Adiabatic quantum computing
and associated models of computation

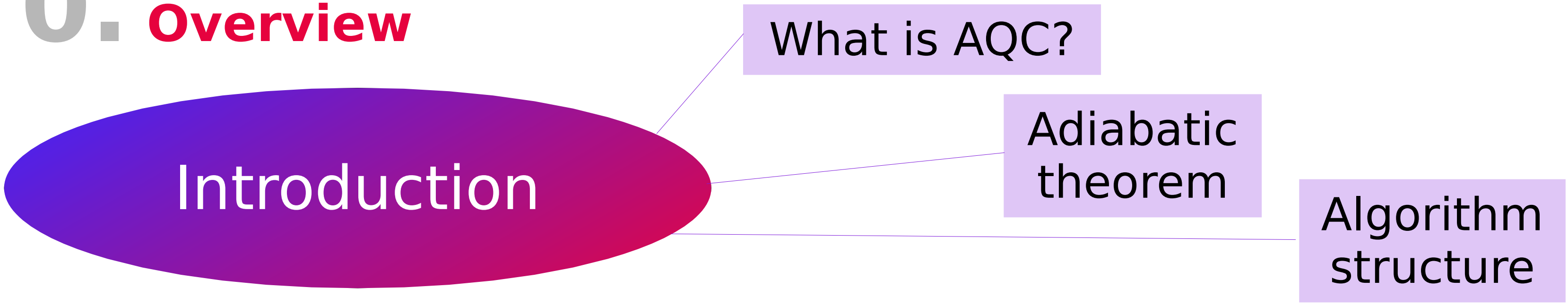
25/04/2024 – Ana Palacios

ana.palacios@qilimanjaro.tech

QILIMANJARO
QUANTUM·TECH

Overview

0. Overview


Introduction
What is AQC?
Adiabatic theorem
Algorithm structure

0. Overview

Introduction

- What is AQC?
- Adiabatic theorem
- Algorithm structure

Applications: general results

- Search for quantum advantage
- Universality of AQC

Section

0. Overview

Introduction
- What is AQC?
- Adiabatic theorem
- Algorithm structure

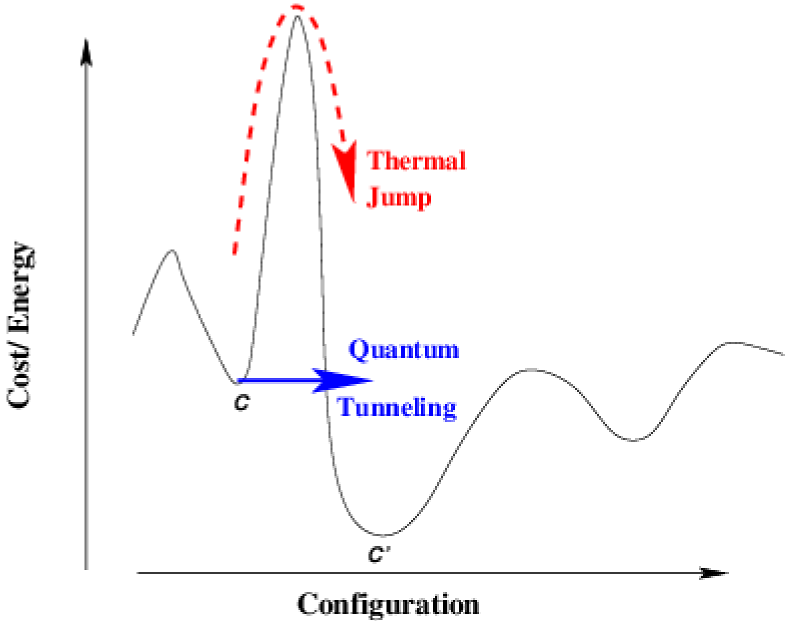
Applications: general results
- Search for quantum advantage
- Universality of AQC

Annealing platforms
- JPAs
- Superconducting flux qubits

Introduction

1. Introduction

Basic idea



- A. Das and B. K. Chakrabarti. *Colloquium : Quantum annealing and analog quantum computation.* Reviews of Modern Physics 80 (2008): 1061-1081.

Before we start throwing them left and right:

Quantum Annealing (QA): the algorithm

Adiabatic Quantum Computing (AQC): the computational paradigm

1. Introduction

<u>Seminal works of annealing and Adiabatic Quantum Computation (AQC)</u>

- T. Kadowaki and H. Nishimori, *Quantum annealing in the transverse Ising model*, Phys. Rev. E 58, 5355 (1998)
- Edward Farhi, et al., *Quantum computation by adiabatic evolution*, arXiv:quant-ph/0001106 (2000)

adimensional time scale, usually ~t/T

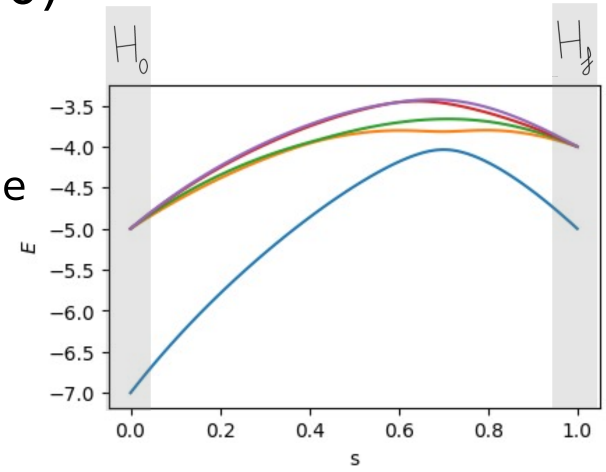
Hamiltonian space

GS easy to prepare in the lab

GS encodes solution to the problem

$$H(s) = (1-s)H_0 + sH_f$$

$s \in [0,1]$



1. Introduction

Seminal works of annealing and Adiabatic Quantum Computation (AQC)

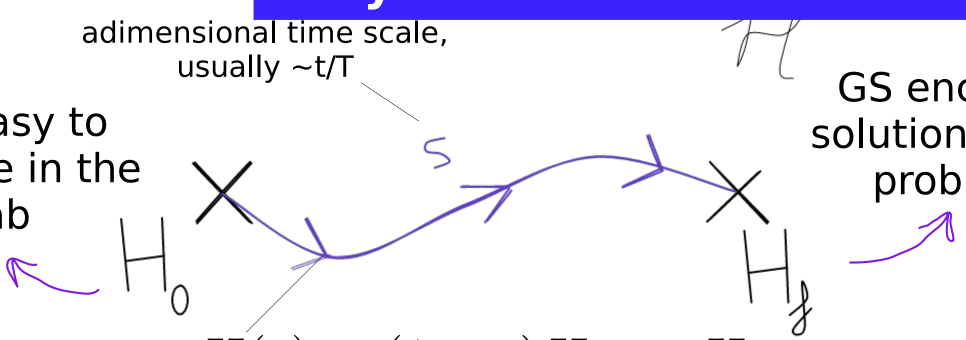
- T. Kadowaki and H. Nishimori, *Quantum annealing in the transverse Ising model*, Phys. Rev. E 58, 5355 (1998)
- Edward F… *…y adiabatic evolution…*

Adiabatic theorem

If the process is slow enough, the system will remain in the GS

adimensional time scale, usually ~t/T

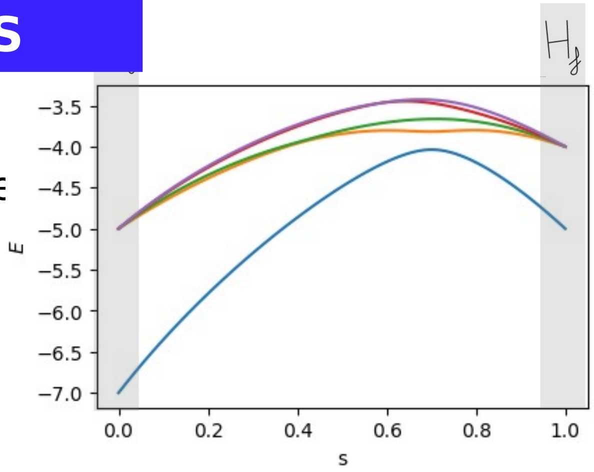
GS easy to prepare in the lab



GS encodes solution to the problem

$$H(s) = (1-s)H_0 + sH_f$$

$s \in [0, 1]$



1.1. Adiabatic theorem

- M. Born and V. Fock. *Beweis des adiabatensatzes*. Z. Physik, 51:165–180 (1928)
- T. Kato. *On the adiabatic theorem of quantum mechanics*. J. Phys. Soc. Jpn. 5 (6), 435 (1950)

Δ : gap between GS and 1st excited state

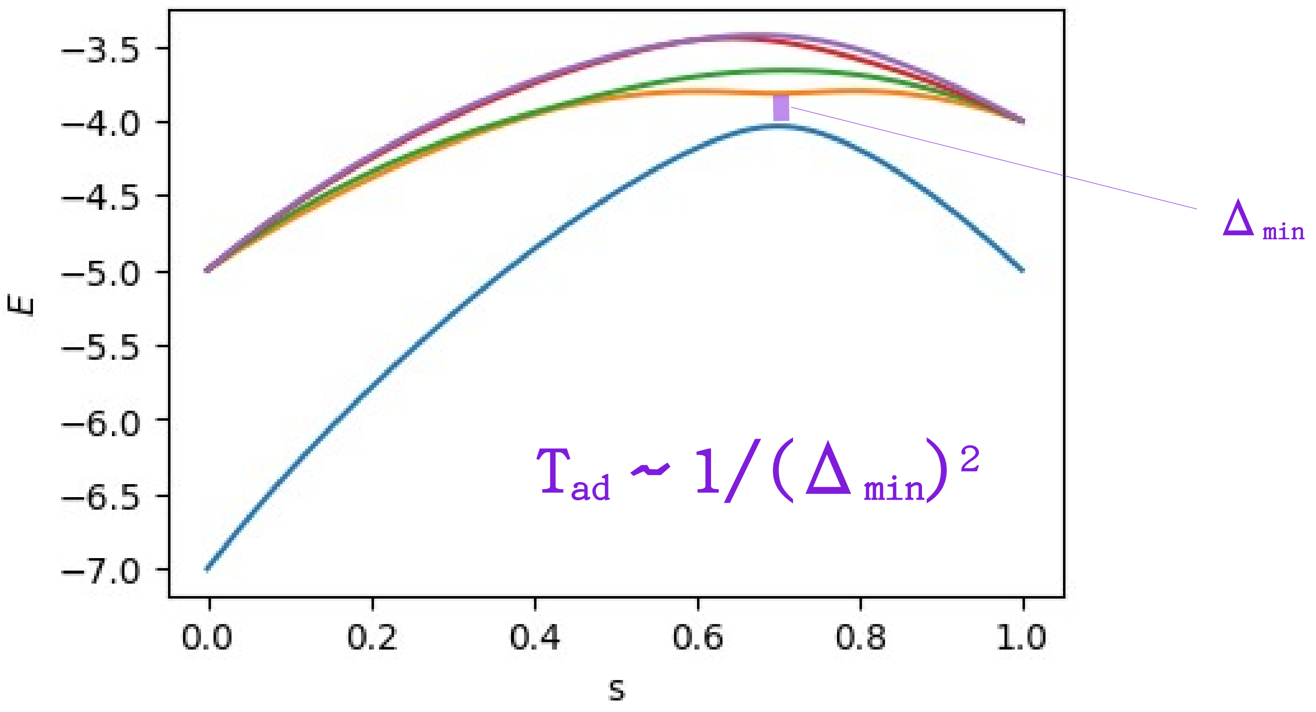
$$T_{ad} = \frac{\max_s ||\partial_s H||}{[\min_s \Delta]^2} \cdot \max_s ||H||$$

