Exploring the Saturation effect via the heavy quark pair photo-production

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- Cyrille Marquet, YS, in preparation

High energy QCD: from the LHC to the EIC
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Outline

1) Introduction
2) TMD evolution of WW gluon
3) Sudakov resummation in heavy quark pair photo-production

Two Gluon Distributions in the small-x

- In small-x physics, there are two gluon distributions which are different:

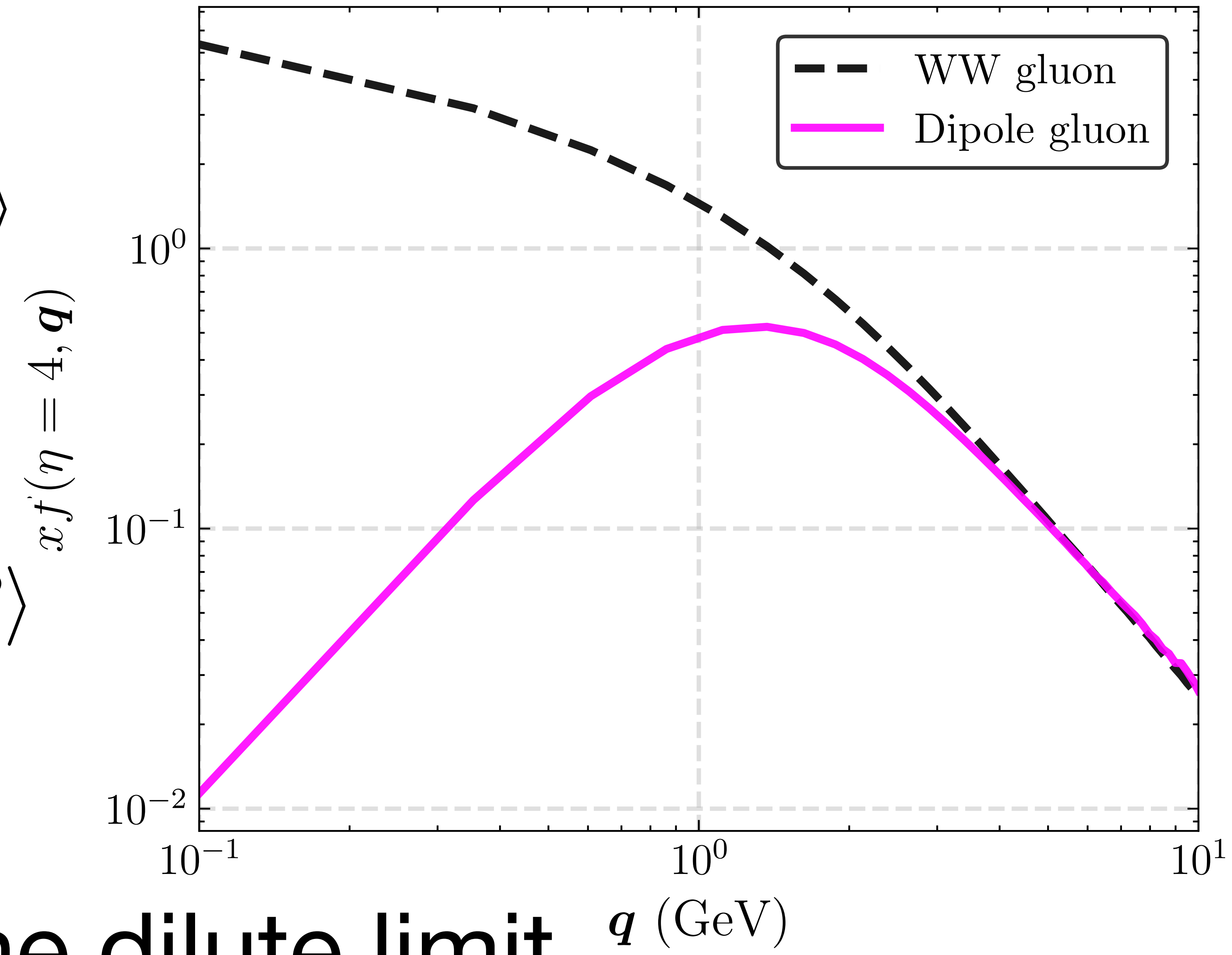
- Weizsacker Williams (WW) gluon distribution

$$xG_{\rm WW}(x,k_\perp) = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} {\rm Tr}\langle P|F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[+]\dagger}F^{+i}(0)\mathcal{U}^{[+]}|P\rangle$$

- Color Dipole gluon distribution

$$xG_{\rm DP}(x,k_\perp) = 2\int \frac{d\xi^- d\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp} {\rm Tr}\langle P|F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[-]\dagger}F^{+i}(0)\mathcal{U}^{[+]}|P\rangle$$

$$\mathscr{U}^{[\pm]} = U[0,\pm\infty;0]U[\pm\infty,\xi^-;\xi_\perp]$$



- They differ markedly in the saturation limit but coincide in the dilute limit.

	Inclusive	Single Inc	DIS dijet	γ +jet	dijet in pA
$xG_{\rm WW}$	$\times$	$\times$	$\surd$	$\times$	$\surd$
$xG_{\rm DP}$	$\surd$	$\surd$	$\times$	$\surd$	$\surd$

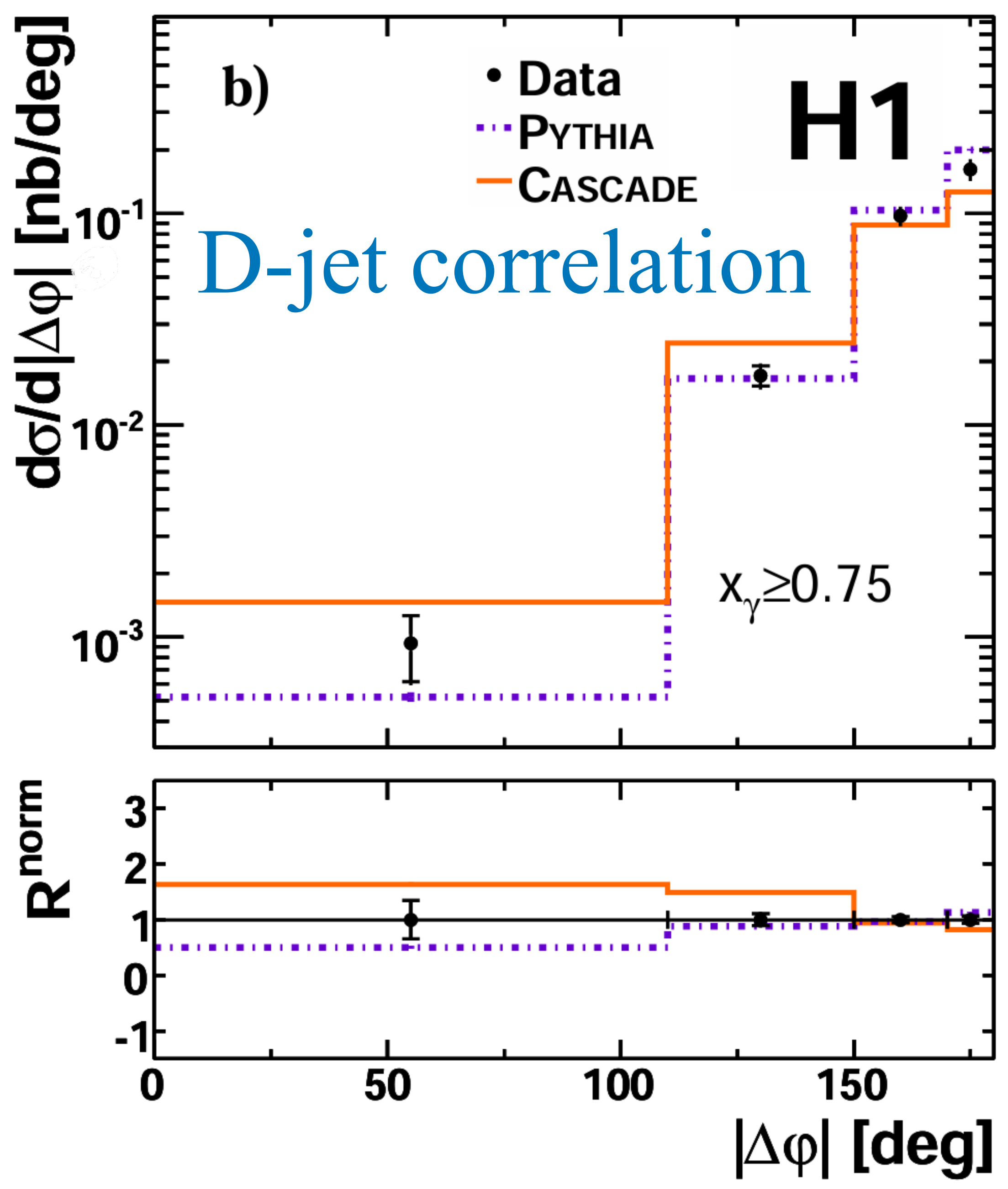
[Kovchegov, Mueller, 98; Kharzeev, Kovchegov, Tuchin; 03; Dominguez, Marquet, Xiao, Yuan, 11]

