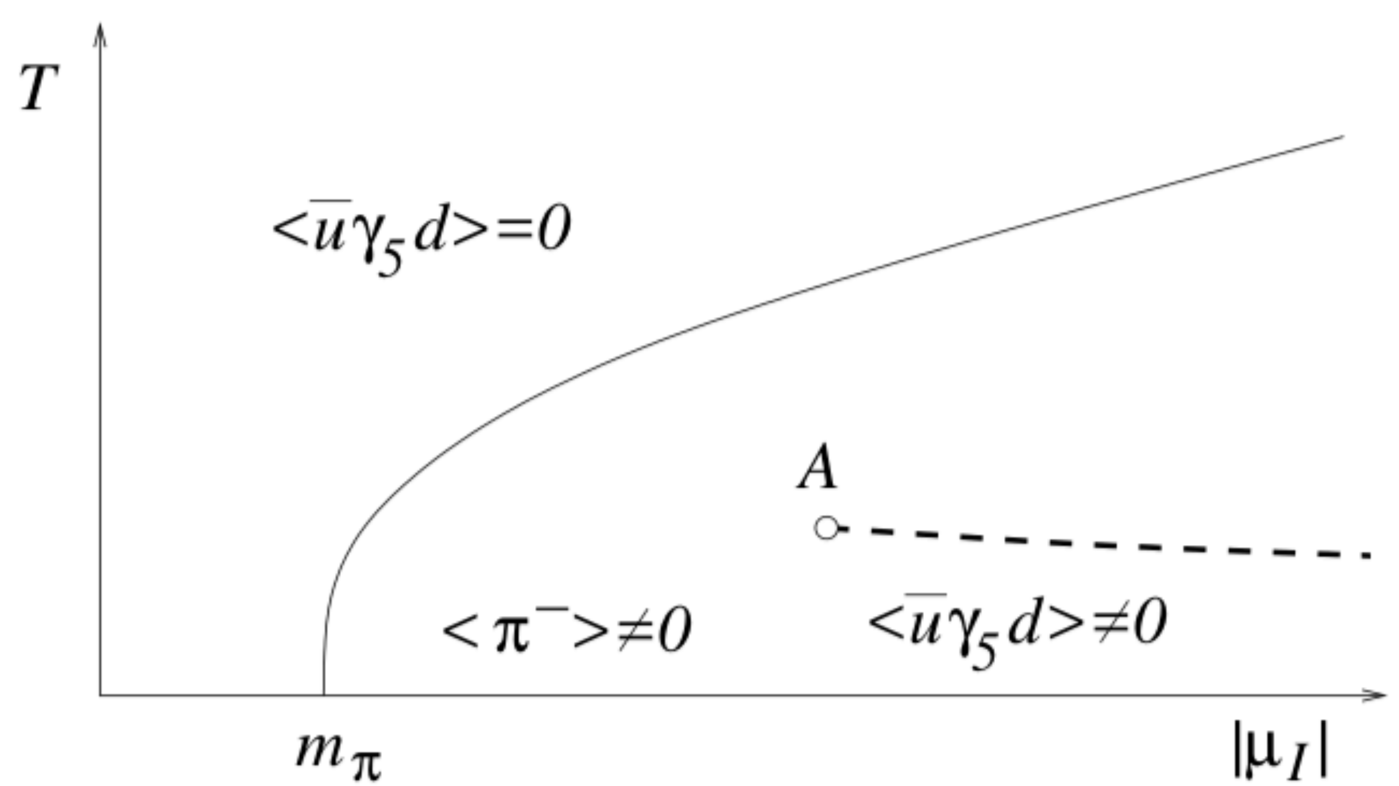
- Low energy QCD: degrees of freedom → pions

Chpt

- Chemical potential for charged pions μ_π

- At T=0 $\mu_\pi \leq m_\pi$ → **Vacuum**

  $\mu_\pi \geq m_\pi$ → **BEC**



Son and Stephanov, PRL 86, 592 (2001)

Isospin chemical potential

- Grand canonical ensemble

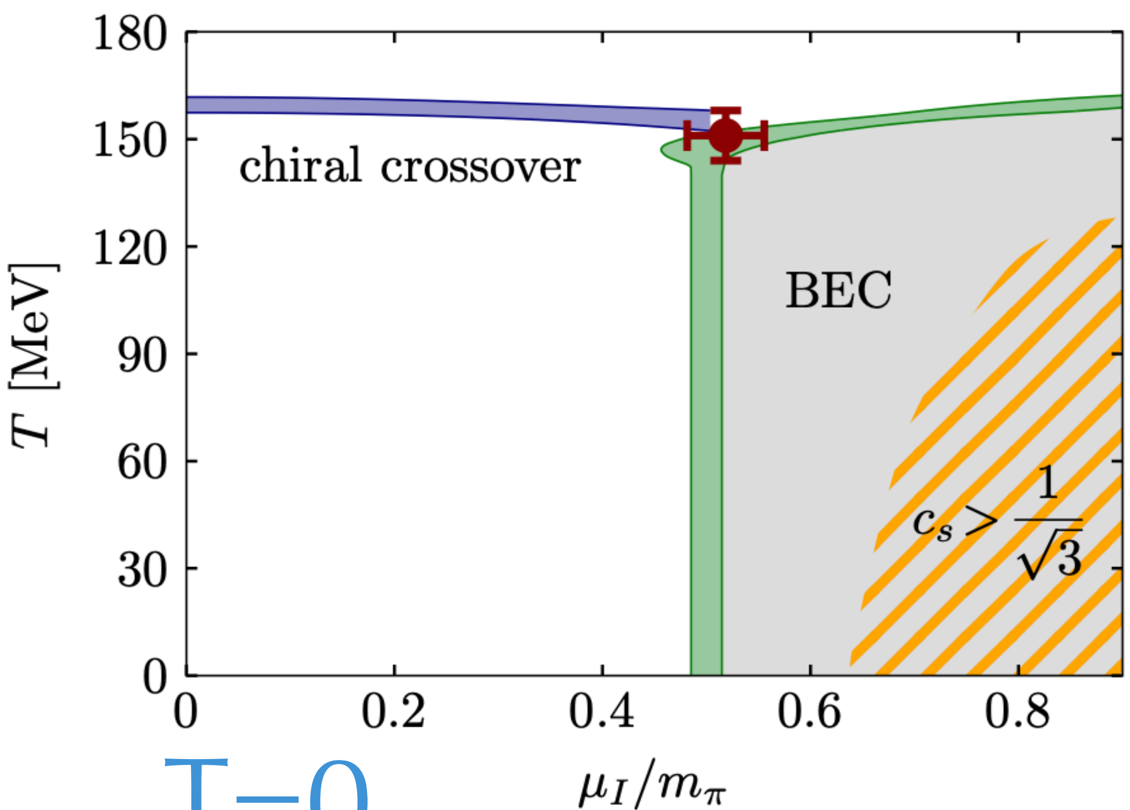
- Chemical potentials
$$\mu_u = \frac{\mu_B}{3} + \mu_I$$
$$\mu_d = \frac{\mu_B}{3} - \mu_I$$

- Zero baryon number:
$$\begin{aligned}\mu_u &= \mu_I \\ \mu_d &= -\mu_I\end{aligned}$$

