things that could go wrong just before your talk:

- you find out it was the wrong day
- a huge blackout takes down the country ✓
- you forgot to upload the slides

Inflationary Fossils beyond perturbation theory

R. Impavido in collaboration with N. Bartolo (UniPD)

The inflationary scenario

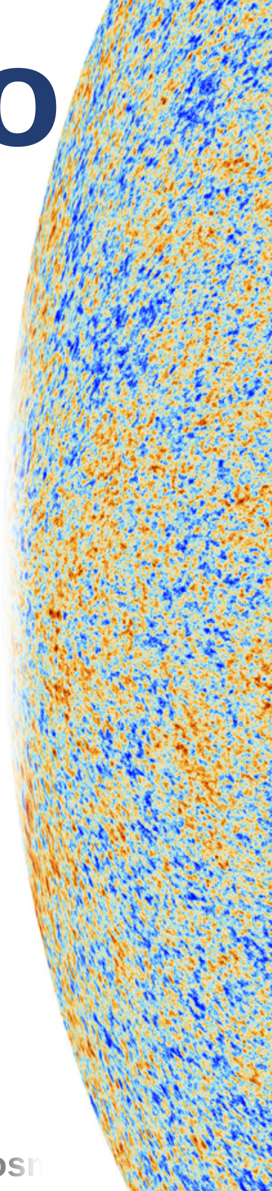
- The early universe is approximately a de Sitter space, expanding at an accelerated rate

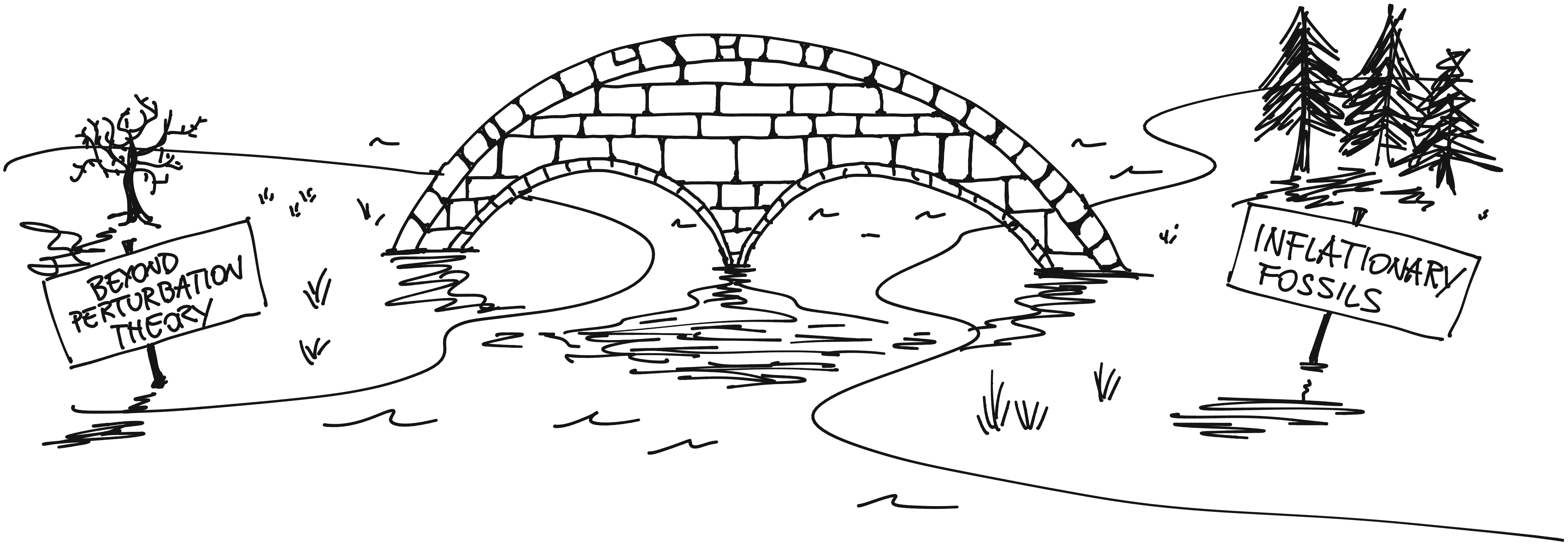
- The accelerated expansion washes away inhomogeneities, explaining the great homogeneity of, e.g. the CMB

- Massless fields in de Sitter are frozen, at late times, by the cosmological expansion, at a value $\simeq H$

- The accelerated expansion is sustained by the v.e.v. of the inflaton

- Quantum fluctuations of the inflaton are responsible for the inhomogeneities of the CMB





M. Celoria, P. Creminelli, G. Tambalo, and V. Yingcharoenrat - JCAP 2021.06

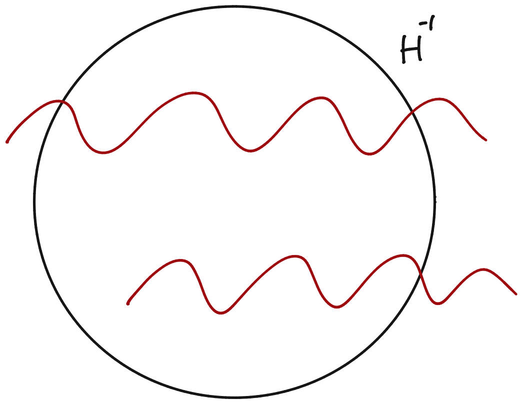
D. Jeong and M. Kamionkowski - Phys. Rev. Lett. 108 (2012), and related works.

