



Quantum Computation and Simulation with Trapped Ions

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1. Trapping and Cooling Ions

1.1 How to trap an ion

1.2 Ion strings for quantum computation

1.3 Choosing an ion

1.4 Laser-ion interaction

1.5 Laser cooling in ion traps

1.6 Gate Operations & Decoherence

1.7 Entanglement



2. Quantum Computation and Simulation

Recall: Laser-ion interaction

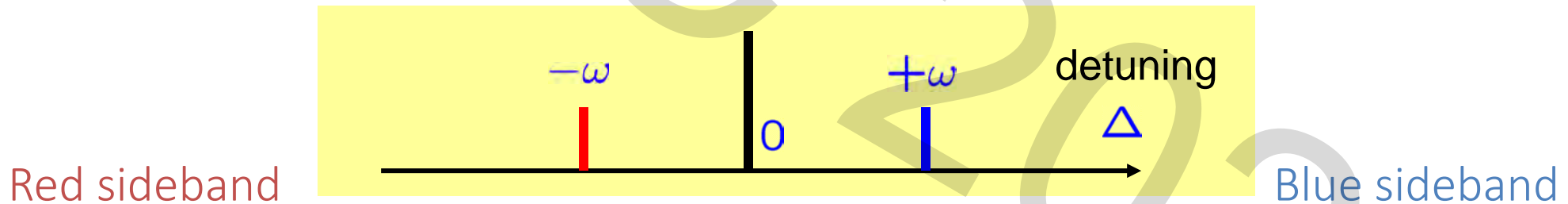
In the Lamb-Dicke regime $\eta^2(2n+1) \ll 1$

we expand $\exp(i\eta(\hat{a}^\dagger + \hat{a})) = 1 + i\eta(\hat{a}^\dagger + \hat{a}) + \mathcal{O}(\eta^2)$

Carrier

$$\Omega_{n,n} = \Omega(1 - \eta^2 n)$$

$$H_I = \frac{1}{2} \hbar \Omega_{n,n} (\sigma^+ + \sigma^-)$$



$$\Omega_{n-1,n} = \eta \sqrt{n} \Omega$$

$$H_I = \frac{1}{2} i \hbar \Omega_{n-1,n} (\hat{a} \sigma^+ - \hat{a}^\dagger \sigma^-)$$

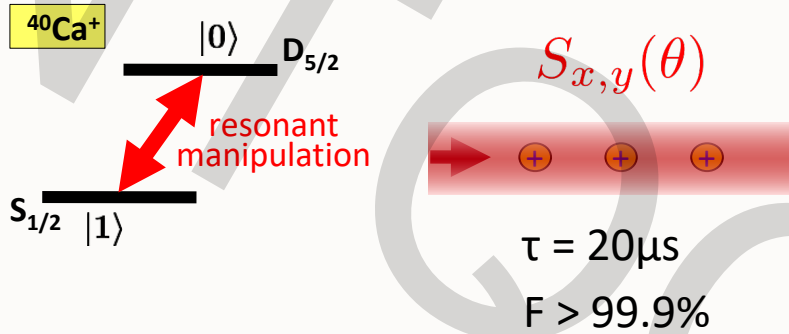
$$\Omega_{n+1,n} = \eta \sqrt{n+1} \Omega$$

$$H_I = \frac{1}{2} i \hbar \Omega_{n+1,n} (\hat{a}^\dagger \sigma^+ - \hat{a} \sigma^-)$$

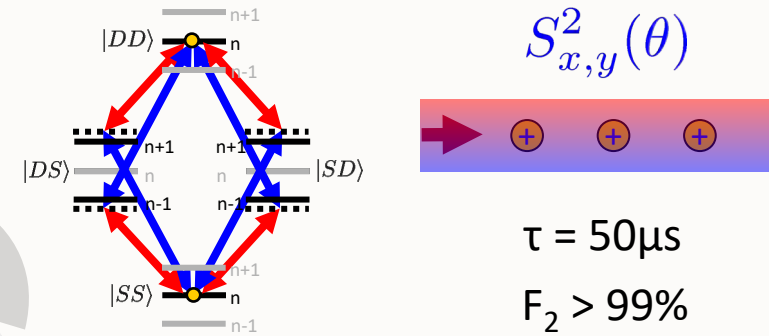
Quantum computing with global and local operations

Global

Collective Local Operations

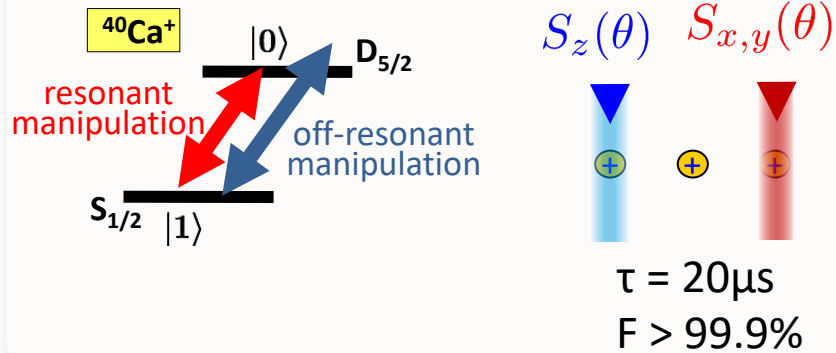


Global Mølmer-Sørensen entangling gate

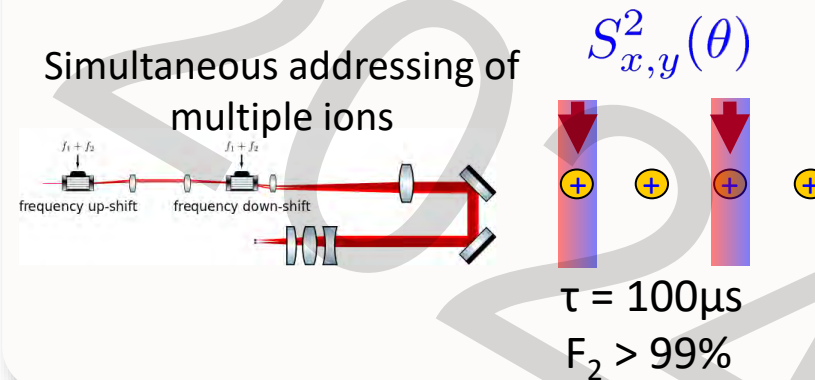


Local

Individual (and parallel) local operations



Local Mølmer-Sørensen entangling gate



2. Quantum Computation and Simulation



2.1 New Generation

2.2 Operations beyond Qubits

2.3 Quantum Error Correction

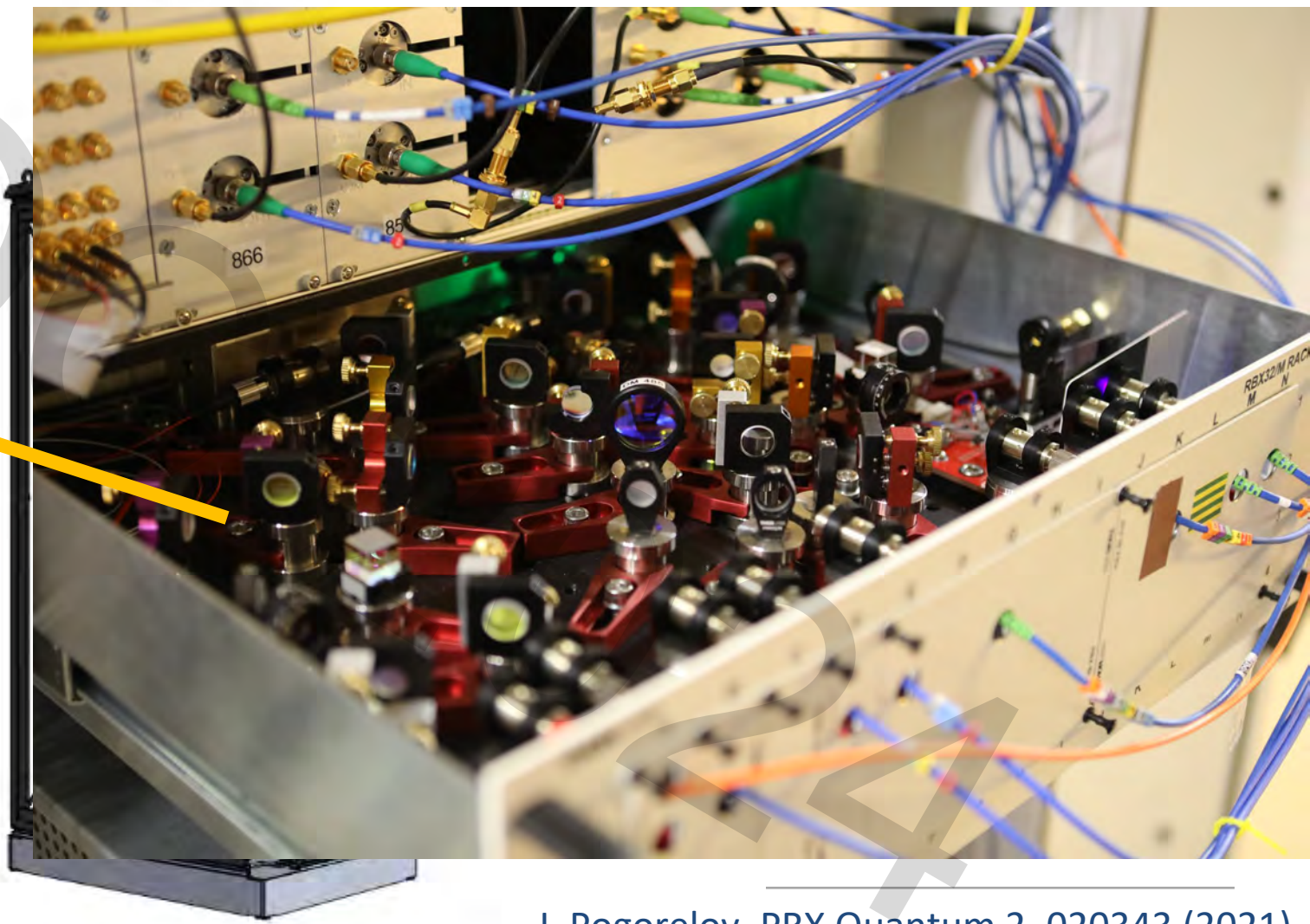
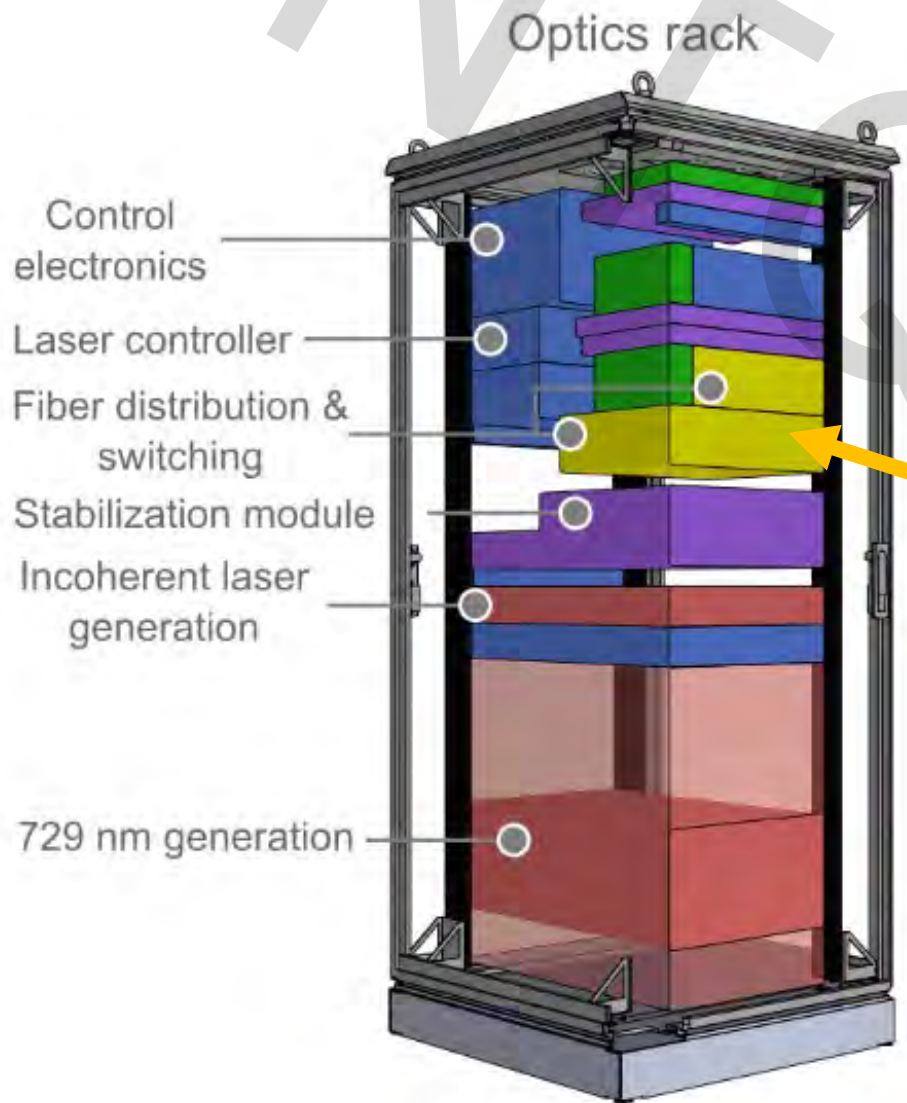
2.4 QIP with Qudits

2.5 Digital Quantum Simulation

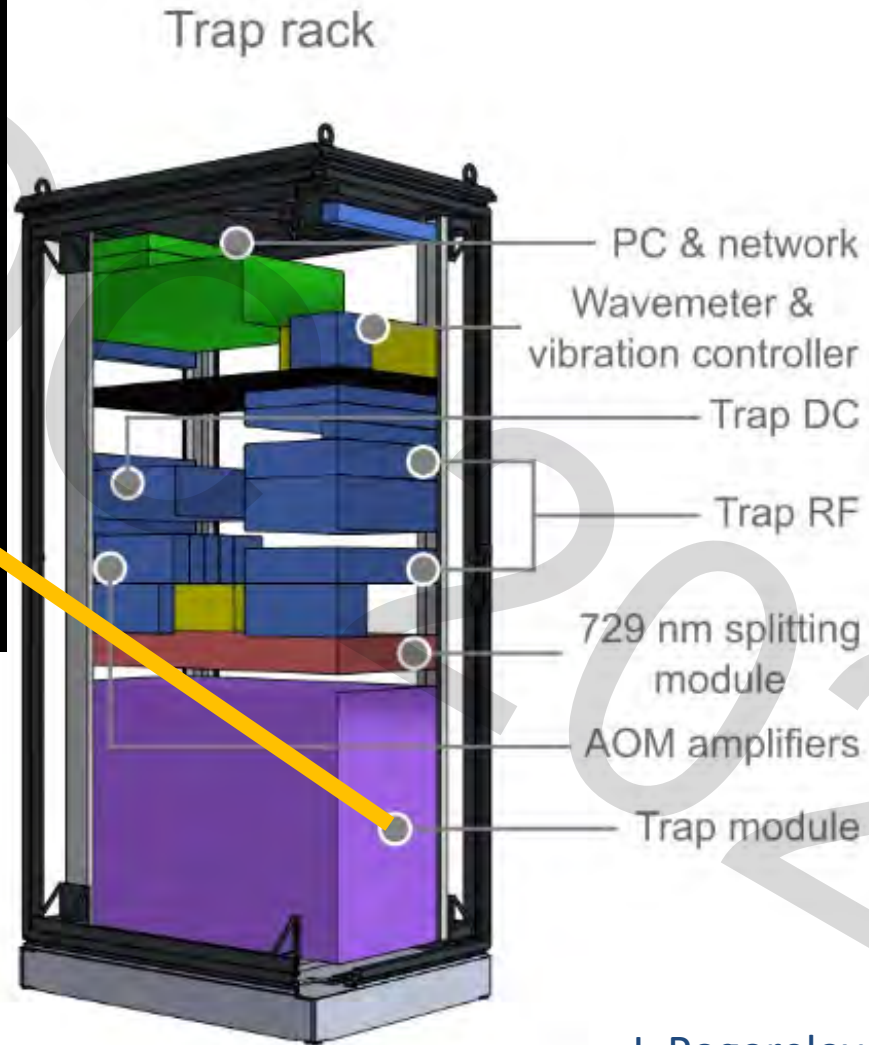
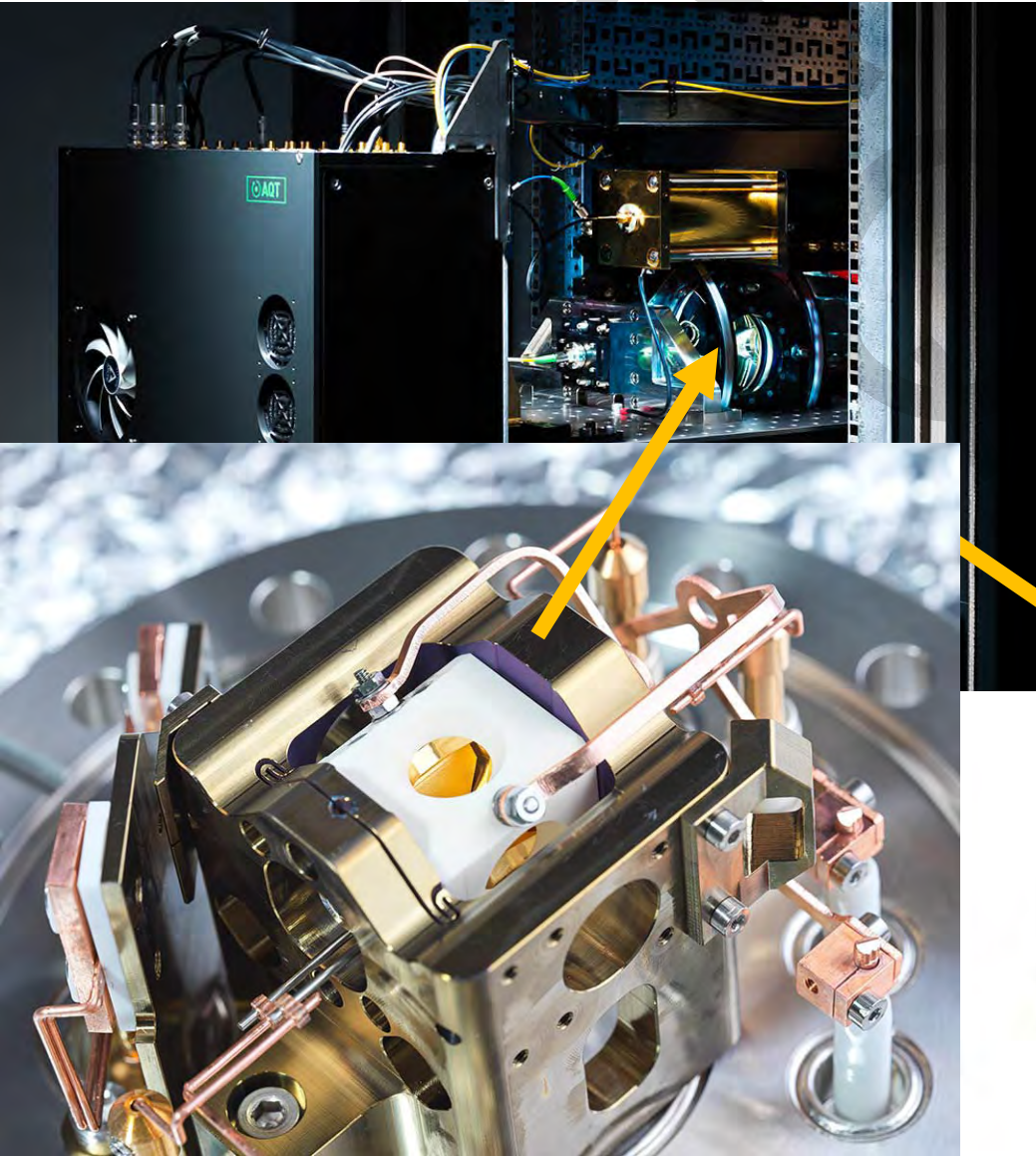
2.6 Scaling



To a Compact, Modular System



To a Compact, Modular System



2. Quantum Computation and Simulation

2.1 New Generation



2.2 Operations beyond Qubits

2.3 Quantum Error Correction

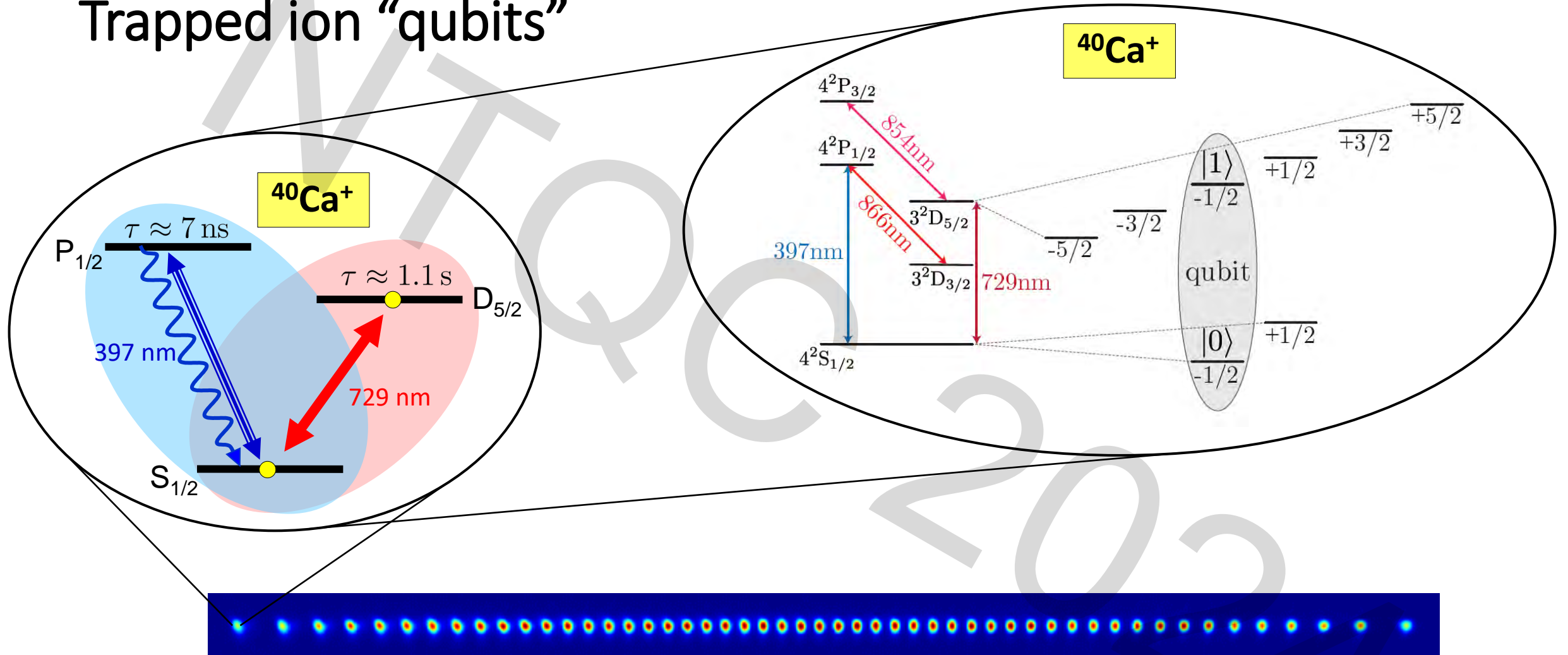
2.4 QIP with Qudits

2.5 Digital Quantum Simulation

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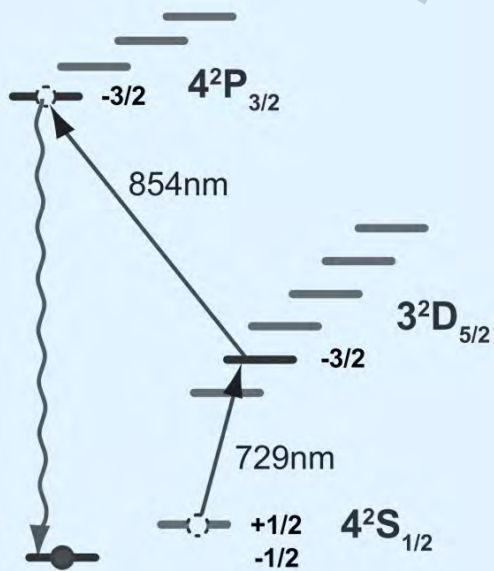