- Pion chemical potential: $\mu_\pi = \mu_u - \mu_d = 2\mu_I$

$$n_I = n_u - n_d$$

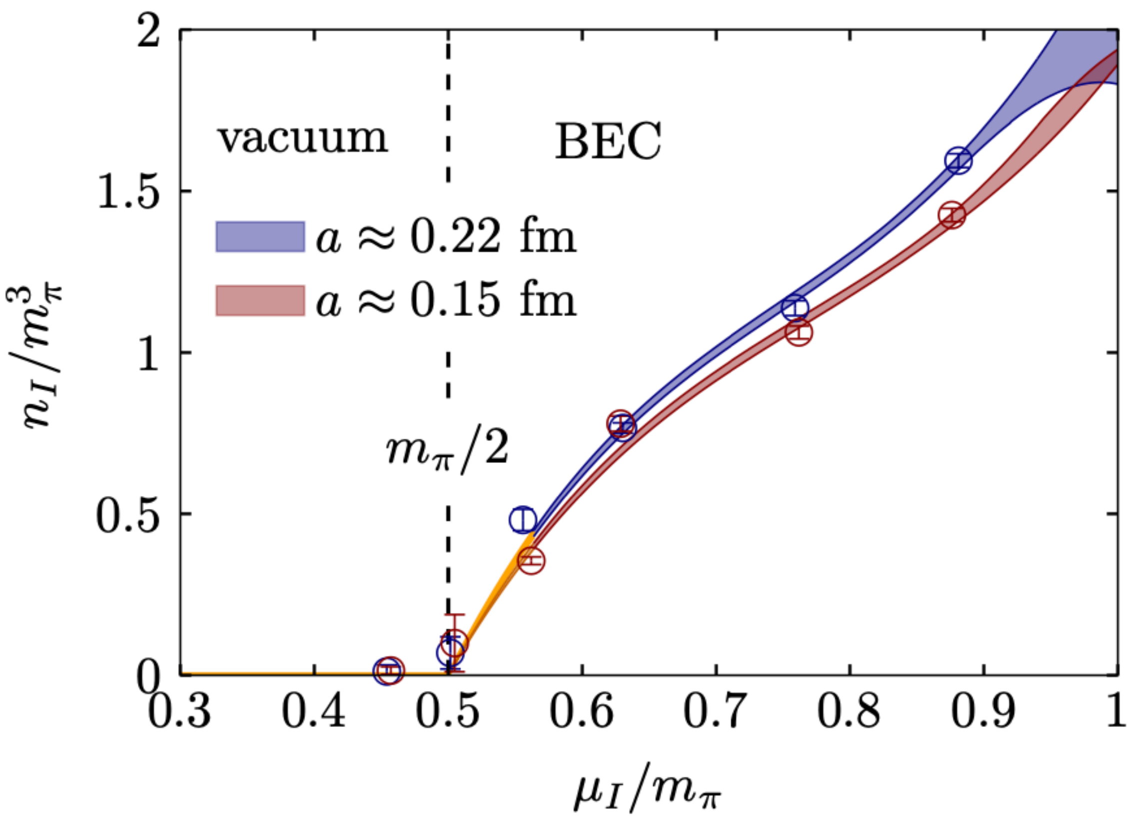
Recent LQCD results



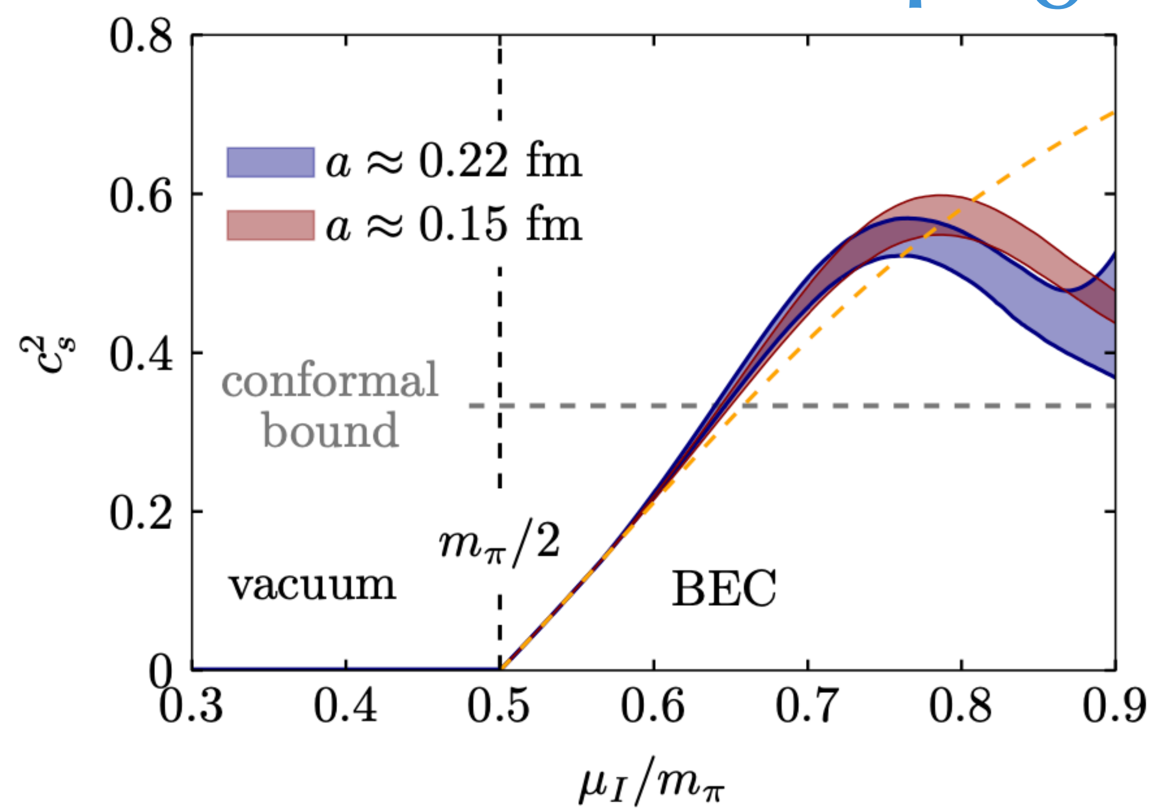
$$\epsilon = -P + \mu_I n_I$$

$$c_s^2 = \frac{\partial P}{\partial \epsilon}$$

T=0

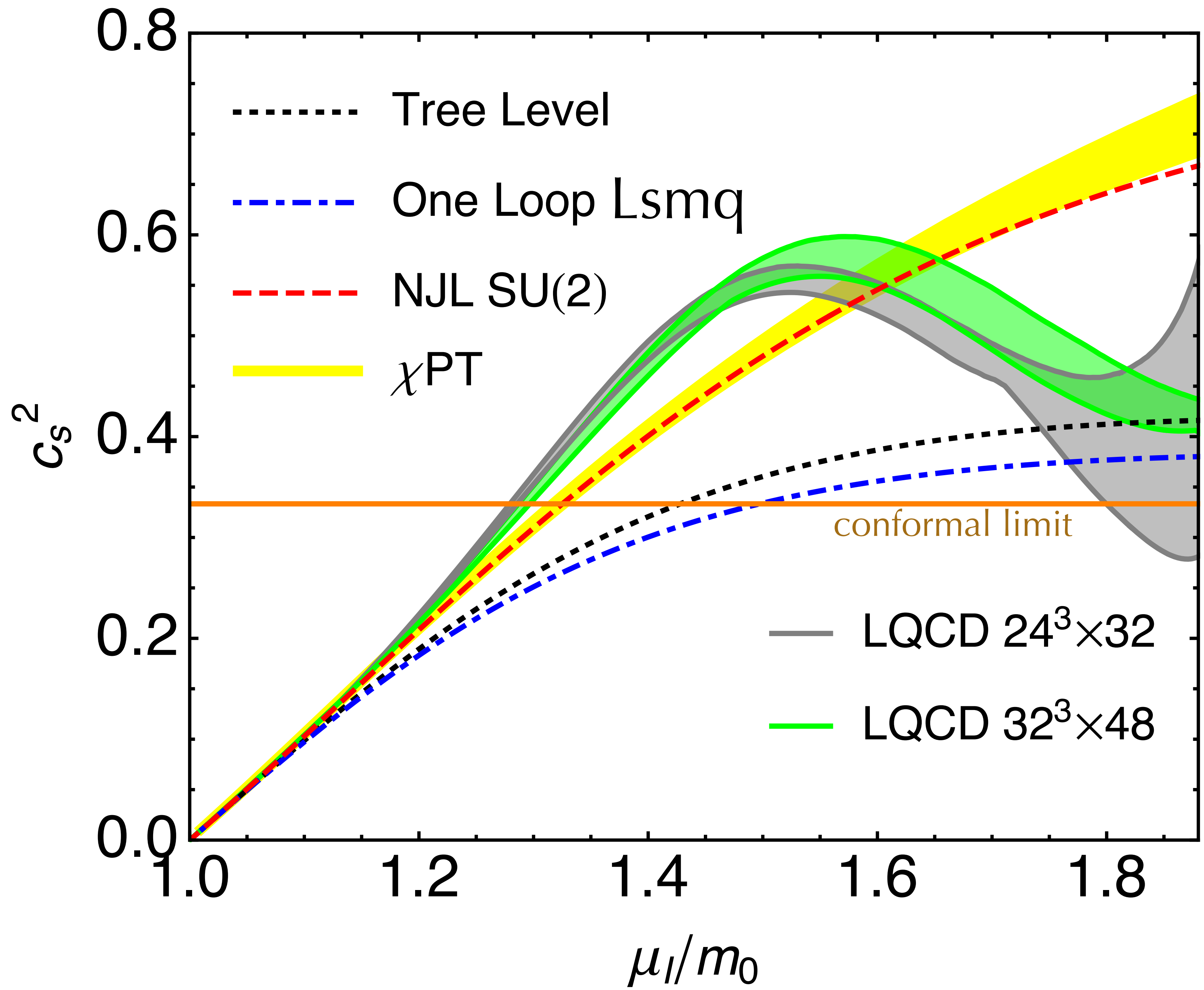


T=0



B.B. Brandt, F. Cuteri, G. Endrodi, JHEP 07 (2023) 055

Motivation



R.L.S.Farias, A. Ayala, A. Bandyopadhyay, L.A. Hernandez and J.L. Hernandez, *Phys.Rev.D* 107 (2023) 7, 074027

B.B. Brandt, F. Cuteri, G. Endrodi, JHEP 07 (2023) 055

SU(2) NJL model

$$\mathcal{L}_{NJL} = \bar{\psi}\left(i\partial_\mu\gamma^\mu - m\right)\psi + G\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\tau_3\psi\right)^2 + 2\left(\bar{\psi}i\gamma_5\tau_+\psi\right)\left(\bar{\psi}i\gamma_5\tau_-\psi\right)\right]$$

τ's are the generator matrices for the pseudoscalar interactions, which corresponds to the pionic excitations π_1, π_2, π_3 or equivalently π_+, π_-, π_3, with $\tau_\pm = (\tau_1 \pm \tau_2)/\sqrt{2}$.

Now we can introduce the chiral condensate $\sigma = -2G\langle\bar{\psi}\psi\rangle$ and pion condensates

$$\begin{aligned}
\sqrt{2}\pi_+ &= -2\sqrt{2}G\langle\bar{\psi}i\gamma_5\tau_+\psi\rangle = \Delta e^{i\theta}, \\
\sqrt{2}\pi_- &= -2\sqrt{2}G\langle\bar{\psi}i\gamma_5\tau_-\psi\rangle = \Delta e^{-i\theta},
\end{aligned}$$

where the phase factor θ indicates the direction of the $U_I(1)$ symmetry breaking.

SU(2) NJL model

In the mean-field approximation, the thermodynamic potential is given by

$$\Omega_{\mathrm{NJL}} = \frac{\sigma^2 + \Delta^2}{4G} - 2N_c \int_\Lambda \frac{d^3k}{(2\pi)^3} \left(E_k^+ + E_k^-\right)$$

where $E_k^\pm = \sqrt{\left(E_k \pm \frac{\mu_I}{2}\right)^2 + \Delta^2}$, $E_k = \sqrt{k^2 + M^2}$

From these equations we obtain

$$\sigma = 4GN_cM\ I_\sigma$$
$$\Delta = 4GN_c\Delta\ I_\Delta$$

with the definitions

$$I_\sigma = \sum_{s=\pm1} \int_\Lambda \frac{d^3k}{(2\pi)^3} \frac{1}{E_k} \frac{E_k + s\mu_I}{\sqrt{(E_k + s\mu_I)^2 + \Delta^2}}$$
$$I_\Delta = \sum_{s=\pm1} \int_\Lambda \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{(E_k + s\mu_I)^2 + \Delta^2}}$$

TRS scheme

Medium Separation Scheme

$$I_\Delta = \frac{1}{\pi} \sum_{j=\pm 1} \int_{-\infty}^{+\infty} dx \int_\Lambda \frac{d^3k}{(2\pi)^3} \frac{1}{x^2 + (E_k + j\mu_I)^2 + \Delta^2}$$

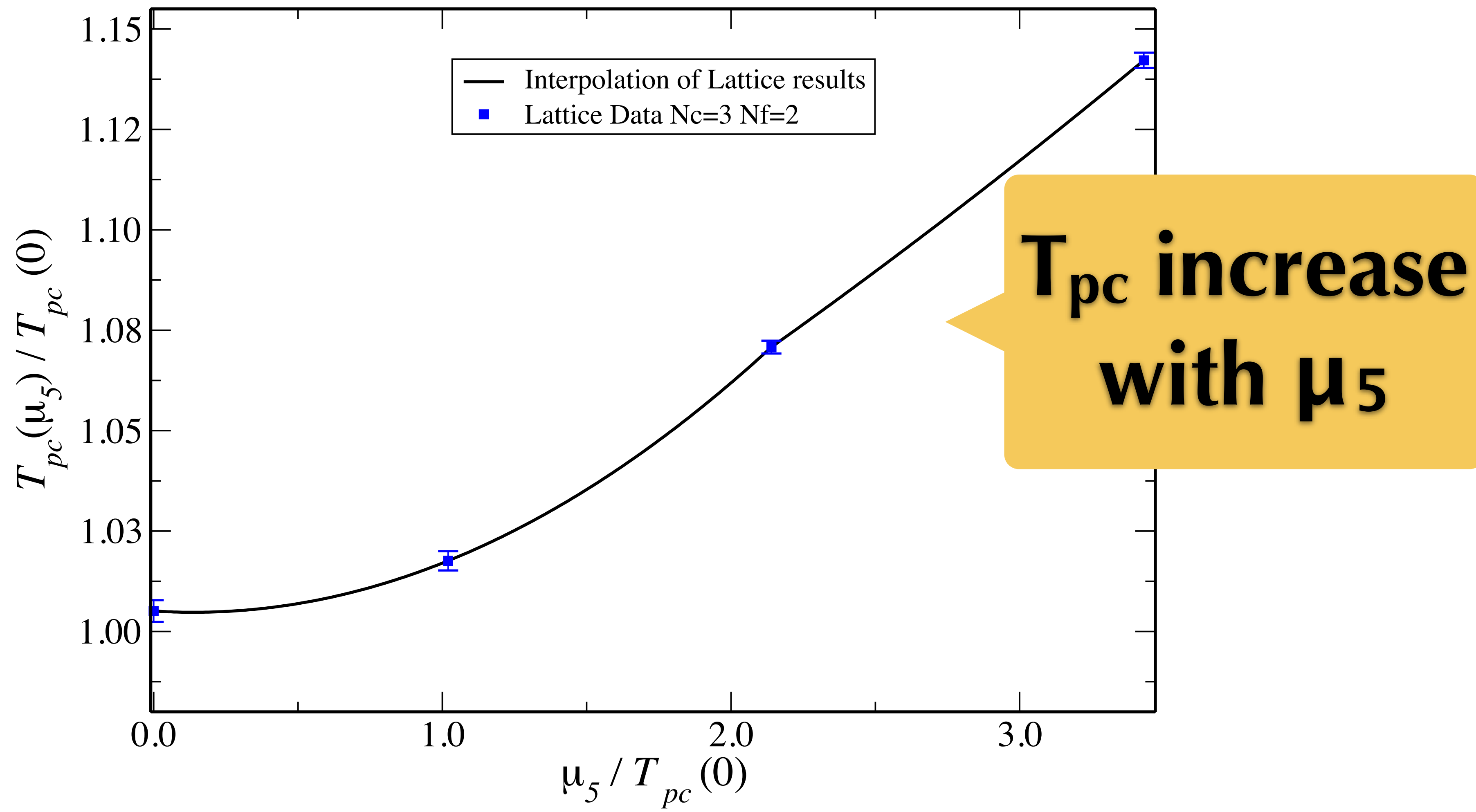
Using the identity

$$\frac{1}{x^2 + (E_k + j\mu_I)^2 + \Delta^2} = \frac{1}{x^2 + k^2 + M_0^2} + \frac{M_0^2 - \Delta^2 - \mu_I^2 - M^2 - 2j\mu_I E_k}{(x^2 + k^2 + M_0^2)[x^2 + (E_k + j\mu_I)^2 + \Delta^2]}$$

M_0 is the vacuum quark mass!

