Gravitational Waves
TAE 2024 - Benasque

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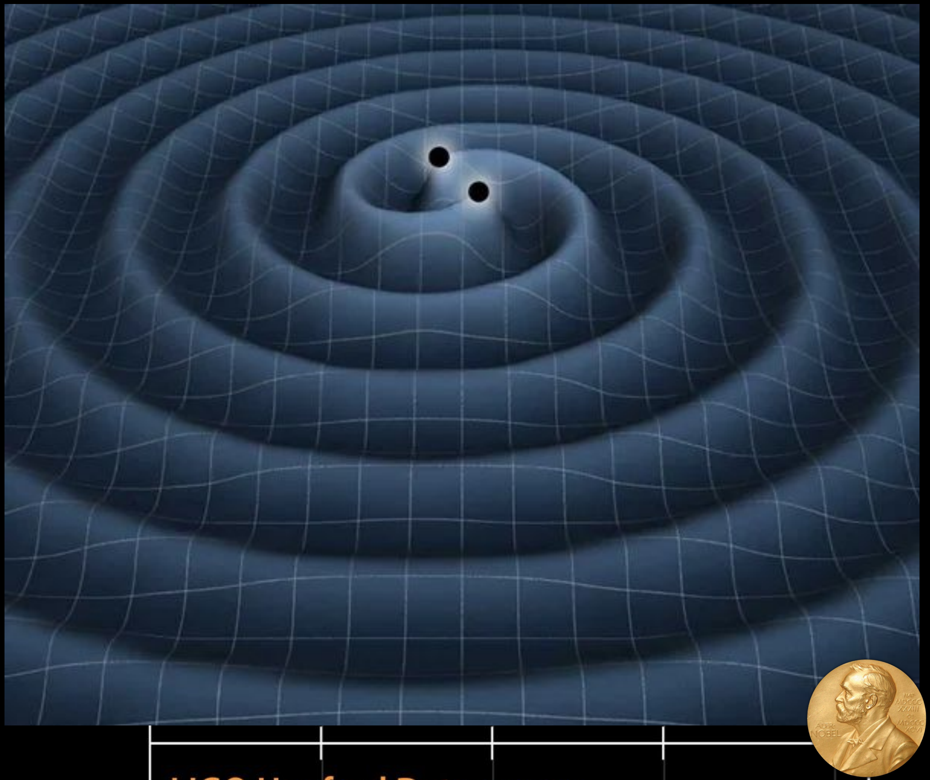
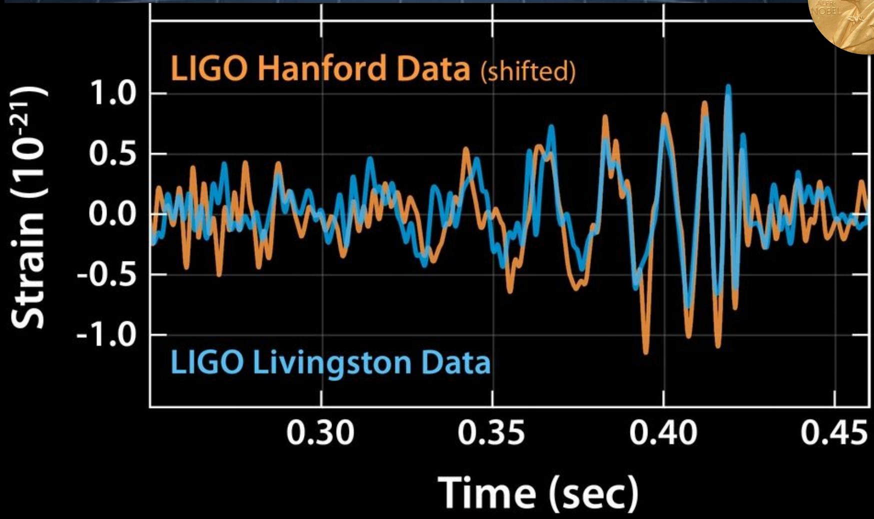
Gravitational Waves (GWs)

0. Introduction
1. Theory
2. Detectors & Sources
3. Applications
4. Exercises (Stefano Savastano)





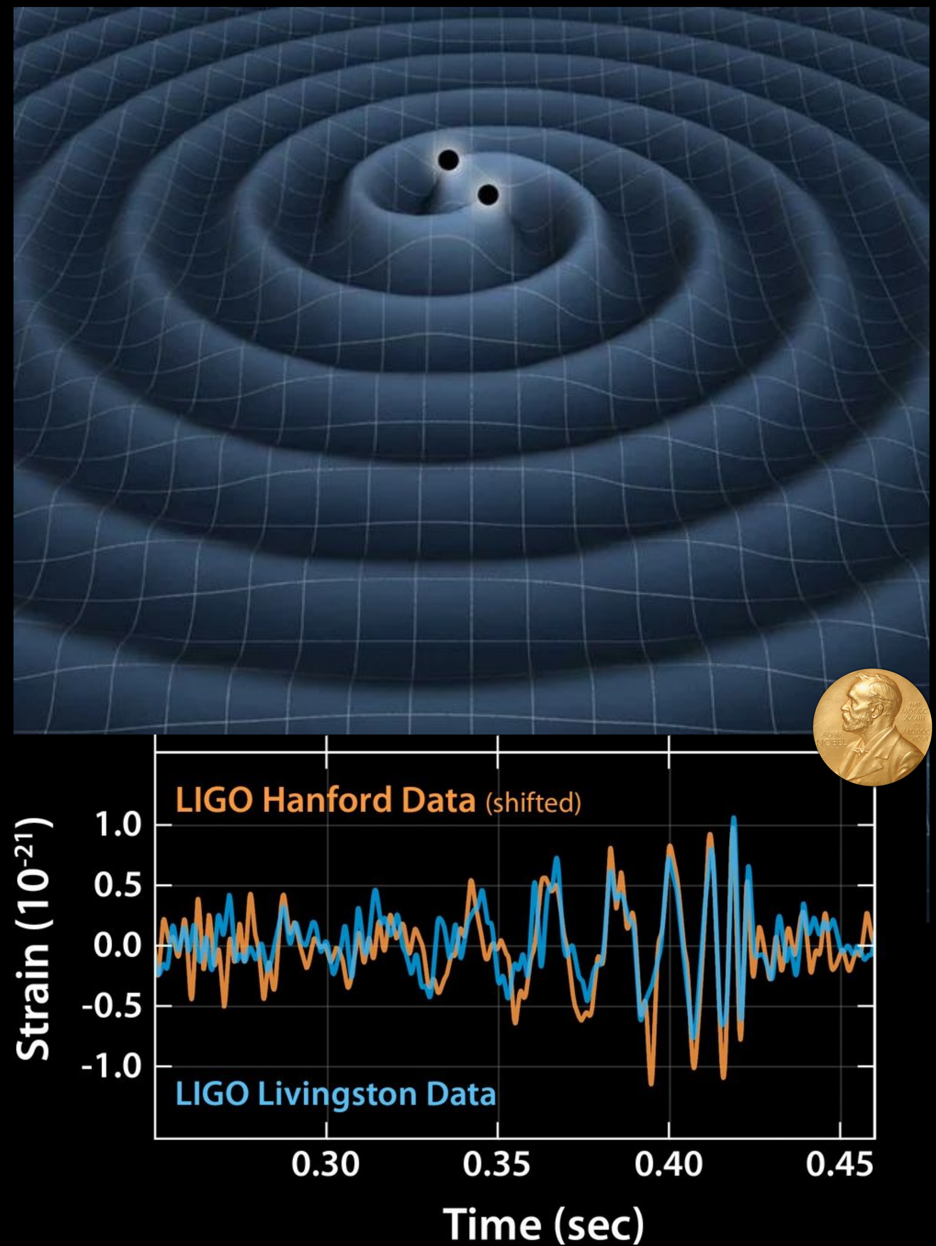
Discussion: What is your interest in GWs?

Activities:

Thinking (alone)

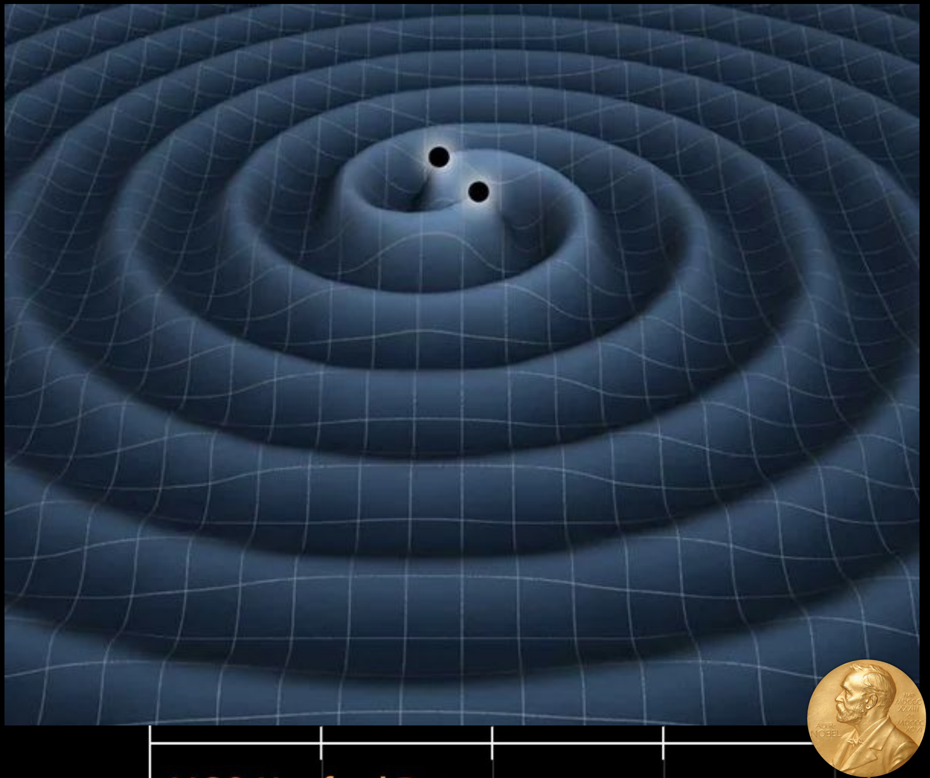
Discussing (2-3 people)

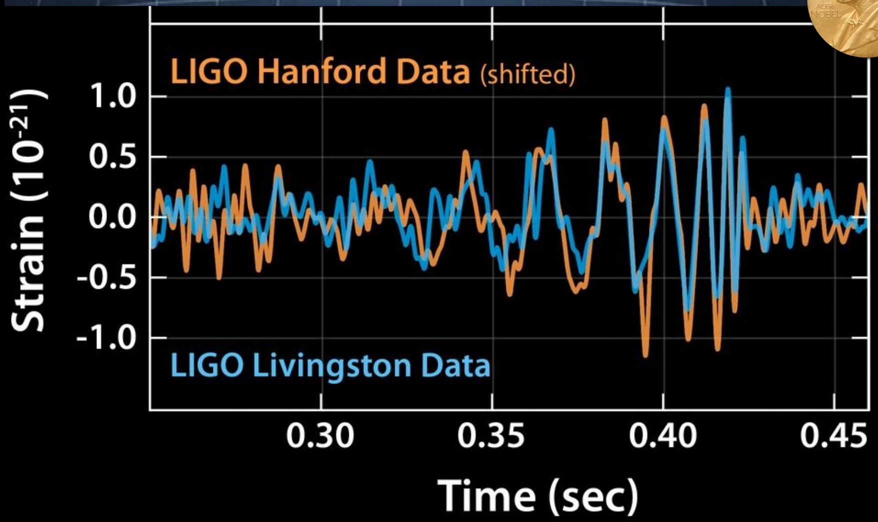
Sharing (all)



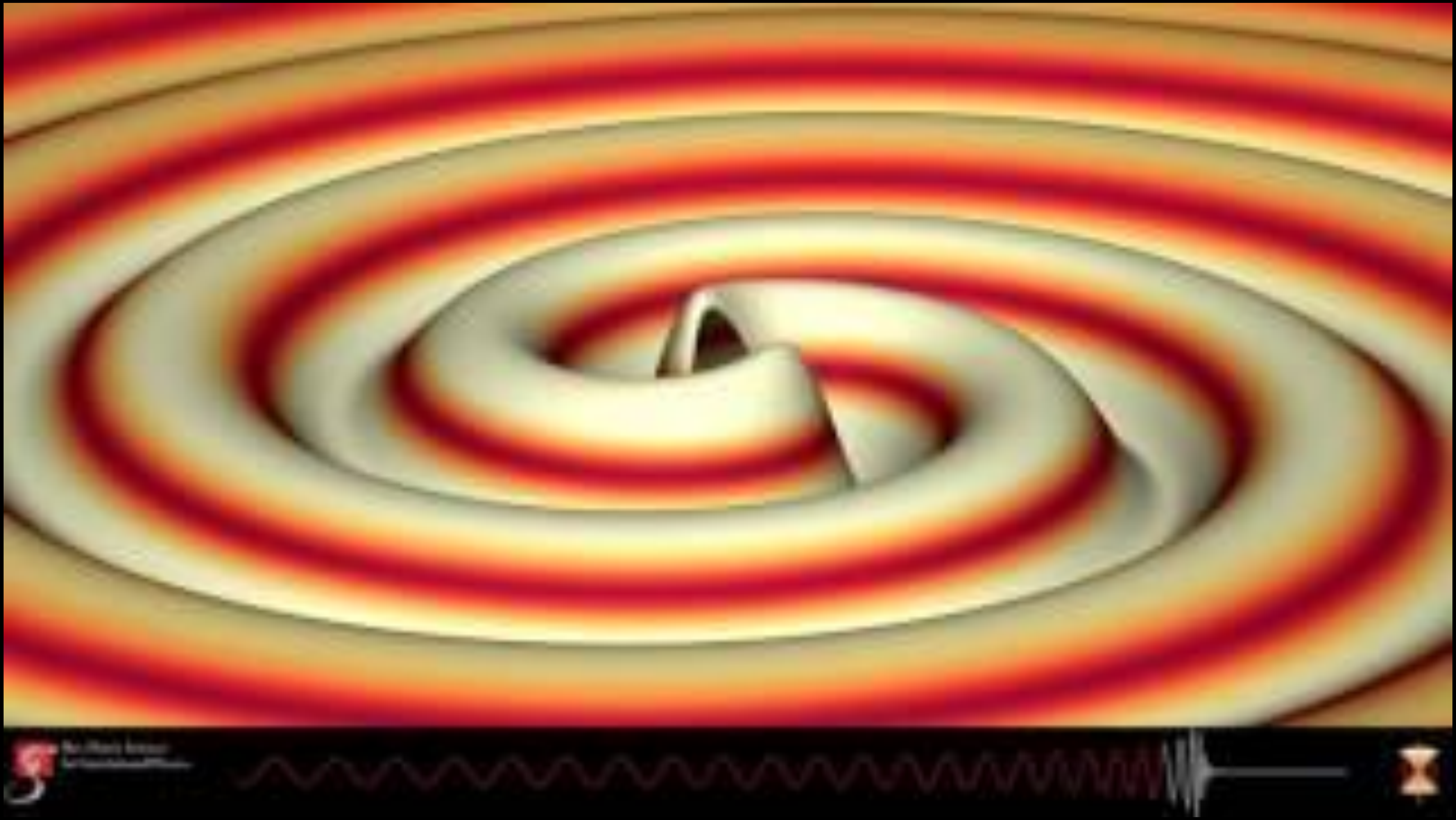
A new window into the universe

- Discover compact objects:
  - Black holes
  - Neutron stars
- Test Gravity:
- Ultra-dense matter
- Fundamental fields
- Cosmology
- …





Basics of Gravitational Waves



Credit: AEI & SXS https://youtu.be/_GhkWuIDzpc

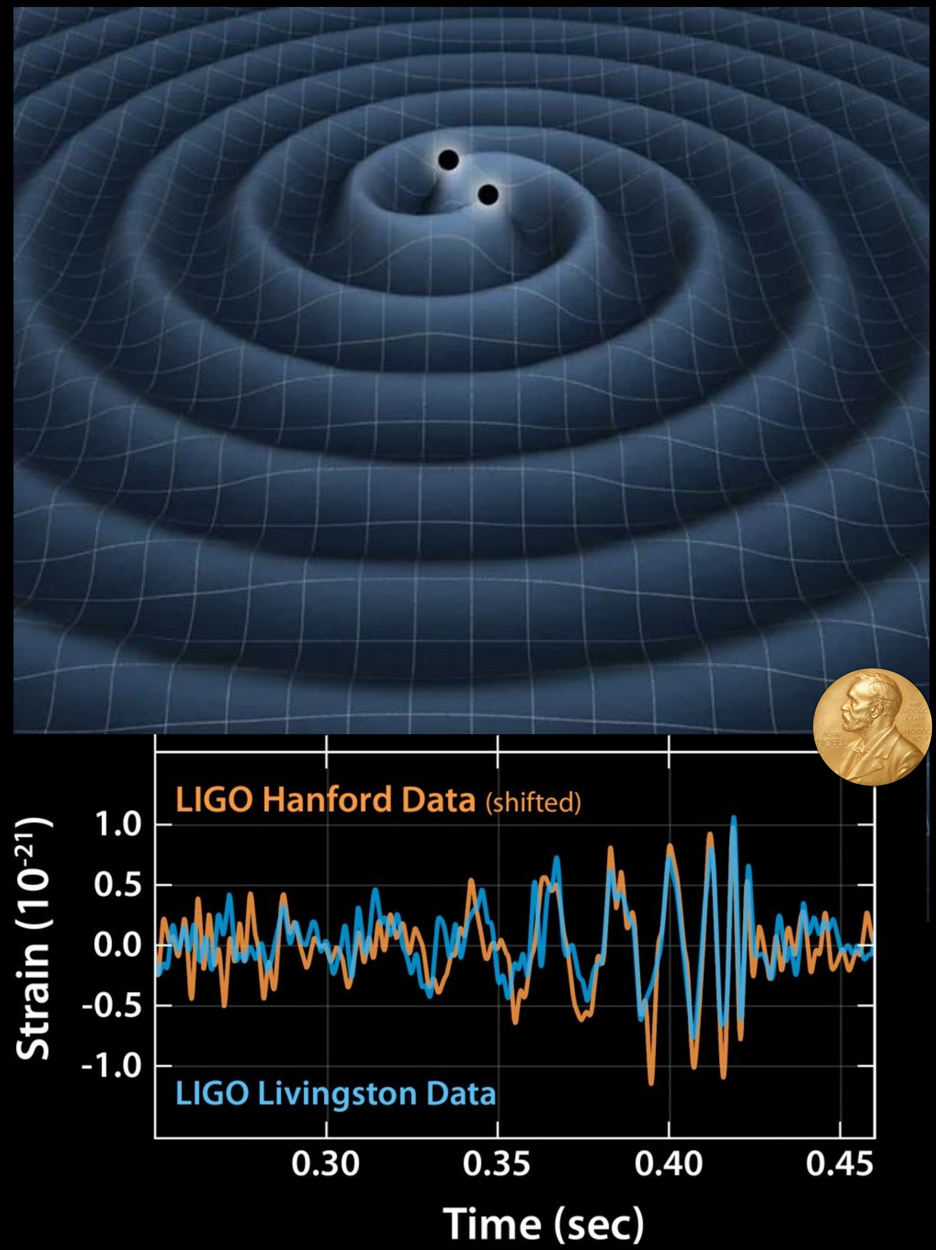
Discussion: Differences with EM radiation?