Trapped ion "qubits"



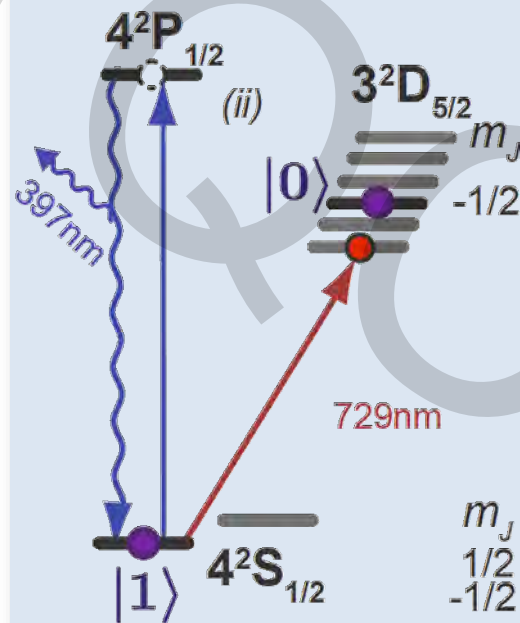
Capabilities beyond Qubits

Optical pumping



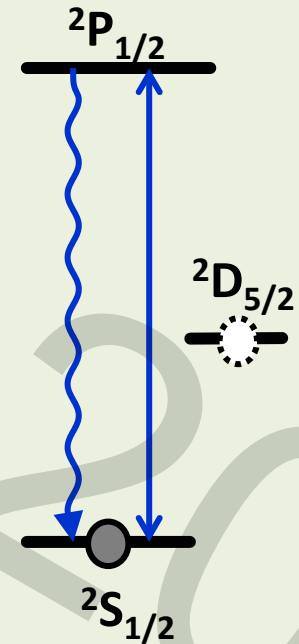
Initializes qubit in one Zeeman state

Decoupling



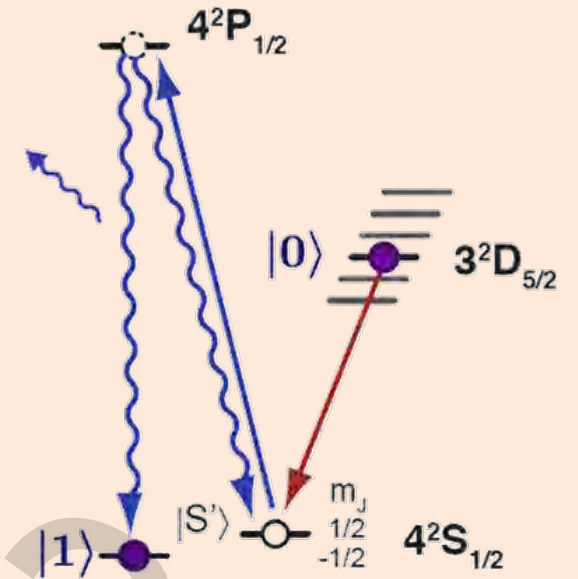
reduces, enlarges the computational subspace

Dephasing



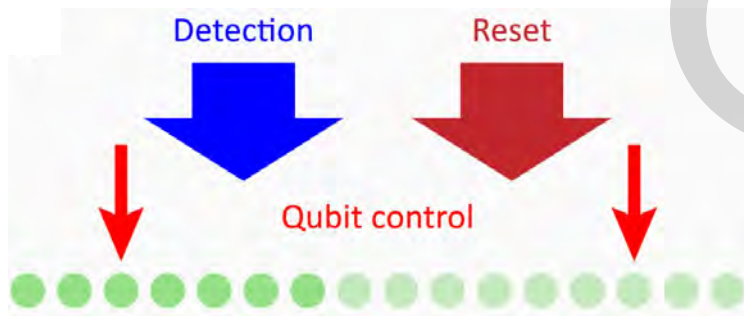
controlled dissipation

Resetting

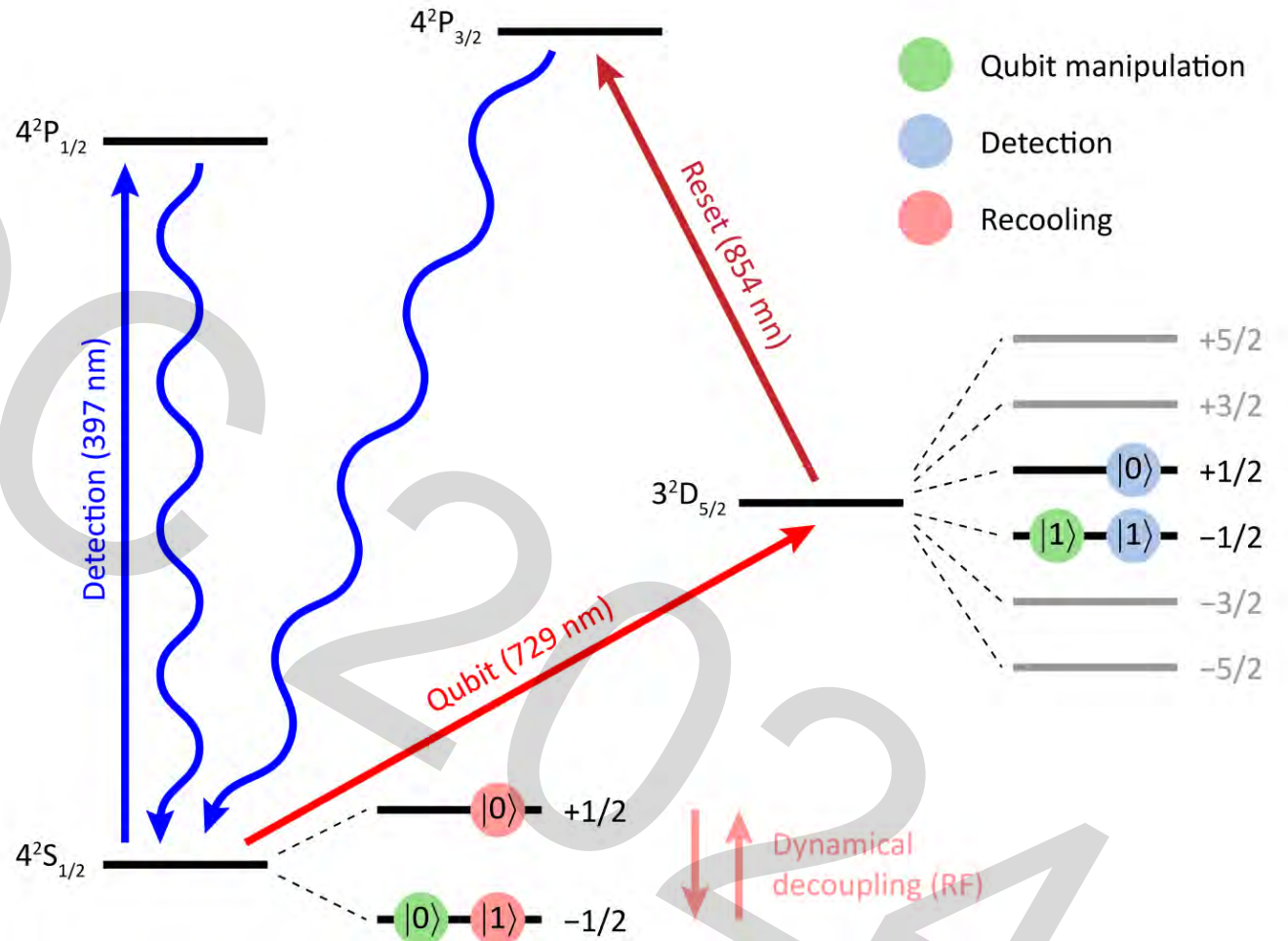


resets the qubit

Mid-Circuit Measurement & Reset



Measure & Reset some qubits
Leave the others alone



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2.6 Scaling



Quantum error correction will be mandatory for large scale quantum computing!

Why small-scale QEC?

- Validate assumptions on errors
- Inform future trap architectures

Ion trap QEC implementations:

J. Chiaverini, Nature 432, 602 (2004)
P. Schindler, Science 332, 1059 (2011)
D. Nigg, Science 345, 302 (2014)
N. Linke Sci. Adv. 3, e1701074 (2017)
C. Flühmann, Nature 566, 513 (2019)
A. Erhard, Nature 589, 220 (2021)

C. Ryan-Anderson, PRX 11, 041058 (2021)
L. Egan, Nature 598, 281 (2021)
De Neeve, Nature Physics 18, 296 (2022)
J. Hilder, PRX 12, 011032 (2022)
Da Silva, arxiv: 2404.02280 (2024)
→ See also: Work by Mainz (Schmidt-Kaler)

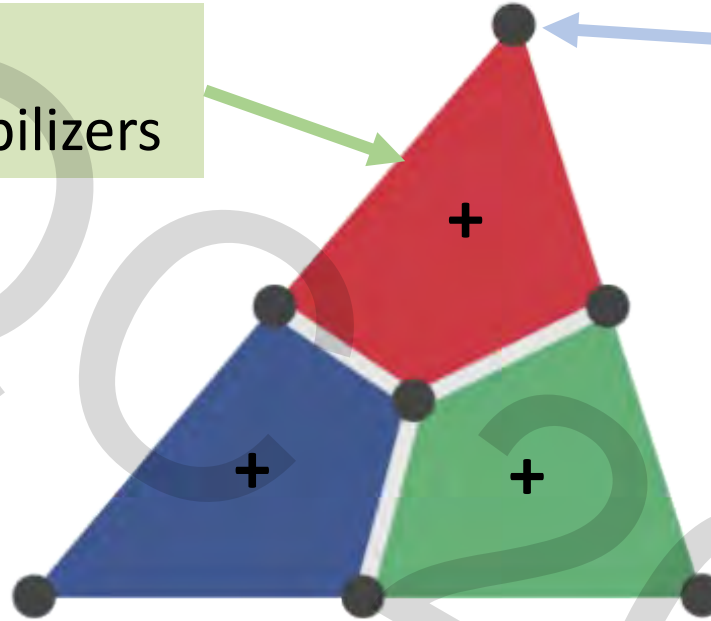


The 7-qubit color code (Steane code)

Plaquette with two associated 4-body stabilizers

$$S_x^{(1)} = X_1 X_3 X_5 X_7$$

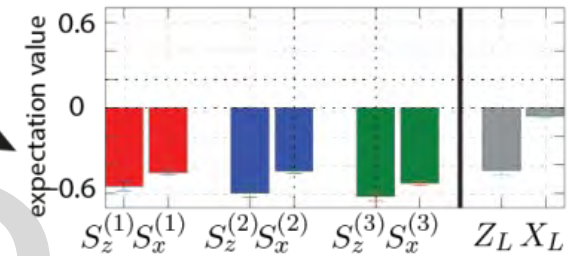
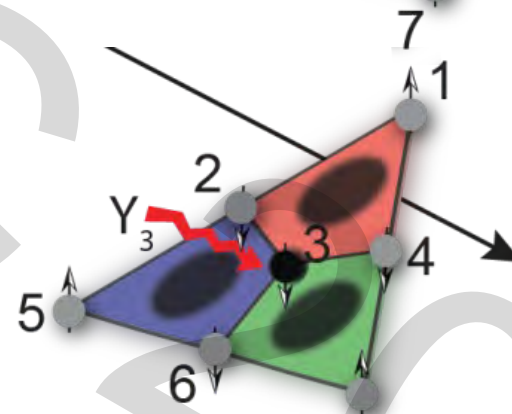
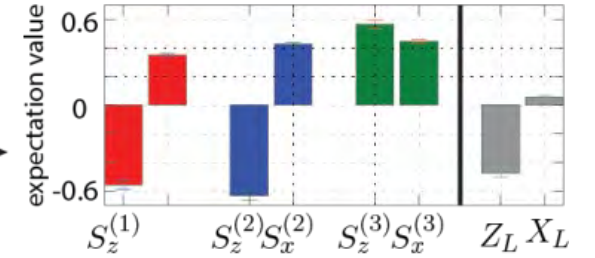
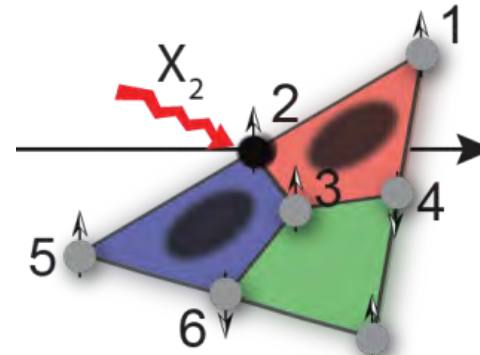
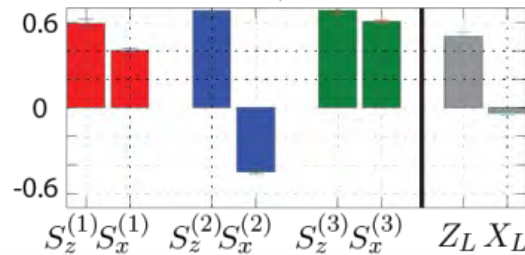
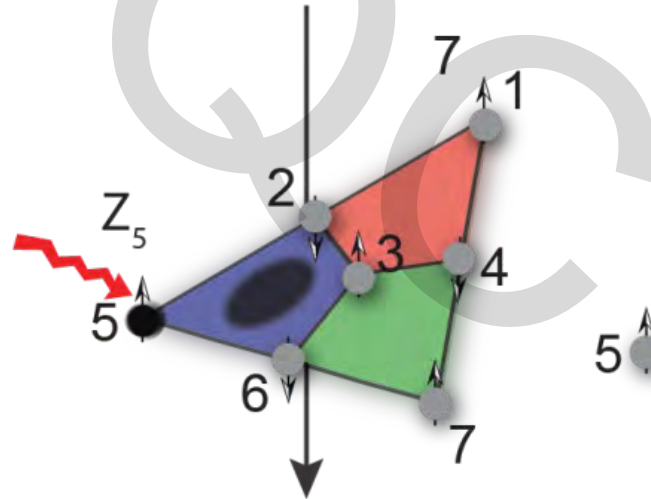
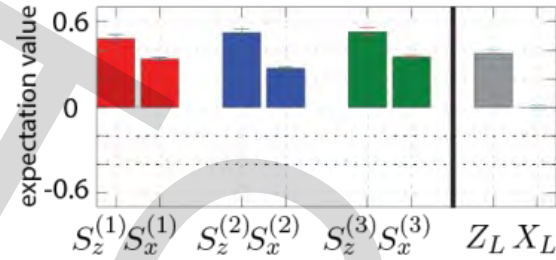
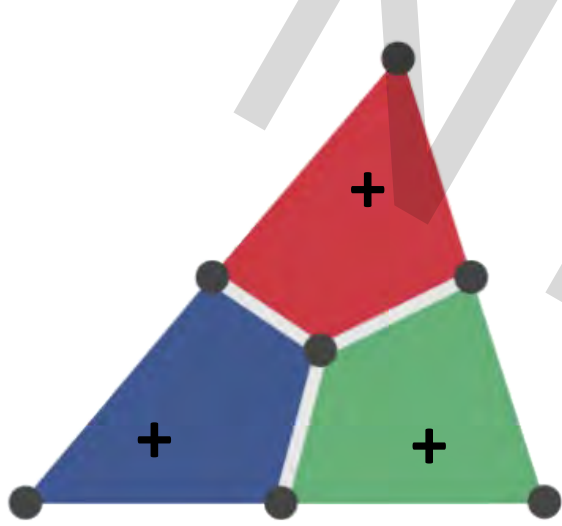
$$S_z^{(1)} = Z_1 Z_3 Z_5 Z_7$$



Physical qubit

Initializing the logical qubit requires the system to be in the +1 eigenstate of each stabilizer

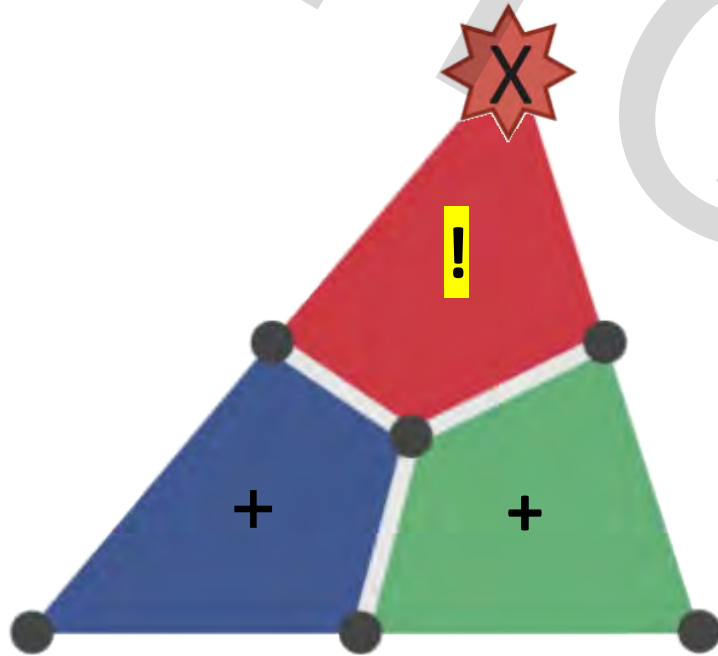
Error Syndromes



- CSS: X/Z error manifest as Z/X errors
- Y error is equiv. to simultaneous X&Z error
- Affected plaquettes flip sign
- Can correct one arbitrary error

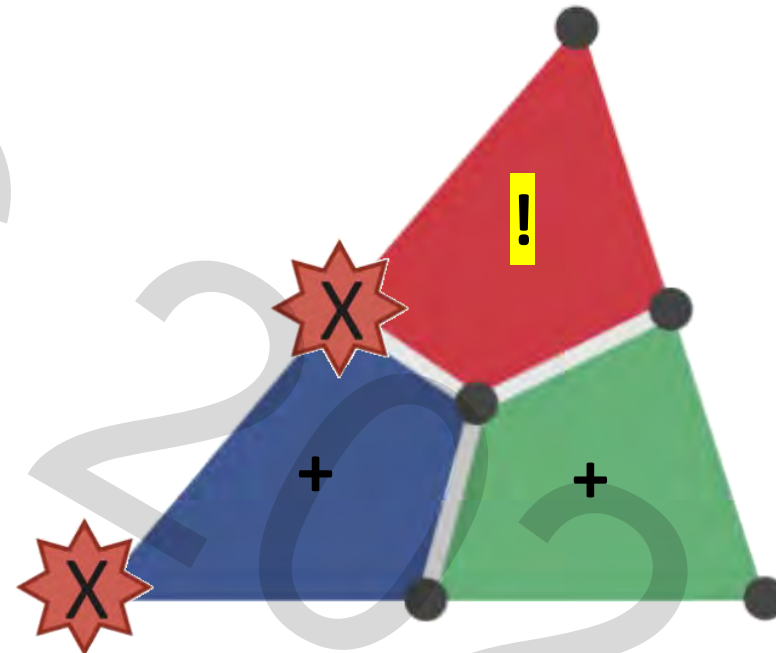
Correctable and Uncorrectable Errors

Any single error can be detected and corrected



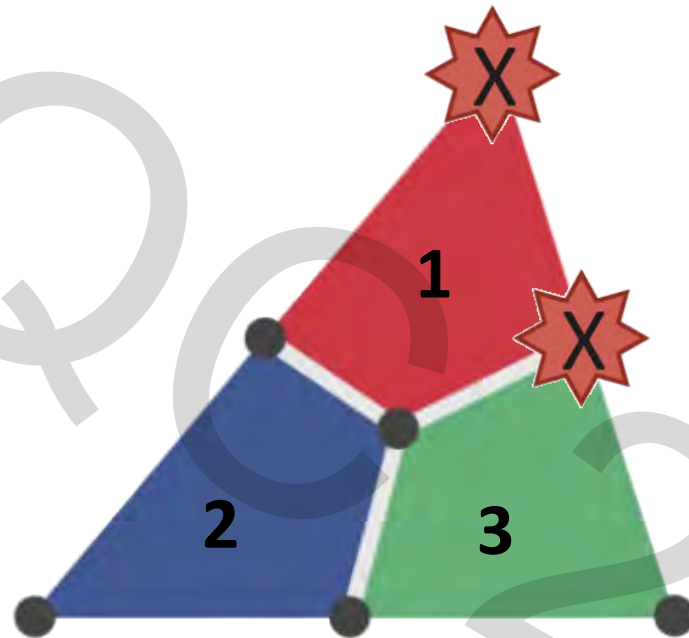
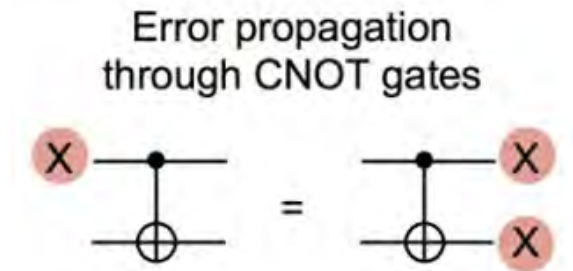
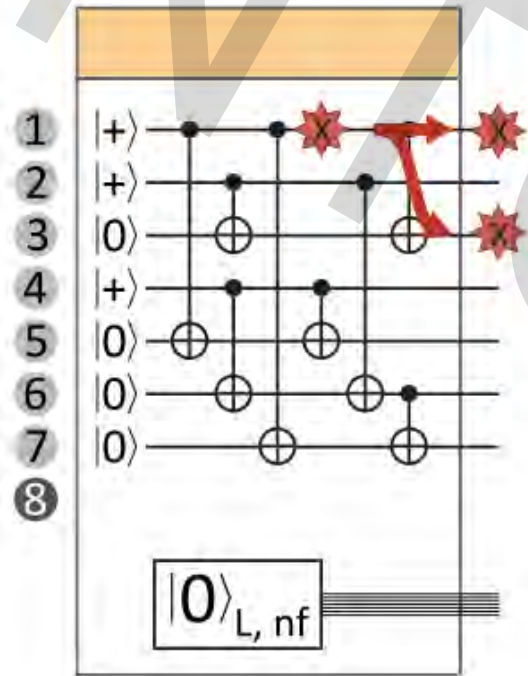
Correctable error

Two physical errors cannot be detected correctly



Uncorrectable error

Error Propagation and Fault Tolerance

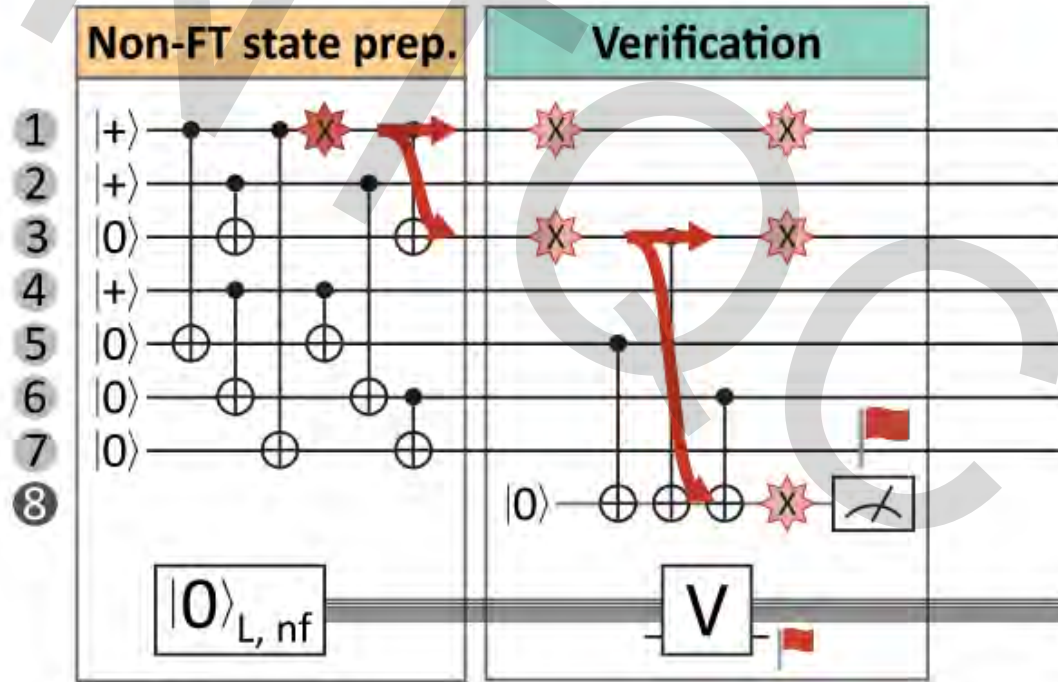


A single physical error can cause an uncorrectable logical error



Non fault-tolerant

Error Propagation and Fault Tolerance



Add a single "flag qubit" to herald uncorrectable errors when preparing the logical ground state