MSS Applications:

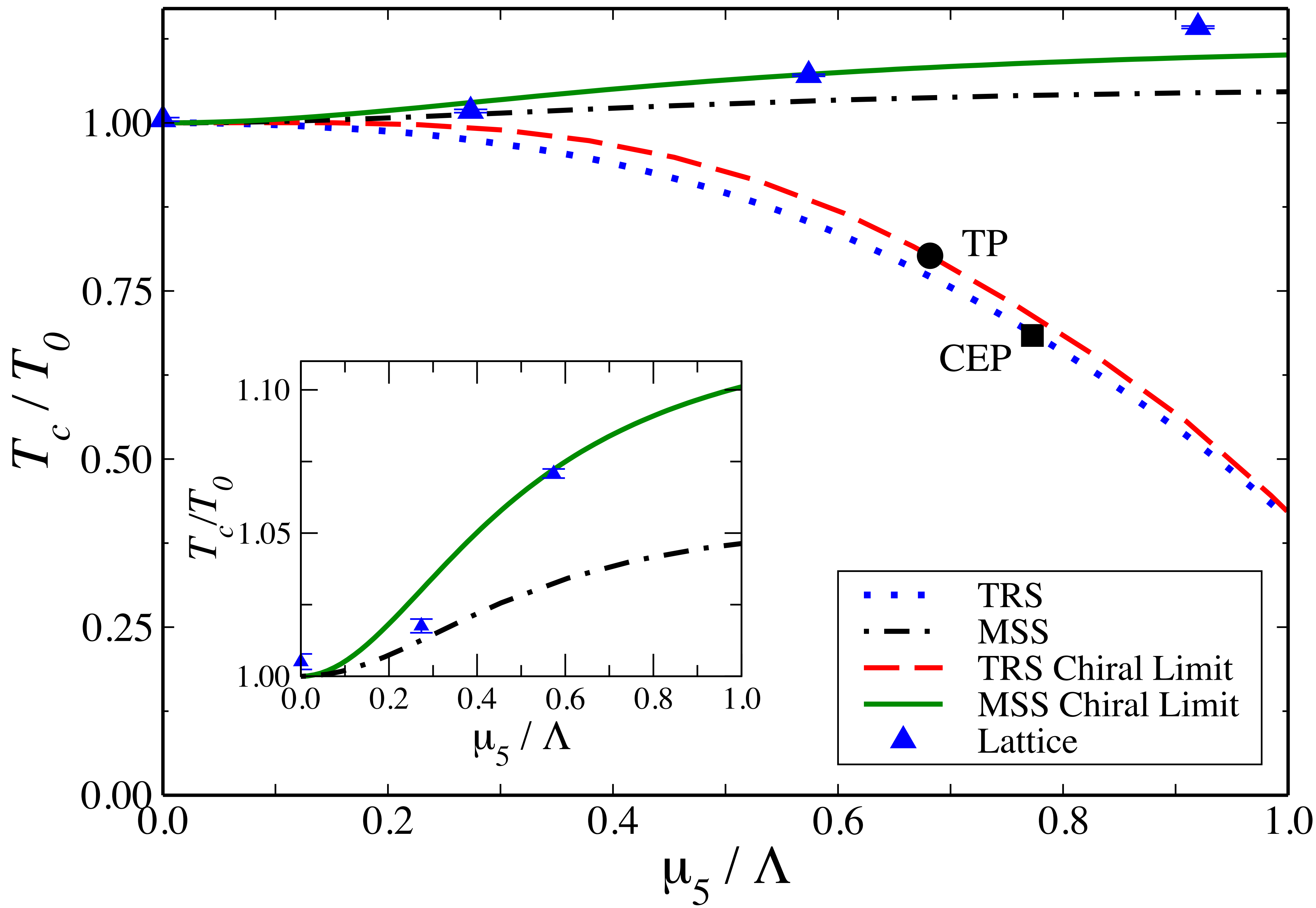
Lattice Results: Nc=3 and Nf=2



- No CP
- Always crossover transition!

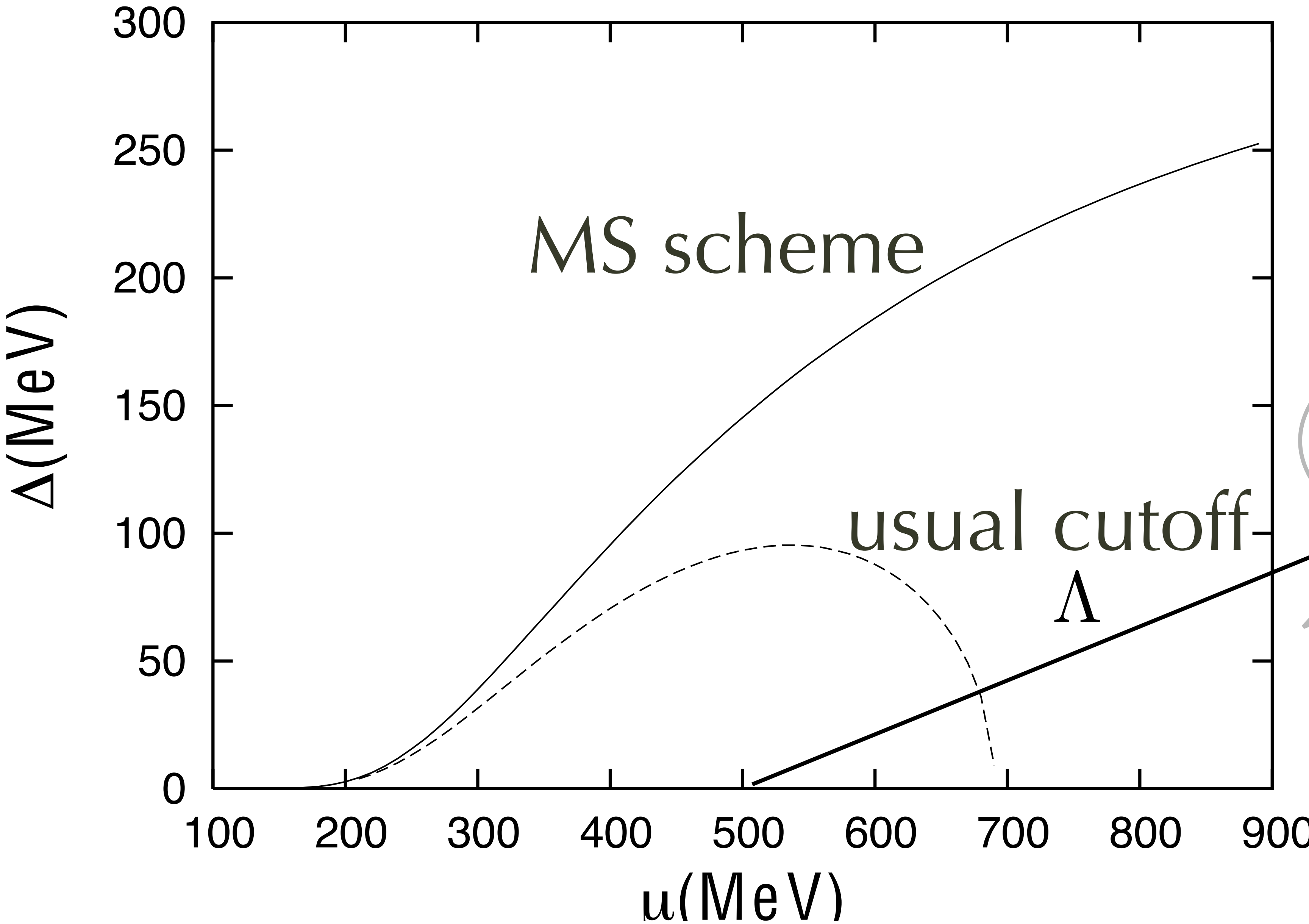
V.V. Braguta, E.M. Ilgenfritz, A. Yu. Kotov, B. Peterson and S.A. Skinderev, Phys. Rev. D, 93, 034509 (2016)

Medium Separation Scheme X TRS



RLSF, D.C. Duarte, G.Krein and R.O.Ramos, PRD 94, 074011 (2016)

SU(2) NJL + Color Superconductivity



RLSF, G. Krein, et al
Phys.Rev. C73
(2006), 018201

neutron star densities!

$$\Delta \sim \frac{\mu}{g^5} \exp\left(-\frac{3\pi^2}{\sqrt{2}g}\right)$$

Superconductivity by long-range color magnetic interaction in high-density quark matter

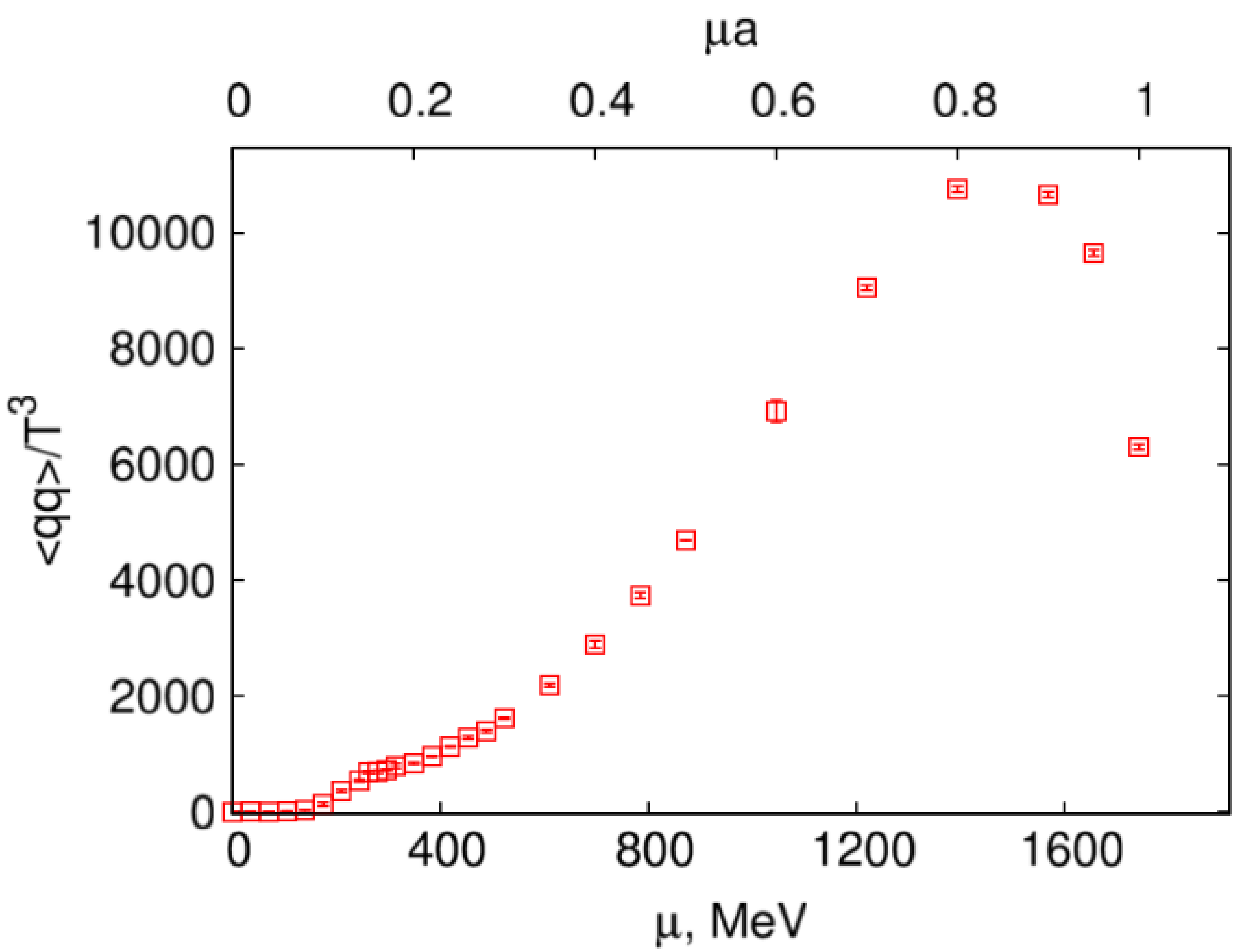
D. T. Son

Phys. Rev. D **59**, 094019 – Published 6 April 1999

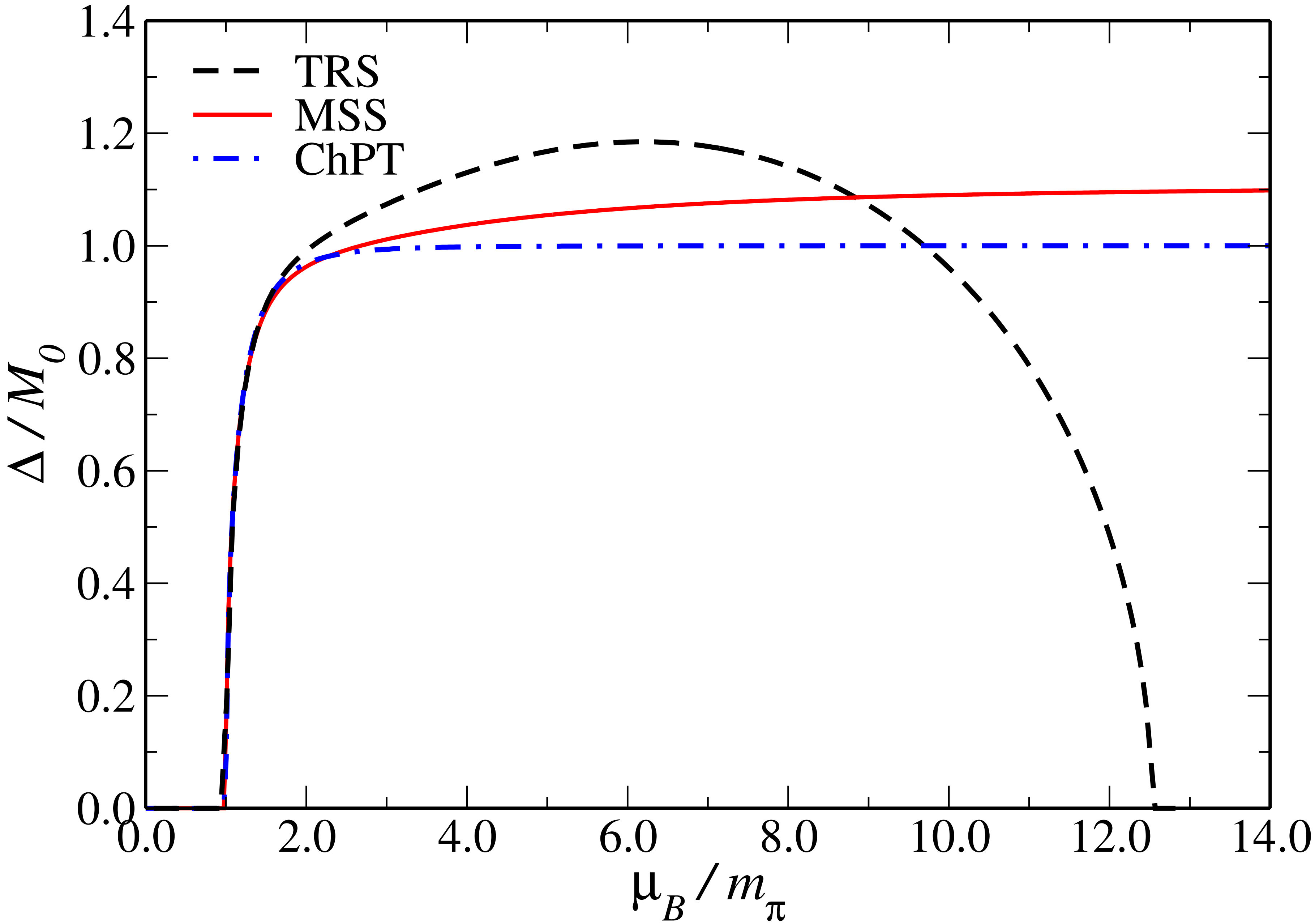
Nc=2 no signal problem!