* Exact shape of the bound is a debated topic, but the inverse dependence on the gap is a common factor

See nice review of this in:

- T. Albash and D. A. Lidar, *Adiabatic Quantum Computation*, Rev. Mod. Phys. 90, 015002 (2018)

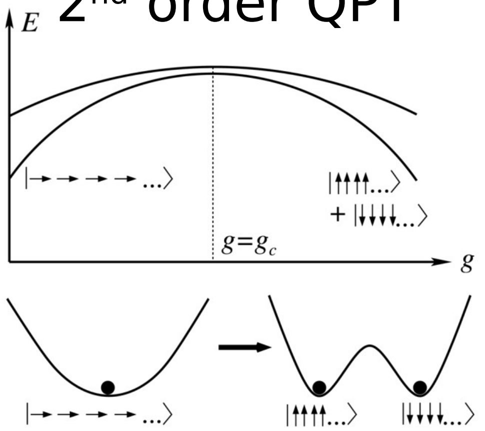
1.1. Adiabatic theorem



1.1. Adiabatic theorem: scenarios

$$\Delta \sim \mathcal{O}(N^{-\alpha})$$

2nd order QPT*



$$T_{ad} \sim \mathcal{O}(N^{\tilde{\alpha}})$$

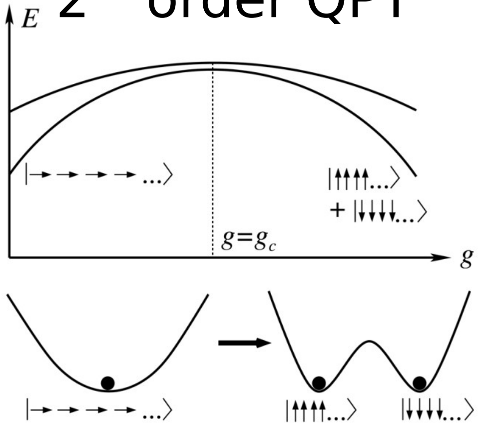
Good news

* in the thermodynamic limit, but we abuse language for finite size

1.1. Adiabatic theorem: scenarios

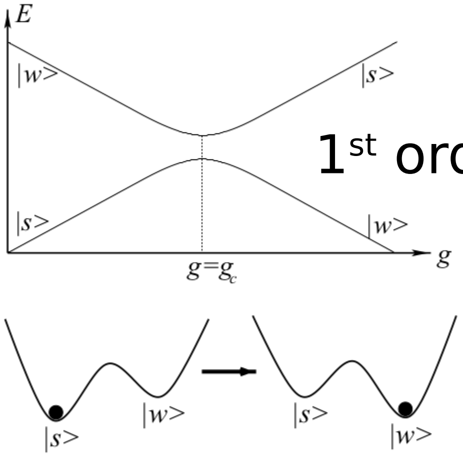
$$\Delta \sim \mathcal{O}(N^{-\alpha})$$

2nd order QPT



$$T_{ad} \sim \mathcal{O}(N^{\tilde{\alpha}})$$

Good news



1st order QPT

Bad news

$$T_{ad} \sim \mathcal{O}(e^{N})$$

$$\Delta \sim \mathcal{O}(e^{-N})$$

1.2. General scheme

$$H(s) = A(s)H_0 + B(s)H_f$$

Initial/driver Hamiltonian

- Must contain some overlap with solution
- Easy to prepare
- Standard choice:

$$H_0 = -\sum_i \sigma_i^x$$

1.2. General scheme

$$H(s) = A(s) H_0 + B(s) H_f$$

Initial/driver Hamiltonian

- Must contain some overlap with solution
- Easy to prepare
- Standard choice:

$$H_0 = -\sum_i \sigma_i^x$$

Schedule

- A(0) = 1, B(0) = 0
- A(1) = 0, B(1) = 1
- Should have smaller derivative around smaller gap (for adiabaticity)

General scheme

$$H(s) = A(s) H_0 + B(s) H_f$$

Initial/driver Hamiltonian

- Must contain some overlap with solution
- Easy to prepare
- Standard choice:

$$H_0 = -\sum_i \sigma_i^x$$

Problem Hamiltonian

- Encoding of the solution
- Choice of a Hamiltonian with such ground state

Schedule

- A(0) = 1, B(0) = 0
- A(1) = 0, B(1) = 1
- Should have smaller derivative around smaller gap (for adiabaticity)

Example of problem encoding in H_f

- **QUBO** (Quadratic Unconstrained Binary Optimisation)

$$f(x) = x^T Q x + P x = \sum_{i=1}^{N} \sum_{j=1}^{N} q_{ij} x_i x_j + \sum_{i=1}^{N} p_i x_i$$

$$x_i = \{0, 1\} \; \forall i$$

Solution: minimising {x}

$$2x_i - 1 \rightarrow \sigma_i^z$$

$$H[\sigma] = \sum_{i=1}^{N} \sum_{j=1}^{N} J_{ij} \sigma_i^z \sigma_j^z + \sum_{i=1}^{N} h_i \sigma_i^z$$

1.2.2. Problem encoding: constraint satisfaction

Example of problem encoding in H_f

- **QUBO** (Quadratic Unconstrained Binary Optimisation)

$f(x)$ → constraint satisfaction problem (CSP)

$$
\begin{aligned}
a)\quad & x_1 + x_2 + x_3 = 1 \\
b)\quad & x_4 - x_2 - x_3 = 0 \\
c)\quad & x_3 + x_4 - x_1 = 0
\end{aligned}
\longrightarrow
\begin{aligned}
f_a(x) &= x_1 + x_2 + x_3 - 1 \\
f_b(x) &= x_4 - x_2 - x_3 \\
f_c(x) &= x_3 + x_4 - x_1
\end{aligned}
\longrightarrow
\begin{aligned}
h_a(x) &= (x_1 + x_2 + x_3 - 1)^2 \\
h_b(x) &= (x_4 - x_2 - x_3)^2 \\
h_c(x) &= (x_3 + x_4 - x_1)^2
\end{aligned}
$$

$2x_i - 1 \to \sigma_i^z$

$$H = h_a + h_b + h_c$$

$$H = \frac{1}{4}[2\sigma_1^z\sigma_3^z - 2\sigma_1^z\sigma_4^z + 4\sigma_2^z\sigma_3^z - 2\sigma_3^z\sigma_4^z + 4\sigma_1^z + 11\sigma_2^z + 8\sigma_3^z + 2\sigma_4^z + 6]$$

1.2.2. Problem encoding: Nurse Scheduling Problem

A more down-to-earth application:

Encoding: $\# \text{ qubits} \rightarrow N_n \cdot N_s \Rightarrow \mathrm{x}_{i,t}$

(# nurses: N_n; # shifts: N_s)

Conditions:

- Having X nurses per shift $$h_X = \sum_{t=1}^{N_s} \left(\sum_{i=1}^{N_n} x_{i,t} - X \right)^2$$
- A nurse can't have 2 or more consecutive shifts $$h_{cs} = \sum_{i=1}^{N_n} \sum_{t} x_{i,t} x_{i,t+1}$$
- Every nurse should have roughly the same number of shifts, which we call Y $$h_Y = \sum_{i=1}^{N_n} \left(\sum_{t=1}^{N_s} x_{i,t} - Y \right)^2$$

1.2.2. Problem encoding: Nurse Scheduling Problem

$$h_X = \sum_{t=1}^{N_s} \left(\sum_{i=1}^{N_n} x_{i,t} - X \right)^2 \qquad h_{cs} = \sum_{i=1}^{N_n} \sum_{t} x_{i,t} x_{i,t+1} \qquad h_Y = \sum_{i=1}^{N_n} \left(\sum_{t=1}^{N_s} x_{i,t} - Y \right)^2$$

$$H = \alpha h_X + h_{cs} + \beta h_Y$$

α, β set the relative strength of each of the constraints

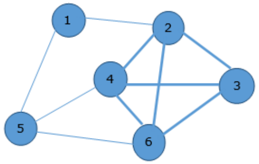
In a similar manner, more information can be added:

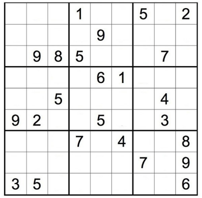
- Nurses' preferences
- Dependence on time of the number of nurses required (X)
- Dependence on the level of effort that can be provided by each nurse (experience, etc)

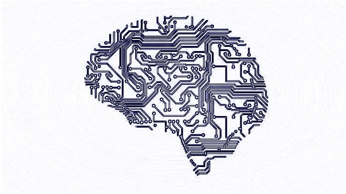
1.2.2. Problem encoding

Common classical optimisation problems that can be formulated in QUBO form

- Network problems (Max-Cut, TSP, ...)
- Scheduling
- Portfolio optimisation (knapsack)
- Satisfiability
- Machine Learning
- ...



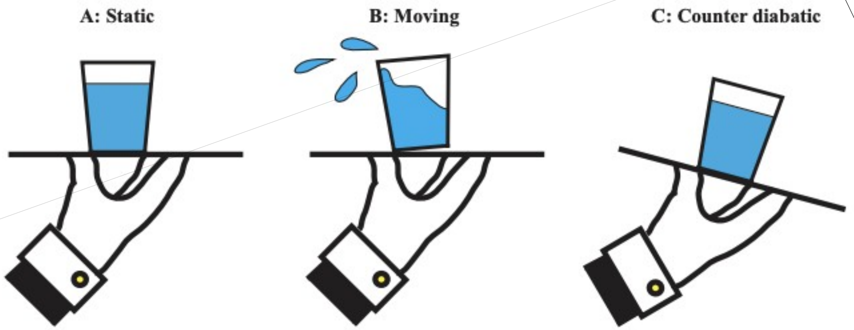




1.2. General scheme

$$H(s) = A(s)H_0 + B(s)H_f + C(s)H_c$$

- D. Sels and A. Polkovnikov, *Variational principle for CD driving*. PNAS 114 (20) E3909-E3916 (2017)



Catalyst Hamiltonian

- Modify Hamiltonian path to enhance performance
- Counter-diabatic terms

Catalyst schedule

- C(0) = 0, C(1) = 0

1.2.1. General scheme: note #1

AQC can be dumb if you're not smart!