A **single** physical error leads **always** to a correctable error

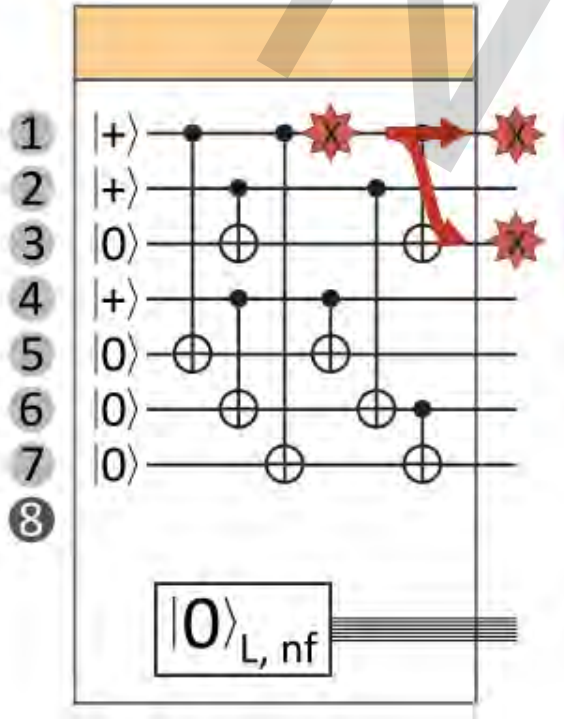


Fault-tolerant

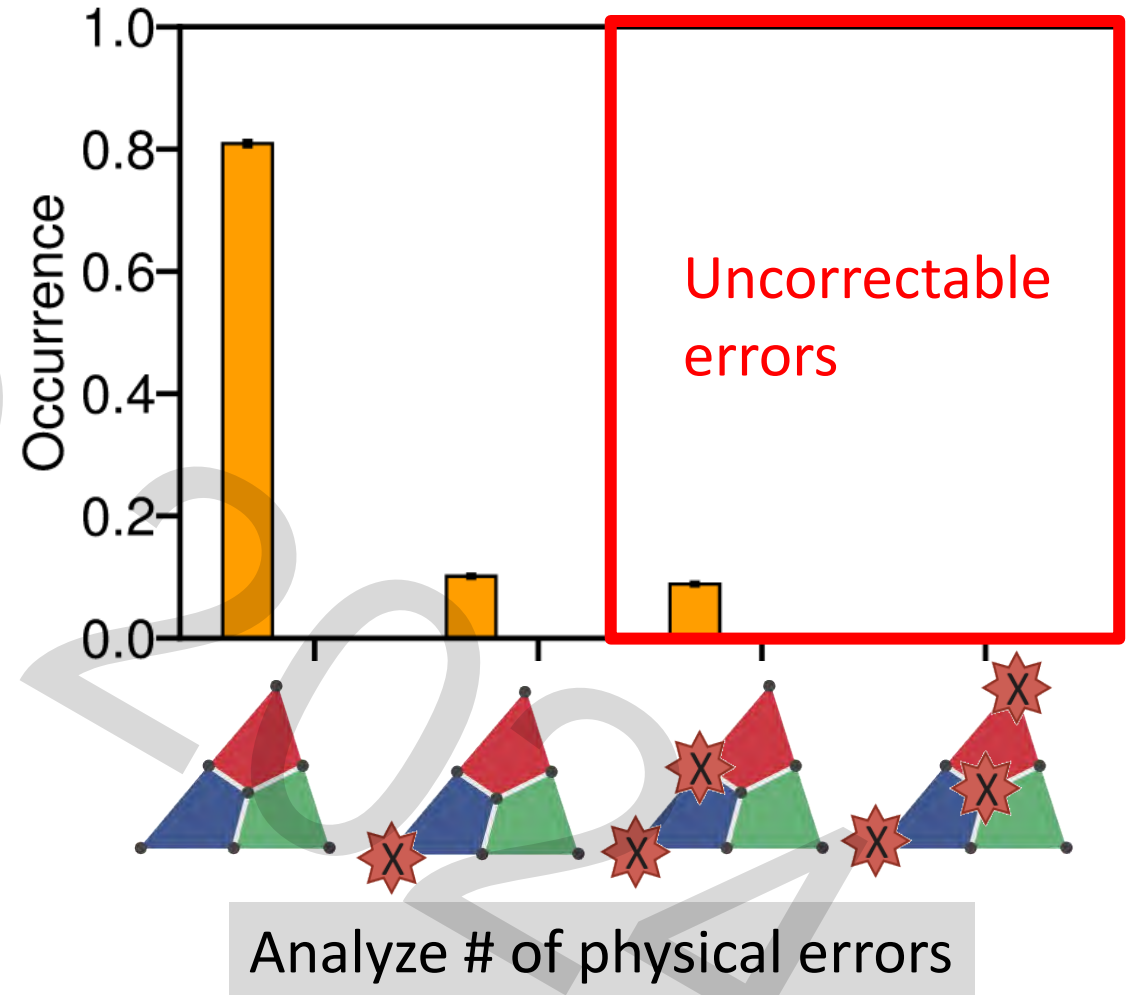


Chao, PRL. 121, 050502 (2018)
 Yoder, Quantum 1, 2 (2017)
 Reichardt, arXiv:1804.06995 (2018)
 C. Chamberland, Quantum 2, 53 (2018).
 C. Chamberland, NJP 22, 023019, (2020)
 C. Chamberland, Quantum 3, 143 (2019)

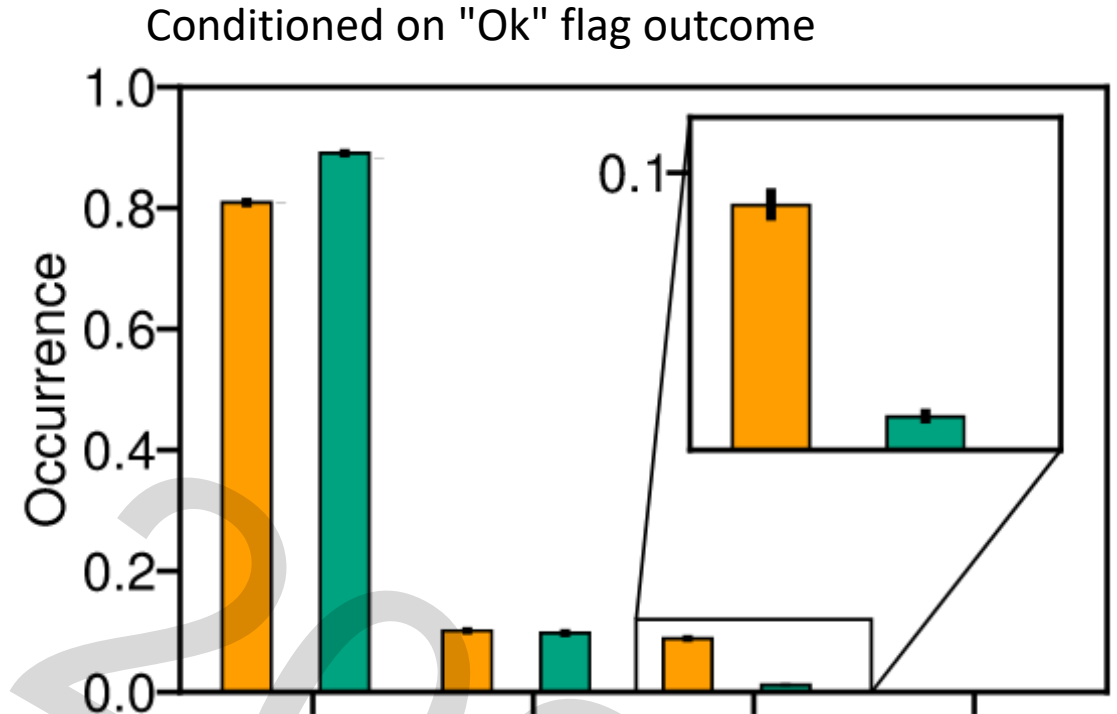
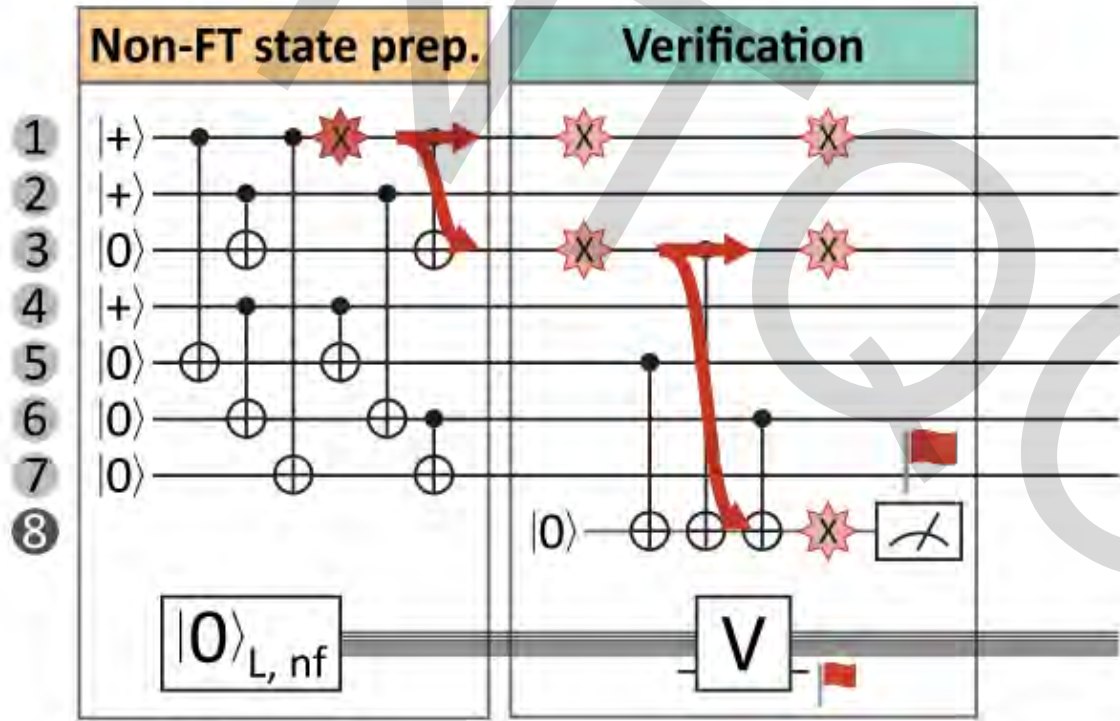
Experimental Results: Non-Fault-Tolerant State Preparation



Correctable and uncorrectable errors occur with similar probability



Experimental Results: Fault-Tolerant State Preparation

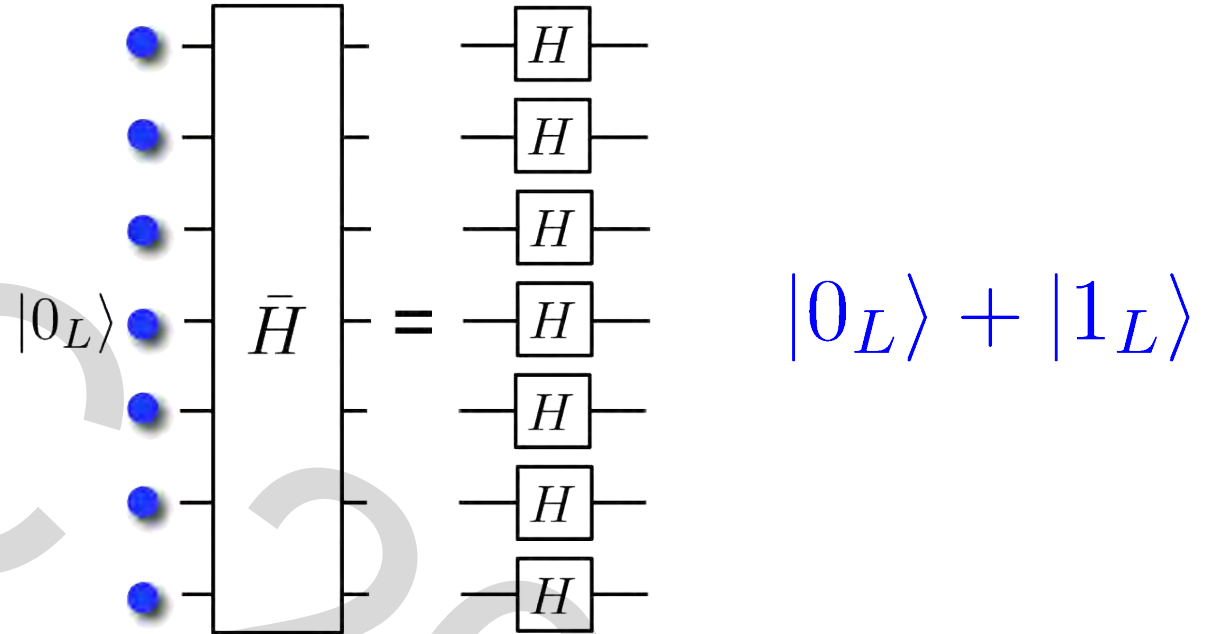
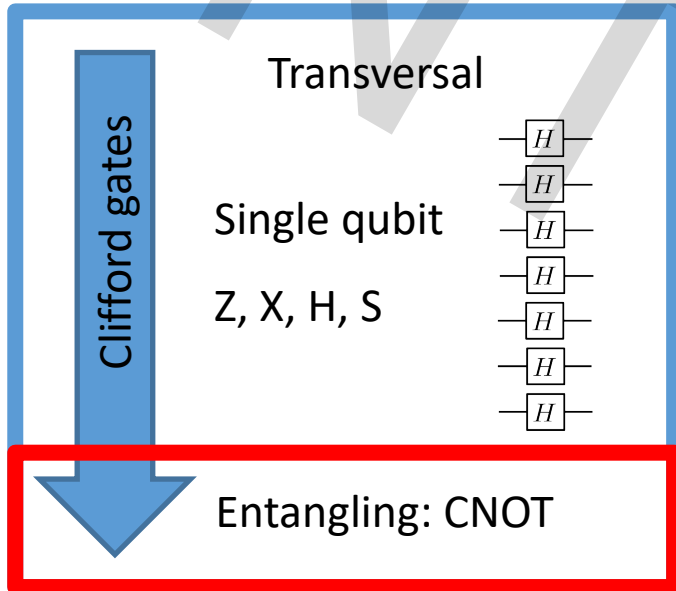


Uncorrectable errors are strongly suppressed

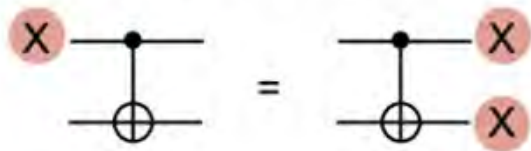
Logical infidelity: 1%
Survival rate: 78%



Operations on Encoded Qubits Need to be Fault-Tolerant!



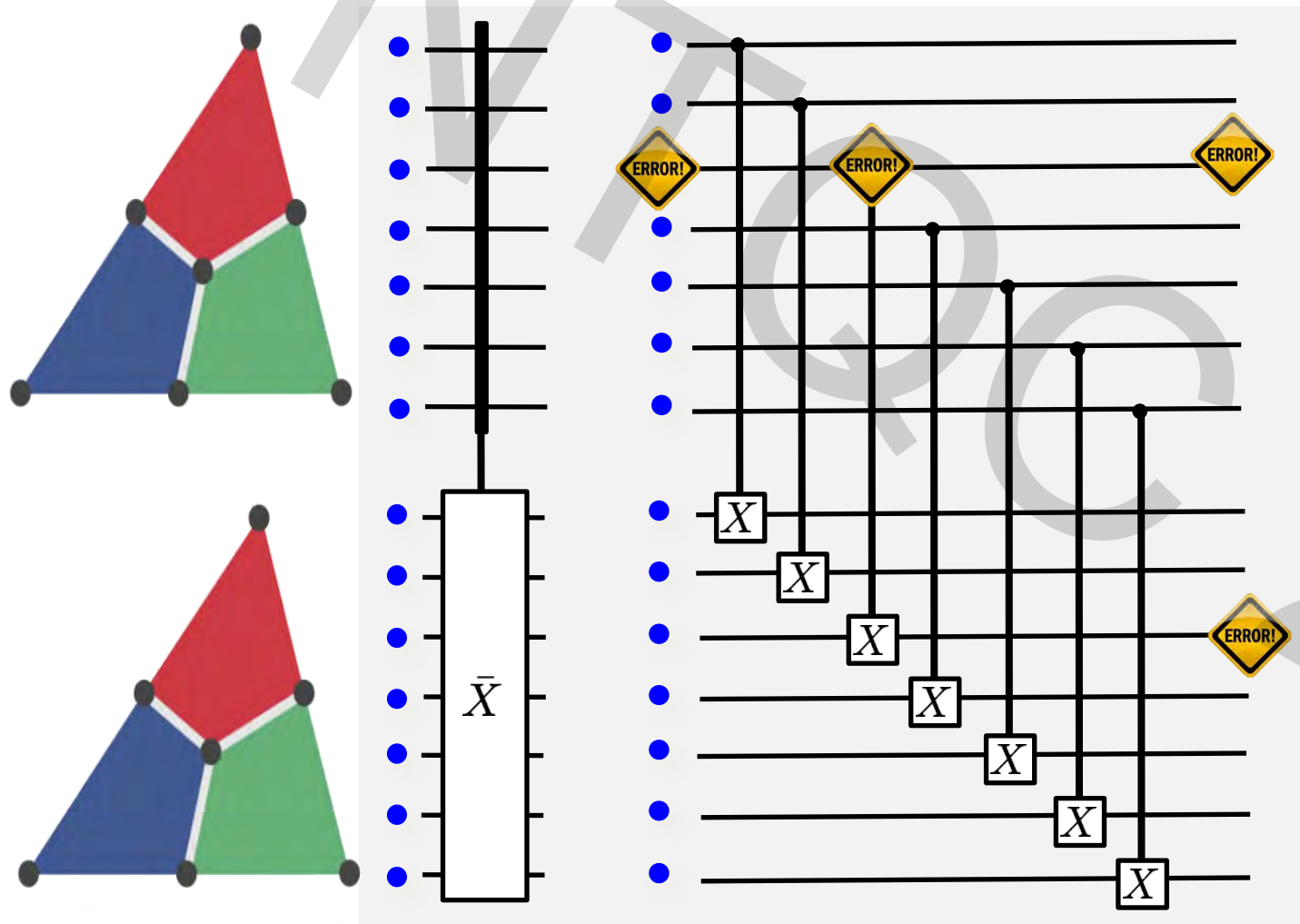
Error propagation through CNOT gates



Transversal operations are fault-tolerant ✓

The entire set of Clifford operations is transversal for the color code

Fault-Tolerant CNOT

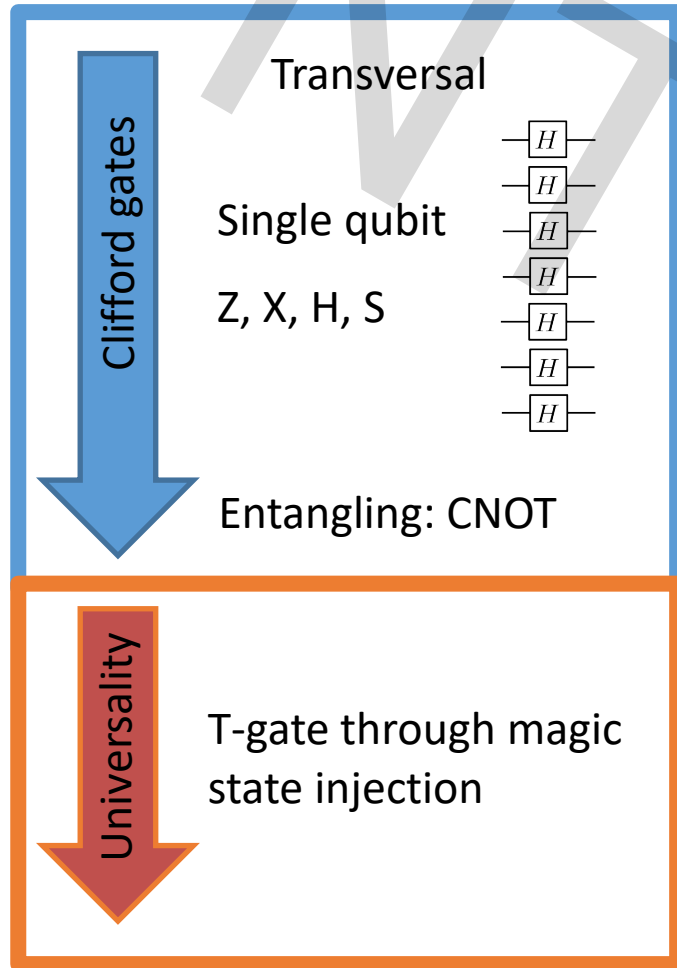


Single physical error maps onto two physical errors on different logical qubits!