Study of the phase diagram of dense two-color QCD within lattice simulation

V. V. Braguta, E.-M. Ilgenfritz, A. Yu. Kotov, A. V. Molochkov, and A. A. Nikolaev
Phys. Rev. D **94**, 114510 – Published 15 December 2016



Nc=2 NJL + MSS



RLSF, D.C. Duarte and R.O.Ramos, PRD 99, 016065 (2019)

gapless 2SC

ELSEVIER

Physics Letters B

Volume 564, Issues 3–4, 10 July 2003, Pages 205-211

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Gapless two-flavor color superconductor

Igor Shovkovy [1], Mei Huang [2]

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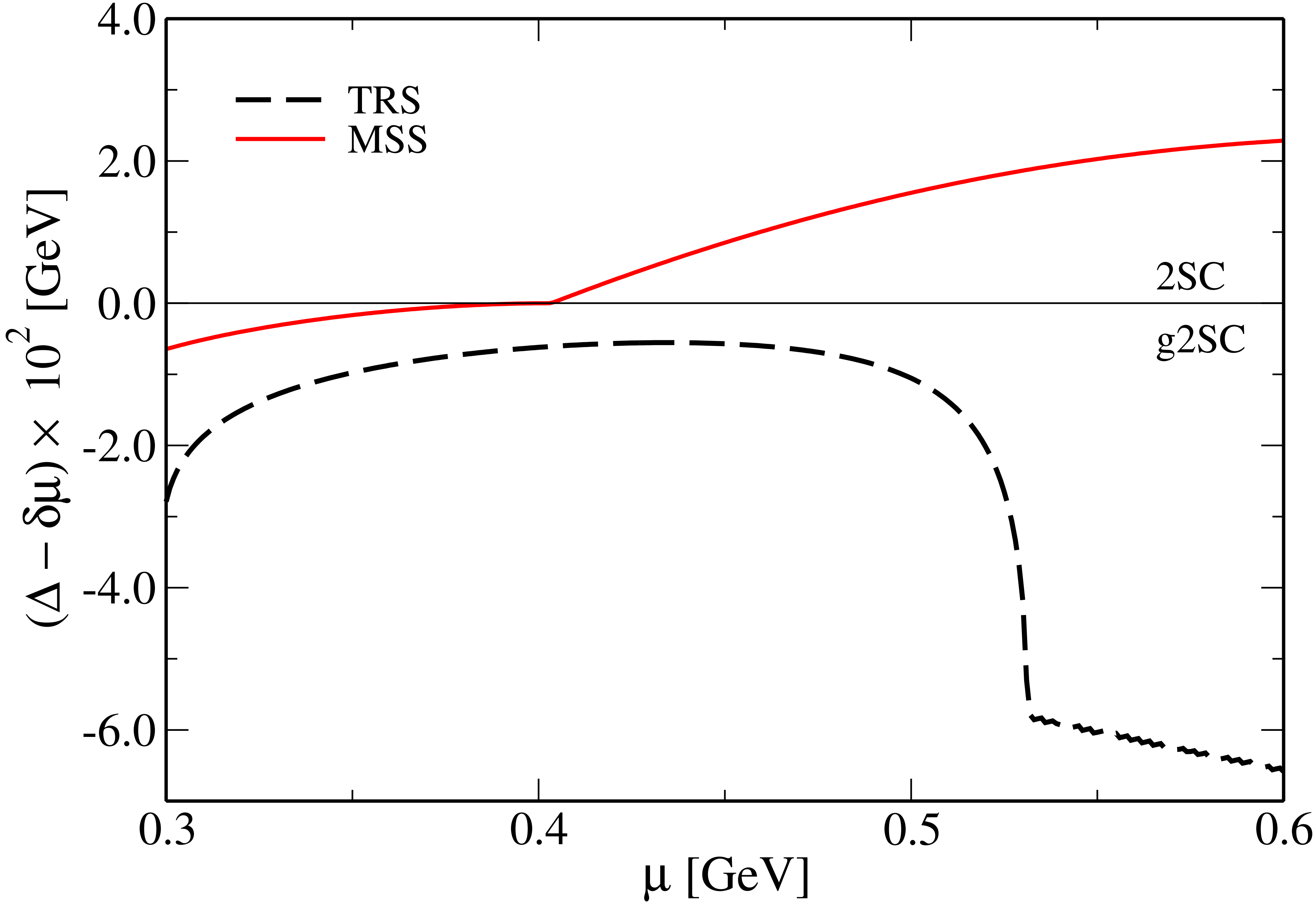
Abstract

A new, stable gapless two-flavor color superconducting phase that appears under conditions of local charge neutrality and β-equilibrium is revealed. In this phase, the symmetry of the ground state is the same as in the conventional two-flavor color superconductor. In the low-energy spectrum of this phase, however, there are only two gapped fermionic quasiparticles, and the other four quasiparticles are gapless. The origin and the basic properties of the gapless two-flavor color superconductor are discussed. This phase is a natural candidate for quark matter in cores of compact stars.

g2SC X 2SC

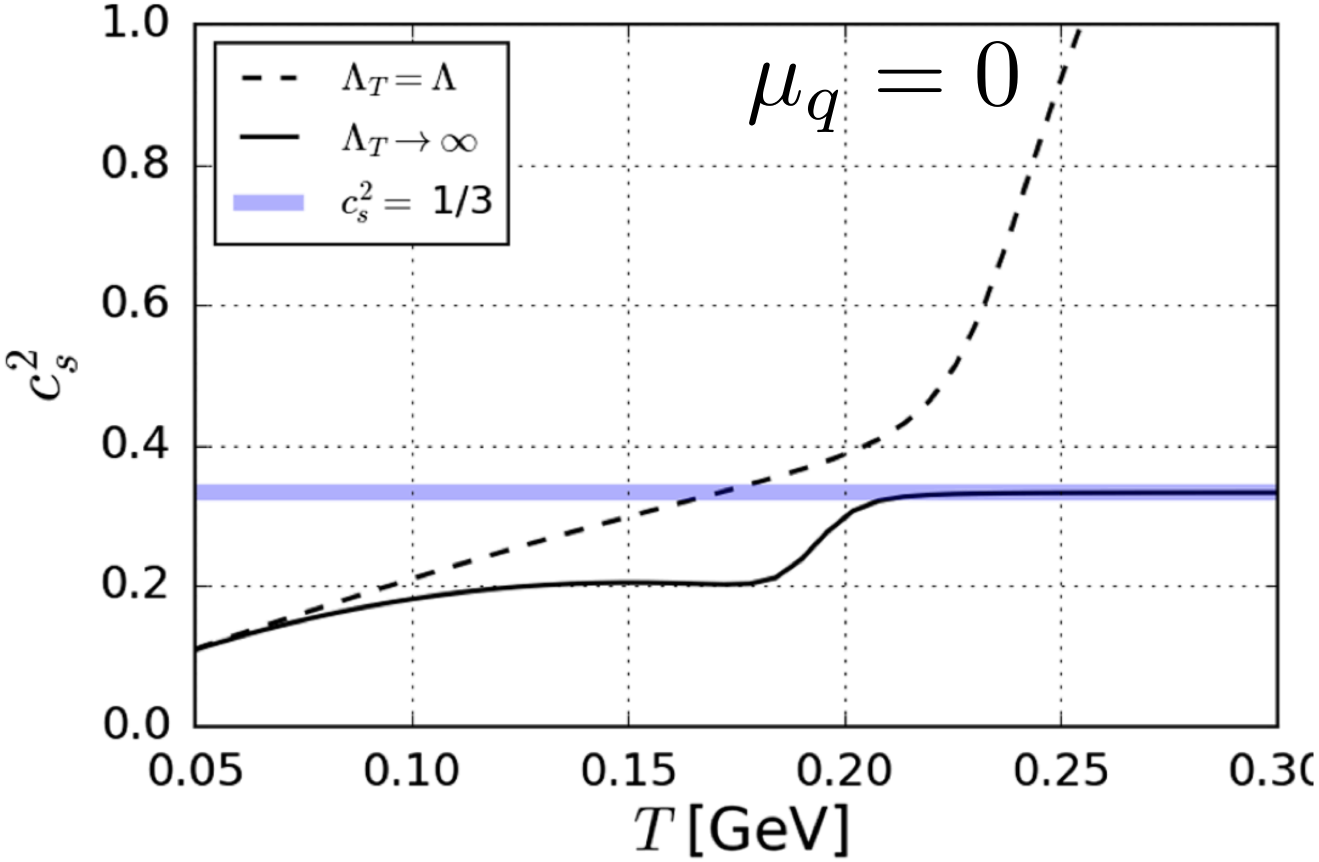


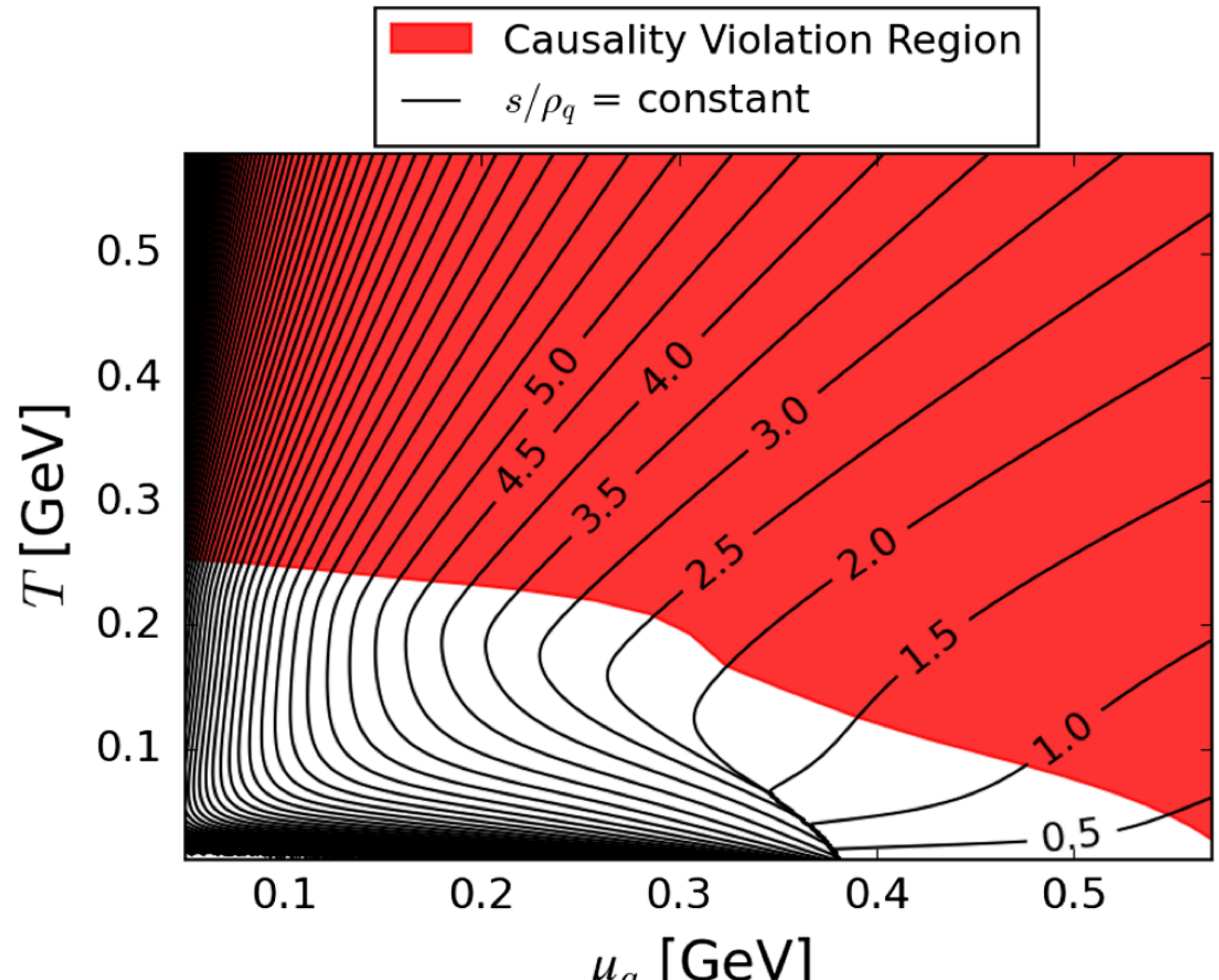
RLSF, D.C. Duarte and R.O.Ramos, PRD 99, 016065 (2019)

