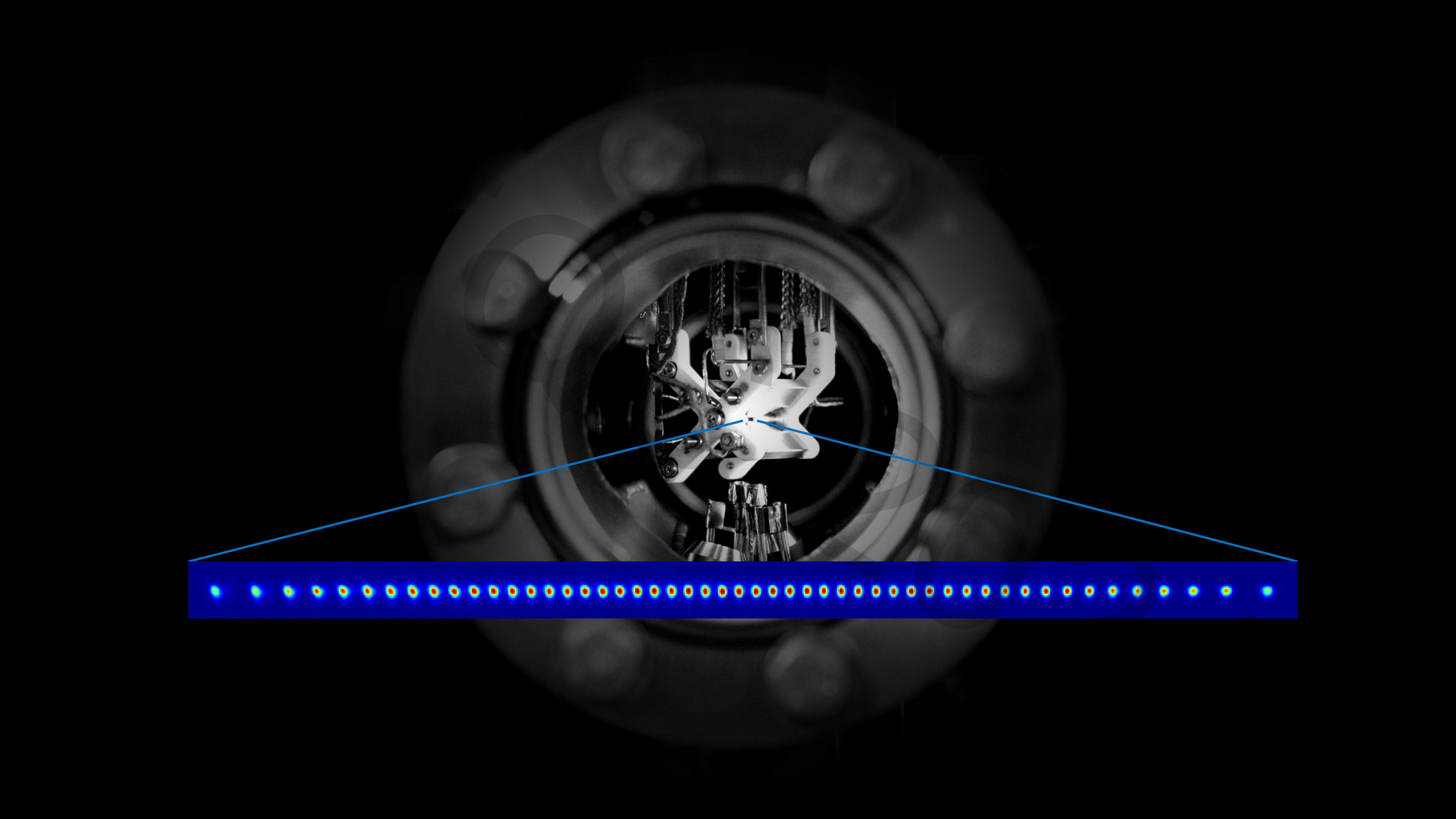
Quantum Computation and Simulation with Trapped Ions

Martin Ringbauer, University of Innsbruck



The requirements for QIP

I. Scalable physical system, well characterized qubits

II. Ability to initialize the state of the qubits

III. Long relevant coherence times, much longer than gate operation time

IV. “Universal” set of quantum gates

V. Qubit-specific measurement capability

VI. Ability to interconvert stationary and flying qubits

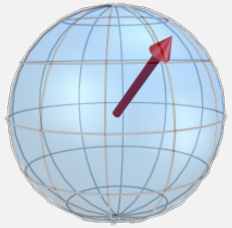
VII. Ability to faithfully transmit flying qubits between specified locations

The seven commandments for QIP

D. P. DiVincenzo, Quant. Inf. Comp. 1 (Special), 1 (2001)

The DV criteria for an experimentalist

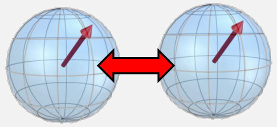
I. Find two-level systems,



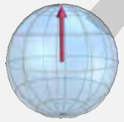
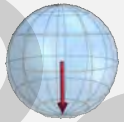
II. that can be individually controlled

III. that are stable and don’t decay while you work on them

IV. that interact to allow for entangling operations

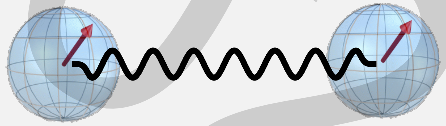


V. that can be efficiently measured

 or  ~100%

VI. Find a way to interconnect remote qubits

VII. Make sure, your interconnection is good



1. Trapping and Cooling Ions

- **1.1** How to trap an ion
- **1.2** Ion strings for quantum computation
- **1.3** Choosing an ion
- **1.4** Laser-ion interaction
- **1.5** Laser cooling in ion traps
- **1.6** Gate Operations & Decoherence
- **1.7** Entanglement

universität innsbruck

AG Quantenoptik und Spektroskopie

Interactions for particle traps

- Magnetic dipole moment: $U \sim \vec{\mu} \cdot \vec{B}$

  Neutral atoms, BEC

- Electric dipole moment: $U \sim \vec{d} \cdot \vec{E}$

  Cold atoms in optical traps

- Electric charge: $U \sim e \cdot \phi(r, t)$

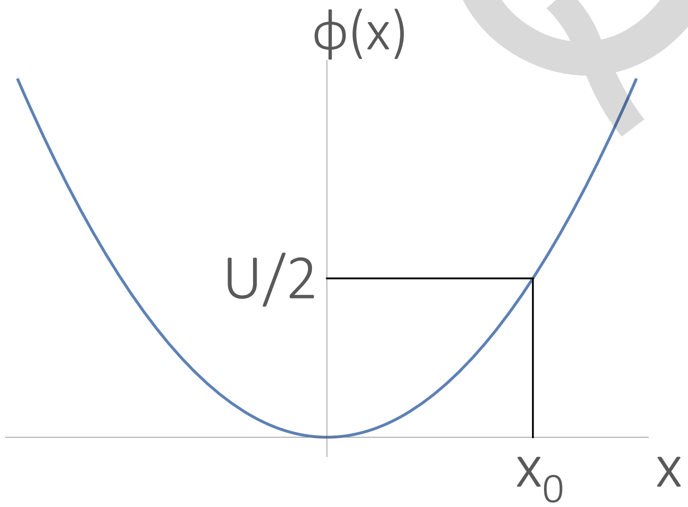
  Atomic or molecular ions

Trapping in electro-static potentials

Ion with mass m, charge e in a 1D harmonic potential

$$\phi = \frac{U}{2}\left(\frac{x}{x_0}\right)^2$$

Potential energy: $E_{pot} = e\phi(x)$



Equation of motion

$$m\ddot{x} = F = -eU\frac{x}{x_0^2}$$

$$\ddot{x} + \omega^2 x = 0$$

$$\omega = \sqrt{\frac{eU}{mx_0^2}}$$

Exercise: Calculate the required voltage for a trap depth of 1eV at x_0=1mm, as well as the trap frequency for a Ca^+ ion

D. Leibfried, et al, Rev. Mod. Phys. 75, 281-324 (2003)

Trapping in 3D

Want: $$\Phi(\mathbf{r}) = \Phi_0 \sum_i \alpha_i (r_i/\tilde{r})^2 \quad , \quad \mathrm{i} = \mathrm{x,y,z}$$

Poisson equation: $$\Delta\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0$$

→ Cannot trap in 3D with static potentials

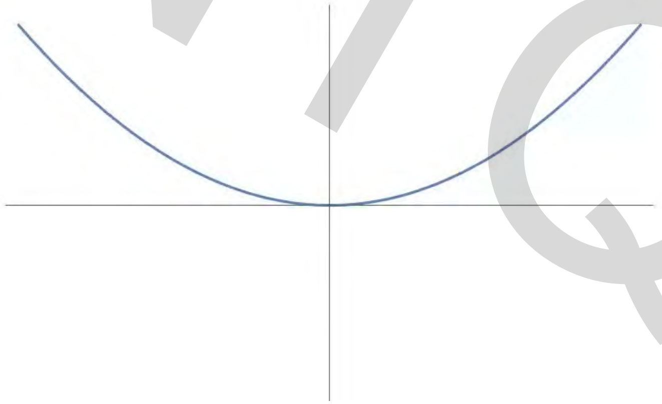
Penning: $$\Phi_0 = U_0 + \text{axial magn. field}$$

Paul: $$\Phi_0 = U_0 + V_0 \cos\Omega t$$

D. Leibfried, et al, Rev. Mod. Phys. 75, 281-324 (2003)

Trapping with dynamic potentials

$$\phi = \frac{U}{2}\left(\frac{x}{x_0}\right)^2 \sin\Omega t$$



Force on charged particle:

$$F(x,t) = -eU\frac{x}{x_0^2}\sin\Omega t$$

If the force was homogenous

$$x(t) = x_i + \frac{eUx_i}{mx_0^2\Omega^2}\sin\Omega t$$

Solve equations of motion in inhomogenous field

$$x(t) \sim \cos(\omega t + \phi)\left(1 + \frac{q}{2}\sin(\Omega t)\right)$$

secular motion: $\cos(\omega t + \phi)$

micromotion: $\frac{q}{2}\sin(\Omega t)$

$$q = \frac{2eU}{mx_0^2\Omega^2}$$

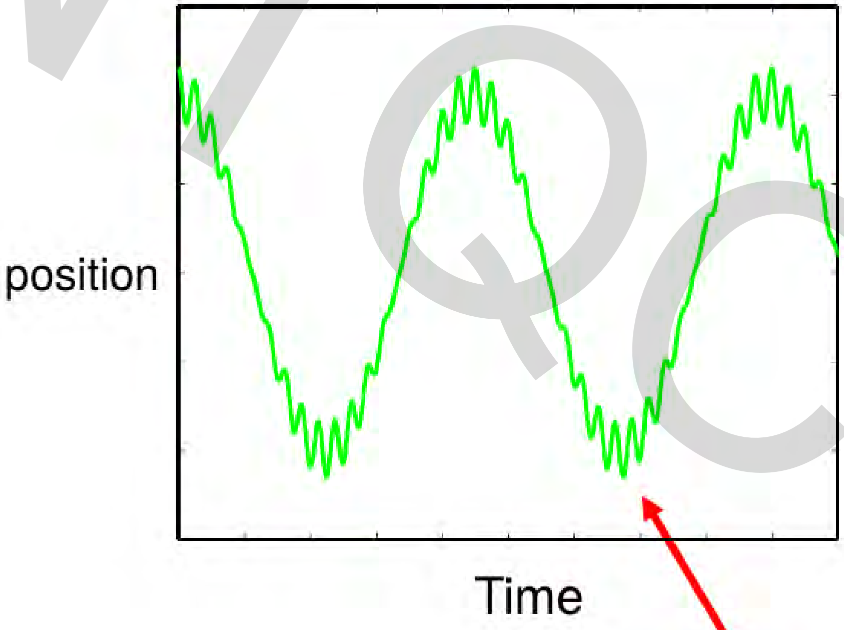
$$\omega = \frac{eU}{\sqrt{2}mx_0^2\Omega}$$

<ins>Exercise</ins>: Calculate the equations of motion in a 1D RF potential

D. Leibfried, et al, Rev. Mod. Phys. 75, 281-324 (2003)

Micromotion

1d-solution of Mathieu equation



Aluminium particle in trap

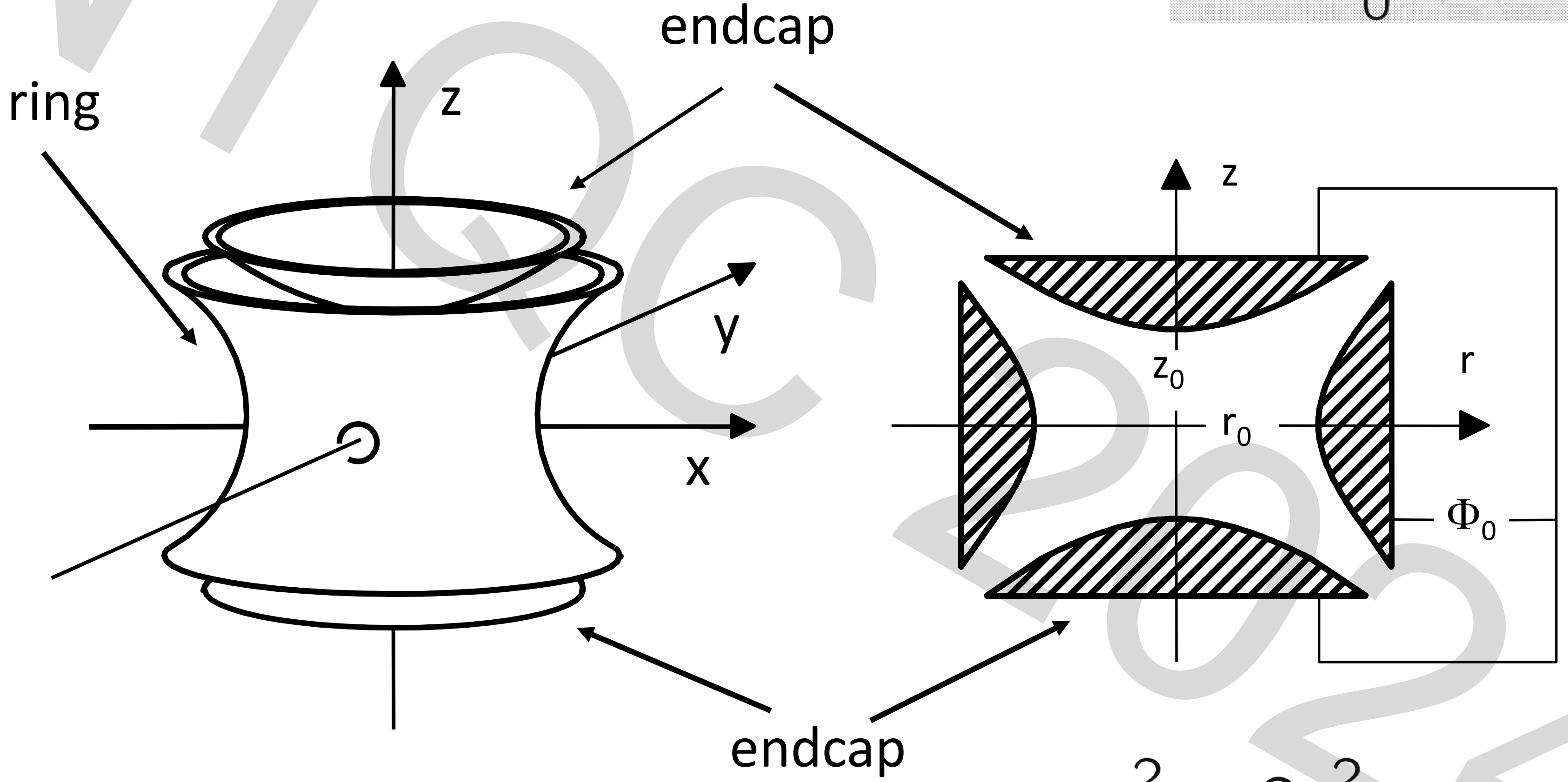
Wuerker, Shelton, Langmuir,
J. Appl. Phys. **30**, 342 (1959)

micromotion

universität innsbruck

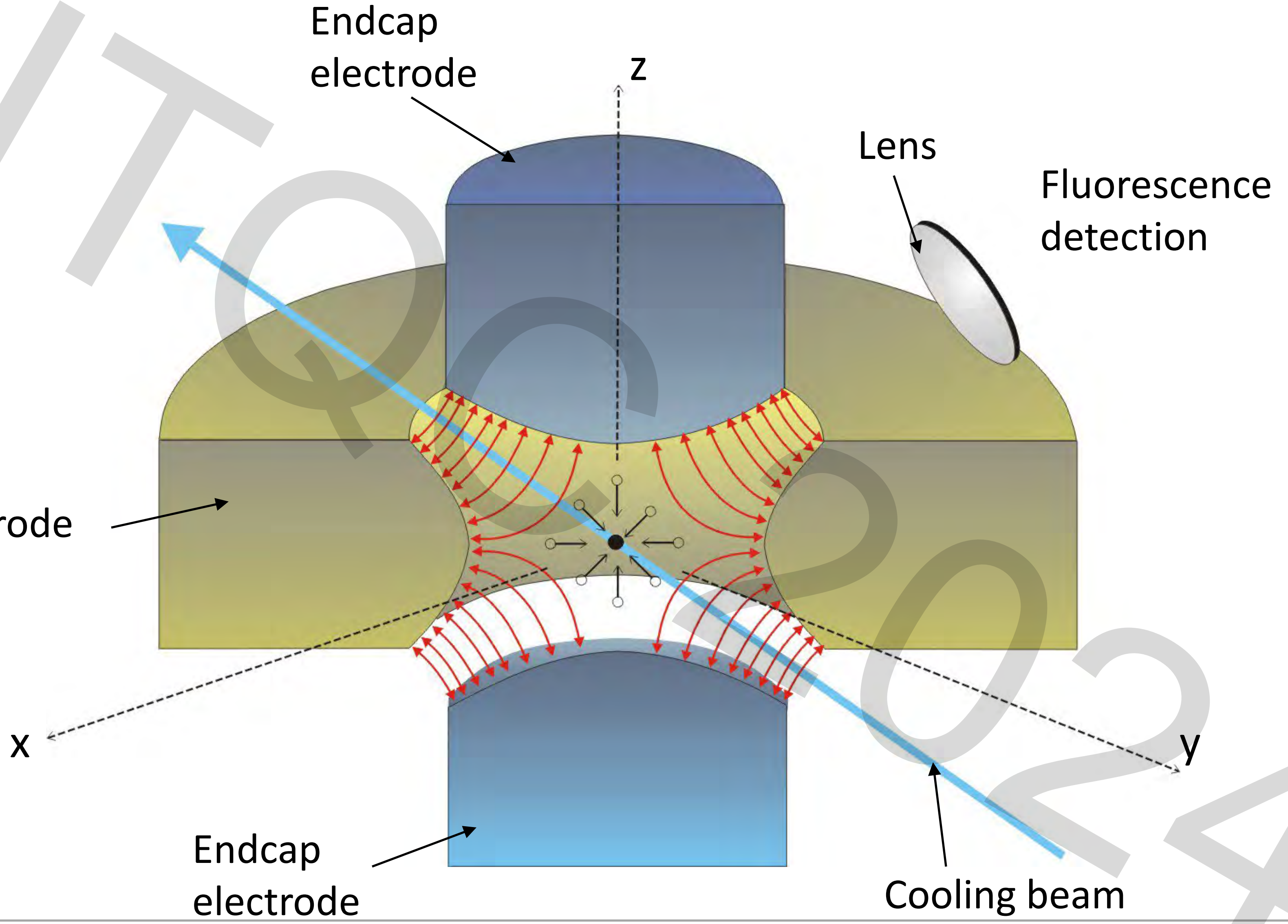
Paul trap (rf – quadrupole trap)

$$\Phi = \frac{\Phi_0}{r_0^2}(x^2 + y^2 - 2z^2)$$



$$r_0^2 = 2z_0^2$$

Paul trap



Endcap
electrode
z
Lens
Fluorescence
detection
Ring
electrode
x
y
Endcap
electrode
Cooling beam

Paul trap: stability diagram

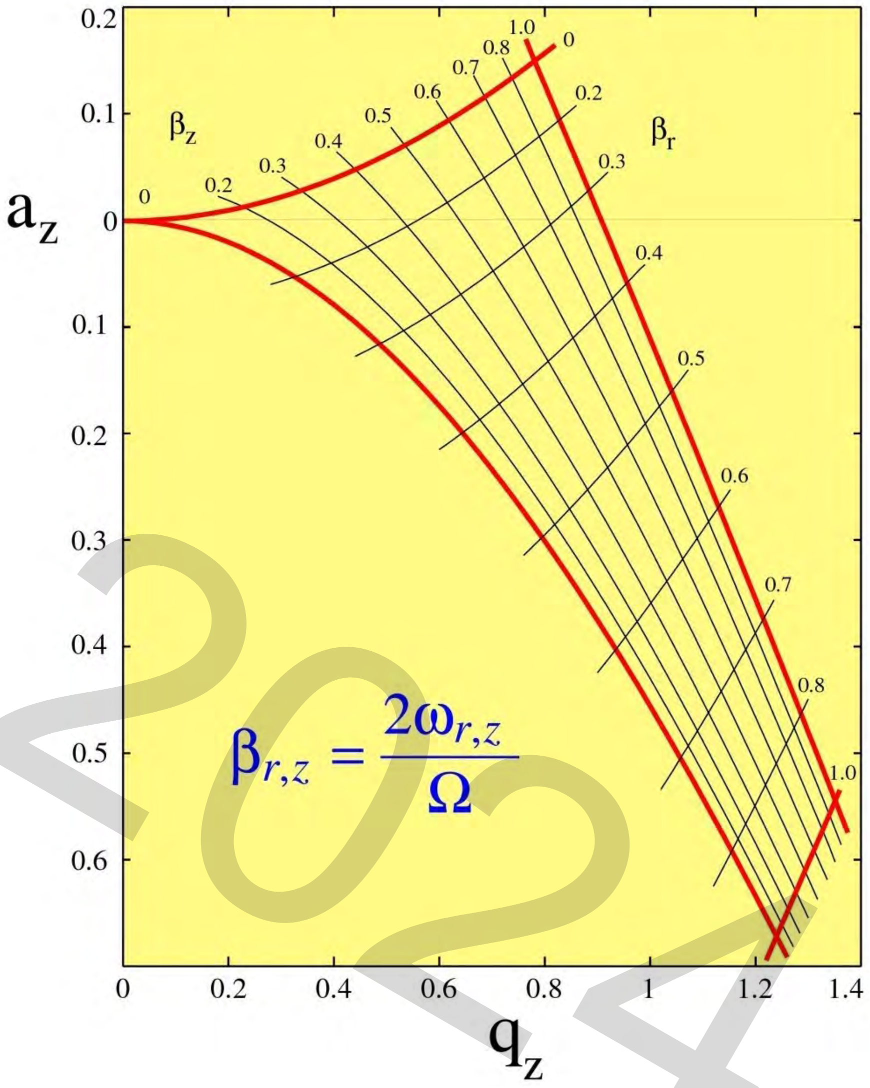
$$a_z = -\frac{8eU_0}{mr_0^2\Omega^2} = -2a_r, \quad r = x, y$$

$$q_z = -\frac{4eV_0}{mr_0^2\Omega^2} = -2q_r, \quad r = x, y$$

$$x_i(t) = C\left(1 - \frac{q_i}{2}\cos\Omega t\right)\cos\omega_i t$$
$$i = x, y, z$$

$$\omega_i \ll \Omega(a_i, q_i \ll 1)$$

$$\beta_i^2 = a_i + \frac{q_i^2}{2}$$



Quantum mechanical motion

$$x_i(t) = C\left(1 - \frac{q_i}{2}\cos\Omega t\right)\cos\omega_i t,\quad i \in \{x, y, z\}$$

classical ion motion = micromotion + secular motion

secular approximation $a_i,\ q_i \ll 1\ (\rightarrow\ \omega_i \ll \Omega)$

neglects micromotion and interprets motion as generated by a „pseudo-potential“

$$e\Psi = \frac{1}{2}\sum_i m\omega_i^2 x_i^2,\quad i \in \{x, y, z\}$$

Thus, we define

$$a_i^\dagger = \sqrt{\frac{m\omega_i}{2\hbar}}x_i + \frac{i}{\sqrt{2m\hbar\omega_i}}p_i$$

$$a_i = \sqrt{\frac{m\omega_i}{2\hbar}}x_i - \frac{i}{\sqrt{2m\hbar\omega_i}}p_i$$

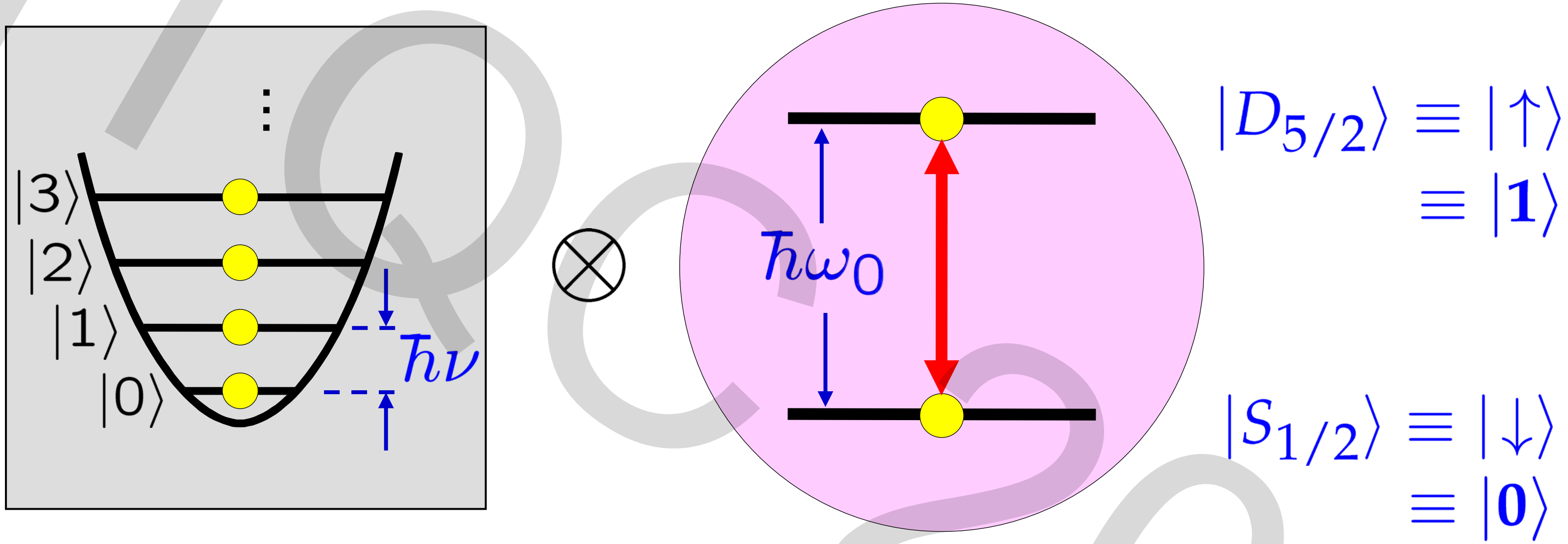
and obtain the Hamiltonian

$$H = \sum_i \hbar\omega_i\left(a_i^\dagger a_i + \frac{1}{2}\right)$$

Single trapped ion

Harmonic oscillator

Quantum bit



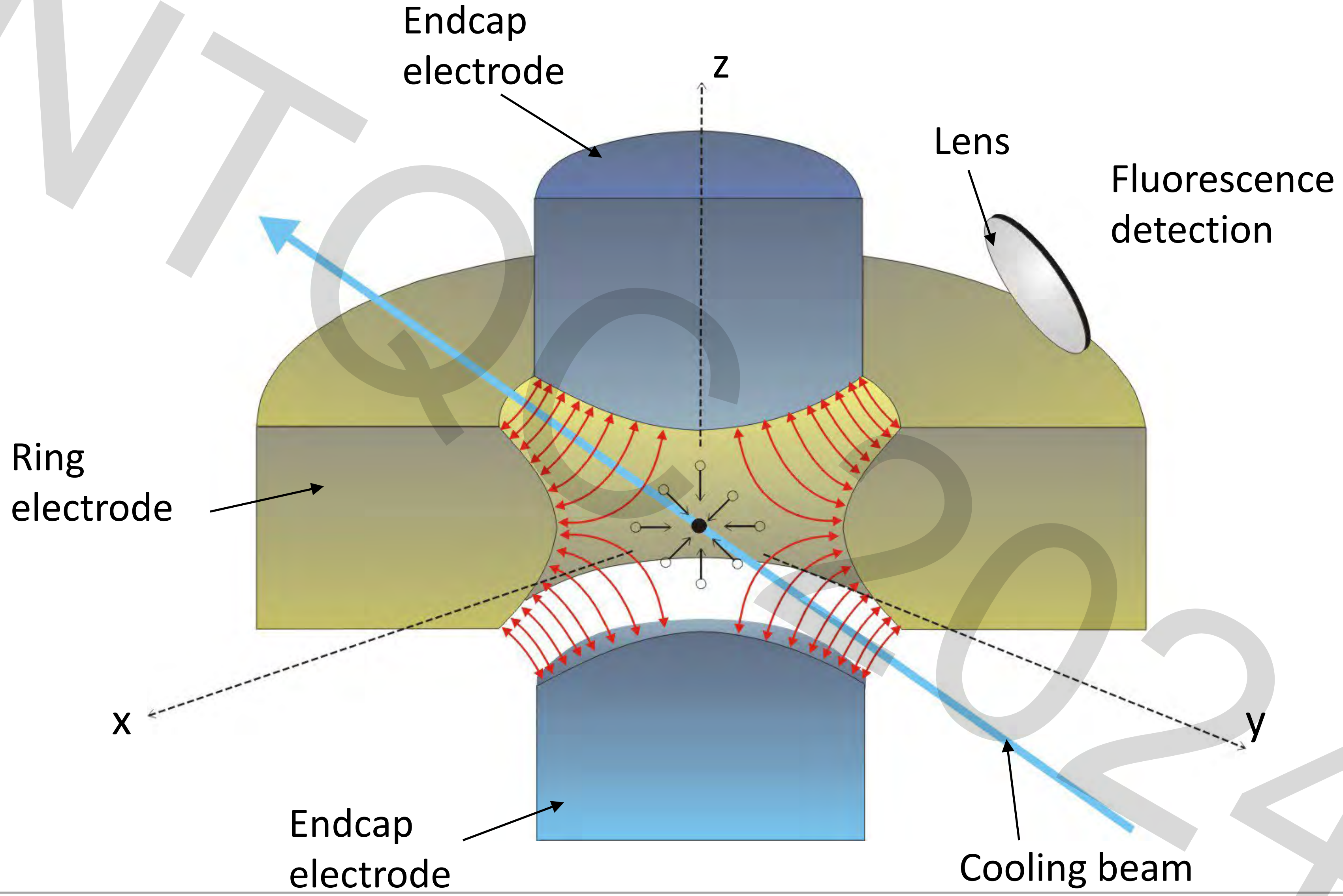
motional states

$|0\rangle, |1\rangle, |2\rangle, |3\rangle, \ldots$

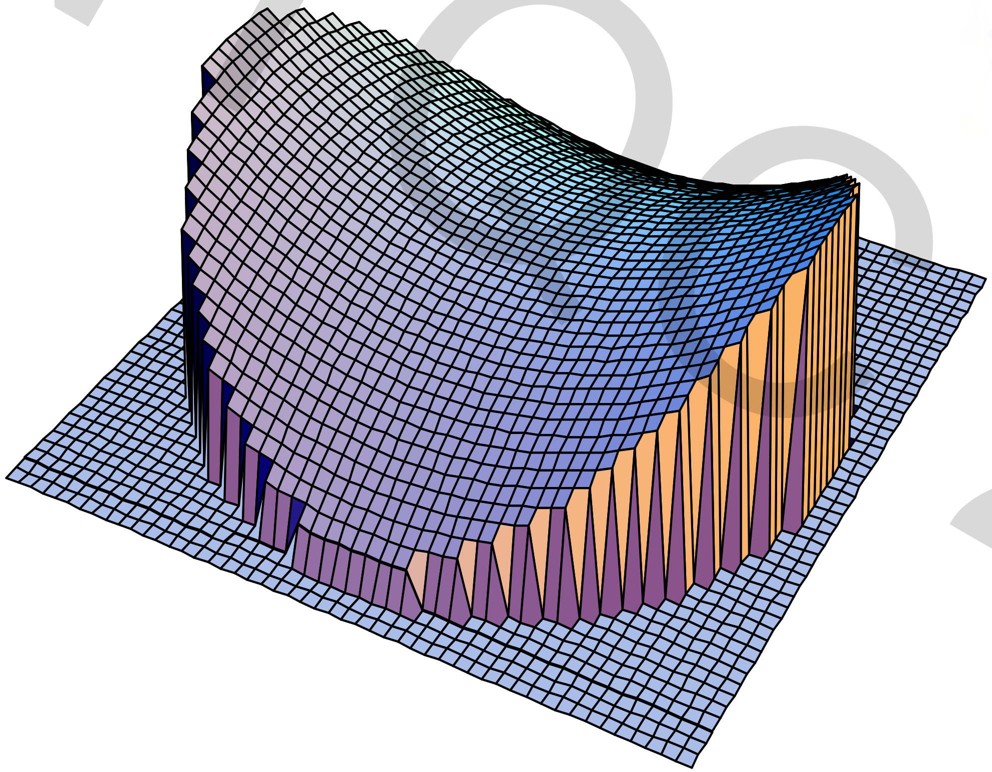
internal states

$|\uparrow\rangle, |\downarrow\rangle$

Paul trap

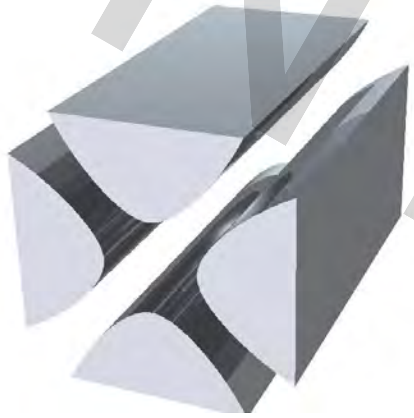

Endcap electrode
z
Lens
Fluorescence detection
Ring electrode
x
y
Endcap electrode
Cooling beam

