GENERAL FRAMEWORK FOR QUANTUM ASSOCIATIVE MEMORIES

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Classifying input data into a set of categories is a fundamental task in machine learning. To this end, we explore a neuro-inspired approach called associative memory (AM), where a system is able to dynamically retrieve a set of pre-stored information. AM is a particular example of attractor networks whose temporal evolution settles on stable solutions. Here, a system stores a set of memory states in the form of stable fixed points. Through its dynamics, the system identifies the stored pattern that is most similar to the clue, according to a properly defined distance. The most prominent example in the classical domain is the Hopfield neural network (HNN), which consists of a network of all-to-all connected binary neurons [1].

In view of the success of classical AM, a number of research contributions have addressed the issue of modelling quantum versions of AM. Among the different approaches, we can distinguish between digital or circuit-based models, based on Grover's search algorithm [2], and analogue or dynamical-based models, based on the original idea of Hopfield [3]. The latter are mostly direct generalisations of the classical HNN, where binary neurons are replaced by qubits and the classical dynamics is encoded in the jump operators of an open quantum system. However, these models suffer from the same limitations as their classical counterparts, since the learning rule used to encode the memory states is the same [4]. Additional works focus on unleashing the storage of quantum patterns by exploring different systems such as quantum walks [5] or single driven-dissipative resonators [6, 7]. Here, recent models that are compatible with generic quantum neural networks seem to identify a potential quantum advantage [8]. However, clarifying the applicability of these results in the quantum AM scenario remains a challenge. Indeed, within the analogue approach, and despite the many instances of quantum models of AM mentioned above, a general paradigm defining these systems is still lacking.

The goal of our work is to define a general theory for open quantum AM, that includes the existing quantum models and through which their strengths and limitations can be analyzed. In assuming a general approach, our starting point are the necessary properties that a generic open quantum system must show to be regarded as an associative memory. This will allow us to draw further considerations upon the amount of quantum states that can be stored by these kind of systems. We demonstrate the feasibility of encoding non-orthogonal states and achieving enhanced storage capacity in contrast to classical models. Nonetheless, challenges arise from the retrieval of classical information through measurement processes and the inherent linearity of quantum mechanics, potentially limiting storage capacity. Our findings lay the groundwork for advancing quantum associative memory systems, holding promise for applications in quantum machine learning or quantum error correction.

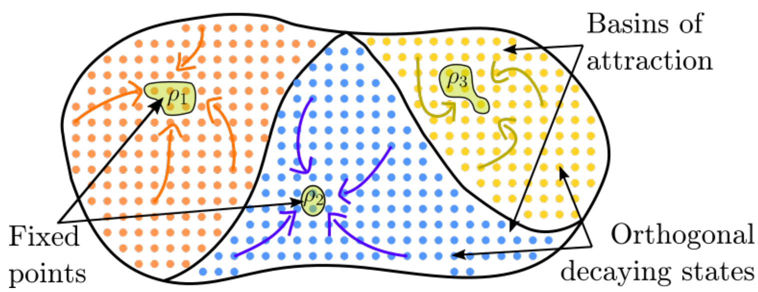

***Figure 1** – Quantum model for AM. The points represent the elements in a basis of the Hilbert space, and the quantum states {ρ_j} represent the patterns. The figure highlights the division of the Hilbert space into basins of attraction, containing the initial conditions that converge towards the corresponding pattern.*

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