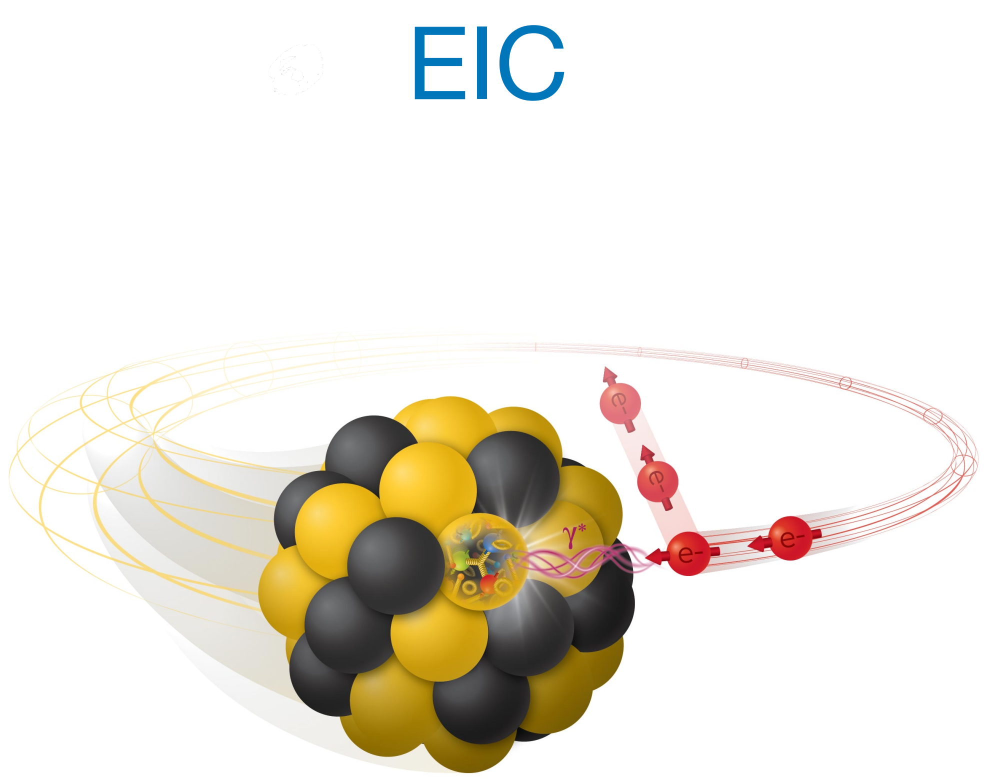
I will discuss WW gluon

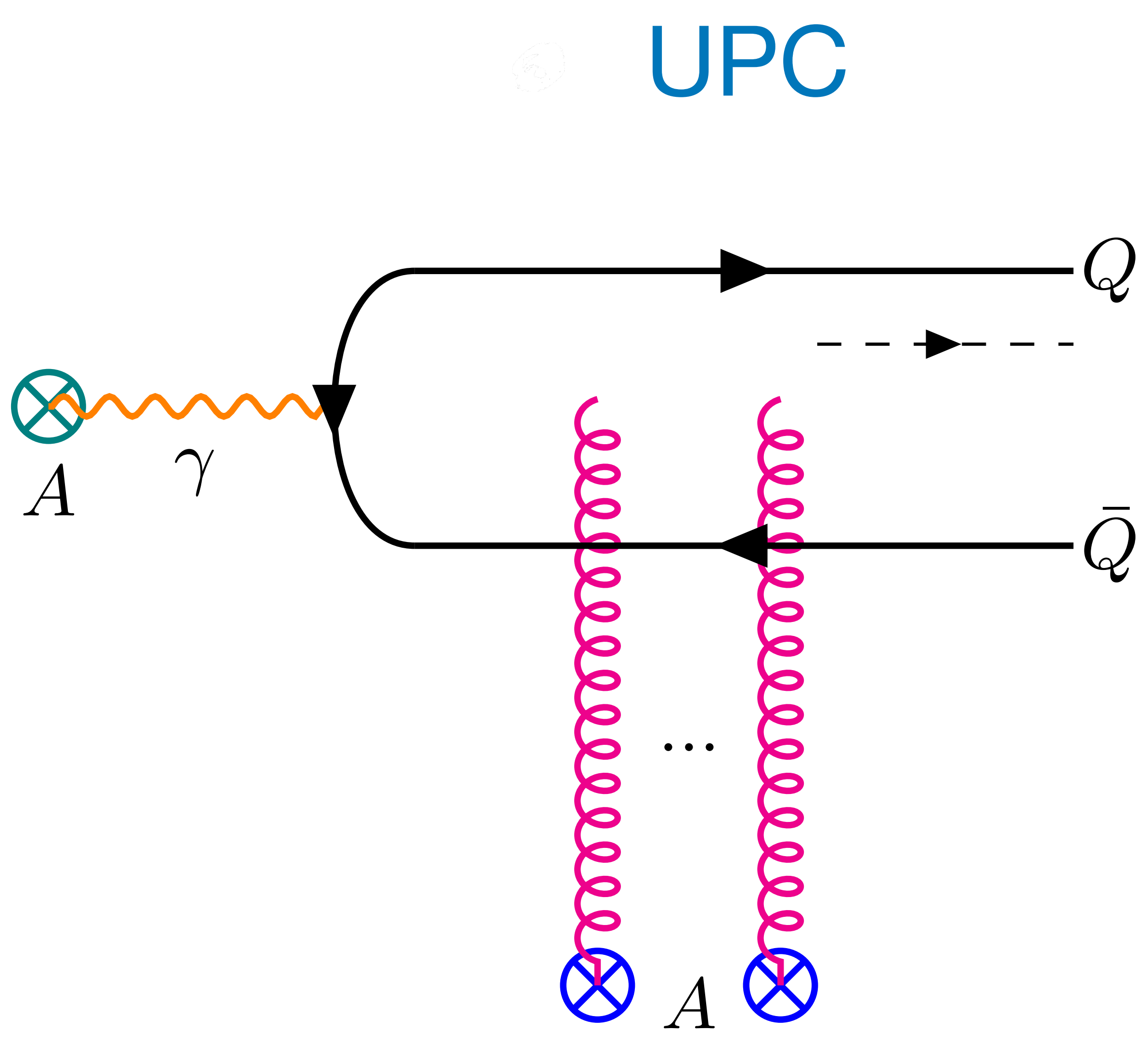
The heavy quark pair in HERA, UPC, future EIC