Activities:

Thinking (alone)

Discussing (2-3 people)

Sharing (all)



Seeing and Hearing the Universe

Wave-like excitations of fundamental fields, travel at the speed of light, deflected by gravitational fields…

Electromagnetic waves

- Substantial interaction
- Microscopic emission
- Incoherent (typically)
- Small wavelength (<30 m)
- Great angular resolution
- Narrow field of view
- Sources poorly modelled

Gravitational waves:

- Very weak interaction
- Macroscopic emission
- Phase coherent
- Long wavelength (>1000 km)
- Poor angular resolution
- All-sky field of view (almost)
- ∃ Ab-initio emission models

Resources

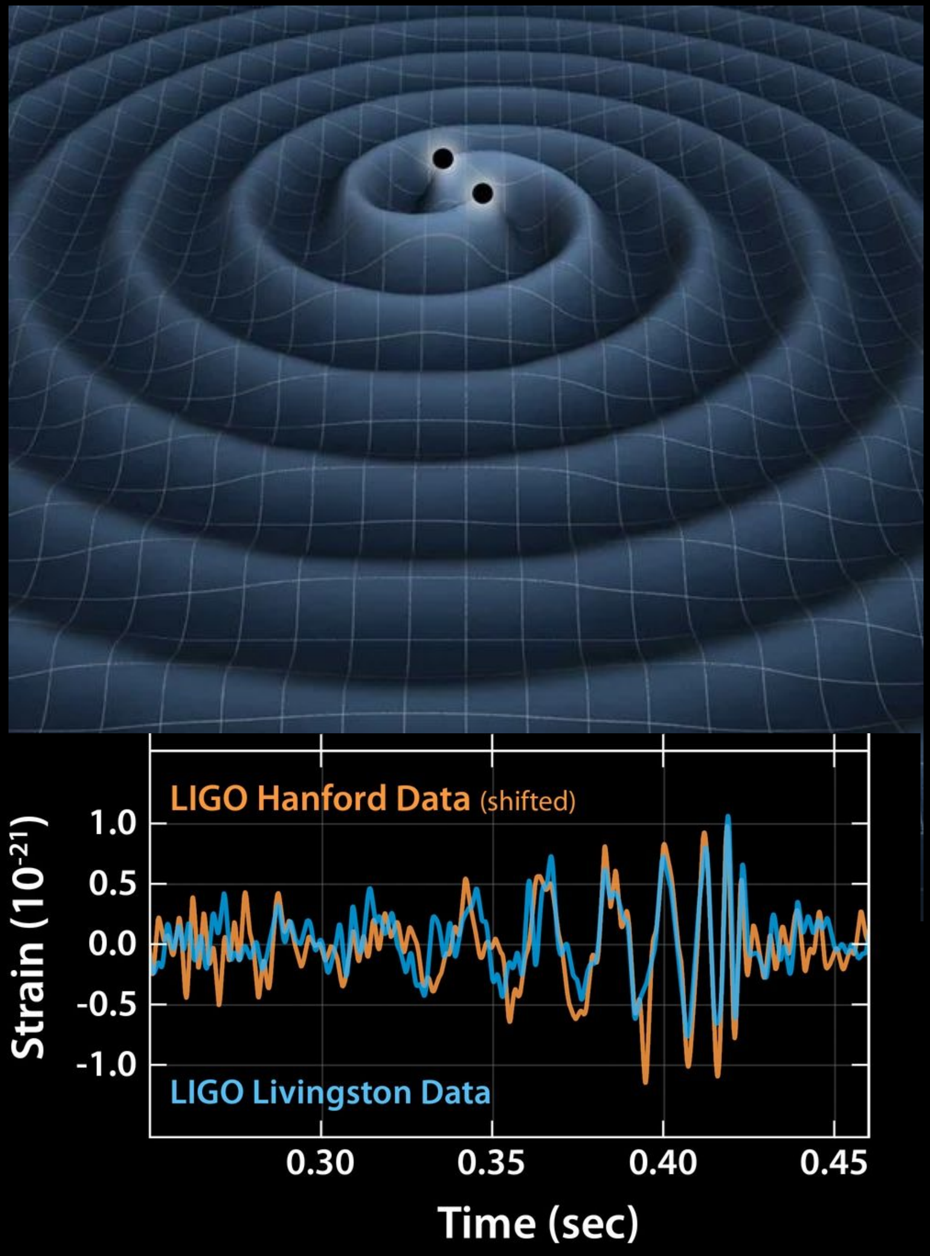
- "The basics of gravitational wave theory" Flanagan & Hughes 2005
  https://arxiv.org/abs/gr-qc/0501041 **Hereafter FH**
- "The basic physics of the binary black hole merger GW150914"
  LIGO-Virgo https://arxiv.org/abs/1608.01940
- "Gravitational waves. Volume 1: theory and experiments" M. Maggiore
  Oxford University Press 2007
- GW Open Science Center (tutorials & workshops) https://gwosc.org/
- PyCBC tutorials https://github.com/gwastro/PyCBC-Tutorials

Ask me about:

- Gravitational Waves
- Gravity Theories & Tests
- Cosmology
- Dark energy & Dark Matter
- Gravitational lensing
- (anything, really)

Part I: Theory

1. Linearized General Relativity
2. Quadrupole emission
3. Inspiral, merger and ringdown
4. Orbital effects and waveforms



Linearized General Relativity

Summation

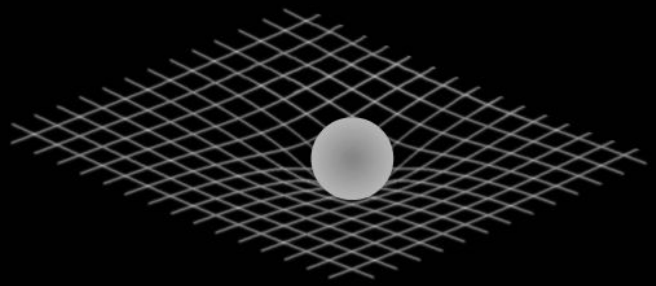
$$u^a v_a \equiv \sum_a u^a v_a.$$

Minkoswki metric (flat spacetime)

$$\eta_{ab} = \mathrm{diag}(-1, 1, 1, 1)$$

Weak field expansion (on Minkoski)

$$g_{ab} = \eta_{ab} + h_{ab}, \qquad \|h_{ab}\| \ll 1.$$



Trace-reversed metric

$$\bar{h}_{ab} = h_{ab} - \tfrac{1}{2}\eta_{ab} h$$

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Einstein tensor

$$G_{ab} = \tfrac{1}{2}(\partial_c \partial_b \bar{h}^c{}_a + \partial^c \partial_a \bar{h}_{bc} - \Box \bar{h}_{ab} - \eta_{ab} \partial_c \partial^d \bar{h}^c{}_d)$$

Linearized Wave Equation I

Gauge transformation

$$x^{a\prime} = x^a + \xi^a \qquad h'_{ab} = h_{ab} - 2\partial_{(a}\xi_{b)}$$

$$\bar{h}'_{ab} = h'_{ab} - \tfrac{1}{2}\eta_{ab}h' = \bar{h}_{ab} - 2\partial_{(b}\xi_{a)} + \eta_{ab}\partial^c\xi_c$$

Lorentz Gauge (transverse)

$$\partial^a \bar{h}_{ab} = 0 \qquad \partial^a \bar{h}'_{ab} = \partial^a \bar{h}_{ab} - \Box \xi_b .$$

Wave equation

$$G_{ab} = -\tfrac{1}{2}\Box\bar{h}_{ab} = 0 \longrightarrow \boxed{\bar{h}_{ab}(\mathbf{x}, t) = \mathrm{Re}\int \mathrm{d}^3k\, A_{ab}(\mathbf{k})\mathrm{e}^{\mathrm{i}(\mathbf{k}\cdot\mathbf{x}-\omega t)}}$$

Linearized Wave Equation II

Wave equation

$$G_{ab} = -\tfrac{1}{2}\Box\bar{h}_{ab} = 0 \longrightarrow \boxed{\bar{h}_{ab}(\mathbf{x}, t) = \mathrm{Re}\int \mathrm{d}^3k A_{ab}(\mathbf{k})\mathrm{e}^{\mathrm{i}(\mathbf{k}\cdot\mathbf{x}-\omega t)}}$$

10x independent component (symmetric)

Lorentz Gauge (Transverse)

$$\boxed{\partial^a \bar{h}_{ab} = 0} \qquad \partial^a \bar{h}'_{ab} = \partial^a \bar{h}_{ab} - \boxed{\Box\xi_b}. \longrightarrow$$

Residual gauge **FH p9**

4x constraints

spatial & Traceless

$$h_{tt} = h_{ti} = 0 \qquad h = h_i{}^i = 0.$$

$$h_{ab}^{TT} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & & 0 \end{pmatrix} e^{ik(z-t)}$$

3 + 1 new constraints

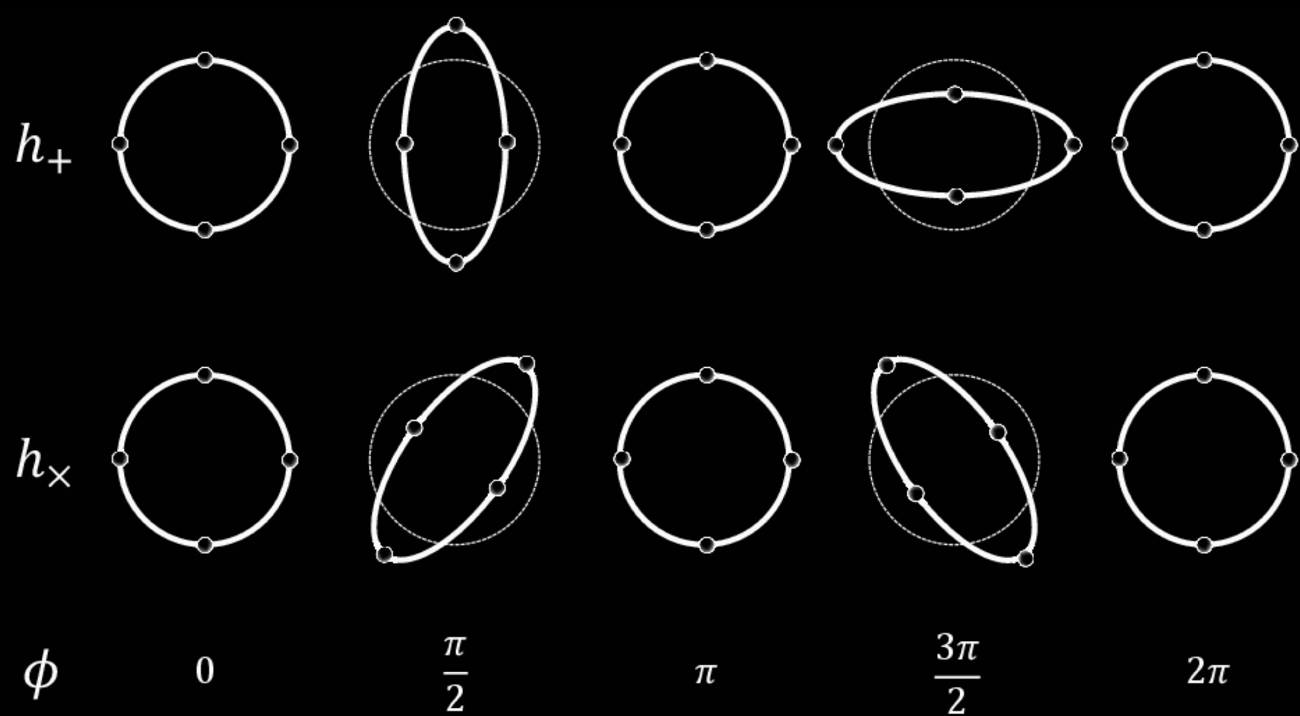
(only in Minkoswki)

Linearized Wave Equation III

Wave equation

$$G_{ab} = -\frac{1}{2}\Box\bar{h}_{ab} = 0 \longrightarrow h^{TT}_{ab} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & & 0 \end{pmatrix} e^{ik(z-t)}$$

2x degrees of freedom



Emission of Gravitational Waves I

Wave equation

$$\Box\bar{h}_{ab} = -16\pi T_{ab}$$

$$h_{ab}^{TT} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & & 0 \end{pmatrix} e^{ik(z-t)}$$

Green's function, $h_{a0} = 0$, multipole expansion ($r \gg r_{src}$)

$$\bar{h}_{ij}(t, \mathbf{x}) = \frac{4}{r}\int \mathrm{d}^3x' \, T^{ij}(t-r, \mathbf{x}')$$

<u>Quadrupole formula</u>

$$\boxed{\bar{h}_{ij}(t, \mathbf{x}) = \frac{2}{r}\frac{\mathrm{d}^2 I_{ij}(t-r)}{\mathrm{d}t^2}}$$

define inertia tensor

$$I_{ij}(t) = \int \mathrm{d}^3x' \, \rho(t, \mathbf{x}')x'^i x'^j$$

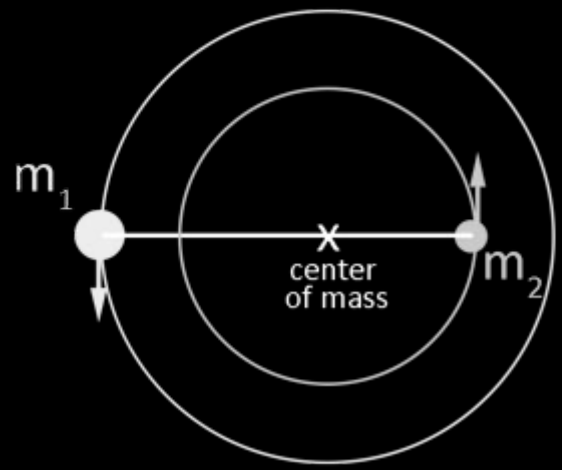
+ transverse-traceless projection (FH 4.23)

Emission by binary system

Moment of inertia

$$\mathcal{I}_{ij} = \mu(x_i x_j - \tfrac{1}{3}R^2\delta_{ij})$$

$$\mu = m_1 m_2/(m_1+m_2)$$



Amplitude

$$h_{ij}^{\mathrm{TT}} = \frac{2\ddot{\mathcal{I}}_{ij}}{r} \propto \Omega^2 \mu R^2 \begin{bmatrix} \cos 2\Omega t & \sin 2\Omega t & 0 \\ -\sin 2\Omega t & -\cos 2\Omega t & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Kepler's law (G=1)

$$R^3\Omega^2 = M$$

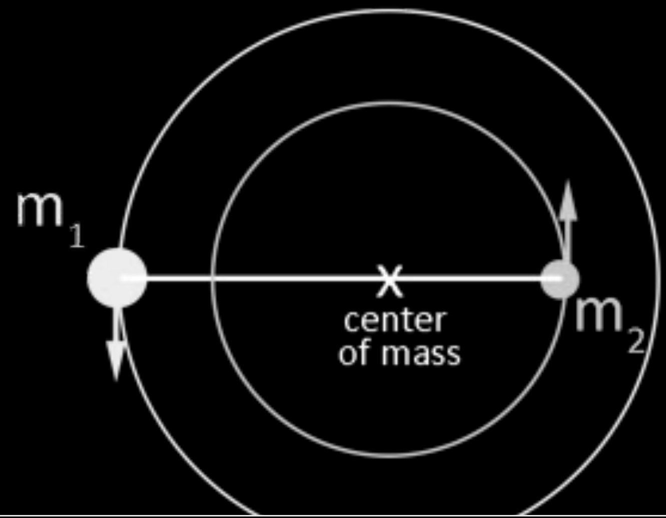
$$h = \frac{4\mu\Omega^2 R^2}{r} \simeq 10^{-22}\left(\frac{M}{2.8M_\odot}\right)^{5/3}\left(\frac{0.01\ \mathrm{second}}{P}\right)^{2/3}\left(\frac{100\mathrm{megaparsecs}}{r}\right)$$

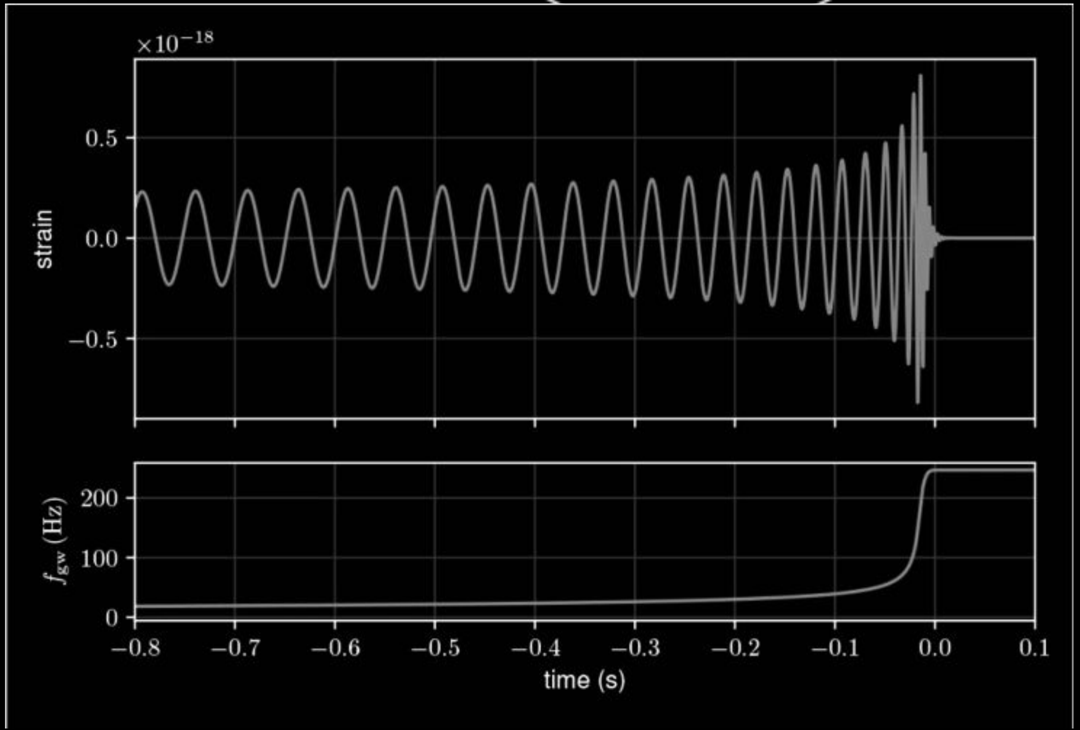
Emission of Gravitational Waves III

GW take energy from binary

=> frequency evolution

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{96}{5}\pi^{8/3}\left(\frac{G\mathcal{M}}{c^3}\right)^{5/3} f^{11/3}$$

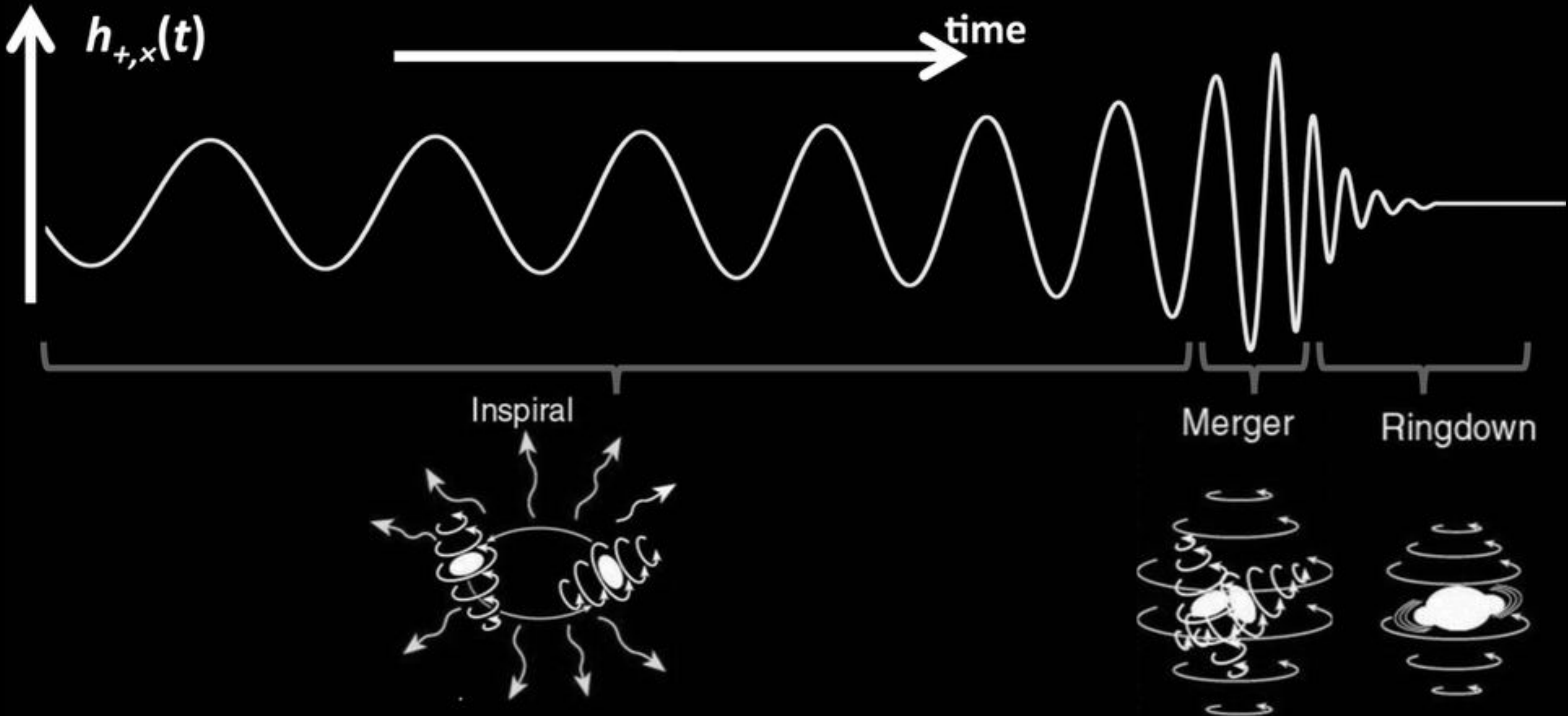




Chirp mass:

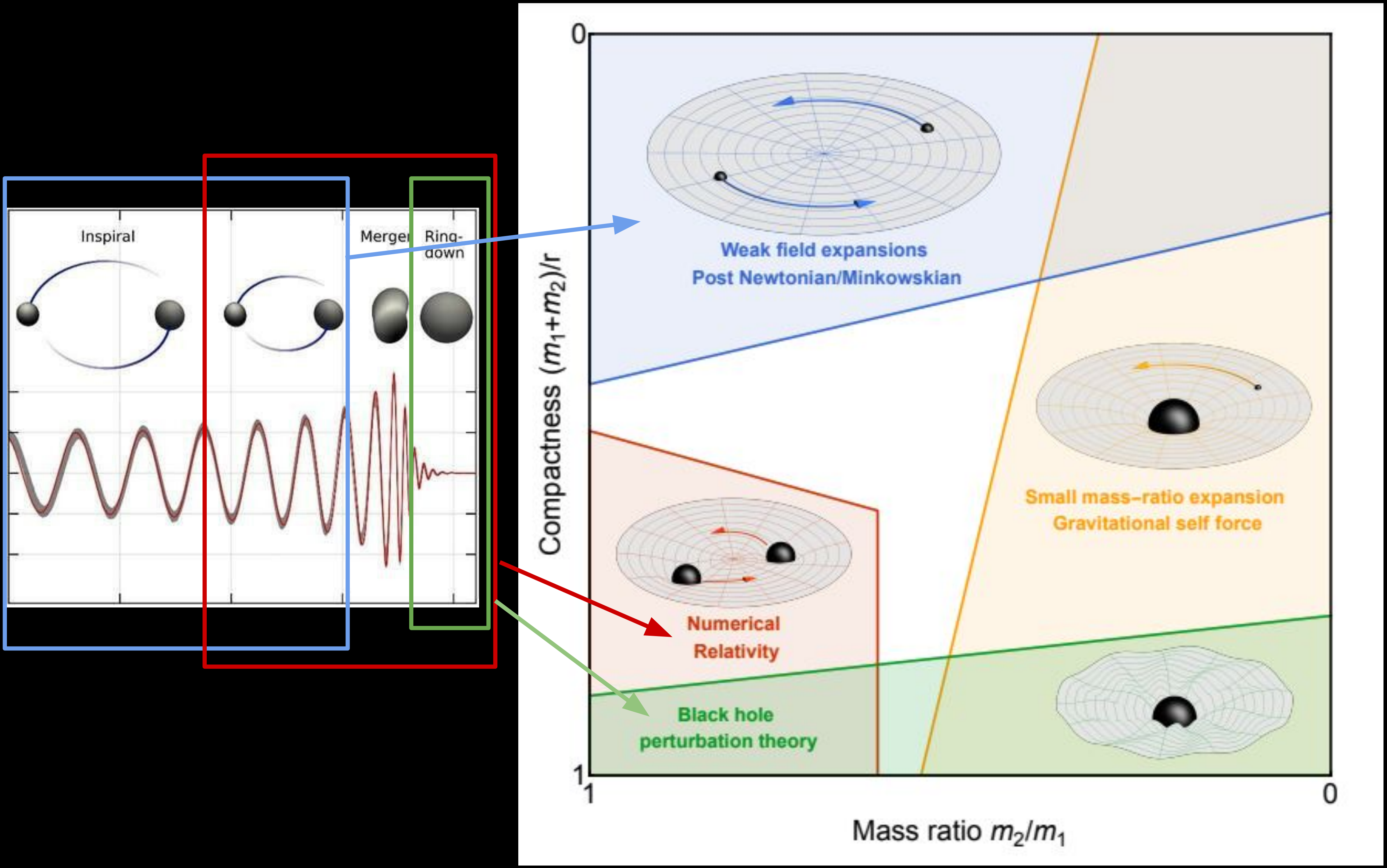
$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

Inspiral, Merger & Ringdown

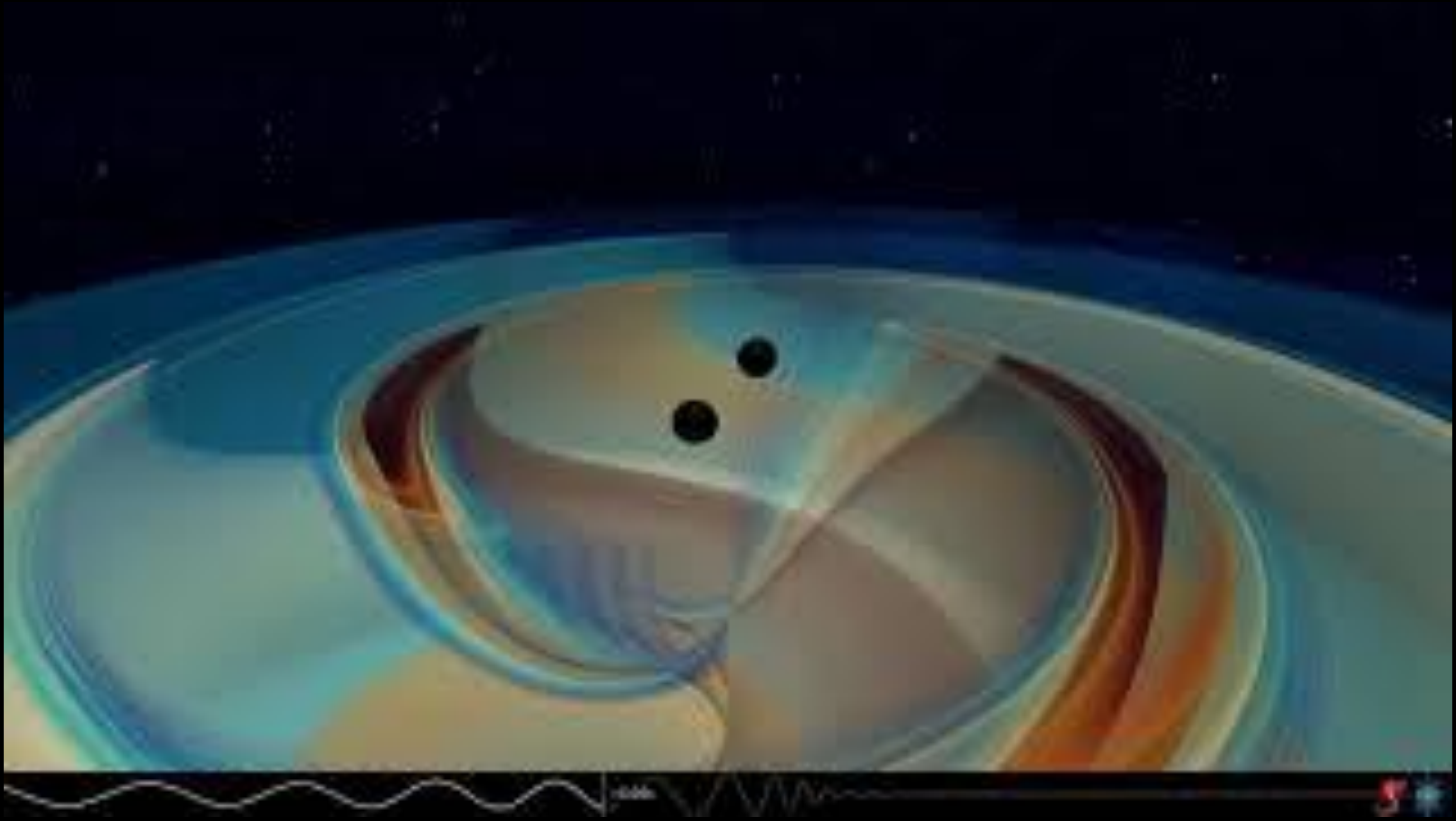


APS, LISA waveform WG 2311.01300

Regimes for GW emission (LISA waveform WG 2311.01300)

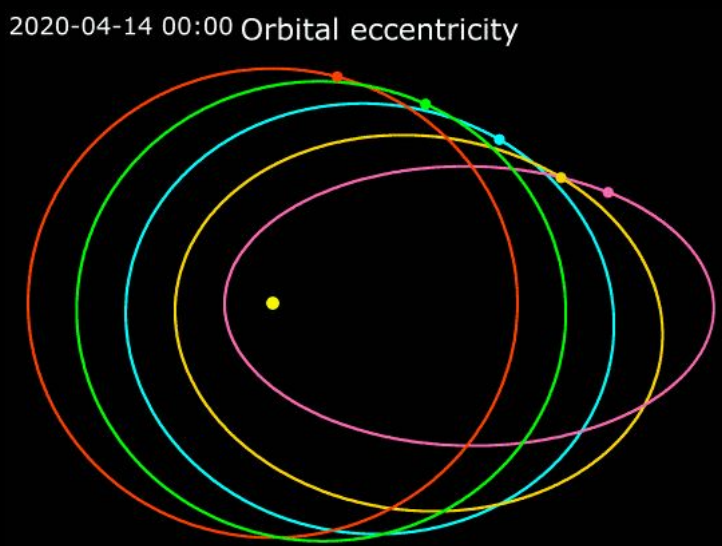


Numerical Relativity 85+66 $M_\odot$ (GW190521)



Credit: AEI https://youtu.be/zRmwtL6lvIM

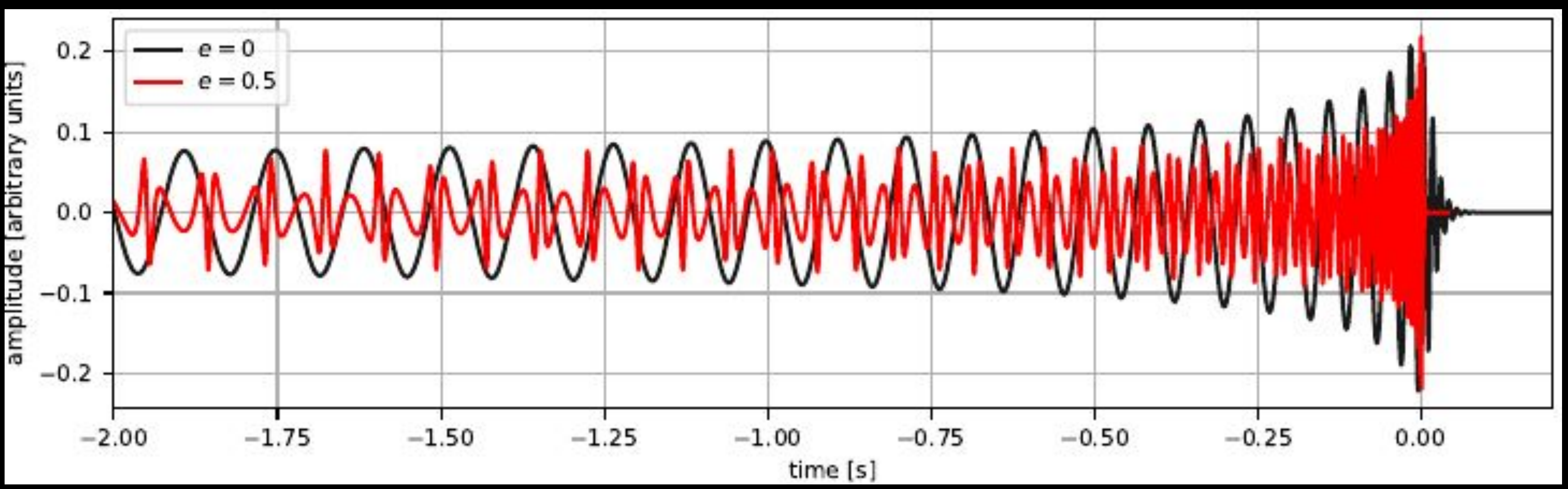
Orbital Effects: Eccentricity



higher harmonics (l,m) -> multiples of frequency

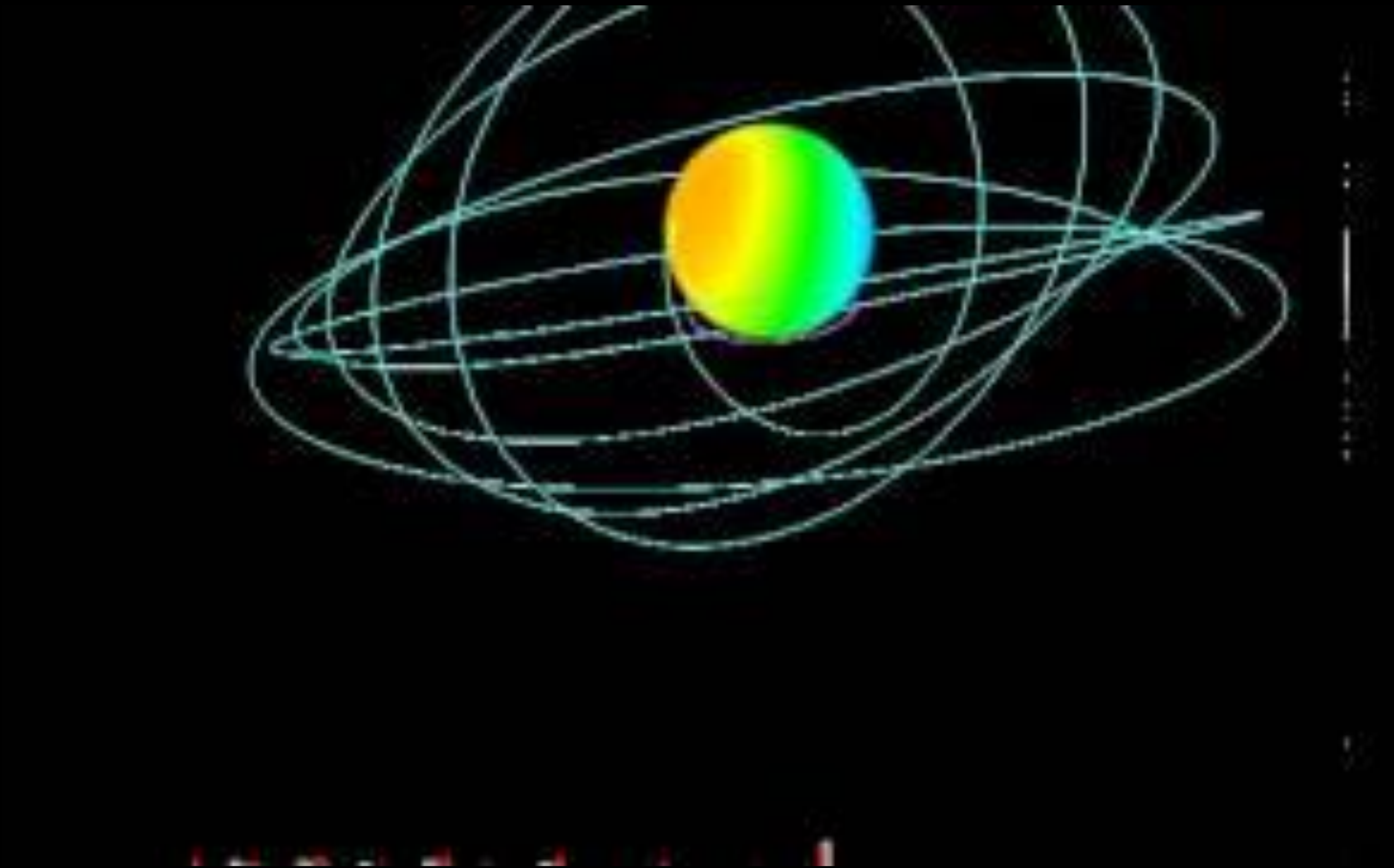
$$h_+(t) - i h_\times(t) = \sum_{\ell,m} {}_{-2}Y_{\ell m}(\iota,\, \phi_0) h_{\ell m}(t;$$

spin-weighted spheroidal harmonics



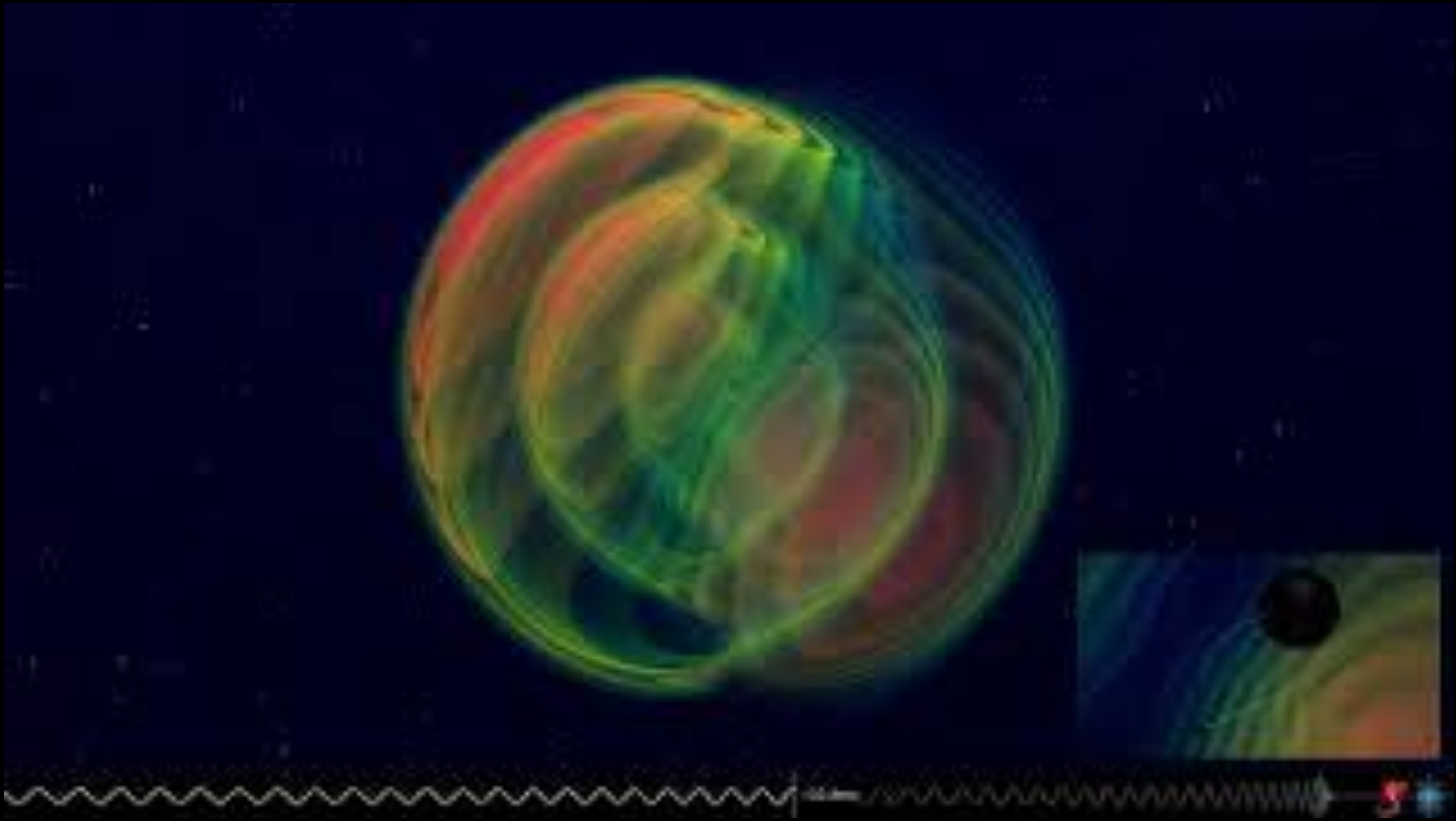
Wiki, LIGO

Orbital Effects: Spin Precession



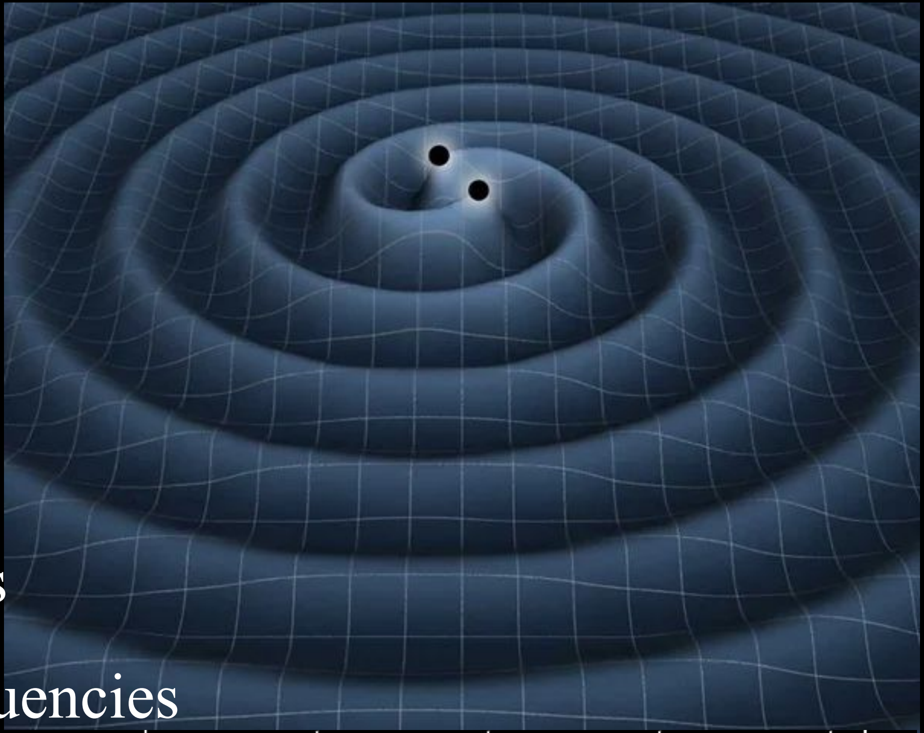
Credit: SXS https://youtu.be/grA5KfDlsAY?si=sMrcyqn1YntmkU-j

Mass ratio & precession 30+8 $M_\odot$ (GW190412)