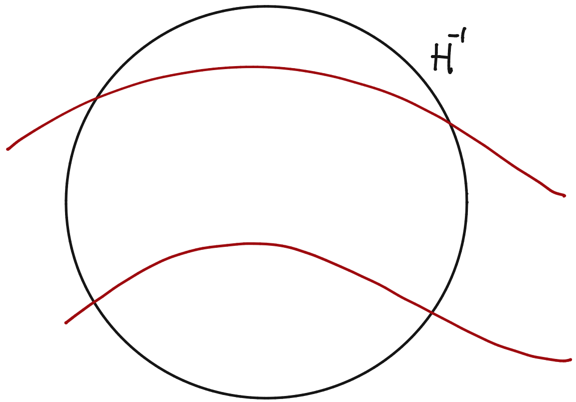
More recently: E. Dimastrogiovanni, M. Fasiello, L. Pinol – JCAP 2022.09

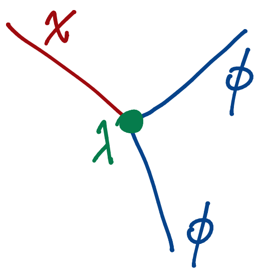
Inflationary fossils

How is it possible to characterize long modes of a field that interacted with the inflaton?

$$\lambda\phi\phi\chi$$







$$\langle\phi\phi(k)\rangle\,|_{\chi_L} = \langle\phi\phi(k)\rangle + \lim_{q\to 0}\frac{\bar{\chi}\,\langle\phi\phi(k)\chi(q)\rangle}{\langle\chi\chi(q)\rangle}$$

Inflationary fossils

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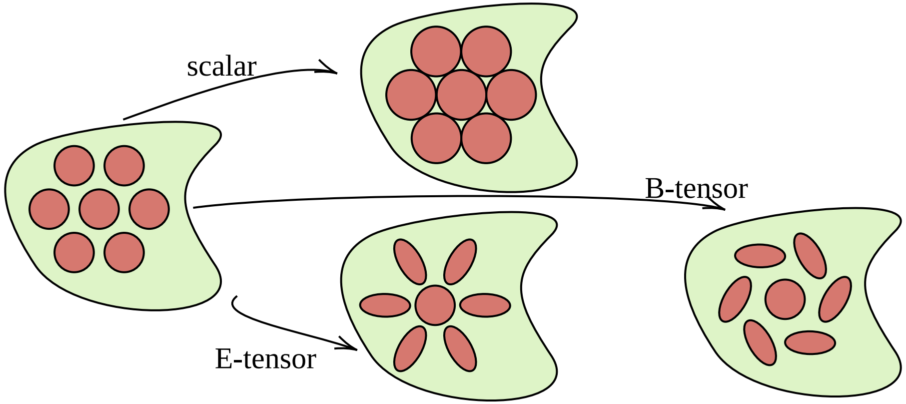

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Beyond perturbation theory

Is it possible to solve nonperturbatively, in some approximation, a theory of two interacting fields in (quasi) de Sitter space?

$$S = \int d\eta d^3\mathbf{x} \frac{1}{2\eta^2 H^2}\left(\phi'^2 - (\partial_i\phi)^2 + \chi'^2 - (\partial_i\chi)^2\right) - \frac{\lambda}{2\eta^4 H^4} H\chi\phi^2$$

$\chi :$ **free part +** $\lambda\phi^2 = 0$

$\phi :$ **free part +** $\lambda\chi\phi = 0$

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approximation(s)

$\lambda \ll 1$ **weak coupling**

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χ : **free part +** $\cancel{\lambda\phi^2} = 0$

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approximation(s)

$\lambda \ll 1$	**weak coupling**
$k_\chi \ll k_\phi$	**long mode of** χ
$\bar{\chi} \gg H$	**large mode of** χ

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Beyond perturbation theory

Is it possible to solve nonperturbatively, in some approximation, a theory of two interacting fields in (quasi) de Sitter space?

$$S_{\rm eff} = \int d\eta d^3\mathbf{x} \frac{1}{2\eta^2 H^2}\left(\phi'^2 - (\partial_i \phi)^2 - \frac{m^2}{\eta^2 H^2}\phi^2\right) \qquad m^2 = H\bar{\chi}\lambda$$

$$\chi: \quad \text{free part} + \cancel{\lambda\phi^2} = 0$$

$$\phi: \quad \text{free part} + \lambda\bar{\chi}\phi = 0$$

approximation(s)

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$$m^2 = H\bar{\chi}\lambda$$

$$\langle\phi\phi(k)\rangle_{\mathrm{eff}} = \langle\phi\phi(k)\rangle\,(-k\eta)^{3-2\sqrt{9/4-m^2/H^2}}$$