Causality violation

$$\Omega(T,\mu_q) = \frac{(M-m_0)}{4G} - 2N_cN_f\int_\Lambda \frac{d^3k}{(2\pi)^3}\,\omega(k)$$
$$-4N_cN_fT\int_{\Lambda_T}\frac{d^3k}{(2\pi)^3}\left[\log\left(1+e^{-(\omega(k)+\mu_q)/T}\right)\right.$$
$$\left.+\log\left(1+e^{-(\omega(k)-\mu_q)/T}\right)\right]$$

where $\omega(k) = (k^2+M^2)^{1/2}$



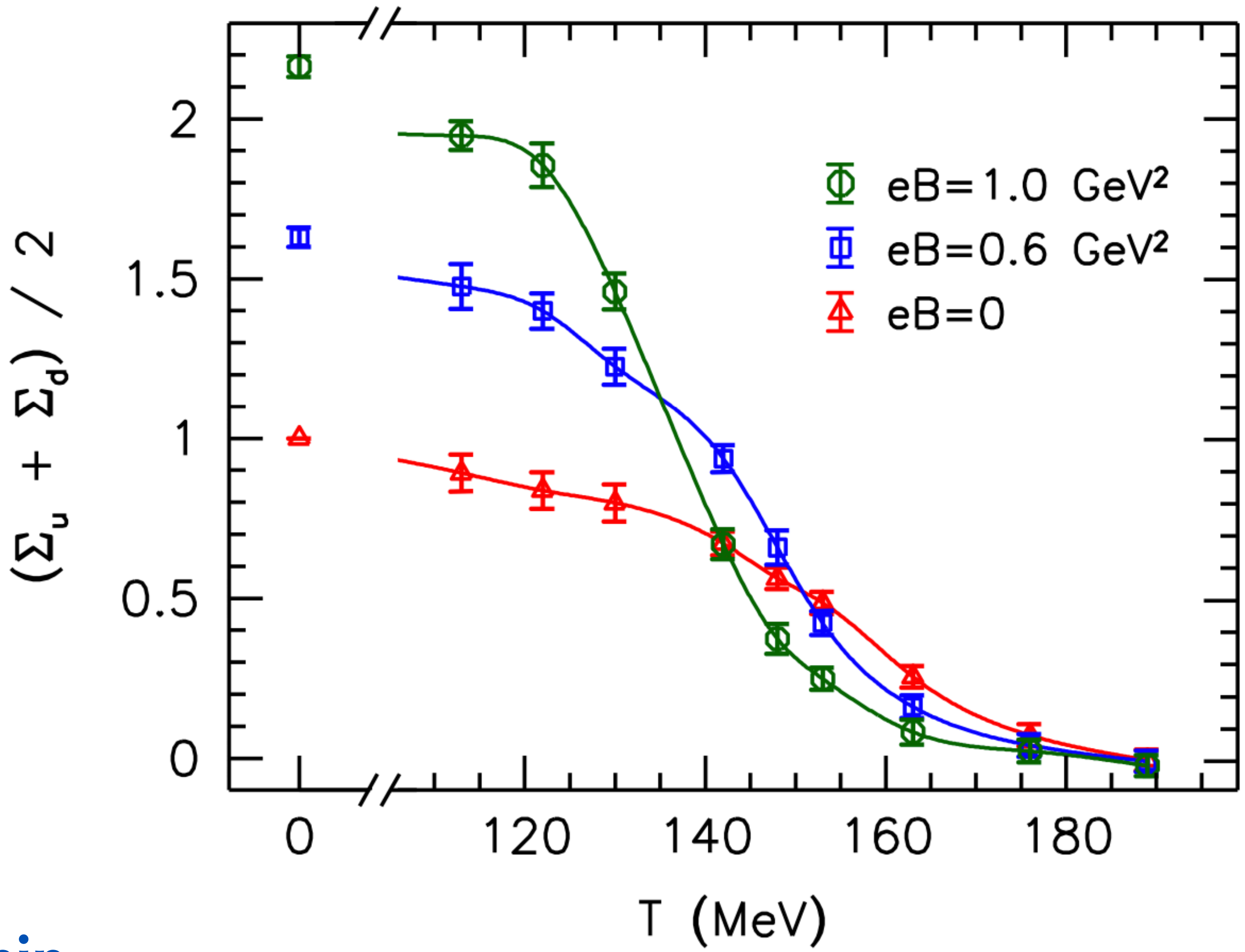


RLSF, A.E.B.Pasqualotto, W.R. Tavares, S. Avancini and G. Krein, Phys. Rev. D 107, 096017 (2023)

Medium dependent coupling constants

B Effects on QCD phase transitions

$$\Lambda_{\rm QCD}^2 \sim (200\,{\rm MeV})^2 \sim 2\times 10^{18}\,{\rm G}$$



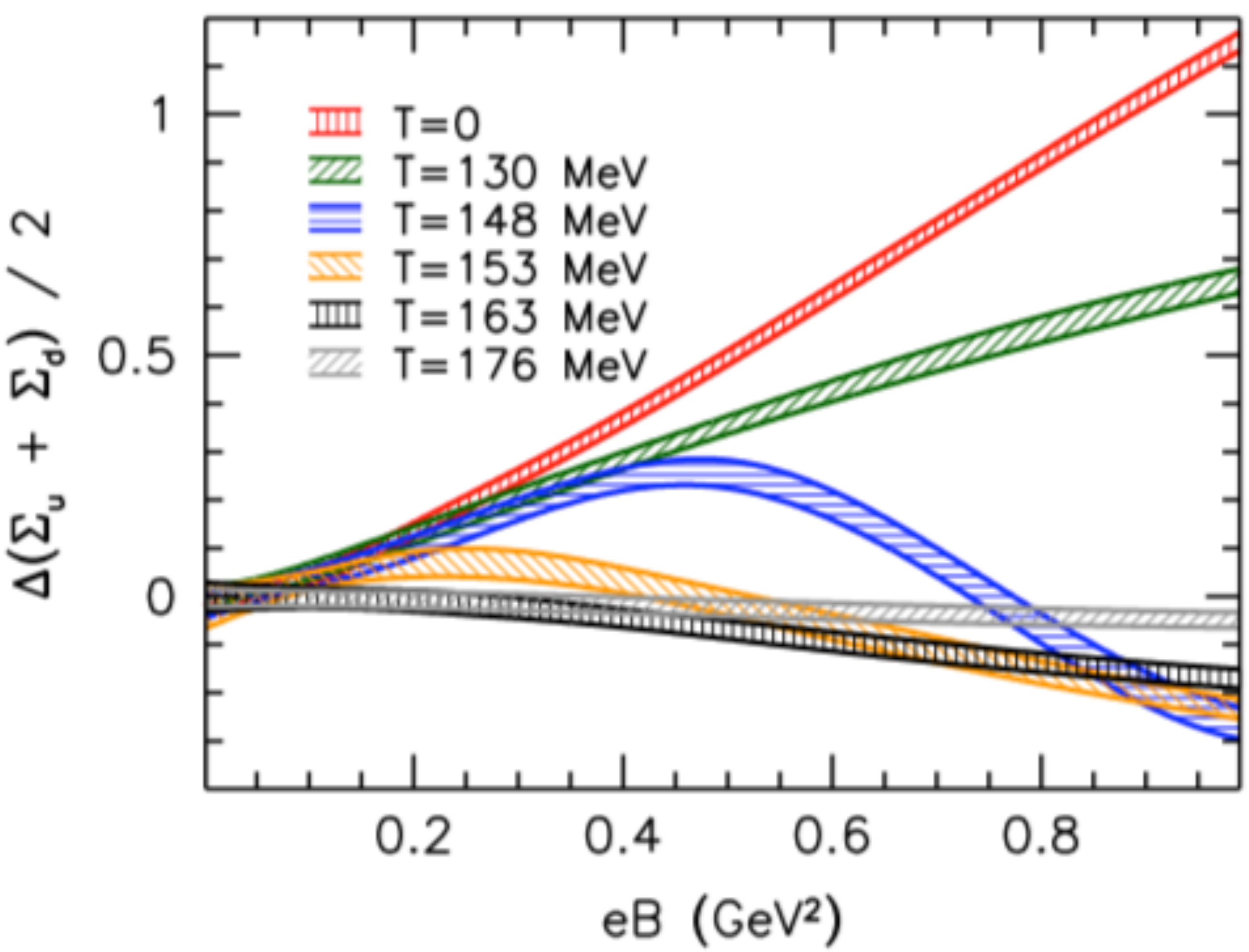
IMC: Bali, Bruckmann, Endrodi, Fodor, Katz et al.
JHEP 02 (2012) 044
Phys.Rev.D 86 (2012) 071502

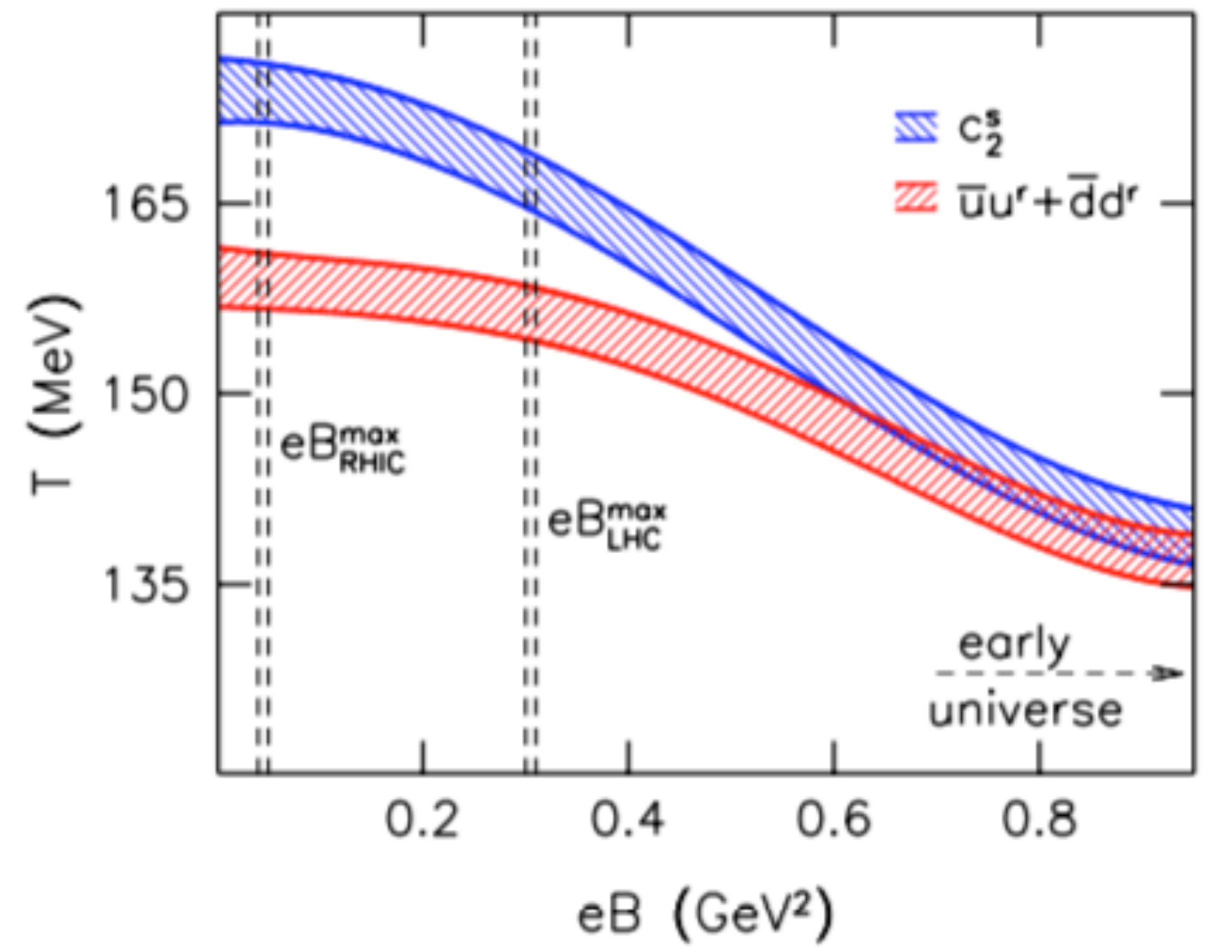
MC: V.P. Gusynin, V.A. Miransky, I.A. Shovkovy,
Nucl. Phys. B 462 249 (1996)

$$\Sigma_f(B,T) = \frac{2m_f}{m_\pi^2 f_\pi^2}\left[\langle\bar\psi_f\psi_f\rangle - \langle\bar\psi_f\psi_f\rangle_0\right] + 1$$