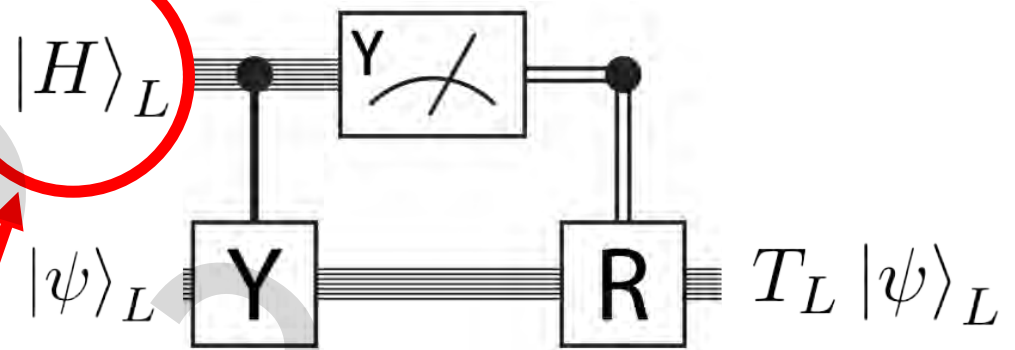
A **single** physical error leads **always** to a correctable error



A Universal Gate-Set



Solution: Gate teleportation with a fault-tolerant magic state

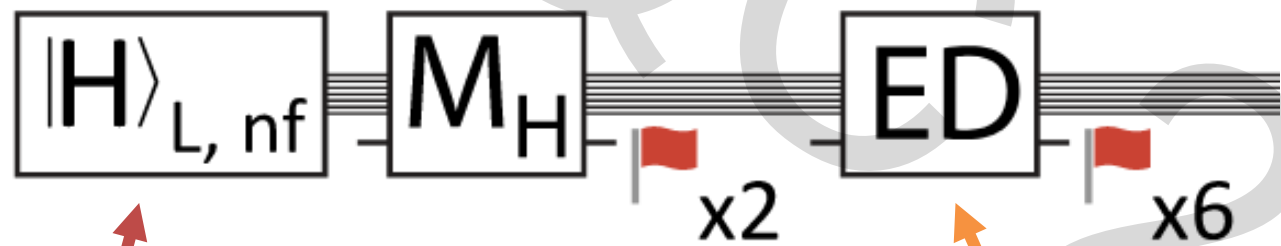


Challenge: Requires fault tolerant operations on two logical qubits

Magic state – hard to create

Fault-Tolerant Magic State Preparation

Project in eigenbasis
to reject -1 eigenstates

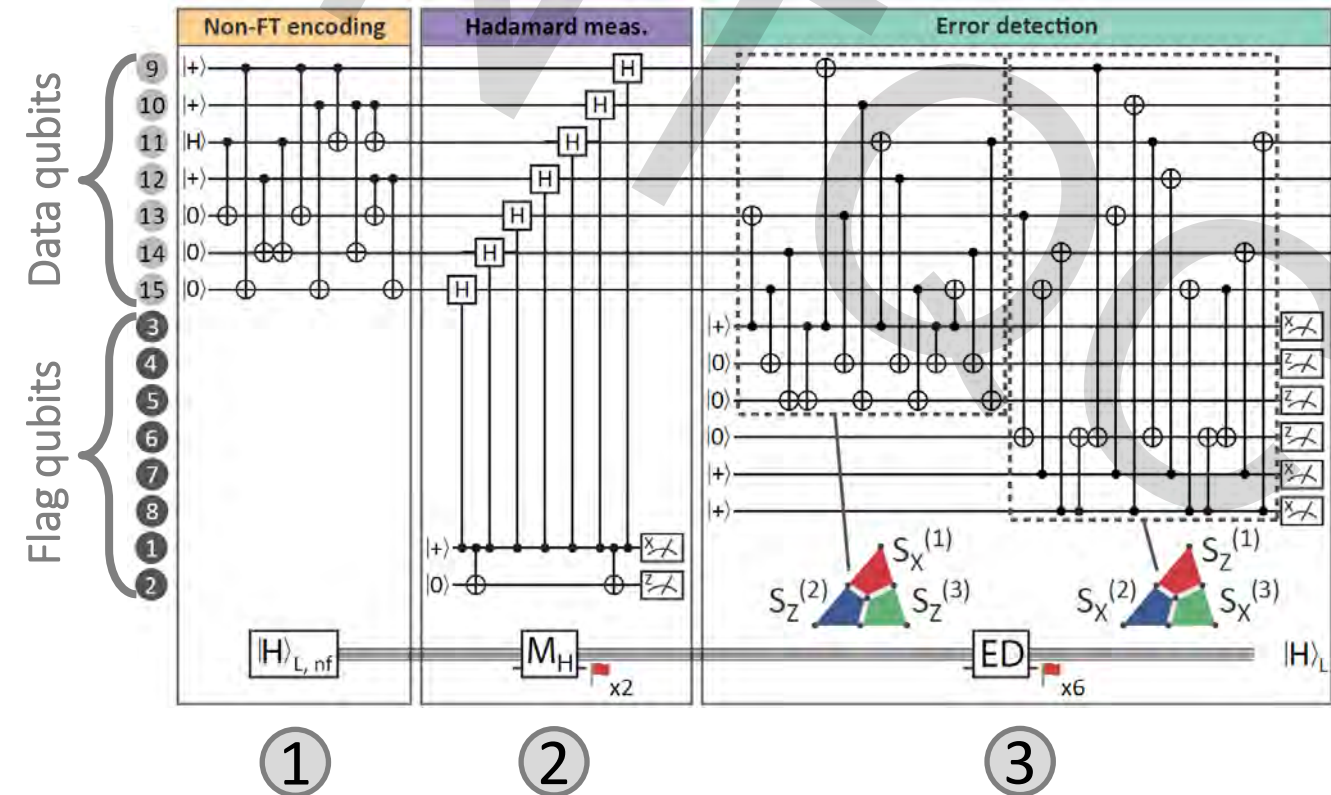


Guaranteed
magic state with
at most one error

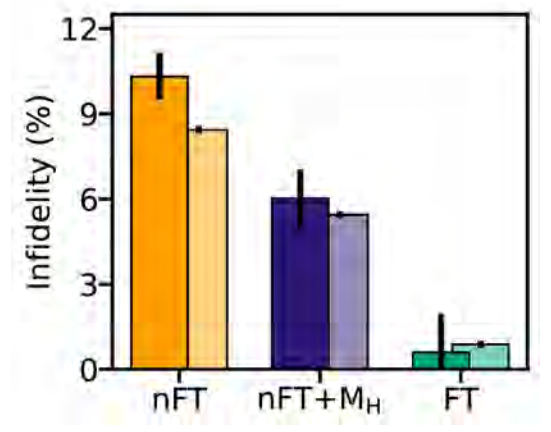
Non-FT preparation

Error detection of
all 6 stabilizers

Fault-Tolerant Magic State Preparation



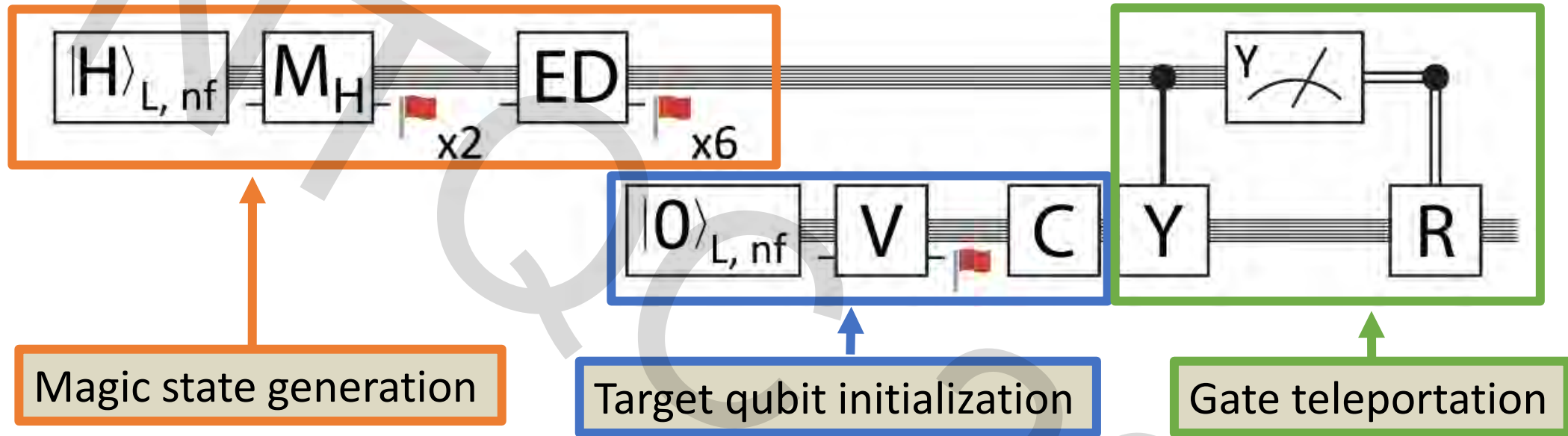
- 1 Non-FT prepatate magic state
- 2 Measure Hadamard: Magic state is +1-eigenstate
- 3 Error detection



$$\mathcal{F}_{\text{FT}} = 0.994_{-14}^{+5}$$

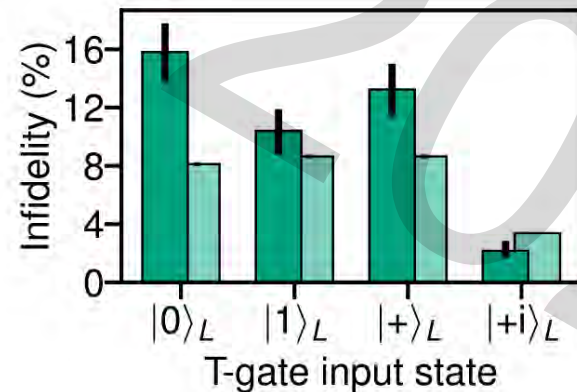
Survival rate = 13%

And Gate teleportation



Required resources:

- 2 logical qubits (14 physical qubits)
- 9 Flag-qubits
- 66 entangling operations



Logical error: 10%
Survival rate: 9%

2. Quantum Computation and Simulation

2.1 New Generation

2.2 Operations beyond Qubits

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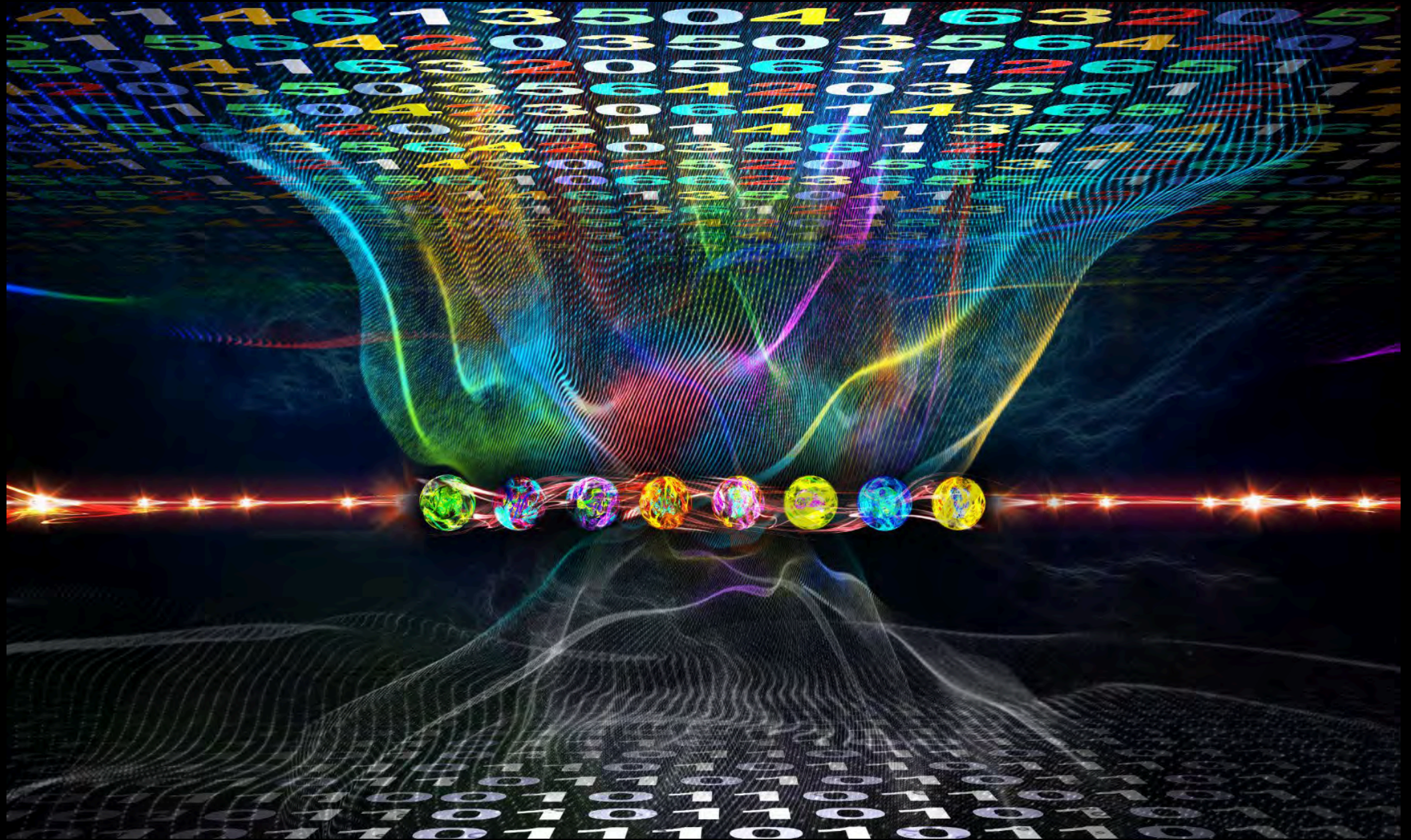
2.4 QIP with Qudits

2.5 Digital Quantum Simulation

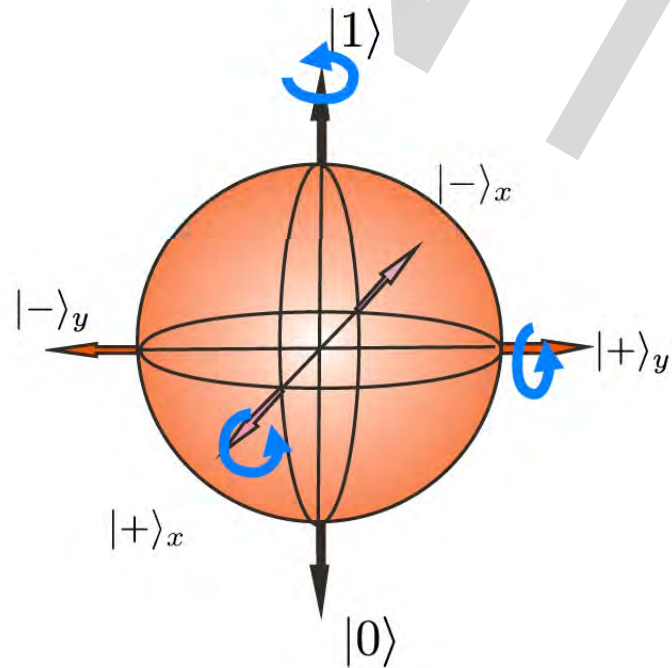
2.6 Scaling



Quantum Information Processing with trapped-ion qubits



Universal Gate Set



Operations on qubits are elements of $SU(2)$.

The Lie algebra is generated by

$$X_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad Y_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad Z_2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

→ Pauli group: $\langle X_2, Y_2, Z_2 \rangle$

→ Clifford group: $\langle S_2, H_2, CNot \rangle$ + $T_2 = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$
Universal !

$$S_2 = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \quad H_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Operations on a qudit

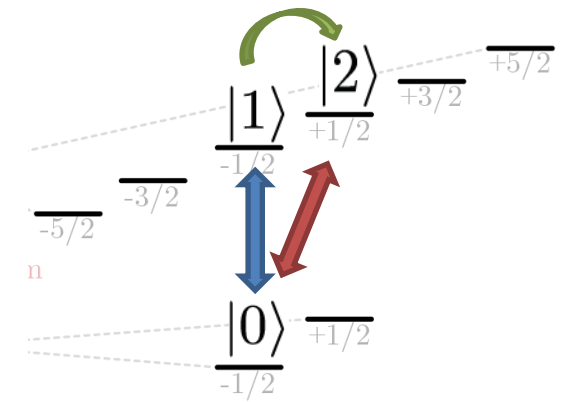
Qudit operations are described by $SU(d)$

For $d=3$ the Lie algebra is generated by the Gell-Mann matrices:

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix},$$

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$



$su(2)$ sub-algebras

$$\{\lambda_1, \lambda_2, \lambda_3\}$$

$$\{\lambda_4, \lambda_5, (\lambda_3 + \sqrt{3}\lambda_8)/2\}$$

$$\{\lambda_1, \lambda_2, (-\lambda_3 + \sqrt{3}\lambda_8)/2\}$$

Qudit operations

“Pauli”

$$Z_d = \omega_d^j |j\rangle \langle j|$$

$$Z_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{\frac{2\pi i}{3}} & 0 \\ 0 & 0 & e^{\frac{4\pi i}{3}} \end{pmatrix}$$

$$X_d = |j + 1 \pmod{d}\rangle \langle j|$$

$$X_3 = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

Clifford

$$S_d = \sum_j \omega_d^{j(j+1)/2} |j\rangle \langle j|$$

$$S_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{\frac{2\pi i}{3}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$H_d = \frac{1}{\sqrt{d}} \sum_{j,k} \omega_d^{jk} |k\rangle \langle j|$$

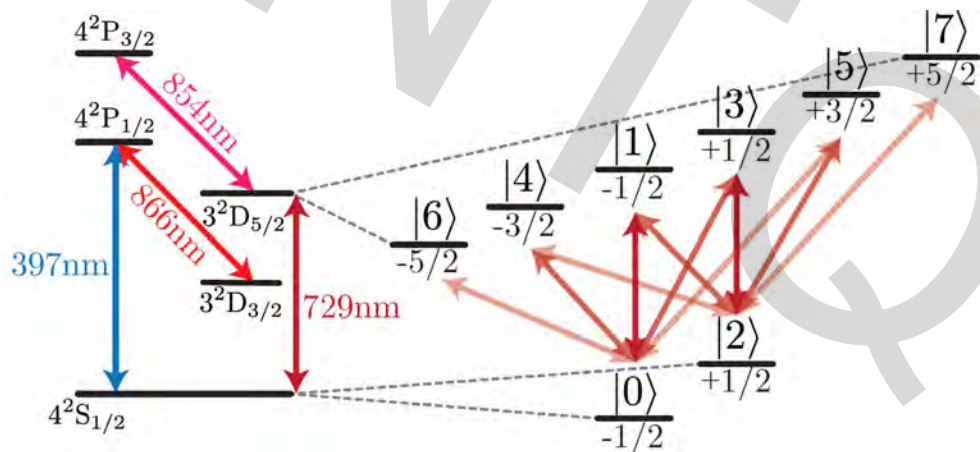
$$H_3 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & e^{\frac{2\pi i}{3}} & e^{-\frac{2\pi i}{3}} \\ 1 & e^{-\frac{2\pi i}{3}} & e^{\frac{2\pi i}{3}} \end{pmatrix}$$

+

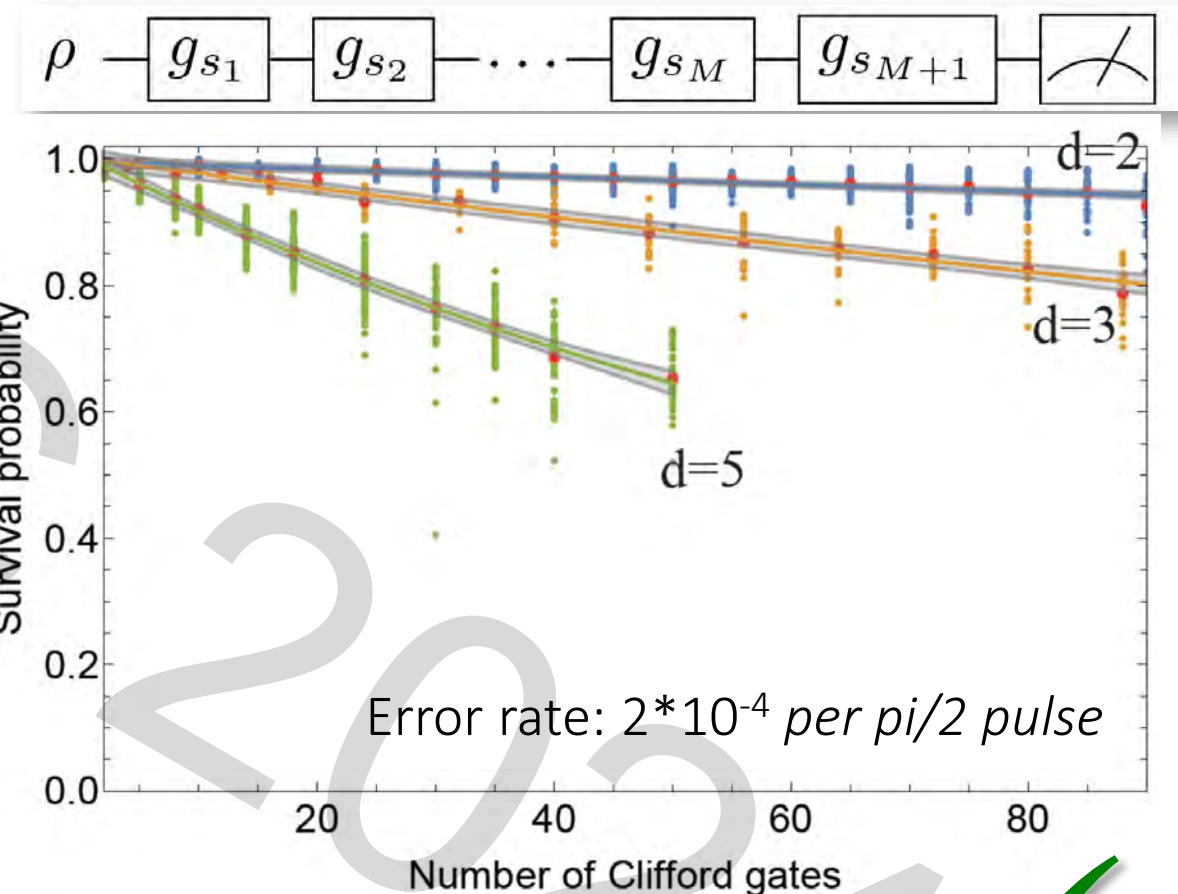
$$T_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{\frac{2\pi i}{9}} & 0 \\ 0 & 0 & e^{-\frac{2\pi i}{9}} \end{pmatrix}$$

Universal

Benchmarking Single Qudit Operations



Efficient decomposition of all local gates
 Universal computation with Clifford + T

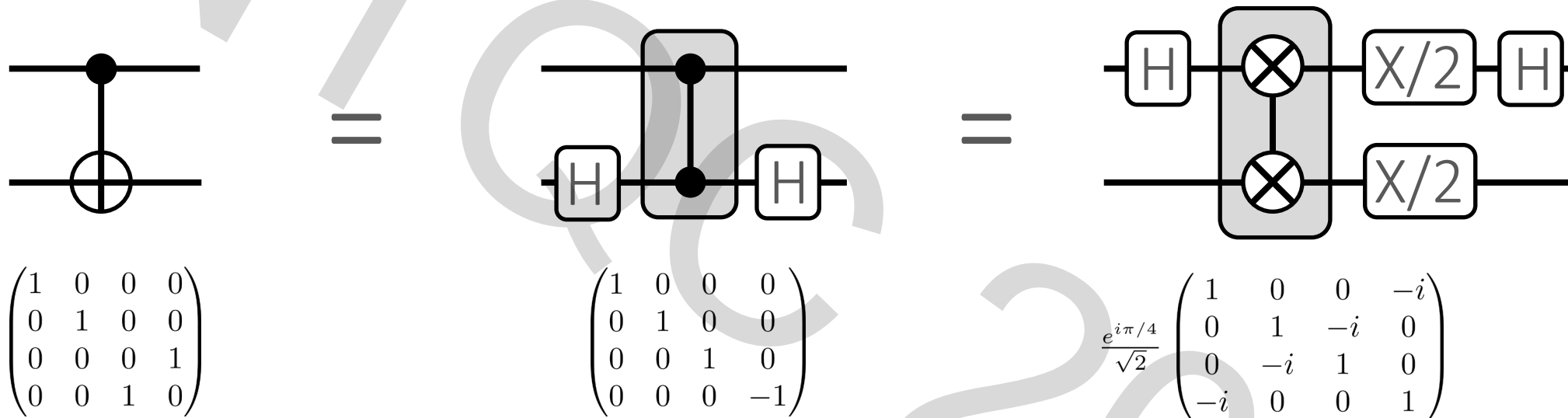


Consistent performance for all d !



Entangling gates

Universal qubit/qudit QC requires arbitrary local gates + any 1 entangling gate!