$\chi :$ free part + $\cancel{\lambda\phi^2} = 0$

$\phi :$ free part + $\lambda\bar{\chi}\phi = 0$

approximation(s)

$\lambda \ll 1$ weak coupling

$k_\chi \ll k_\phi$ long mode of χ

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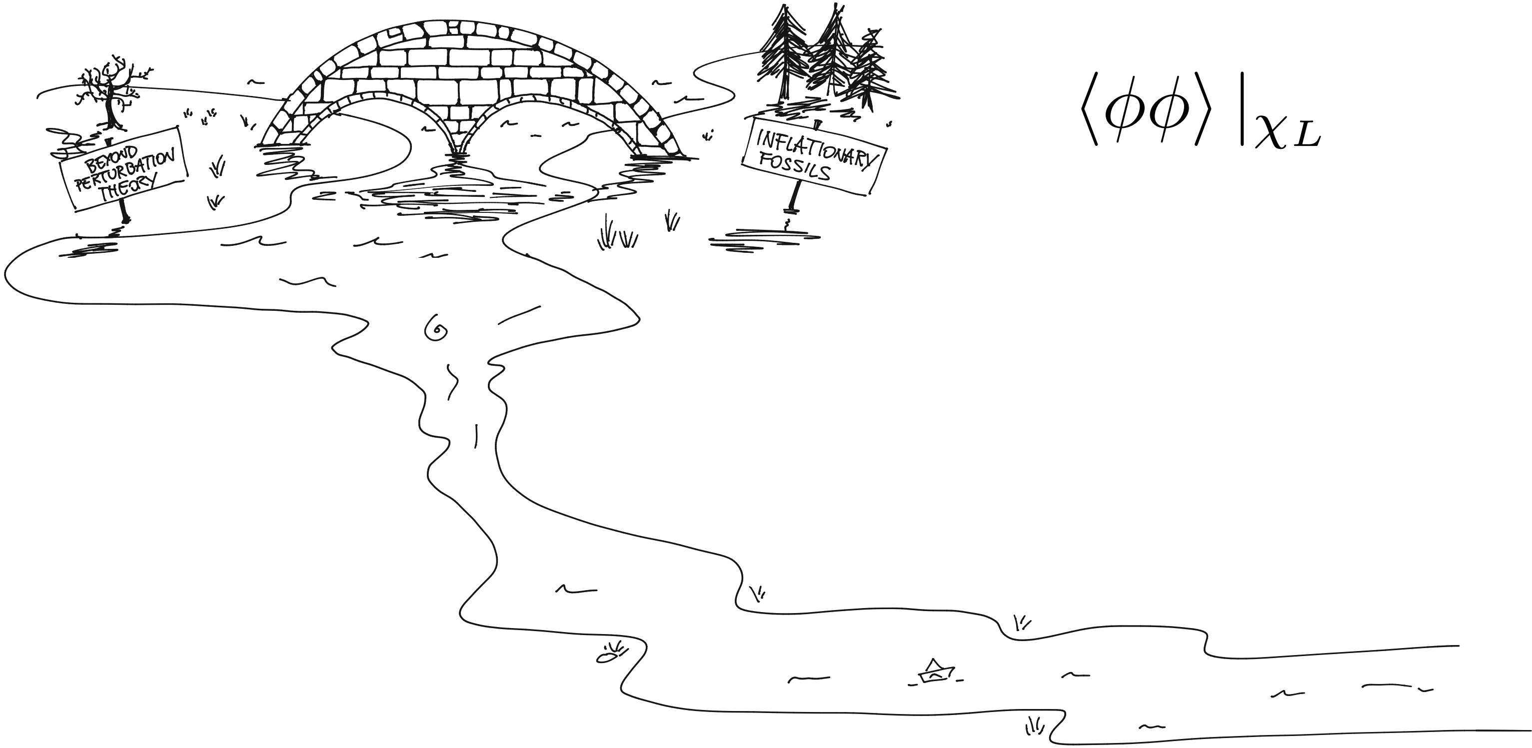
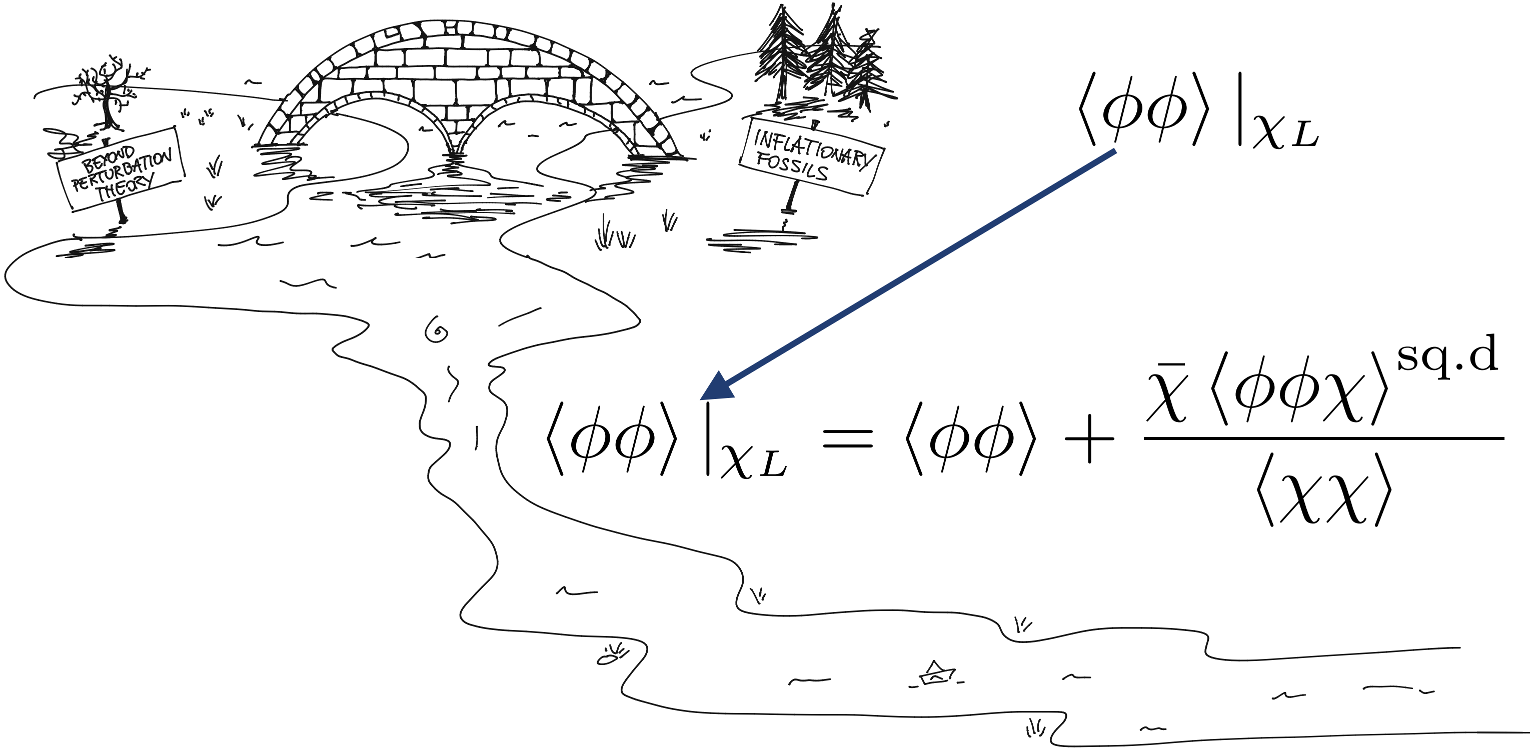
$$\langle\phi\phi\rangle_{\rm eff}$$