Example: **p-spin model**

$$H_f = -N\left(\frac{1}{N}\sum_i \sigma_i^z\right)^p$$

$$H(t) = \Gamma(t)\left(-\sum_i \sigma_i^x\right) - N\left(\frac{1}{N}\sum_i \sigma_i^z\right)^p$$

$$\Gamma(t) \in (\infty, 0]$$

- T. Jörg et al., *Energy gaps in quantum first-order mean-field–like transitions: The problems that quantum annealing cannot solve*, EPL 89, 40004 (2010)

1.2.1. General scheme: note #1

AQC can be dumb if you're not smart!

Example: **p-spin model**

$$H_f = -N\left(\frac{1}{N}\sum_i \sigma_i^z\right)^p$$

$$H(t) = \Gamma(t)\left(-\sum_i \sigma_i^x\right) - N\left(\frac{1}{N}\sum_i \sigma_i^z\right)^p$$

$$\Gamma(t) \in (\infty, 0]$$

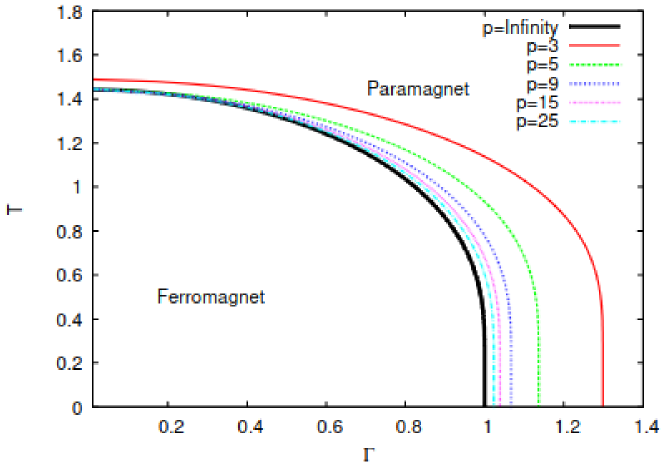


Fig. 1: Phase diagram of the ferromagnetic p-spin ferromagnet for different values of p. A first-order transition separates the ferromagnetic and quantum paramagnetic phases.

- T. Jörg et al., *Energy gaps in quantum first-order mean-field–like transitions: The problems that quantum annealing cannot solve*, EPL 89, 40004 (2010)

1.2.1. General scheme: note #1

AQC can be dumb if you're not smart!

Exam... -spin model

Bad choice of path in Hamiltonian space

$$H_f = -N \left(\ldots \right)^p$$

$$H(t) = \Gamma(t)\left(-\sum_i \sigma_i^x\right) - N\left(\frac{1}{N}\sum_i \sigma_i^z\right)^p$$

$$\Gamma(t) \in (\infty, 0]$$

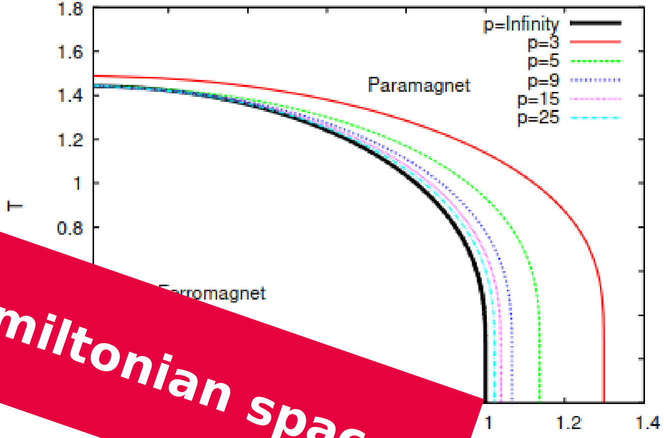


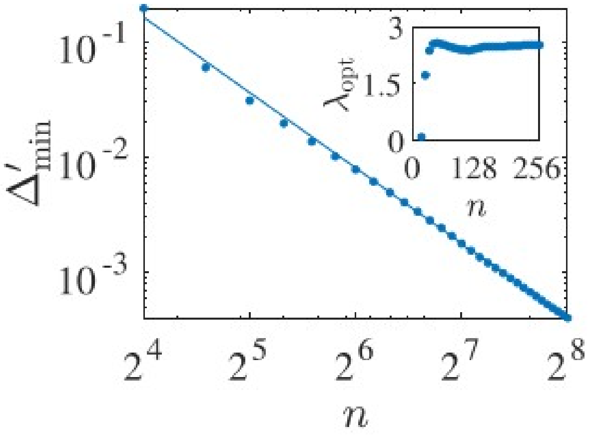
Fig. 1: Phase diagram of the ... tic p-spin ferromagnet for different values of p. A first-ord... transition separates the ferromagnetic and quantum paramagnetic phases.

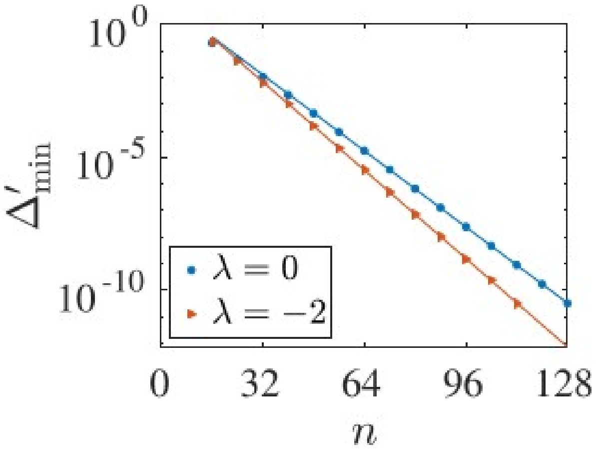
- T. Jörg et al., *Energy gaps in quantum first-order mean-field–like transitions: The problems that quantum annealing cannot solve*, EPL 89, 40004 (2010)

1.2.1. General scheme: note #1

Gap can be opened through introduction of catalysts

$$H(t) = (1-s)\left(-\sum_i \sigma_i^x\right) - sN\left(\frac{1}{N}\sum_i \sigma_i^z\right)^p + \lambda\frac{s(1-s)}{N}\left(\sum_i \sigma_i^x\right)^2$$





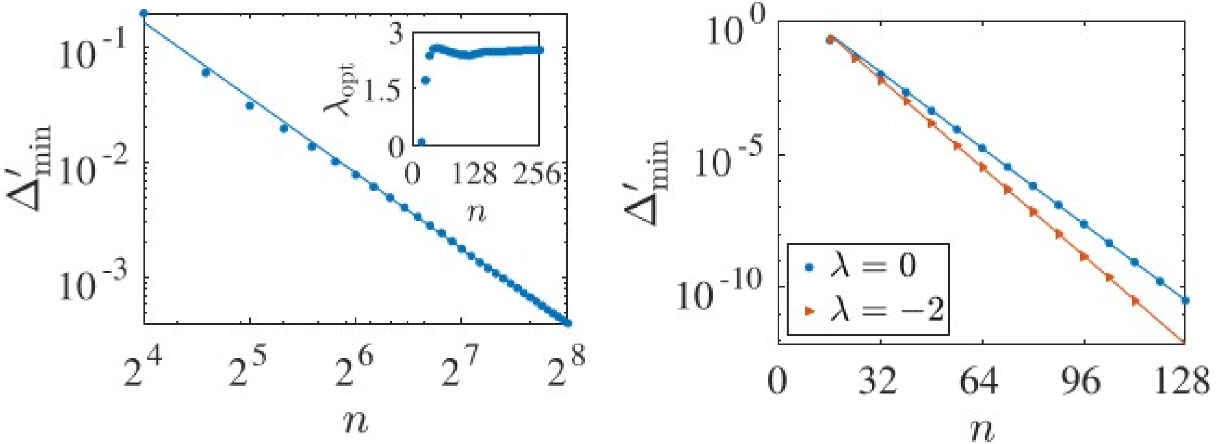
- Fig. 1 from T. Albash, *Role of nonstoquastic catalysts in quantum adiabatic optimization*. Phys. Rev. A 99, 042334 (2019)

1.2.1.

Better choice of path in Hamiltonian space

Gap can

$$H(t) = (1-s)\left(-\sum_i \sigma_i^x\right) - sN\left(\frac{1}{N}\sum_i \sigma_i^z\right)^p + \lambda \frac{s(1-s)}{N}\left(\sum_i \sigma_i^x\right)^2$$

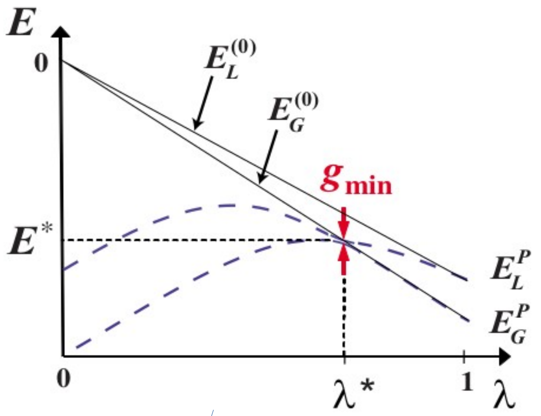


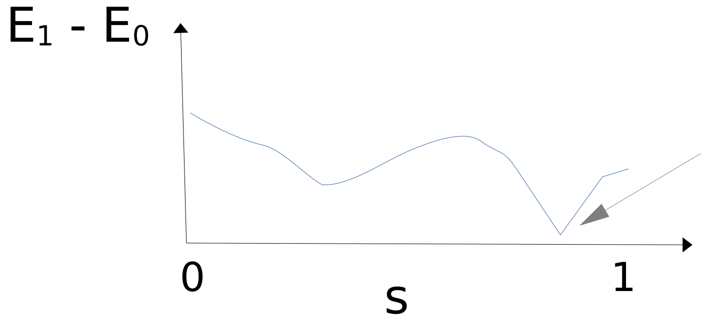
- Fig. 1 from T. Albash, *Role of nonstoquastic catalysts in quantum adiabatic optimization*. Phys. Rev. A 99, 042334 (2019)

1.2.1. General scheme: note #2

Common issue in classical optimisation problems:

Perturbative anticrossings



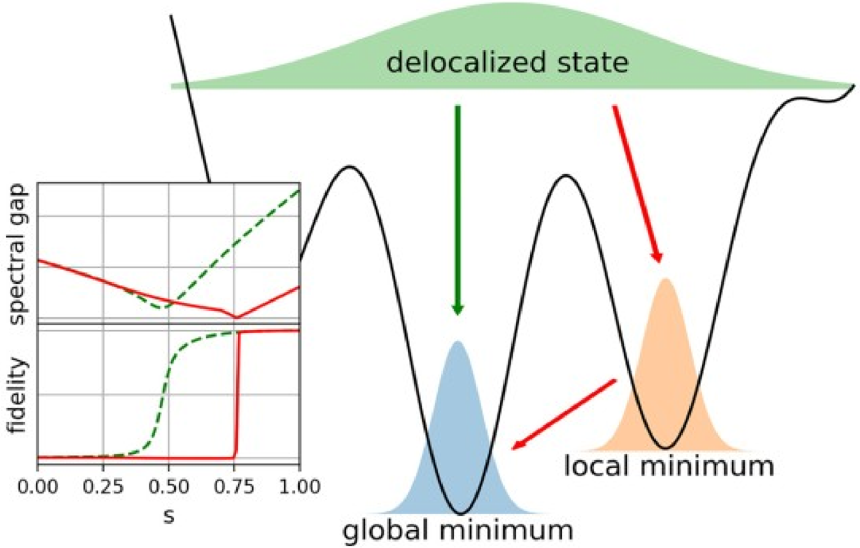


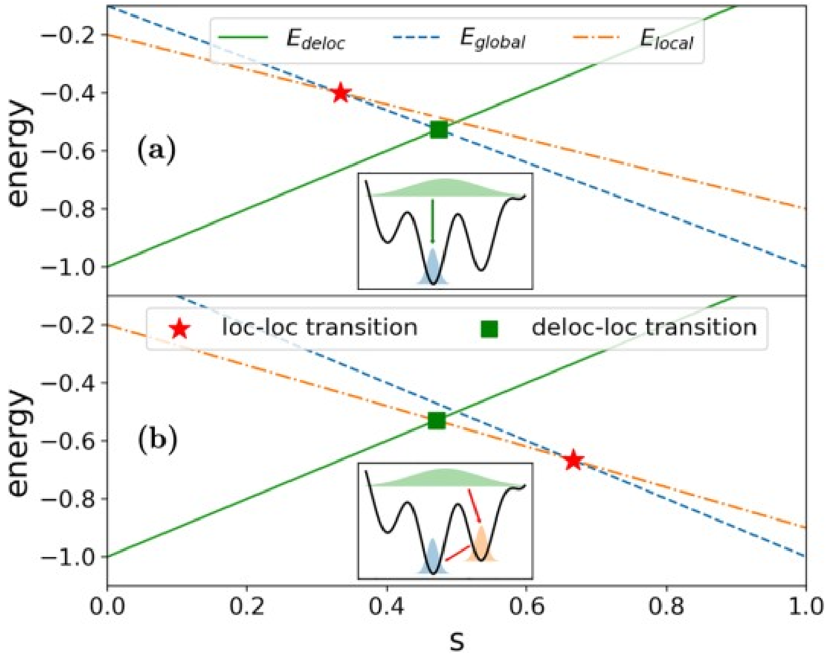
Size of gap dependent on number of spin flips between GS and 1exc (Hamming distance)

- Fig. 1 from M. H. S. Amin and V. Choi, *First order quantum phase transition in adiabatic quantum computation*. Phys. Rev. A 80, 062326 (2009)

1.2.1. General scheme: note #2

An intuitive picture





- Figs. 1 and 3 from M. Werner et al., *Bounding first-order quantum phase transitions in adiabatic quantum computing*. Phys. Rev. Res. 5, 042334 (2023)

1.2. General scheme

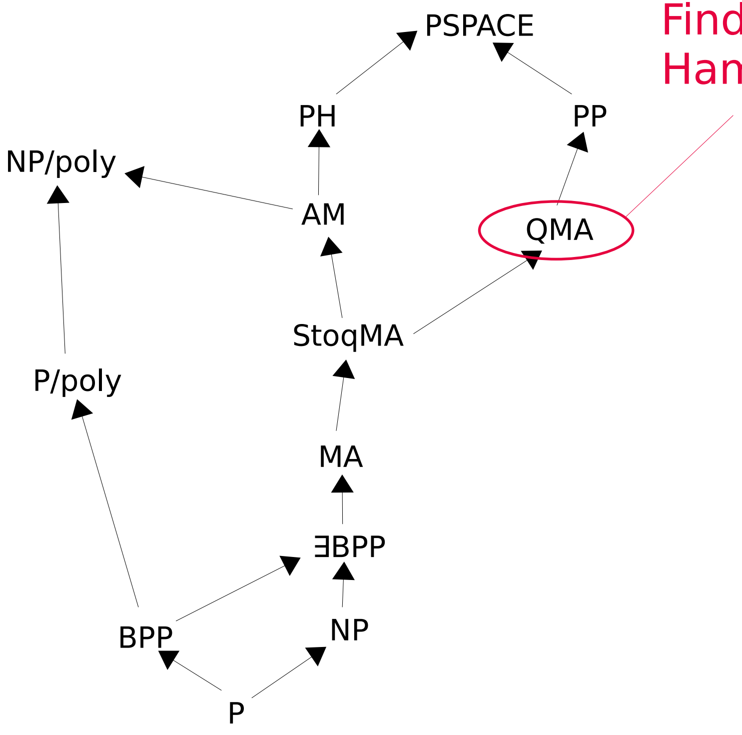
$$H(s) = A(s)H_0 + B(s)H_f$$

Success of the algorithm depends on structure of H_0 and H_f → the more you know about your solution, the better you can tailor your anneal

Conserved quantities enhance performance if leveraged, ruin it if unknown/uncontrolled

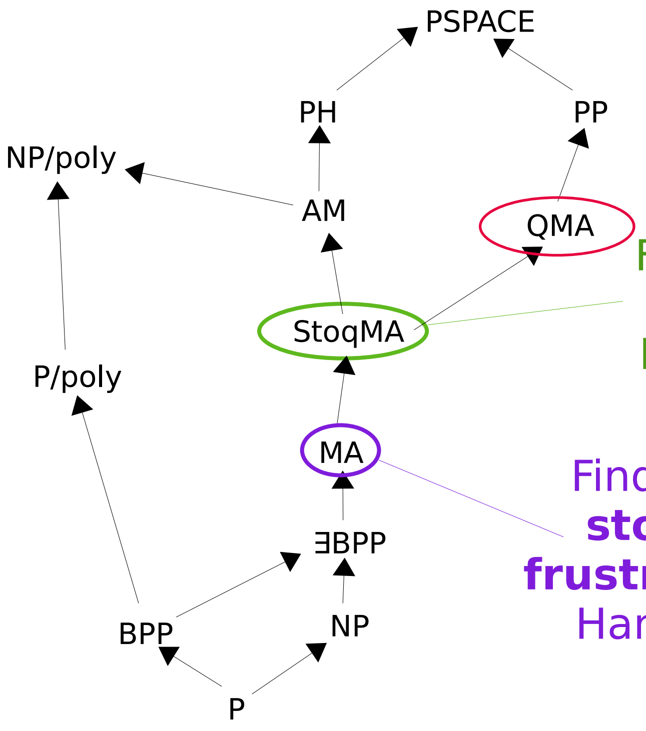
Applications: general results

2. On AQC and the search for quantum advantage



Finding GS of k-local Hamiltonian is QMA-complete

2. On AQC and the search for quantum advantage



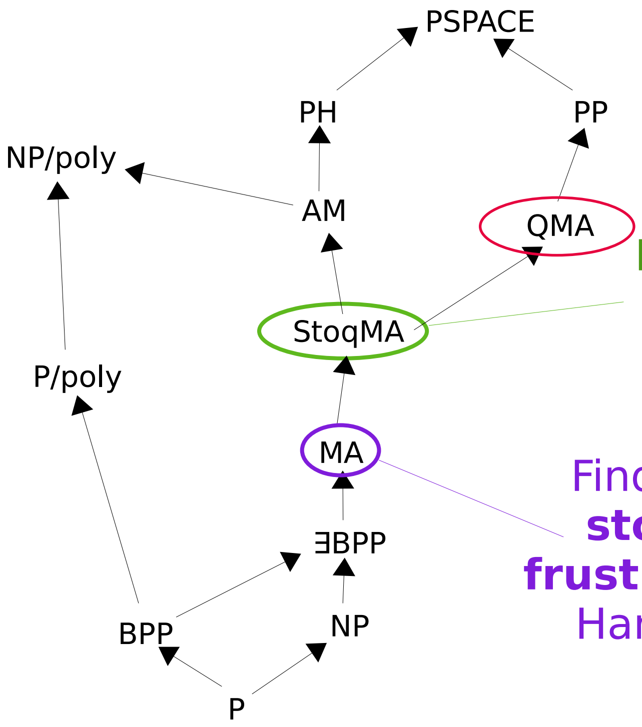
Finding GS of **stoquastic** Hamiltonian

Finding GS of **stoquastic frustration-free** Hamiltonian

Includes Hamiltonians encoding classical optimisation problems, anything encoded in an Ising model

All off-diagonal entries strictly non-positive

2. On AQC and the search for quantum advantage



Finding GS of **stoquastic** Hamiltonian

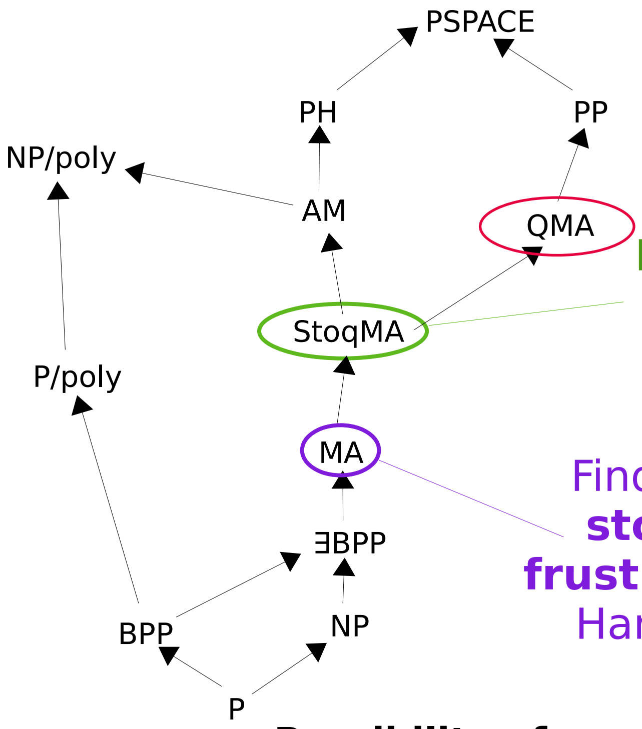
Finding GS of **stoquastic frustration-free** Hamiltonian