B Effects on QCD phase transitions

MC and IMC



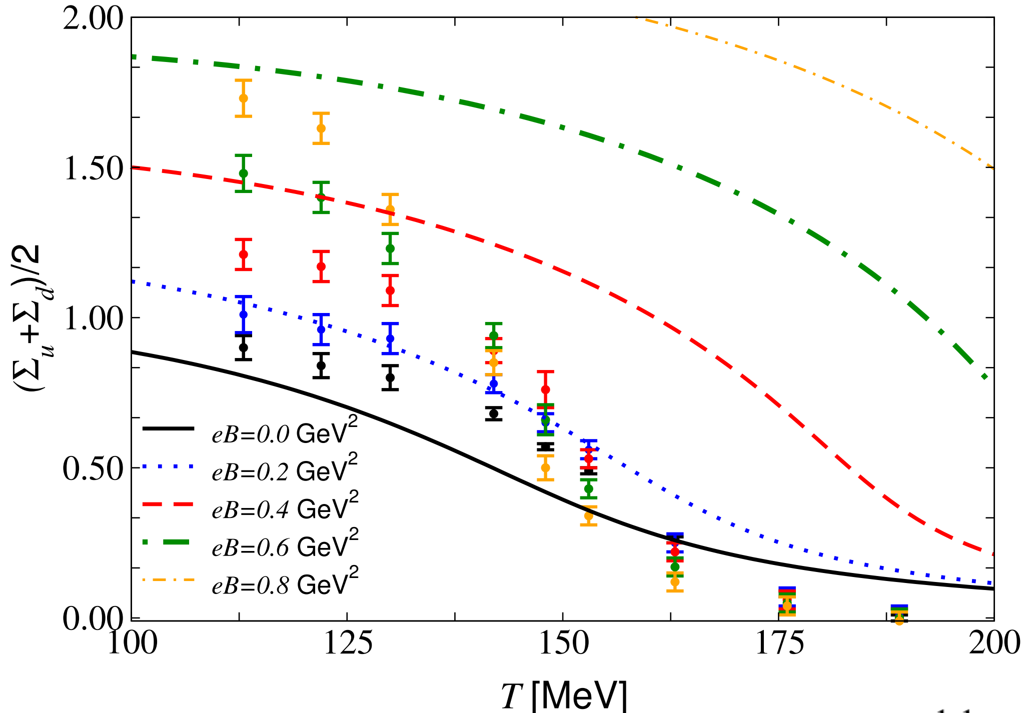


Failure of ALL effective models in providing inverse magnetic catalysis!

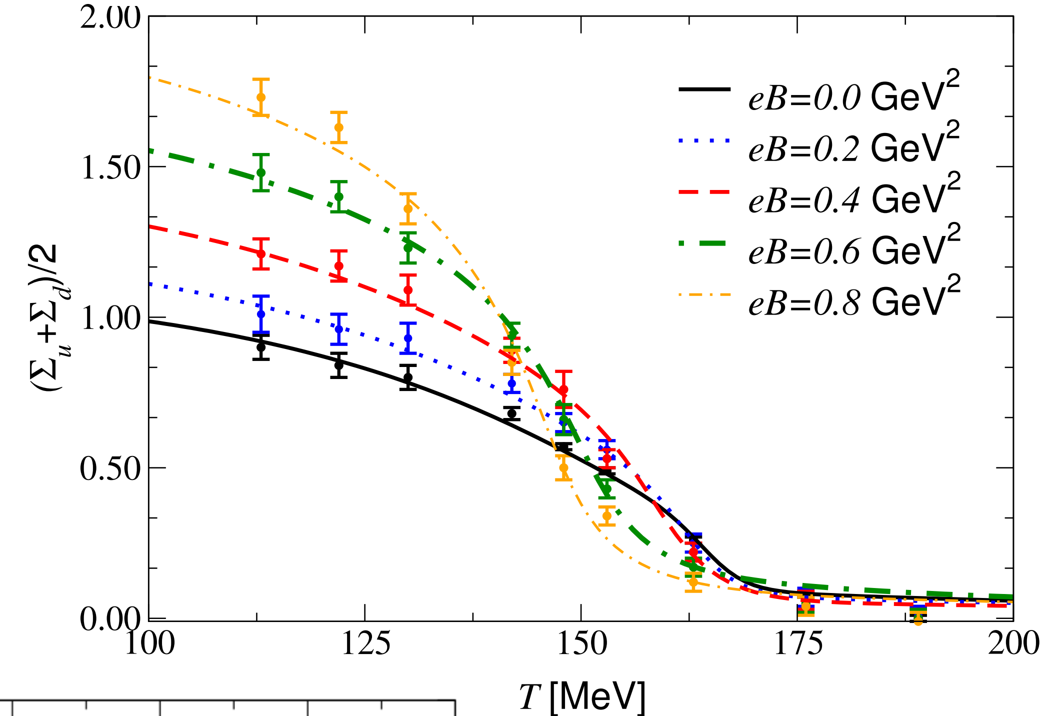
G. S. Bali et al., JHEP **1202**, 044 (2012)

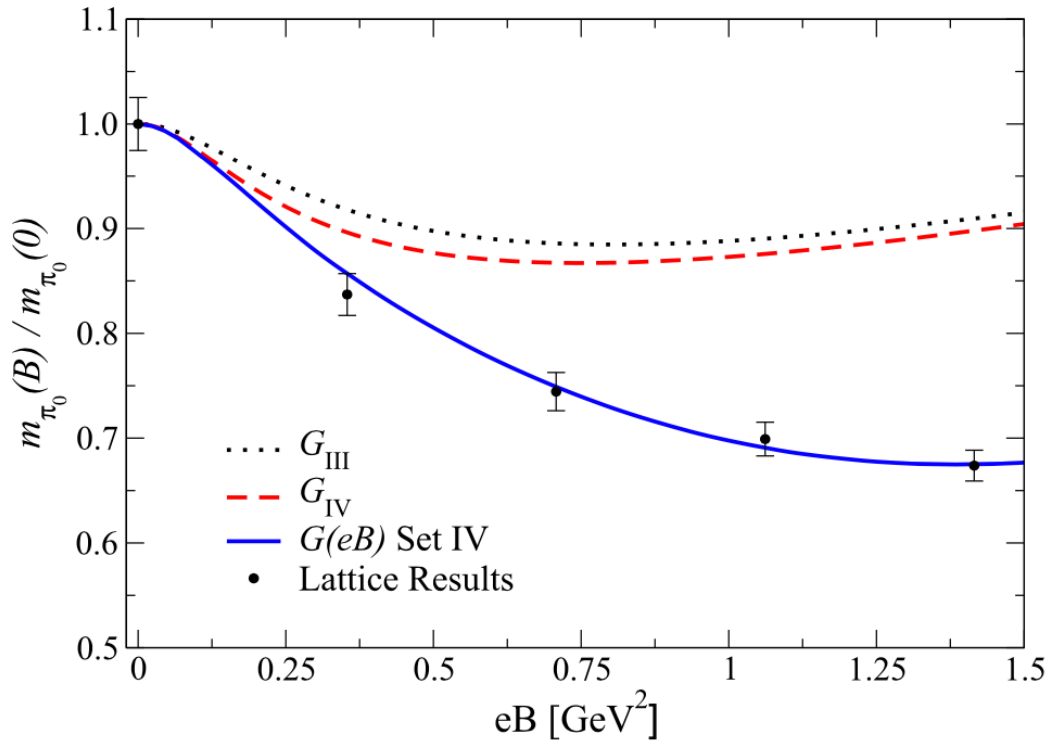
SU(2) NJL + Thermo-Magnetic effects G(B,T)

G(0,0)