$$\langle\phi\phi\rangle\big|_{\chi_L}$$

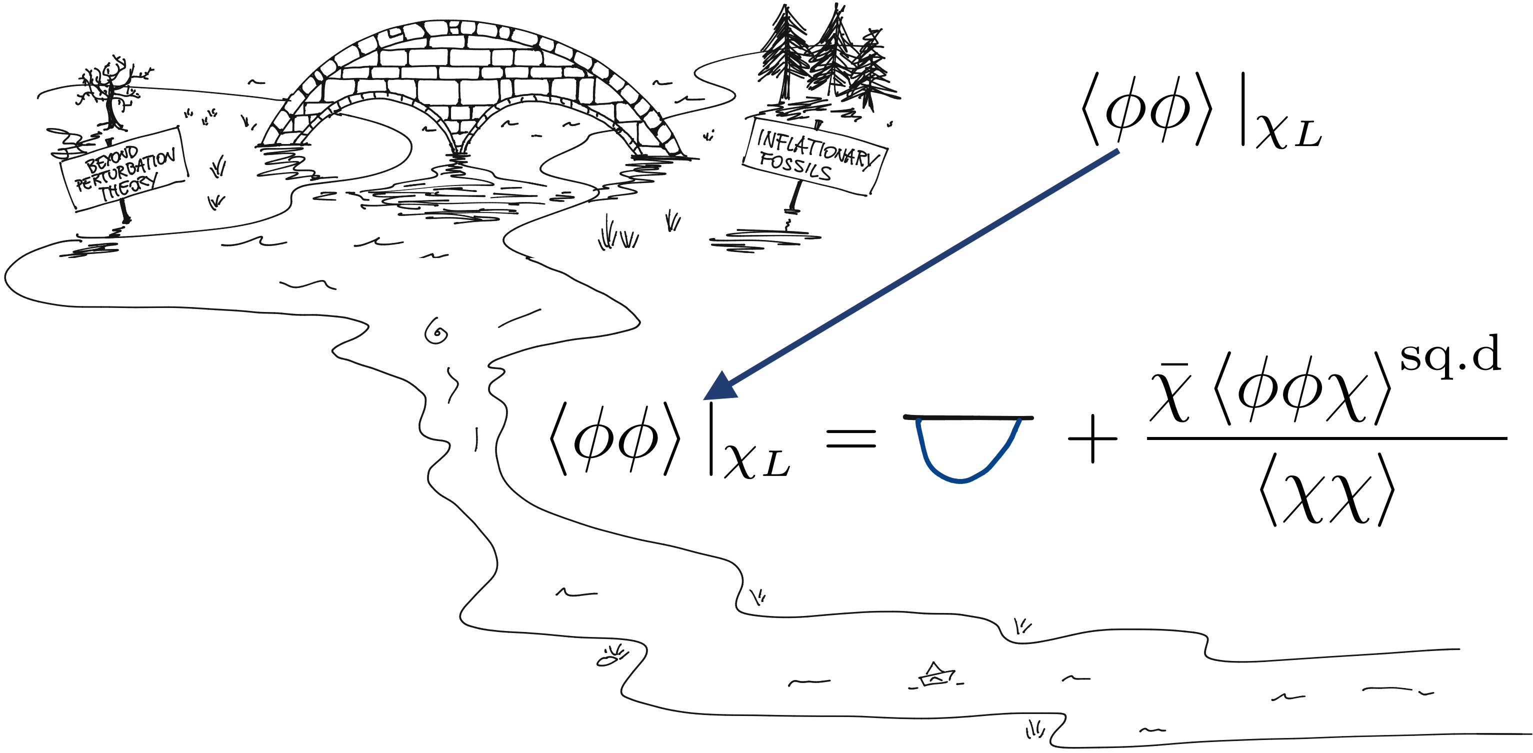
$$\langle \phi\phi \rangle_{\rm eff}$$



$$\langle \phi\phi \rangle \big|_{\chi_L}$$

$$\langle \phi\phi \rangle \big|_{\chi_L} = \langle \phi\phi \rangle + \frac{\bar{\chi} \, \langle \phi\phi\chi \rangle^{\rm sq.d}}{\langle \chi\chi \rangle}$$

$$\langle\phi\phi\rangle_{\mathrm{eff}}$$



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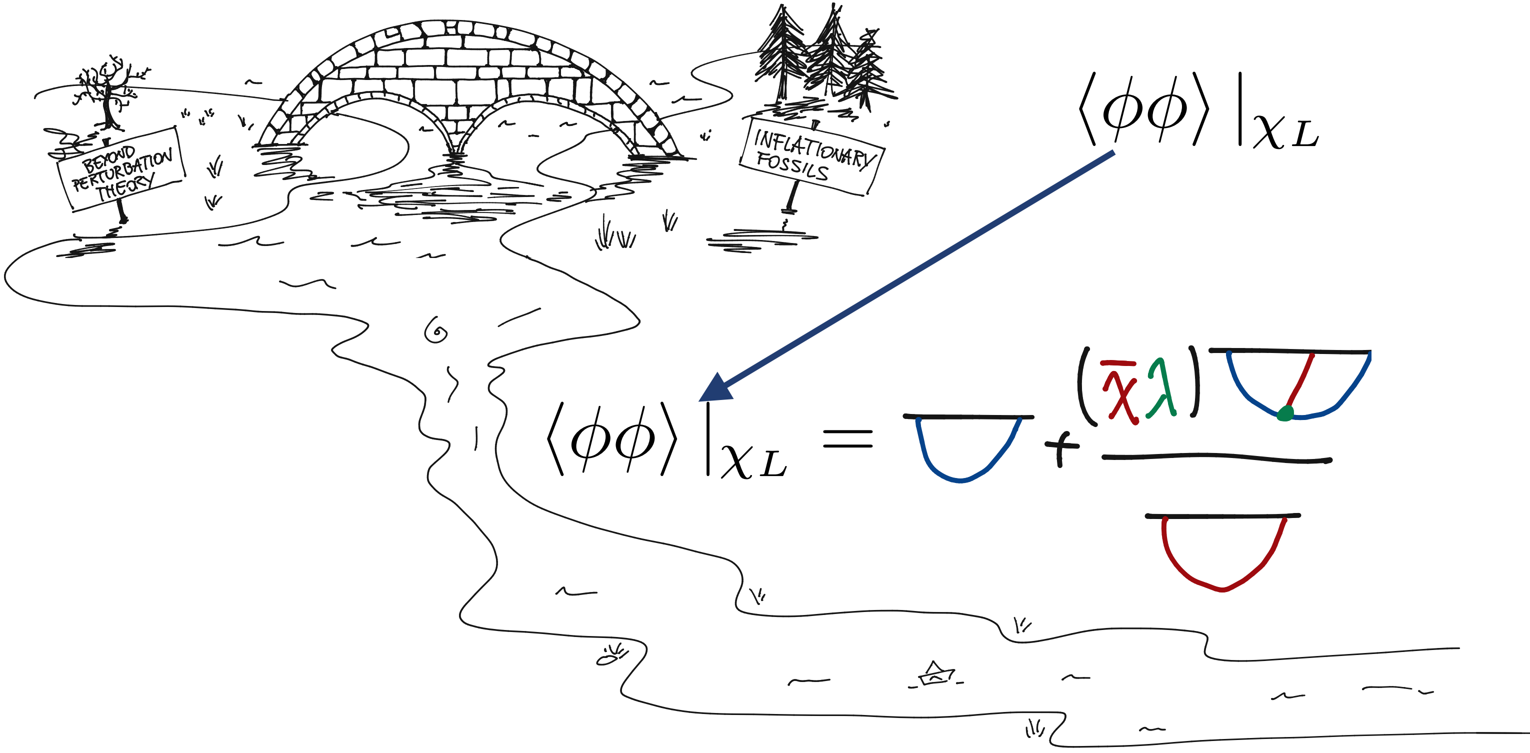