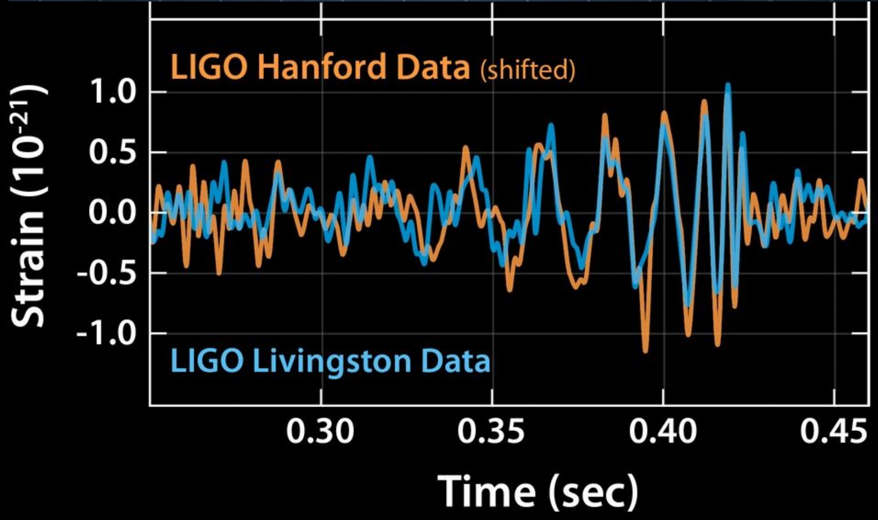


Credit: AEI & SXS https://youtu.be/5AkT4bPk-00?si=gAlkXSlW1mbN4fCW

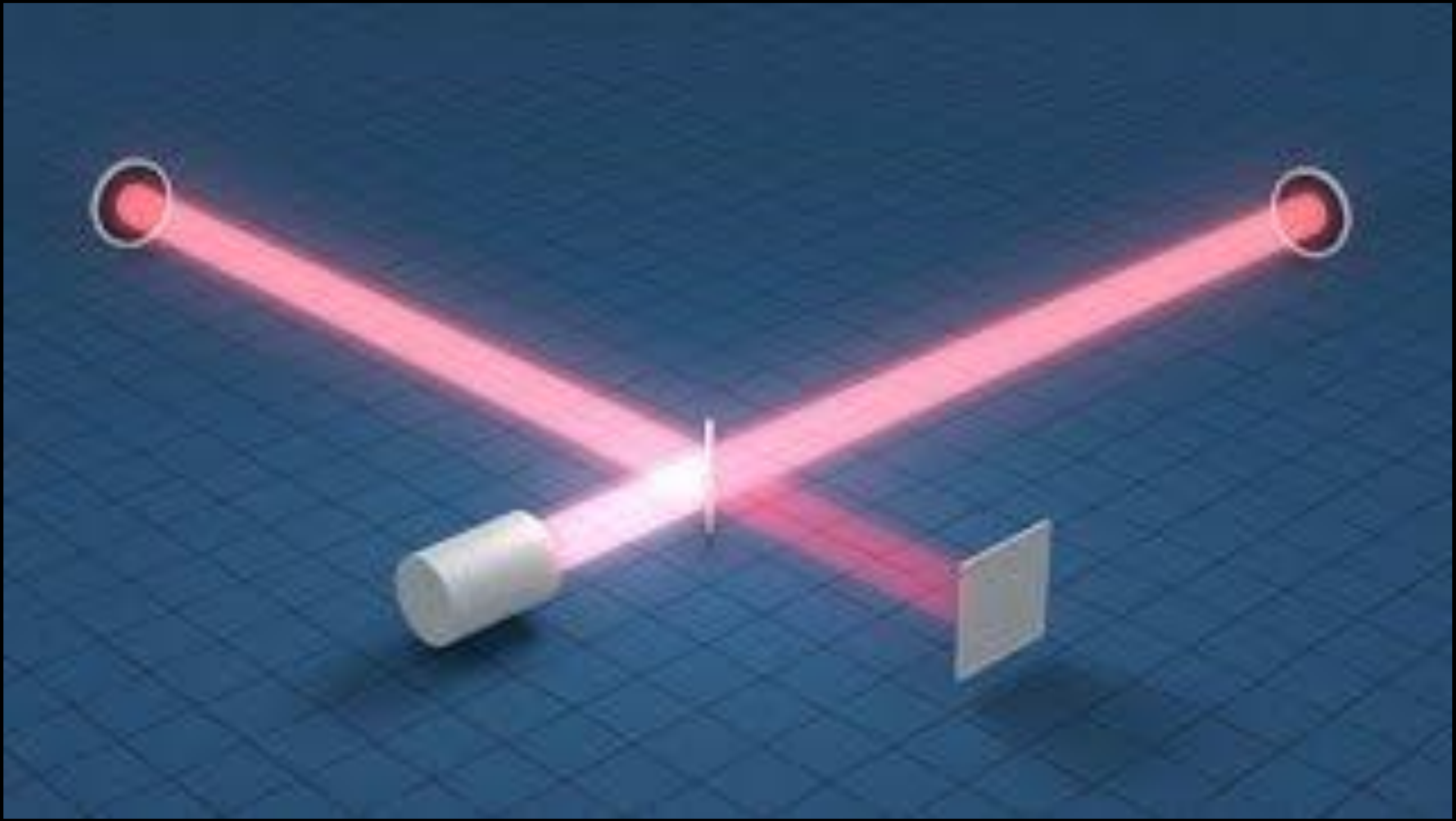
Part II: Detectors & Sources



1. Interferometers & Detection
2. Antenna pattern & sky localization
3. Signal-to-noise ratio & Searches
4. Sources for Ground Based detectors
5. Detectors and sources at lower frequencies



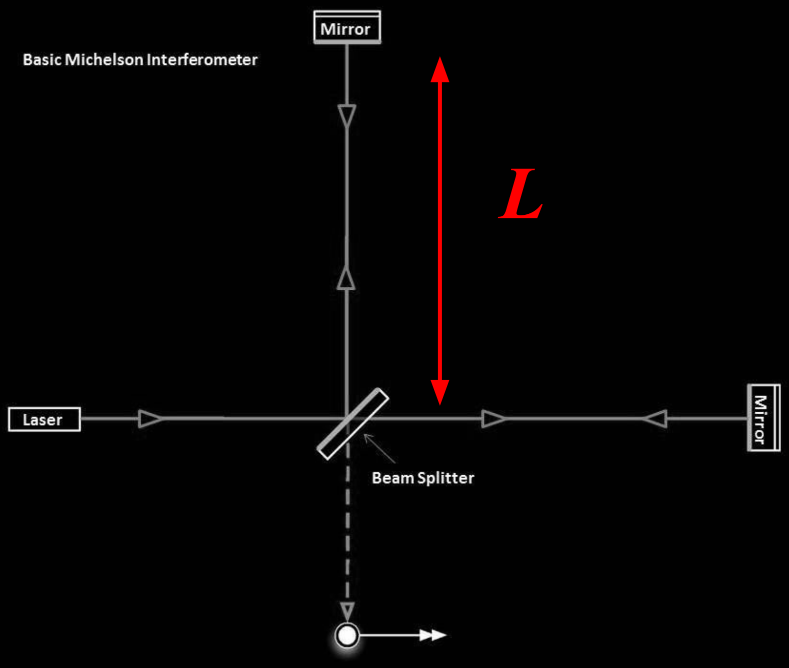
Interferometers



Credit LIGO https://www.youtube.com/watch?v=tQ_teIUb3tE

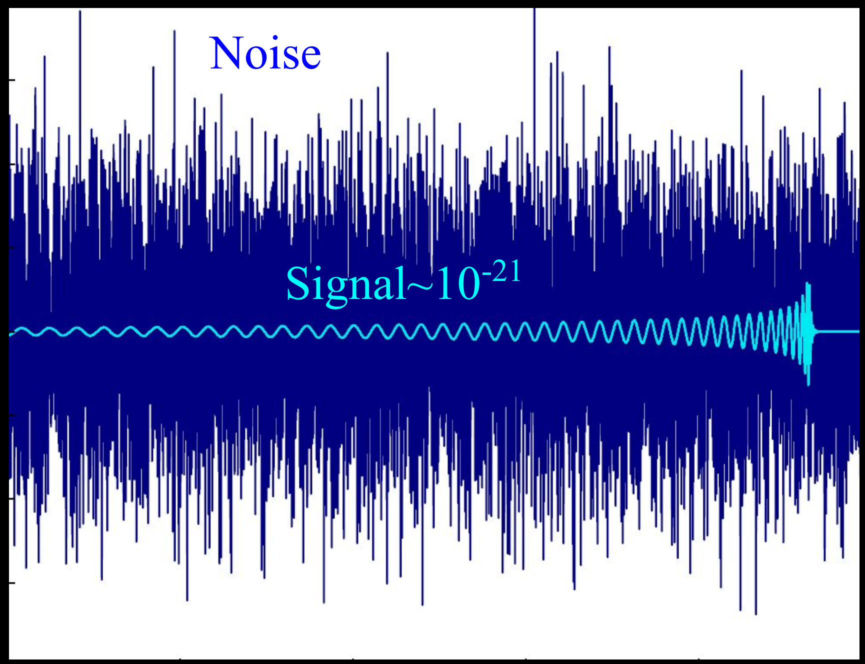
GW Detection principle

$$h_{ab} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & & 0 \end{pmatrix} e^{ik(z-t)}$$



Strain ↔ Change in length **(FH eq 3.12)**

$$\delta L^i(t) = \frac{1}{2} h_{ij}^{\rm TT}(t) L_0^j.$$



∃ corrections L/λ **(FH p16)**

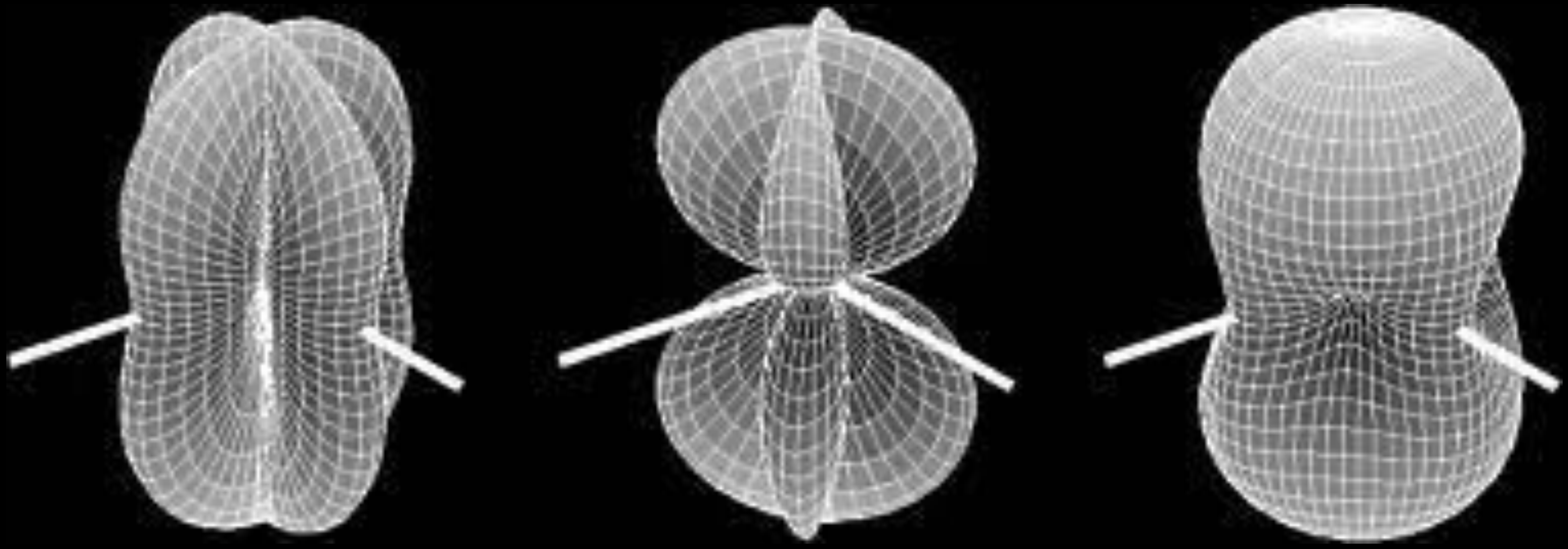
(important for space-detectors)

Antenna Pattern

Angular-dependence of detector response

$$h(t) = F^{+}(\theta,\phi,\psi)A_{+}\cos(2\pi f t) + F^{\times}(\theta,\phi,\psi)A_{\times}\cos(2\pi f t + \Delta\phi)$$

$+$ $\quad\times\quad$ unpolarized

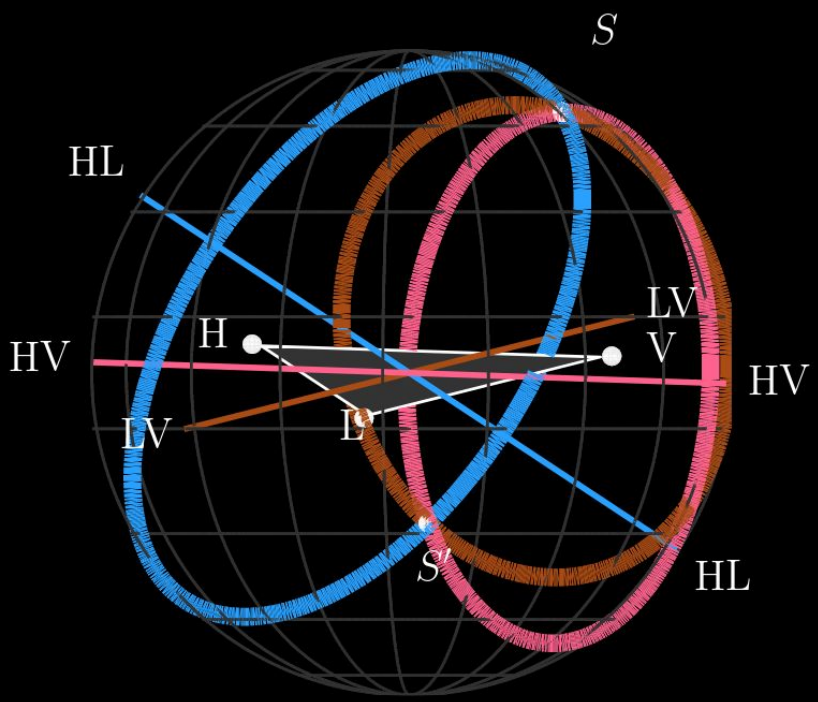


Sky Localization ~ 100 deg^2

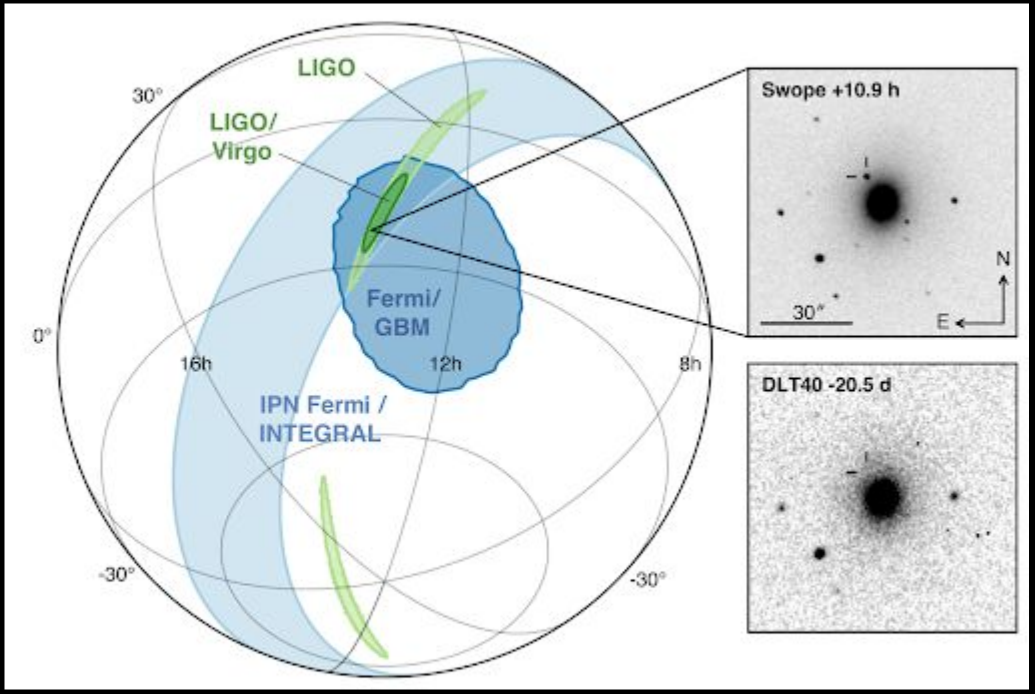
Angular-dependence of detector response

$$h(t) = F^{+}(\theta, \phi, \psi) A_{+} \cos(2\pi f t) + F^{\times}(\theta, \phi, \psi) A_{\times} \cos(2\pi f t + \Delta\phi)$$

Time delay between detectors



search for electromagnetic counterpart



LIGO-Virgo

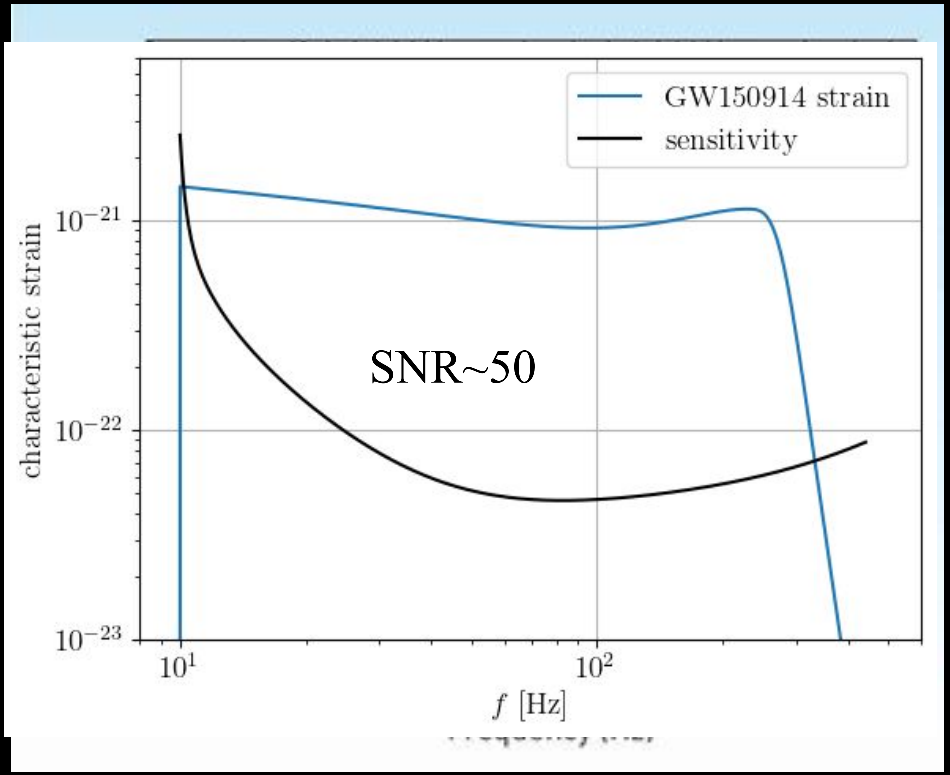
Signal vs Noise

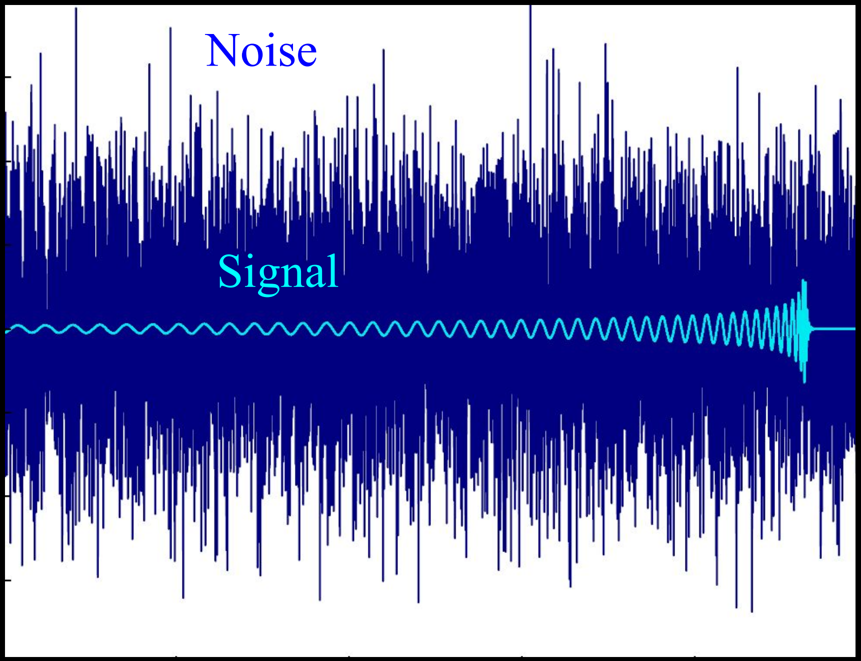
Gravitational-wave sensitivity curves 1408.0740
A guide to LVK detector noise & extraction of transient GW signals 1908.11170

Strain = signal+noise

$$h(t) = F^{+}(\theta, \phi, \psi) A_{+} \cos(2\pi f t) + \cdots$$

(~gaussian)