For qubits all entangling gates are equal

For qudits, some are more equal than others

Qudit entangling gates

Embedded qubit gates



States such as
 $|00\rangle + |11\rangle$

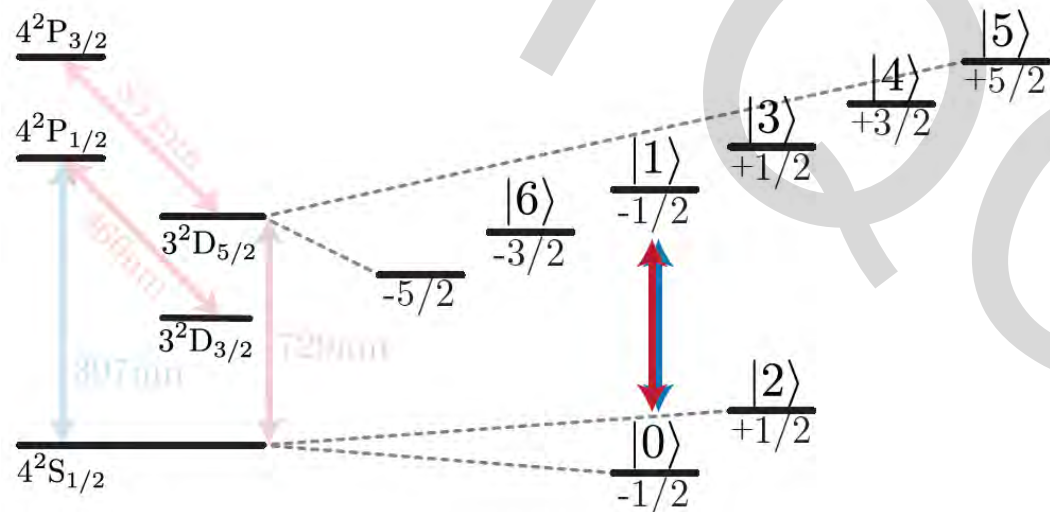
Genuine qudit gates



States such as
 $|00\rangle + |11\rangle + |22\rangle$

Qudit entangling gates

Embedded qubit gates



Two-level entanglement in qudit Hilbert space

→ No drop in fidelity due to larger Hilbert space ✓

Genuine qudit gates

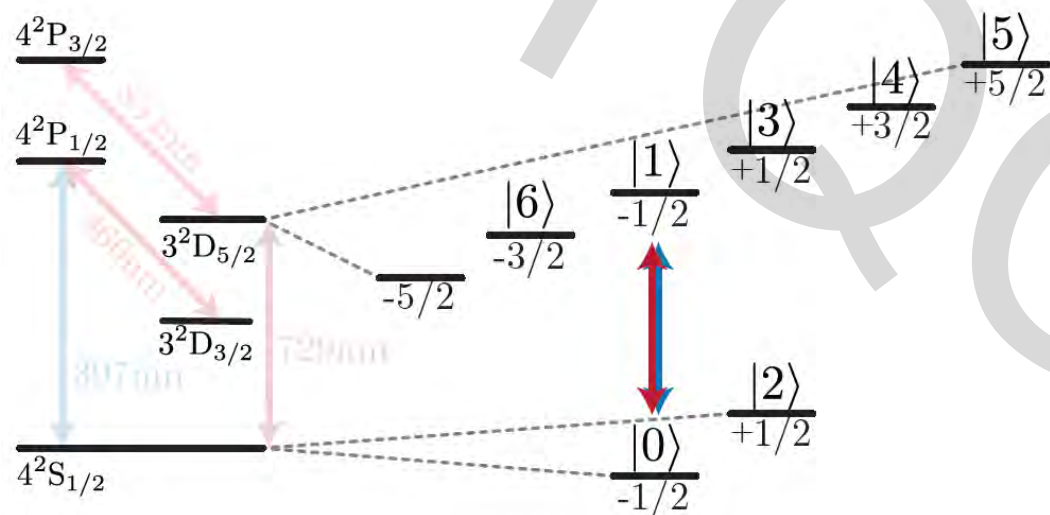


States such as

$$|00\rangle + |11\rangle + |22\rangle$$

Qudit entangling gates

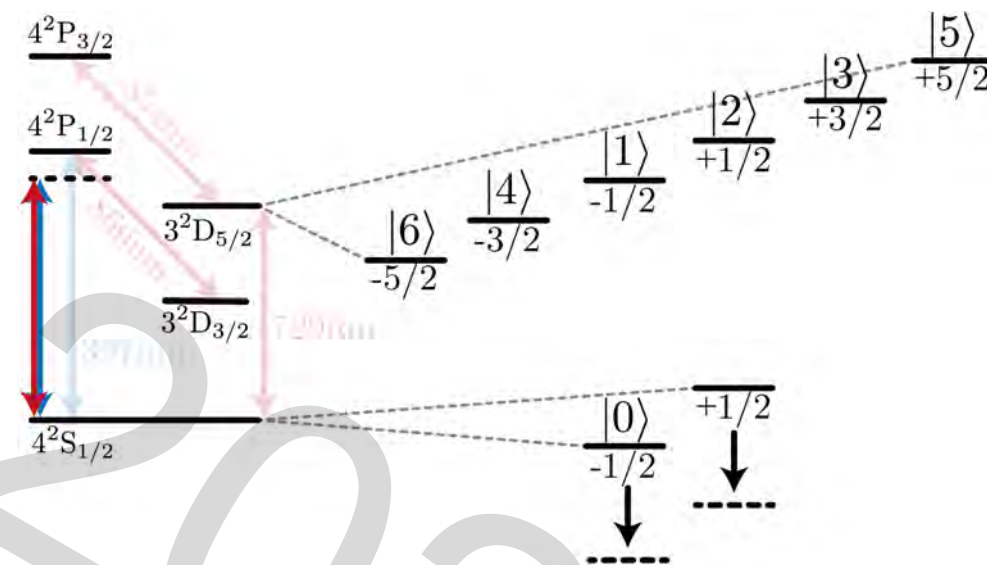
Embedded qubit gates



Two-level entanglement in qudit Hilbert space

→ No drop in fidelity due to larger Hilbert space ✓

Genuine qudit gates



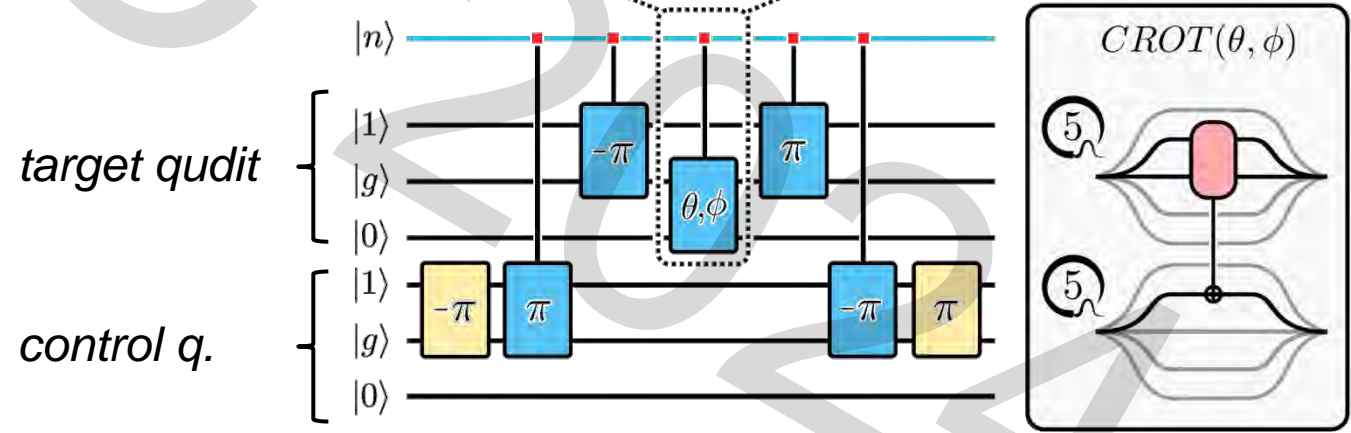
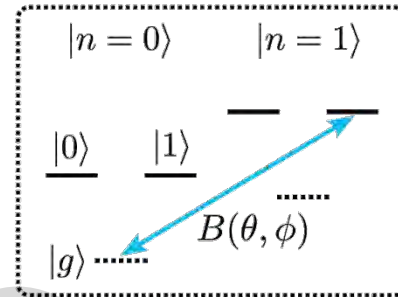
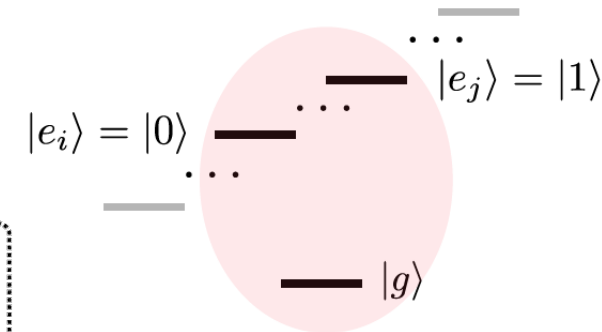
Genuine qudit gate with $F=0.990(4)\%$

→ One control parameter independent of dim ✓

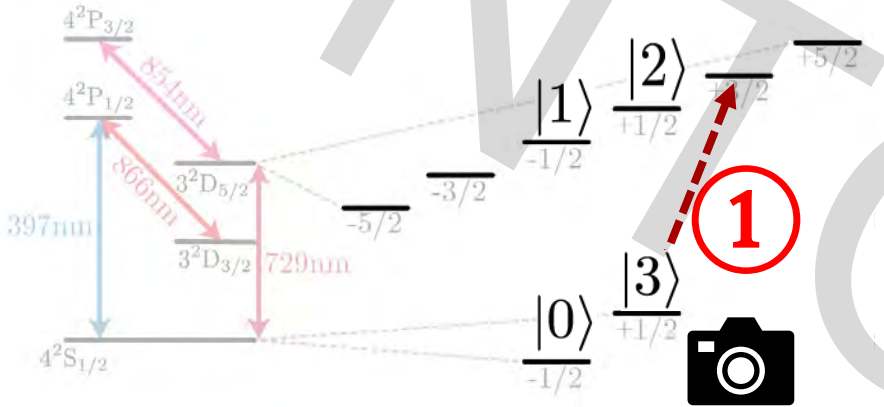
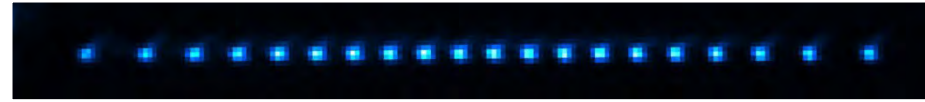
Mixed-dimensional controlled-rotation gates

Encode qudit in excited state manifold of $^{40}\text{Ca}^+$
 ground state is (generally) not populated
 local blue sideband pulses to mediate interaction

- Any qudit state can serve as control state
- Arbitrary rotation on target qudit
- Mediated via motional mode



Qudit Measurement

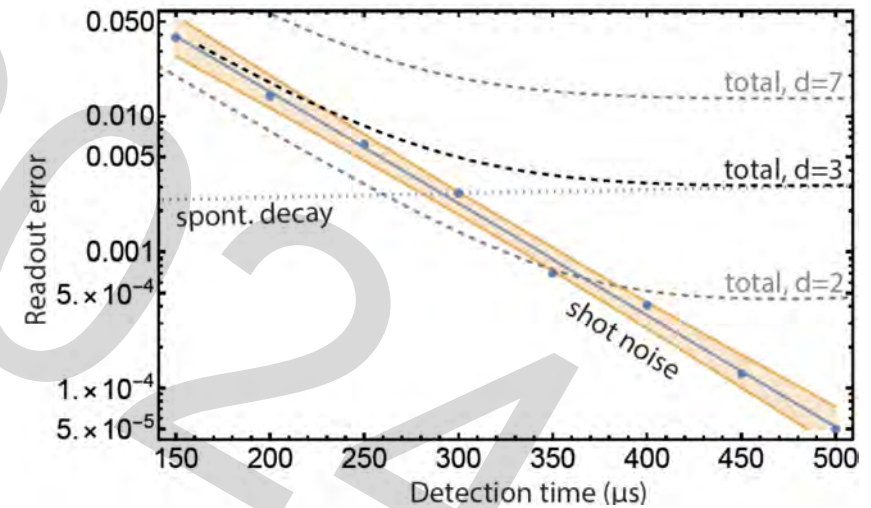
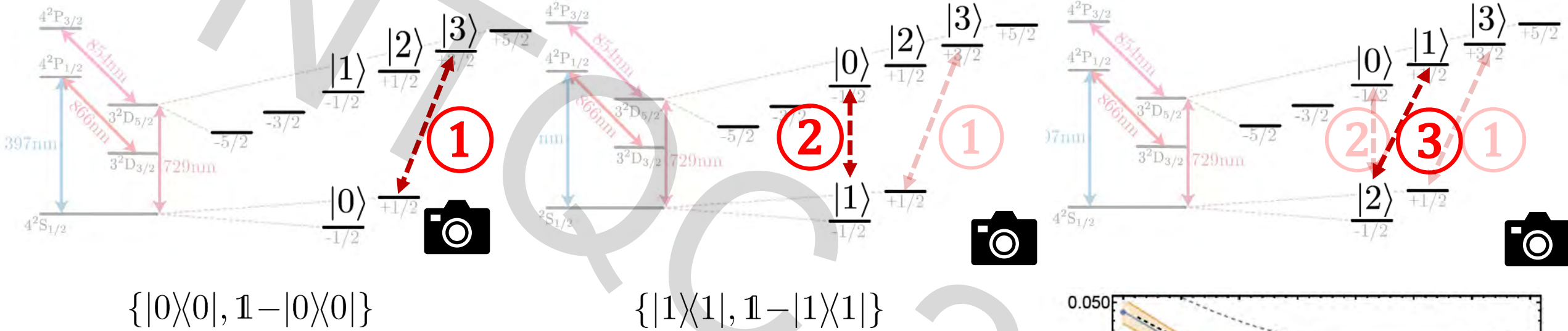
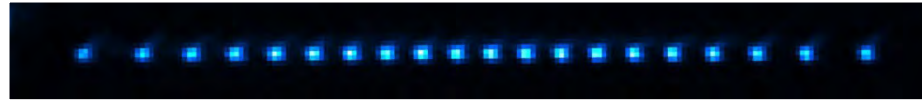


$$\{|0\rangle\langle 0|, \mathbb{1} - |0\rangle\langle 0|\}$$

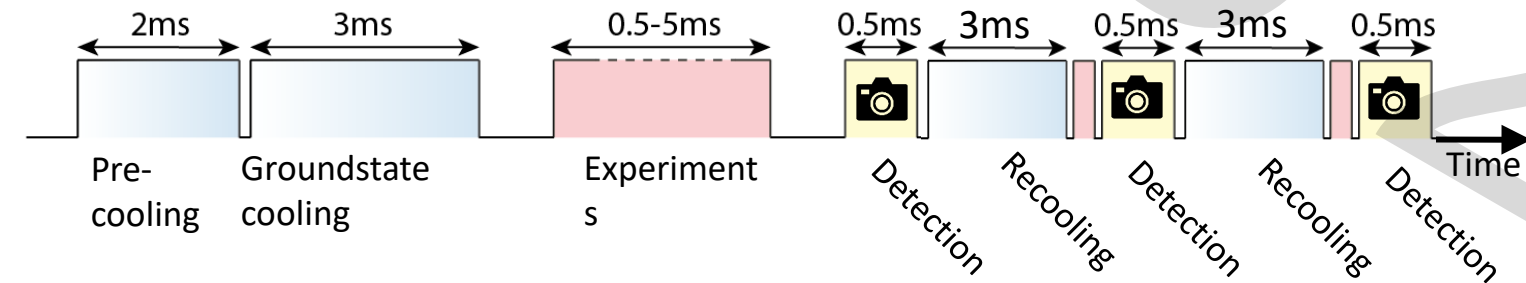
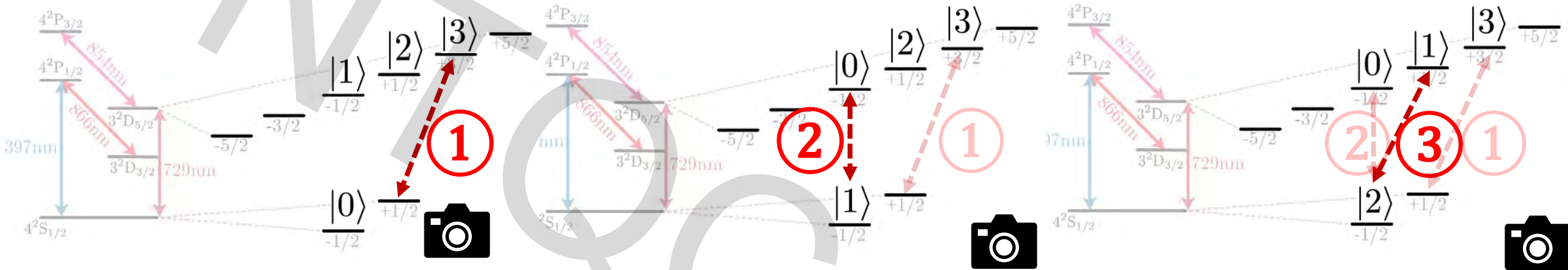
$$\{|1\rangle\langle 1|, \mathbb{1} - |1\rangle\langle 1|\}$$

$$\{|2\rangle\langle 2|, \mathbb{1} - |2\rangle\langle 2|\}$$

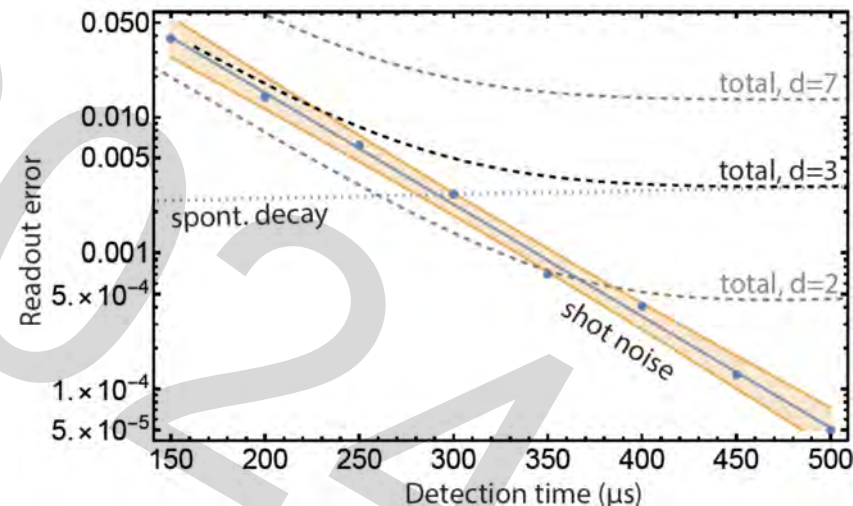
Qudit Measurement



Qudit Measurement

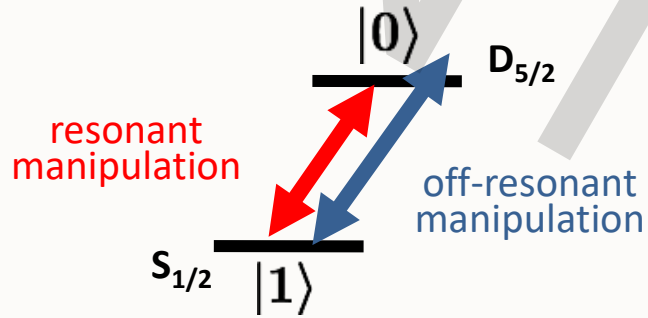


Requires fast detection & recooling!

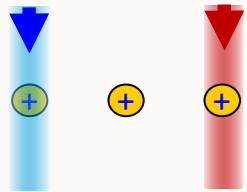


Quantum computing with qudits

Local Operations

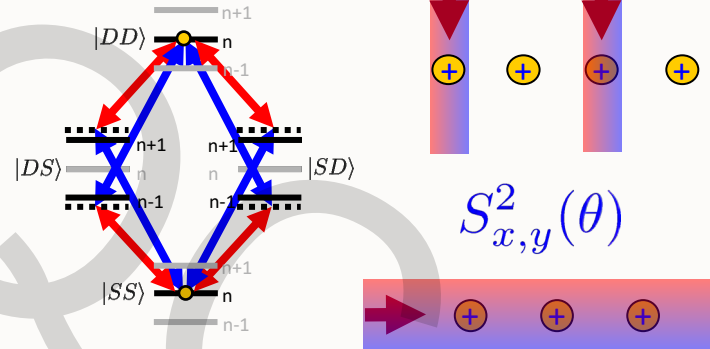


$S_z(\theta)$ $S_{x,y}(\theta)$



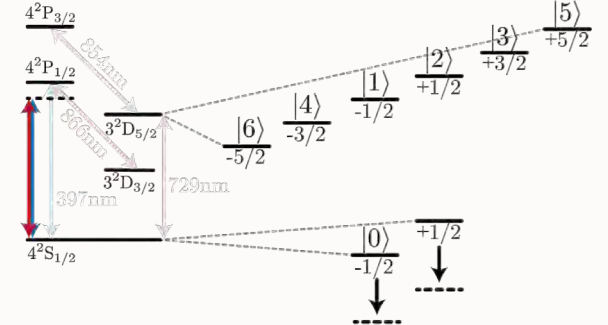
$\tau = 20\mu\text{s}$
 $F > 99.9\%$

Qubit Entanglement



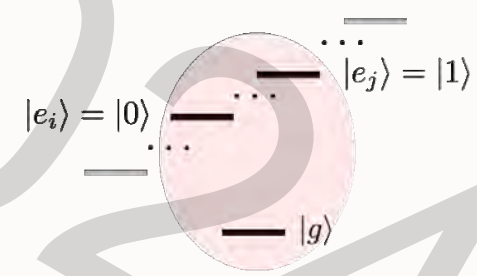
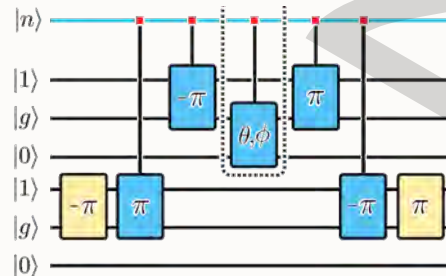
$F_2 > 99\%$ $\tau = 100\mu\text{s}$

Qudit Entanglement



$F_2 > 99\%$ $\tau = 100\mu\text{s}$

Mixed-dimensional Entanglement



$F_2 > 99\%$
 $\tau = 100\mu\text{s}$

2. Quantum Computation and Simulation

2.1 New Generation

2.2 Operations beyond Qubits

2.3 Quantum Error Correction

2.4 QIP with Qudits

➔ **2.5 Digital Quantum Simulation**

2.6 Scaling



Digital Simulation – Universal Quantum Simulator

$$H = \sum_k h_k$$

← model of some local system to be simulated for a time t

1) build each local evolution operator separately, for small time steps

$$u_k = e^{-ih_k t/n}$$

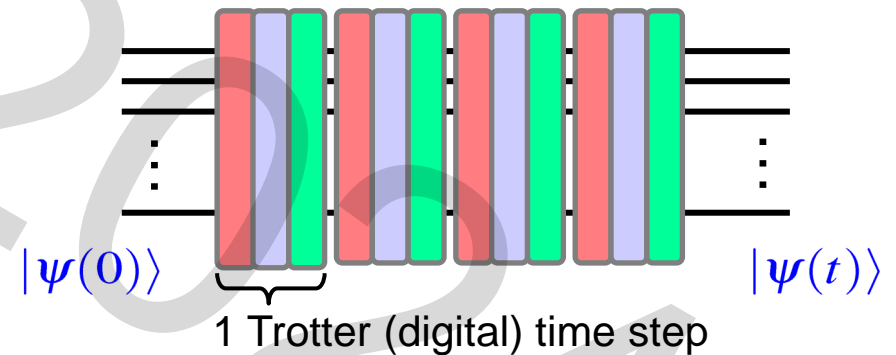
2) approximate global evolution operator using the Trotter approximation

$$U = e^{-iHt} \approx \left(e^{-ih_1 \frac{t}{n}} e^{-ih_2 \frac{t}{n}} e^{-ih_3 \frac{t}{n}} \dots e^{-ih_k \frac{t}{n}} \right)^n$$

“Efficient for local quantum systems”

S. Lloyd,
Science **273**, 1073 (1996)

Discretization errors are well behaved
M. Heyl et al, Sci. Adv. **5**, eaau8342 (2019)