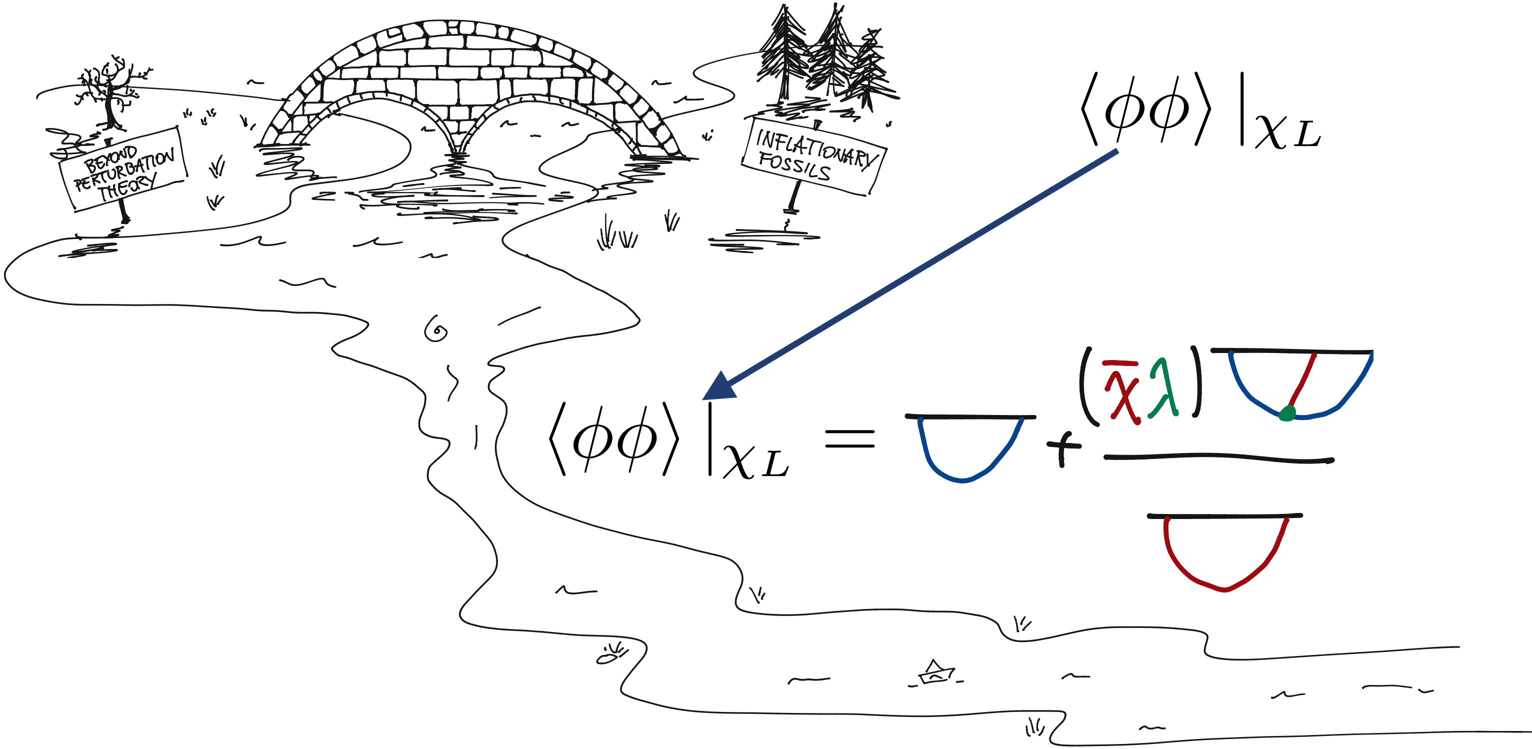
$$\langle\phi\phi\rangle\big|_{\chi_L}$$