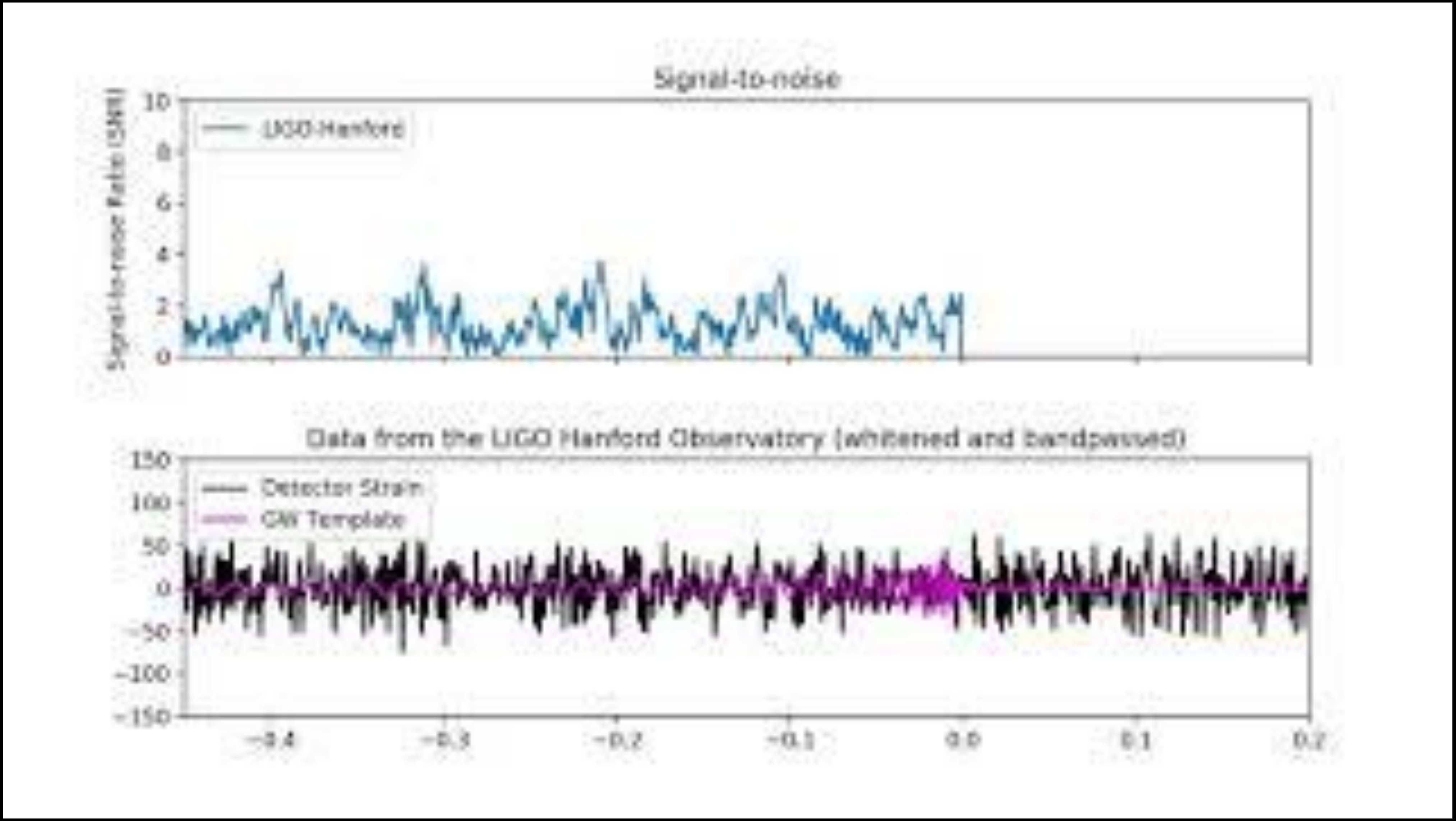




Signal-to-noise ratio

$$\mathrm{SNR}^2 \equiv 4 \int \frac{df}{S_n(f)} |\tilde{h}(f)^2|$$

Searches: Matched filtering



Credit: Alex Nitz https://youtu.be/bBBDR5jf9oU

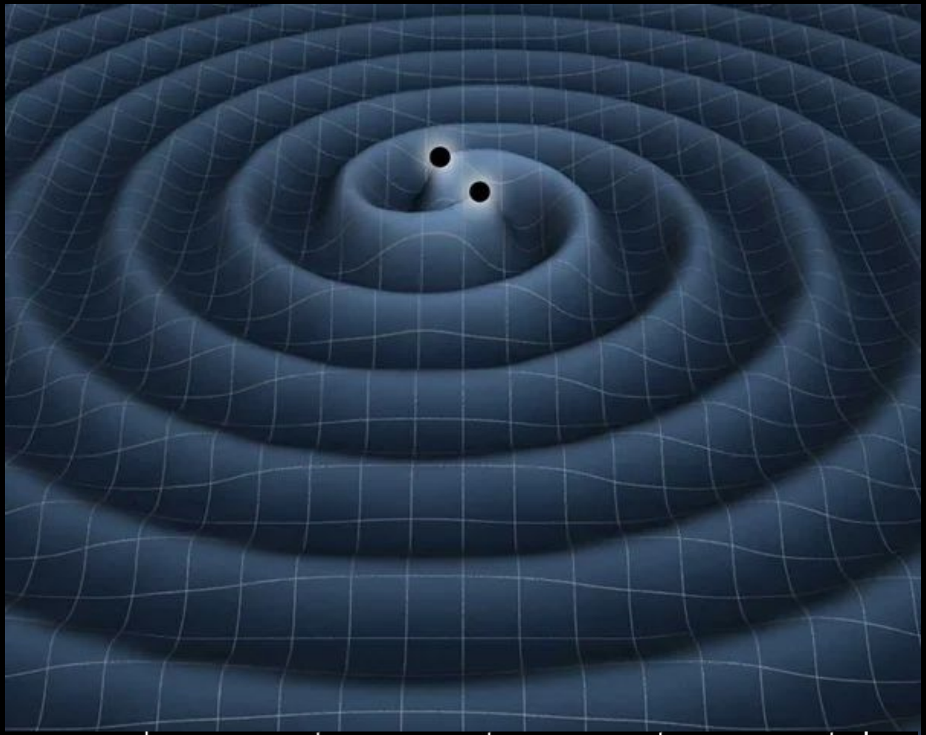
Discussion: What sources can we detect?

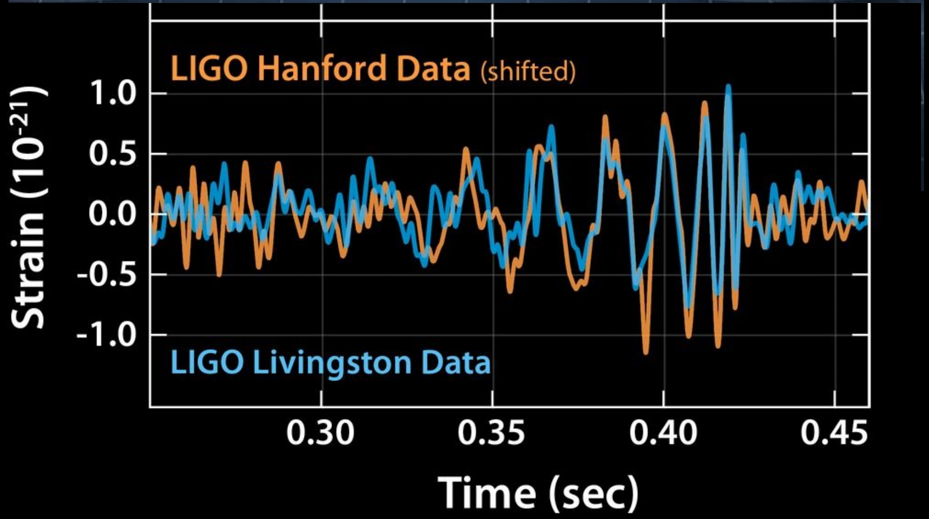
<u>Activity:</u>

Thinking (alone)

Discussing (2-3 people)

Sharing (all)

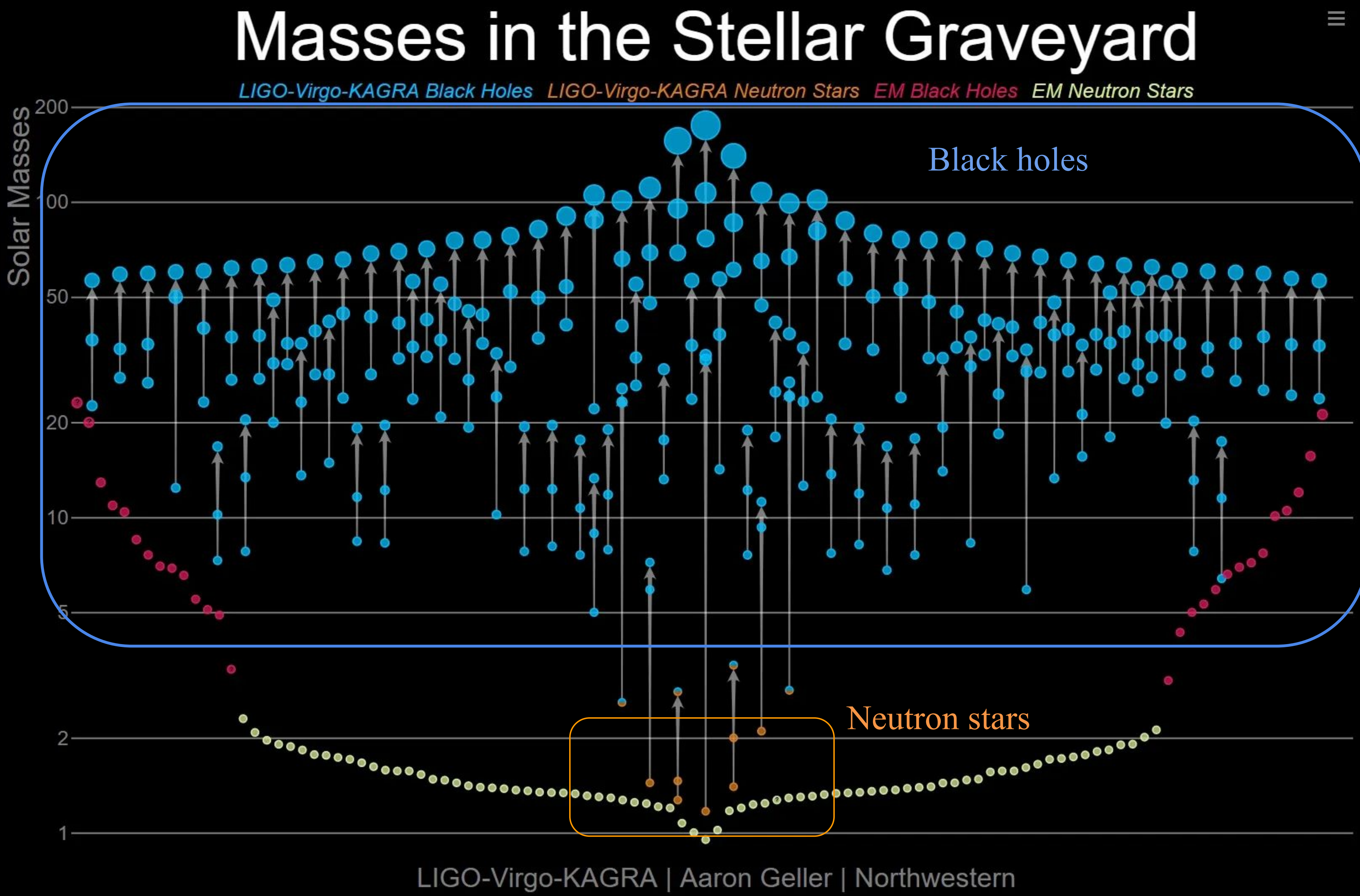




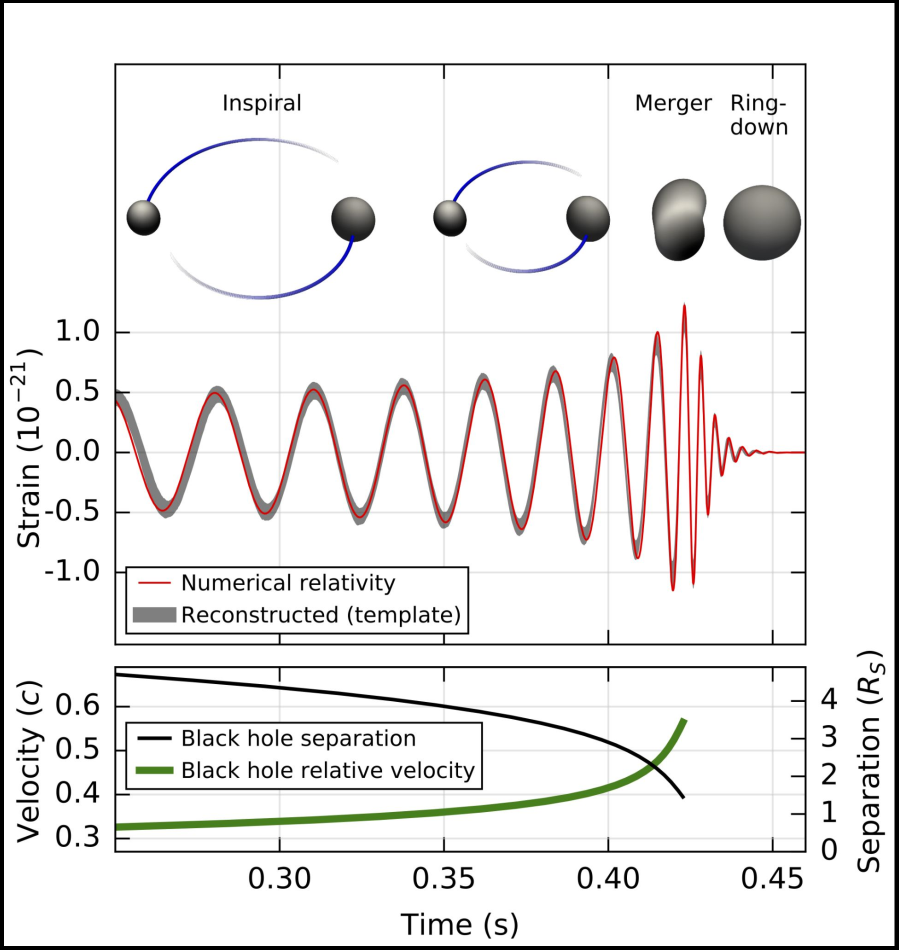
GW sources

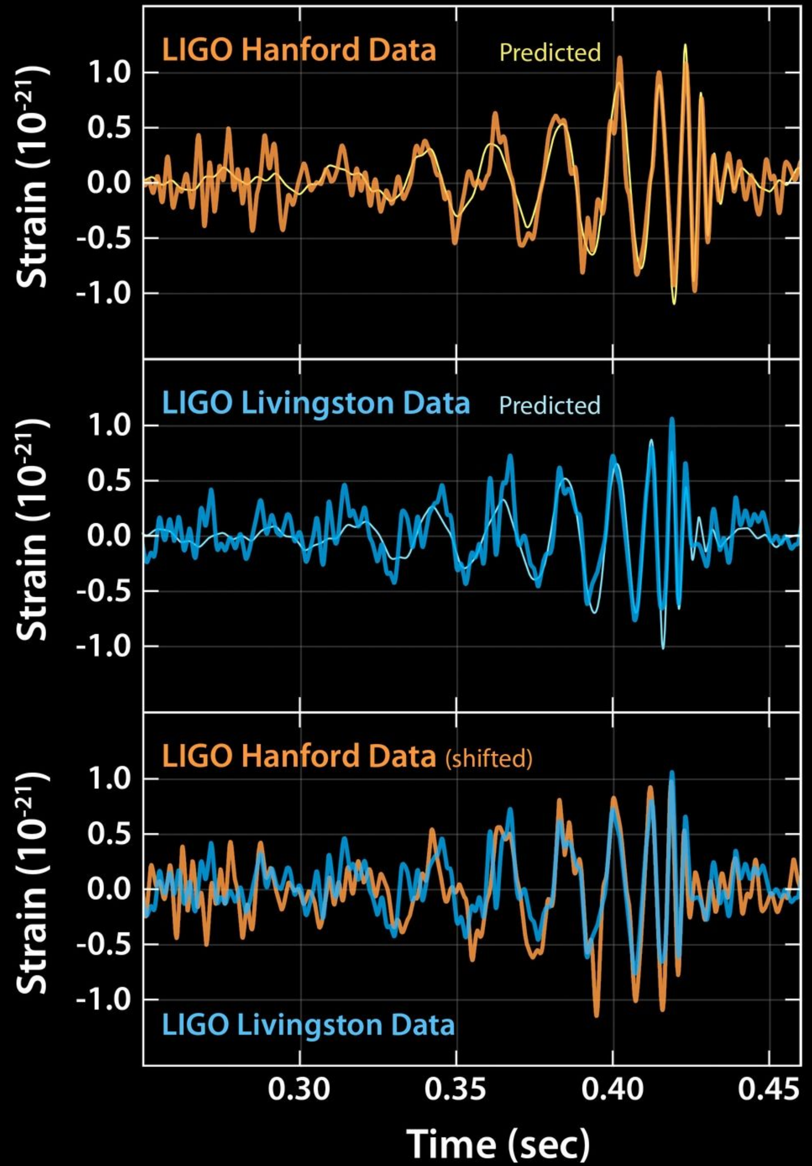
	Transient	Persistent
Modelled	Binary Mergers ~ 90 events many to come	Continuous Waves (e.g ms Pulsars)
Unmodeled	Bursts (e.g. Supernovae)	Stochastic evidence @ nHz (unresolved sources, early universe)