[Eur. Phys. J. C 50, 251 (2007),

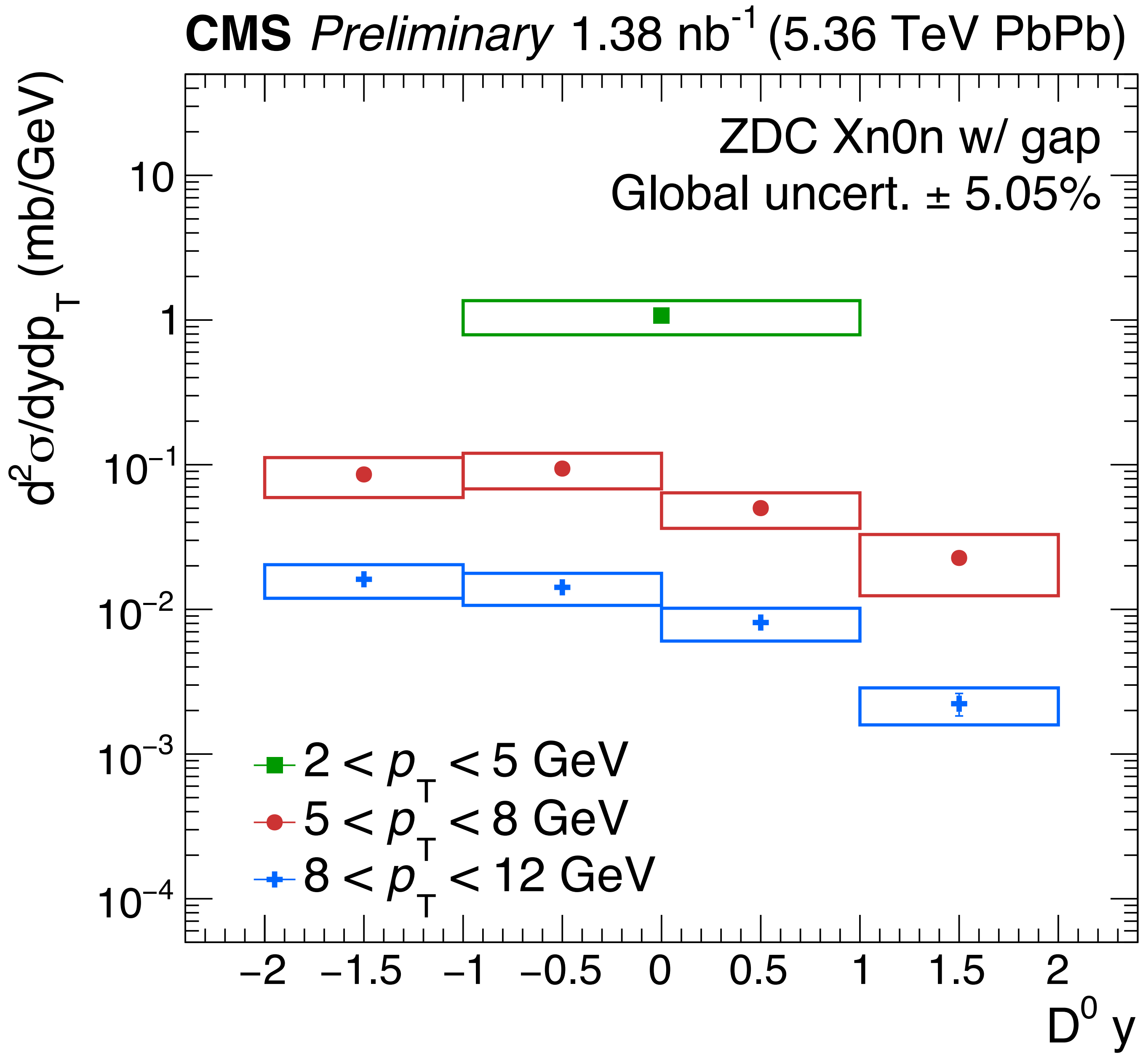
Eur. Phys. J.C 72 (2012)]





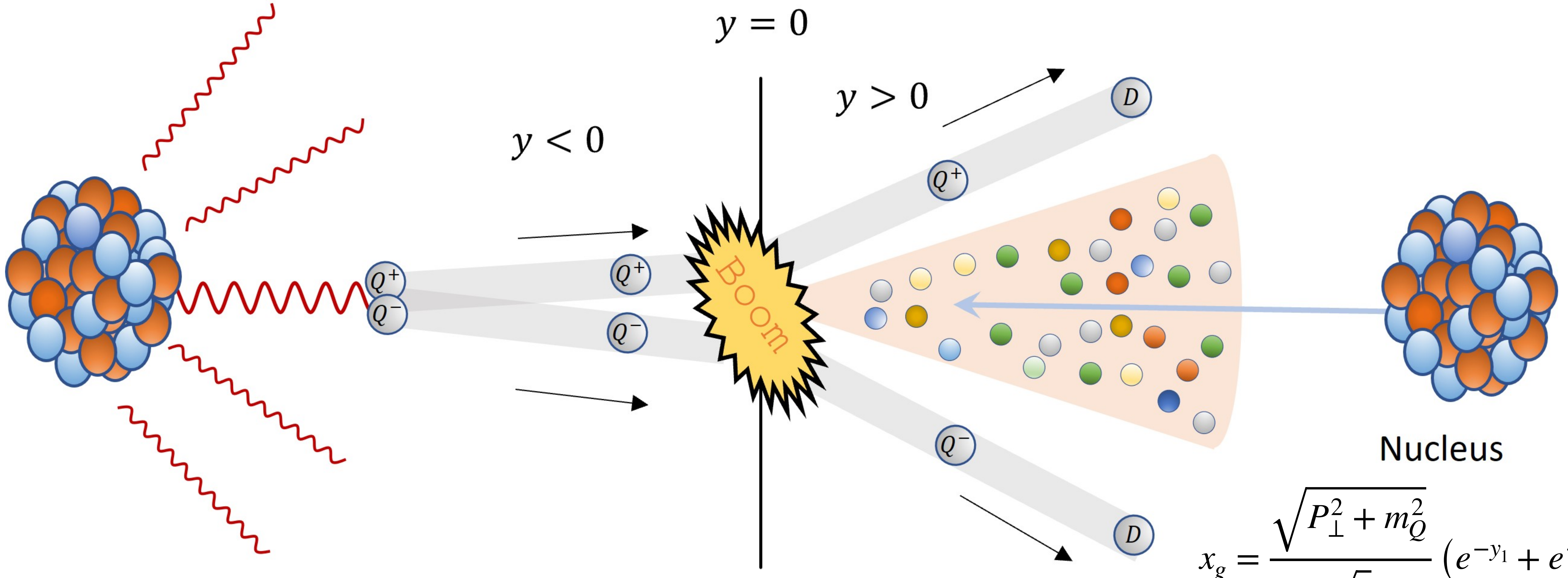


[CMS, Quark matter 2025]
Inclusive D photo-production



- D-jet correlations in the H1 data still lack a satisfactory pQCD explanation.
- CMS will measure heavy quark di-jets/hadrons with higher-statistics Run 3 data.
- More experimental data will be available in the near future !!!