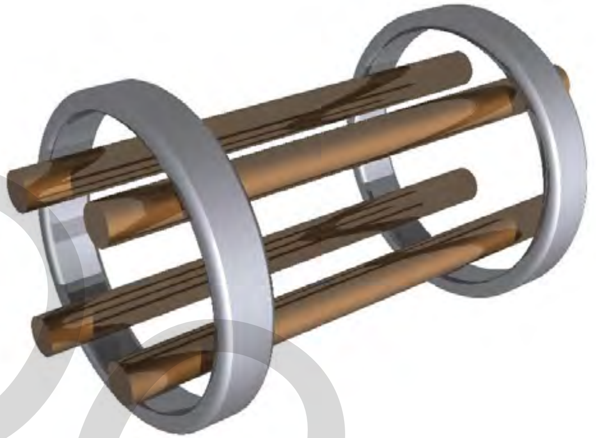
2D linear Paul trap

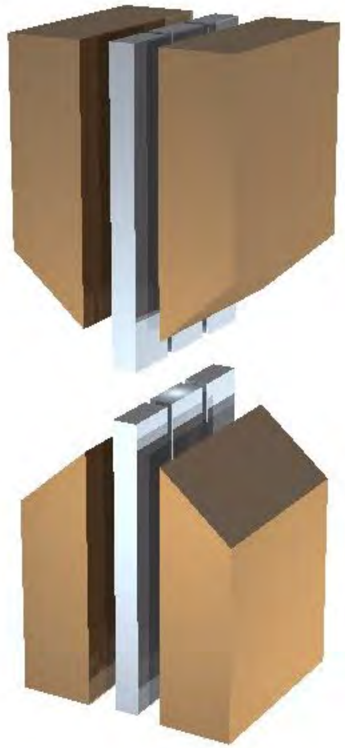


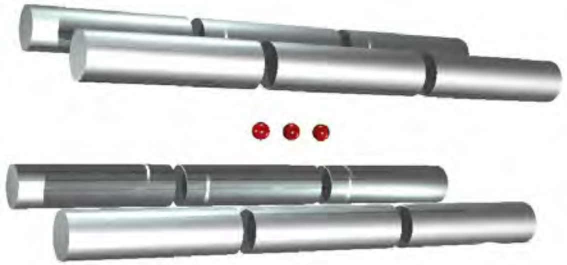
$\Phi \sim (x^2 - y^2) \sin \Omega t$ & $\Phi_S \sim 2z^2 - (x^2 + y^2)$

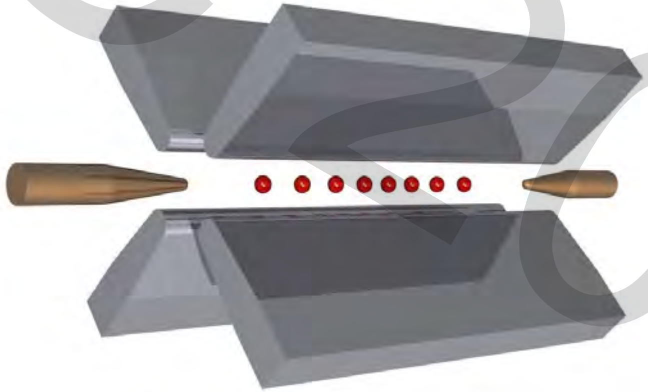
Linear ion traps











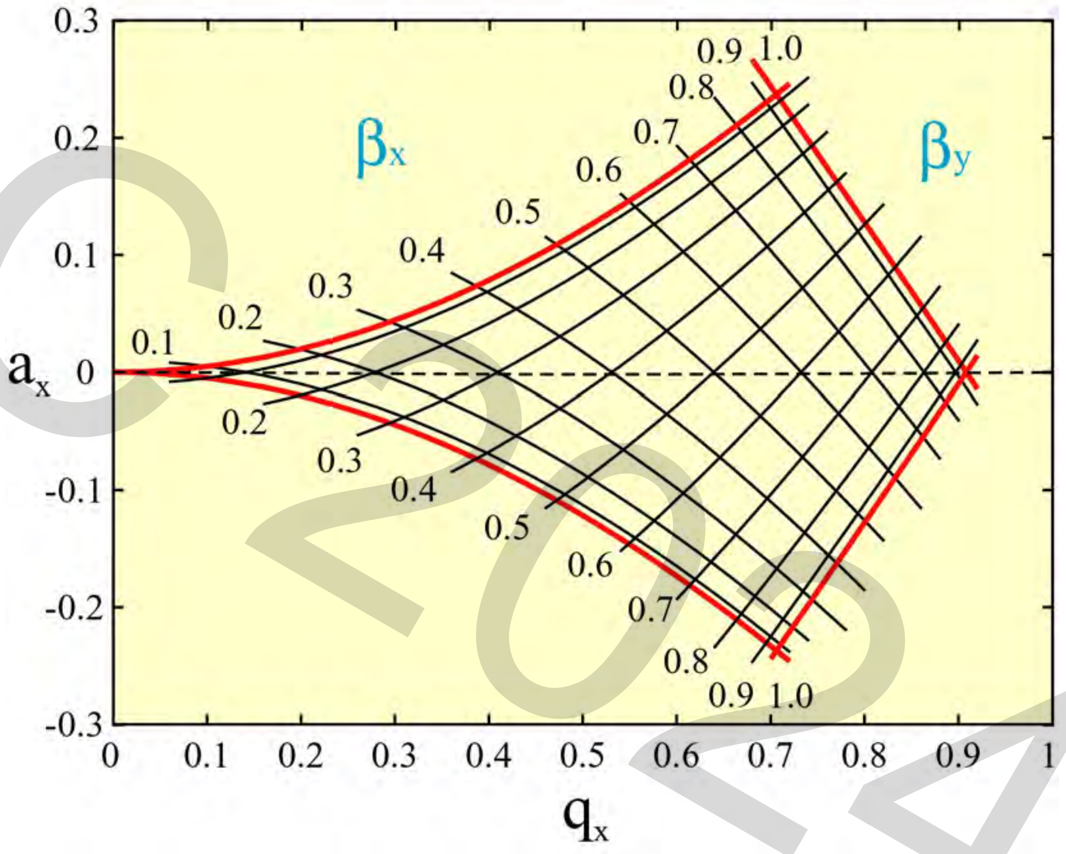
Trap designs differ primarily in effective distance & optical access

Linear Paul trap: Stability diagram

$$\omega_r \approx \frac{eV_0}{\sqrt{2}m\omega^2 r_0^2}$$

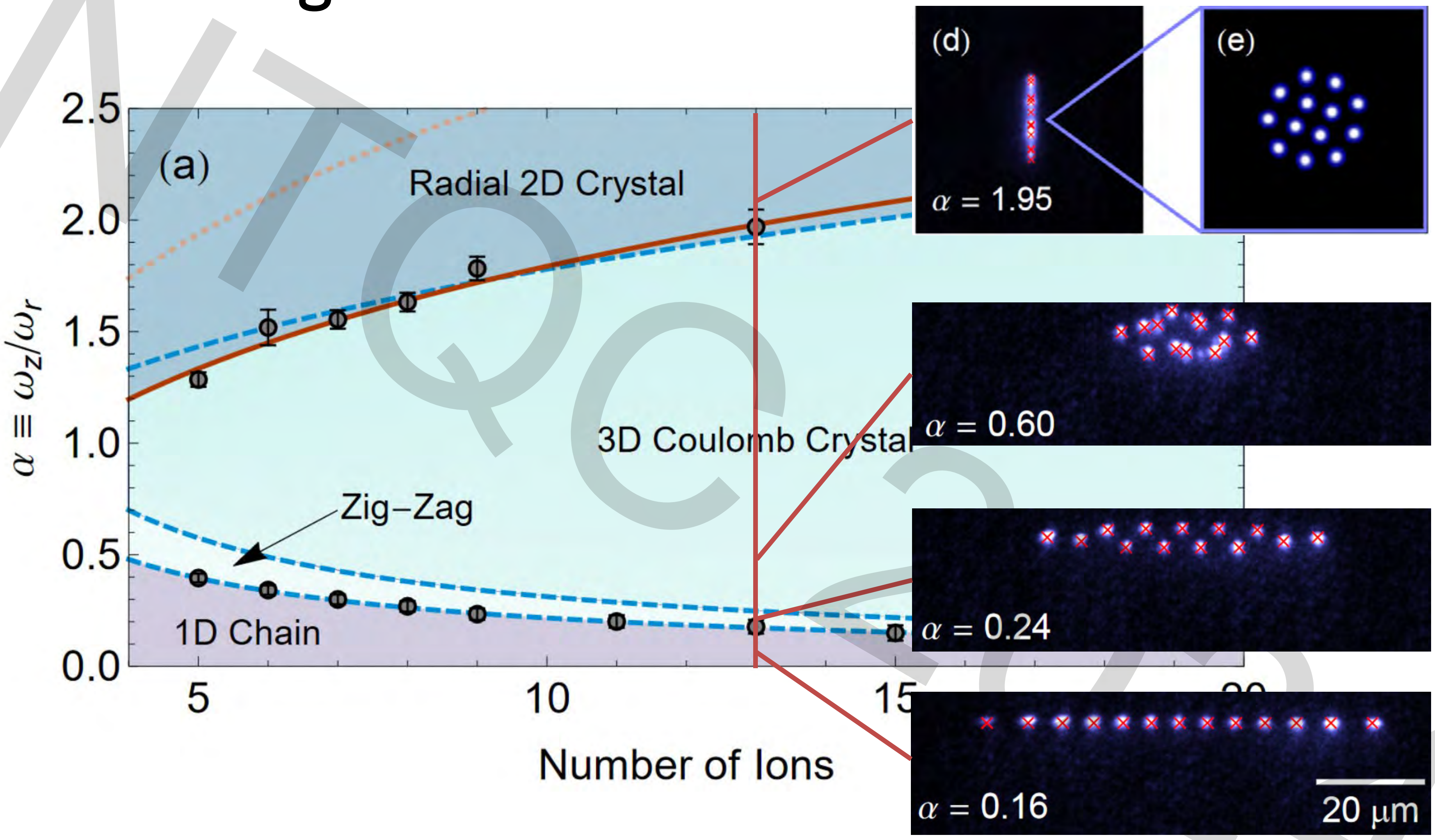
$$\omega_z \approx \sqrt{\frac{2\kappa U_{cap}}{mz_0^2}}$$

κ: geometry factor



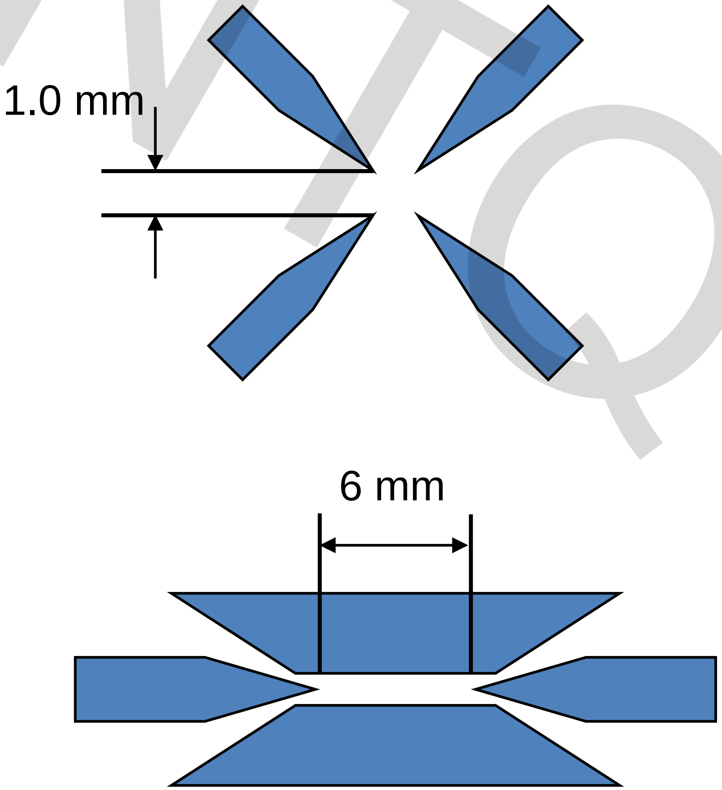
D. Leibfried et al., Rev. Mod. Phys. 75, 281 (2003)

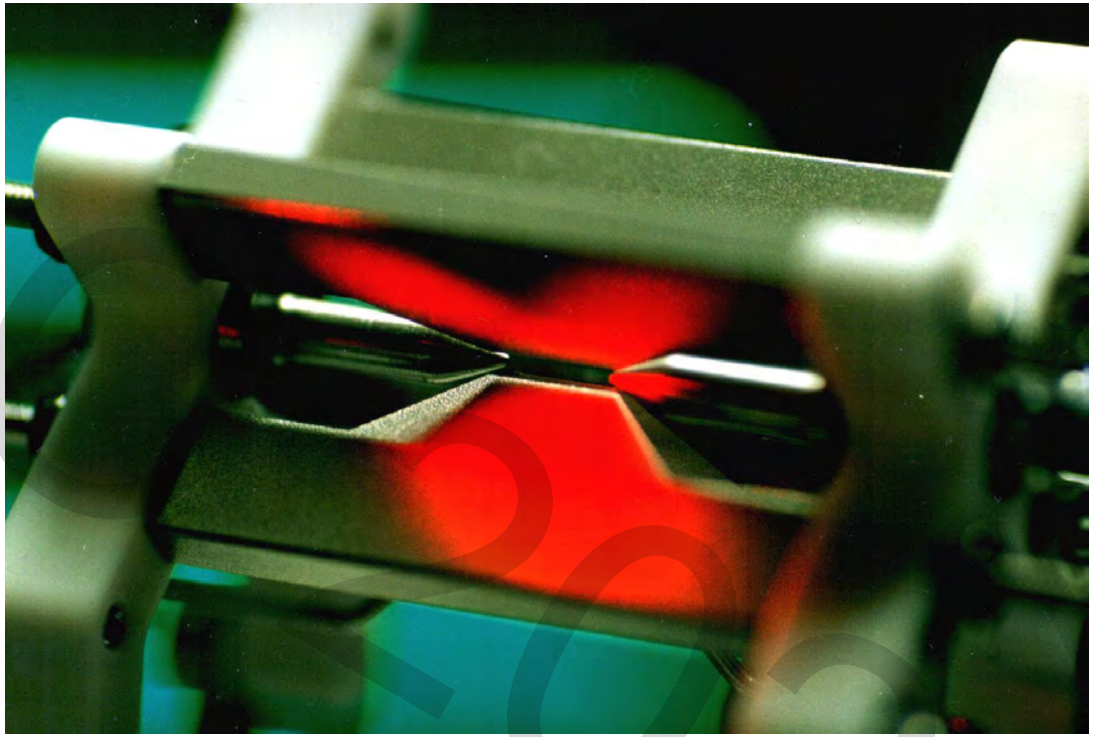
Non-linear configurations



M. D'Onofrio et al, arxiv:2021.12766 (2020)

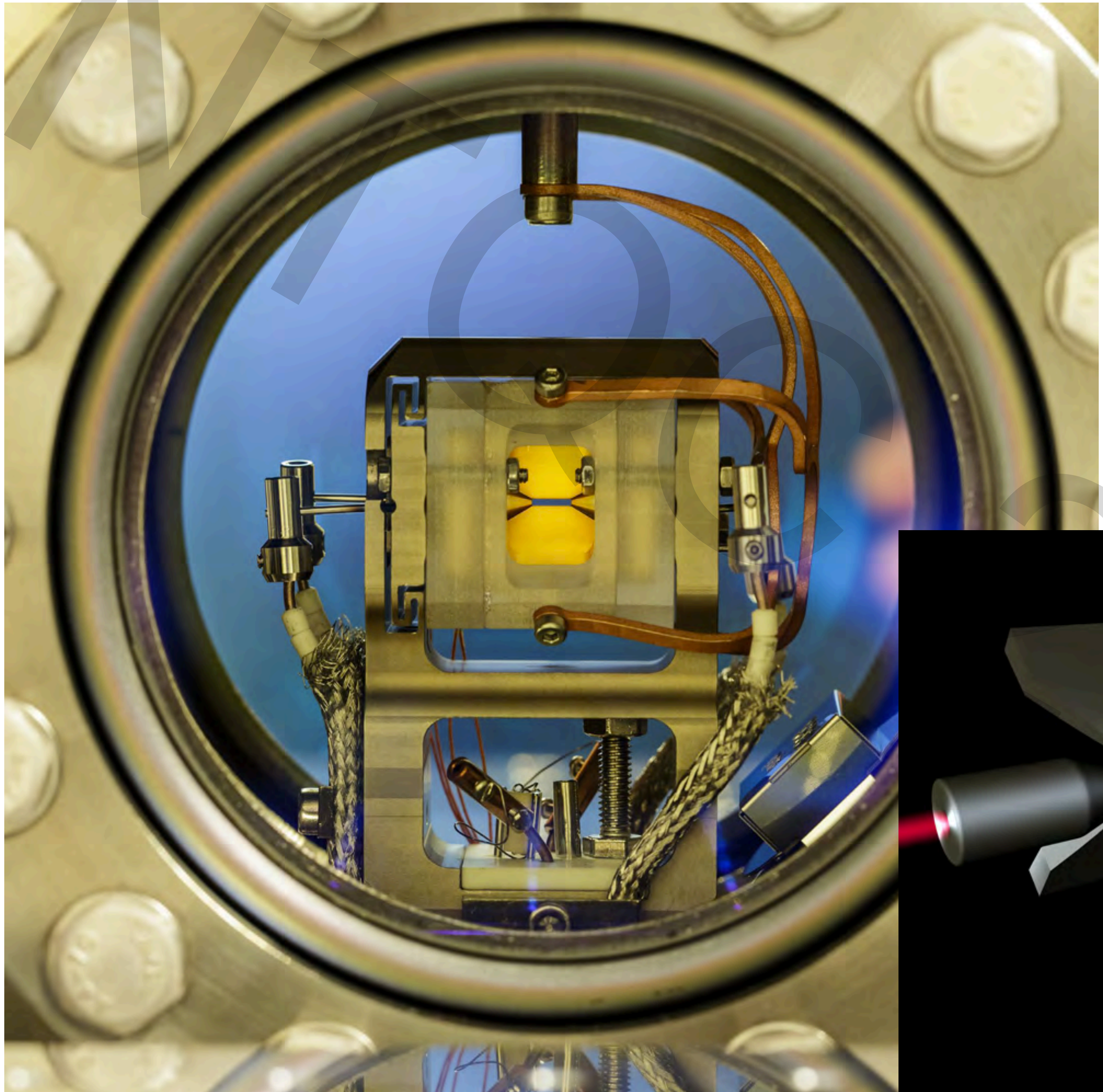
Innsbruck linear ion trap (2000)

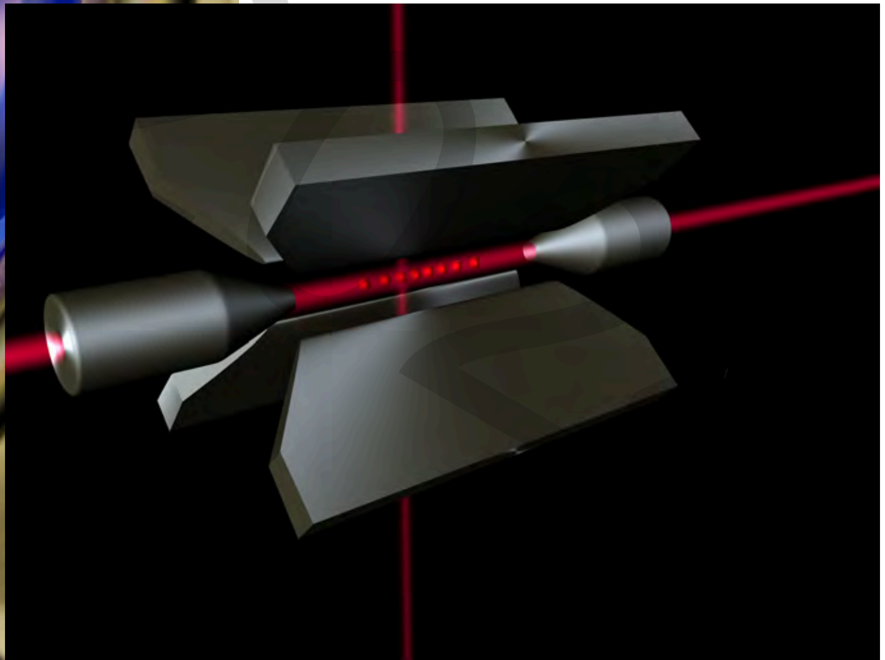




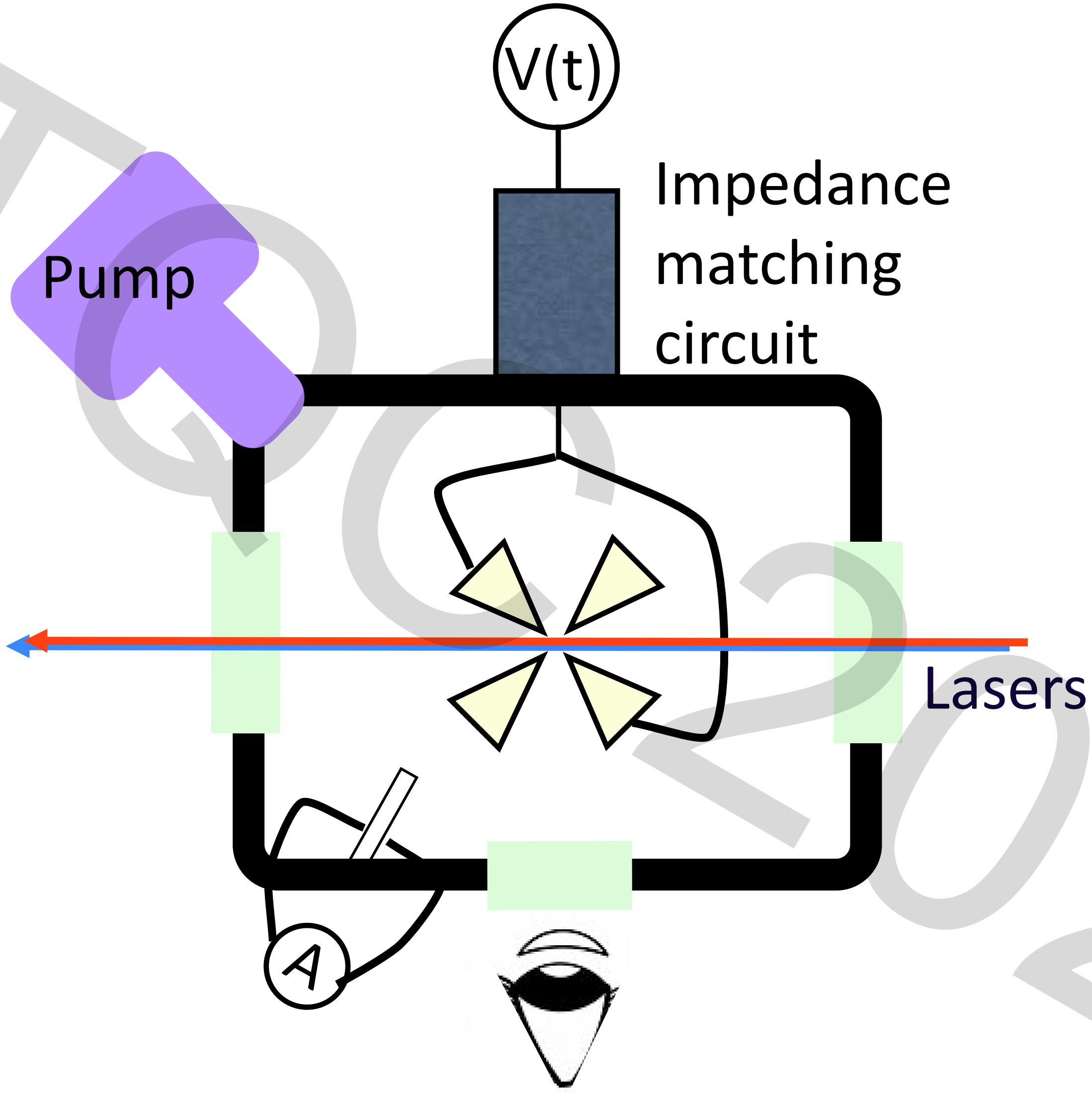
$\omega_z \approx 0.7 - 2\,\text{MHz}$ $\omega_{x,y} \approx 1.5 - 4\,\text{MHz}$

How does it look like?