$$\langle\phi\phi\rangle\big|_{\chi_L} = \quad + \; (\bar{\chi}\lambda)$$

$$\langle\phi\phi\rangle_{\text{eff}}$$

$$\langle\phi\phi\rangle_{\text{eff}}\,(\lambda\bar{\chi})$$

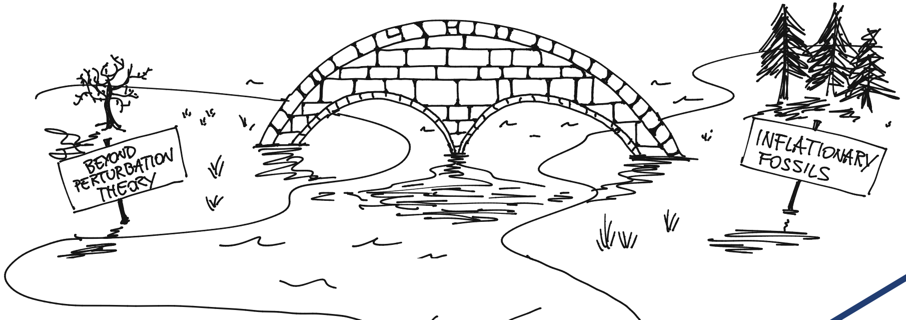
$$\langle\phi\phi\rangle\,|_{\chi_L}$$



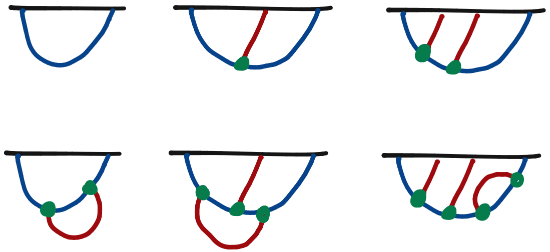
$$\langle\phi\phi\rangle_{\text{eff}}$$

$$\downarrow$$

$$\langle\phi\phi\rangle_{\text{eff}}(\lambda\bar{\chi})$$



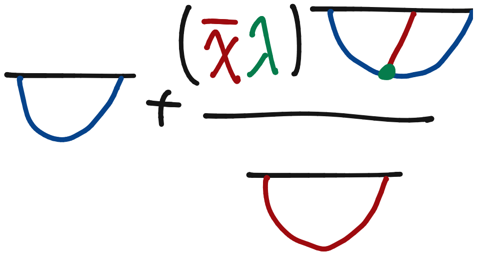
$$\langle\phi\phi\rangle|_{\chi_L}$$



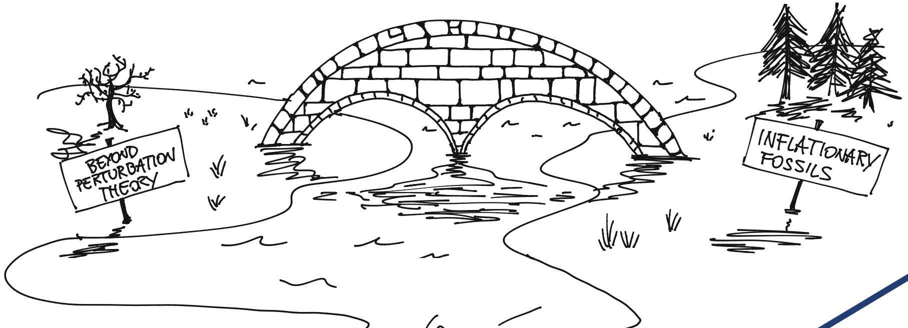
$\mathcal{O}(\bar{\chi}\lambda)^n$

$\mathcal{O}\lambda^2(\bar{\chi}\lambda)^n$

$$\langle\phi\phi\rangle|_{\chi_L} =$$

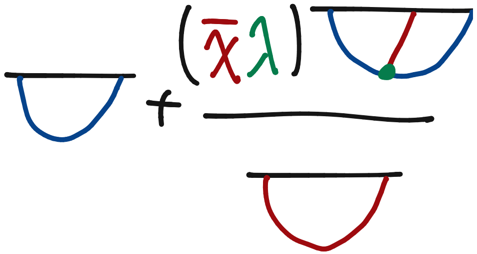


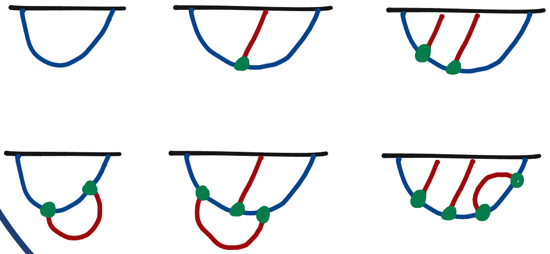
$\langle\phi\phi\rangle_{\text{eff}}$



$\langle\phi\phi\rangle|_{\chi_L}$

$\langle\phi\phi\rangle|_{\chi_L} =$

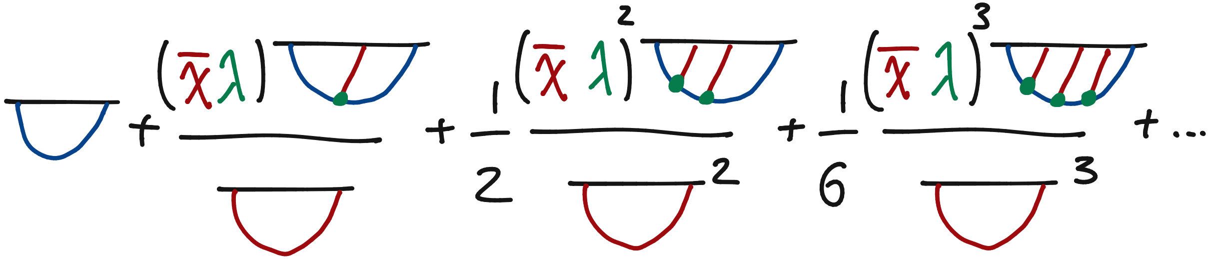




$\mathcal{O}((\bar{\chi}\lambda)^n)$

$\mathcal{O}(\lambda^2(\bar{\chi}\lambda)^n)$

$\langle\phi\phi\rangle_{\text{eff}} =$



Results: all matchings

We have checked the matching between Fossils and BPT to first order for five different couplings