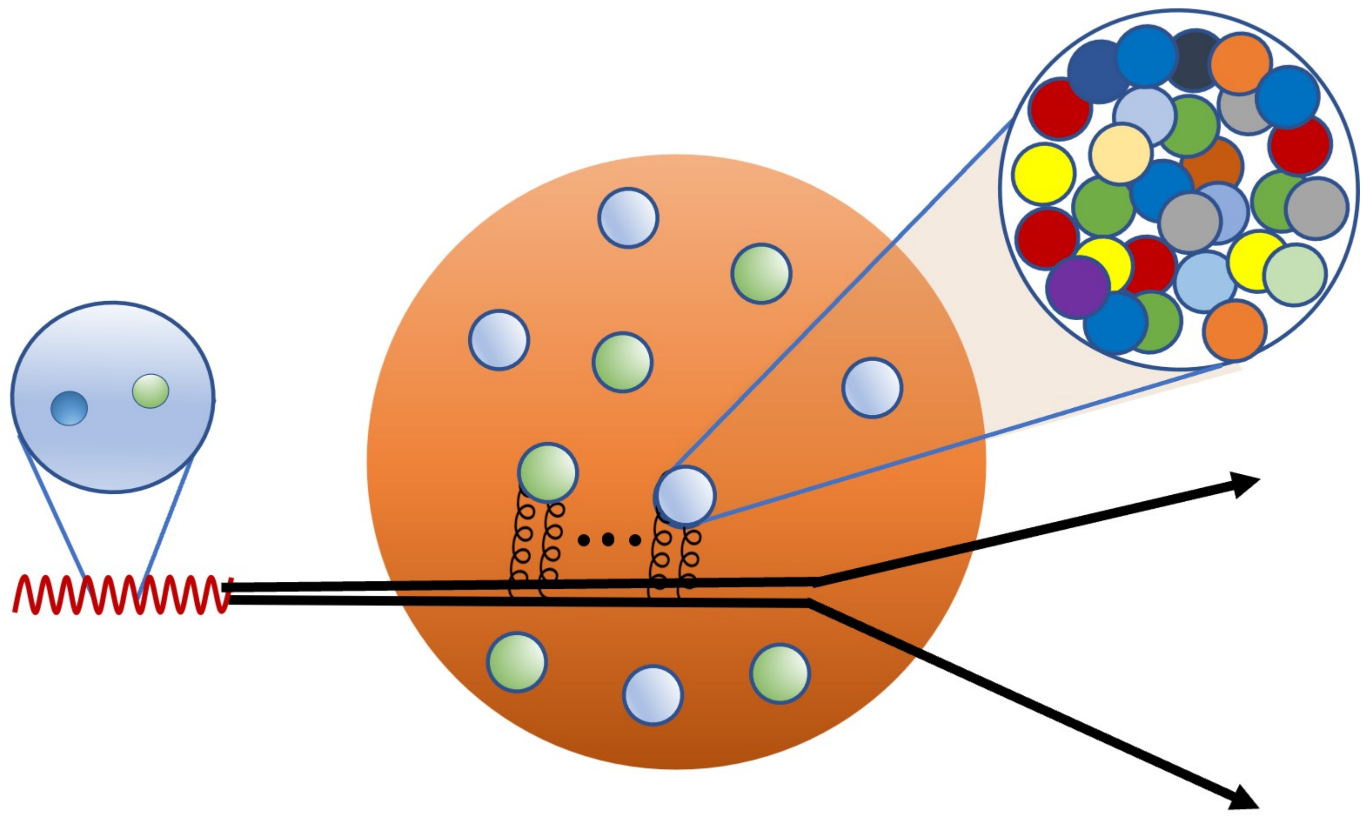
The heavy quark pair photo-production



$$x_g = \frac{\sqrt{P_\perp^2 + m_Q^2}}{\sqrt{s}}\left(e^{-y_1} + e^{-y_2}\right) \ll 1$$

quasi-real photon   heavy quark pair   small-x gluon! (Dense)

ultra-dense gluonic matter



nucleus

- This process can help us to study WW gluon or the gluon saturation effect in the small-x regime.

The heavy quark pair in UPC

In the back-to-back limit ($\boldsymbol{q} \ll \boldsymbol{P}$), it can be factorized as TMD hard factor ⊗ WW gluon dis

$$\frac{d\sigma_{\rm LO}^{\gamma A \to Q\bar{Q}X}}{d^2\boldsymbol{P} d^2\boldsymbol{q}} = x_\gamma f(x_\gamma) \otimes H_{\rm TMD}^{ij}(\boldsymbol{P}) \otimes xG_{ij}(x_g, \boldsymbol{q})$$