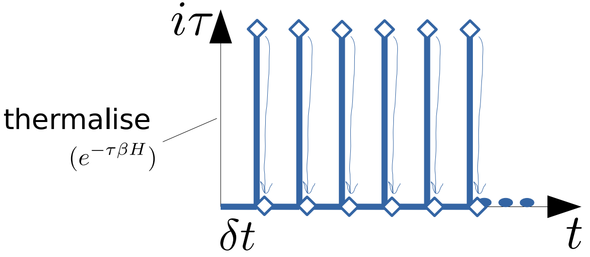
Includes Hamiltonians encoding classical optimisation problems, anything encoded in an Ising model

All off-diagonal entries strictly non-positive

Simulable with quantum Monte Carlo (QMC) methods (no **sign problem**)

2. On AQC and the search for quantum advantage





Includes Hamiltonians encoding classical optimisation problems, anything encoded in an Ising model

Finding GS of **stoquastic** Hamiltonian

Finding GS of **stoquastic frustration-free** Hamiltonian

All off-diagonal entries strictly non-positive

Simulable with quantum Monte Carlo (QMC) methods (no **sign problem**)

Possibility of any quantum advantage comes into question here

2.1. The sign problem

$$Z = \mathrm{Tr}[e^{-\beta H}]$$

$$\langle A \rangle = \frac{\mathrm{Tr}[A e^{-\beta H}]}{Z}$$

If Z is too small, small oscillations explode and the algorithm fails to converge

2.1. The sign problem

$$Z = \mathrm{Tr}[e^{-\beta H}]$$

divide system into
K "time slices"

the continuous extension
of this is path integral MC

$$\mathrm{Tr}[e^{-\beta H}] = \sum_c \langle \psi(c) | e^{-\beta H} | \psi(c) \rangle = \sum_{\{c_i\}} \prod_{i=1}^{K} \langle \psi(c_{i+1}) | e^{-\beta H/K} | \psi(c_i) \rangle$$

sum over all possible
trajectories for all *c*

$$(c_{K+1} = c_1)$$

$$\langle \psi(c_{i+1}) | e^{-\beta H/K} | \psi(c_i) \rangle \approx \delta_{c_{i+1}, c_i} - \frac{\beta}{K} \langle \psi(c_{i+1}) | H | \psi(c_i) \rangle$$

2.1. The sign problem

$$Z = \mathrm{Tr}[e^{-\beta H}]$$

divide system into K "time slices"

the continuous extension of this is path integral MC

$$\mathrm{Tr}[e^{-\beta H}] = \sum_c \langle\psi(c)|e^{-\beta H}|\psi(c)\rangle = \sum_{\{c_i\}} \prod_{i=1}^{K} \langle\psi(c_{i+1})|e^{-\beta H/K}|\psi(c_i)\rangle$$

sum over all possible trajectories for all *c*

$$(c_{K+1} = c_1)$$

$$\langle\psi(c_{i+1})|e^{-\beta H/K}|\psi(c_i)\rangle \approx \delta_{c_{i+1},c_i} - \frac{\beta}{K}\langle\psi(c_{i+1})|H|\psi(c_i)\rangle$$

Simplest way to ensure $Z > \delta$ → ensure all positive contributions

$\langle\psi(c_{i+1})|H|\psi(c_i)\rangle < 0$ → **stoquastic**

2.1. The sign problem

- Key points
  - Basis-dependent, but finding a basis change that cures it is NP-complete in general
    - M. Marvian, D. A. Lidar, and I. Hen. *On the computational complexity of curing non-stoquastic hamiltonians.* Nature Communications, 1 (2019)
  - Not the only reason QMC may fail, it can still be inefficient
    - M. B. Hastings and M. H. Freedman. *Obstructions to classically simulating the adiabatic algorithm*. Quantum Information and Computation, 13:1038 (2013)

2.2. Current status

- Regarding the competition with QMC
  - Super-polynomial oracle separation between AQC with no sign problem and classical computation
    - M. B. Hastings. The power of adiabatic quantum computation with no sign problem. arXiv:2005.03791 (2020)

2.2. Current status

- Regarding the competition with QMC
  - Super-polynomial oracle separation between AQC with no sign problem* and classical computation
    - M. B. Hastings. The power of adiabatic quantum computation with no sign problem. arXiv:2005.03791 (2020)
  - Shift from stoquastic to VGP (vanishing geometric phase)
    - I. Hen. Determining quantum monte carlo simulability with geometric phases. Phys. Rev. Research, 3:023080 (2021)

*according to stoquastic/nonstoquastic classification

2.2. Current status

- Regarding competition with other classical methods that don’t present the sign problem
  - **Recurrent neural networks** (RNNs) sometimes used as benchmark, but costly to train
  - **Tensor networks** currently state of the art benchmark
  - **Parallel tempering** algorithms (Markov chain Monte Carlo techniques) also state of the art benchmark for classical optimisation problems

2.2. Current status

- More recent arguments for quantum advantage
  - Scaling advantage in coherent regime for the simulation of TFIM
  - Large scale simulation of quenches in TFIM beyond classical capabilities

Short coherence times → study of nonequilibrium dynamics

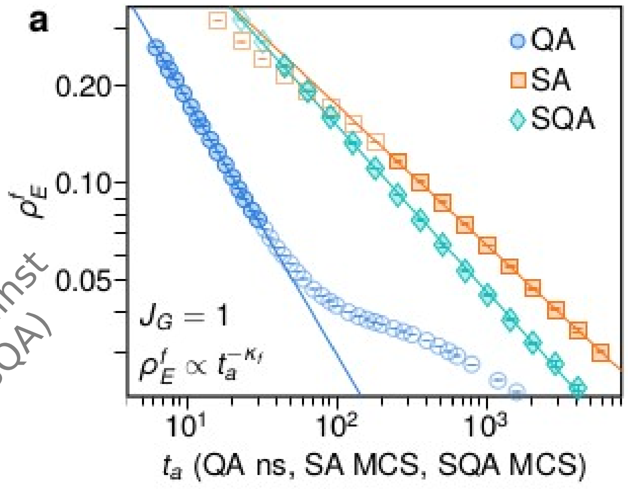
$$\mathcal{H}(t) = \Gamma(t/t_a)\mathcal{H}_D + \mathcal{J}(t/t_a)\mathcal{H}_P,$$

$$\mathcal{H}_D = -\sum_i \sigma_i^x, \qquad \mathcal{H}_P = \sum_{i<j} J_{ij}\sigma_i^z\sigma_j^z.$$

(Study of different topologies)

- A. D. King et al. *Quantum critical dynamics in a 5000-qubit programmable spin glass.* Nature 617, 61–66 (2023)
- A. D. King et al. *Computational supremacy in quantum simulation*. arXiv:2403.00910 (2024)

2.2. Current status



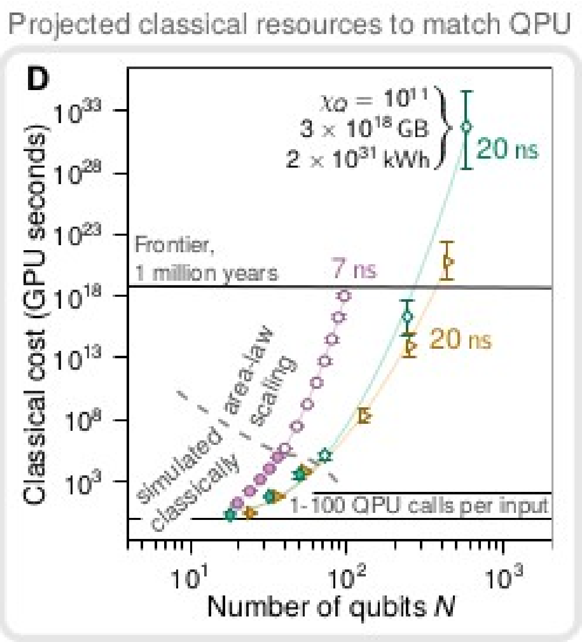
Critical scaling of final residual energy

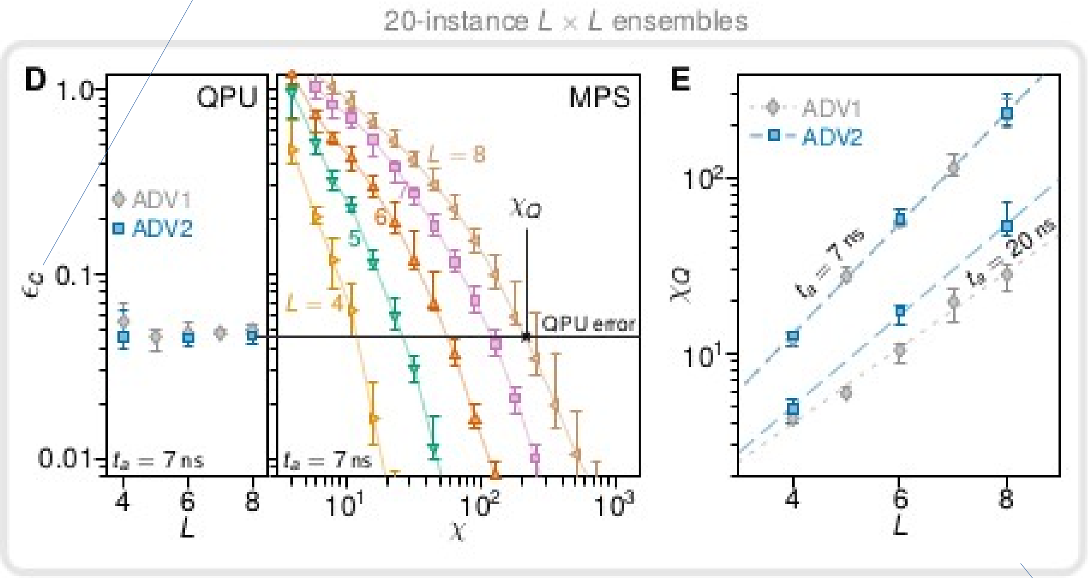
Benchmark against SA, QMC (SQA)

- Fig. 4a from A. D. King et al. *Quantum critical dynamics in a 5000-qubit programmable spin glass.* Nature 617, 61–66 (2023)

2.2. Current status

Residual energy





Benchmark against TN (MPS)

- Figs. 5d, 2d and 2e from A. D. King et al. *Computational supremacy in quantum simulation*. arXiv:2403.00910 (2024)

- Regarding all else
  - Grover's quadratic speed-up for quantum search also found in AQC
    - J. Roland and N. J. Cerf. Quantum search by local adiabatic evolution. Phys. Rev. A 65, 042308 (2002)
  - **Search beyond adiabaticity**

    → allowing for diabatic passages enables universal computation
    - S. P. Jordan, D. Gosset, and P. J. Love. *Quantum-Merlin-Arthur–complete problems for stoquastic Hamiltonians and Markov matrices*. Phys. Rev. A 81, 032331 (2010)
    - E. J. Crosson and D. A. Lidar. *Prospects for quantum enhancement with diabatic quantum annealing*. Nature Reviews Physics 3, 466–489 (2021)

3. On QS and universality

Canonical proof of universality of AQC based on equivalence to gate model (GM) though Kitaev’s history state

one can approximate the other **efficiently**

GM → AQC : depth $poly(NT)$

AQC → GM : running time $poly(L)$

qubits

annealing time

circuit depth

► A. Kitaev, A. Shen, and M. Vyalyi, *Classical and Quantum Computation*, Number 47 in Graduate Series in Mathematics. AMS, Providence, RI (2002)

Current focus: experimental feasibility

Efficient maps in terms of qubits and energy scale

- Rigorous definition of Hamiltonian simulation

  - Toby S. Cubitt, Ashley Montanaro and Stephen Piddock, *Universal Quantum Hamiltonians*, Proceedings of the National Academy of Sciences 115, 9497 (2018).