LIGO-Virgo-KAGRA mergers

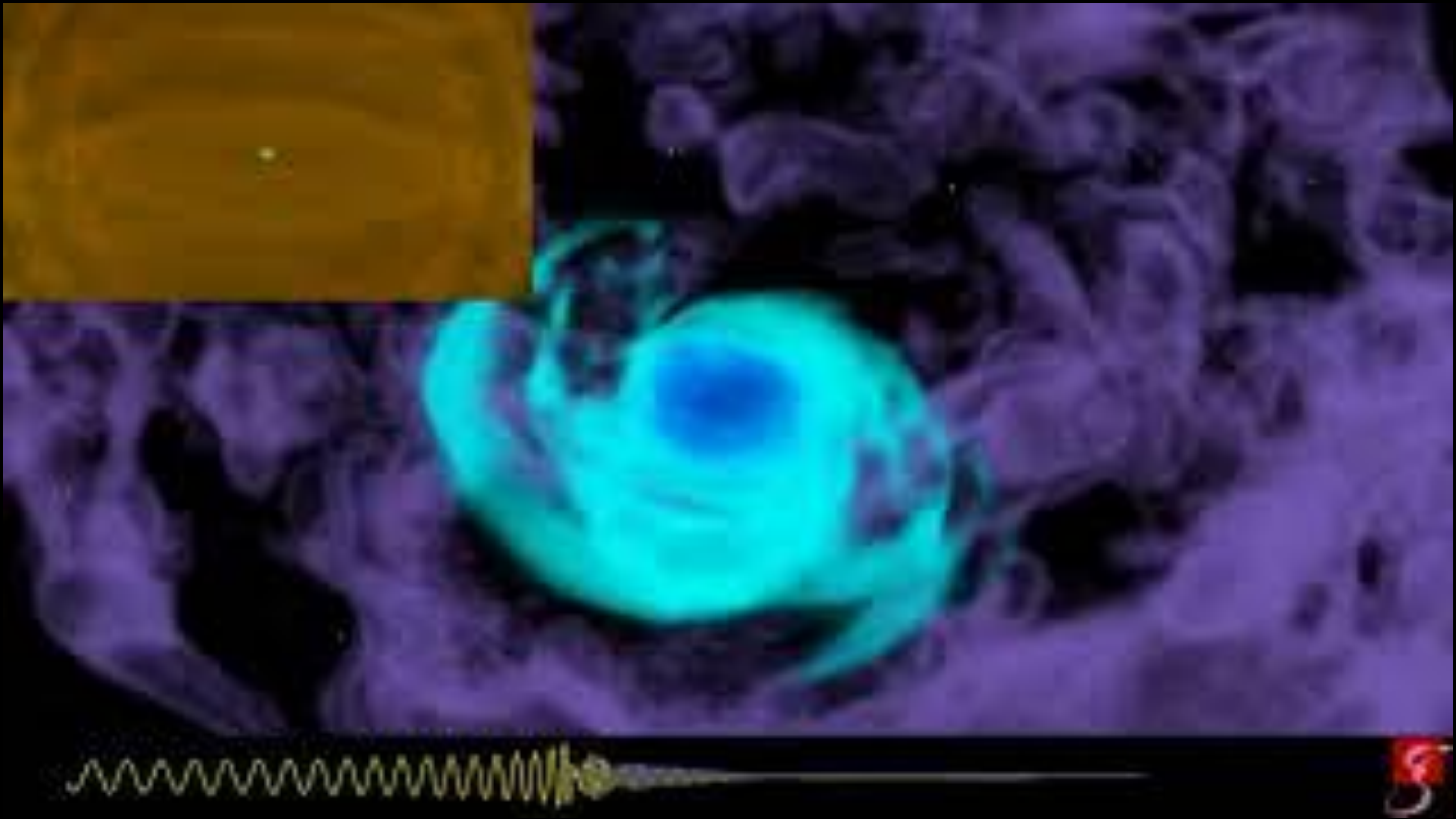


GW150914 - The first detection



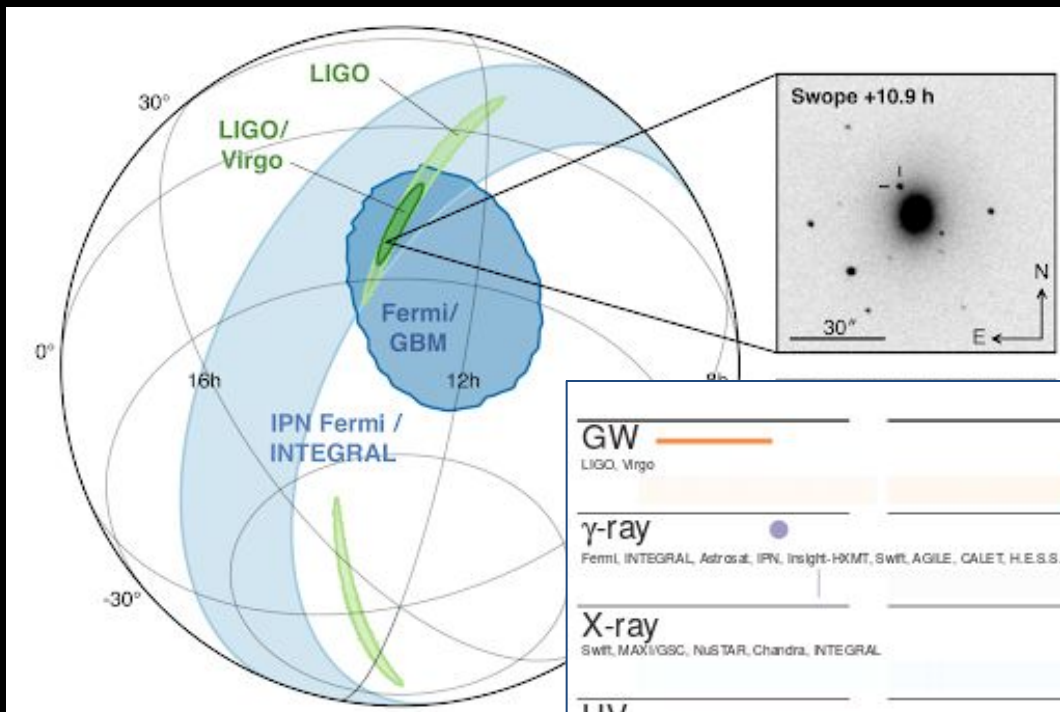


GW170817 - First neutron star + electromagnetic

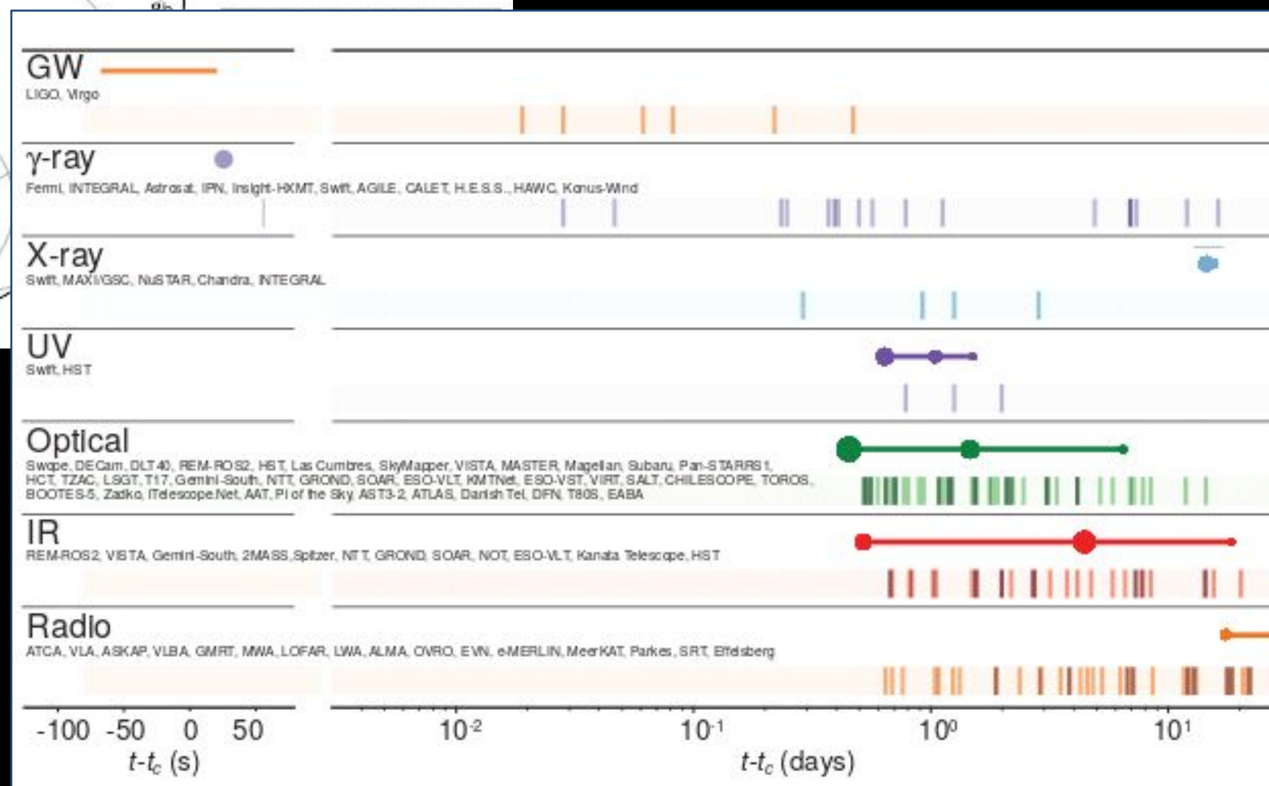


Credit: AEI https://youtu.be/V6cm-0bwJ98?si=wd7zWgMiSX8Kzsnu

GW170817 - First neutron star + electromagnetic

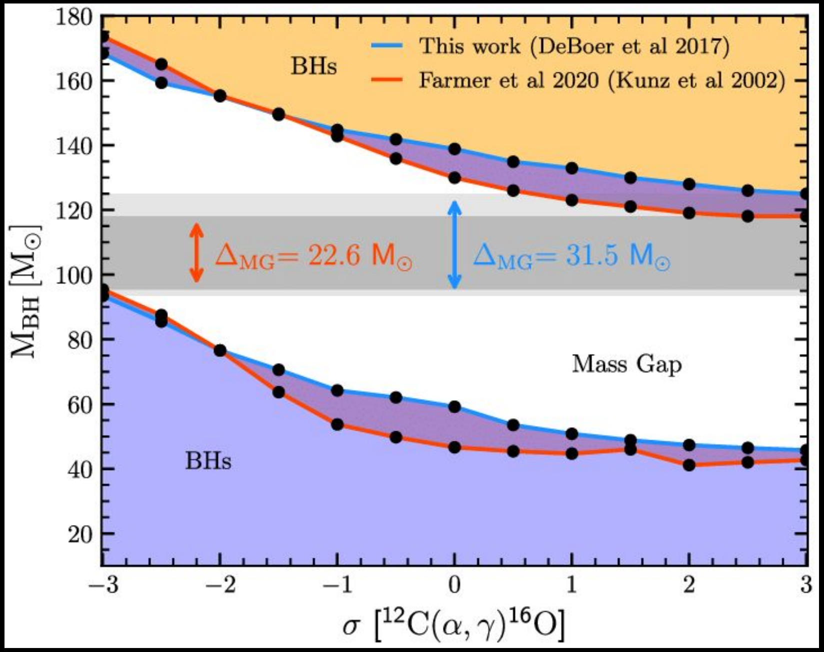


1710.05833

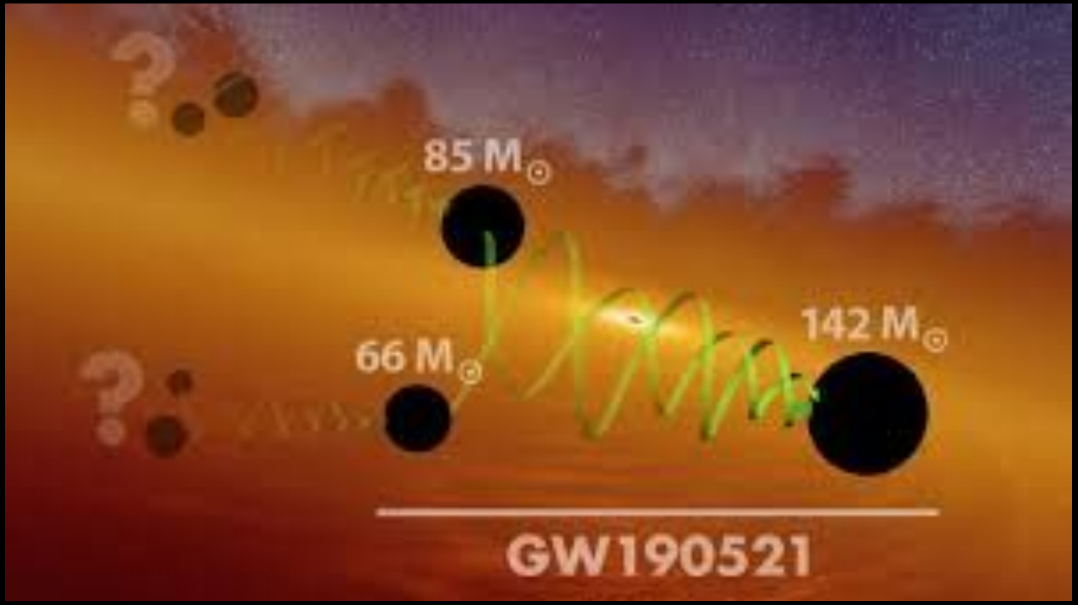


GW190521 - Heaviest binary

Mass gap: parent star unstable because $\gamma \rightarrow e^{-}$-e^{+} production



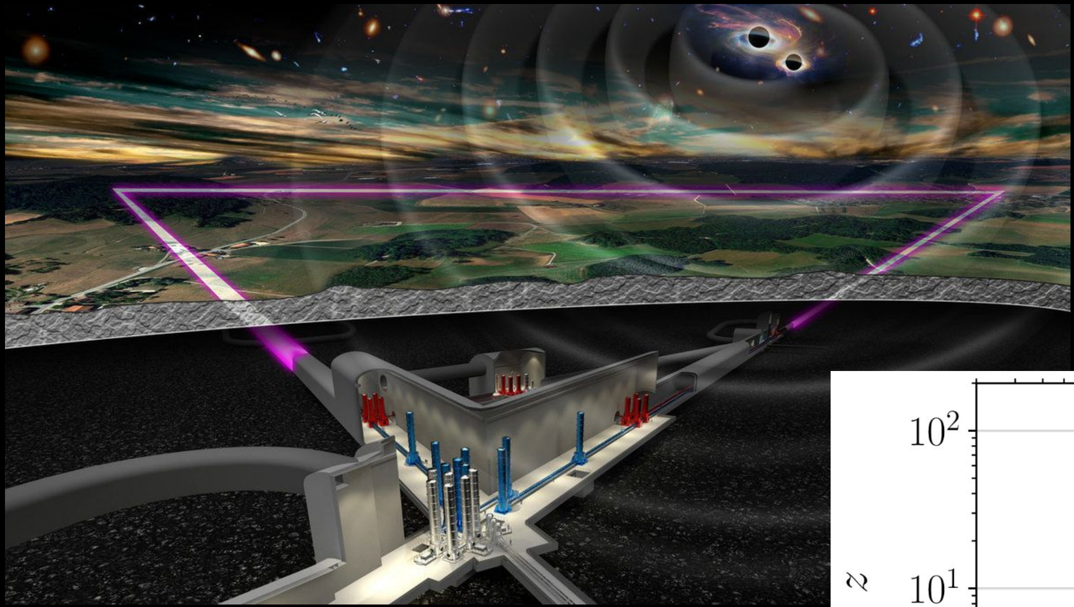
2nd generation black holes?



Credit: 2105.06366 (Kumar-Mehta+), LIGO

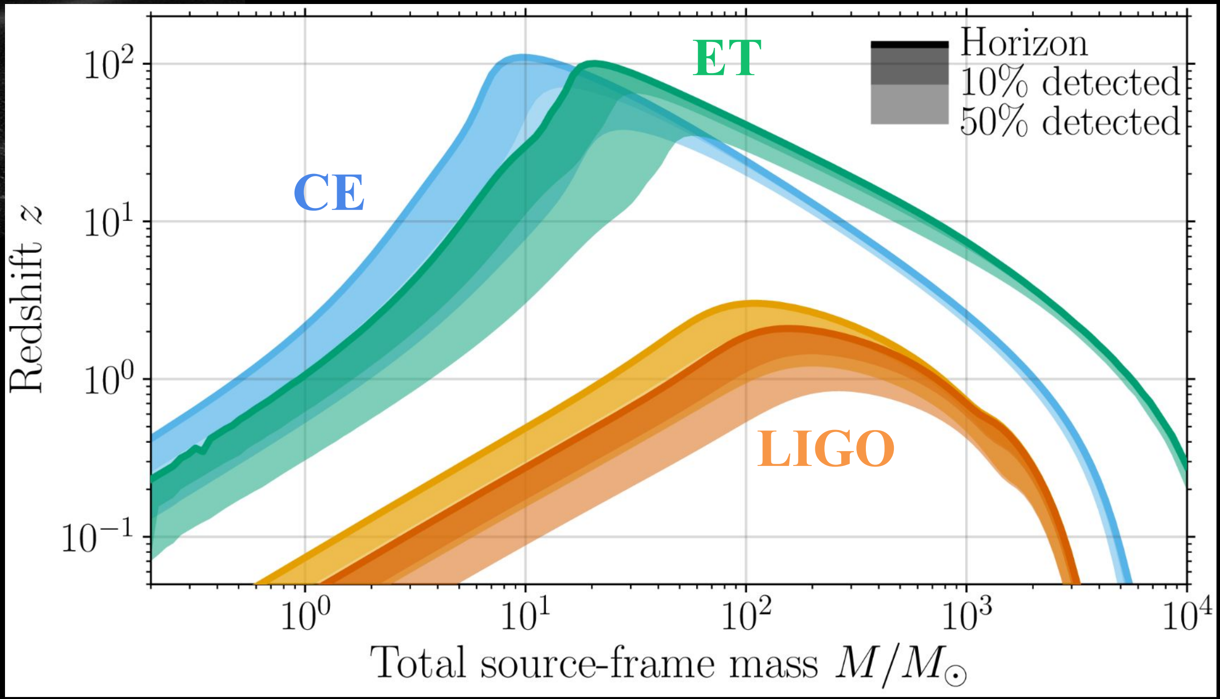
Future ground-based detectors (~2035)

Einstein Telescope ~10km (Europe, TBD)



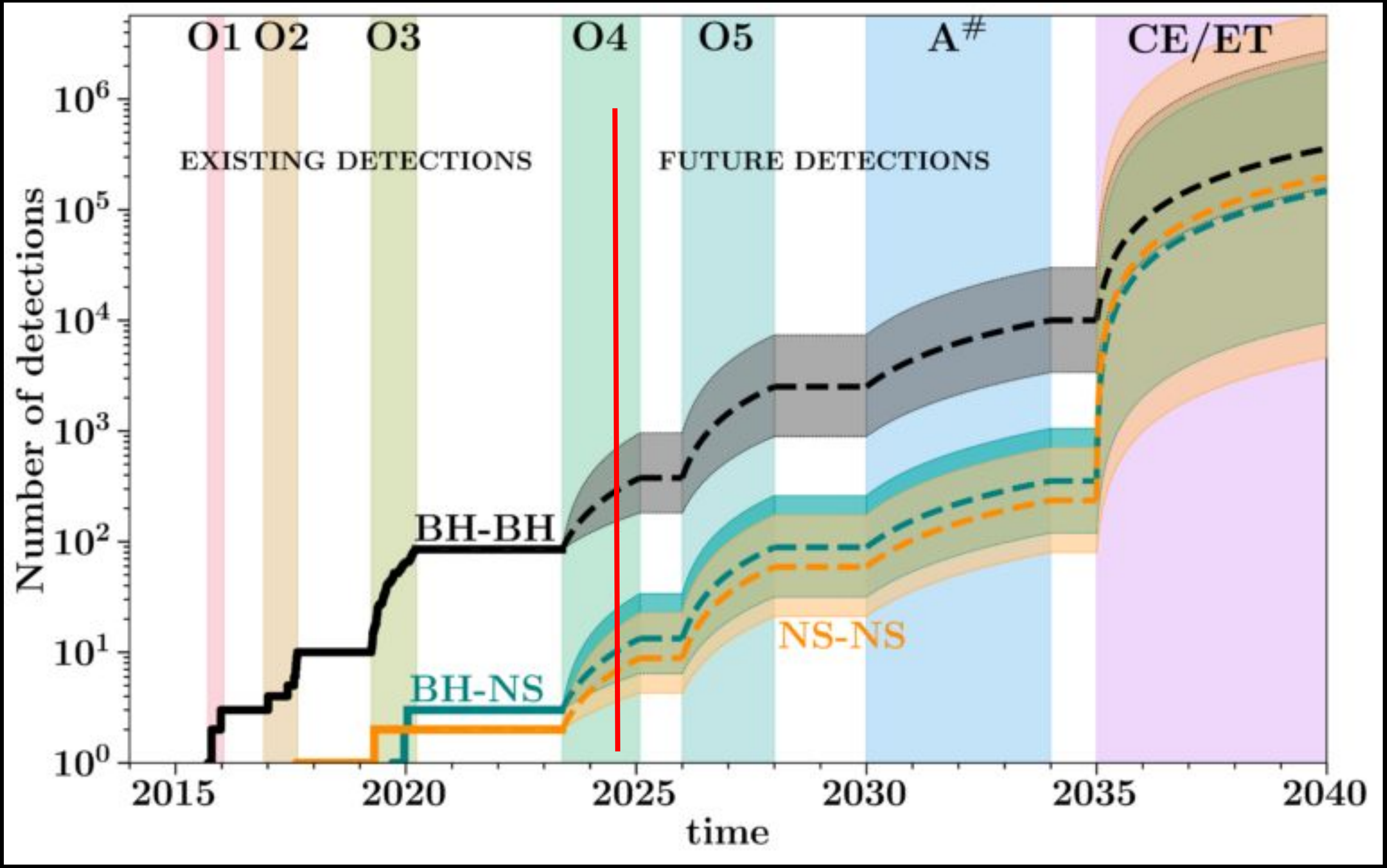
Cosmic Explorer ~40km (US, TBD)

Detect every BH in the universe!