$$\propto xf(x_g, \boldsymbol{q}) + \cos(2\phi_{Pq}) xh(x_g, \boldsymbol{q})$$

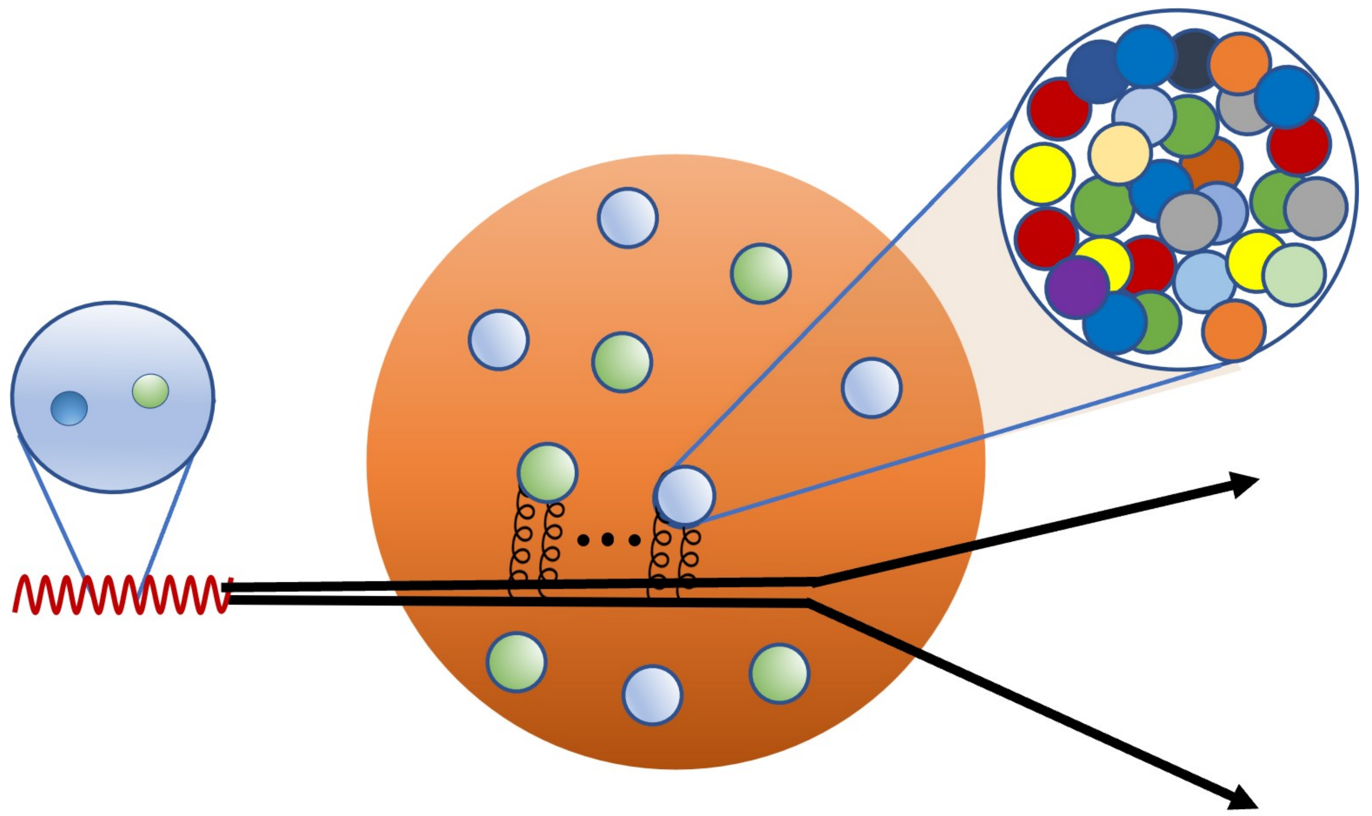
WW gluon

$$xG^{ij}(x_g, \boldsymbol{q}) = -\frac{4}{g_s^2}\int \frac{d^2\boldsymbol{x} d^2\boldsymbol{y}}{(2\pi)^3} e^{-\boldsymbol{q}\cdot(\boldsymbol{x}-\boldsymbol{y})} \langle {\rm Tr}\left[(\partial_i U_x) U_y^\dagger (\partial_i U_y) U_x^\dagger\right]\rangle$$

$$= \frac{\delta^{ij}}{2} xf(x_g, \boldsymbol{q}) + \left(\frac{\boldsymbol{q}_i \boldsymbol{q}_j}{\boldsymbol{q}^2} - \frac{\delta^{ij}}{2}\right) xh(x_g, \boldsymbol{q})$$

un-polarized linearly-polarized

ultra-dense gluonic matter



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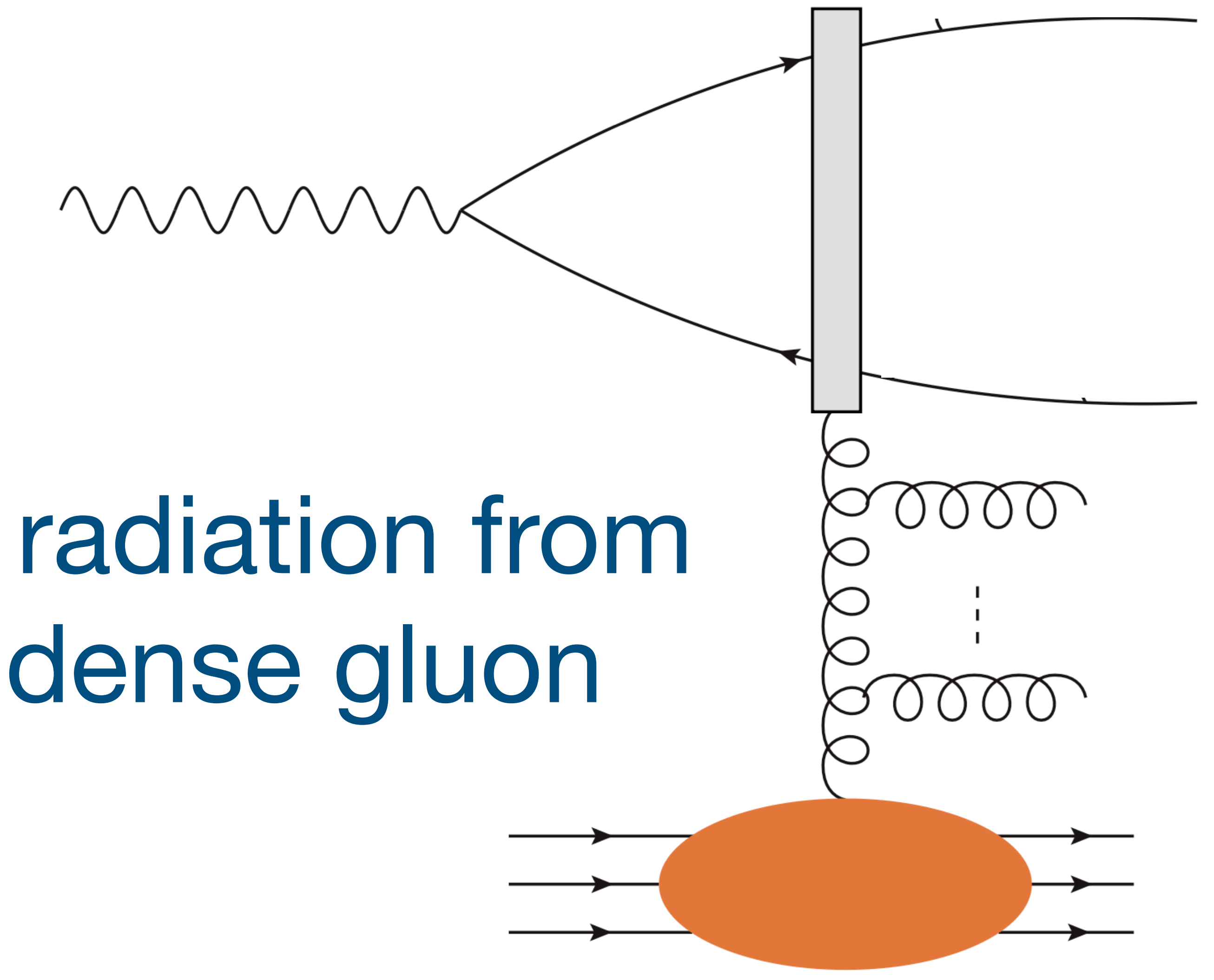
- This process can help us to study WW gluon or the gluon saturation effect in the small-x regime.

[Dominguez, Marquet, Xiao, Yuan, Phys.Rev.D 83 (2011) 105005; Metz, Zhou, Phys.Rev.D 84 (2011) 051503 ; Dominguez, Qiu, Xiao, Yuan, Phys.Rev.D 85 (2012) 045003]