Simulation efficient for target Hamiltonians with local interactions in the same (or lower) spatial dimension

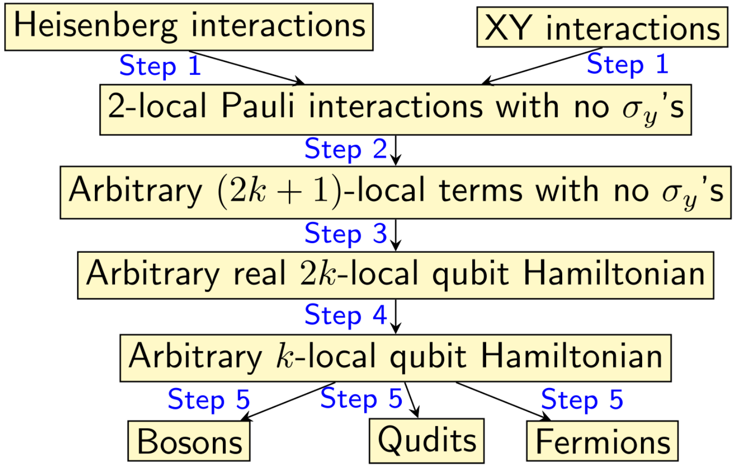


Fig. 2. Part of the sequence of simulations used in this work. An arrow from one box to another indicates that a Hamiltonian of the first type can simulate a Hamiltonian of the second type.

Current focus: experimental feasibility

Efficient maps in terms of qubits and energy scale

- Rigorous definition of Hamiltonian simulation
  - Toby S. Cubitt, Ashley Montanaro and Stephen Piddock, *Universal Quantum Hamiltonians*, Proceedings of the National Academy of Sciences 115, 9497 (2018).
- Proof of existence of efficient simulators regardless of target graph
  - Leo Zhou and Dorit Aharonov, *Strongly Universal Hamiltonian Simulators*, arXiv:2102.02991 (2021)
- Connection between the universality of Hamiltonians and their complexity class
  - Tamara Kohler et al., *General Conditions for Universality of Quantum Hamiltonians*, PRX Quantum 3, 010308 (2022)

...

3. On QS and universality

Efficient maps in terms of qubits and energy scale

- Rigorous definition of Hamiltonian simulation
  - Toby S. Cubitt, Ashley Montanaro and Stephen Piddock, *Universal Quantum Hamiltonian*… 18).
- Proof of exi… t graph
  - Leo Zhou … s, arXiv:210…

In summary:
Practical, precise universality recipes are currently under development

- Connection between the universality of Hamiltonians and their complexity class
  - Tamara Kohler et al., *General Conditions for Universality of Quantum Hamiltonians*, PRX Quantum 3, 010308 (2022)

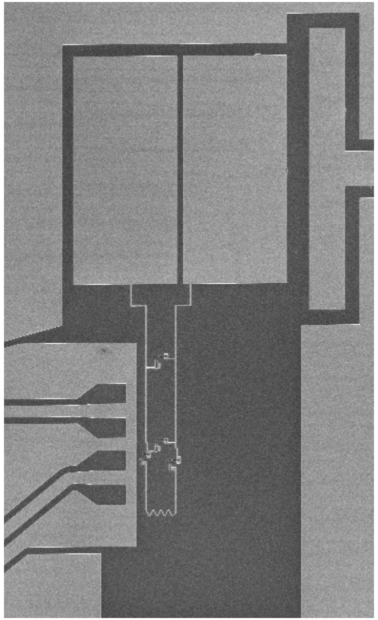
…

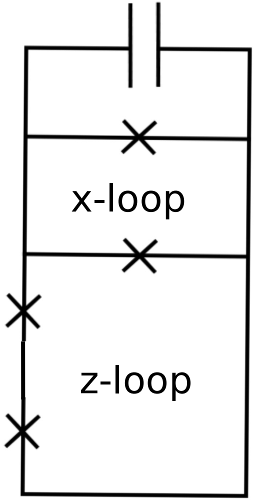
Annealing platforms (superconducting)

Section

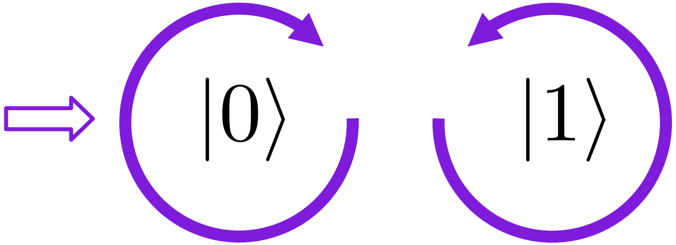
4. QA platforms: superconducting flux qubits

Capacitively shunted flux qubit (CSFQ)





Computational basis states: direction of the current in the z-loop



- SEM image of Qilimanjaro chip

Section

4. QA platforms: superconducting flux qubits

Circuit QED Hamiltonian

$$H = \frac{e^2}{2C_{\mathrm{sh}} + (4\alpha + 1)C_{\mathrm{z}}}(2\hat{n}_1 + \hat{n}_2)^2 + \frac{e^2}{C_{\mathrm{z}}}\hat{n}_2^2$$

kinetic part

$$-2I_{\mathrm{z}}\frac{\Phi_0}{2\pi}\cos(\hat{\varphi}_2 - \hat{\varphi}_1/2)\cos(\hat{\varphi}_1/2 - \varphi_{\mathrm{z}}/2)$$

$$-2\alpha I_{\mathrm{z}}\frac{\Phi_0}{2\pi}\cos(\varphi_{\mathrm{x}}/2)\sqrt{1 + \tan^2(\varphi_{\mathrm{d}})}\cos(\hat{\varphi}_1 - \varphi_{\mathrm{d}})$$

potential part

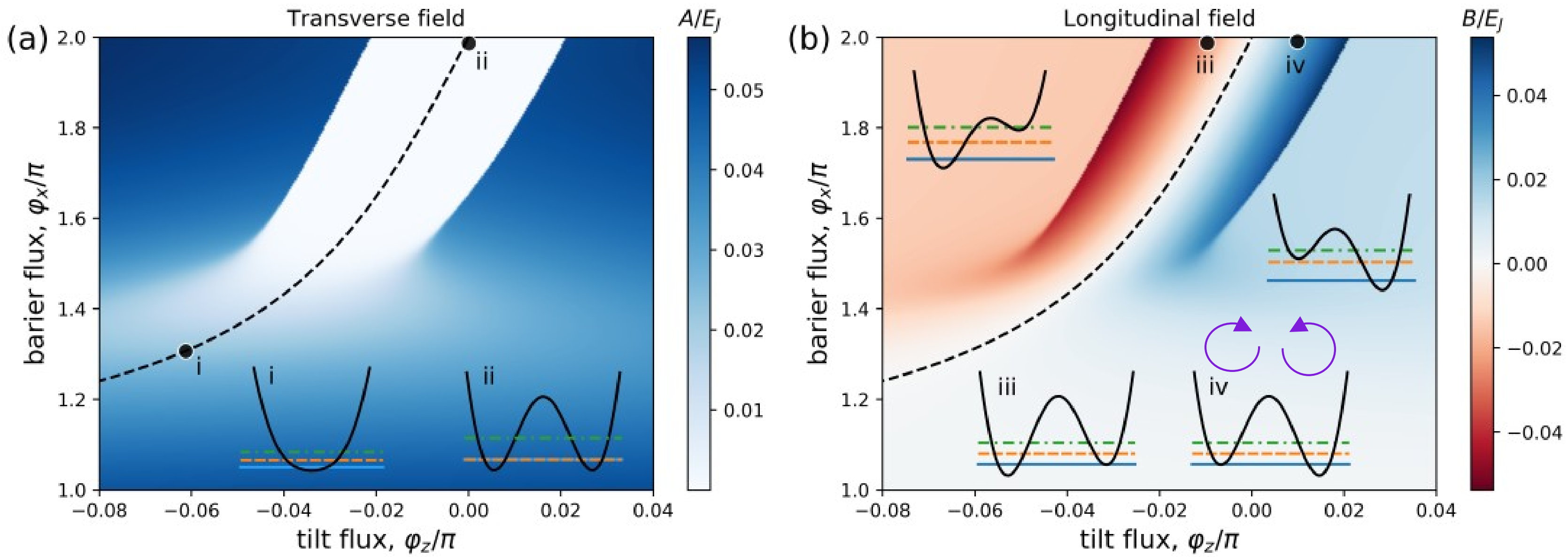
Junction asymmetry

$$\varphi_{\mathrm{d}} = \arctan[d\tan(\varphi_{\mathrm{x}}/2)]$$

- Additional JJ w.r.t. transmon → richer potential landscape
- Higher anharmonicity
- Restriction to low-energy subspace to use as computational space (Schrieffer-Wolff transformation)

4. QA platforms: superconducting flux qubits

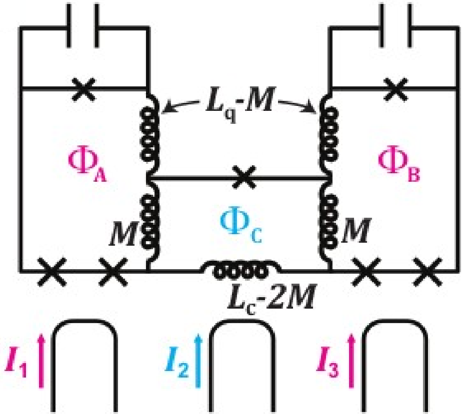
Projected and gauged Hamiltonian: $$H = A\sigma^x + B\sigma^z$$