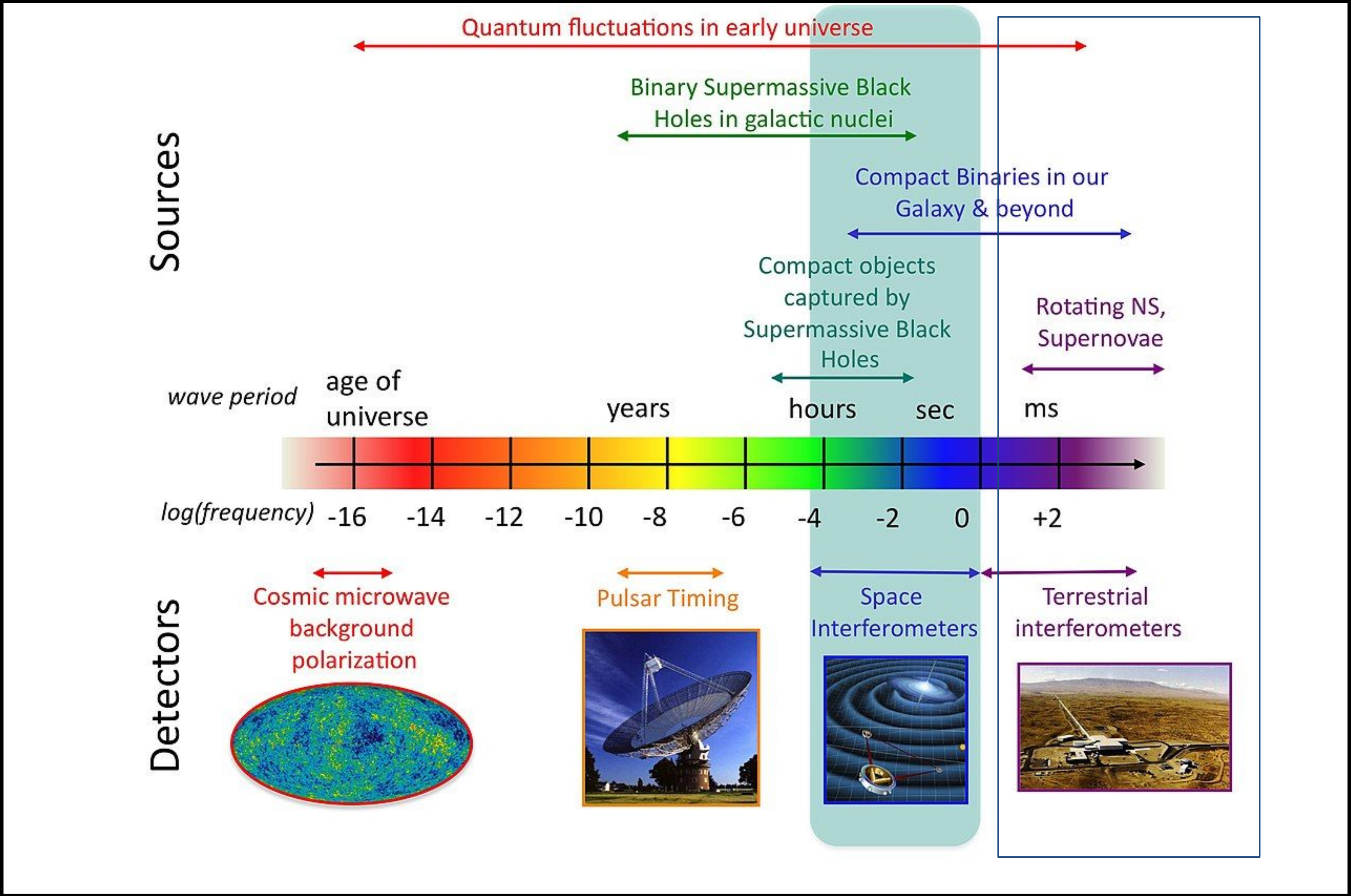


The future for ground detectors

F. Broekgaarden (2303.17628)



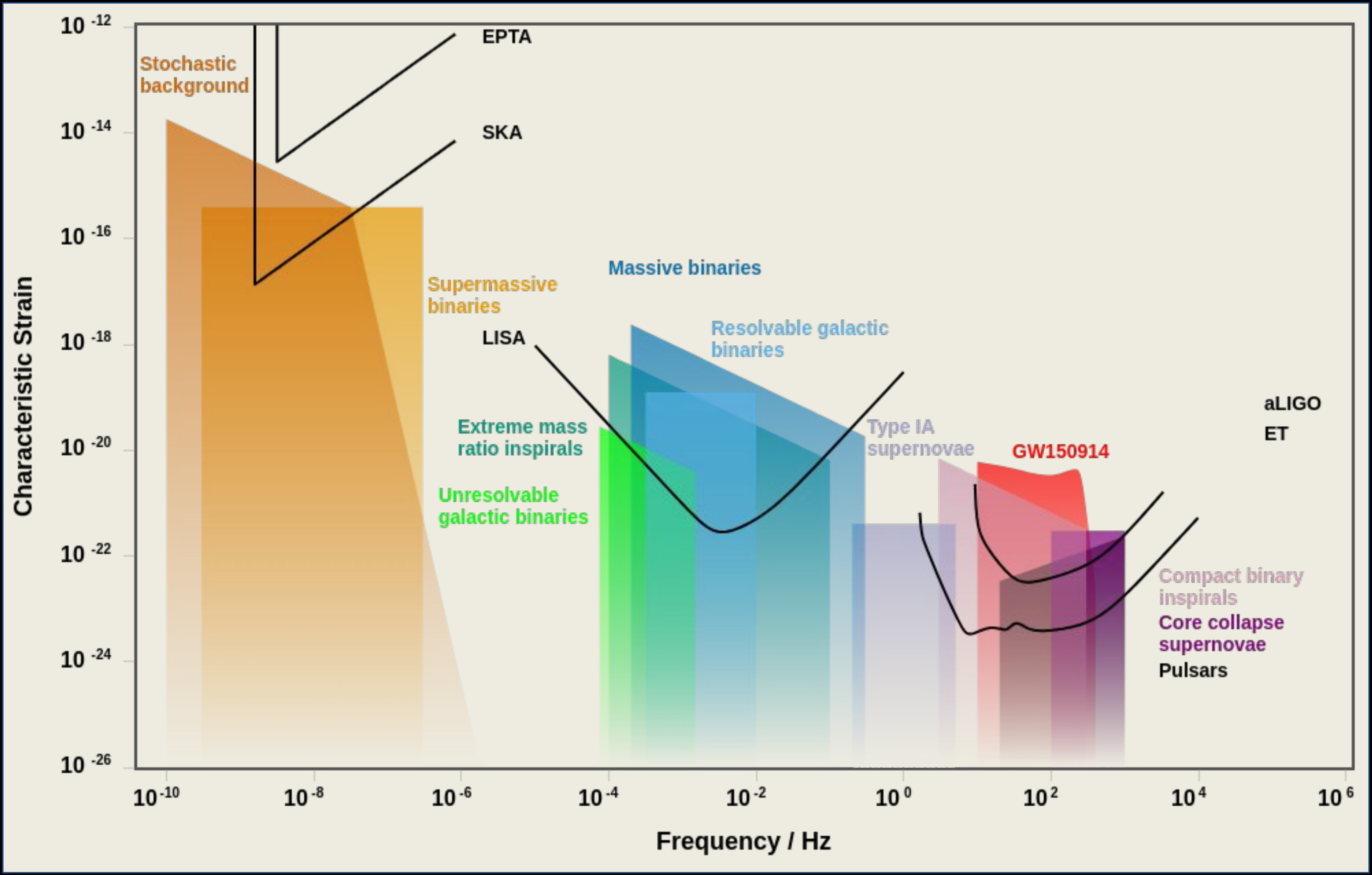
The Gravitational Wave Spectrum



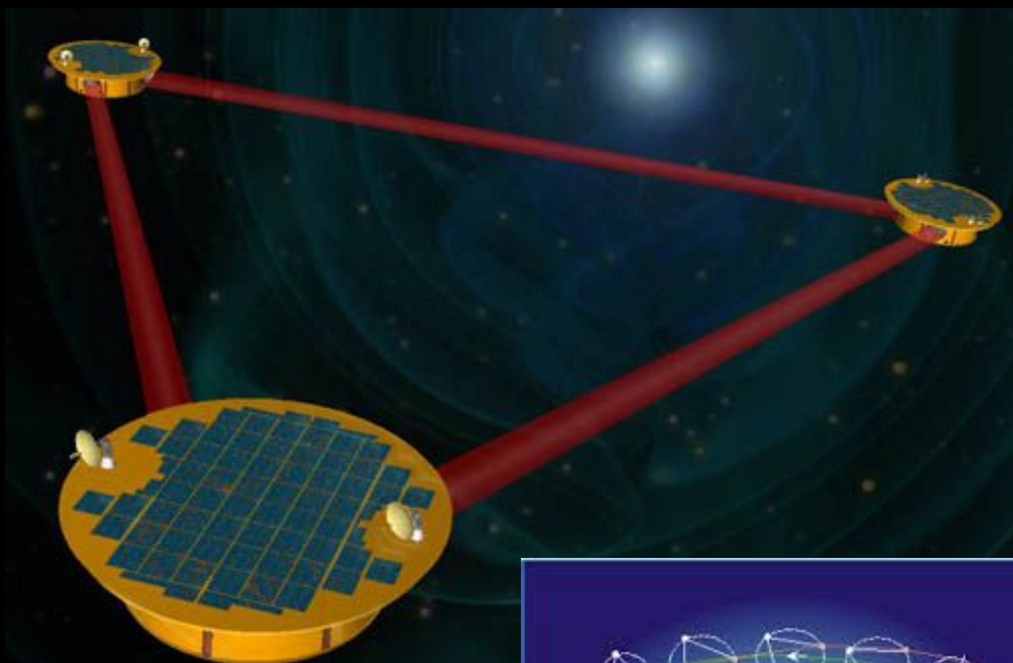
Credit: NASA Goddard Space Flight Center

The GW Spectrum

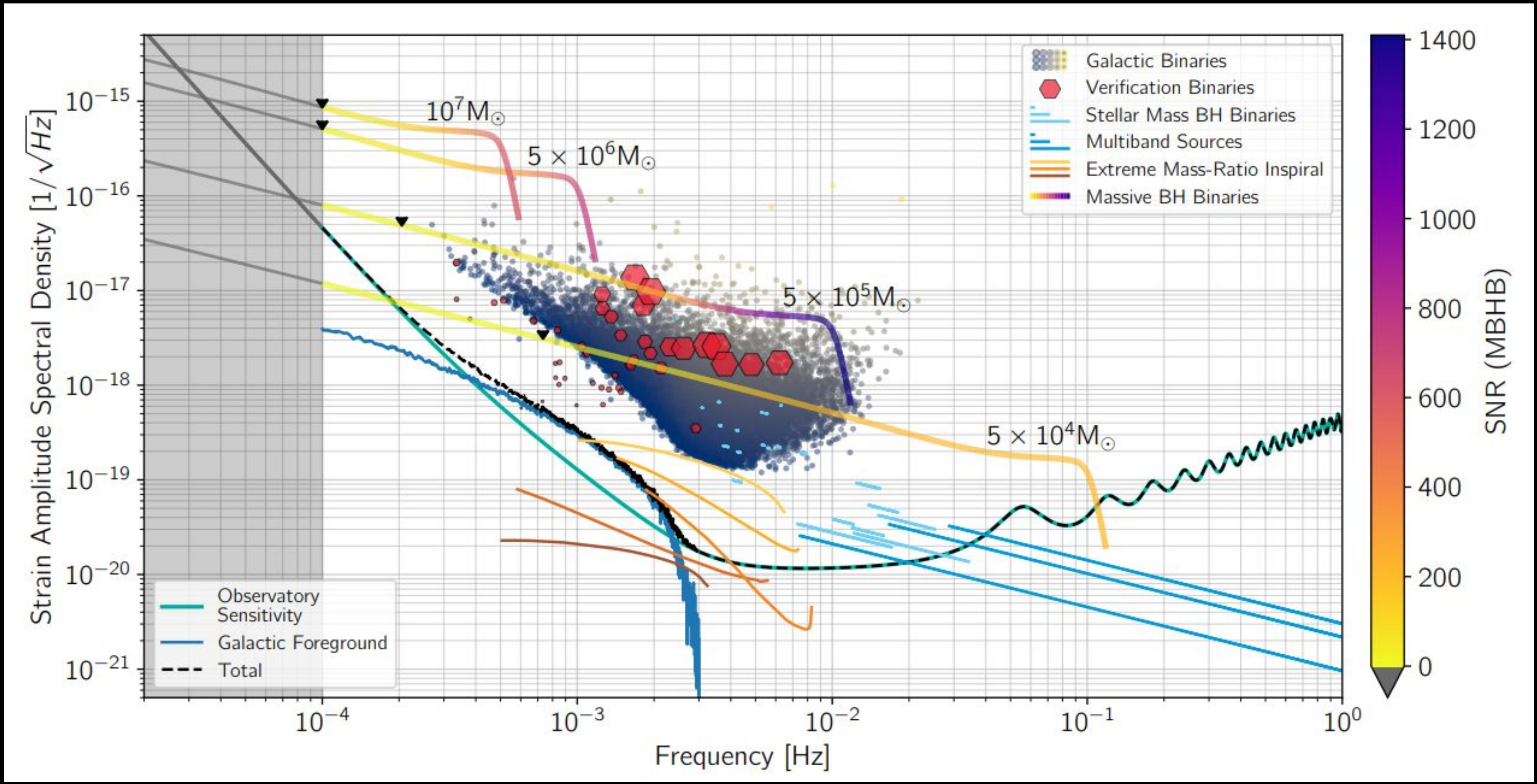
www.sr.bham.ac.uk/~cplb/GWplotter/



Space Interferometers: LISA (~2035)



LISA sources



massive ~10^6 $M_\odot$ Black hole mergers, stellar-massive & stellar-stellar inspirals (well before merger)

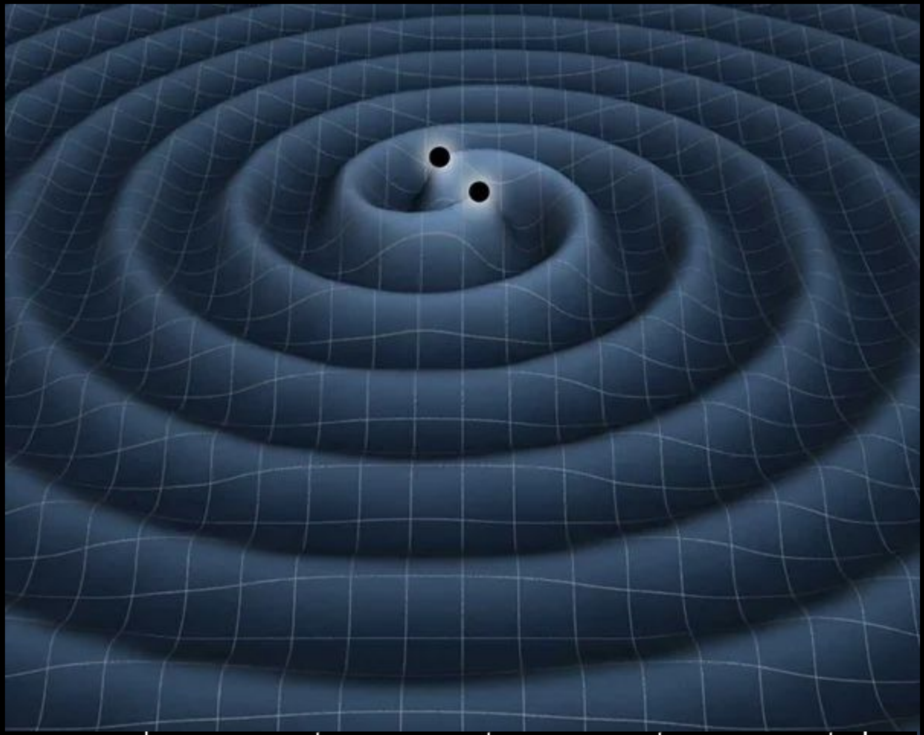
Discussion: What would you do with GWs?

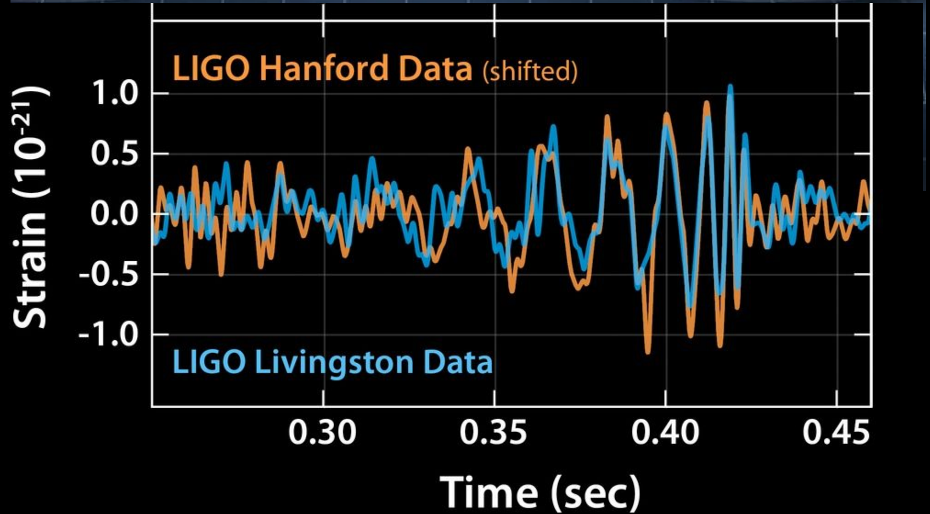
<u>Activity:</u>

Thinking (alone)

Discussing (2-3 people)

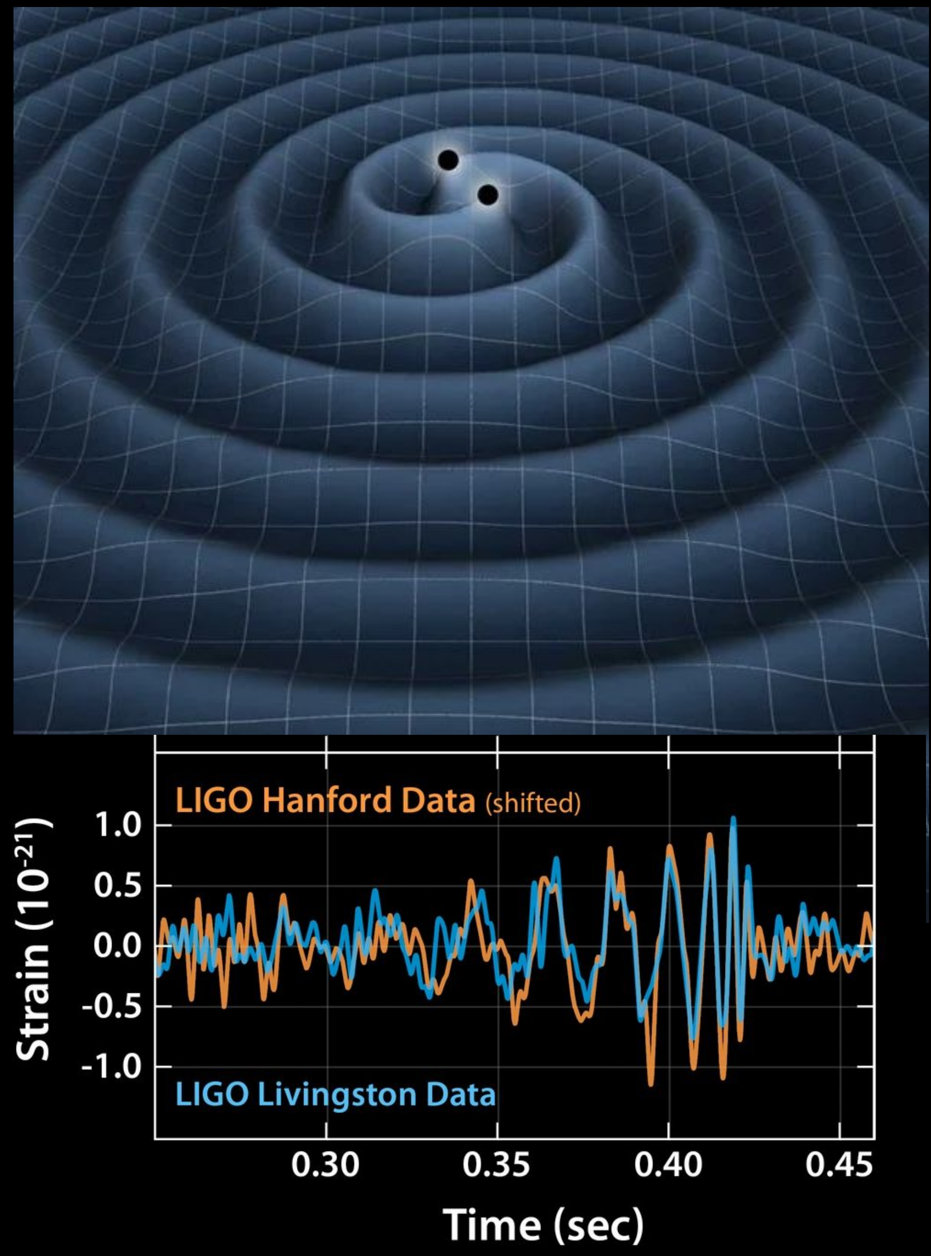
Sharing (all)





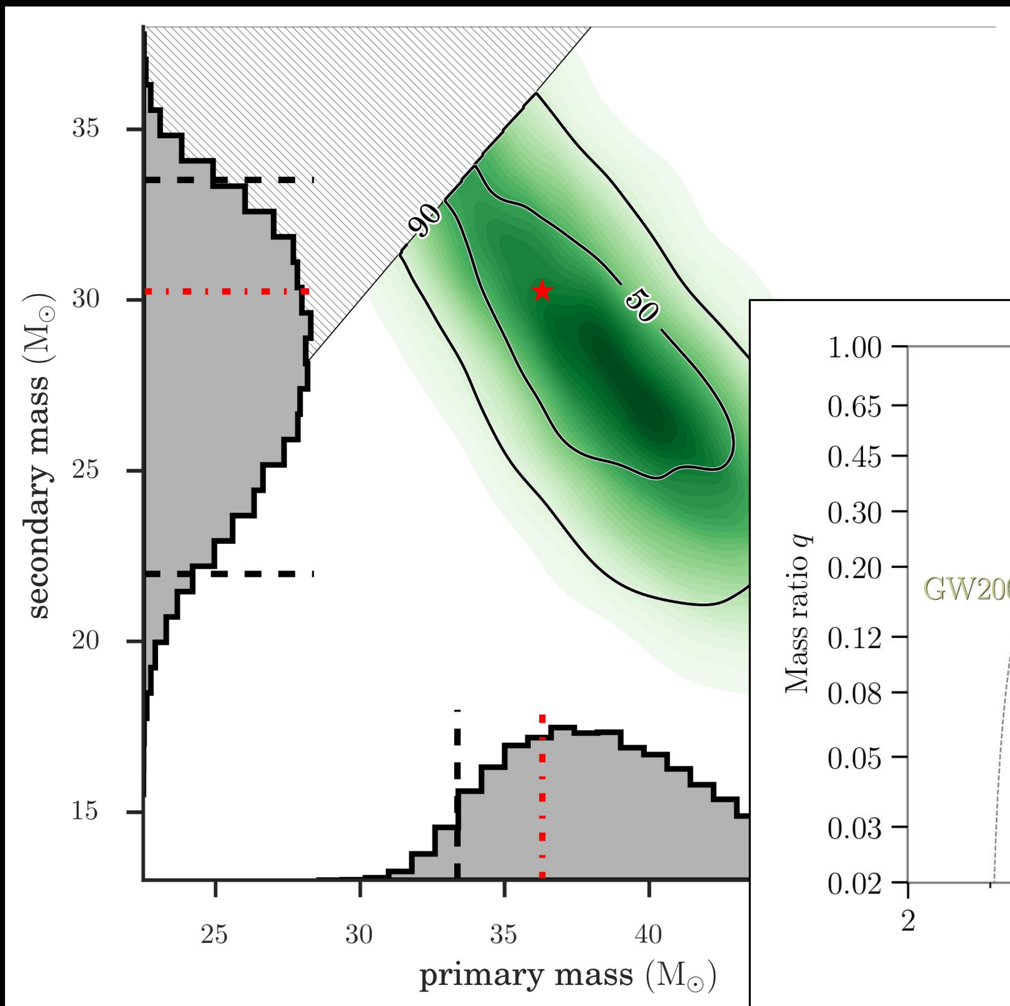
Part III: Applications

1. Astrophysics & populations
2. Extreme matter
3. Tests of gravity
4. Cosmography
5. Gravitational lensing
6. Ultra-light fields
7. Early universe

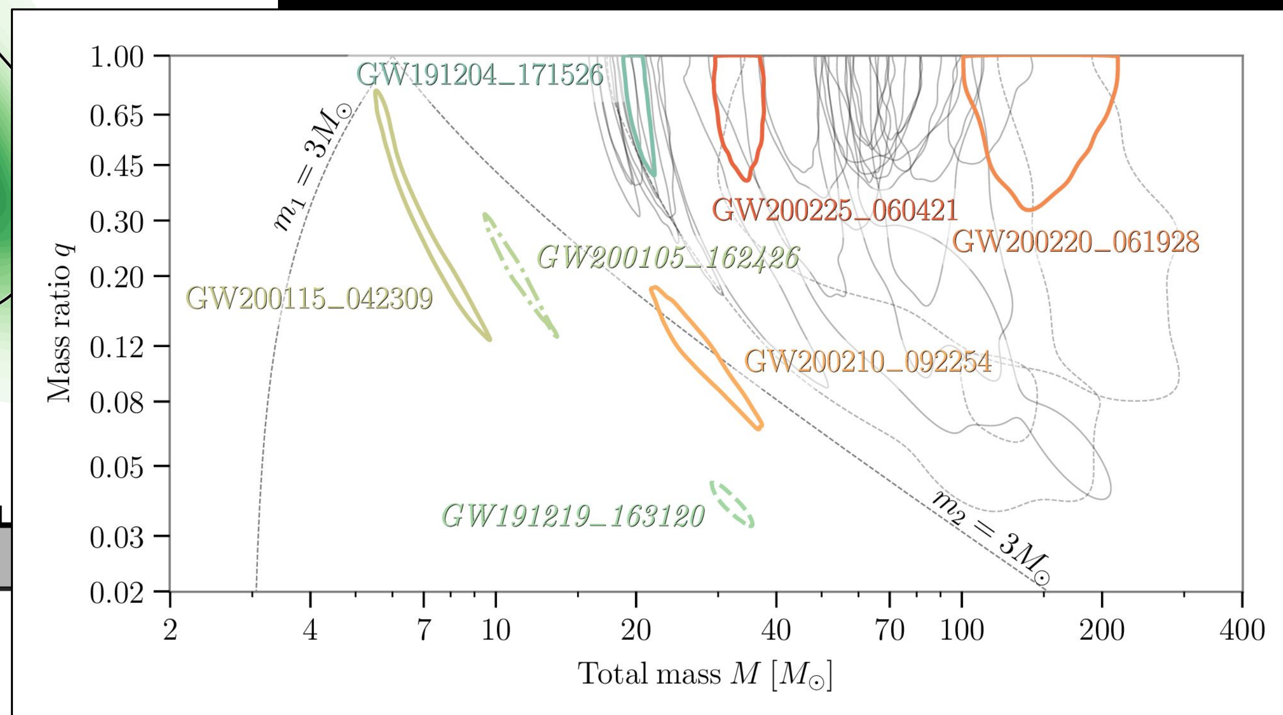


Binary Parameters

GW150914 (first event)