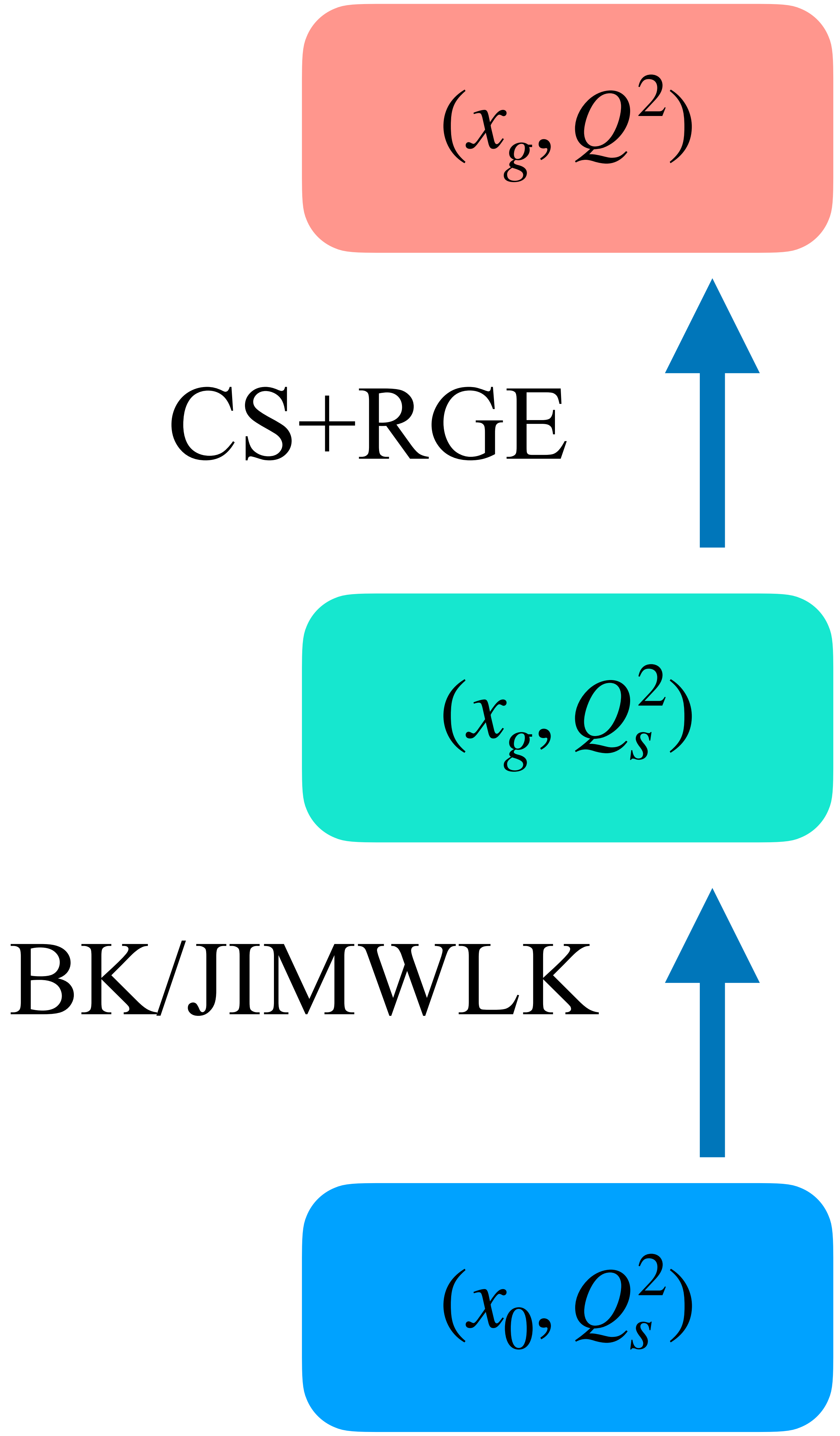
2) TMD evolution of WW gluon

TMD evolution of WW gluons in the small-x region

- In the back-to-back limit ($\boldsymbol{q} \ll \boldsymbol{P}$), large logs emerge. (e.g. in dijet or heavy-quark-pair)
- multiple well-separated hard scales:
- the invariant mass Q^2
- Momentum imbalance $\boldsymbol{q}$
- the momentum fraction x_g



Initial radiation from the dense gluon



$$P_{gg}(\xi) = 2C_A \frac{(\xi^2 - \xi + 1)^2}{\xi(1-\xi)}$$

$\xi \to 0$ $\quad \ln\left(1/x_g\right)$ ✓Small-x evolution equation

$\xi \to 1$ $\quad \ln^2\left(Q^2/\boldsymbol{q}^2\right) \quad \ln\left(Q^2/\boldsymbol{q}^2\right)$ ✓TMD evolution equation

TMD evolution of WW gluons in the small-x region

- Three methods to counting the multi-soft-gluon emission：

- 1. Resummation in coordinate space. (Sudakov Resummation)

[Mueller, Xiao, Yuan, PRL (12), PRD (13); Zheng, Aschenauer, Lee, Xiao, PRD, 14]

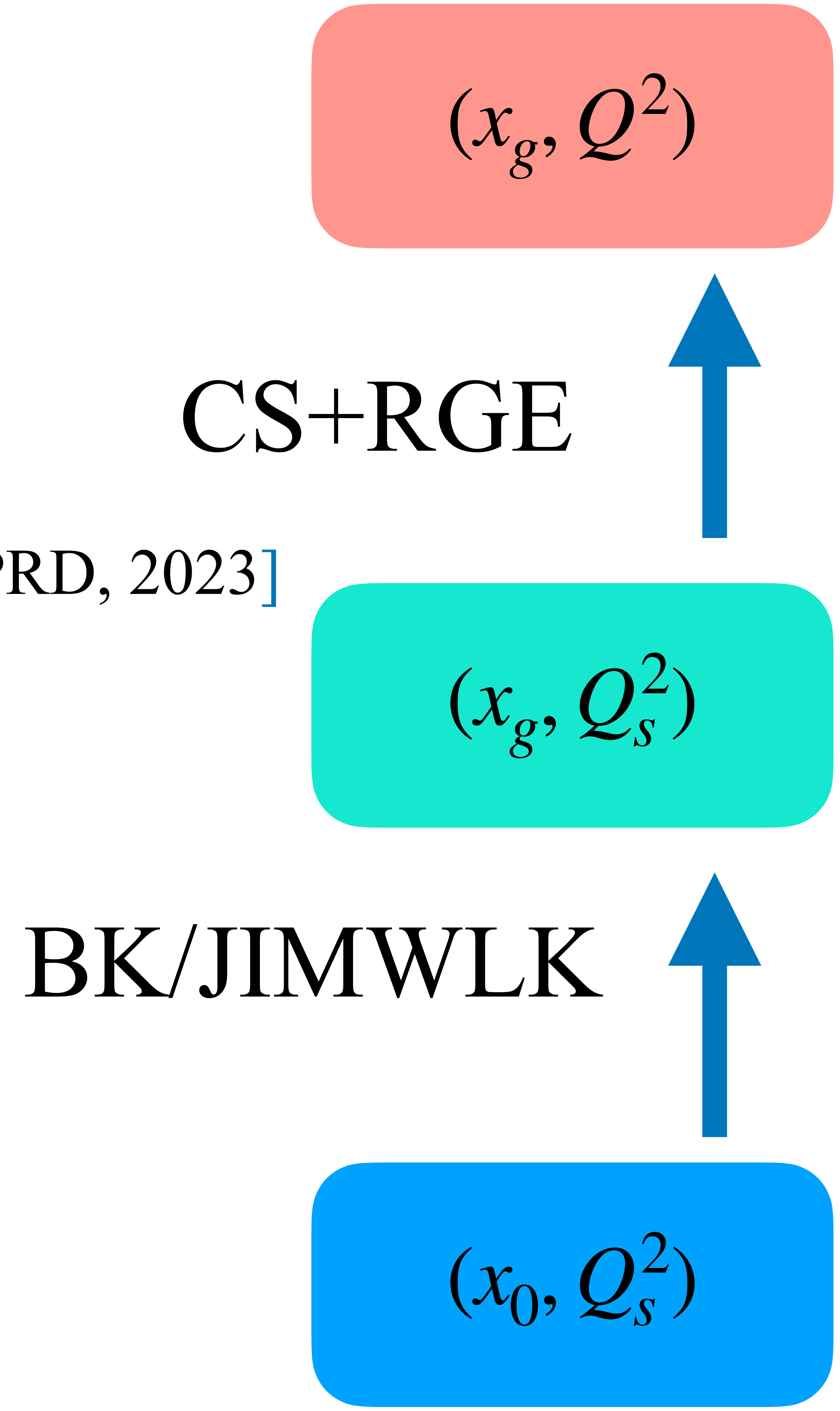
$$N(Q^2,\eta,k_\perp)=\int\frac{d^2b_\perp}{(2\pi)^2}e^{ik_\perp\cdot b_\perp}e^{-S(\mu_b^2,Q^2)}\int d^2l_\perp e^{-il_\perp\cdot b_\perp}N(\eta,l_\perp)$$