$$G(B,T) = c(B)\left[1 - \frac{1}{1+e^{\beta(B)[T_a(B)-T]}}\right] + s(B).$$



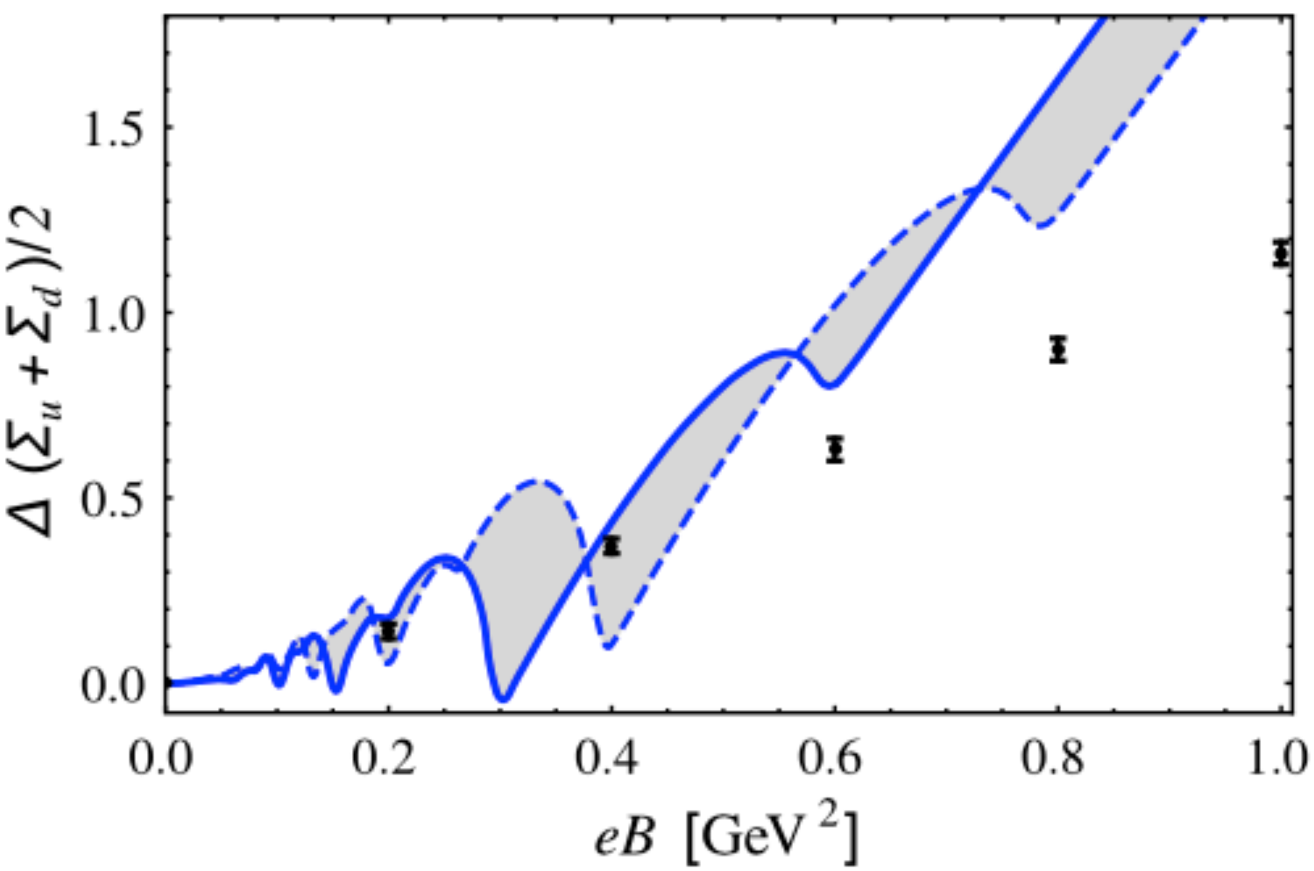


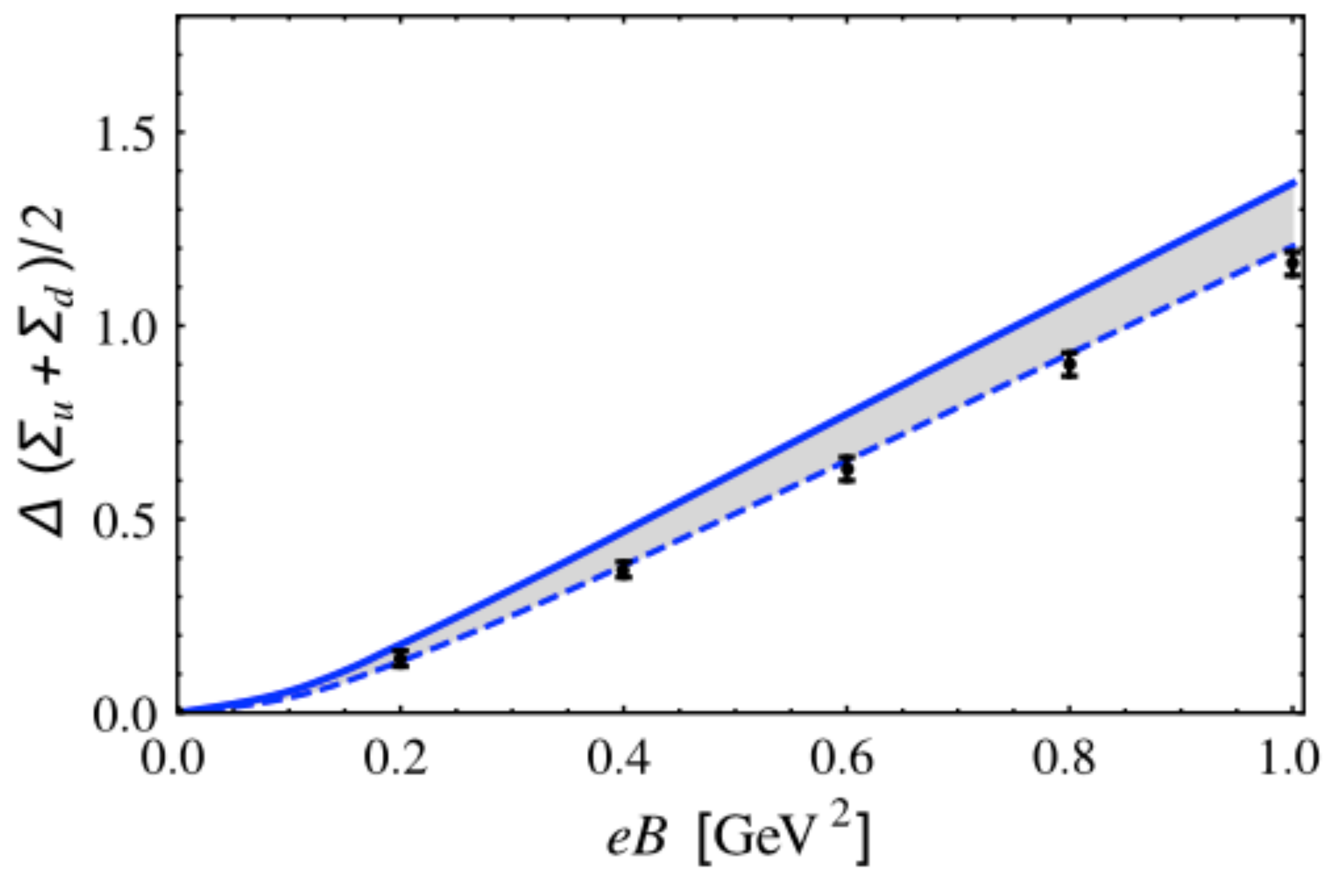
RLSF, K.P. Gomes, M.B. Pinto, G. Krein, Phys. Rev. C 90, 025203 (2014); **RLSF**, S.S. Avancini, M.B.Pinto and V.S. Timoteo, Phys. Lett. B 767, 247 (2017); **RLSF**, V.S. Timoteo, S.S. Avancini, M.B. Pinto and G. Krein Eur. Phys. J. A (2017) 53: 101; **RLSF**, W. Tavares, S.S. Avancini, V.S. Timoteo, G. Krein and M.B. Pinto, Eur. Phys. J. A (2021) 57: 278

Fermi-Dirac Form Factor

$$U_\Lambda^{\mathrm{FD}}(x) = \frac{1}{2}\left[1 - \tanh\left(\frac{\frac{x}{\Lambda} - 1}{\alpha}\right)\right]$$

$$245\,\mathrm{MeV} < -\bar{\Phi}_0^{1/3} < 260\,\mathrm{MeV}$$

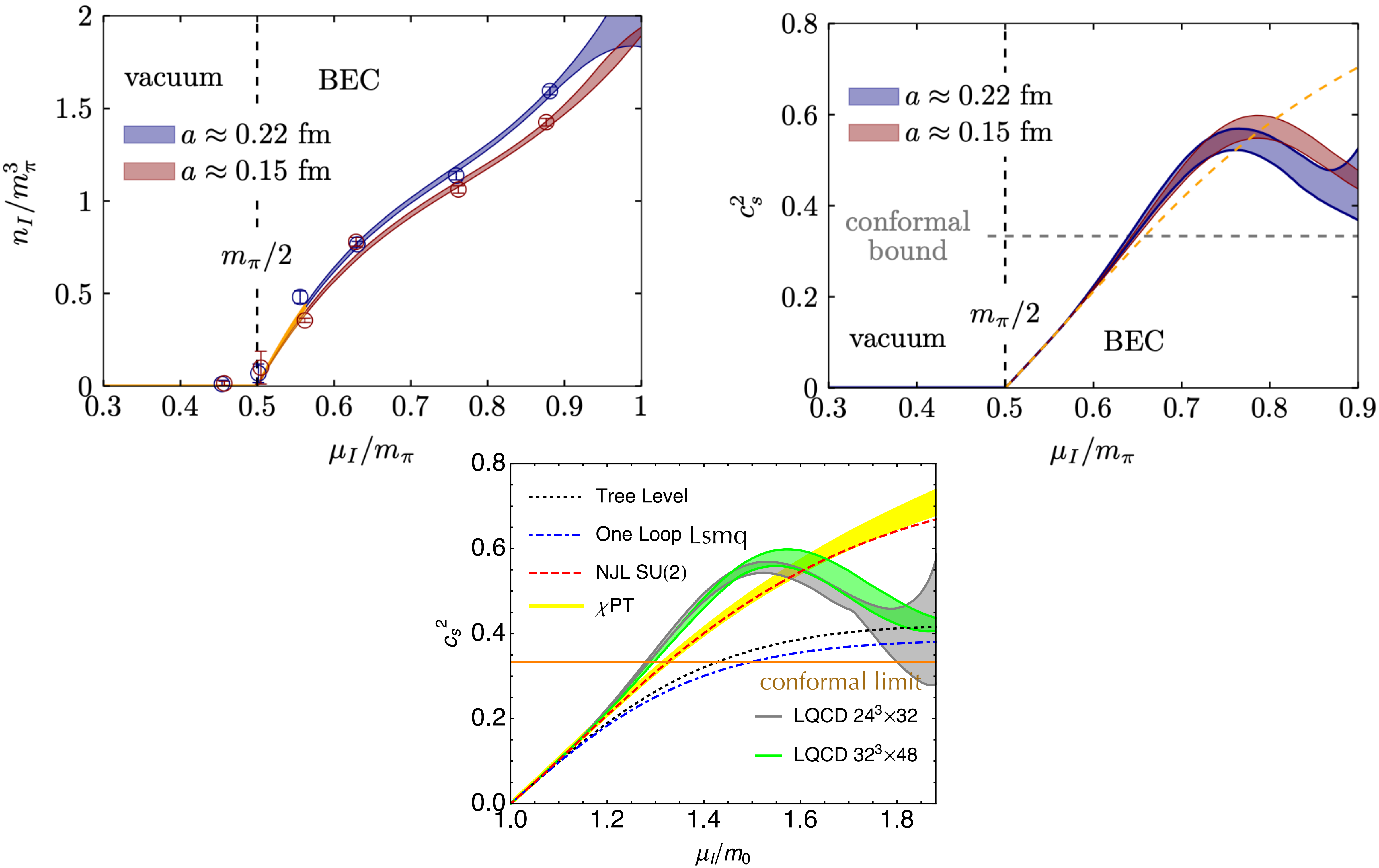




Lattice data: G. S. Bali, F. Bruckmann, G. Endrodi, Z. Fodor, S. D. Katz and A. Schafer, Phys. Rev. D **86**, 071502(R) (2012)

RLSF, S.S. Avancini, N. Scoccola, W.R. Tavares, PRD **99**, 116002 (2019).

Effective Models X LQCD



R.L.S.Farias, A. Ayala, A. Bandyopadhyay, L.A. Hernandez and J.L. Hernandez, *Phys.Rev.D* 107 (2023) 7, 074027

B.B. Brandt, F. Cuteri, G. Endrodi, JHEP 07 (2023) 055

Thermodynamic Consistency

$$
\begin{aligned}
P(T,\mu) &= \frac{T}{V} \ln \mathrm{Tr}\left[e^{-\beta(H-\mu N)}\right] , \\
\varepsilon(T,\mu) &= \frac{1}{V}\frac{1}{Z(T,\mu,V)} \mathrm{Tr}\left[He^{-\beta(H-\mu N)}\right] , \\
n(T,\mu) &= \frac{1}{V}\frac{1}{Z(T,\mu,V)} \mathrm{Tr}\left[Ne^{-\beta(H-\mu N)}\right] ,
\end{aligned}
$$

When in a model represented by H a given parameter, say a coupling constant G, depends on the chemical potential, namely $G = G(\mu)$, then $H \to H(\mu)$ and the number density, obtained by means of the thermodynamic relation

$$
n = \left(\frac{\partial P}{\partial \mu}\right)_T ,
$$

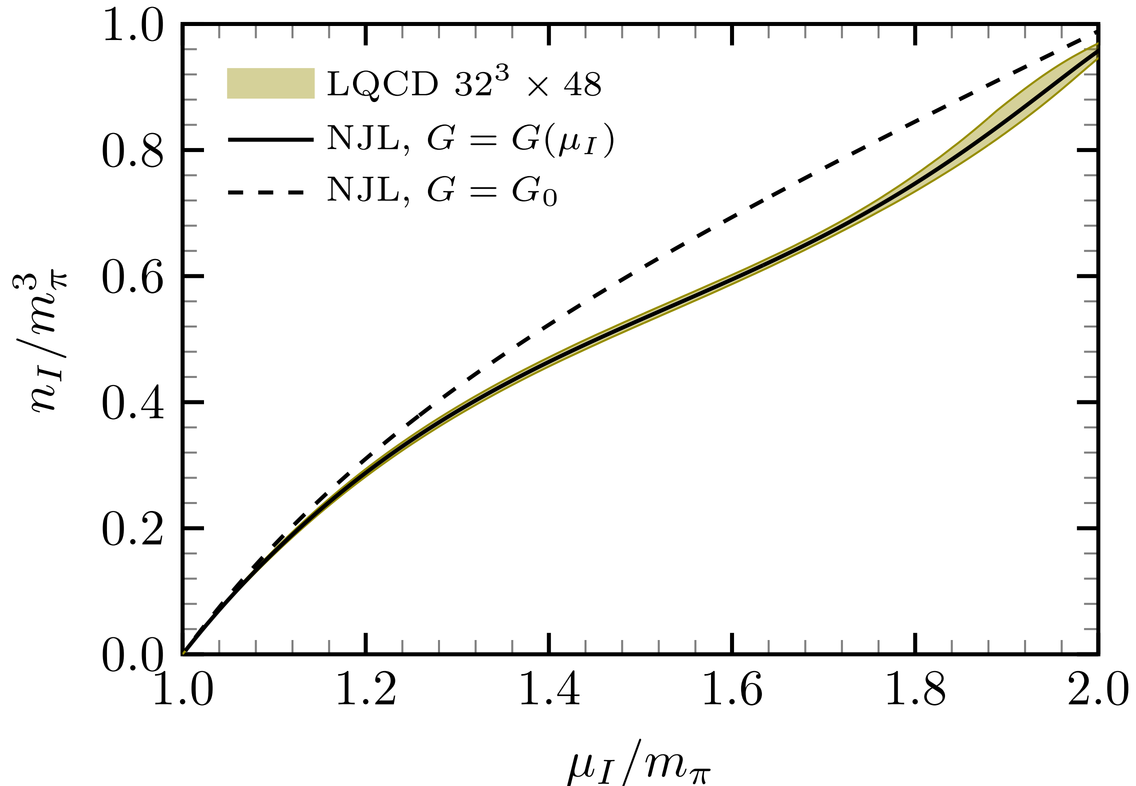
no longer coincides with the definition above.