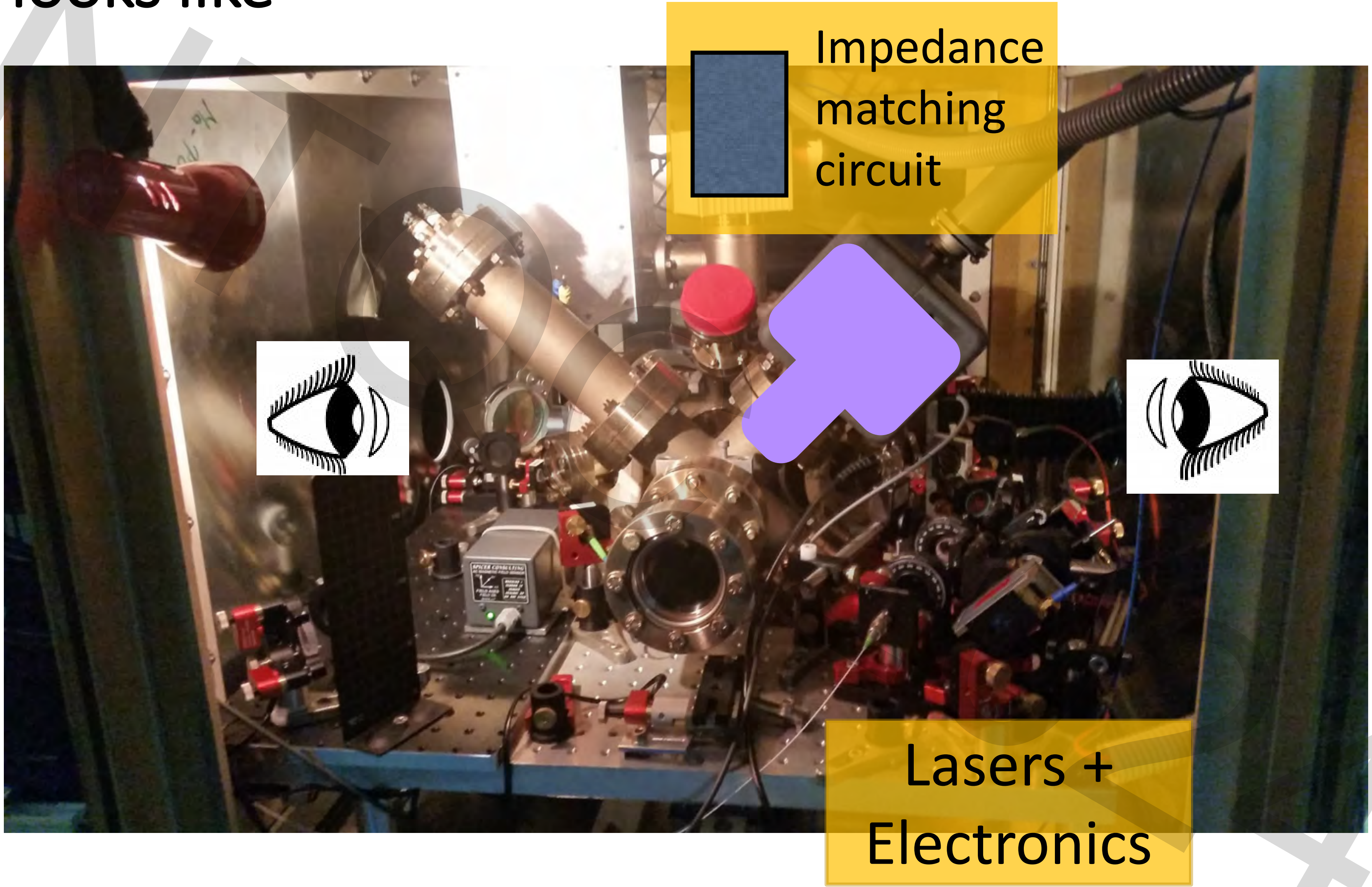




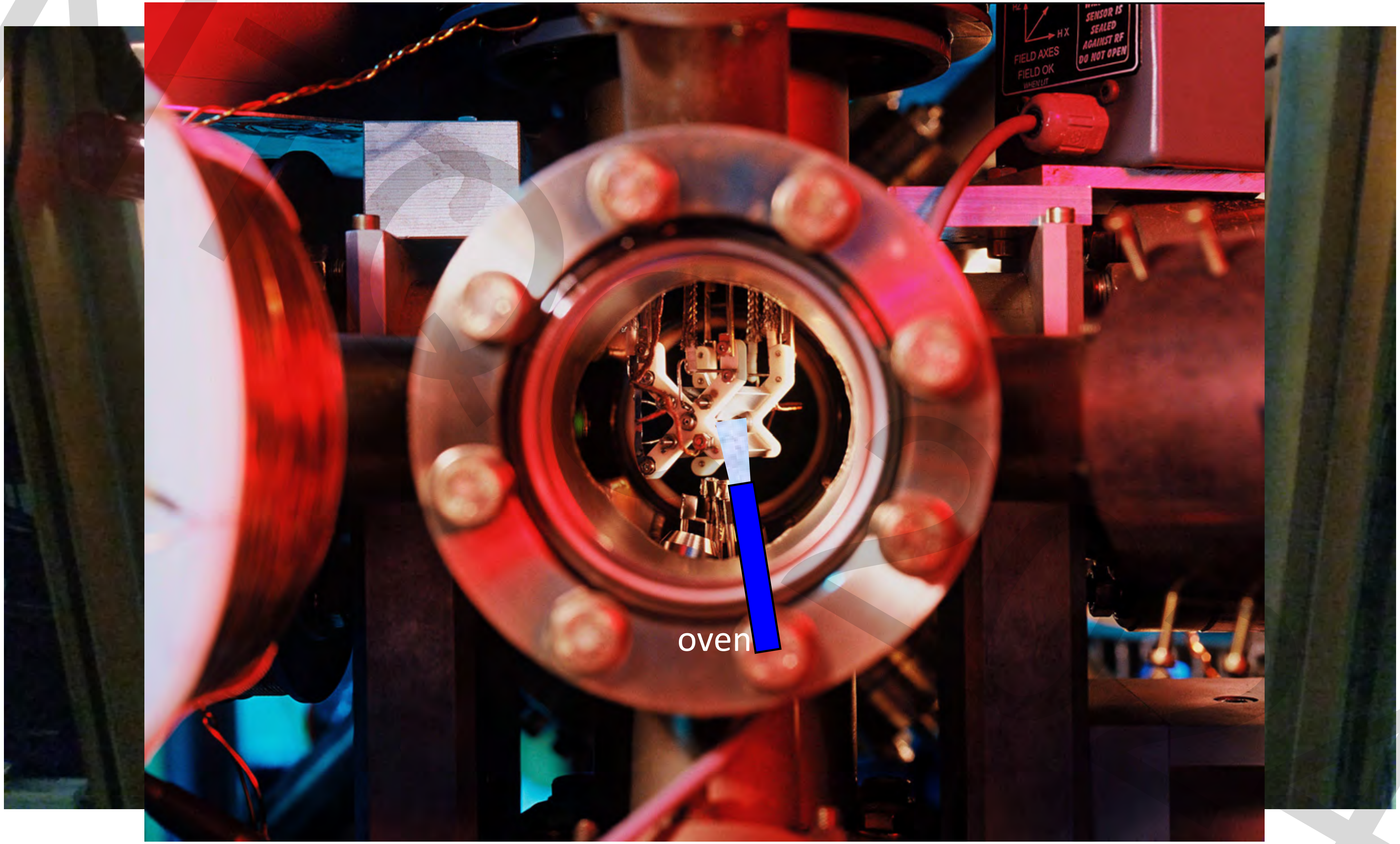
What equipment do I need?



How it looks like



How it looks like



Ion loading

1) An oven (or laser ablation) produces a weak atomic beam of neutral atoms crossing the trap

2) Atoms are ionized within the trap by

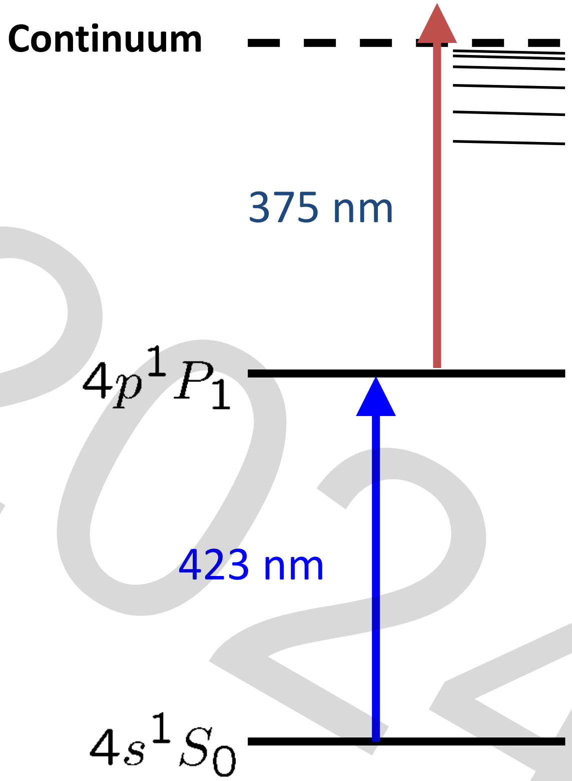
- electron bombardement
- photoionization

(experimentally demonstrated for Mg^+, Ca^+, Cd^+, Yb^+)

Advantages of photoionization:

- higher cross-section
- isotope-selective loading

2-step photoionization of neutral calcium



Summary

✓ Charge particles cannot be trapped in 3D by static fields

✓ Radio-frequency Paul traps are 3D harmonic oscillators

✓ Motion of particle: Mathieu equation have stability region

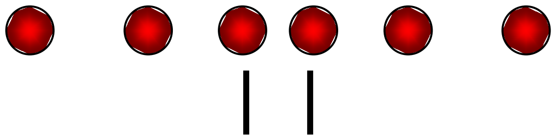
1. Trapping and Cooling Ions

Ion crystals

Equilibrium positions: Minimize potential energy of ions in a linear chain:

$$V=\frac{m\omega_z^2}{2}\sum_{i=1}^{N} z_i(t)^2+\frac{(Ze)^2}{8\pi\varepsilon_0}\sum_{\substack{j,i=1\\ n\neq i}}^{N}\frac{1}{|z_j(t)-z_i(t)|}$$

Coulomb repulsion defines a length scale



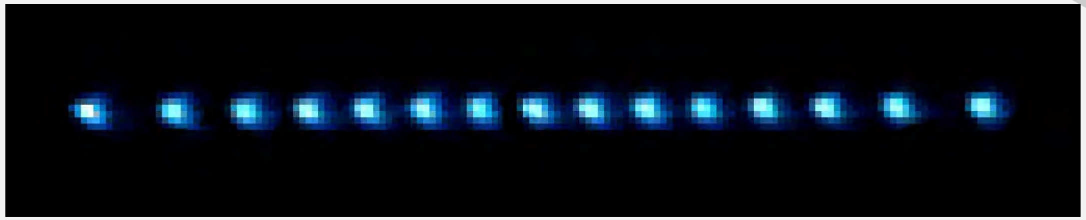
Δz_{min}

$$\Delta z_{min}\approx 2.0 z_s N^{-0.57}$$

$$z_s=\left(\frac{e^2}{4\pi\varepsilon_0 m\omega_z^2}\right)^{\frac{1}{3}}$$

$$^{40}Ca^+,\omega_z=2\pi\cdot 1\mathrm{MHz}$$

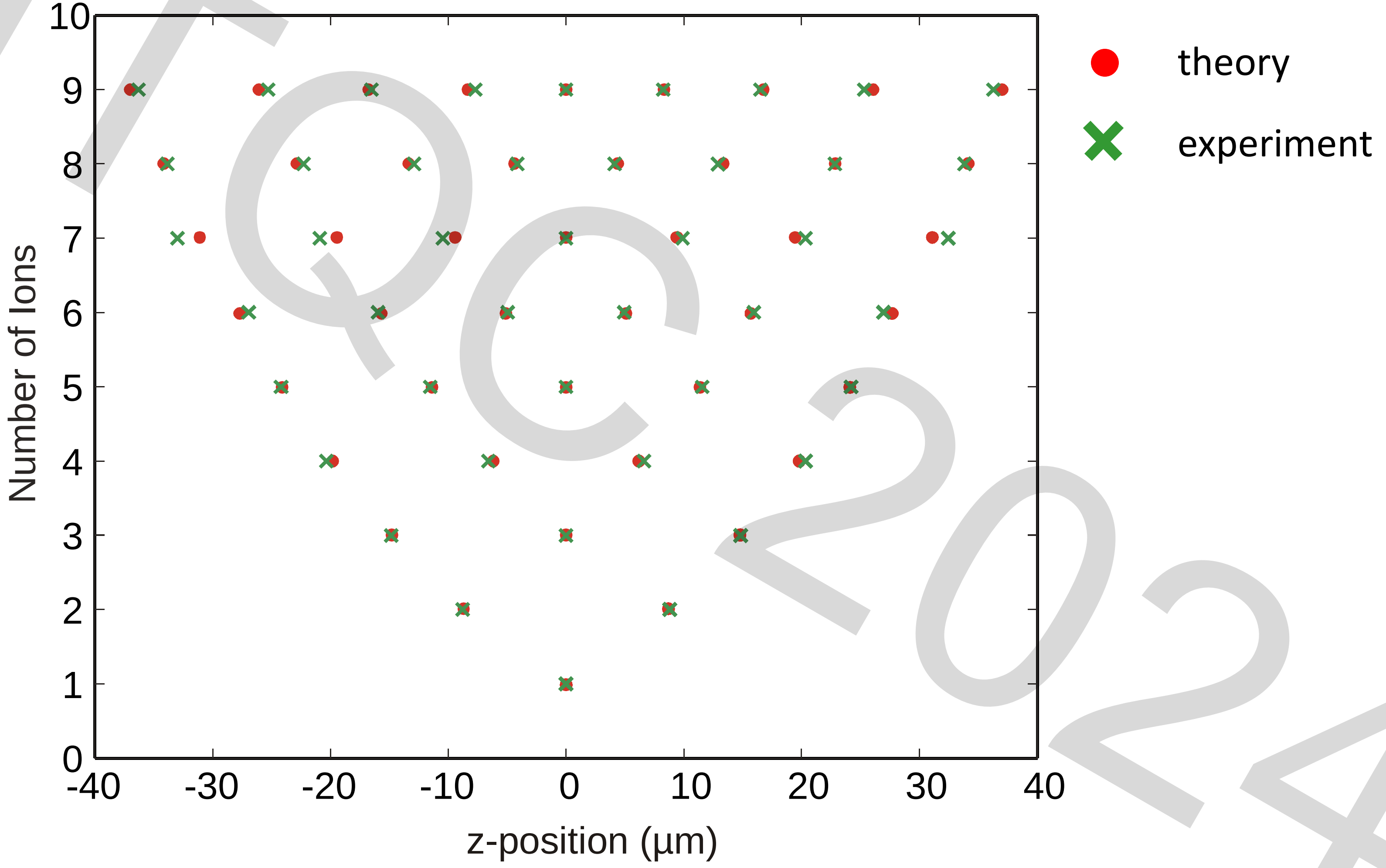
$$\longrightarrow\quad z_s=4.5\mu m$$



universität innsbruck

PhD thesis, Petar Jurcevic
www.quantumoptics.at

Ion strings: experimental positions

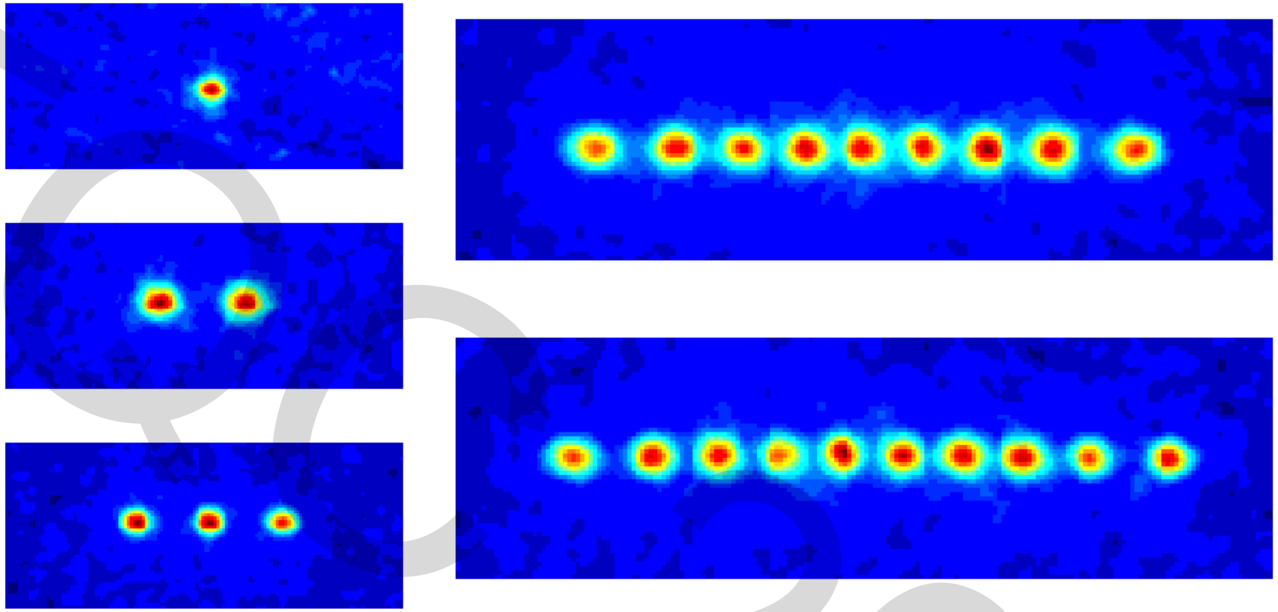


H.C. Nägerl et al., Appl. Phys. B 66, 603 (1998)

Ion strings as quantum registers

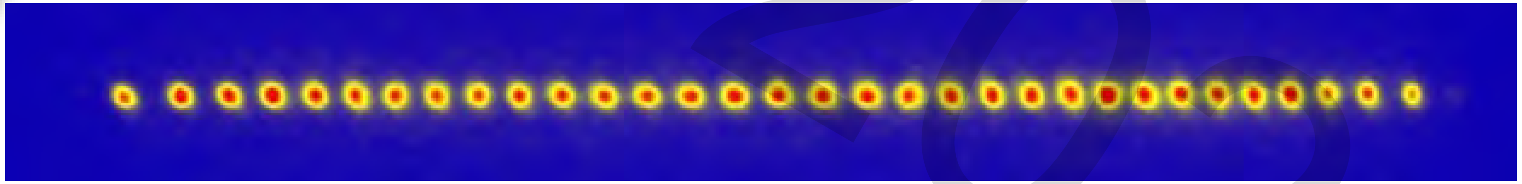
1996 – 2006

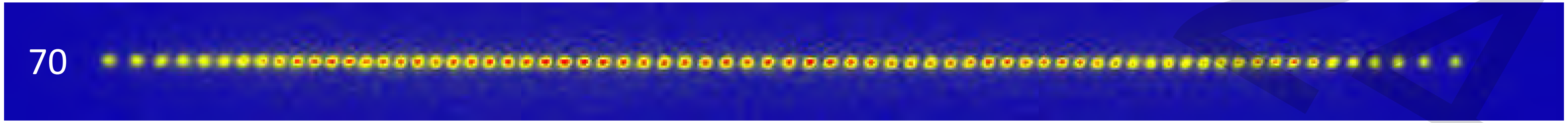
1 - 10 qubits



2007 - 2010

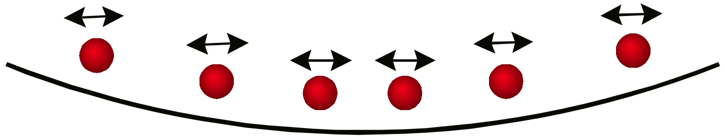
32 qubits





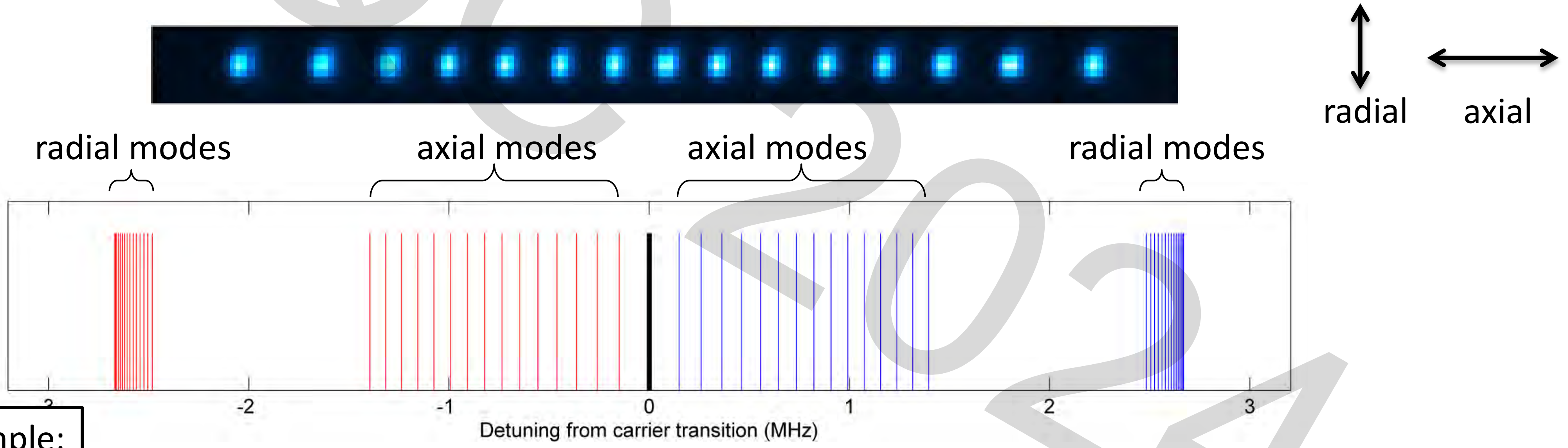
Normal modes of motion

At low temperatures, ions oscillate around their equilibrium positions



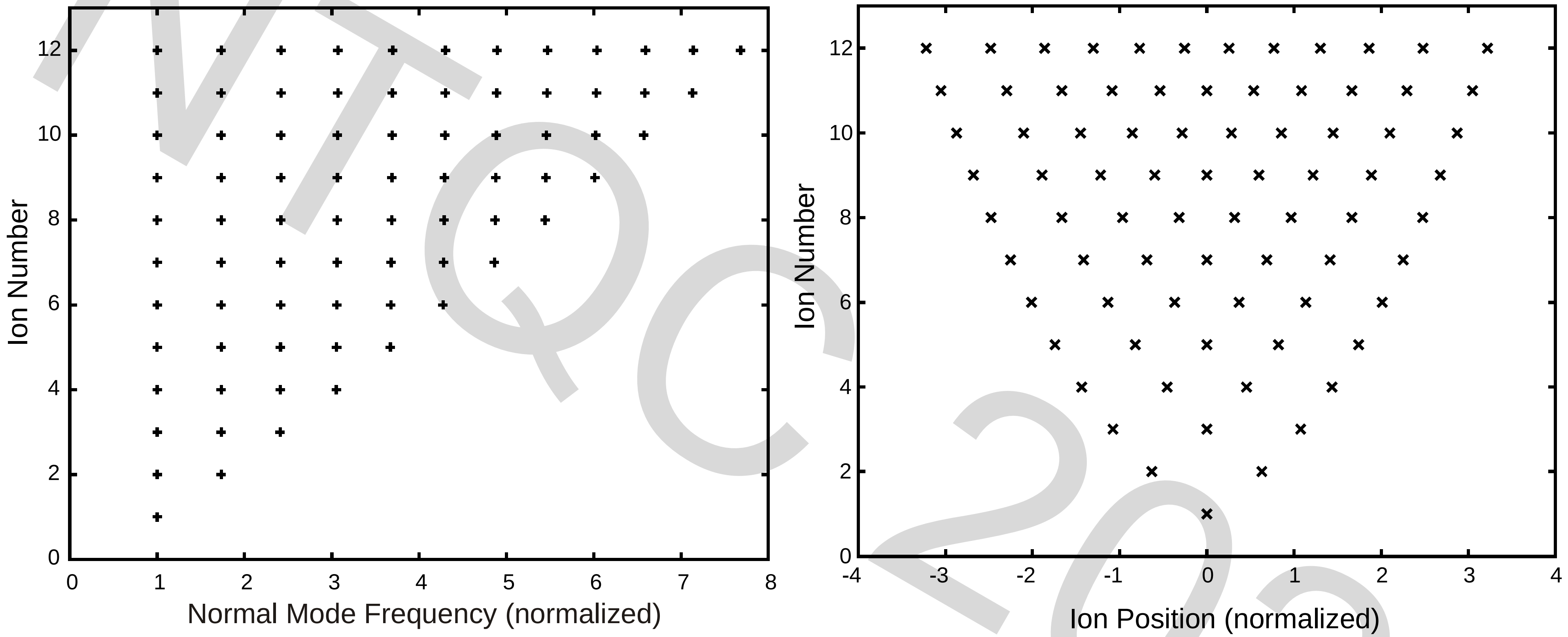
Coulomb interaction: coupling of ion motion

➡ small excitations: collective normal modes



Example:
15 ions

Ion strings: mode frequencies and positions

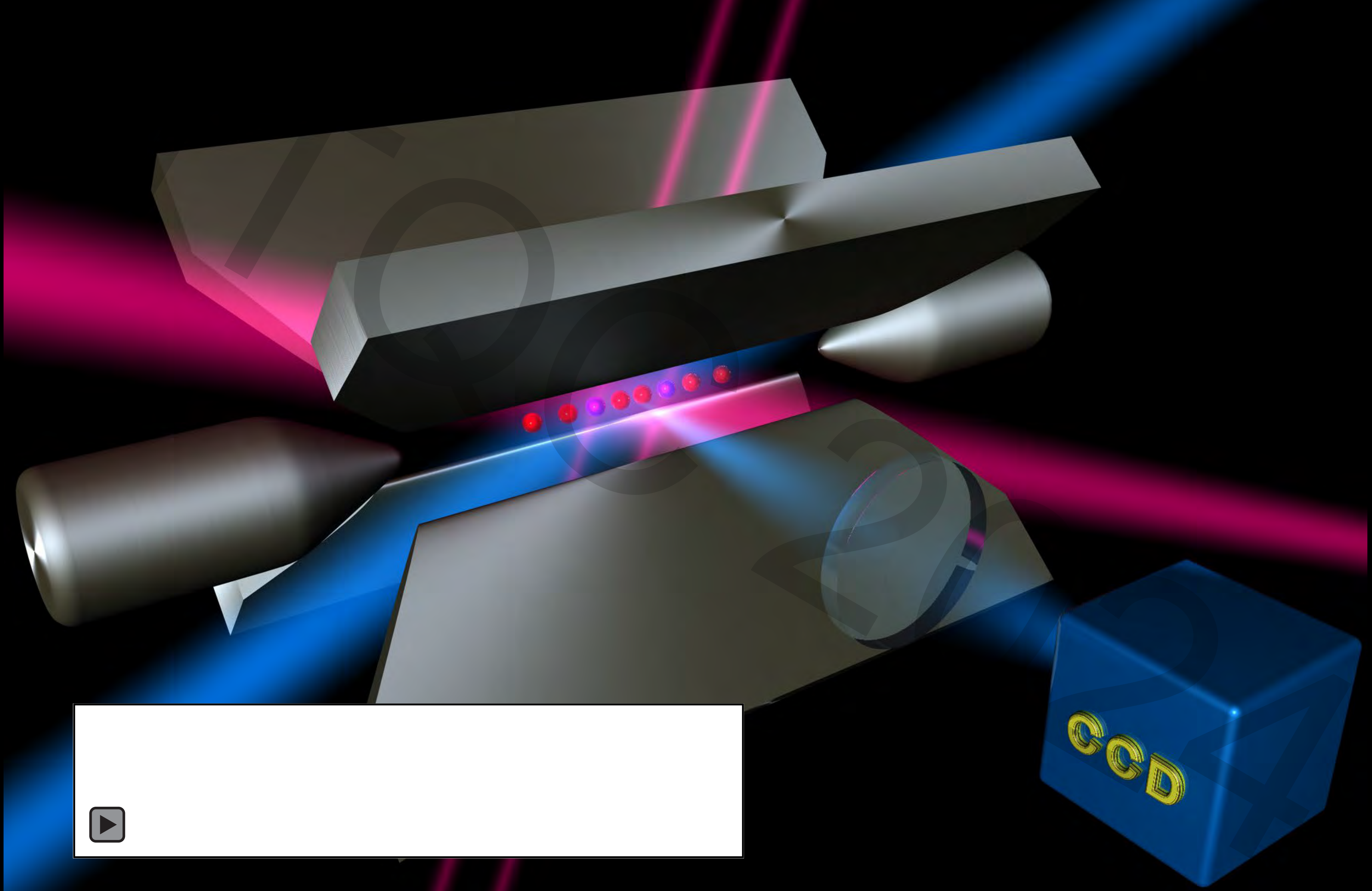


Mode frequencies are nearly independent of ion number N

$$\nu_n = \nu\{1, \sqrt{3}, \sqrt{29/5}, 3.05, 3.67, 4.23, 4.86, 5.44, \ldots\}$$

A. Steane, Appl. Phys. B 64, 623 (1997)
D. James, Appl. Phys. B 66, 181 (1998)

The Innsbruck quantum computer



P. Schindler et al., New. J. Phys. 15, 123012 (2013)

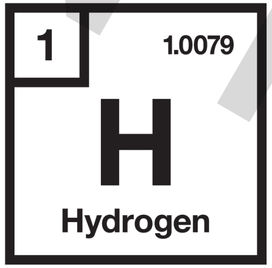
Summary

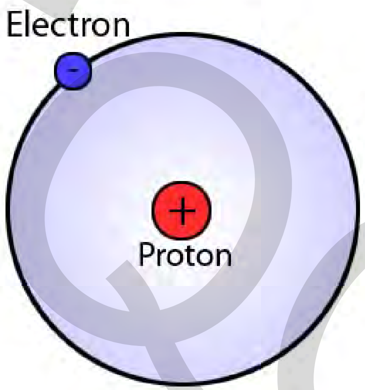
- ✓ Ions in the chain act as coupled oscillators with normal modes
- ✓ Mode frequencies are nearly independent of ion number
- ✓ Ion spacing decreases with ion number

1. Trapping and Cooling Ions

AG Quantenoptik und Spektroskopie

Physicists like it simple

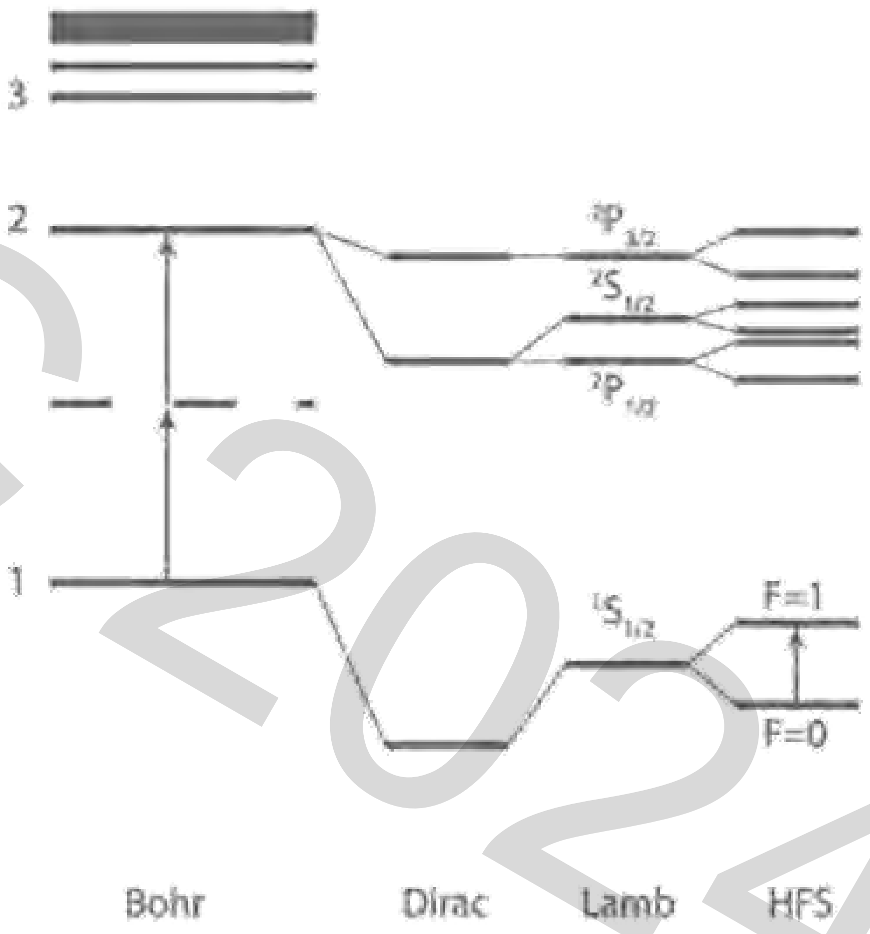




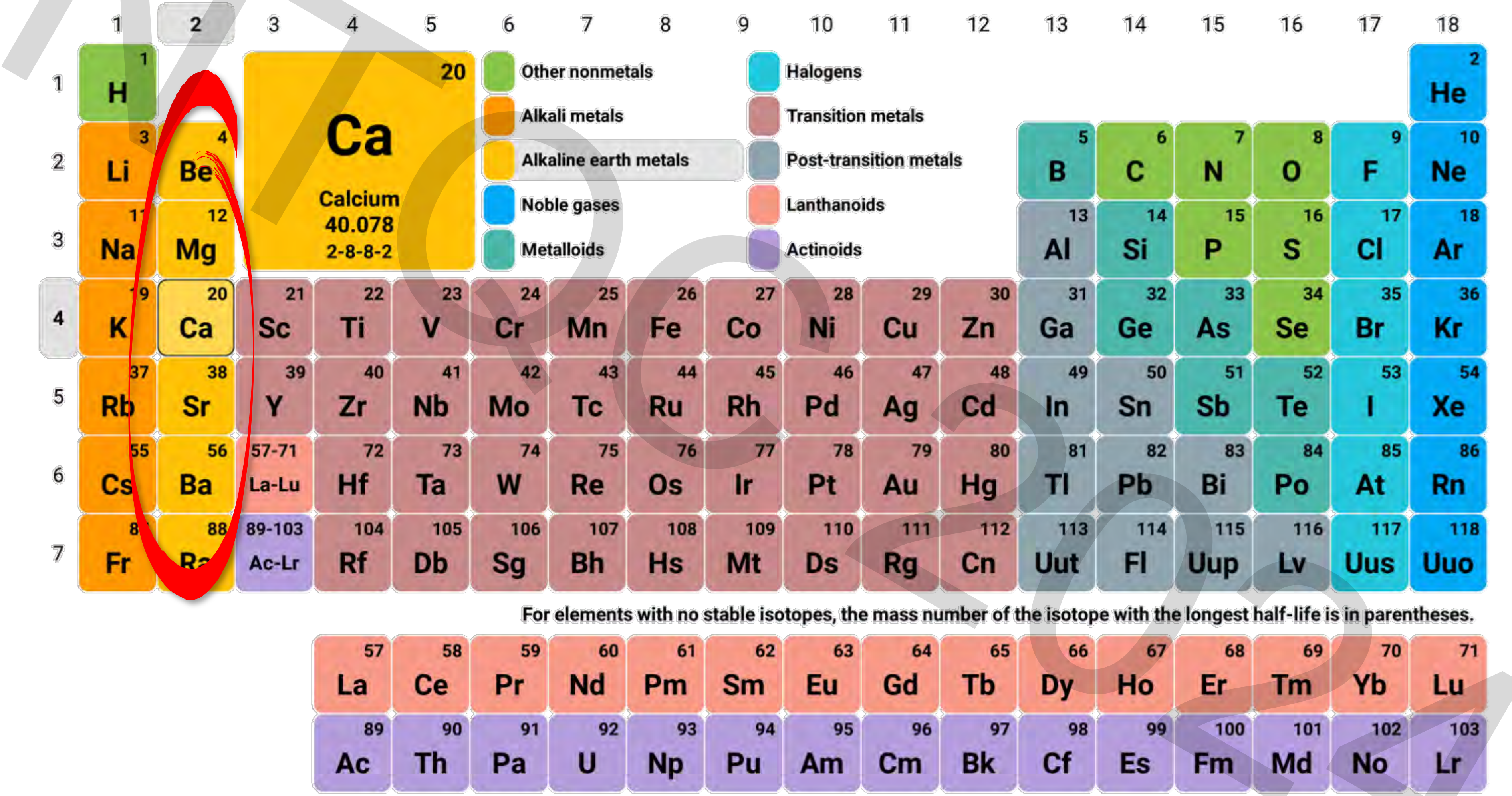
As simple as it gets:

- Only one electron
- No rotations
- No vibrations

Energy levels



Ion trappers' favorites

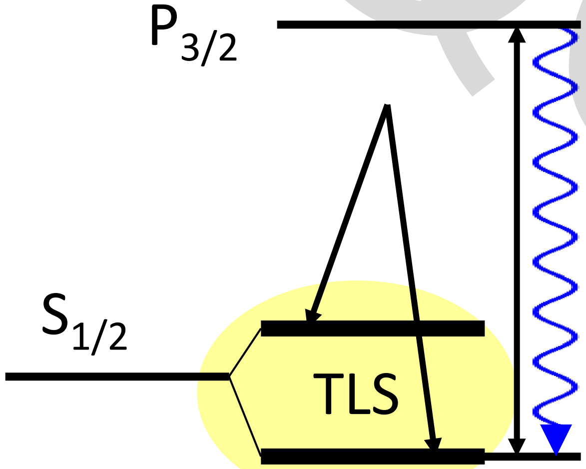


universität innsbruck

Possible qubits

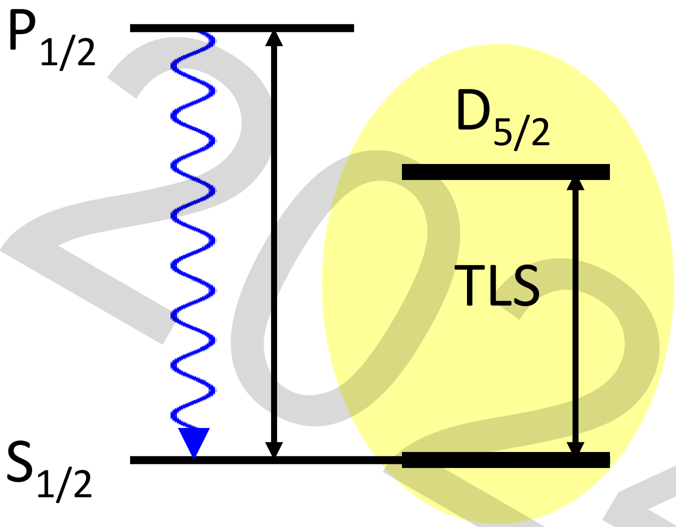
Storing and keeping quantum information requires long-lived atomic states:

- microwave transitions (hyperfine, Zeeman) alkaline earths: $^{9}Be^{+}$, $^{25}Mg^{+}$, $^{43}Ca^{+}$, $^{87}Sr^{+}$, $^{137}Ba^{+}$, $^{111}Cd^{+}$, $^{171}Yb^{+}$



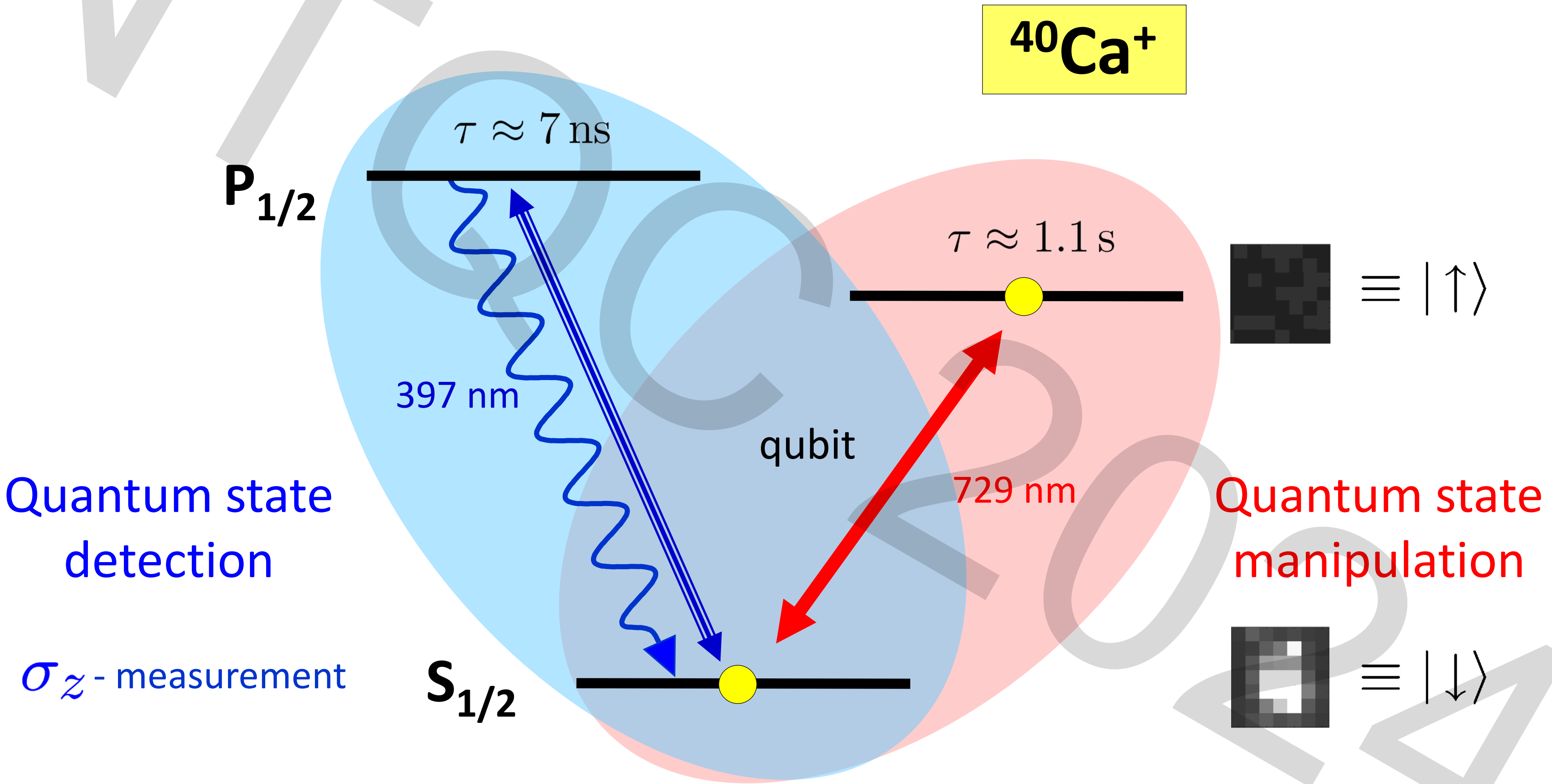
Boulder $^{9}Be^{+}$; Michigan $^{111}Cd^{+}$; Innsbruck $^{43}Ca^{+}$, Oxford $^{43}Ca^{+}$; Maryland $^{171}Yb^{+}$;

- optical transition frequencies (forbidden transitions, intercombination lines) S – D transitions in alkaline earths: Ca^{+}, Sr^{+}, Ba^{+}, Ra^{+}, (Yb^{+}, Hg^{+}) etc.

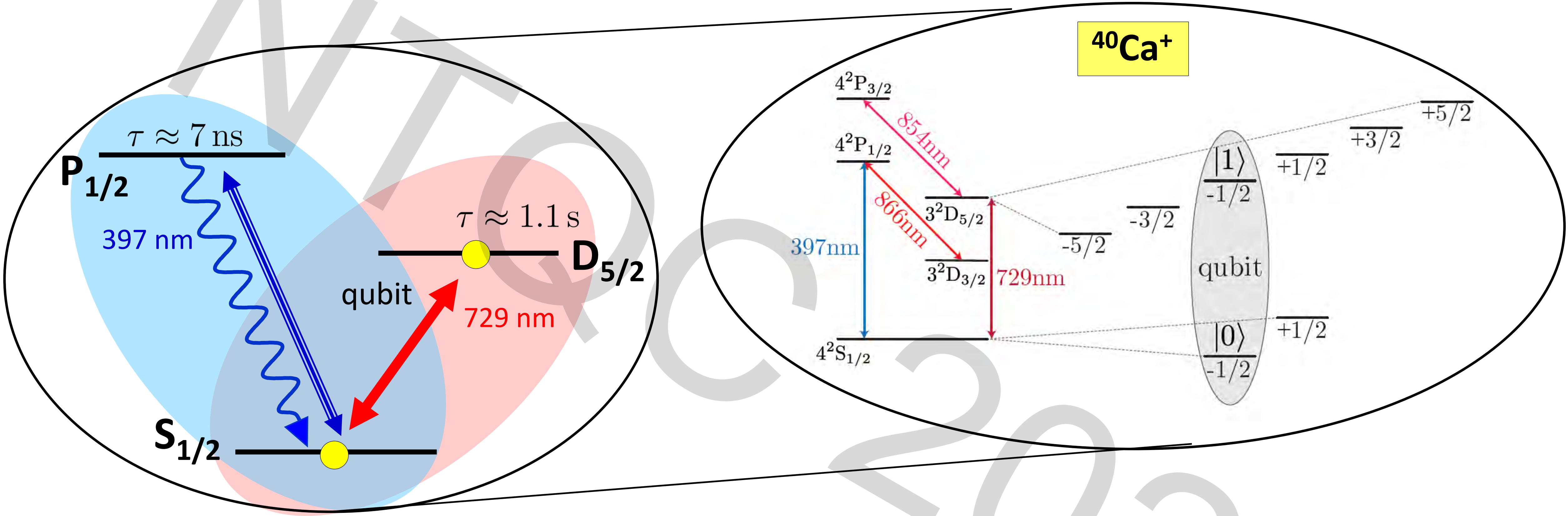


Innsbruck $^{40}Ca^{+}$

Our ion of choice



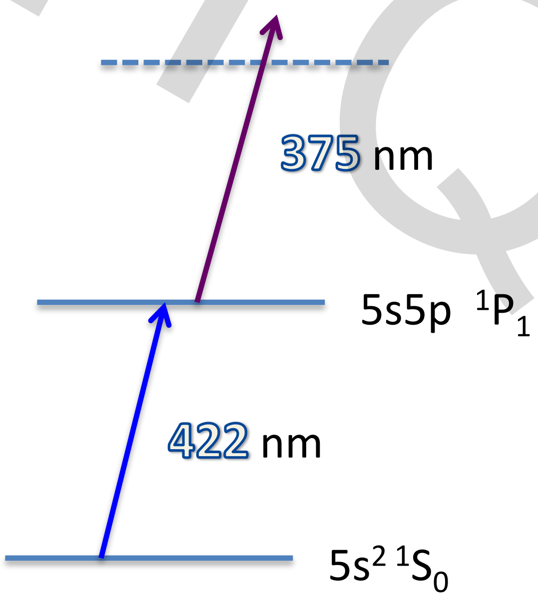
It's a two-level system?



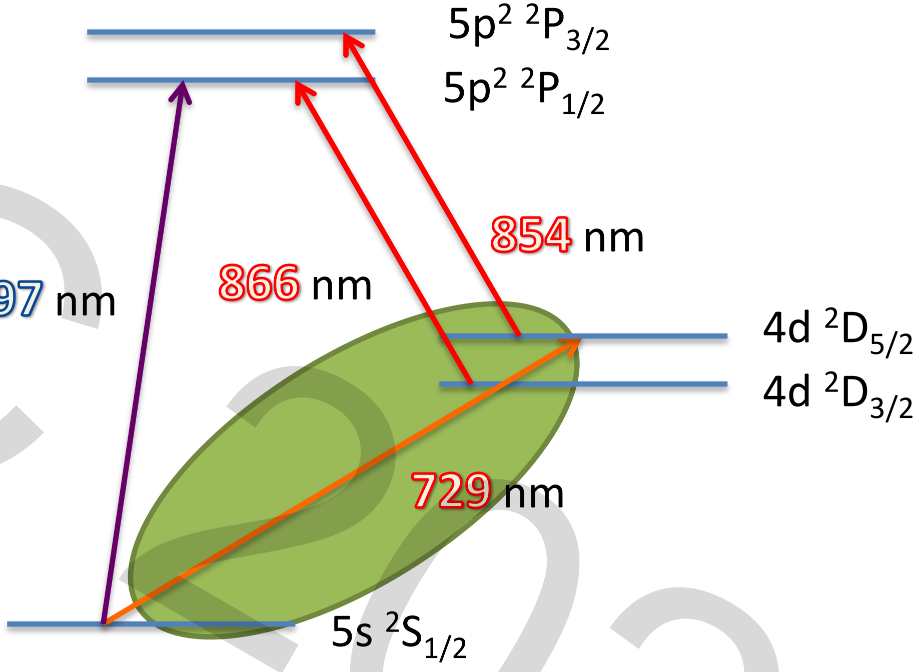
+ motional frequencies

Required lasers

Ca energy levels

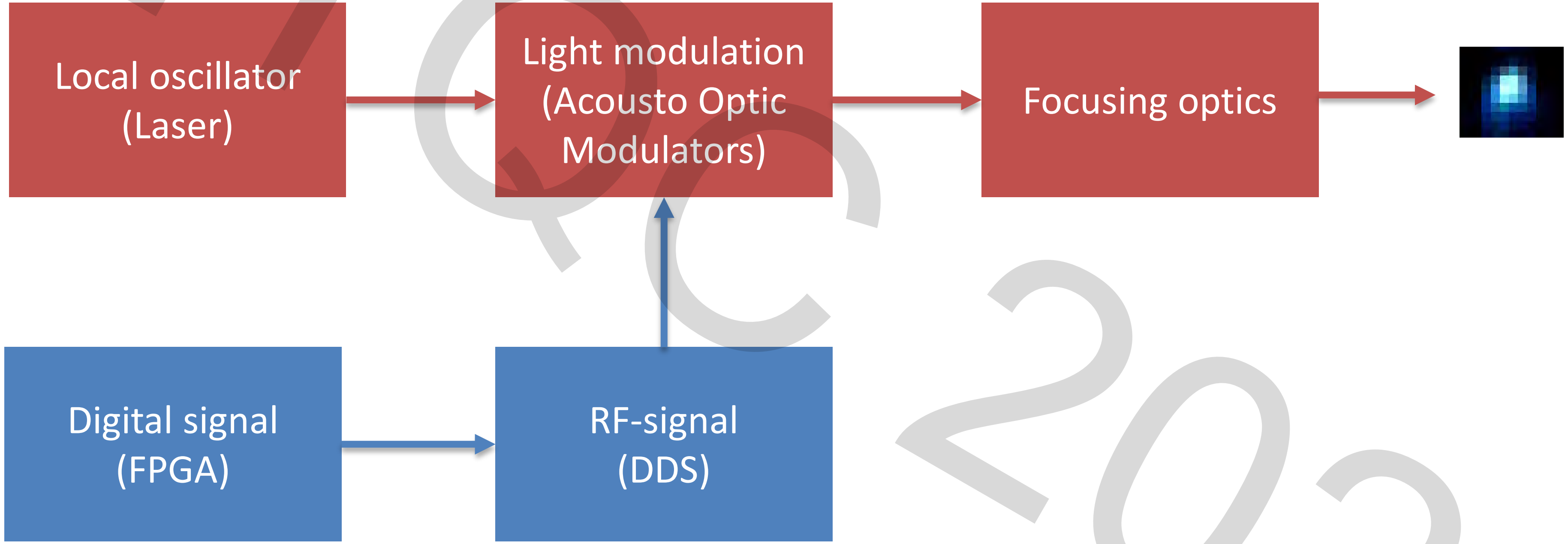


Ca$^+$ energy levels

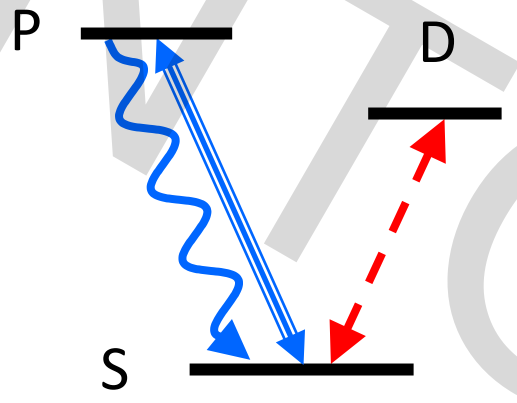


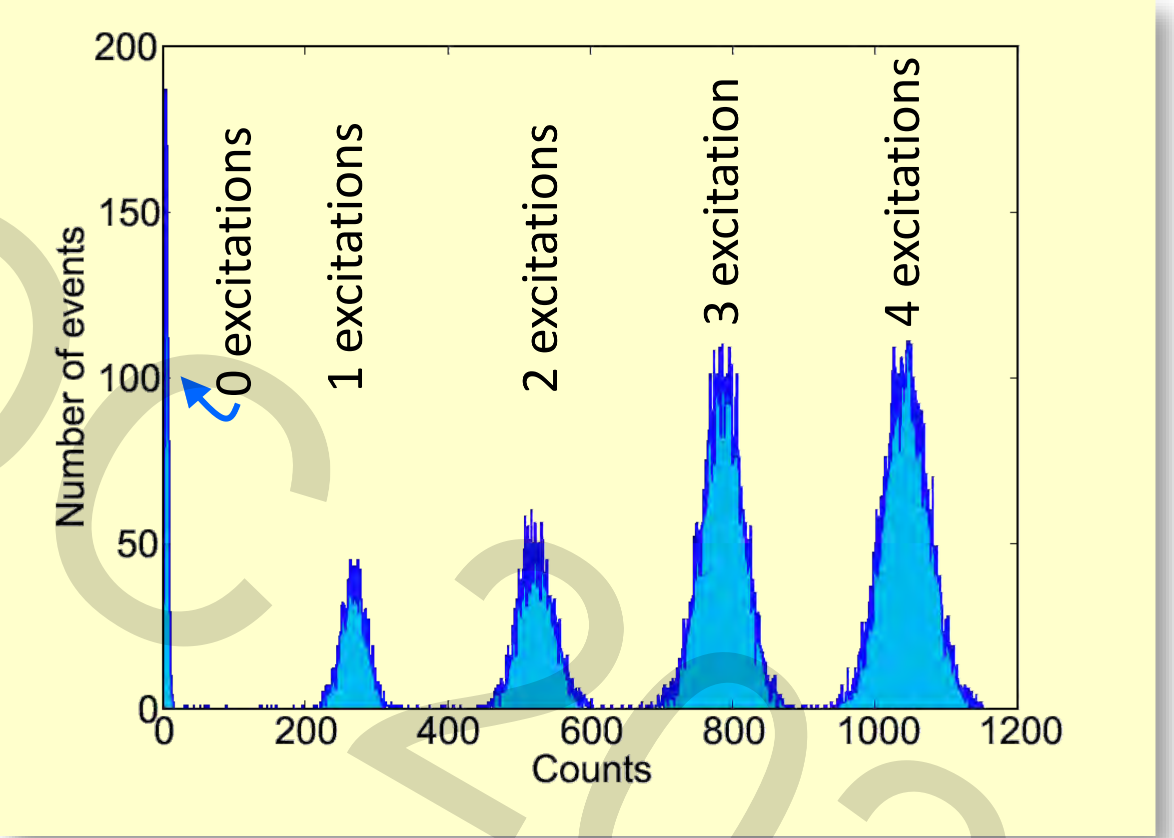
6 laser systems required

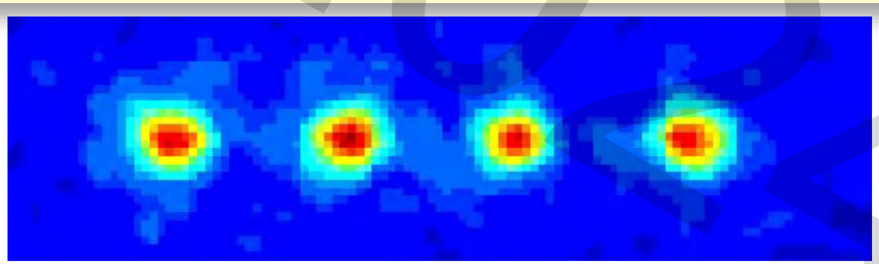
Lasers and Electronics



Qubit measurement







Detection:
Quantum Jumps

- Projection of ions to either S or D states,

Summary

- ✓ Alkali-earth ions are particularly simple
- ✓ There are different possibilities for encoding qubits into ions
- ✓ All ions are multi-level systems

1. Trapping and Cooling Ions

AG Quantenoptik und Spektroskopie

universität innsbruck

Laser-ion interaction

A single ion in an harmonic potential interacting with single-mode laser

$$H = H_0 + H_1$$

$$H_0 = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 + \frac{1}{2}\hbar\nu\sigma_z$$

$$H_1 = \frac{1}{2}\hbar\Omega(\sigma^+ + \sigma^-)\left(e^{i(kx-\nu_L t+\phi)} + e^{-i(kx-\nu_L t+\phi)}\right)$$

k, v, ϕ:wavenumber, frequency and phase of laser radiation

m: mass of the ion

$$\sigma^{\pm} = (\sigma_x \pm i\sigma_y)/2$$

D. Leibfried, C. Monroe, R. Blatt, D. Wineland Rev. Mod. Phys. 75, 281 (2003)

Laser-ion interaction – Lamb-Dicke Parameter

Define Lamb-Dicke parameter $\eta = kx_0 = k\sqrt{\langle (a+a^\dagger)^2 \rangle} = k\sqrt{\frac{\hbar}{2m\omega}}$

$$H_I = \frac{1}{2}\hbar\Omega(\sigma^+ + \sigma^-)\left(e^{i(kx-\nu_L t+\phi)} + e^{-i(kx-\nu_L t+\phi)}\right)$$

$$H_I = \frac{1}{2}\hbar\Omega(\sigma^+ + \sigma^-)\left(e^{i(\eta(a+a^\dagger)-\nu_L t+\phi)} + e^{-i(\eta(a+a^\dagger)-\nu_L t+\phi)}\right)$$

Laser-ion interaction – Interaction Picture

$$H_I = \frac{1}{2}\hbar\Omega(\sigma^+ + \sigma^-)\left(e^{i(\eta(a+a^\dagger)-\nu_L t+\phi)} + e^{-i(\eta(a+a^\dagger)-\nu_L t+\phi)}\right)$$

Transform to the interaction picture

$$H_I = e^{iH_0 t/\hbar}\, H\, e^{-iH_0 t/\hbar}$$

$$H_0 = \frac{\hbar\omega_0}{2}\sigma_z + \hbar\omega_m(a^\dagger a + \tfrac{1}{2})$$

$$H_I = \frac{1}{2}\hbar\Omega(e^{i\omega_0 t}\sigma^+ + e^{-i\omega_0 t}\sigma^-)\cdot \left(e^{i(\eta(ae^{-i\omega_m t}+a^\dagger e^{i\omega_m t})-\nu_L t+\phi)} + e^{-i(\eta(ae^{-i\omega_m t}+a^\dagger e^{i\omega_m t})-\nu_L t+\phi)}\right)$$

define $\hat{a} = ae^{-i\omega_m t}$

Laser-ion interaction – Rotating Wave Approximation

$$H_I = \frac{1}{2}\hbar\Omega(e^{i\omega_0 t}\sigma^+ + e^{-i\omega_0 t}\sigma^-)\cdot$$
$$\left(e^{i(\eta(ae^{-i\omega_m t}+a^\dagger e^{i\omega_m t})-\nu_L t+\phi)} + e^{-i(\eta(ae^{-i\omega_m t}+a^\dagger e^{i\omega_m t})-\nu_L t+\phi)}\right)$$

Rotating Wave Approximation (drop rapidly oscillating terms)

$$H_I = \frac{\hbar\Omega}{2}\left(e^{i\eta(\hat{a}+\hat{a}^\dagger)}\sigma^+ e^{-i\Delta t}e^{i\phi} + e^{-i\eta(\hat{a}+\hat{a}^\dagger)}\sigma^- e^{i\Delta t}e^{-i\phi}\right)$$

with $\hat{a} = ae^{-i\omega_m t}$

$\Delta = \nu_L - \omega_0$

universität innsbruck

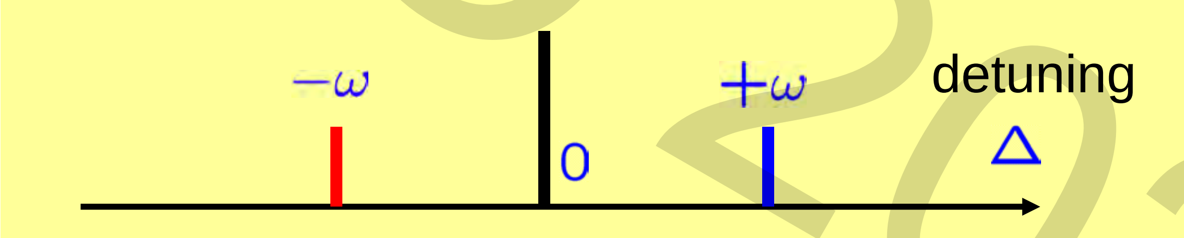
Laser-ion interaction – Lamb-Dicke regime

In the Lamb-Dicke regime $\eta^2(2n+1) \ll 1$

we expand $\exp(i\eta(\hat{a}^\dagger + \hat{a})) = 1 + i\eta(\hat{a}^\dagger + \hat{a}) + \mathcal{O}(\eta^2)$

Carrier

$$\Omega_{n,n} = \Omega(1 - \eta^2 n)$$
$$H_I = \tfrac{1}{2}\hbar\Omega_{n,n}(\sigma^+ + \sigma^-)$$



Red sideband

$$\Omega_{n-1,n} = \eta\sqrt{n}\Omega$$
$$H_I = \tfrac{1}{2}i\hbar\Omega_{n-1,n}(\hat{a}\sigma^+ - \hat{a}^\dagger\sigma^-)$$

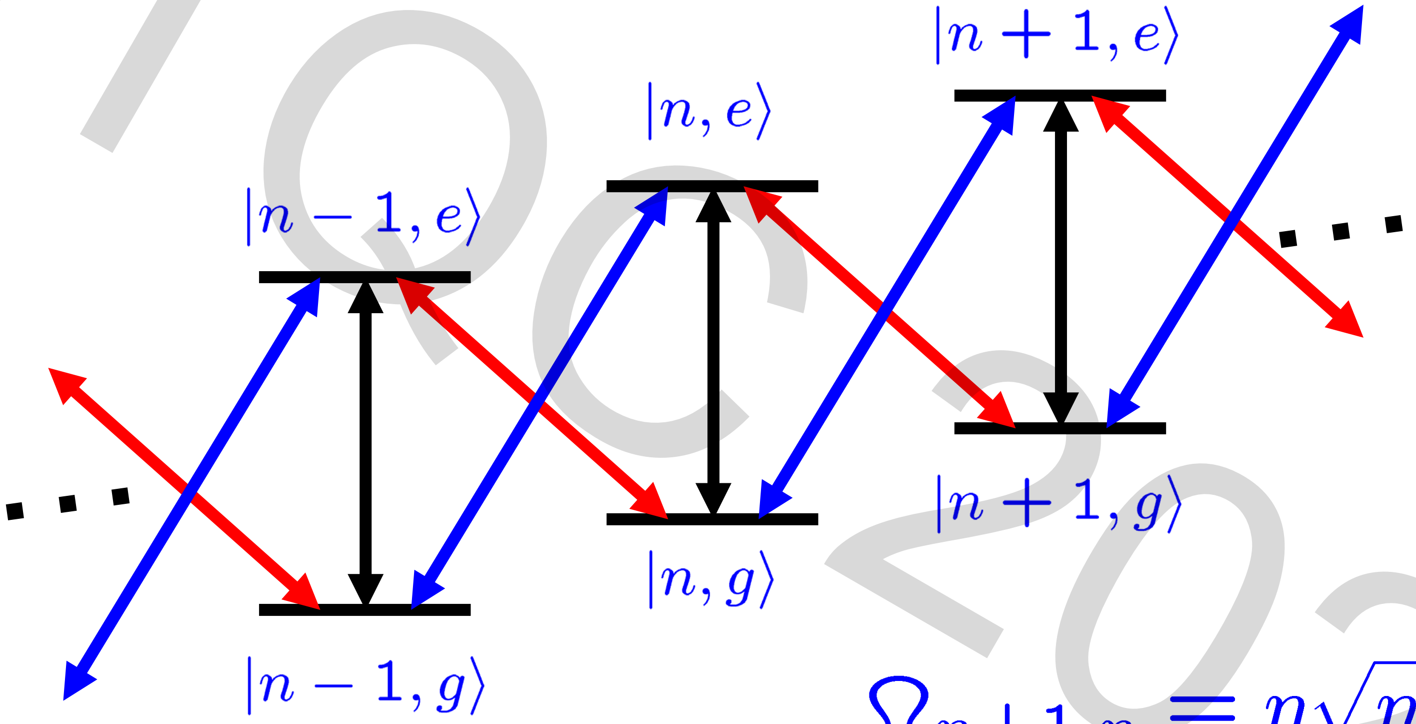
Blue sideband

$$\Omega_{n+1,n} = \eta\sqrt{n+1}\Omega$$
$$H_I = \tfrac{1}{2}i\hbar\Omega_{n+1,n}(\hat{a}^\dagger\sigma^+ - \hat{a}\sigma^-)$$