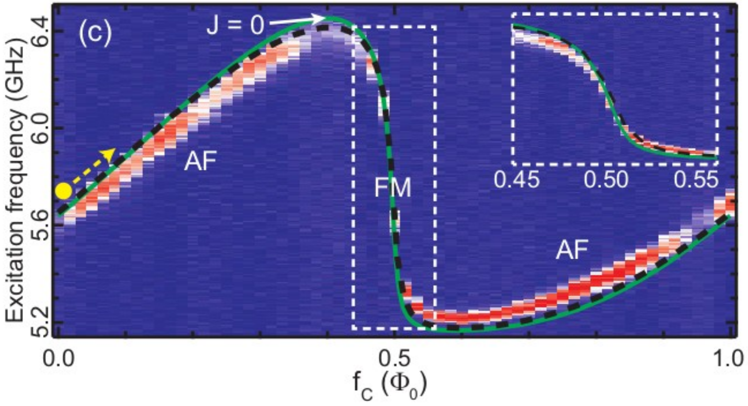


- Fig. 2 from M. Khezri et al., *Anneal-path correction in flux qubits*, npj Quantum Inf 7, 36 (2021)

4. QA platforms: superconducting flux qubits

Couplings:
- ZZ coupling relatively easy to implement → TFIM
- Each coupler is essentially another qubit with a very large gap





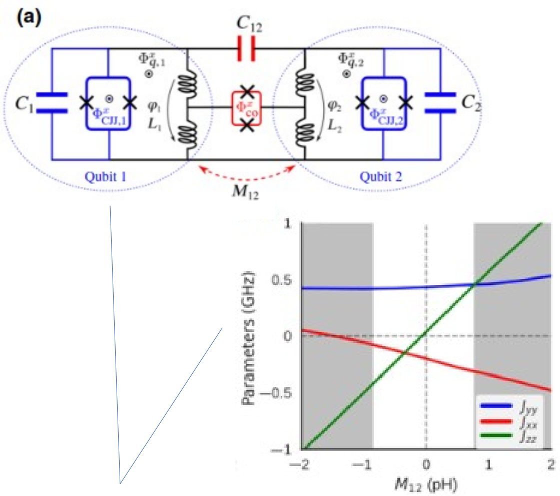
Coupler bias

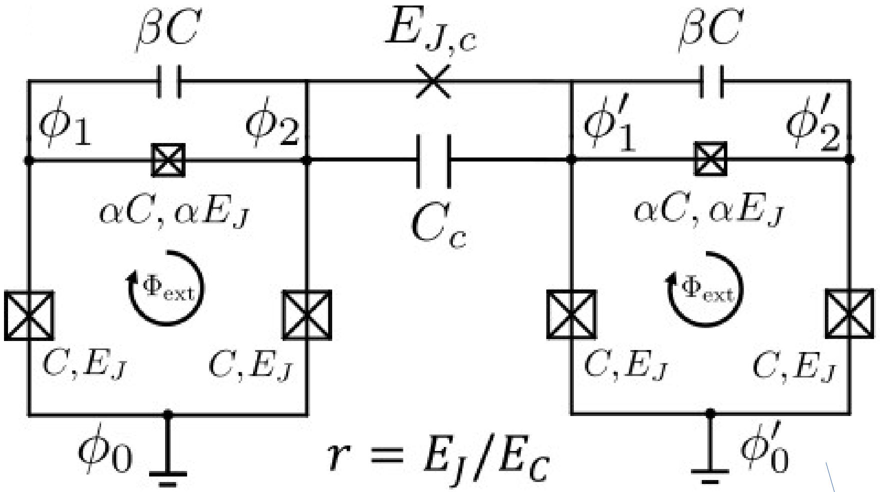
- Figs. 1c and 2c from S. J. Weber et al., *Coherent coupled qubits for quantum annealing*, Phys. Rev. Applied 8, 014004 (2017)

4. QA platforms: superconducting flux qubits

Couplings:

- Capacitive couplings allow to implement XX, YY interactions



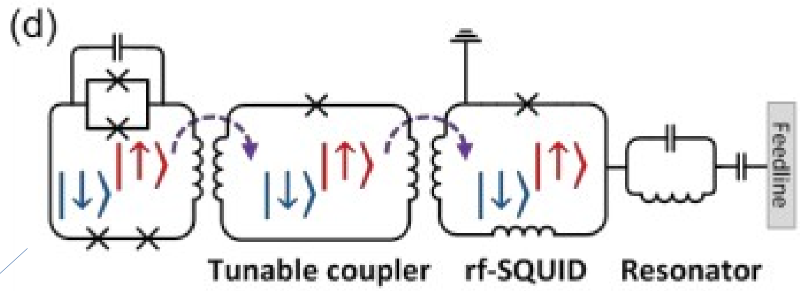


- Figs. 1a and 3d from I. Ofzidan et al., *Demonstration of a nonstoquastic hamiltonian in coupled superconducting flux qubits*, Phys. Rev. Applied 13, 034037 (2020)
- Fig. 1a from María Hita-Pérez, Gabriel Jaumà, Manuel Pino, Juan José García-Ripoll, *Three-Josephson junctions flux qubit couplings*. Appl. Phys. Lett. 29, 119 (22): 222601 (2021)

4. QA platforms: superconducting flux qubits

Persistent current readout:

- Couple qubit to SQUID such that direction of current in the loop can be measured



- Fig. 3a from S. Novikov et al., *Exploring More-Coherent Quantum Annealing*, 2018 IEEE International Conference on Rebooting Computing (ICRC), , McLean, VA, USA, 2018, pp. 1-7

4. QA platforms: superconducting flux qubits

Summary

- High coherence times (~10us)
- TFIM easy to encode, further developments towards XX, YY
- Custom qubit design → high controllability

Challenges

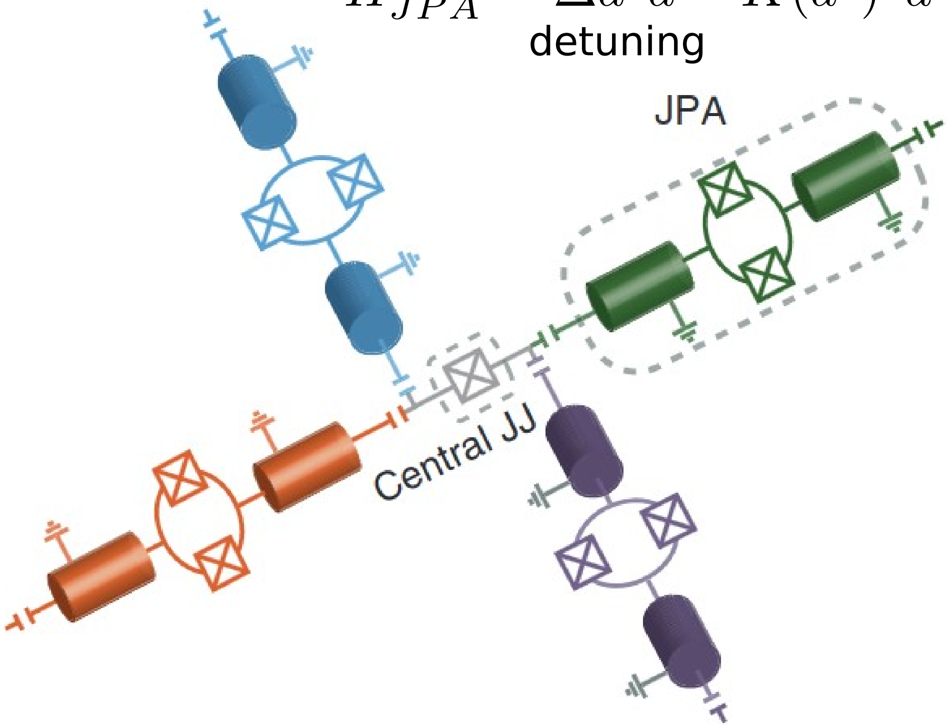
- Connectivity
- Crosstalk compensation

4. QA platforms: Josephson Parametric Amplifiers (JPAs)

Kerr non-linearity — weak 1-ph drive

$$H_{JPA}^{rot} = \Delta a^\dagger a - K(a^\dagger)^2 a^2 + \mathcal{E}_p((a^\dagger)^2 + a^2) + \mathcal{E}_0(a^\dagger + a)$$

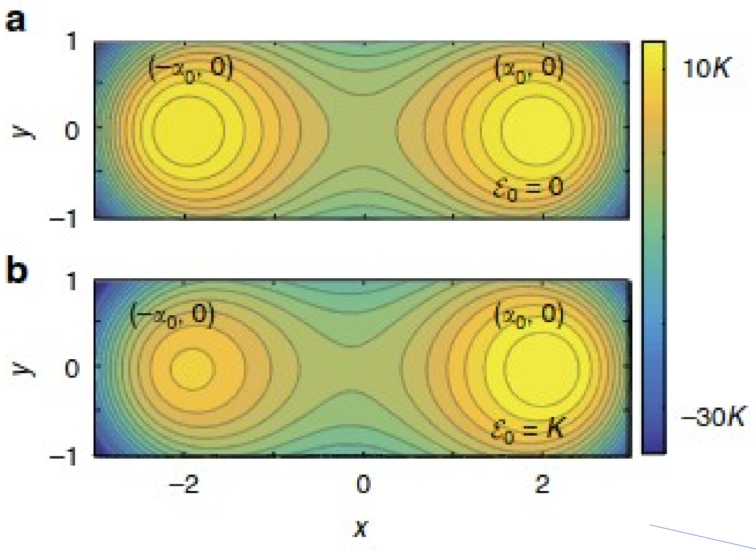
detuning — 2-ph pump drive



In reference frame of pump drive (~2 ω_0), after RWA

- Fig. 4a from S. Puri et al., *Quantum annealing with all-to-all connected nonlinear oscillators*. Nat Commun 8, 15785 (2017)

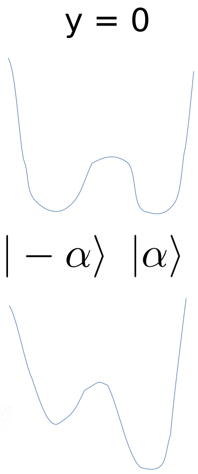
QA platforms: Josephson Parametric Amplifiers (JPAs)



Contour plot of metapotential

$$a \to x + iy$$

$$a^\dagger \to x - iy$$



Degenerate eigenstates $|\alpha_0\rangle, |-\alpha_0\rangle$

$$H = -K(a^\dagger)^2 a^2 + \mathcal{E}_p((a^\dagger)^2 + a^2)$$

$$\alpha_0 = \sqrt{\mathcal{E}_p / K}$$

High α_0, so that they can be considered orthogonal

Non-degenerate eigenstates $|\alpha_0\rangle, |-\alpha_0\rangle$

gap $4\mathcal{E}_0\alpha_0$

$$H = -K(a^\dagger)^2 a^2 + \mathcal{E}_p((a^\dagger)^2 + a^2) + \mathcal{E}_0(a^\dagger + a)$$