Digital Simulators are flexible

Ising

$$J_x \sigma_x^1 \sigma_x^2$$

$$B(\sigma_z^1 + \sigma_z^2)$$



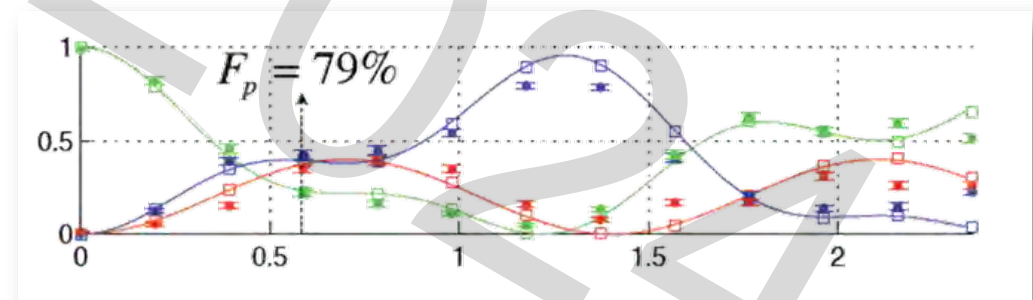
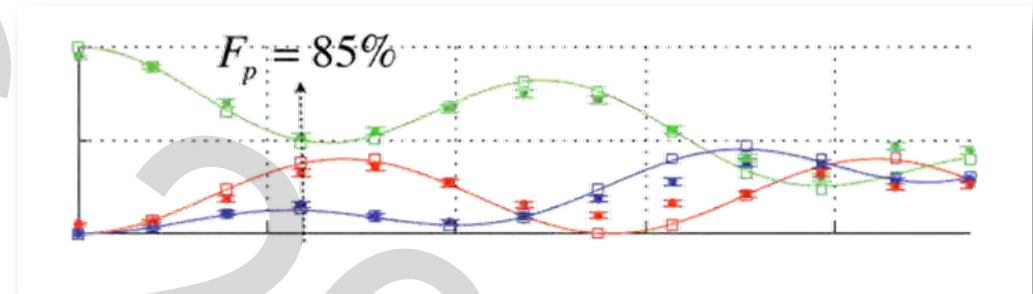
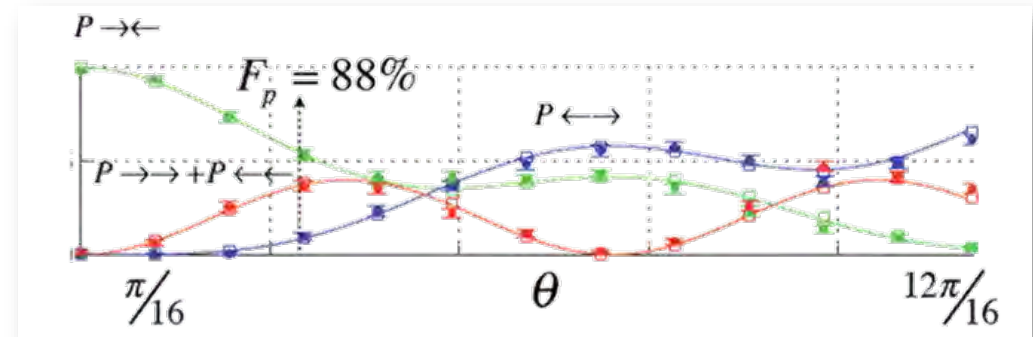
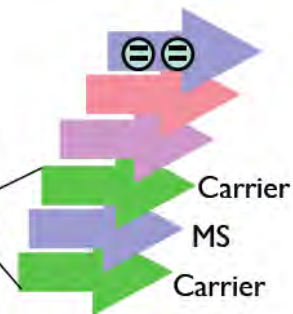
XY

$$\dots + J_y \sigma_y^1 \sigma_y^2$$

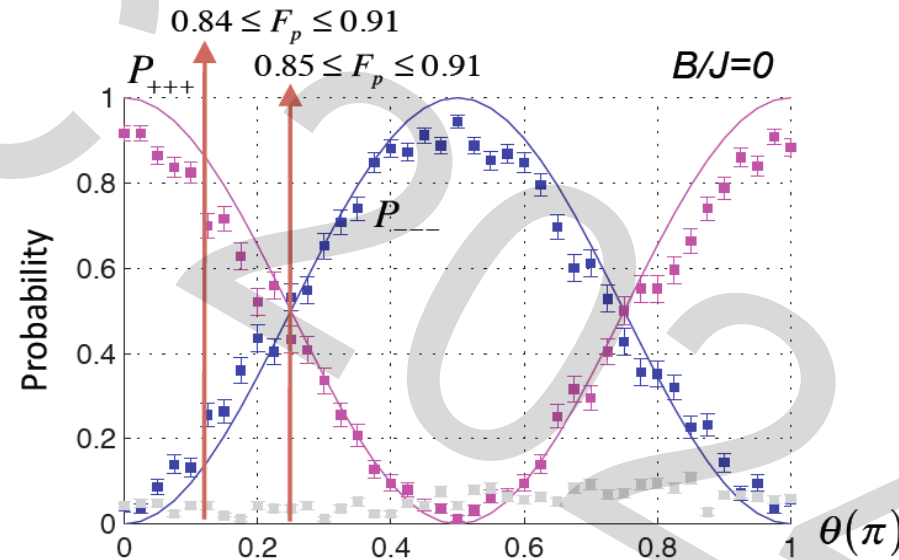
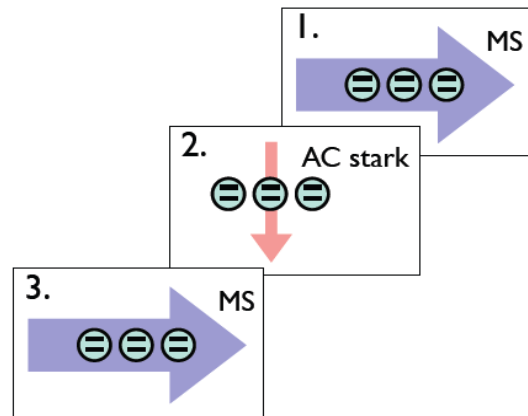
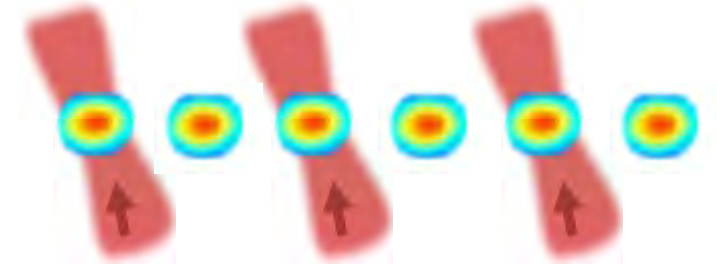
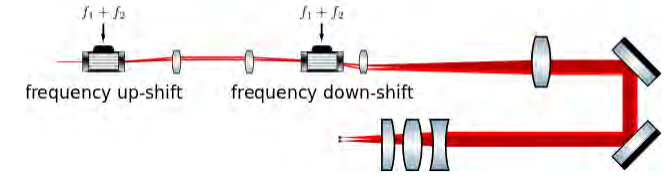
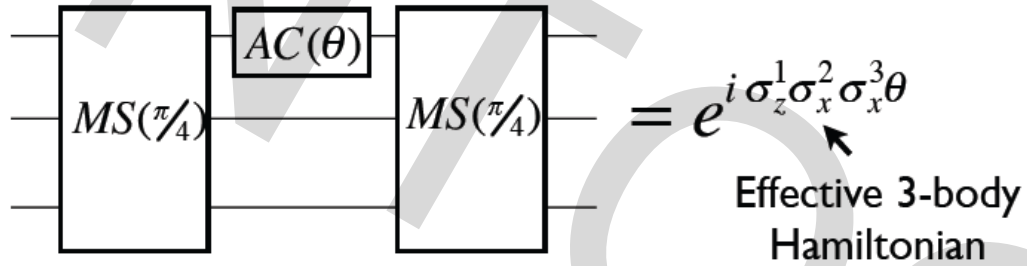


XYZ

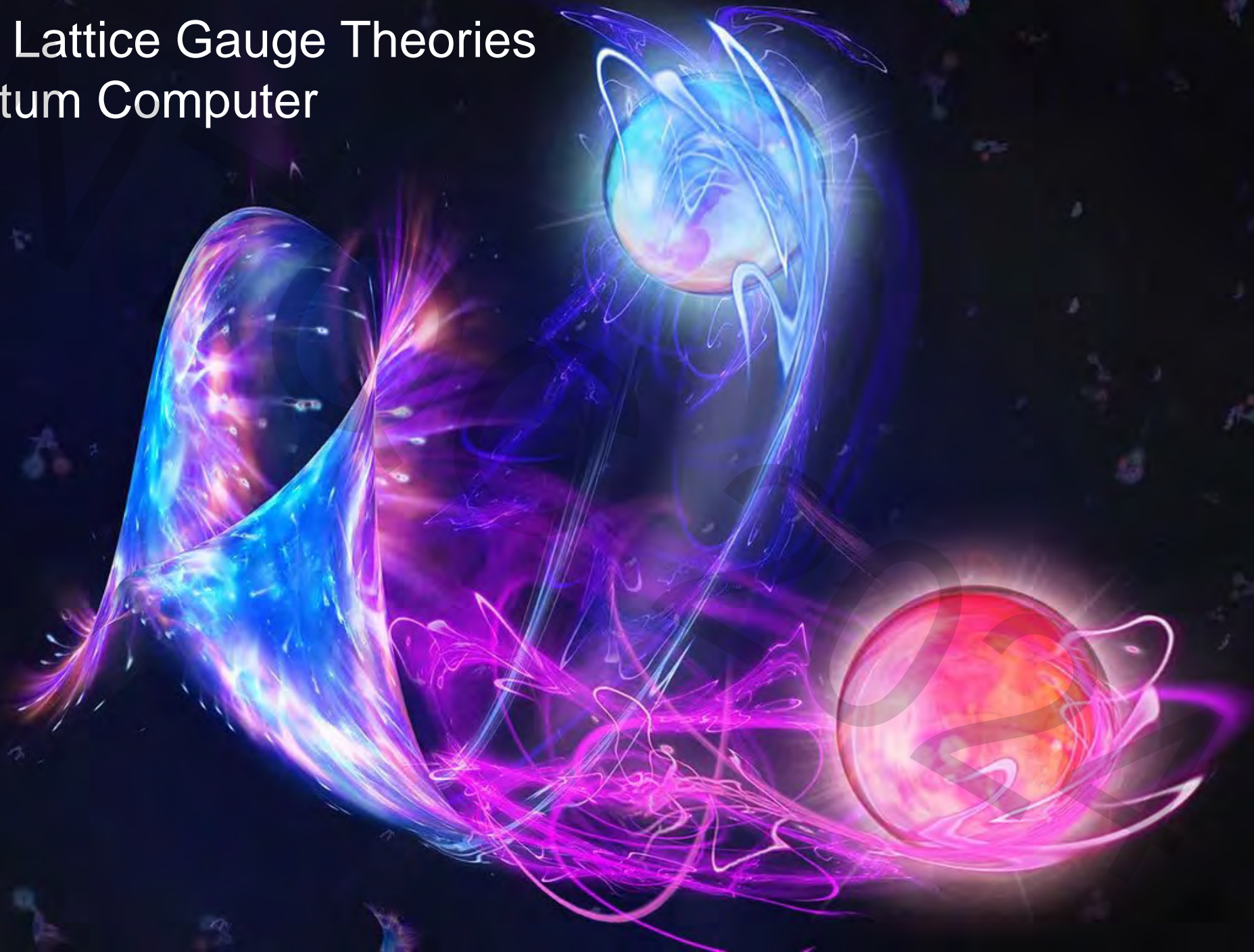
$$\dots + J_z \sigma_z^1 \sigma_z^2$$



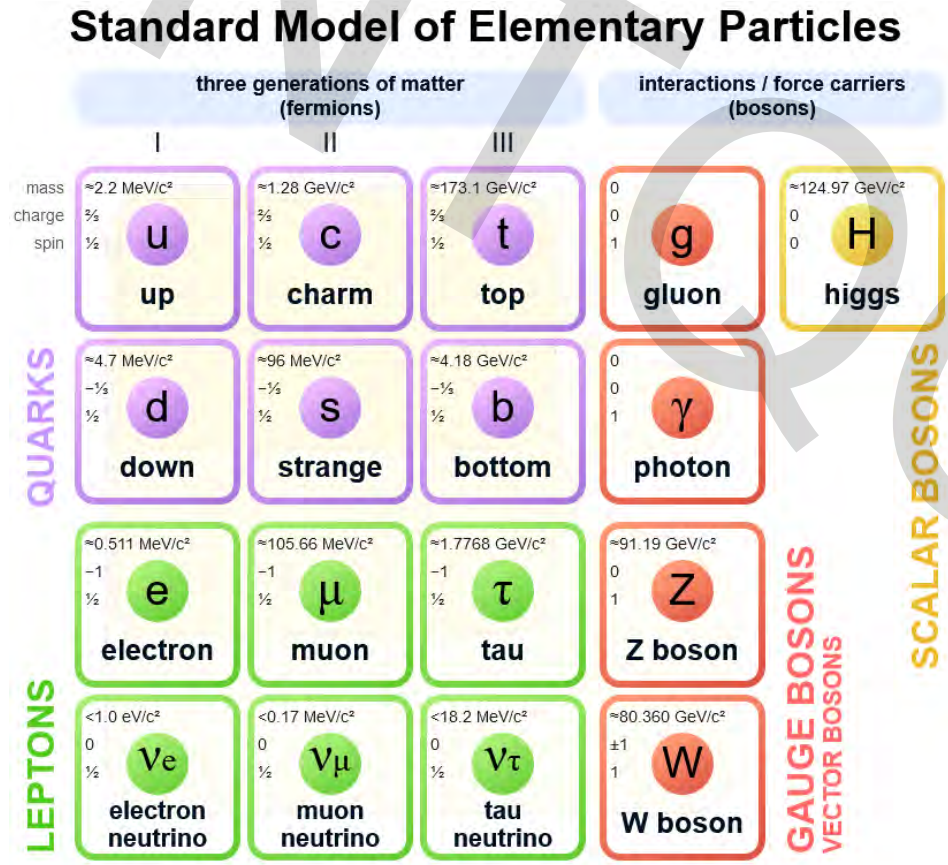
Many-body Interactions



Simulating Lattice Gauge Theories on a Quantum Computer



Gauge Theories



gauge theories describe interactions between particles and forces

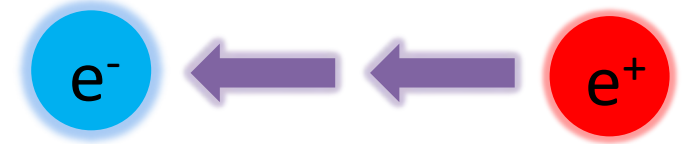
e.g. *Quantum Electrodynamics*

Hard to simulate classically due to sign problems
 ↓
 quantum simulation

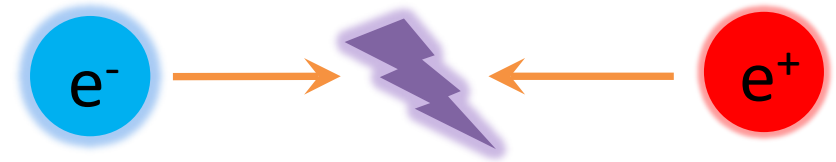
Image source: Uploaded by MissMJ, created by Fermilab, https://en.wikipedia.org/wiki/Standard_Model

Quantum electrodynamics

Charged particles (electrons, e^-) and antiparticles (positrons, e^+) interact via electromagnetic force fields.



Particles and antiparticles can mutually annihilate.

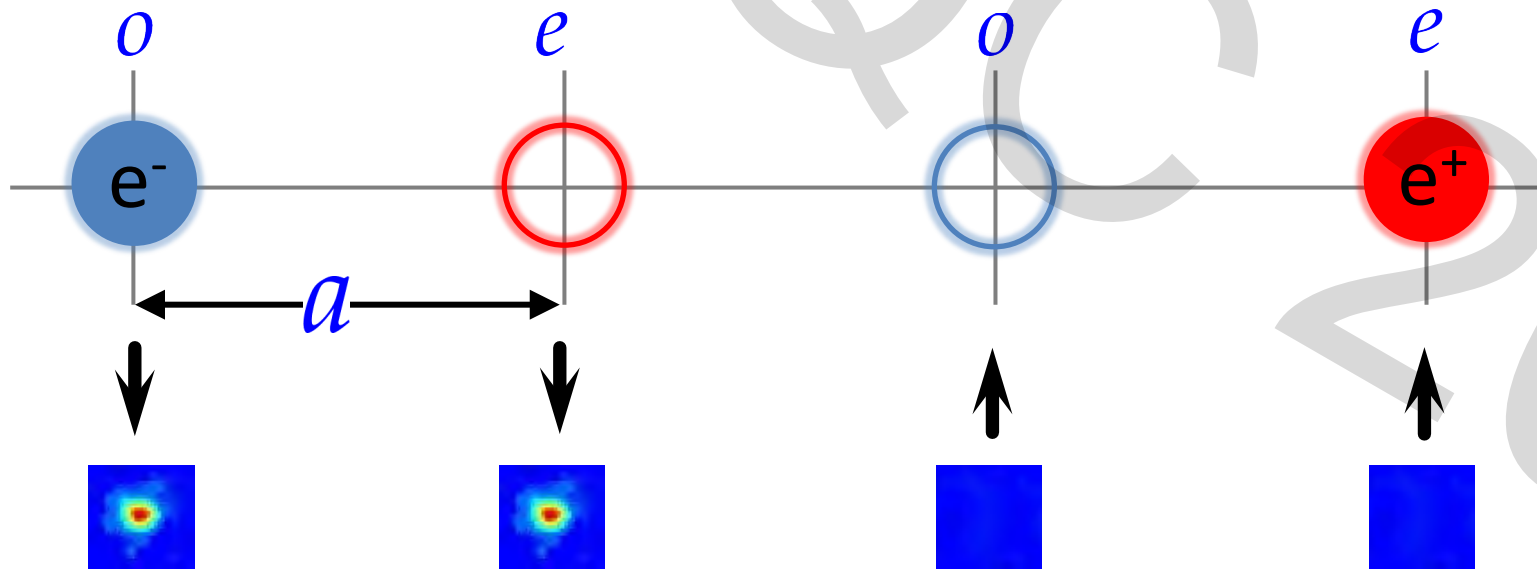


spontaneous creation of particle-antiparticle pairs in strong static fields (Schwinger mechanism).



Encoding Fermions into two-level systems

- Fermions (e^- , e^+) and holes are encoded in two-level systems (of ions)
- Odd(o)** sites: e^- , **even(e)** sites: e^+



Hilbert space

$$|0000\rangle = |\uparrow\downarrow\uparrow\downarrow\rangle$$

$$|e^-e^+00\rangle = |\downarrow\uparrow\uparrow\downarrow\rangle$$

$$|0e^+e^-0\rangle = |\uparrow\uparrow\downarrow\downarrow\rangle$$

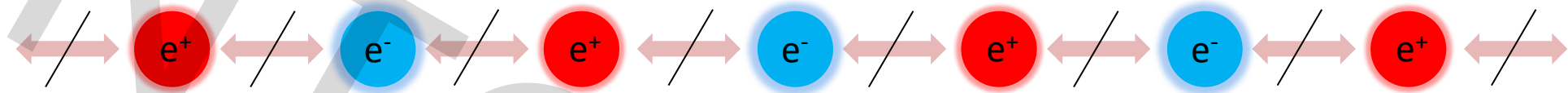
$$|00e^-e^+\rangle = |\uparrow\downarrow\downarrow\uparrow\rangle$$

$$|e^-00e^+\rangle = |\downarrow\downarrow\uparrow\uparrow\rangle$$

$$|e^-e^+e^-e^+\rangle = |\downarrow\uparrow\downarrow\uparrow\rangle$$

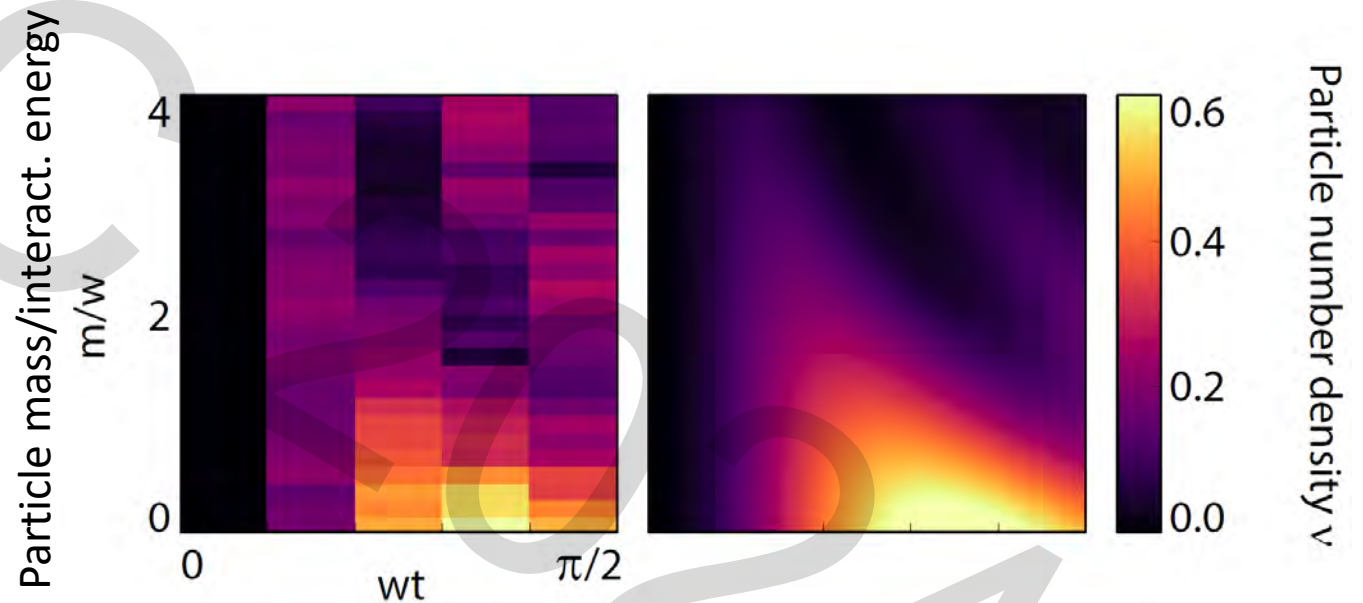
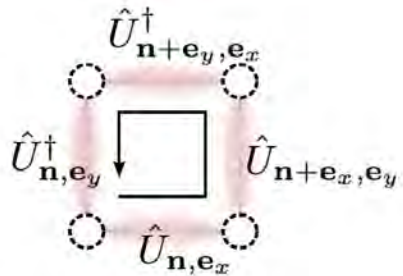
➔ Error detection

Quantum Simulating Lattice Gauge Theories



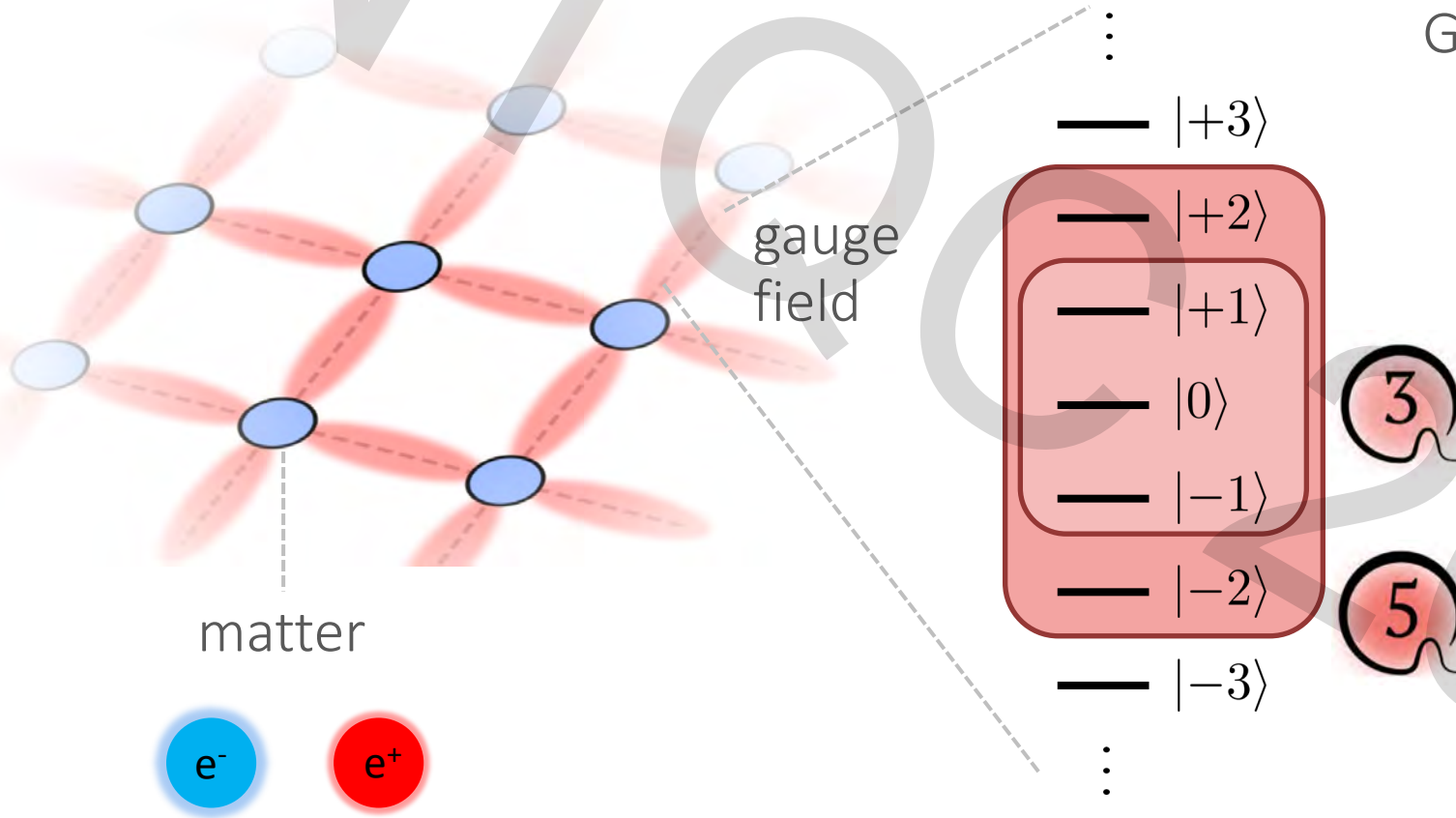
Example: 1D QED

- Gauge fields can be eliminated
- No gauge field dynamics
- No magnetic fields



Physics is different beyond 1D!