D. Leibfried et al, Rev. Mod. Phys. 75, 281-324 (2003)

Interaction in the ladder structure

carrier $\Omega_{n,n} = (1 - \eta^2 n)\Omega$



$\Omega_{n+1,n} = \eta\sqrt{n+1}\Omega$

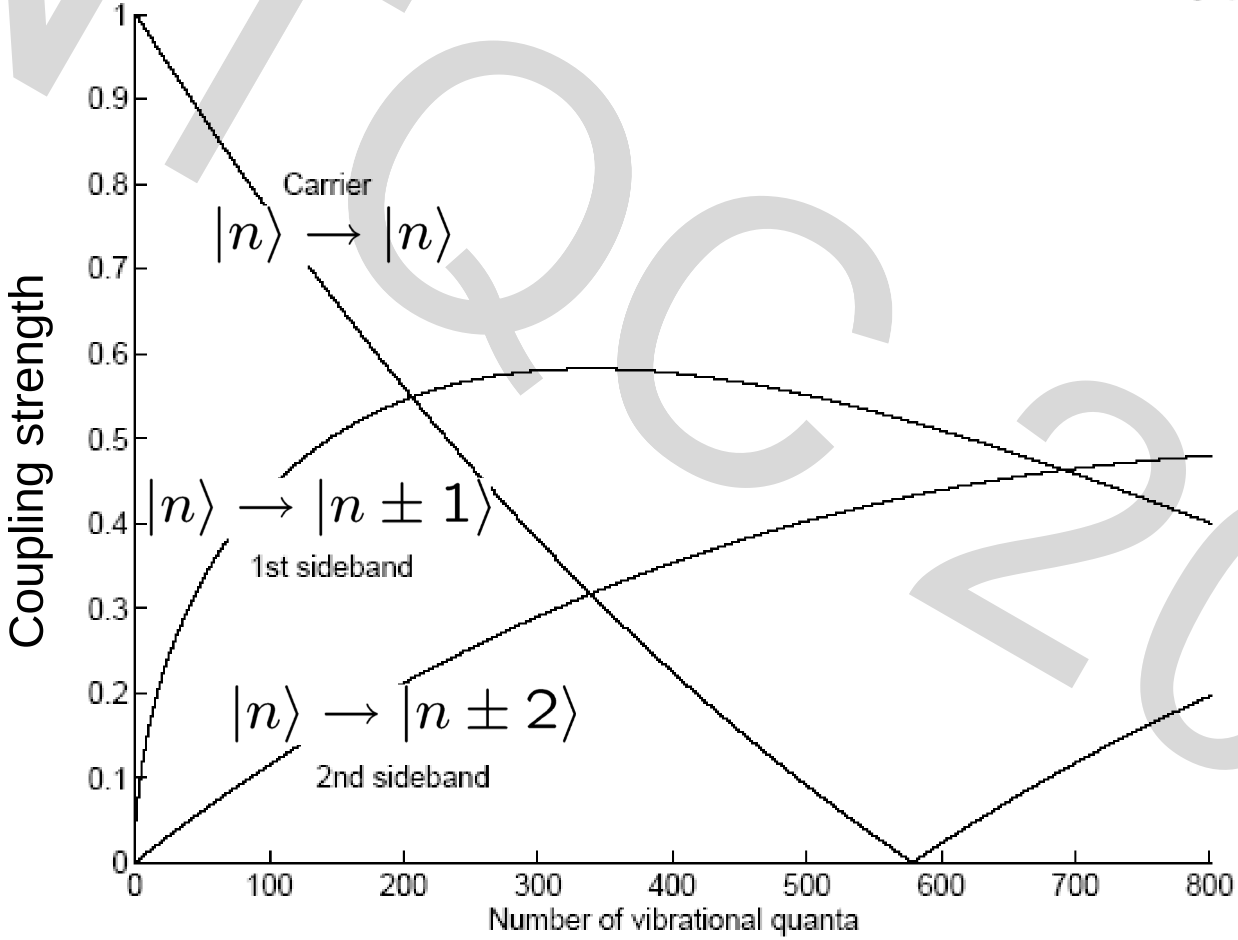
blue sidebands

$\Omega_{n-1,n} = \eta\sqrt{n}\Omega$

red sidebands

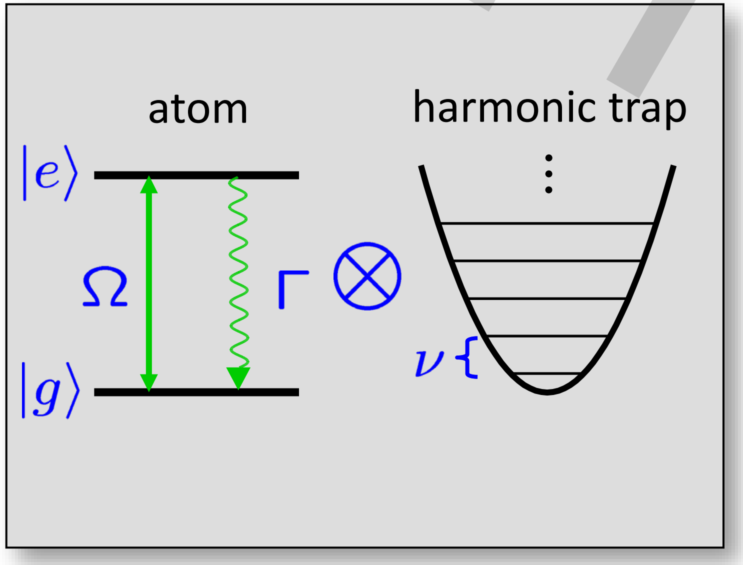
Coupling strength beyond the Lamb Dicke regime

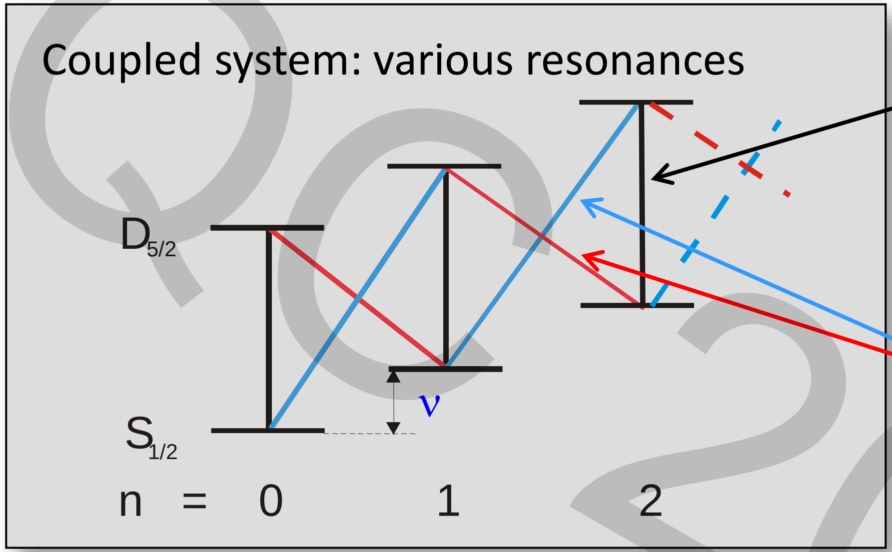
$$\langle n+m|e^{i\eta(\hat{a}+\hat{a}^\dagger)}|n\rangle = \exp(-\frac{\eta^2}{2})\eta^{|m|}L_n^{|m|}(\eta^2)\left(\frac{n!}{(n+m)!}\right)^{\mathrm{sign}(m)/2}$$



D. Leibfried et al, Rev. Mod. Phys. 75, 281-324 (2003)

Quantum state manipulation: Carrier and Sidebands



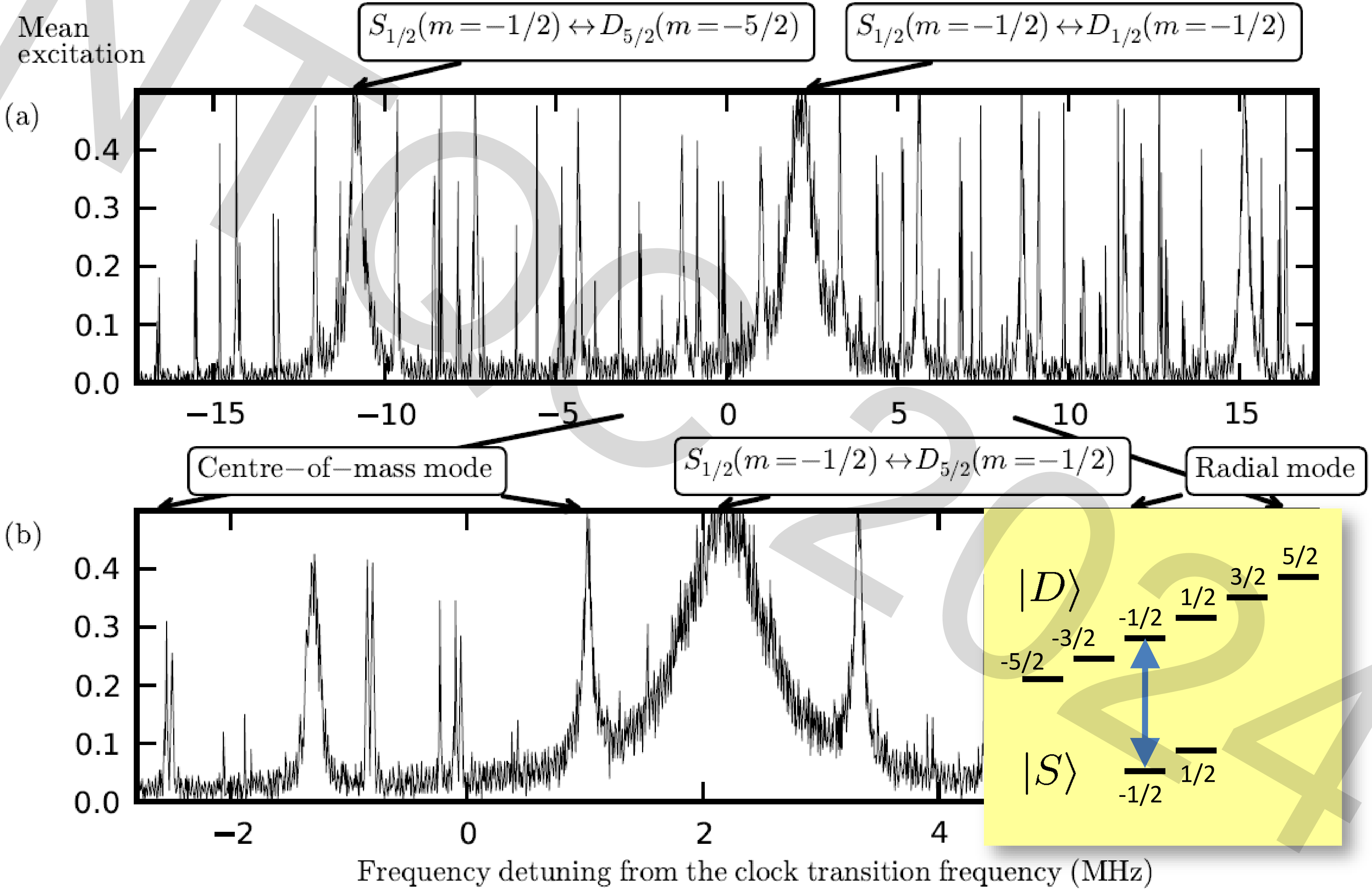


Carrier:
manipulate Q Info
→ internal superpositions

Sidebands:
manipulate motion and Q Info
→ create entanglement

P. Schindler, at al., New. J. Phys. 15, 123012 (2013)

Ca40 Spectroscopy



Summary

Lamb-Dicke regime:

Extension of the ion‘s wave function Ψ much smaller than optical wavelength

$$\eta\sqrt{\langle\Psi|(a+a^\dagger)^2|\Psi\rangle} \ll 1$$

Taylor expansion to first order:

$$H_{int} = \frac{\hbar\Omega}{2}\sigma_+\{\mathbb{1}+i\eta(e^{-i\nu t}a+e^{i\nu t}a^\dagger)\}e^{-i\delta t+i\phi} + h.c.$$

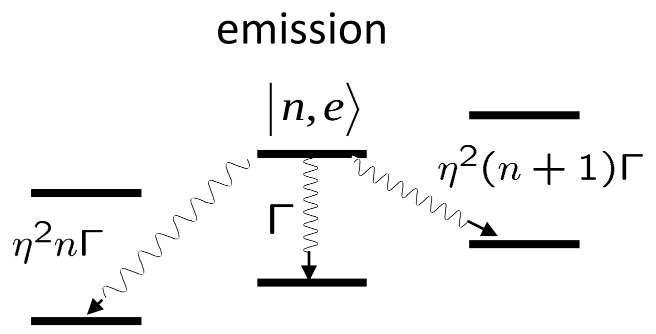
1. Trapping and Cooling Ions

AG Quantenoptik und Spektroskopie

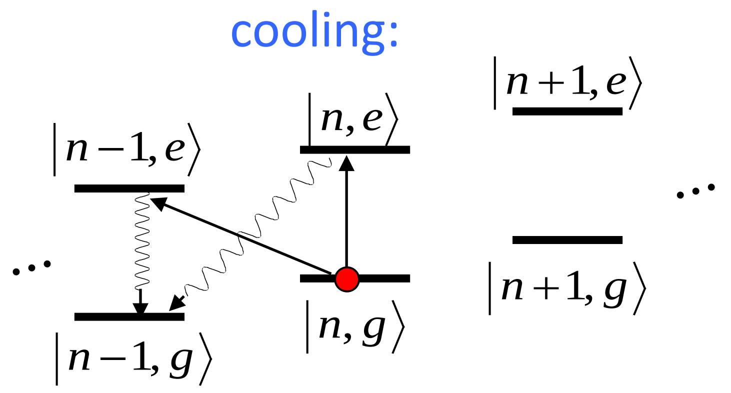
universität innsbruck

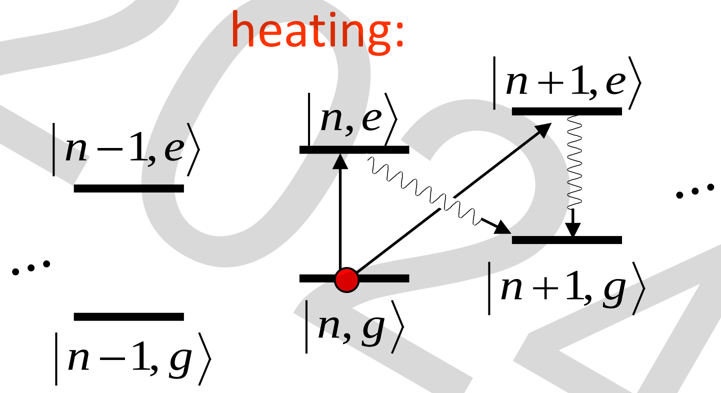
Laser cooling

In the Lamb-Dicke regime,
spontaneous photons
rarely change the motional state |n>:



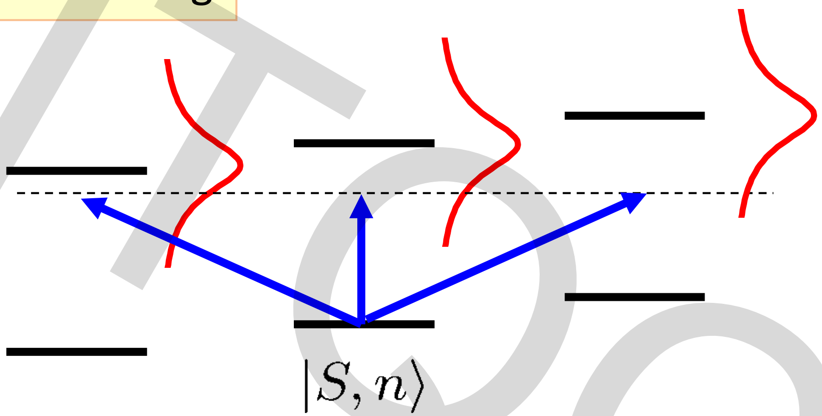
Physical processes that change *n*, in lowest order of *η*





Laser cooling regimes

Doppler cooling

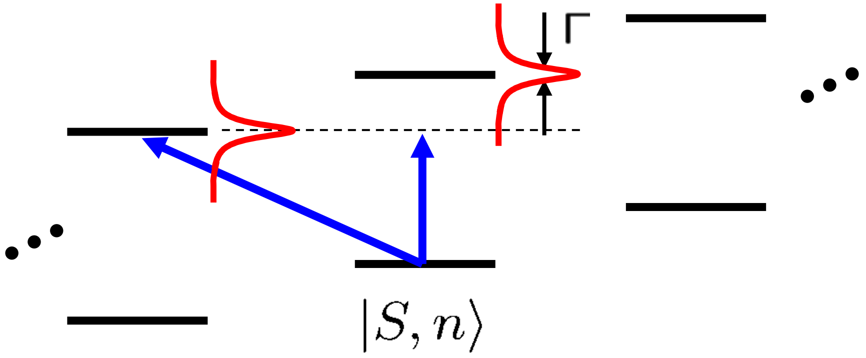


$\nu \ll \Gamma$ **weak** confinement, Doppler cooling

$$\langle n \rangle = \frac{\Gamma}{2\nu} > 1$$

if laser detuned by $\Delta = -\Gamma/2$

Sideband cooling



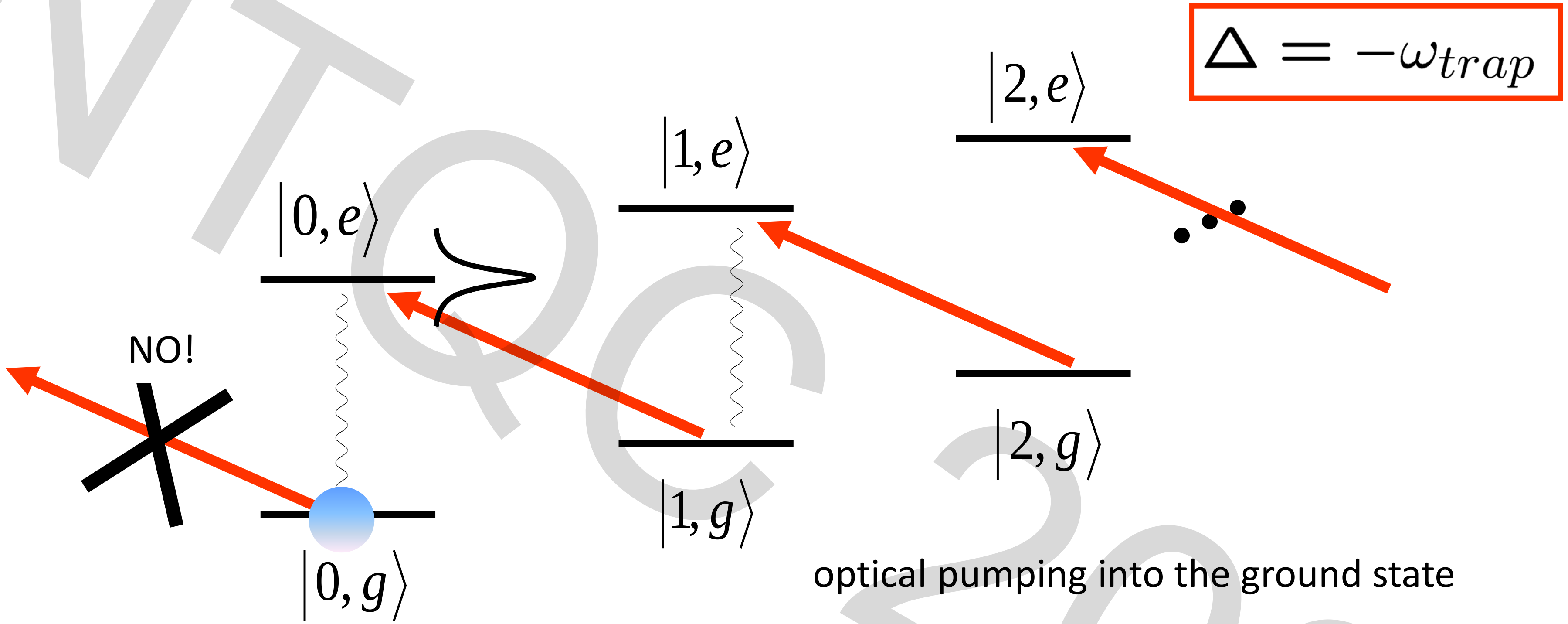
$\nu \gg \Gamma$ **strong** confinement, sideband cooling