Current catalog

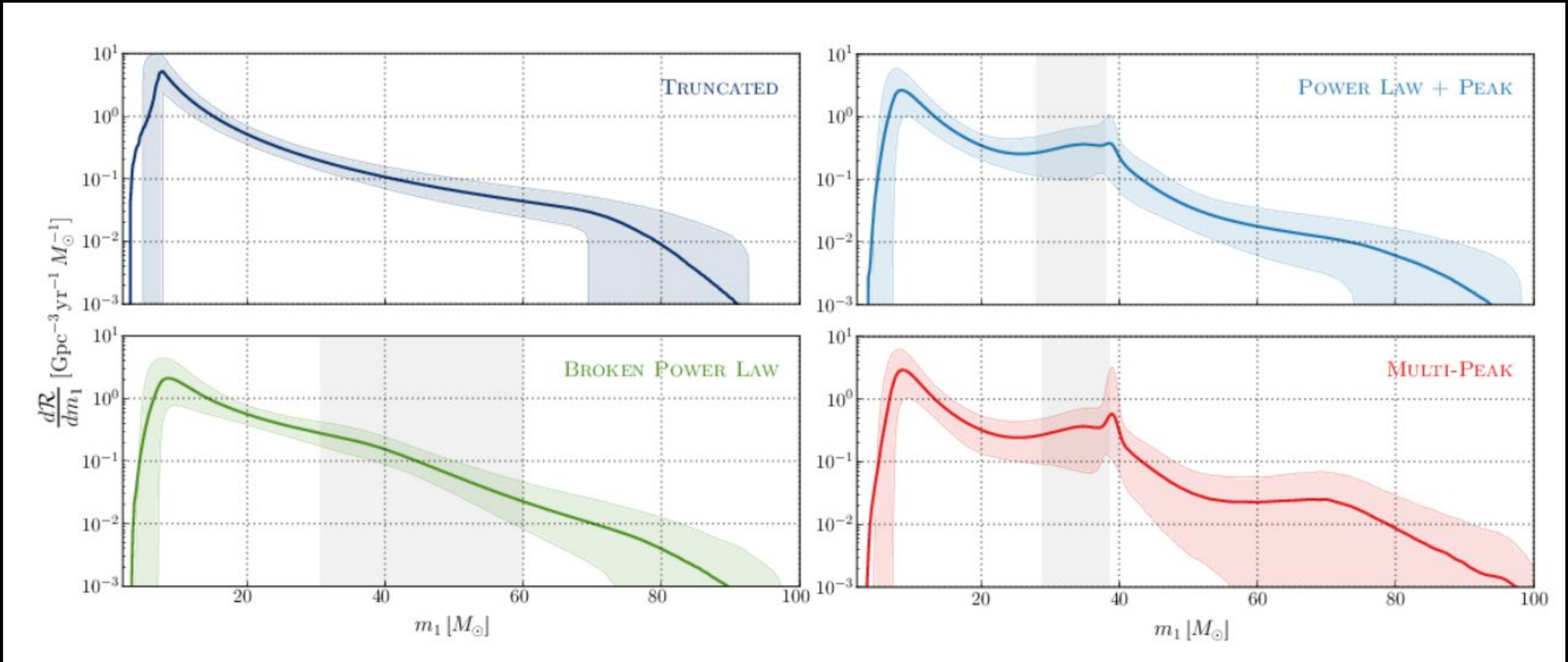


LIGO-Virgo

Population: merger rate & primary mass

Merger rate ~ 30 events/yr/Gpc^3 (black holes)

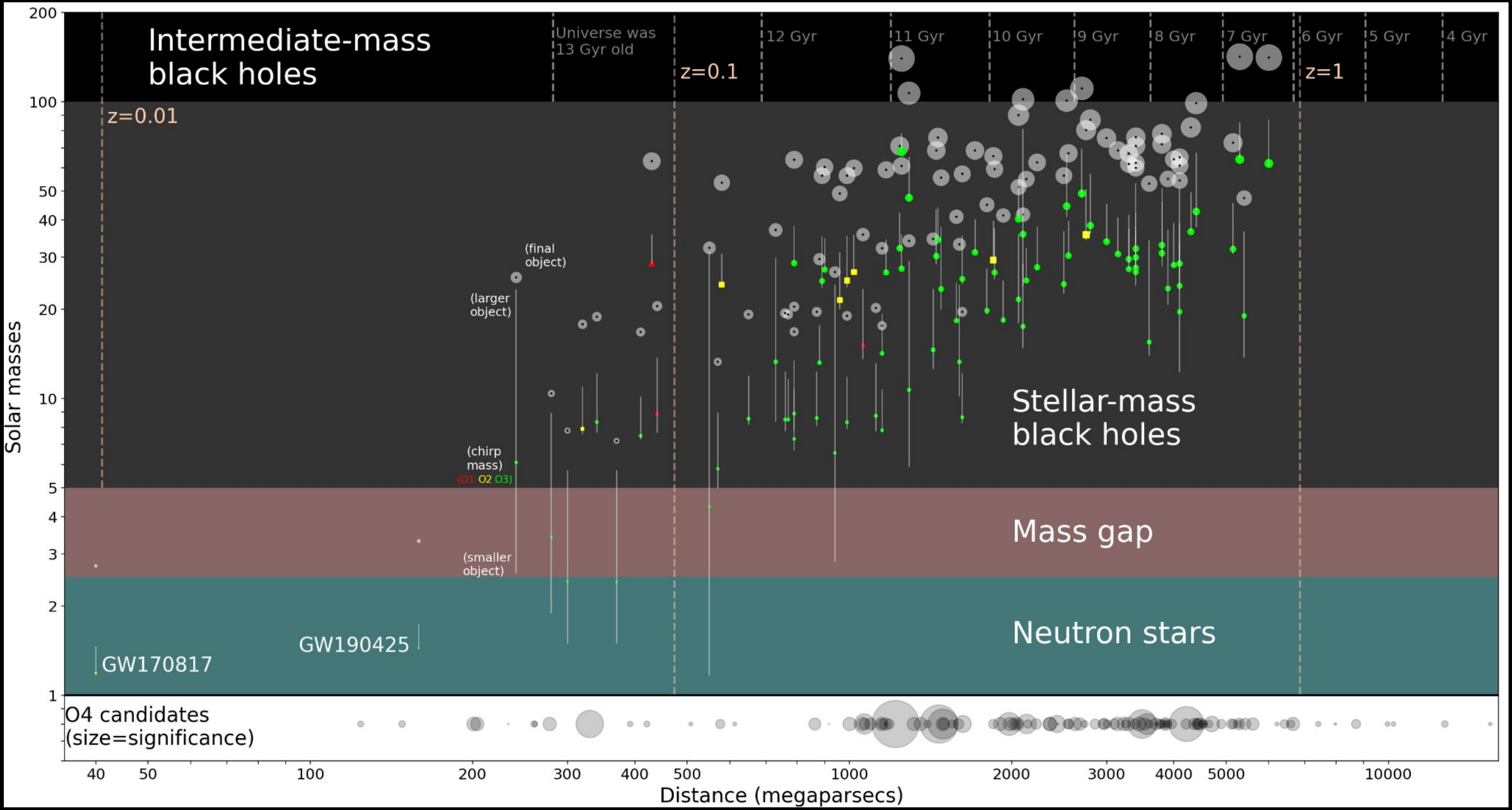
LIGO



Also secondary mass, spin, etc…

plus redshift dependence

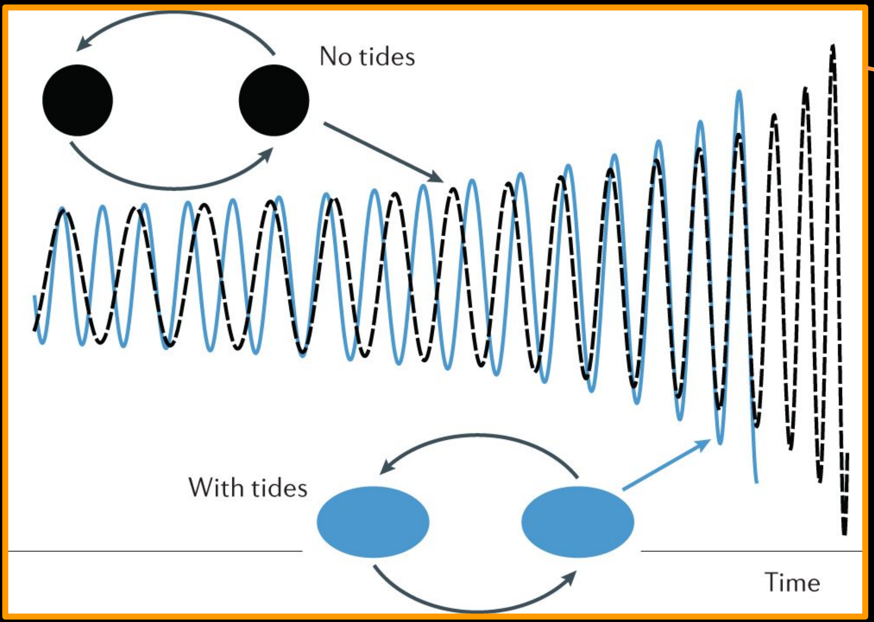
Redshift distribution



Wikipedia

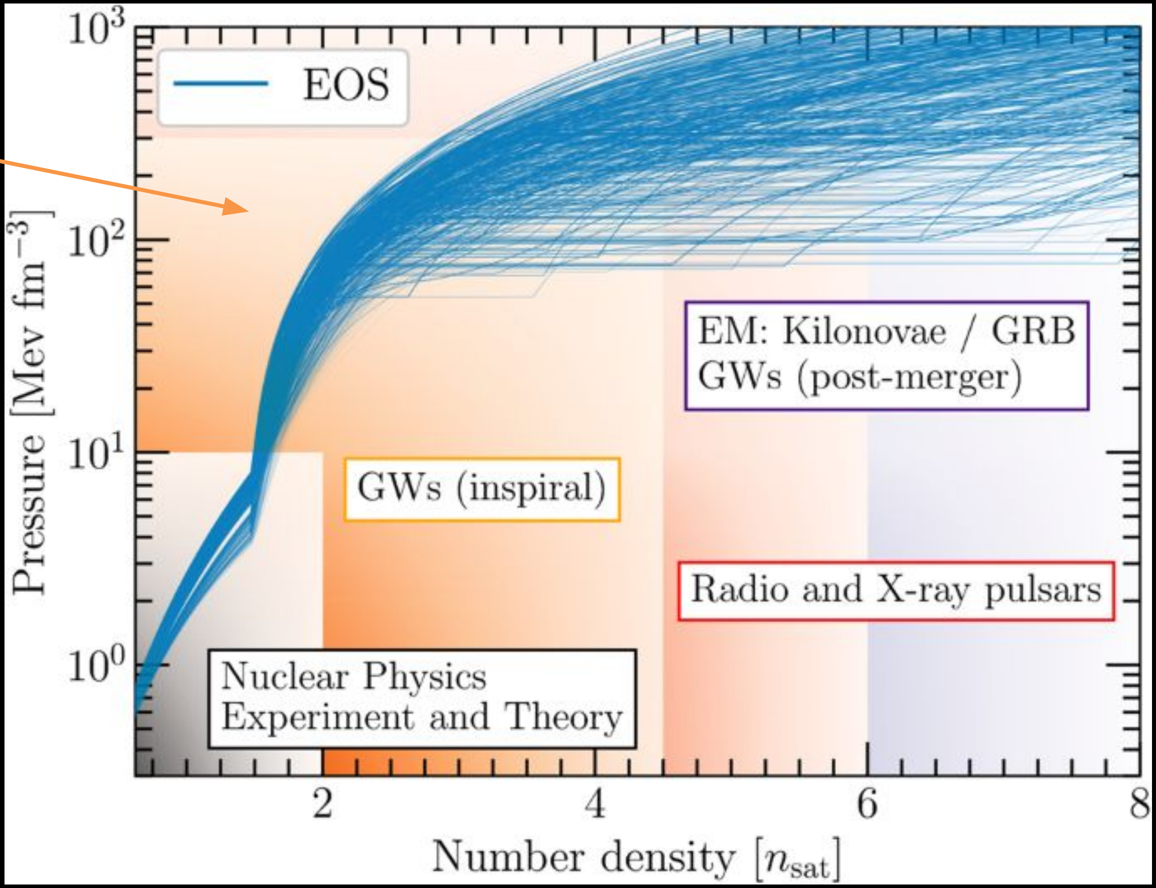
Neutron star equation of state

2202.04117



Matter ↔tidal effects

2205.08513

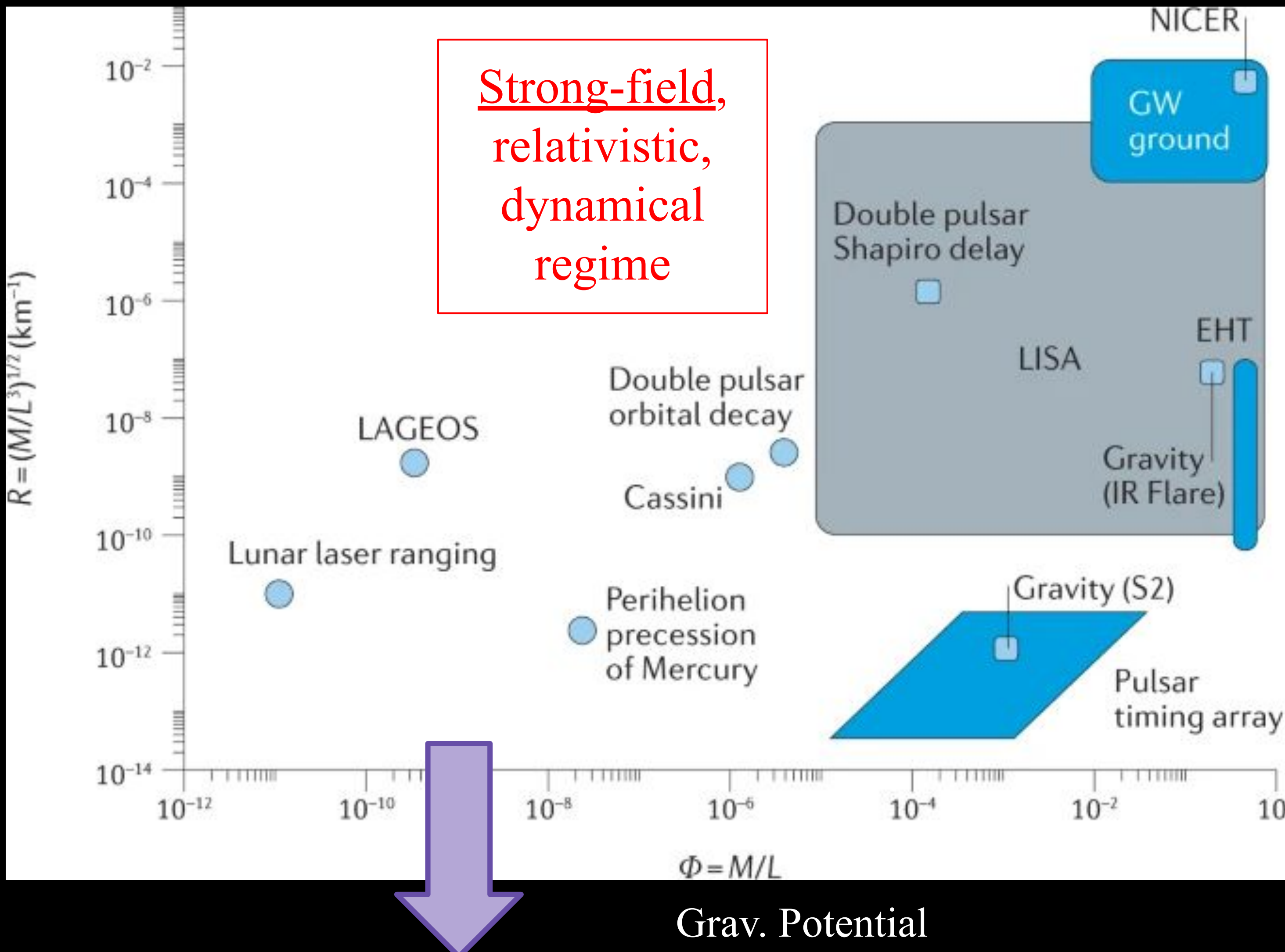


Resources:

https://multi-messenger.physik.uni-potsdam.de/eos_constraints/ → **interactive constraints**

www.computational-relativity.org/resources

Test General Relativity: Regimes



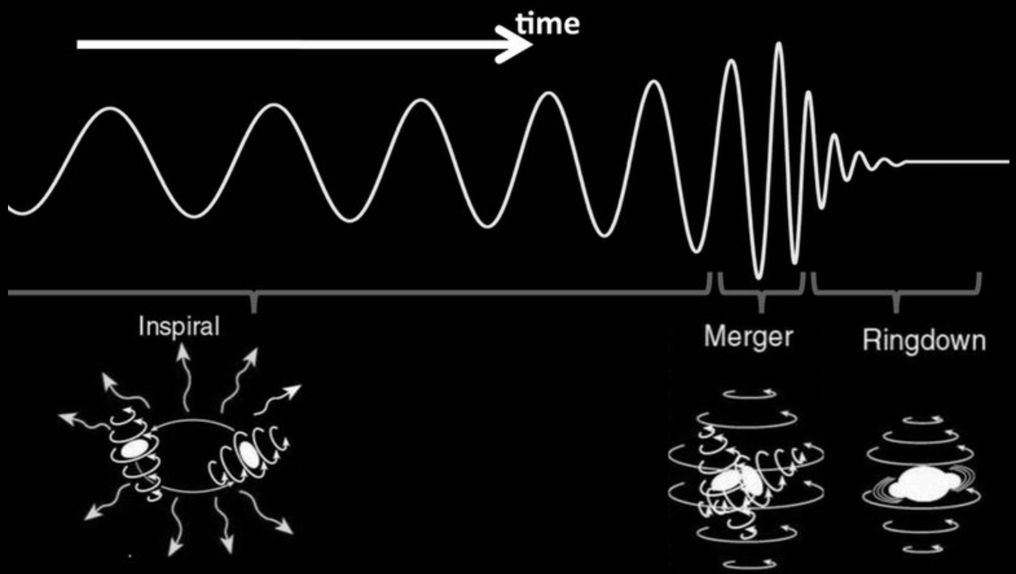
Curvature Scale ~1/λ

Grav. Potential

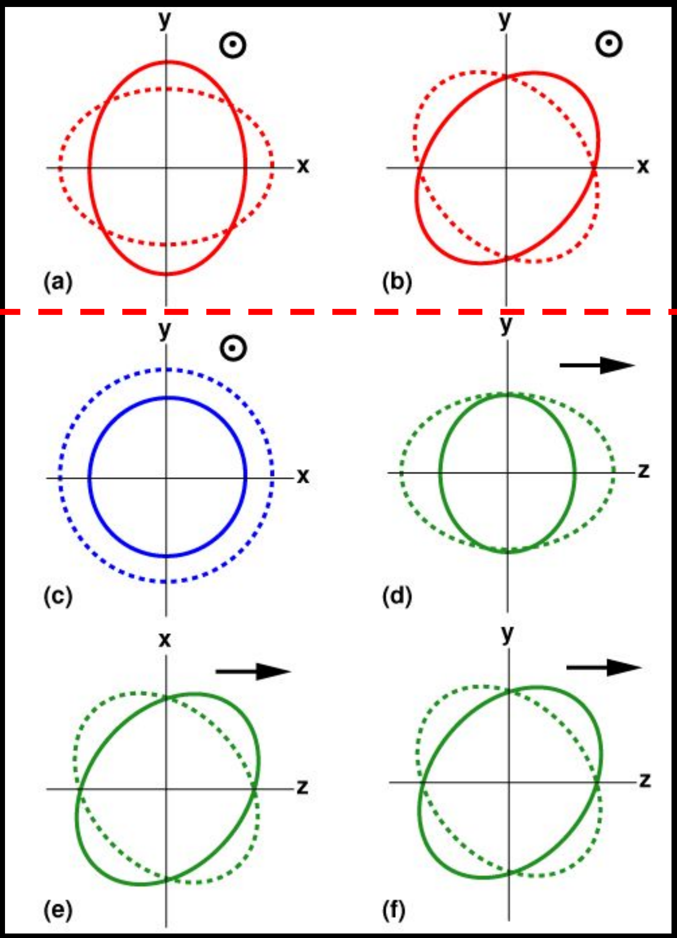
Cosmology

Credit: LIGO

Test General Relativity: Examples



Consistency between Inspiral, Merger and Ringdown (e.g. source masses)?



+,x
(standard)

scalar
(spin 0)

vector
(spin 1)

∃ Additional polarizations?

LIGO

Final Discussion: What have you learned?

<u>Activity:</u>

Thinking (alone)

Discussing (2-3 people)

Sharing (all)

Feedback Please!

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@miguelzuma

https://forms.gle/66P3bEugSqSw8fBi9