- Fig 1 from S. Puri et al., *Quantum annealing with all-to-all connected nonlinear oscillators*. Nat Commun 8, 15785 (2017)

QA platforms: Josephson Parametric Amplifiers (JPAs)

$$H_0 = \Delta a^\dagger a + K(a^\dagger)^2 a^2$$

eigenstates Fock $|0\rangle, |1\rangle$

anneal to

$$H_1 = \mathcal{E}_p((a^\dagger)^2 + a^2) + \mathcal{E}_0((a^\dagger) + a) + K(a^\dagger)^2 a^2$$

eigenstates coherent $|\alpha_0\rangle, |-\alpha_0\rangle$

GS is resilient to photon loss (main source of decoherence) both at beginning and end of anneal

- S. Puri et al., *Quantum annealing with all-to-all connected nonlinear oscillators*. Nat Commun 8, 15785 (2017)
- S. E. Nigg et al., *Robust quantum optimizer with full connectivity*. Sci. Adv. 3, e1602273 (2017)

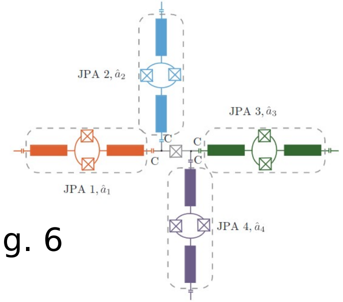
4. QA platforms: Josephson Parametric Amplifiers (JPAs)

Readout:

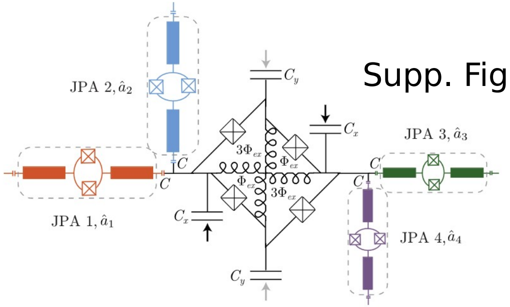
- Homodyne detector (read out phase)

Coupling:

- Linear coupling (capacitive) → tricky to make tuneable
- 4-body coupling → enables LHZ scheme, only local fields to tune
  - S. Puri et al., *Quantum annealing with all-to-all connected nonlinear oscillators*. Nat Commun 8, 15785 (2017)



Supp. Fig. 6

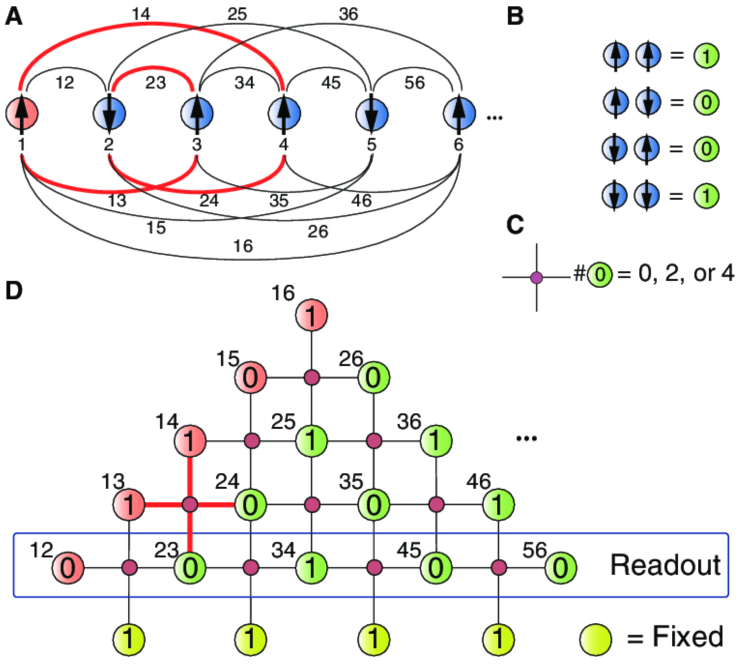


Supp. Fig. 8

4.1. Lechner-Hauke-Zoller (LHZ) scheme

Alternative encoding of Ising problems

- Qubits encode parity between variables
- Local fields implemented as parities between variable and fixed spin
- Constraints enforced in plaquettes to restrict to logical subspace
  - 4-body interactions (planar graph)
  - 2-local interactions with mediating qutrits



- Fig. 1 from W. Lechner et al., *A quantum annealing architecture with all-to-all connectivity from local interactions*. Sci. Adv. 1, e1500838(2015)

QA platforms: Josephson Parametric Amplifiers (JPA)

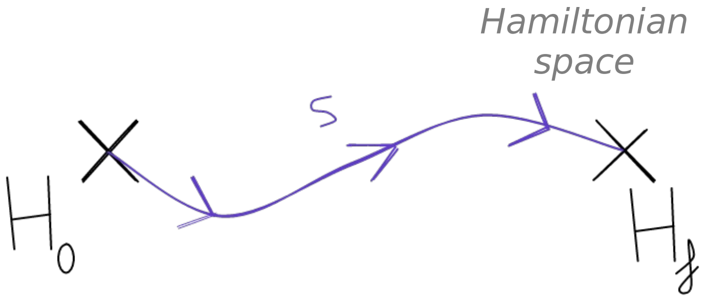
Summary

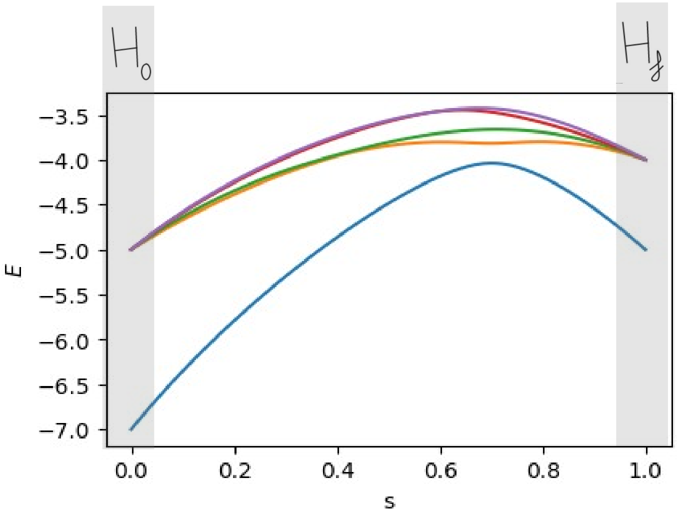
- Some shared features with superconducting qubits
- Qubits potentially more resilient to noise (but no public experimental data as of today)
- Possibility to implement all-to-all Ising models through LHZ

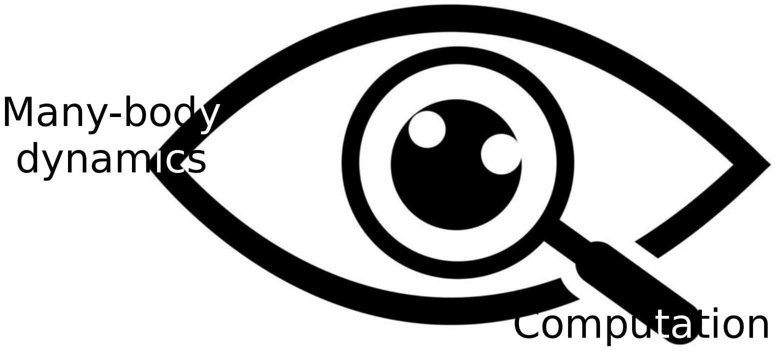
Challenges

- Tunability of capacitive coupler
- Coupler strength (4-body LHZ)

5. Final recap







5. Final recap

Applications

- Optimisation (generally QUBO problems)
- Quantum simulation

▶ Given sufficiently powerful set of interactions universal computation/simulation can be achieved, but no practical recipe

Hardware platforms

- Neutral atoms, superconducting flux qubits, JPAs, trapped ions

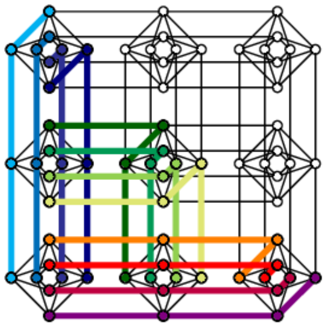
▶ Keys: high coherence, coupling strengths, connectivity, controllability, available interactions

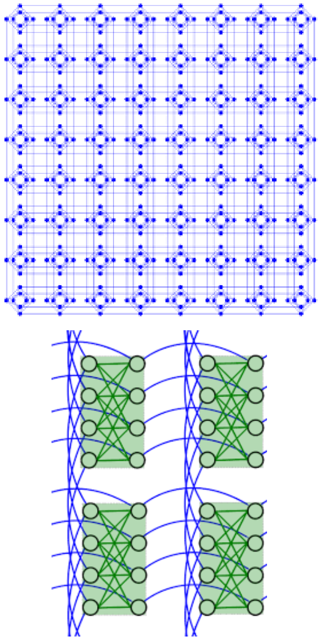
QILIMANJARO
QUANTUM · TECH

Topologies of current superconducting large scale annealing devices

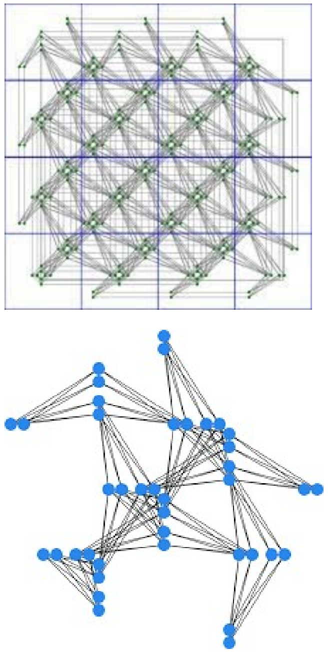
D-Wave
The Quantum Computing Company™

Minor embedding

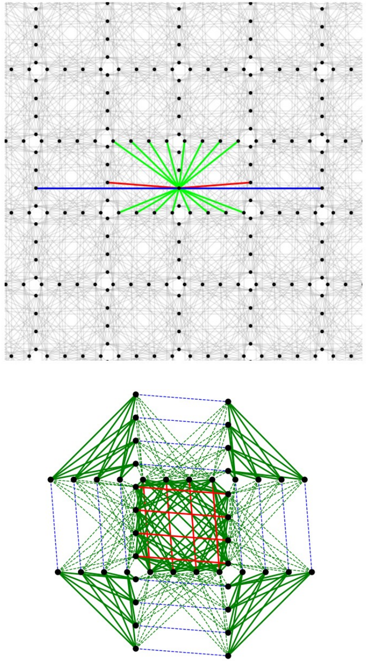




Chimera topology
d = 6



Pegasus topology
d = 15 and native K_4 and $K_{6,6}$ subgraphs



Zephyr topology
d = 20 and native K_4 and $K_{8,8}$ subgraphs