- 2. Evolution equation in momentum space. (CS+RGE)

[Collins, Soper, 81; Collins, Soper, Sterman, 85; Xiao, Yuan, Zhou, NPB, 17; YS, Wei, Zhou, PRD, 2023]

- 3. New Parton shower algorithm. (CS+RGE evolution equation)

[YS, Wei, Zhou, PRD, 2023]



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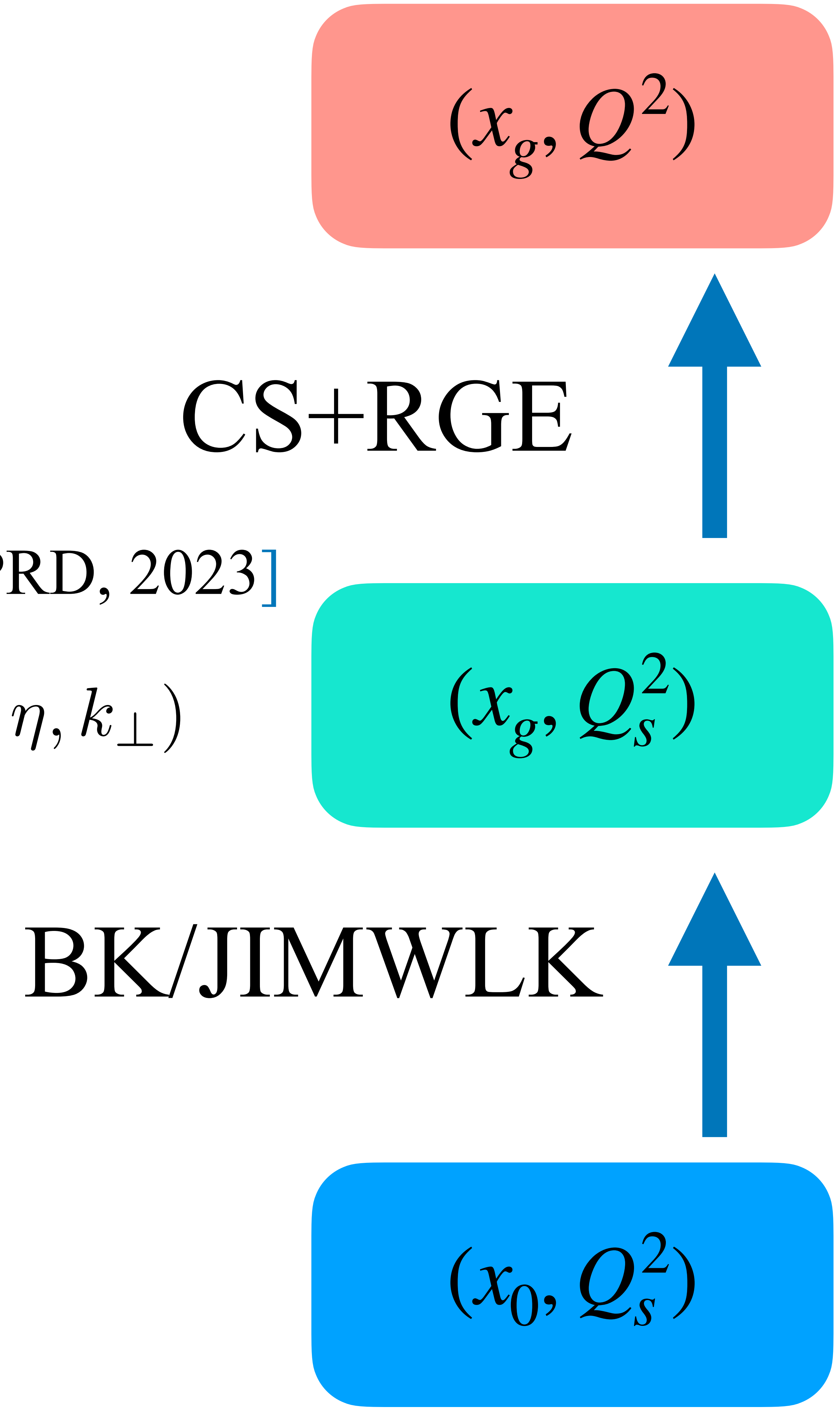
$$\frac{\partial N(Q^2,\eta,k_\perp)}{\partial \ln Q^2} = \frac{\bar{\alpha}_s}{2\pi}\int_0^Q \frac{d^2 l_\perp}{l_\perp^2}\left[N(Q^2,\eta,k_\perp+l_\perp) - N(Q^2,\eta,k_\perp)\right] + \bar{\alpha}_s\beta_0 N(Q^2,\eta,k_\perp)$$

- 3. New Parton shower algorithm. (CS+RGE evolution equation)

  [YS, Wei, Zhou, PRD, 2023]