$$\langle n \rangle = \frac{\Gamma^2}{4\nu^2} \ll 1$$

if laser detuned by $\Delta = -\nu$

S. Stenholm, Rev. Mod. Phys. 58, 699 (1986)

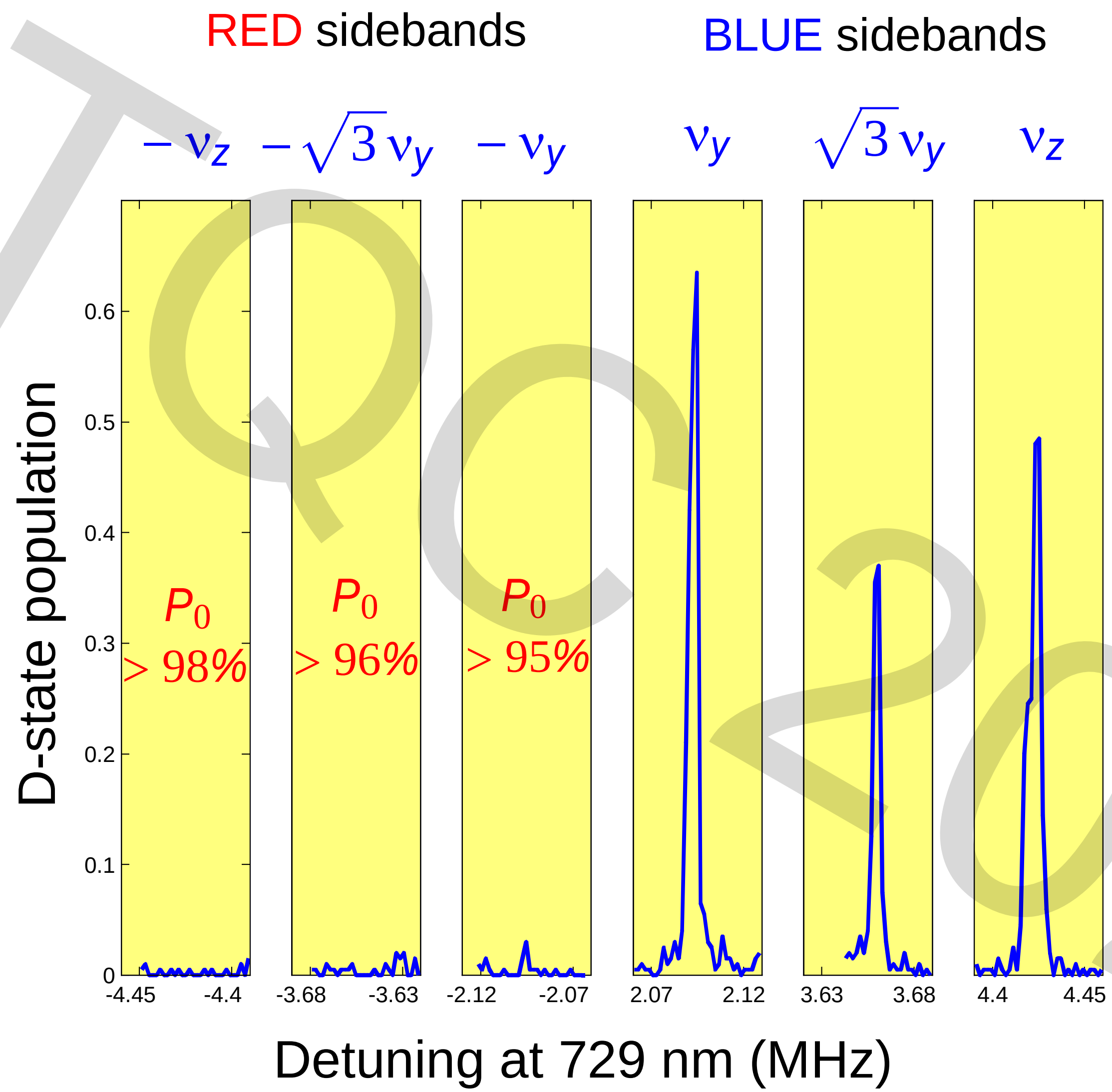
Sideband cooling



optical pumping into the ground state

Signature: no further excitation possible
„dark state“ |0>

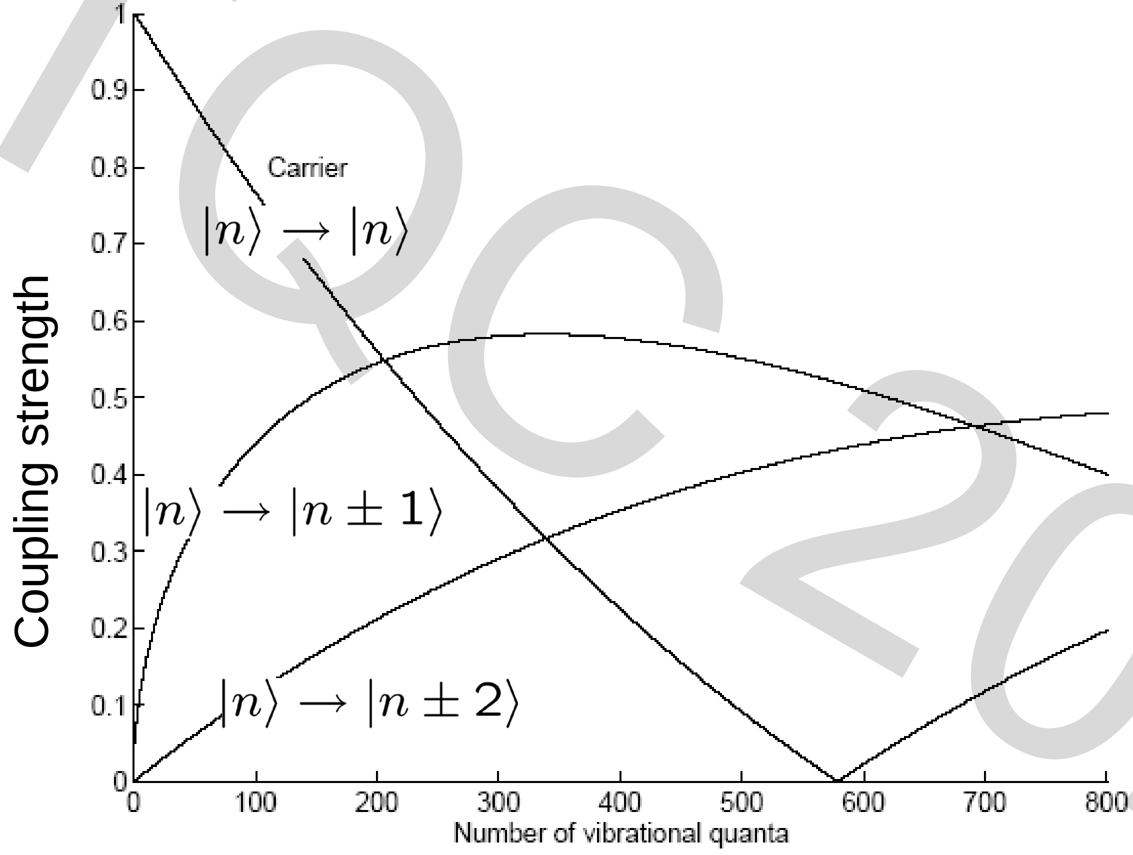
Measuring temperature using sidebands



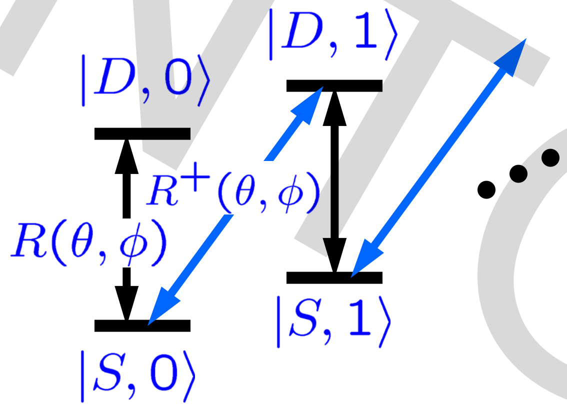
universität innsbruck

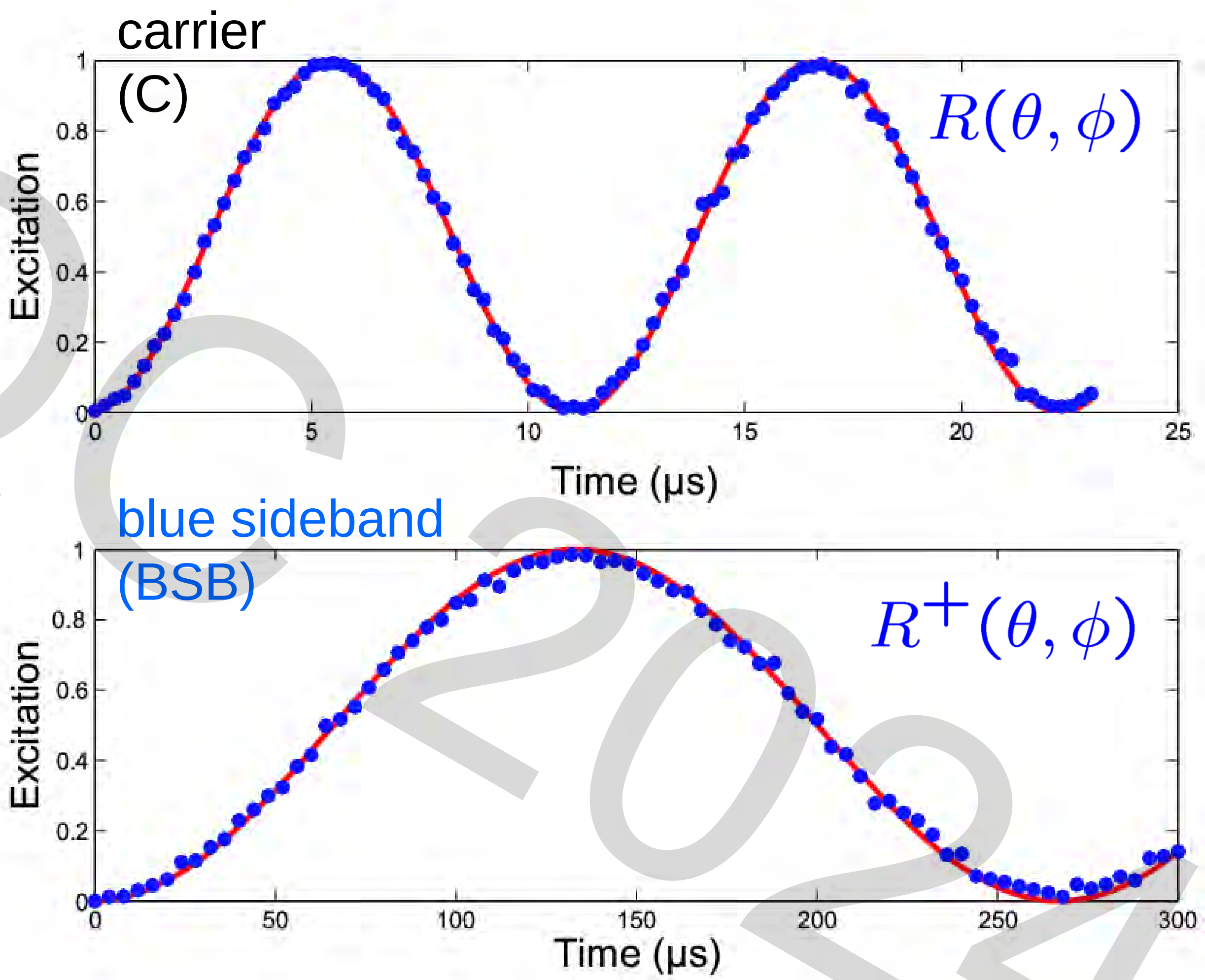
Measuring the temperature of an ion

Recall



Measuring temperature using Rabi flops



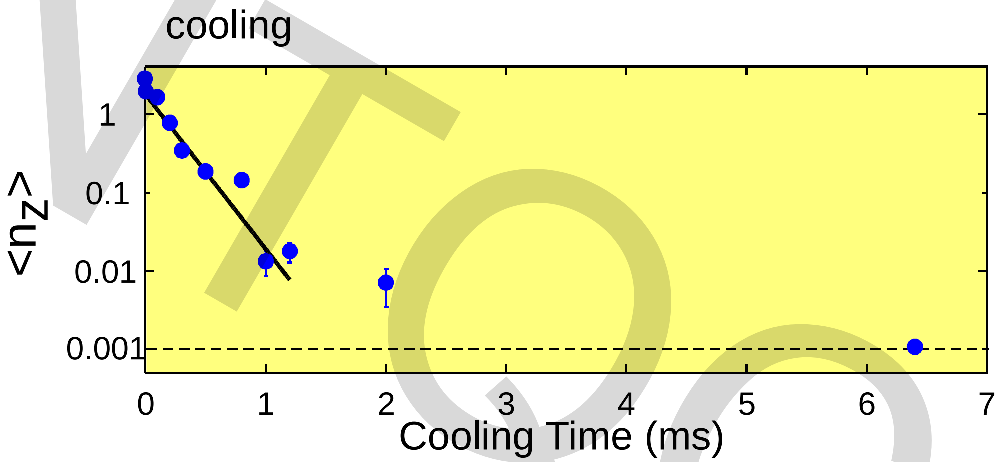


carrier and sideband
Rabi oscillations
with Rabi frequencies

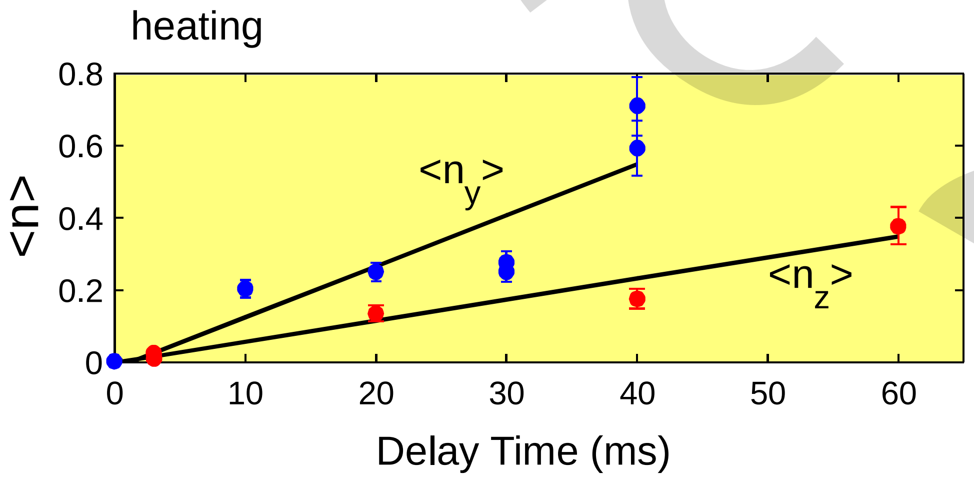
$$\Omega, \quad \eta\Omega\sqrt{n+1}$$

$\eta = kx_0$ Lamb-Dicke parameter

Cooling and Heating



cooling: $0.2 \frac{\text{ms}}{\text{phonon}}$



heating:

radial: $70 \frac{\text{ms}}{\text{phonon}}$

axial: $190 \frac{\text{ms}}{\text{phonon}}$

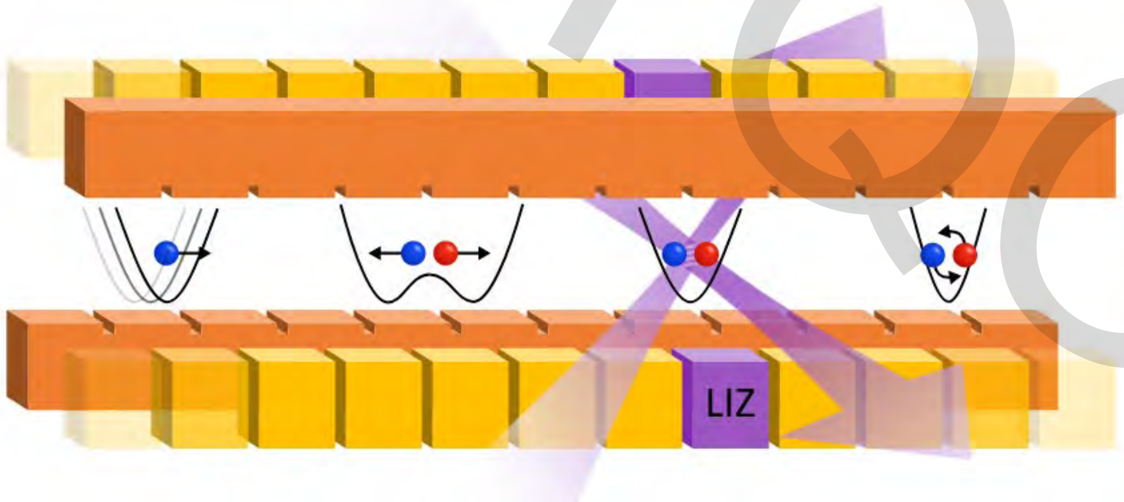
1. Trapping and Cooling Ions

universität innsbruck

AG Quantenoptik und Spektroskopie

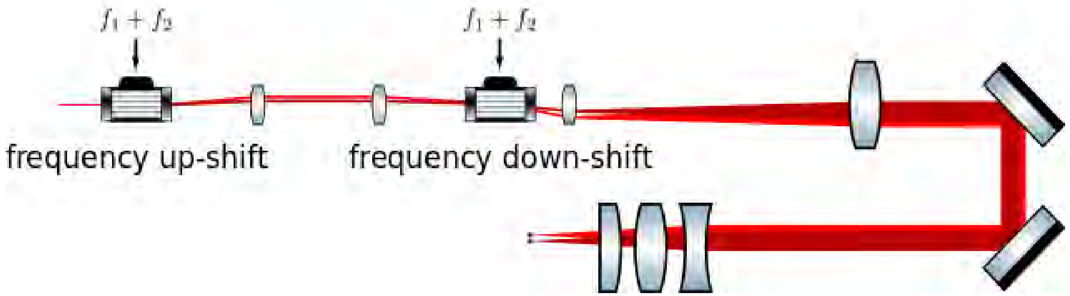
Single ion addressing

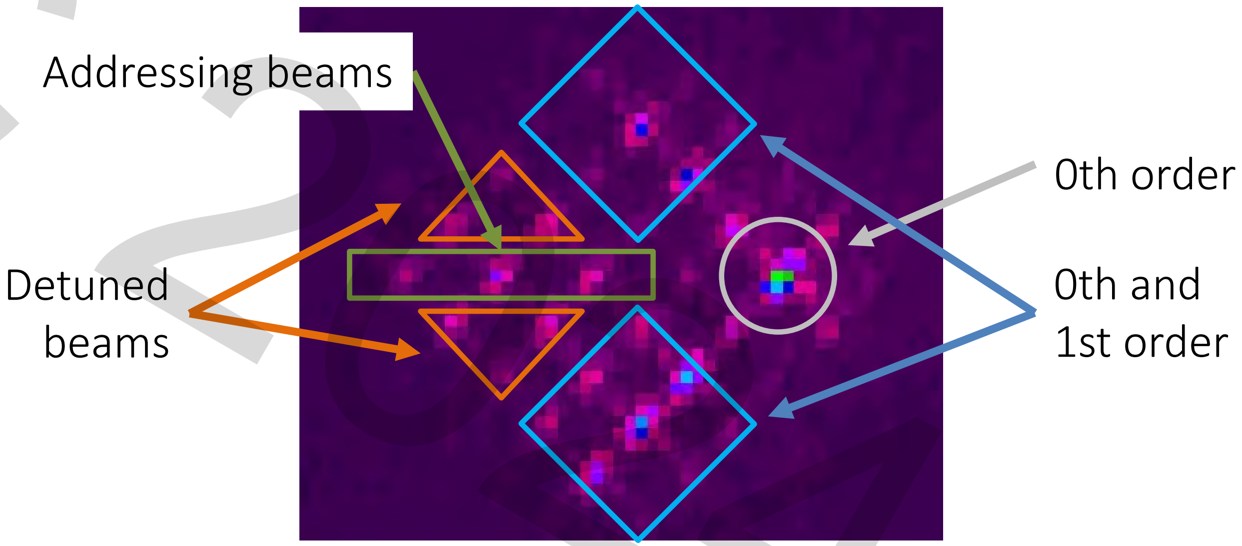
Option 1: Move the ions



V. Kaushal et al, *AVS Quantum Sci.* 2, 014101 (2020)

Option 2: Move the laser



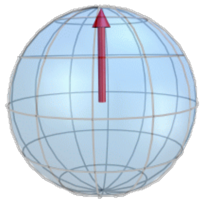


M. Ringbauer et al, Nature Physics 18, 1053 (2022)

The required operations

- **algorithms:**
  sequence of single qubit and two-qubit gate operations

single qubit operation

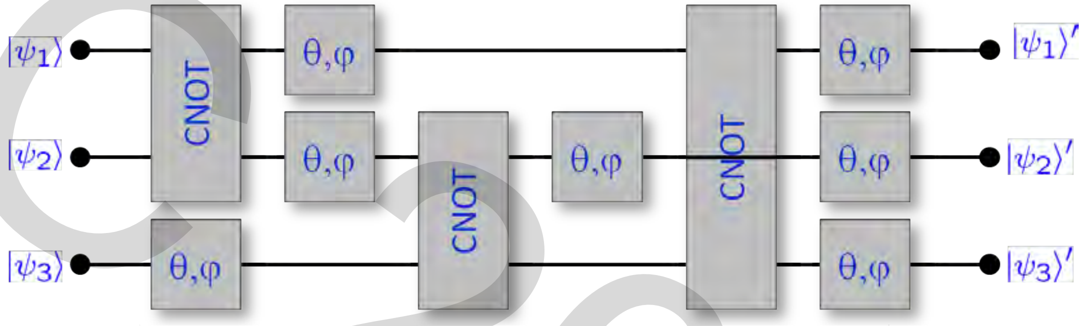


CNOT

$$\begin{cases} |0\rangle|0\rangle \rightarrow |0\rangle|0\rangle \\ |0\rangle|1\rangle \rightarrow |0\rangle|1\rangle \\ |1\rangle|0\rangle \rightarrow |1\rangle|1\rangle \\ |1\rangle|1\rangle \rightarrow |1\rangle|0\rangle \end{cases}$$

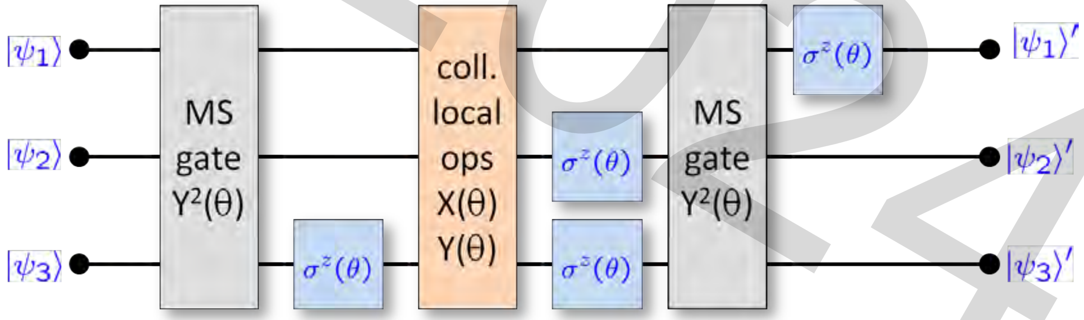
single-qubit (local) operations
two-qubit CNOT gate operations

➔ **universal set**

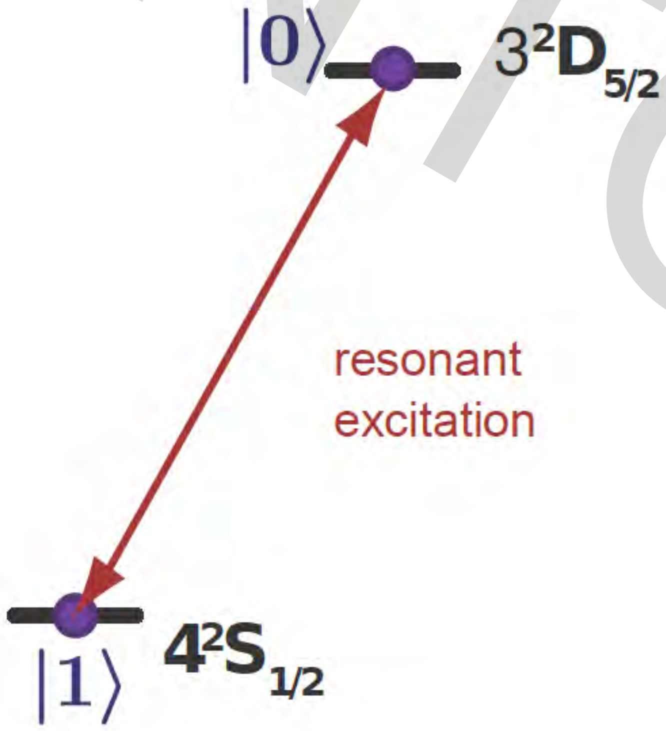


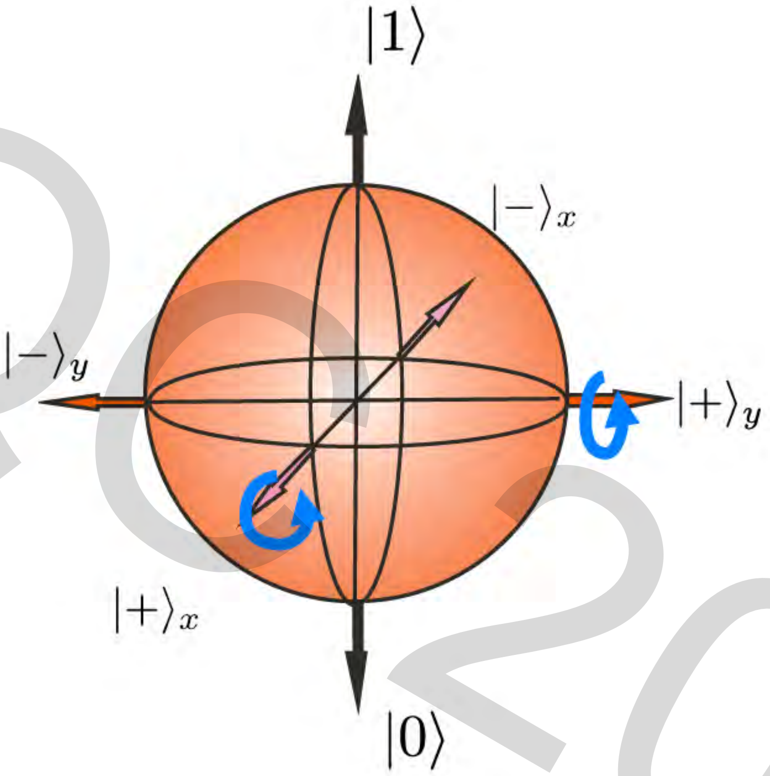
single-qubit (local) operations
two-qubit entangling operations

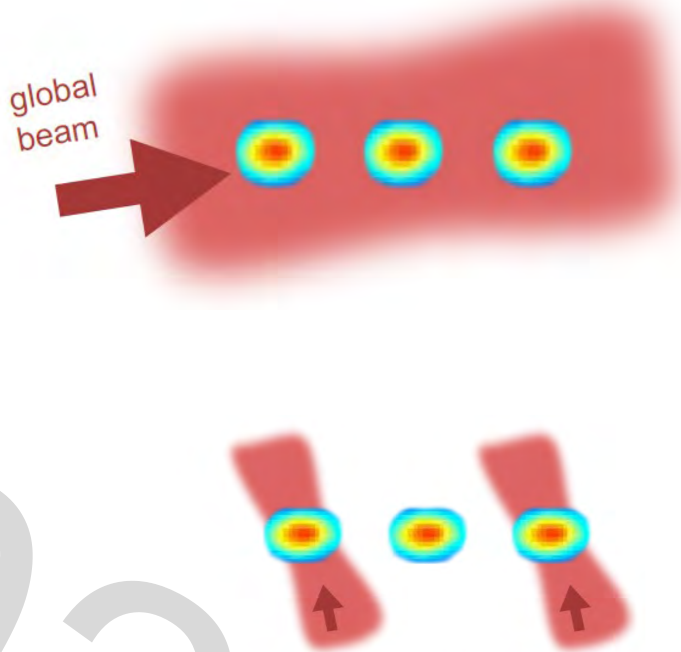
➔ **universal (over-complete) set**



Resonant Operations



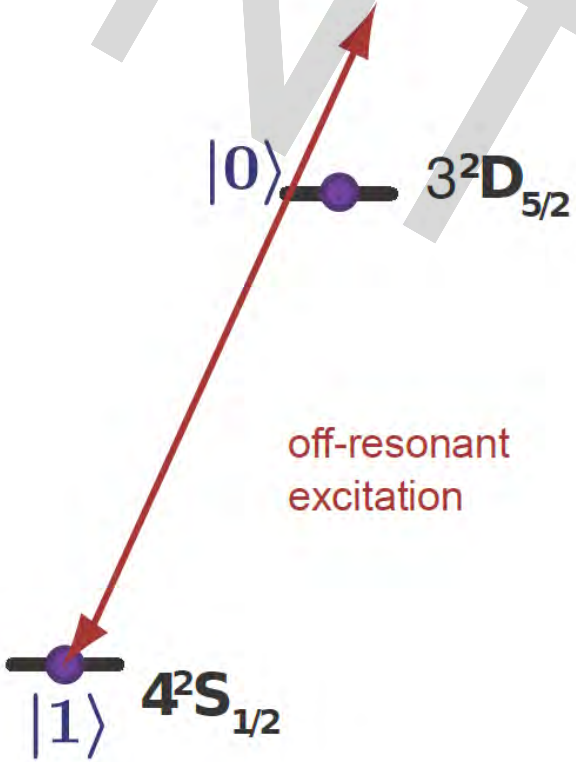


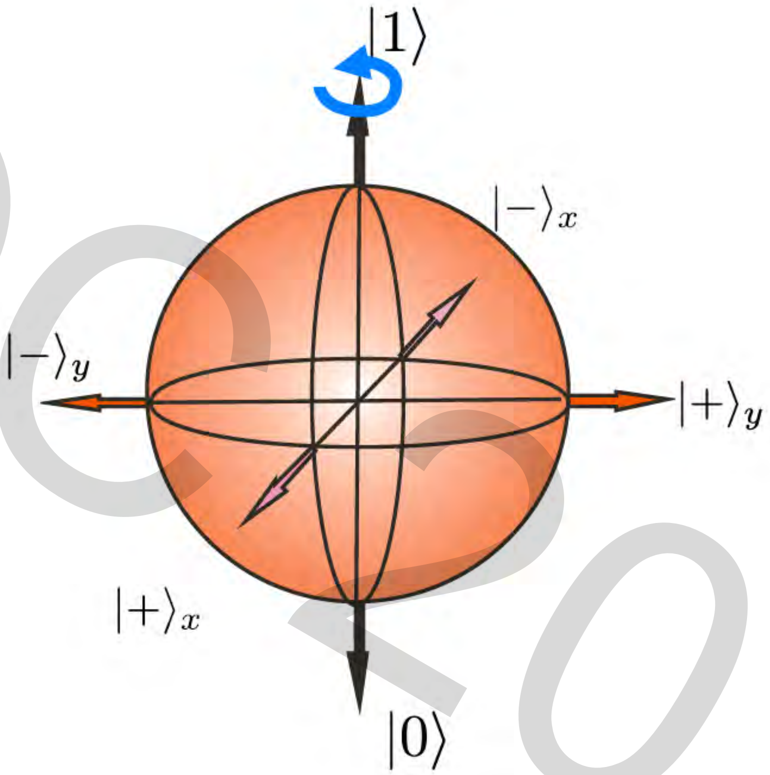


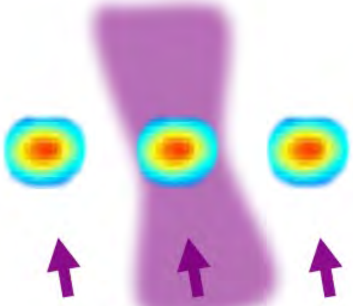
$$R(\theta, \phi) = e^{-i\theta/2(\sigma_x \cos\phi + \sigma_y \sin\phi)}$$

P. Schindler, at al., New. J. Phys. 15, 123012 (2013)

Off-resonant Operations







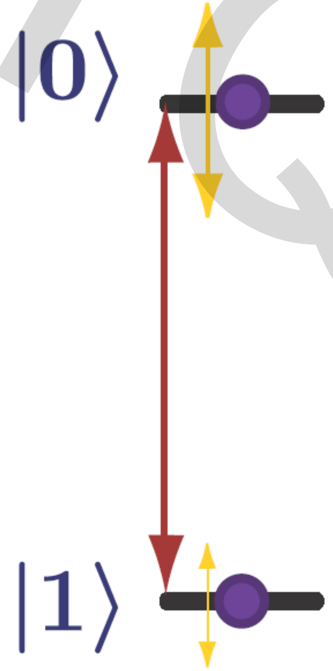
$$R_Z(\theta) = e^{-i\theta/2\sigma_z}$$

universität innsbruck

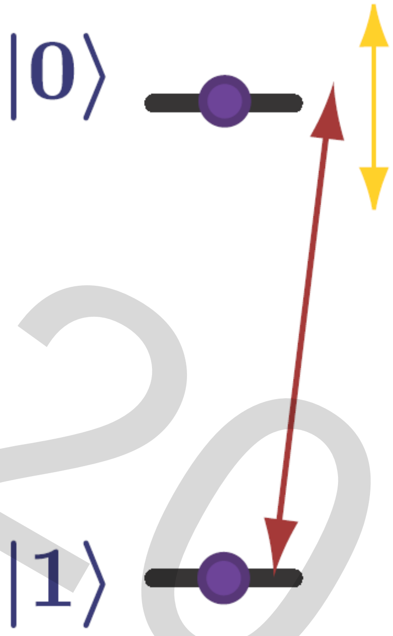
P. Schindler, at al., New. J. Phys. 15, 123012 (2013)

Decoherence – phase damping (T2)

To keep the "quantumness" of the qubit, the phase of the driving laser and the two-level system needs to be preserved.



Level spacing fluctuations
(B-field)



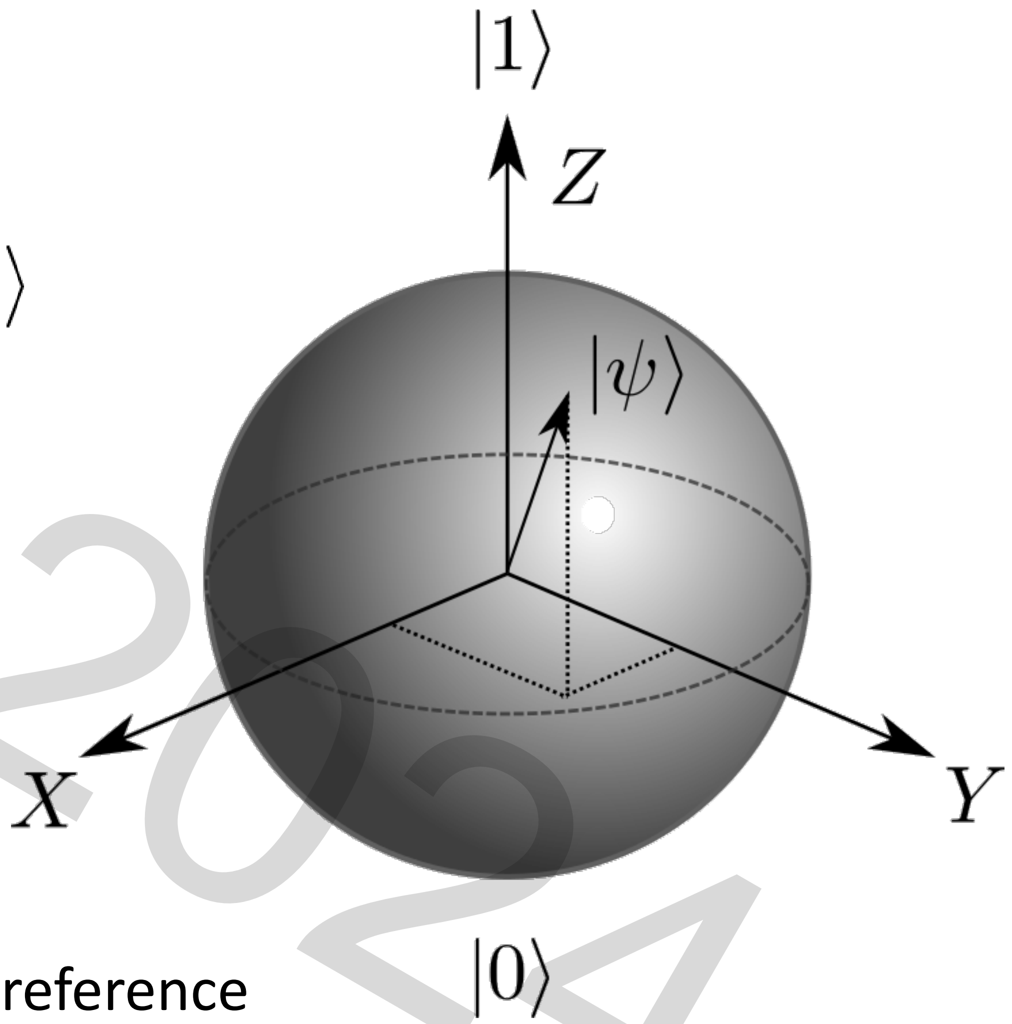
Local oscillator fluctuations
(Laser, RF source)

Single ion as an atomic clock

Schrödinger Equation:

Relative phase evolution $\propto$ energy difference

$$|0\rangle + |1\rangle \rightarrow |0\rangle + \exp(i\ \Delta E\ t)|1\rangle$$



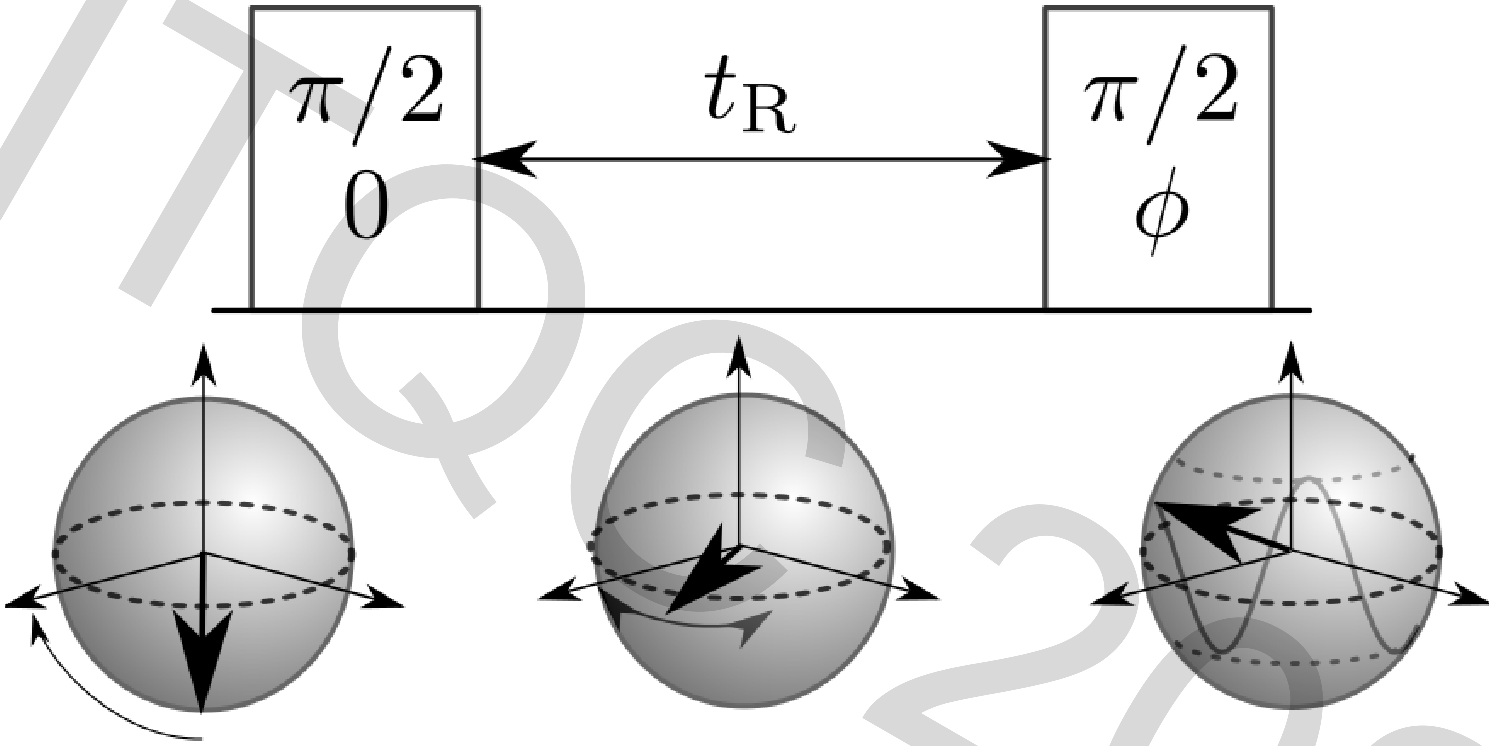
Evolution at about 10^{15} Hz

Linewidth between Hz and mHz

Need to track the clock

→ Use resonant, ultrastable laser as reference

Ramsey experiments

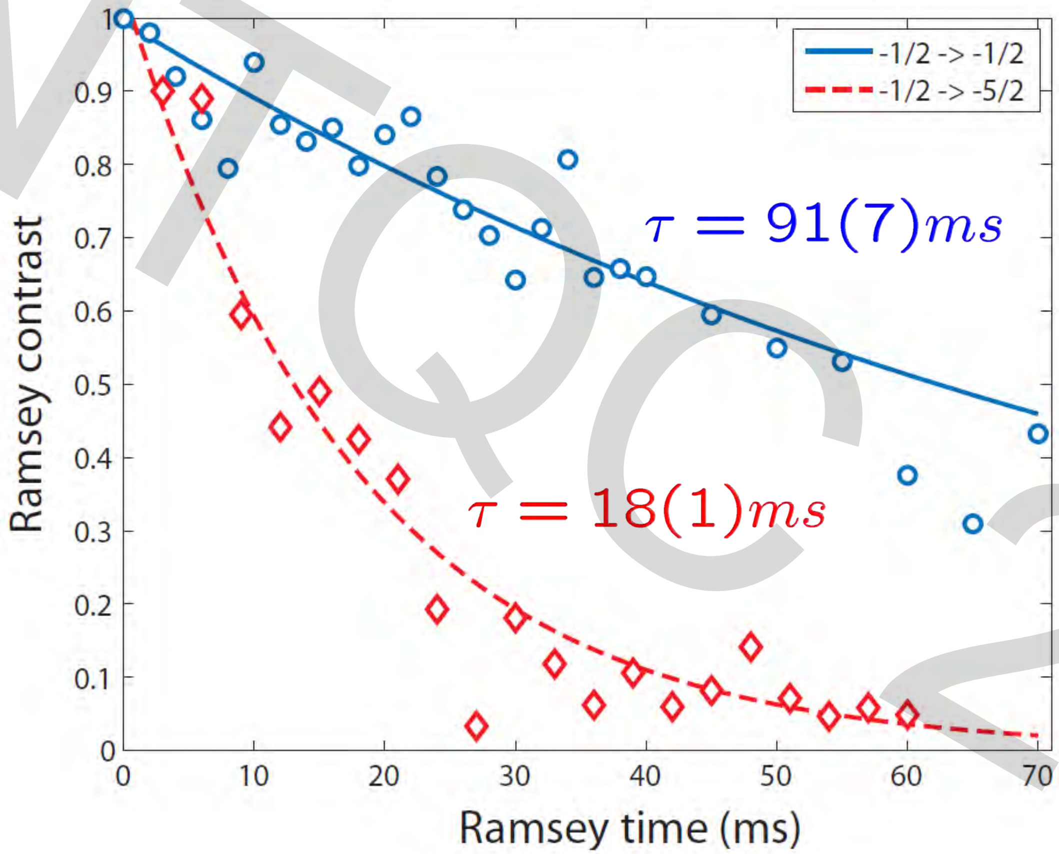


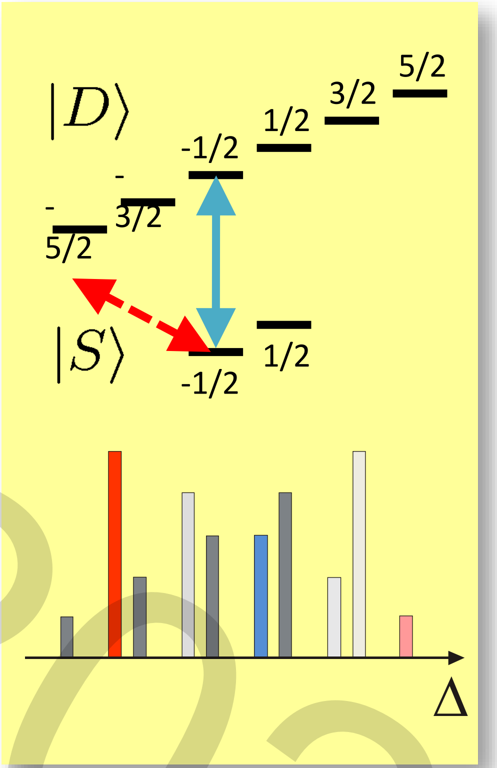
Second Ramsey pulse maps phase into excitation

ν_{Ca+} = 411 042 129 776 393.2(10) Hz

Chwalla et al., Phys. Rev. Lett. 102, 023002 (2009)

Qubit coherence





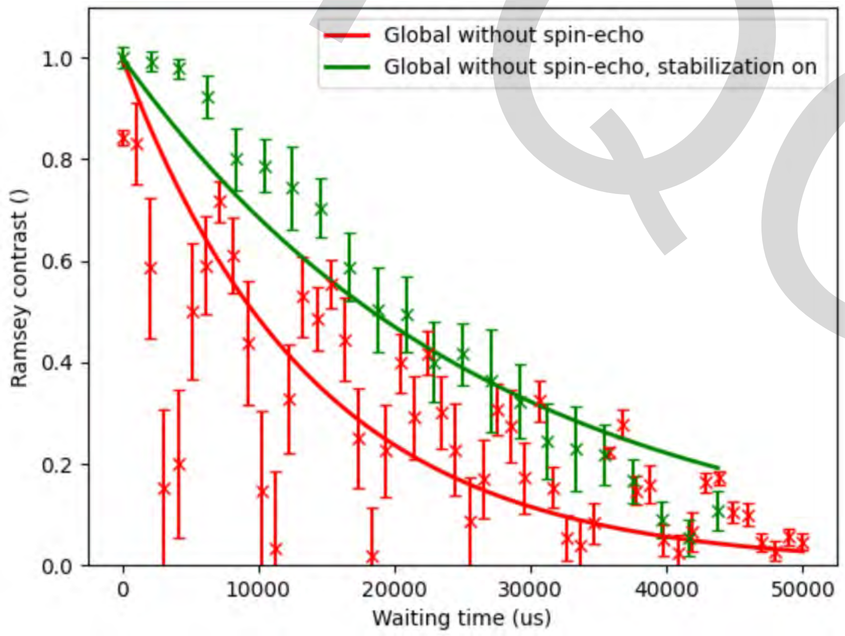
Important limit: $T_2 \leq 2\,T_1$

Tells you when to work on lifetime of the qubit

Magnetic Field Stabilization

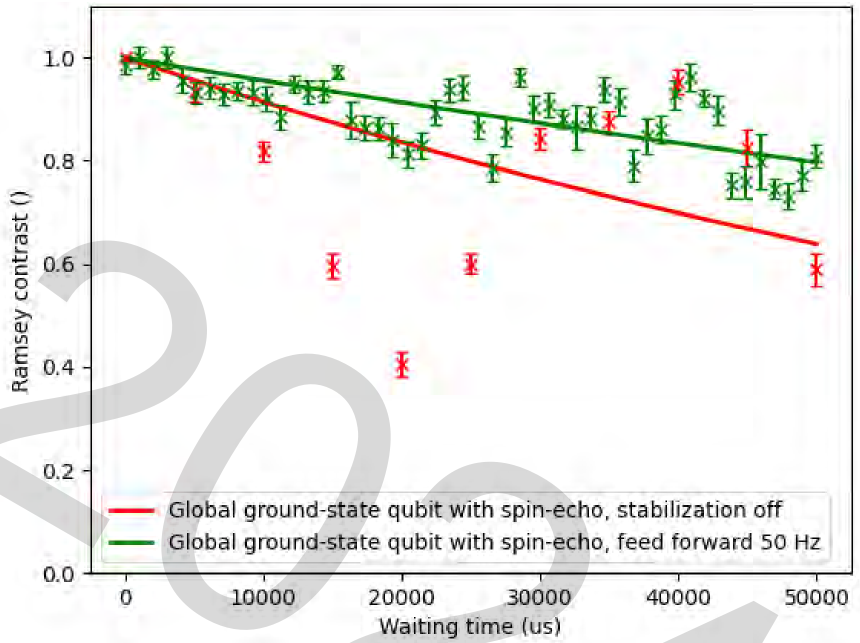
1) µ-metal shield: 2ms → 100ms

2) Magnetic field **feedback**



"challenge" with DC jumps
to test feedback performance

3) Magnetic field **feedforward**



Coherence time ~ 220ms

universität
innsbruck

1. Trapping and Cooling Ions

1.1 How to trap an ion

1.2 Ion strings for quantum computation

1.3 Choosing an ion

1.4 Laser-ion interaction

1.5 Laser cooling in ion traps

1.6 Gate Operations & Decoherence

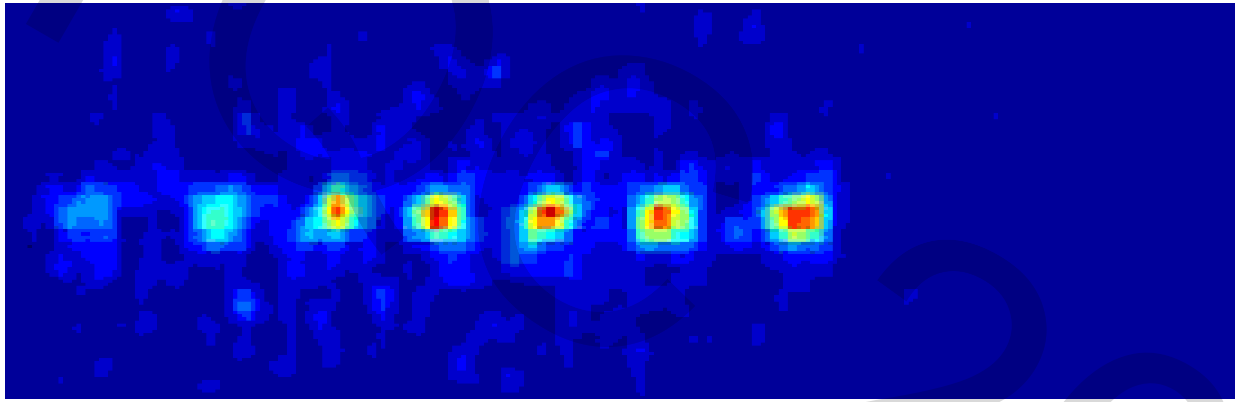
1.7 Entanglement

universität innsbruck

AG Quantenoptik und Spektroskopie

universität innsbruck

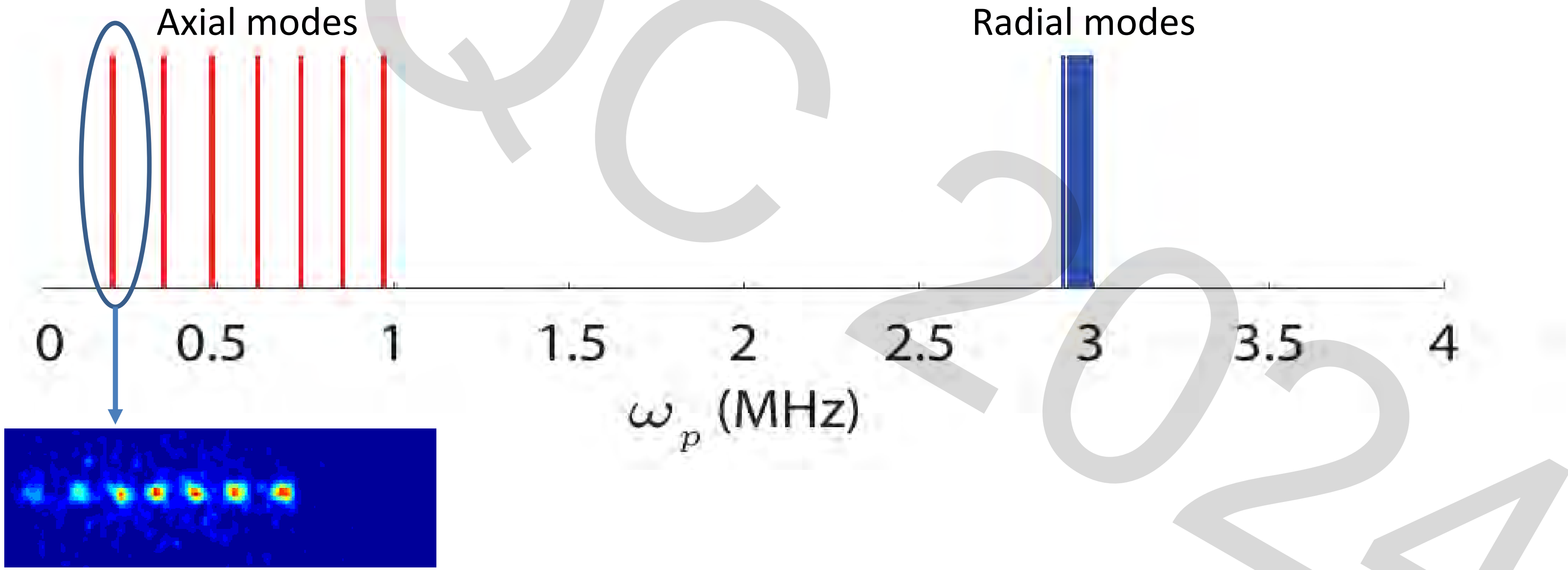
More ions



Normal modes

Perform Taylor expansion around equilibrium positions to find normal modes.

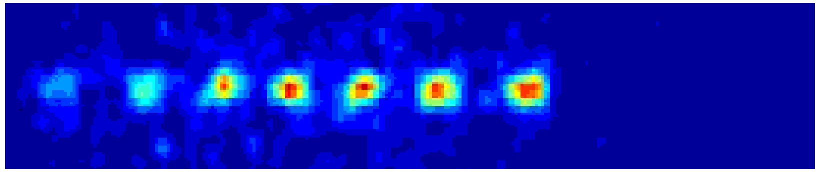
Analogous to 3D classical coupled harmonic oscillator: 3N modes.

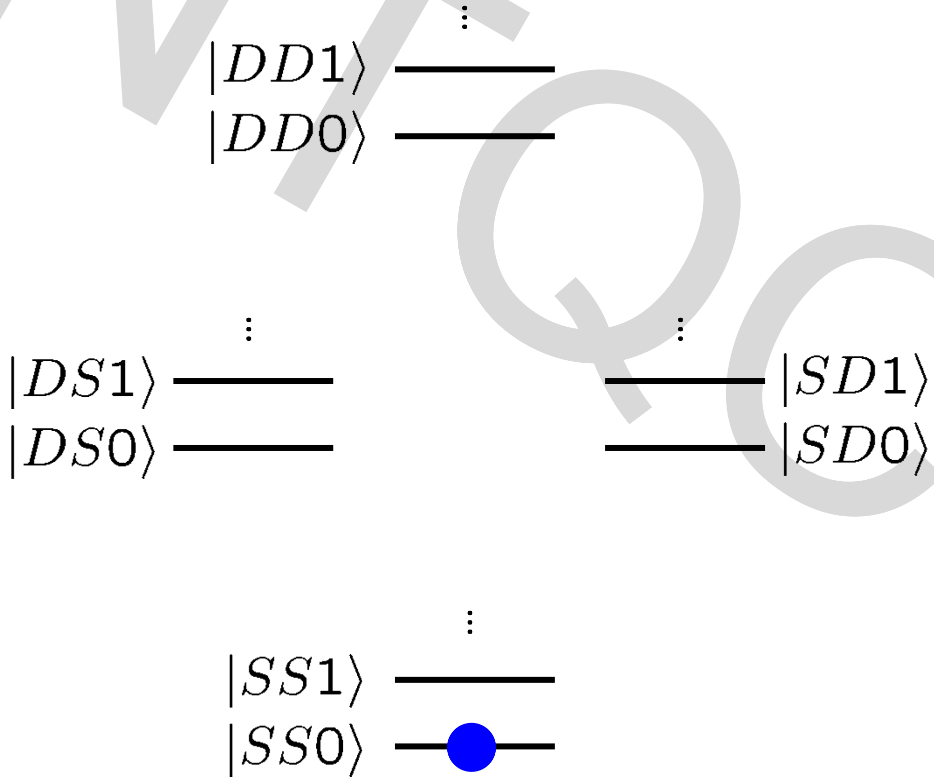


universität innsbruck

C. Marquet, F. Schmidt-Kaler, and D. F. V. James, Applied Physics B 76, 199 (2003)

Generating Entanglement





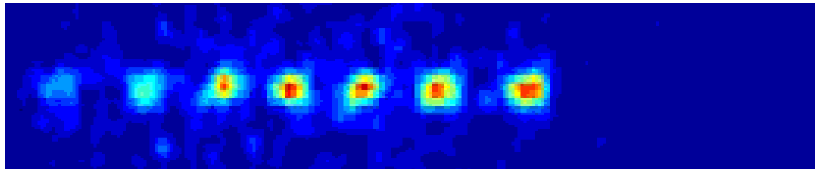
Pulse sequence:

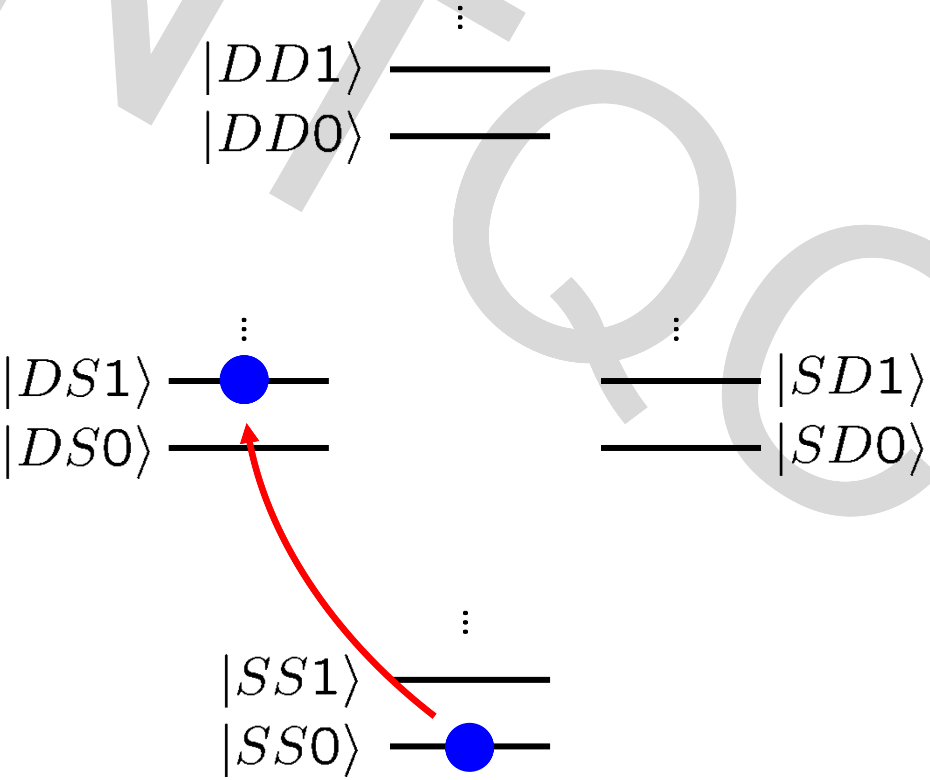
spin motion

$|SS0\rangle$

I. Cirac, P. Zoller, Phys. Rev. Lett. 74, 4091 (1995)

Generating Entanglement





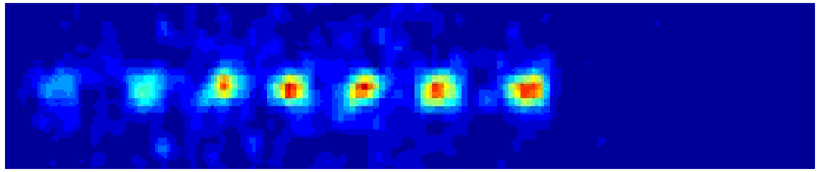
Pulse sequence:

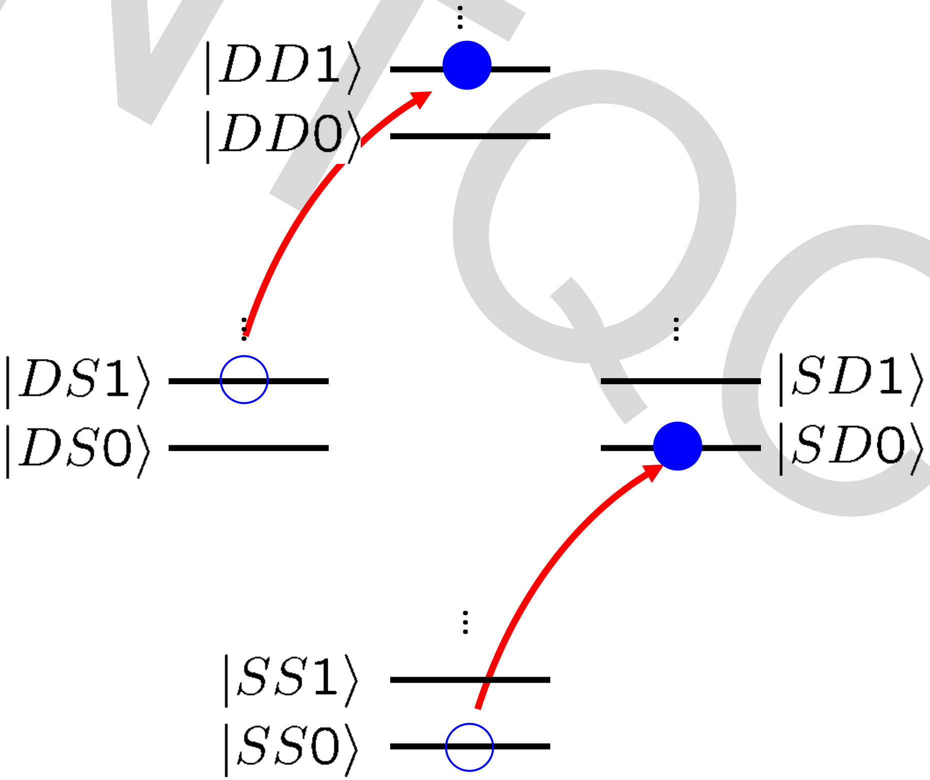
Ion 1: π/2 , blue sideband

$$|SS0\rangle + |DS1\rangle$$

I. Cirac, P. Zoller, Phys. Rev. Lett. 74, 4091 (1995)

Generating Entanglement





Pulse sequence:

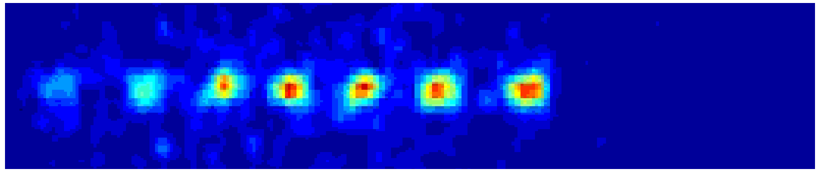
Ion 1: $\pi/2$, blue sideband

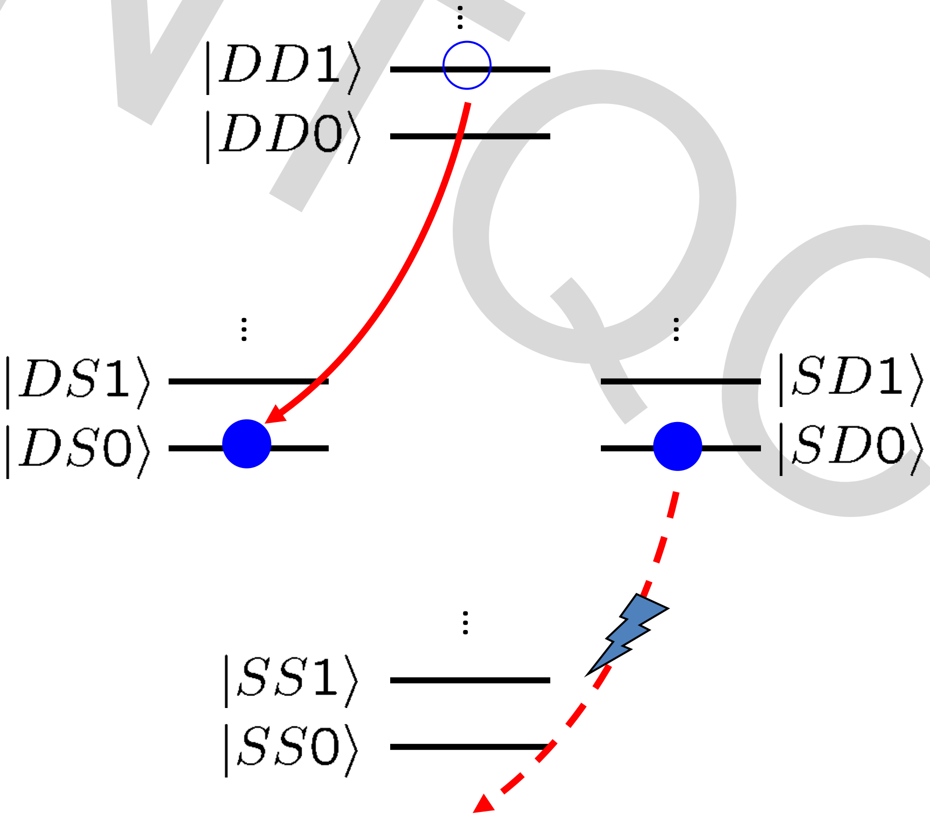
Ion 2: π , carrier

$$|SD0\rangle + |DD1\rangle$$

I. Cirac, P. Zoller, Phys. Rev. Lett. 74, 4091 (1995)

Generating Entanglement





Pulse sequence:

Ion 1: π/2 , blue sideband

Ion 2: π , carrier

Ion 2: π , red sideband

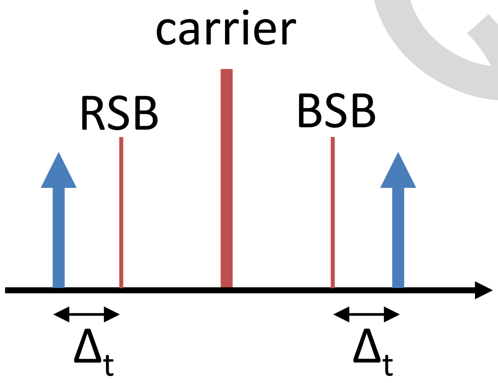
$$(|SD\rangle + |DS\rangle)|0\rangle$$

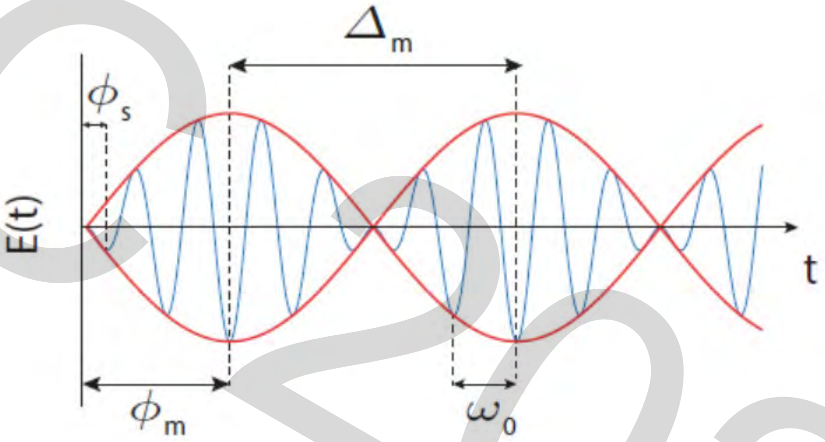
I. Cirac, P. Zoller, Phys. Rev. Lett. 74, 4091 (1995)

Mølmer-Sørensen entangling operation

Recall: in the Lamb-Dicke regime the interaction Hamiltonian becomes

$$H_{int} = \hbar\frac{\Omega}{2}\left\{(e^{-i(\Delta t-\phi_L)}\sigma_+\left[1+i\eta\left(ae^{-i\omega_t t}+a^\dagger e^{i\omega_t t}\right)\right]+h.c.\right\}.$$





<u>Exercise:</u> Derive the interaction Hamiltonian for a bichromatic drive

$$H_{\mathrm{Bic}} = \hbar\eta\Omega\sigma_x\left(ae^{i\Delta_t t}+a^\dagger e^{-i\Delta_t t}\right)$$

C. F. Roos, New Journal of Physics 10, 013002 (2008)
P. Jurcevic, PhD thesis

Mølmer-Sørensen entangling operation

$$H_{\mathrm{MS}} = \hbar\eta\Omega \left(a e^{i\Delta_t t} + a^\dagger e^{-i\Delta_t t}\right)\left(\sigma_x^{(1)} + \sigma_x^{(2)}\right)$$

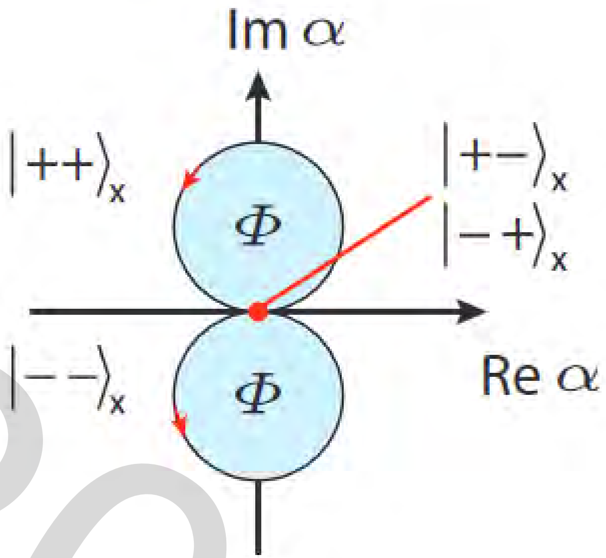
Integrating this Hamiltonian:

$$U^{\mathrm{MS}}(t) = \hat{D}\left(\alpha(t) S_x\right) \exp\left(i\Phi(t) S_x^2\right)$$