Coupling	Type	Effect BPT
$\lambda\chi\sigma^2$	scalar-scalar, linear	effective mass
$\lambda\chi\sigma'\sigma'$	scalar-scalar, derivative	change of kin. term
$\lambda\chi\partial_i\sigma\partial^i\sigma$	scalar-scalar, derivative	change of kin. term
$\epsilon\gamma_{ij}\partial_i\zeta\partial_j\zeta$	scalar-tensor, derivative	change of kin. term
$\epsilon\zeta\left(\gamma'_{ij}\gamma'_{ij}g^{00} - \partial_k\gamma_{ij}\partial_h\gamma_{ij}g^{hk}\right)$	scalar-tensor, der.	change of kin. term

Large (& long mode) scalar *fossils* modfy GW's speed of sound

free part for ζ **free part for γ**

$$S = \int d^3\mathbf{x} d\eta \frac{-\epsilon}{\eta^4 H^4}\left[\zeta'\zeta' g^{00} + \partial_i\zeta\partial_j\zeta g^{ij}\right] - \frac{1}{8\eta^4 H^4}\left[\gamma'_{ij}\gamma'_{ij} g^{00} + \partial_k\gamma_{ij}\partial_h\gamma_{ij} g^{hk}\right]$$

$$\frac{-\epsilon}{8\eta^4 H^4}\left[\zeta\gamma'_{ij}\gamma'_{ij} g^{00} - \zeta\partial_k\gamma_{ij}\partial_h\gamma_{ij} g^{hk}\right] - \frac{1}{4\eta^4 H^4} g^{hk}\left[\gamma'_{ij}\partial_k\gamma_{ij}\partial_h\chi\right]$$

interaction $\gamma\gamma\zeta$ **auxiliary field**

BPT **BPT@1st order = *Fossils***

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interaction $\gamma\gamma\zeta$ **auxiliary field**

$$S_{\text{eff}} = \int d\eta d^3\mathbf{x} \frac{-1}{8\eta^4 H^4}\left[\gamma'_{ij}\gamma'_{ij} g^{00}(1+\epsilon\bar{\zeta}) + \partial_h\gamma_{ij}\partial_k\gamma_{ij} g^{hk}(1-\epsilon\bar{\zeta})\right]$$

$$\langle\gamma_{ij}\gamma_{ij}(k)\rangle' = \frac{4H^2}{k^3}\frac{\sqrt{1+\epsilon\bar{\zeta}}}{(1-\epsilon\bar{\zeta})^{3/2}} \sim \frac{4H^2}{k^3}\left(1+2\epsilon\bar{\zeta}\right)$$

BPT **BPT@1st order = *Fossils***

Sum up + Conclusions

- We have formalized the missing link between the *Inflationary Fossils* and a technique beyond perturbation theory, checking the result at first order in five different cases. **Resumming diagrams = skipping nested IN-IN integrals**

- To go beyond pertubation theory we **integrated out a light field at late time**

- It is now possible to extend the *Fossils* approach to **large primordial fluctuations**

- Interesting phenomenology lies ahead: it could be possible to **characterize large primordial GW or large curvature modes beyond perturbation theory**

Backup slides

“Large” fluctuation?

- BPT approach is **semiclassical**: **no problem** in considering a large field

- *Fossils* approach is **quantum field theoretic** (IN IN formalism), in which we are used to have fluctuations of order H if we wish to match with Bunch-Davies in the UV.
We can however apply the field operator to states that are **coherent** states with a **very large number of particles**: we are therefore considering not a high energy limit, but a **high occupation number limit**.

auxiliary field?

See e.g. Maldacena 2003. GR+scalar field using the ADM formalism.

$$S = \frac{1}{2}\int \sqrt{g}[R - (\nabla\phi)^2 - 2V(\phi)] \qquad ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

$$S = \frac{1}{2}\int \sqrt{h}\left[NR^{(3)} - 2NV + N^{-1}(E_{ij}E^{ij} - E^2) + N^{-1}(\dot{\phi} - N^i\partial_i\phi)^2 - Nh^{ij}\partial_i\phi\partial_j\phi\right]$$

Solve for the constraints in the $\delta\phi = 0$ gauge

$$\nabla_i[N^{-1}(E^i_j - \delta^i_j E)] = 0$$

$$R^{(3)} - 2V - N^{-2}(E_{ij}E^{ij} - E^2) - N^{-2}\dot{\phi}^2 = 0$$

$$N^i = \partial_i\psi + N^i_T \text{ where } \partial_i N^i_T = 0 \text{ and } N = 1 + N_1.$$

$$N_1 = \frac{\dot{\zeta}}{\dot{\rho}}\,, \qquad N^i_T = 0\,, \qquad \psi = -e^{-2\rho}\frac{\zeta}{\dot{\rho}} + \chi\,, \qquad \partial^2\chi = \frac{\dot{\phi}^2}{2\dot{\rho}^2}\dot{\zeta}$$

Expanding the action at cubic order and selecting $\gamma\gamma\zeta$ terms leaves some auxiliary χ dependence. However there is no influence for the BPT procedure