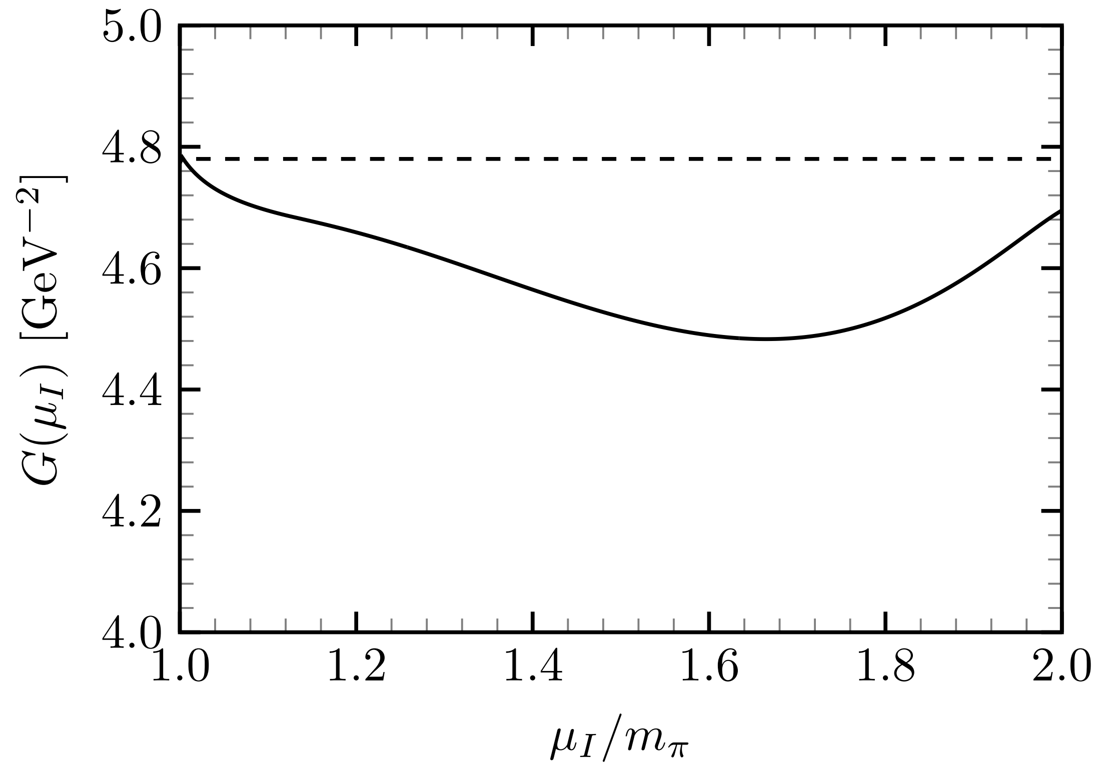
We work at $T = 0$ but with $\mu_I \neq 0$. The dependence of G on μ_I is obtained by fitting the model to the LQCD for the isospin density to find the $G(\mu_I)$ that best describes the data.

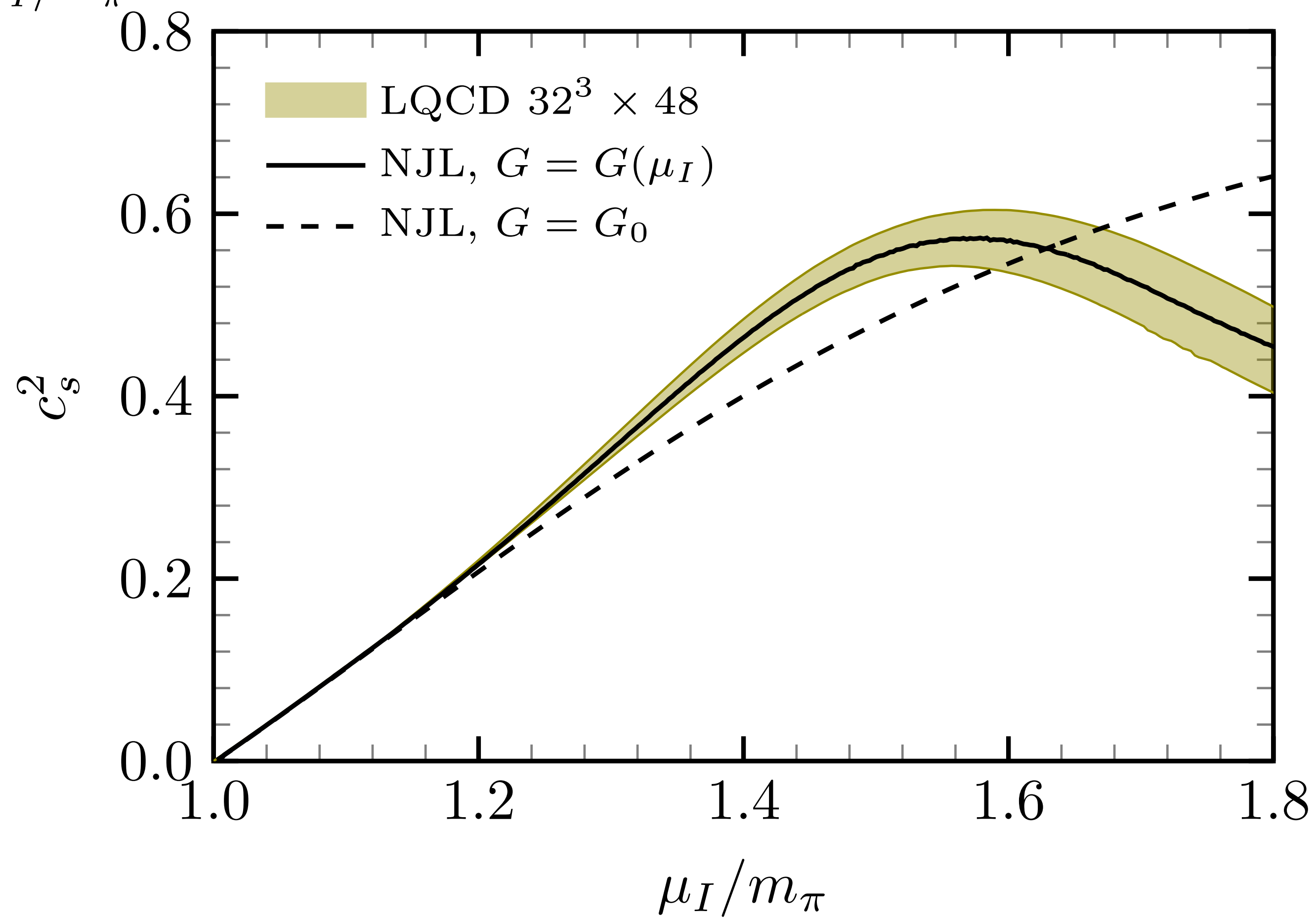
R.L.S.Farias, A. Ayala, B. Lopes and L.C. Parra, e-Print: 2310.13130 [hep-ph]

M.I. Gorenstein and S.N. Yang, Phys. Rev. D 52, 5206 (1995)

NJL results



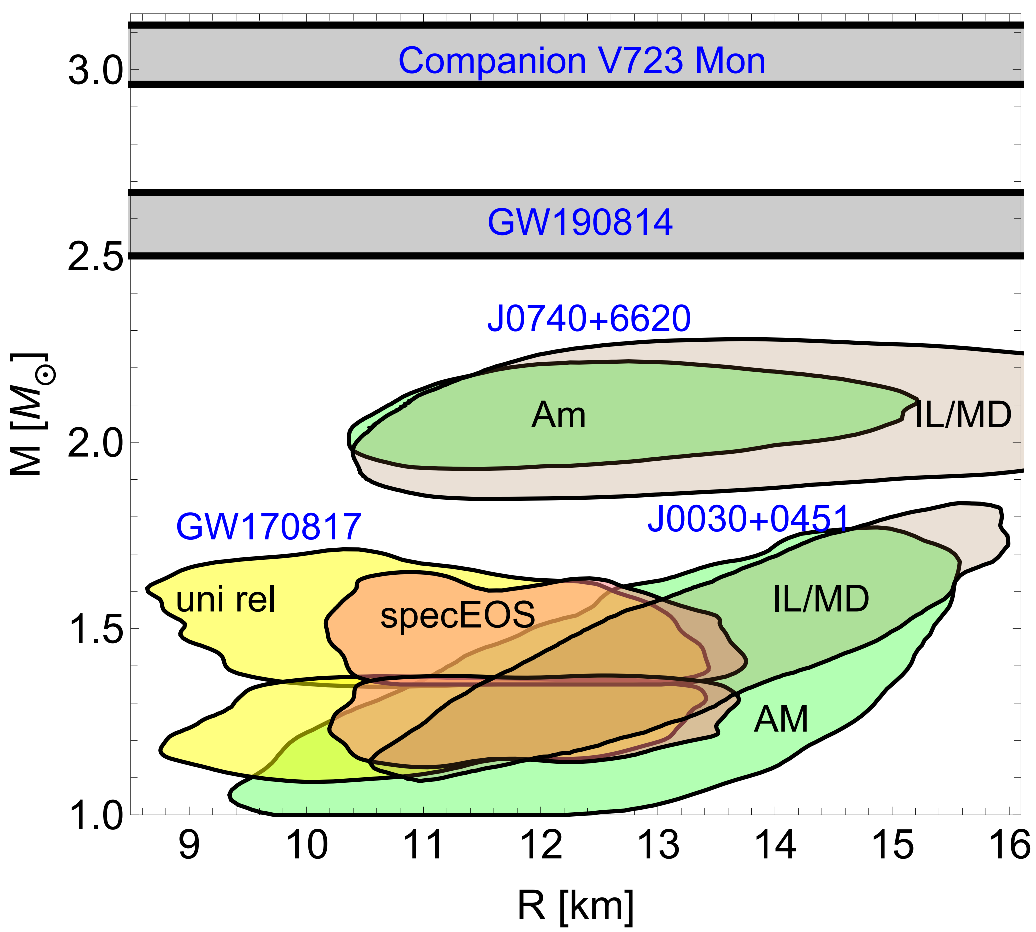




R.L.S.Farias, A. Ayala, B. Lopes and L.C. Parra, e-Print: 2310.13130 [hep-ph]

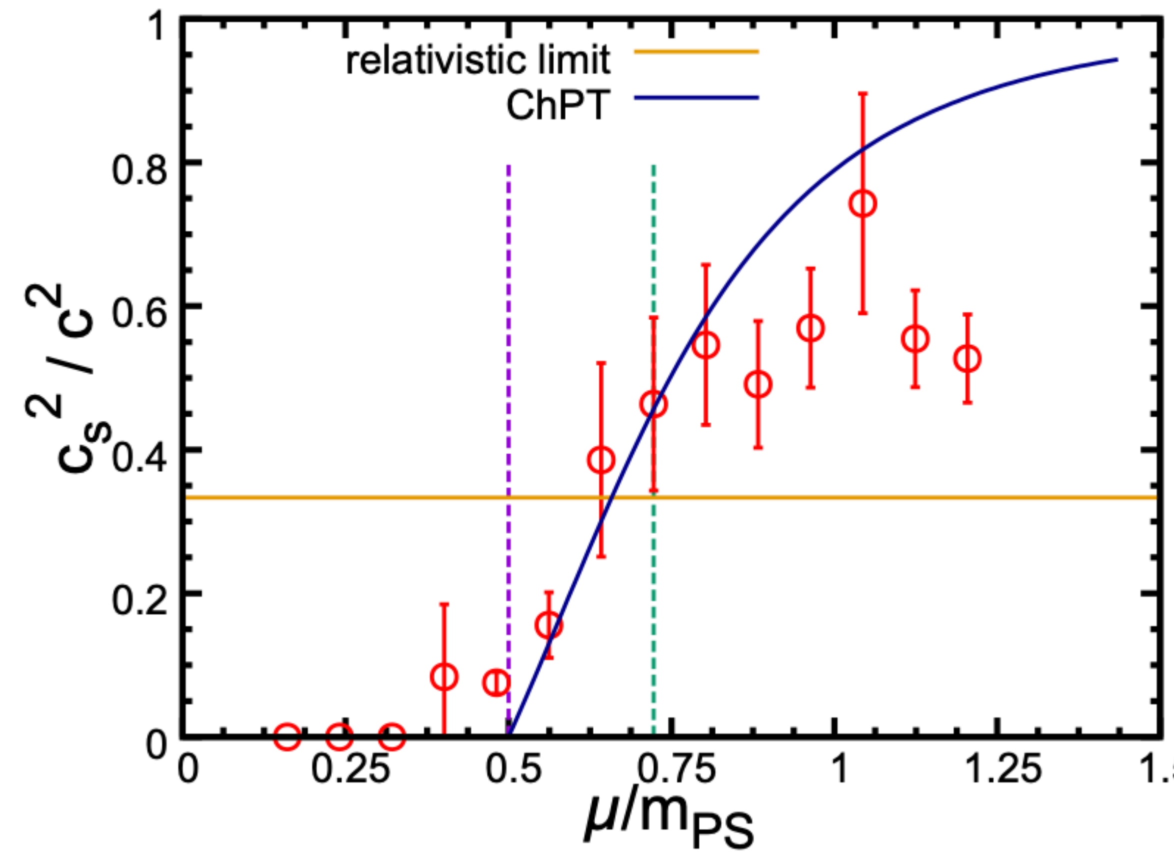
Finite baryon density ?

Observational constraints on the neutron star mass-radius plane from LIGO/Virgo and NICER data.



H. Tan, T.Dore, V.Dexheimer, J.Noronha-Hostler, and N. Yunes, Phys. Rev. D **105**, 023018 (2022).

Nc =2 lattice simulations



Bump of sound velocity in dense 2-color QCD

Kei Lida and Etsuko Itou, *PTEP* 2022 (2022) 11, 111B01

Conclusions:

- We can use different lattice results to verify the predictions from effective models of QCD
- MSS and medium dependent coupling constants -> results in agreement with LQCD

Perspectives:

- Beyond mean field approximation (e.g. O. Ivanytskyi talk last Monday)
- Cross-talk between models
- Finite densities + confinement effects
- eE effects on the chiral symmetry restoration

Thank you for your attention!