Quantum Simulating Gauge Theories



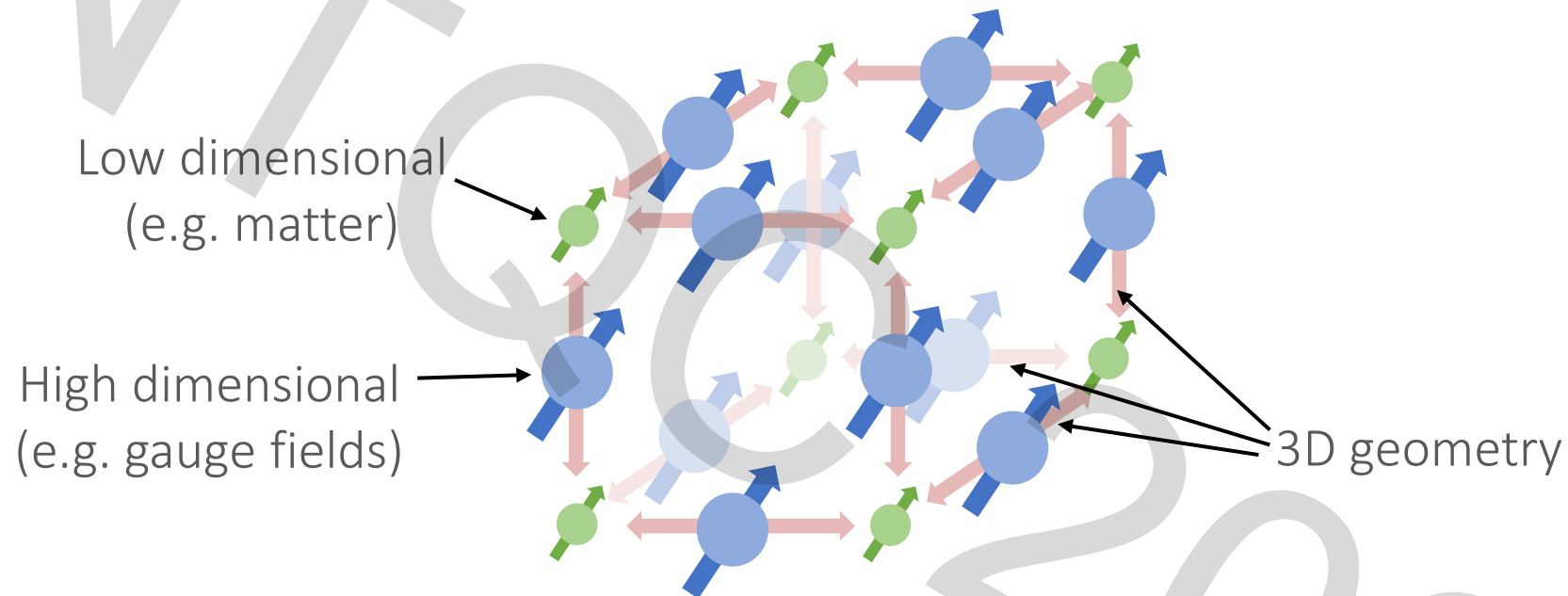
In classical and quantum simulation:
Gauge fields must be truncated

Minimal truncation: $d=3$

- field in pos direction
- zero
- field in neg direction

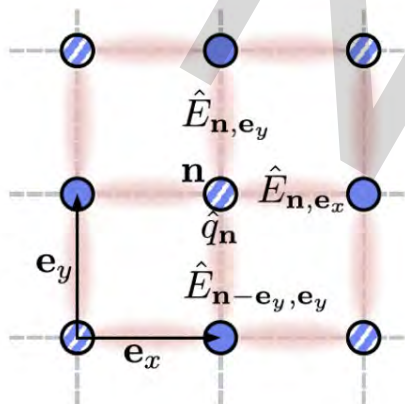
Better truncation: $d=5$

Quantum Simulation of LGTs beyond 1D



- ✓ Native support for mixed-dimensional systems w/o loss of fidelity
- ✓ Arbitrary geometries through all-to-all connectivity

Simulating 2D QED on a Qudit Quantum Computer



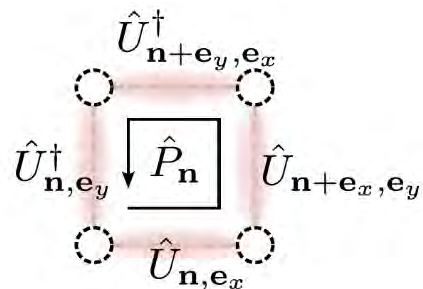
$$\hat{H} = g^2 \hat{H}_E + \frac{1}{g^2} \hat{H}_B + m \hat{H}_m + \Omega \hat{H}_k$$

Electric Field Energy

Particle Rest Mass

Kinetic Energy Term

Magnetic Field Energy

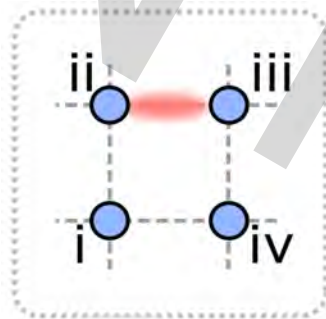


Gauss Law:

$$\hat{G}_{\mathbf{n}} = \sum_{\mu} \left(\hat{E}_{\mathbf{n}, \mathbf{e}_{\mu}} - \hat{E}_{\mathbf{n} - \mathbf{e}_{\mu}, \mathbf{e}_{\mu}} \right) - \hat{q}_{\mathbf{n}}$$

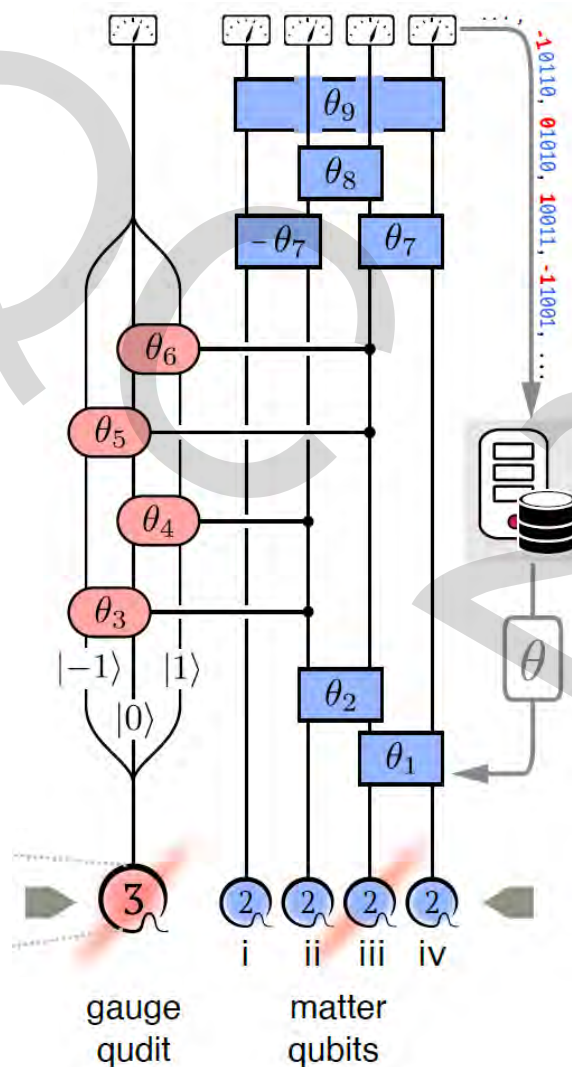
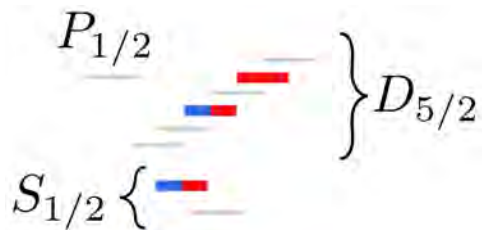
Variational Quantum Simulation of 2D QED

Elementary building block:
Plaquette



Quantum Computer:

- Parametrized circuit
- Energy measurement



$$\hat{H} = g^2 \hat{H}_E + \frac{1}{g} \hat{H}_B + m \hat{H}_m + \Omega \hat{H}_k$$

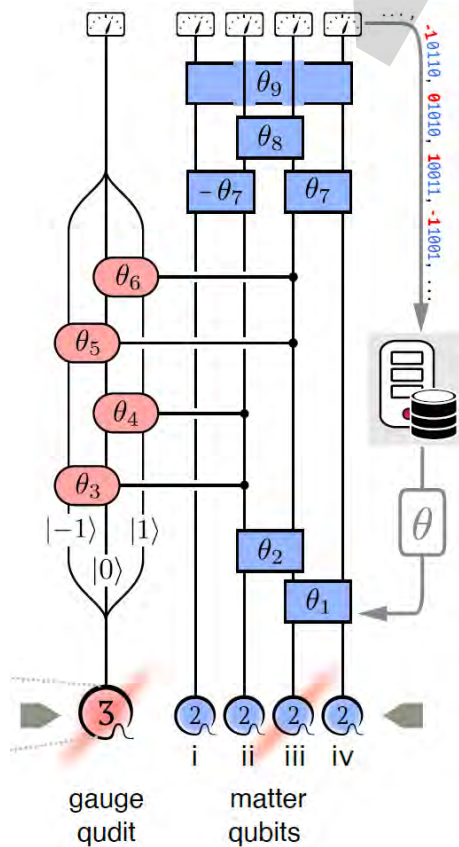
Goal: prepare groundstate of H by variationally minimizing the energy

Classical Computer:

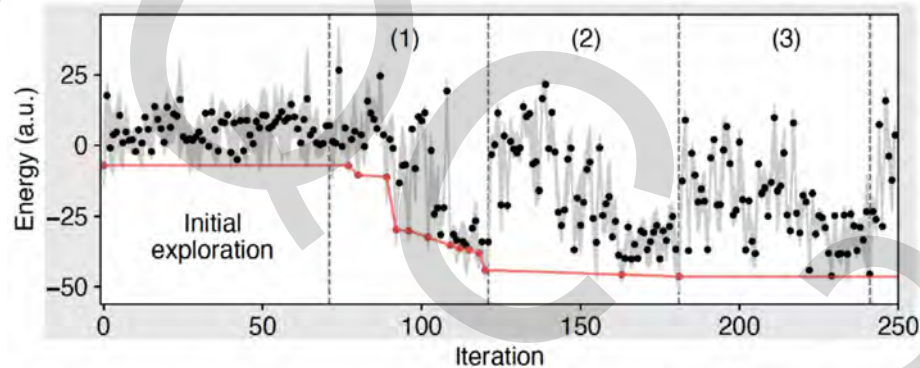
- Hamiltonian
- Parameter optimization

Simulating 2D QED on a qudit quantum computer

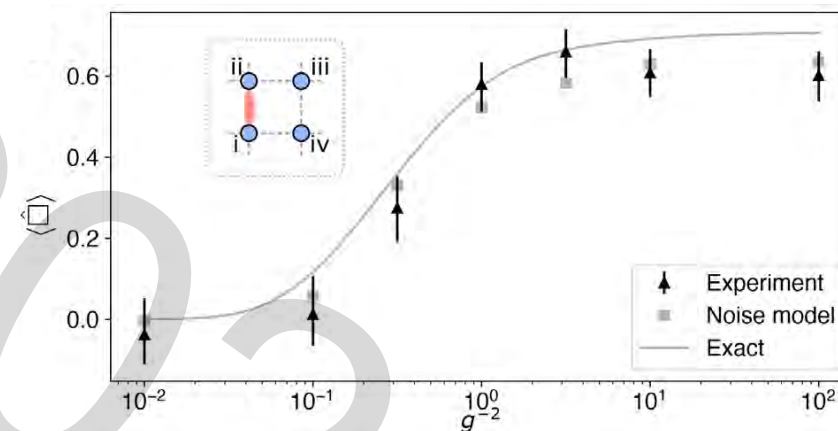
Mixed-dimensional VQE



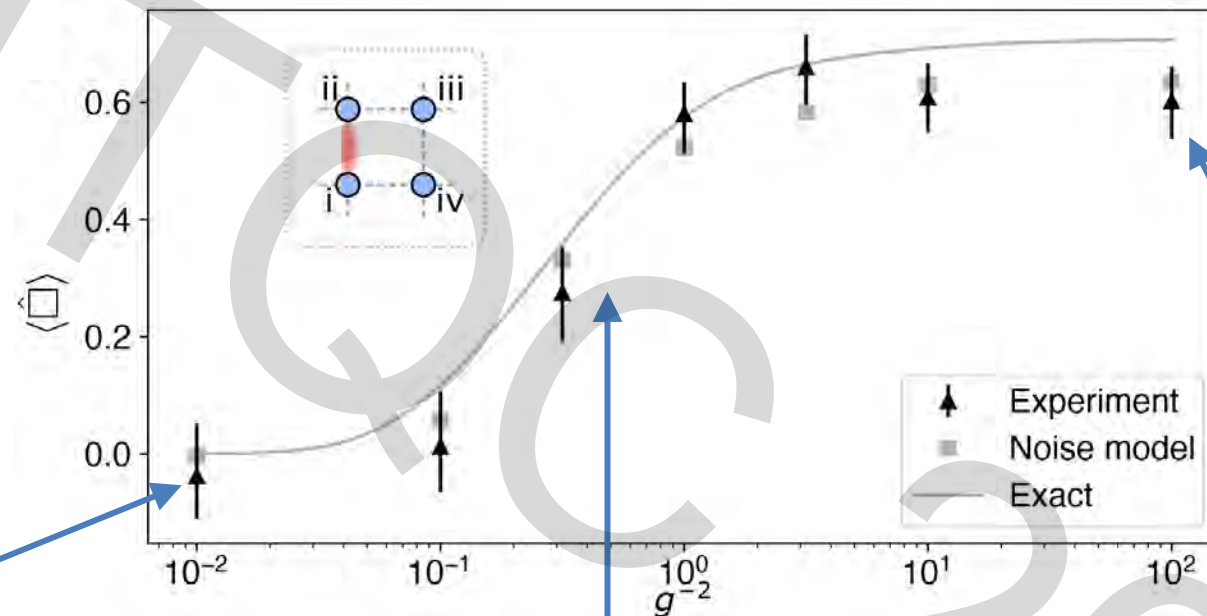
Ground-state preparation



Plaquette expectation value



Simulating 2D QED on a qudit quantum computer



Strong-coupling regime

- Pair generation suppressed
- No Magnetic fields

Intermediate regime

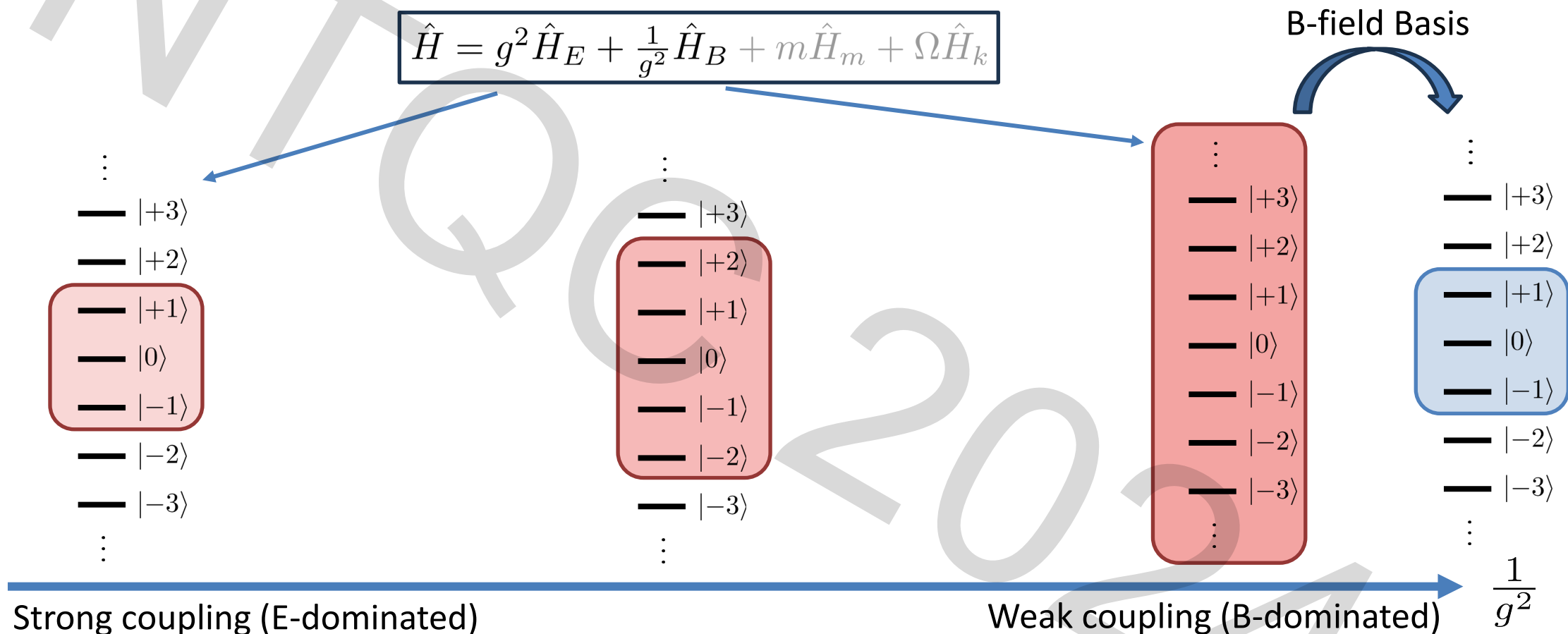
- Competition between pair generation and field energy terms
- Dynamical magnetic fields

Weak-coupling regime

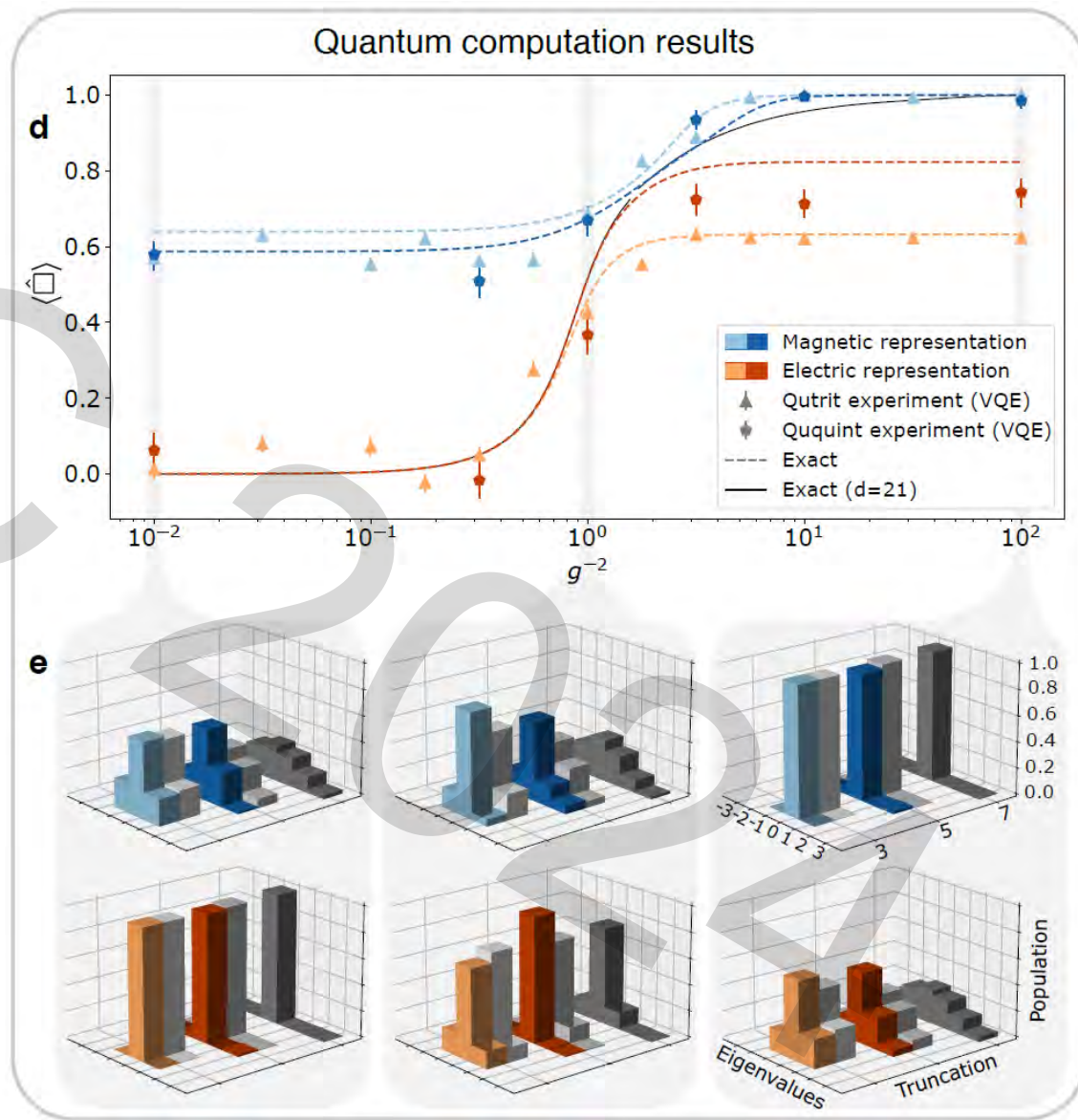
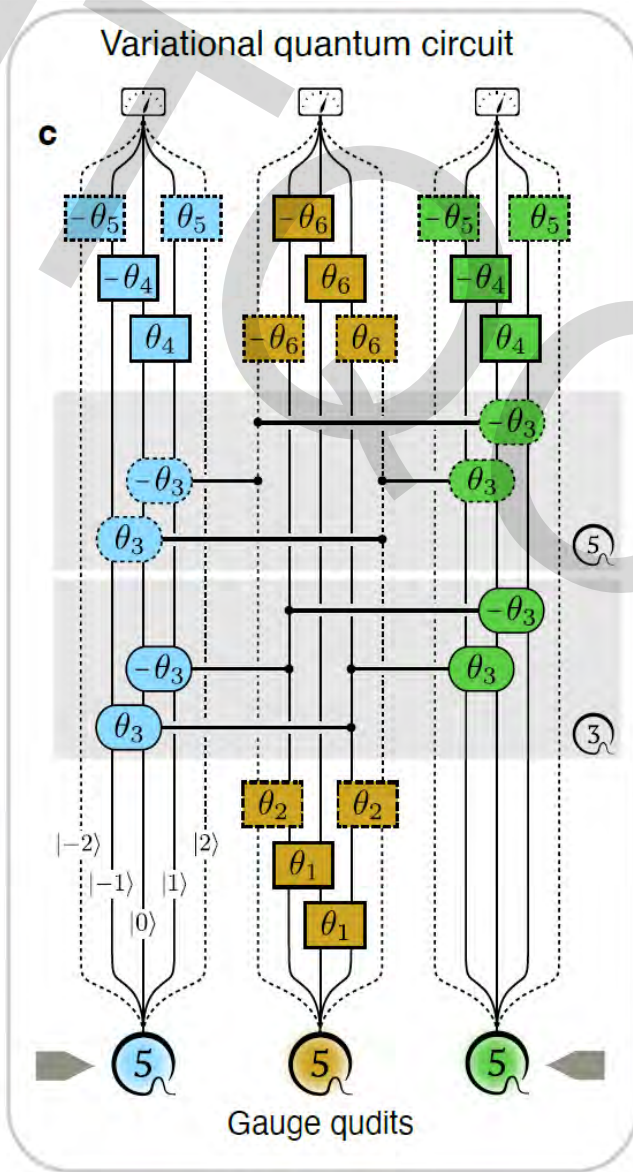
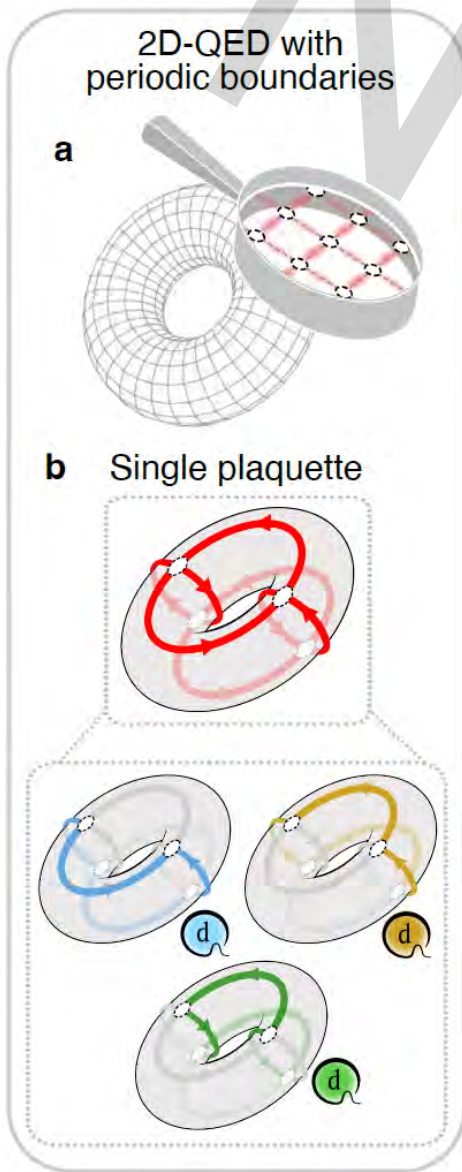
- Magnetic-field dominated
- No entanglement with matter