With displacement operator $\hat{D}(\alpha) = e^{\alpha a^\dagger - \alpha^* a}$

$$\alpha(t) = \frac{\eta\Omega}{\Delta_t}\left(e^{i\Delta_t t} - 1\right)$$

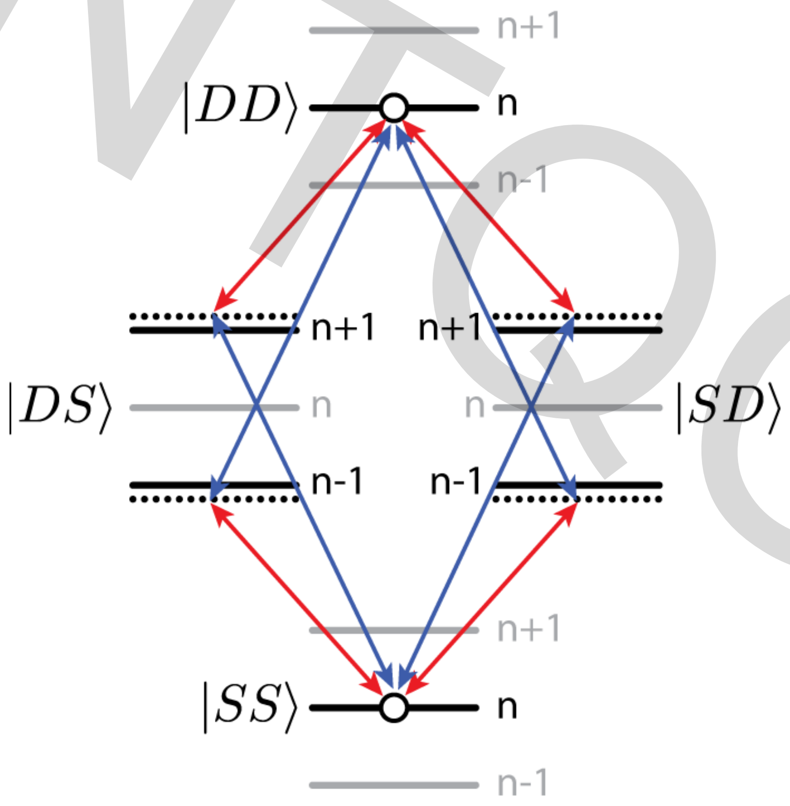
$$\Phi(t) = \left(\frac{\eta\Omega}{\Delta_t}\right)^2 \left(\Delta_t t - \sin(\Delta_t t)\right)$$

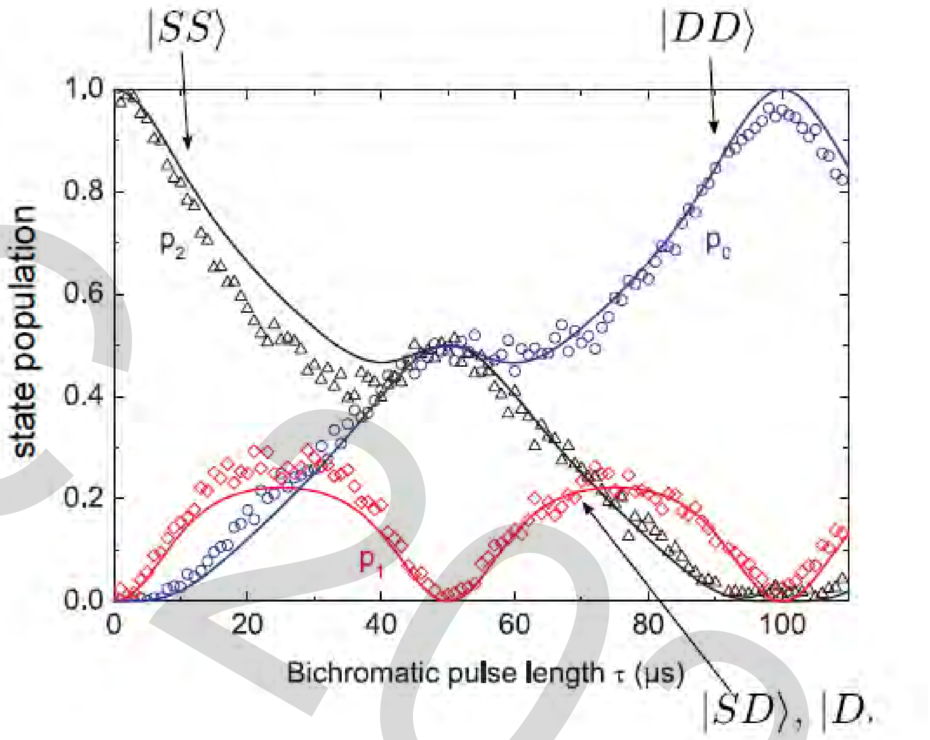


$$\tau_{\mathrm{gate}} = 2\pi/\Delta_t$$

$$\Phi\left(\tau_{\mathrm{gate}}\right) = 2\pi\left(\frac{\eta\Omega}{\Delta_t}\right)^2$$

Mølmer-Sørensen entangling operation



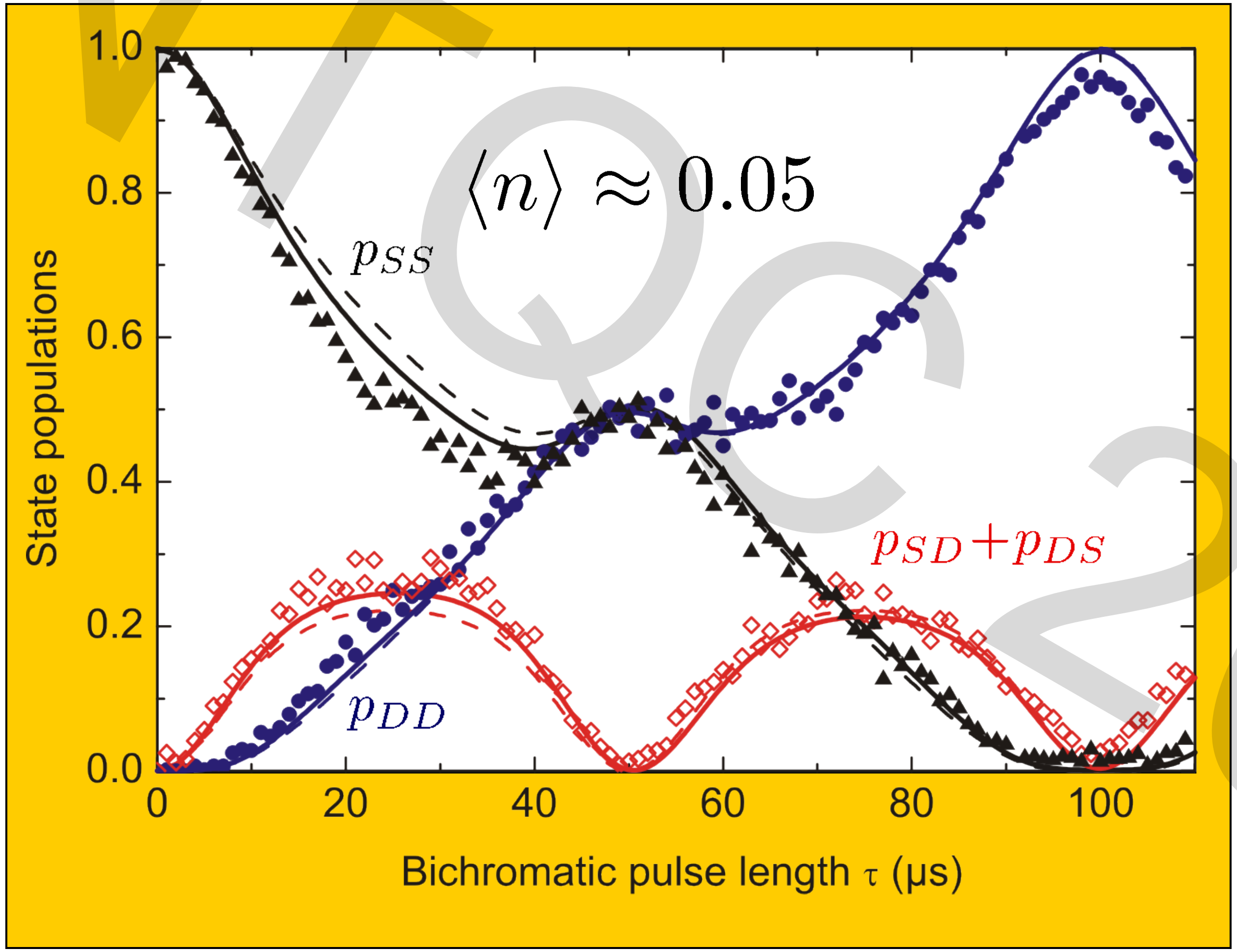


Off-resonant coupling to the sidebands
Unwanted populations interfere destructively

G. Kirchmair, et. al. New. J. Phys. 11, 023002 (2009)
K. Mølmer and A. Sørensen, PRL 82, 1835 (1999)

Mølmer-Sørensen gate: thermal states

Gate operation after ground state cooling



Bell state:

$\Psi = |SS\rangle + i|DD\rangle$

Fidelity :

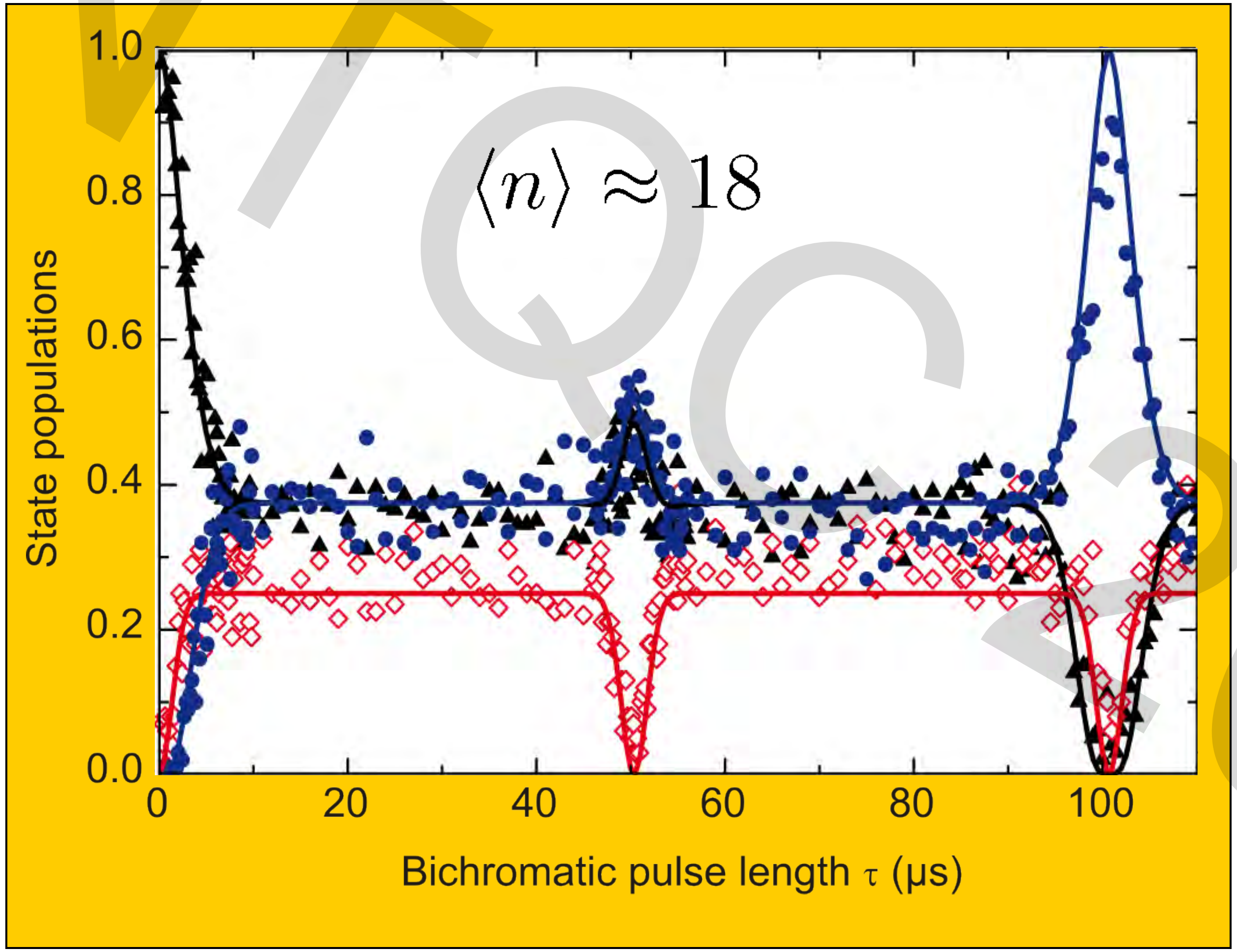
F = 99.3(1) %

universität innsbruck

G. Kirchmair et al., New. J. Phys. 11, 023002 (2009)

Mølmer-Sørensen gate: thermal states

Gate operation after Doppler cooling



Bell state:

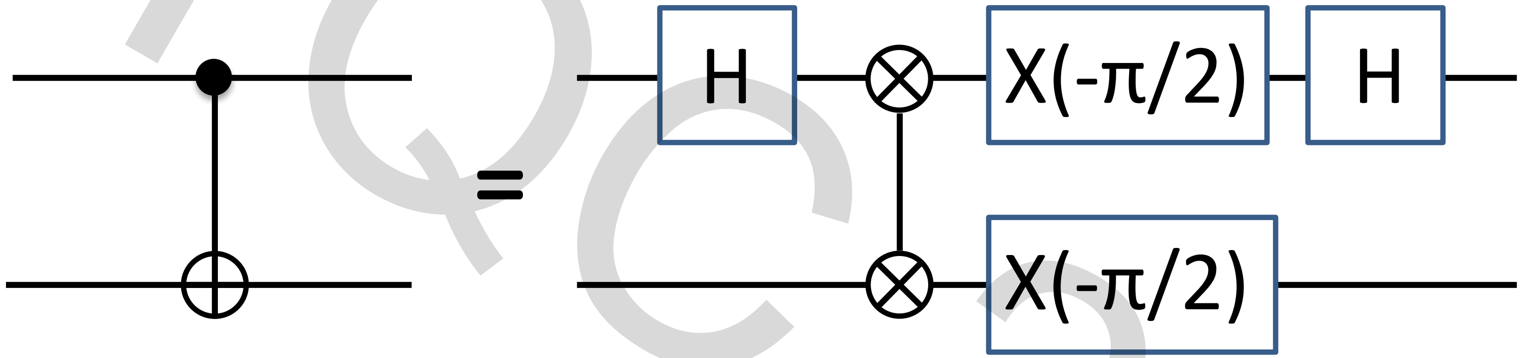
$$\Psi = |SS\rangle + i|DD\rangle$$

Fidelity :

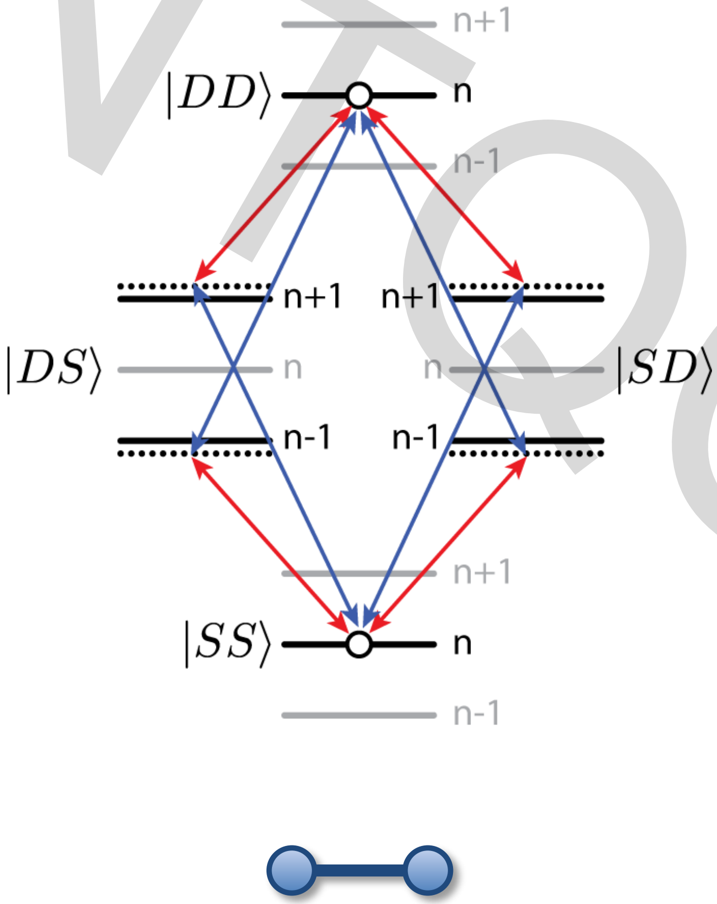
F = 98.0(1) %

G. Kirchmair et al., New. J. Phys. 11, 023002 (2009)

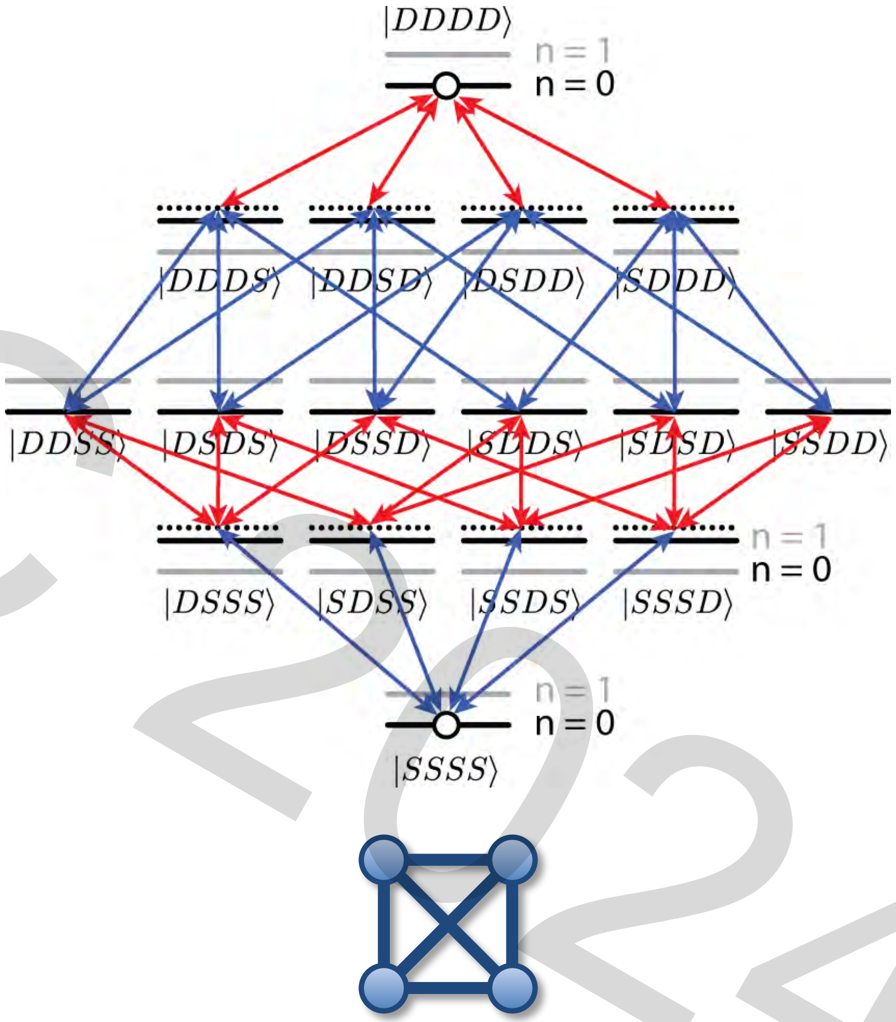
MS vs CNot



Multi path interferometer

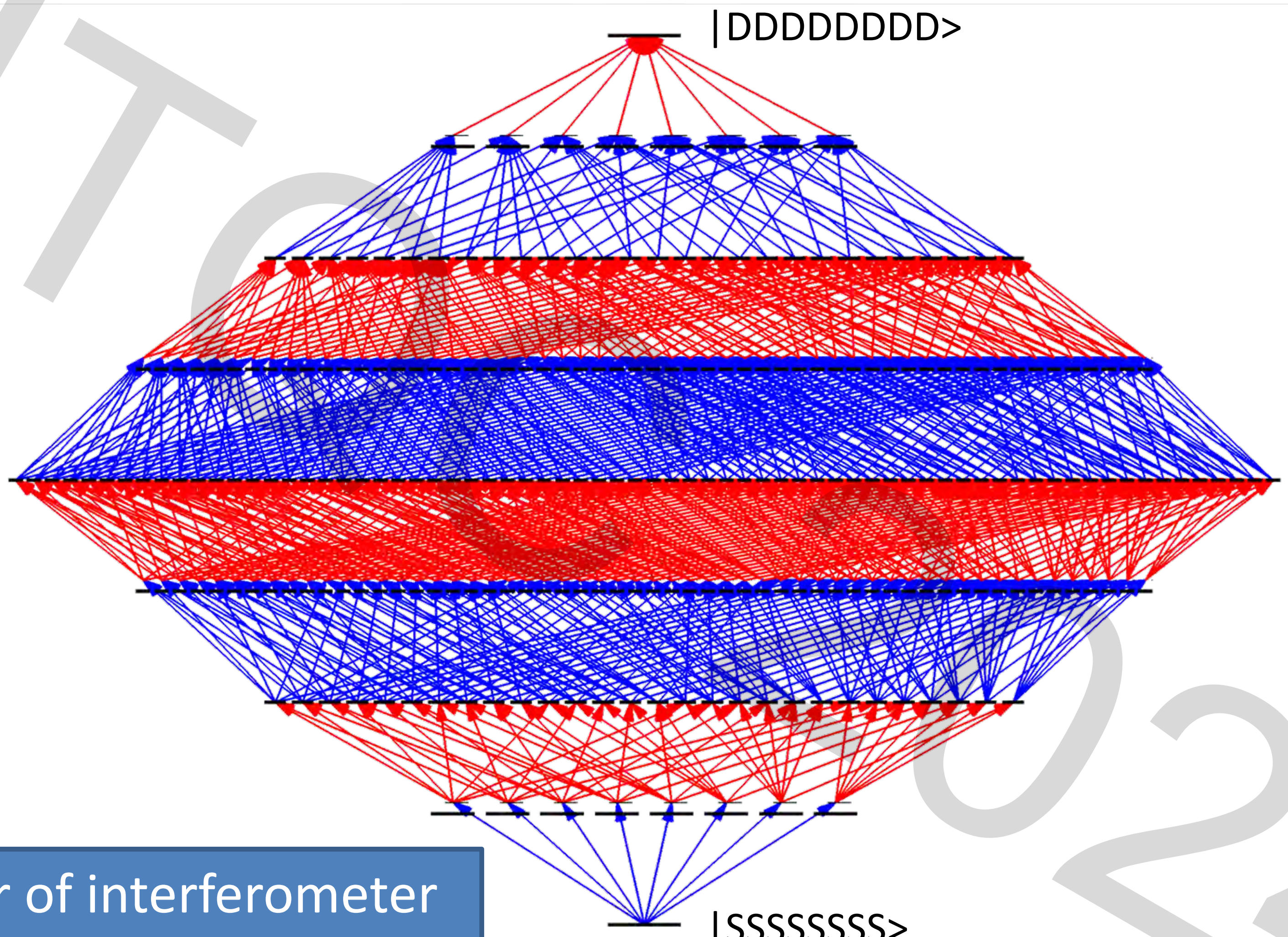


2 qubits



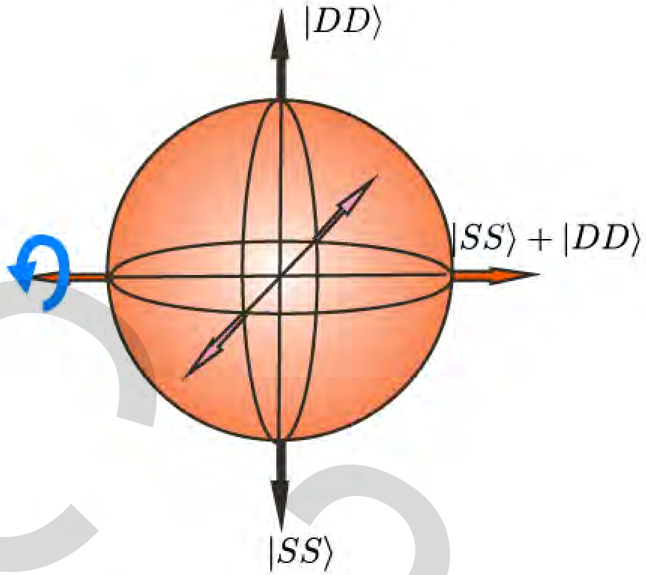
4 qubits

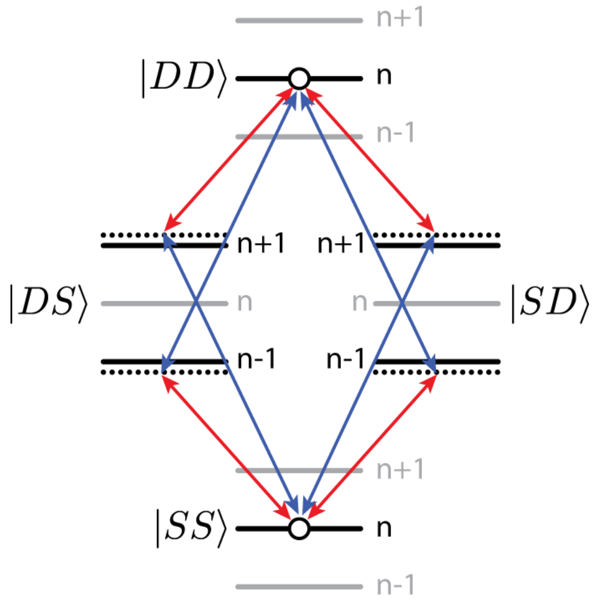
Multi path interferometer – 8 ions



Number of interferometer paths grows exponentially.

Mølmer-Sørensen Entangling Operation

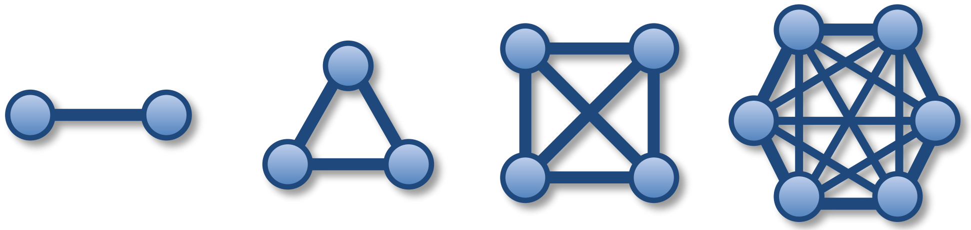


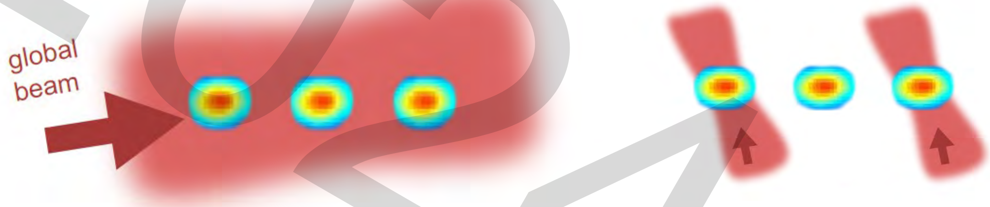


Works for any number of qubits

Effective infinite range 2-body interaction.

Enables arbitrary coupling graph





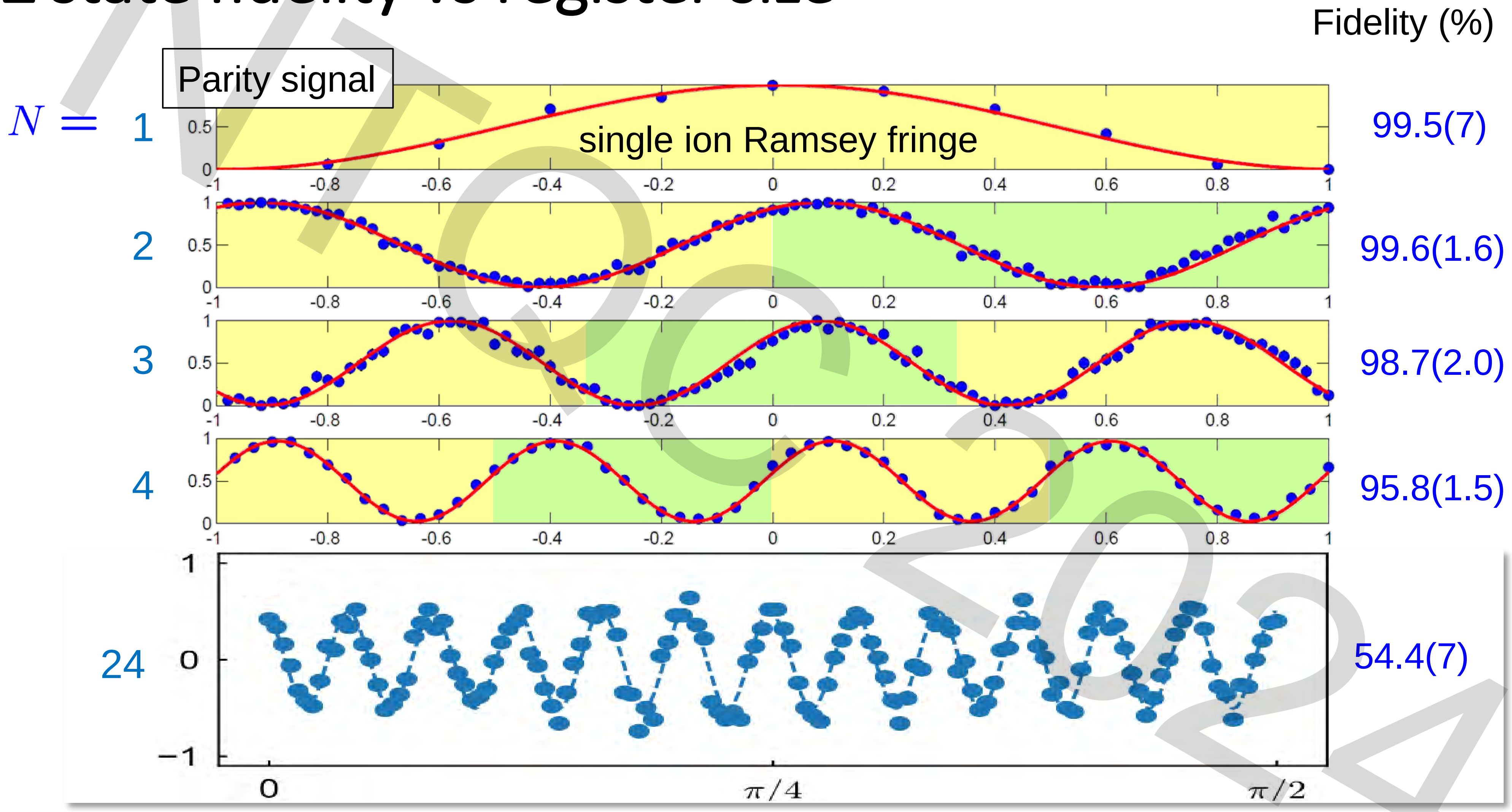
universität innsbruck

T. Monz et al., *PRL.* **106**, 130506 (2011).

K. Mølmer and A. Sørensen, PRL 82, 1835 (1999).

GHZ state fidelity vs register size

$$\Psi = \frac{1}{\sqrt{2}}(|SS\ldots S\rangle + |DD\ldots D\rangle)$$



T. Monz et al., *PRL.* **106**, 130506 (2011).
V. Pogorelov et al., *PRX Quantum* **2**, 020343 (2021).

AG Quantenoptik und Spektroskopie

The Innsbruck Ion Trappers 2023

universität innsbruck

€ FWF SFB erc IQI GmbH eQual QUDITS

FWF ATION IQI $ IARPA NeXST