Truncation strategies for a pure gauge plaquette

$$\hat{H} = g^2 \hat{H}_E + \frac{1}{g^2} \hat{H}_B + m \hat{H}_m + \Omega \hat{H}_k$$



Pure gauge 2D QED



Hardware efficient encoding

	Qudit encoding			Qubit encoding		
dimension d	3	5	7	3	5	7
register size	5	5	5	7	9	11
CNOT count	26	34	42	90	162	234
CNOT fidelity	99%					
approx. circ. fid.	77%	71%	66%	40%	20%	10%
CNOT fidelity	99.5%					
approx. circ. fid.	88%	84%	81%	64%	44%	31%

← Gauge+Matter

Pure gauge →

	Qudit encoding			Qubit encoding		
dimension d	3	5	7	3	5	7
register size	3	3	3	9	15	21
CNOT count	8	8	8	84	108	132
CNOT fidelity	99%					
approx. circ. fid.	92%	92%	92%	43%	34%	27%
CNOT fidelity	99.5%					
approx. circ. fid.	96%	96%	96%	66%	58%	52%

2. Quantum Computation and Simulation

2.1 New Generation

2.2 Operations beyond Qubits

2.3 Quantum Error Correction

2.4 QIP with Qudits

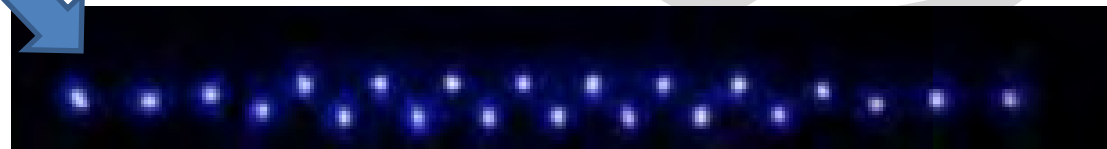
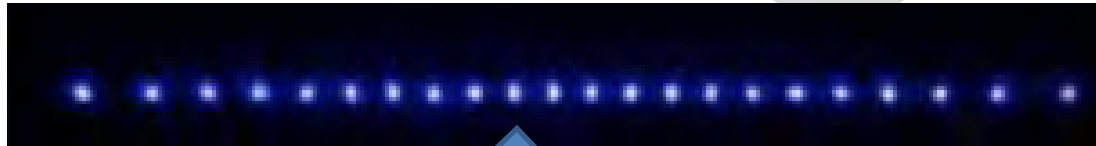
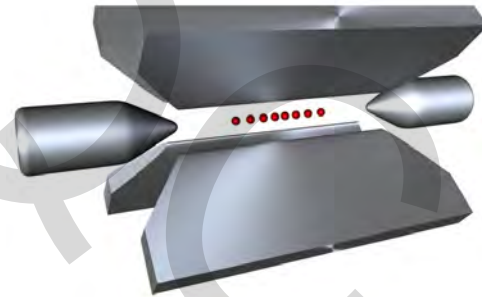
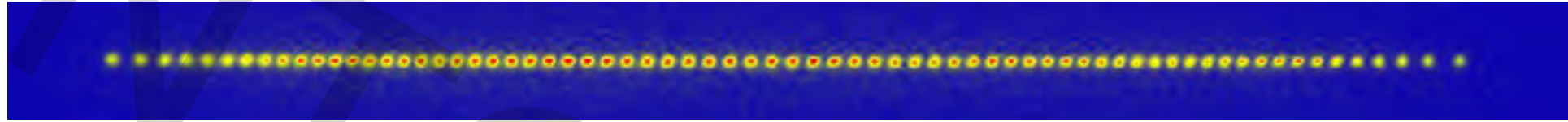
2.5 Digital Quantum Simulation



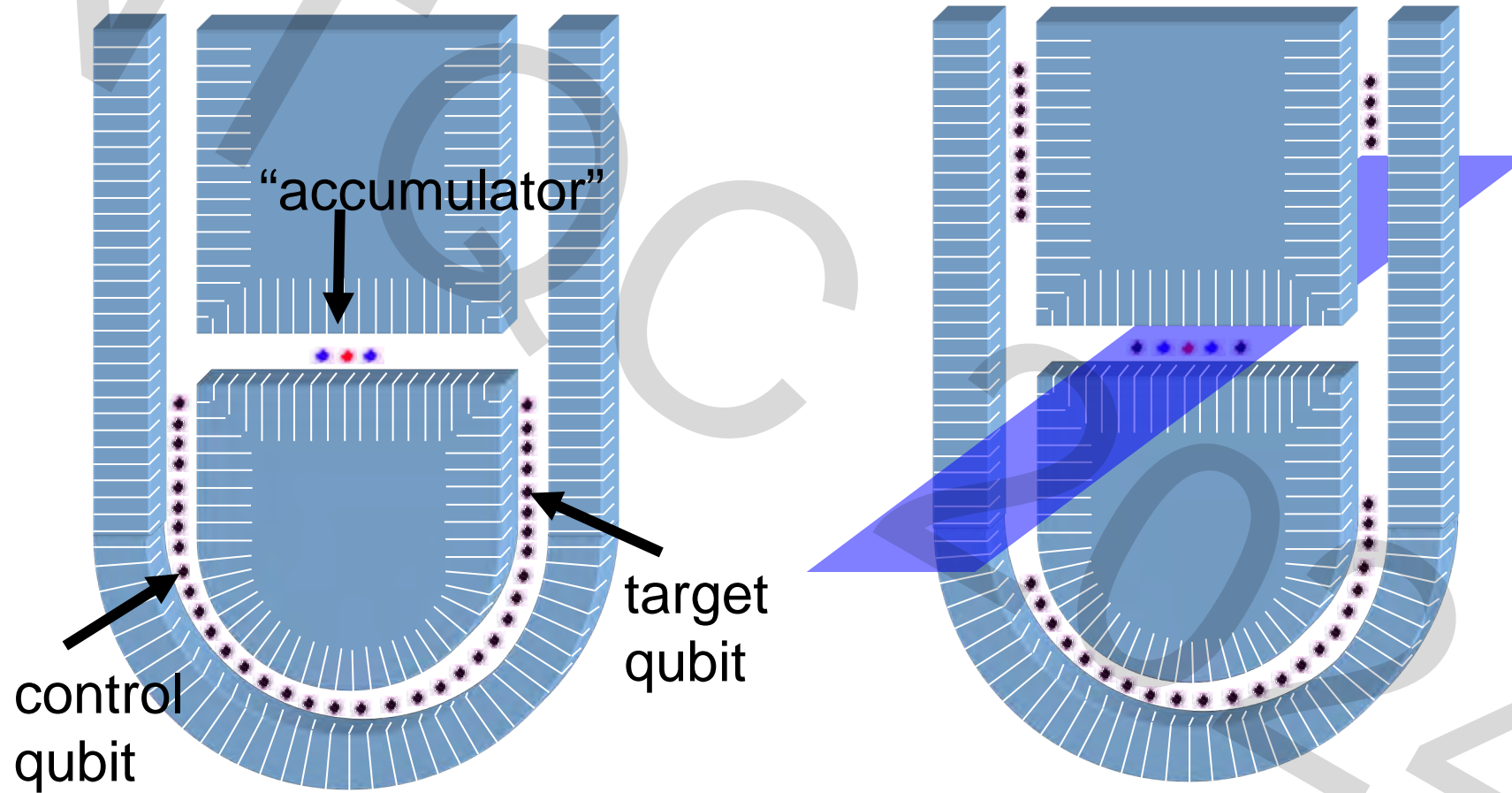
2.6 Scaling



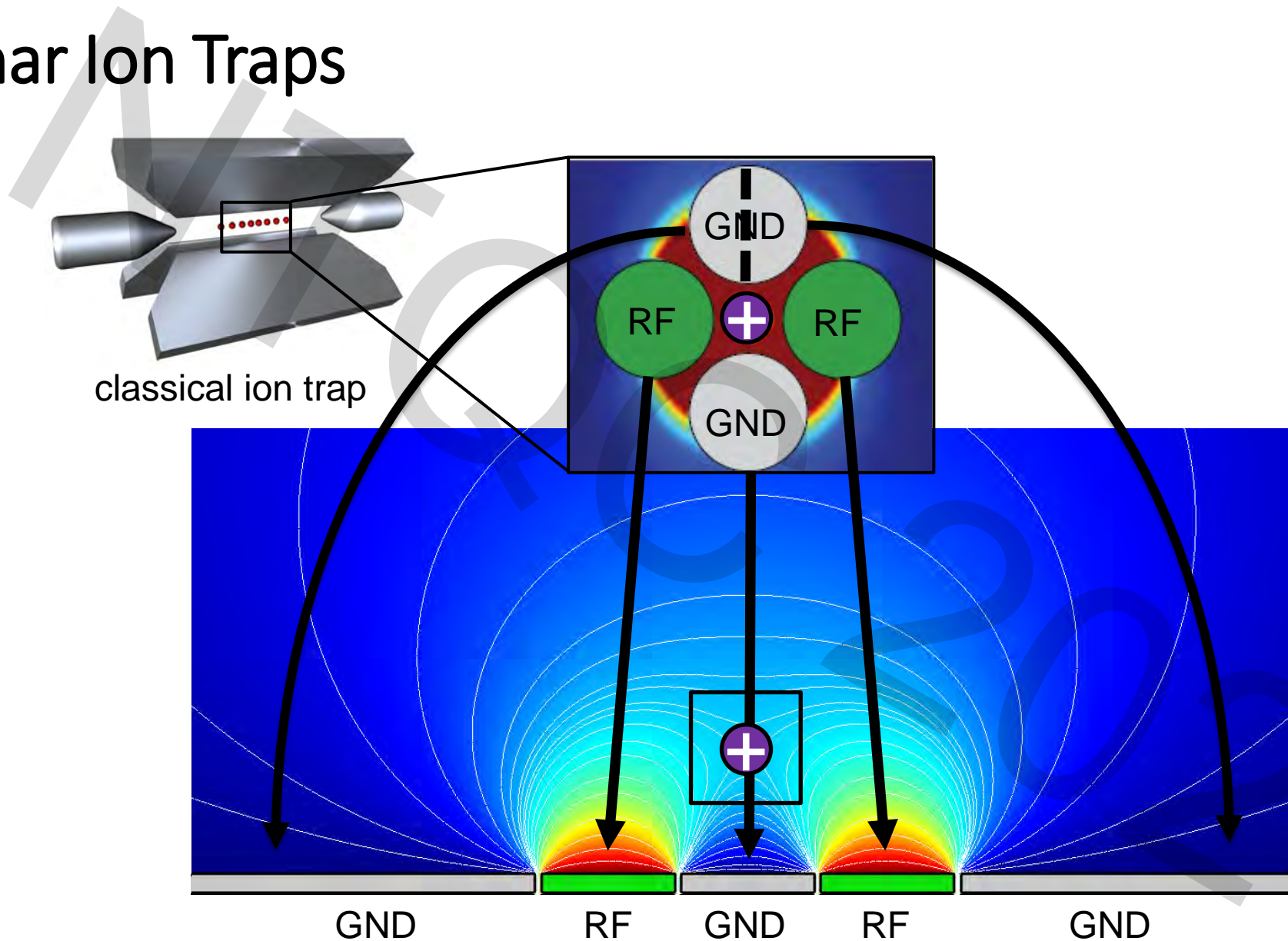
Are ion traps scalable?



Ion shift register (CCD)

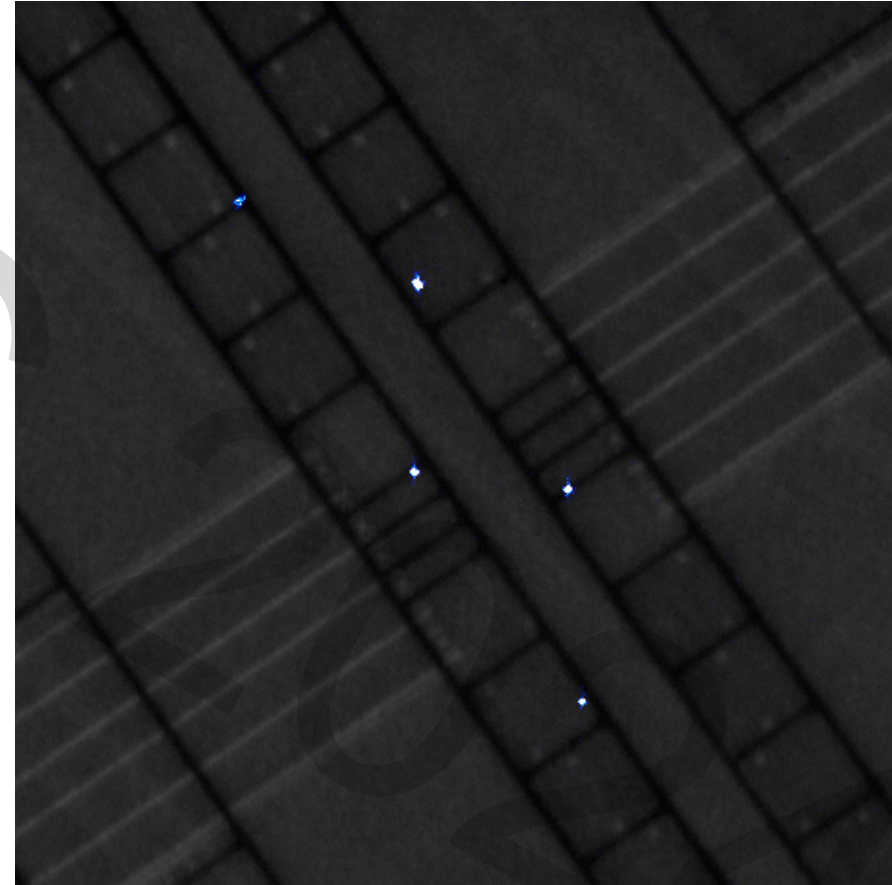
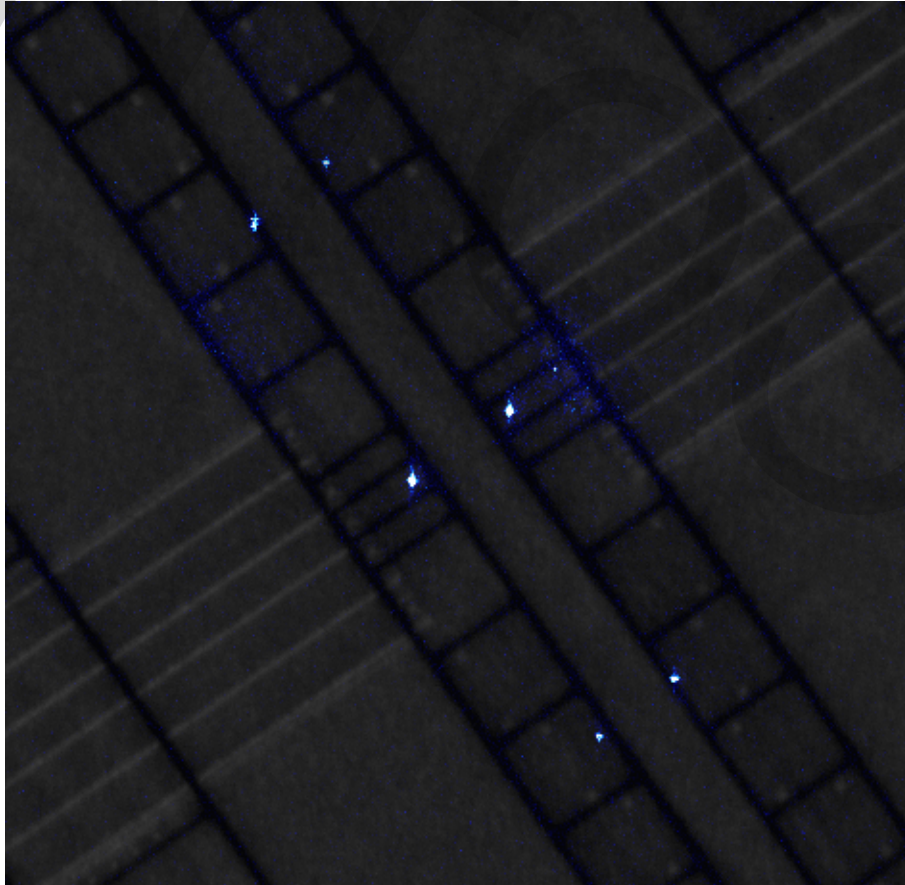


Planar Ion Traps



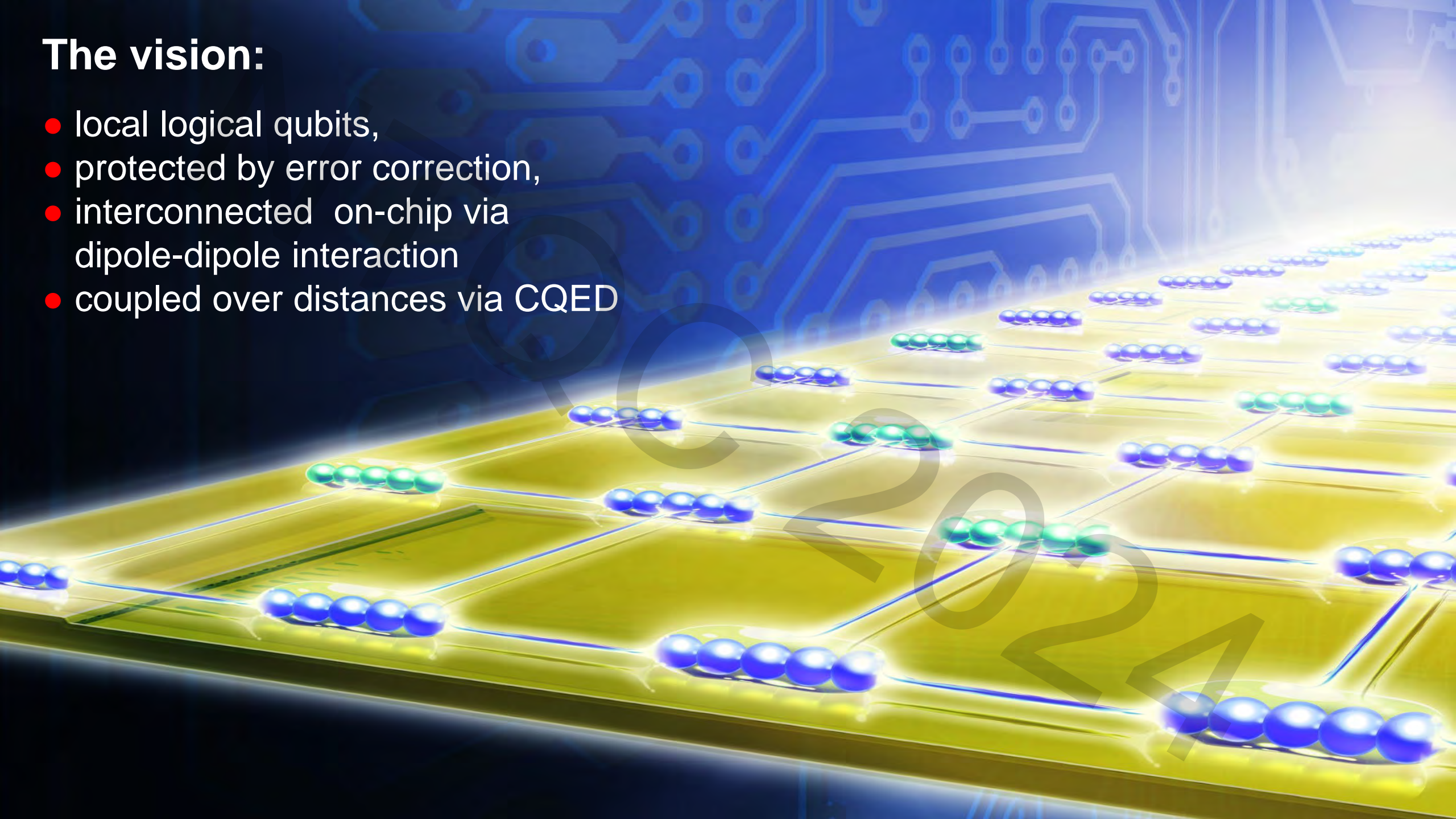
2D ion trap arrays

PEDMONS

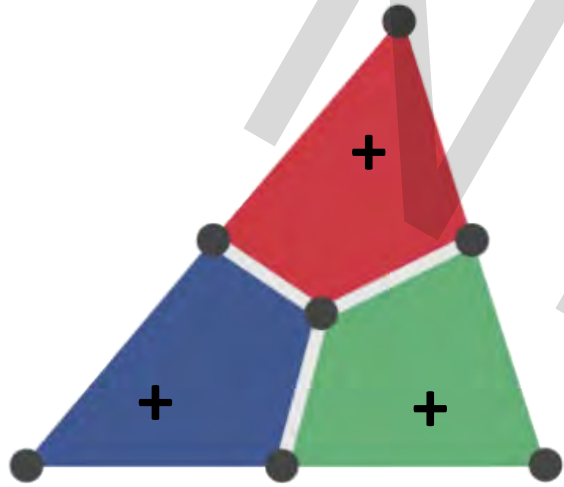


The vision:

- local logical qubits,
- protected by error correction,
- interconnected on-chip via dipole-dipole interaction
- coupled over distances via CQED



Take-home message



The trapped-ion toolbox goes much beyond qubit gates



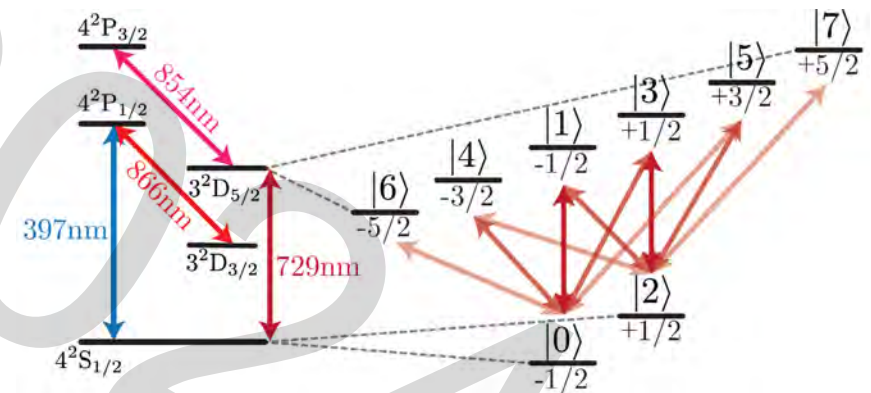
Quantum Error Correction suppresses errors through redundancy



Universal mixed-dimensional quantum computing



Natural platform for gauge theory simulations



The Innsbruck Ion Trappers 2023



PhDs and PostDocs wanted !



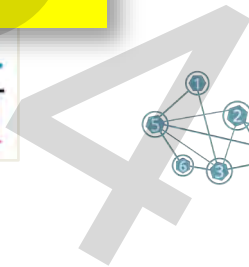
FWF
SFB



IQI
GmbH

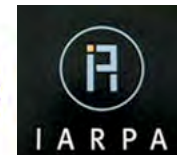


eQual



QUDITS

FWF



NeOST