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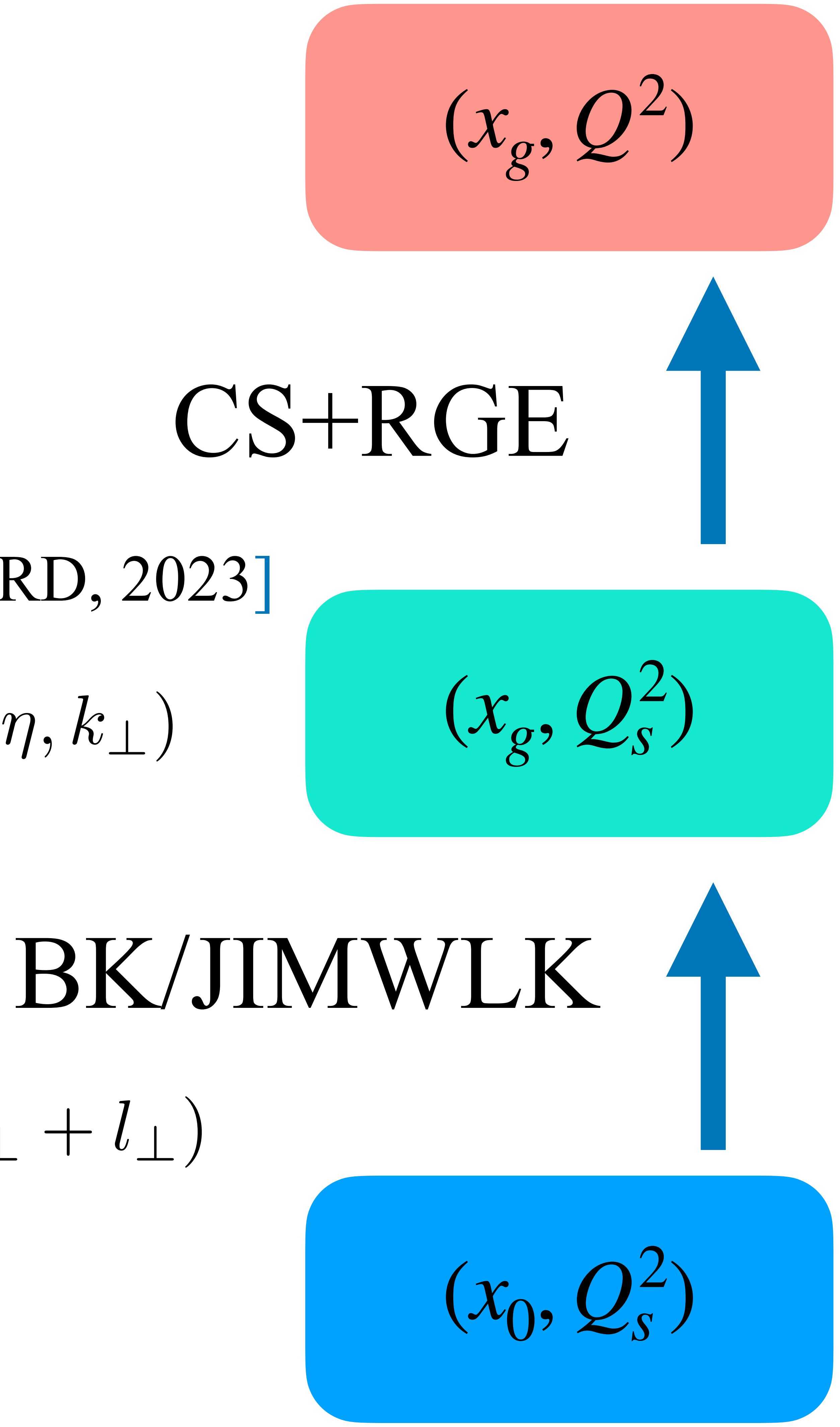
$$\frac{\partial N(Q^2,\eta,k_\perp)}{\partial\ln Q^2}=\frac{\bar\alpha_s}{2\pi}\int_0^Q\frac{d^2l_\perp}{l_\perp^2}\left[N(Q^2,\eta,k_\perp+l_\perp)-N(Q^2,\eta,k_\perp)\right]+\bar\alpha_s\beta_0N(Q^2,\eta,k_\perp)$$

- 3. New Parton shower algorithm. (CS+RGE evolution equation)

[YS, Wei, Zhou, PRD, 2023]

$$N(Q^2,\eta,k_\perp)=N(Q_0^2,\eta,k_\perp)\Delta_s(Q^2)+\int_{Q_0^2}^{Q^2}\frac{dt}{t}\frac{\Delta_s(Q^2)}{\Delta_s(t)}\frac{\bar\alpha_s(t)}{2\pi}\int_{\Lambda_{\rm cut}}^{Q}\frac{d^2l_\perp}{l_\perp^2}N(t,\eta,k_\perp+l_\perp)$$

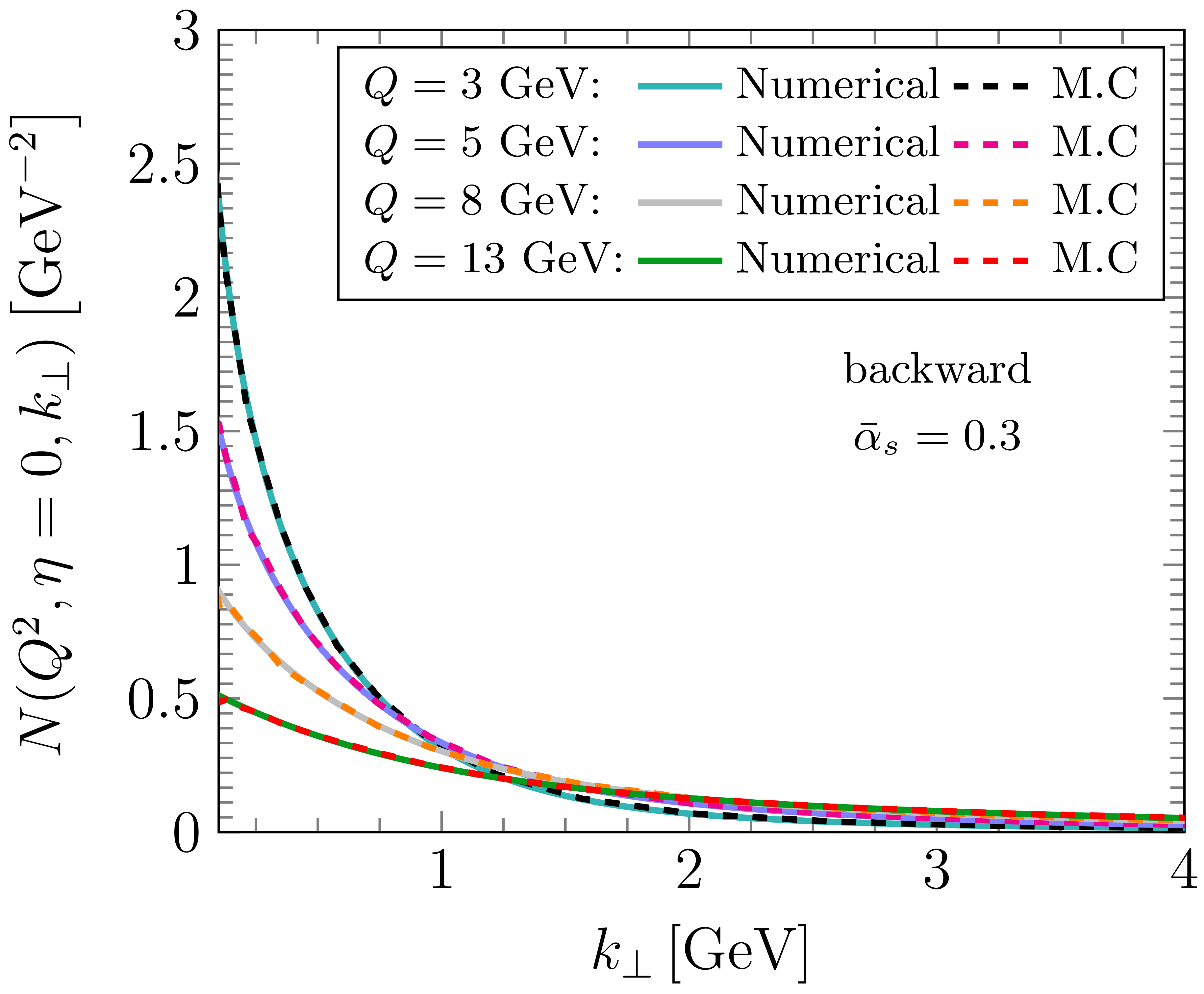
With Sudakov form factor $\Delta_s(Q^2)=\exp\left[-\int_{Q_0^2}^{Q^2}\frac{dt}{t}\frac{\bar\alpha_s(t)}{2}\left(\ln\frac{t}{\Lambda_{\rm cut}^2}-2\beta_0\right)\right]$



TMD evolution of WW gluons in the small-x region

- Three methods to counting the multi-soft-gluon emission：
- 1. Sudakov resummation in coordinate space.
- 2. CS+RGE evolution equation in momentum space.
- 3. New Parton shower algorithm based on CS+RGE evolution equation.

[YS, Wei, Zhou, PRD, 2023]



- Parton shower results agree with the numerical solutions.
- These three methods are equivalent !!!

3) Sudakov resummation in Heavy quark/meson pair photo-production

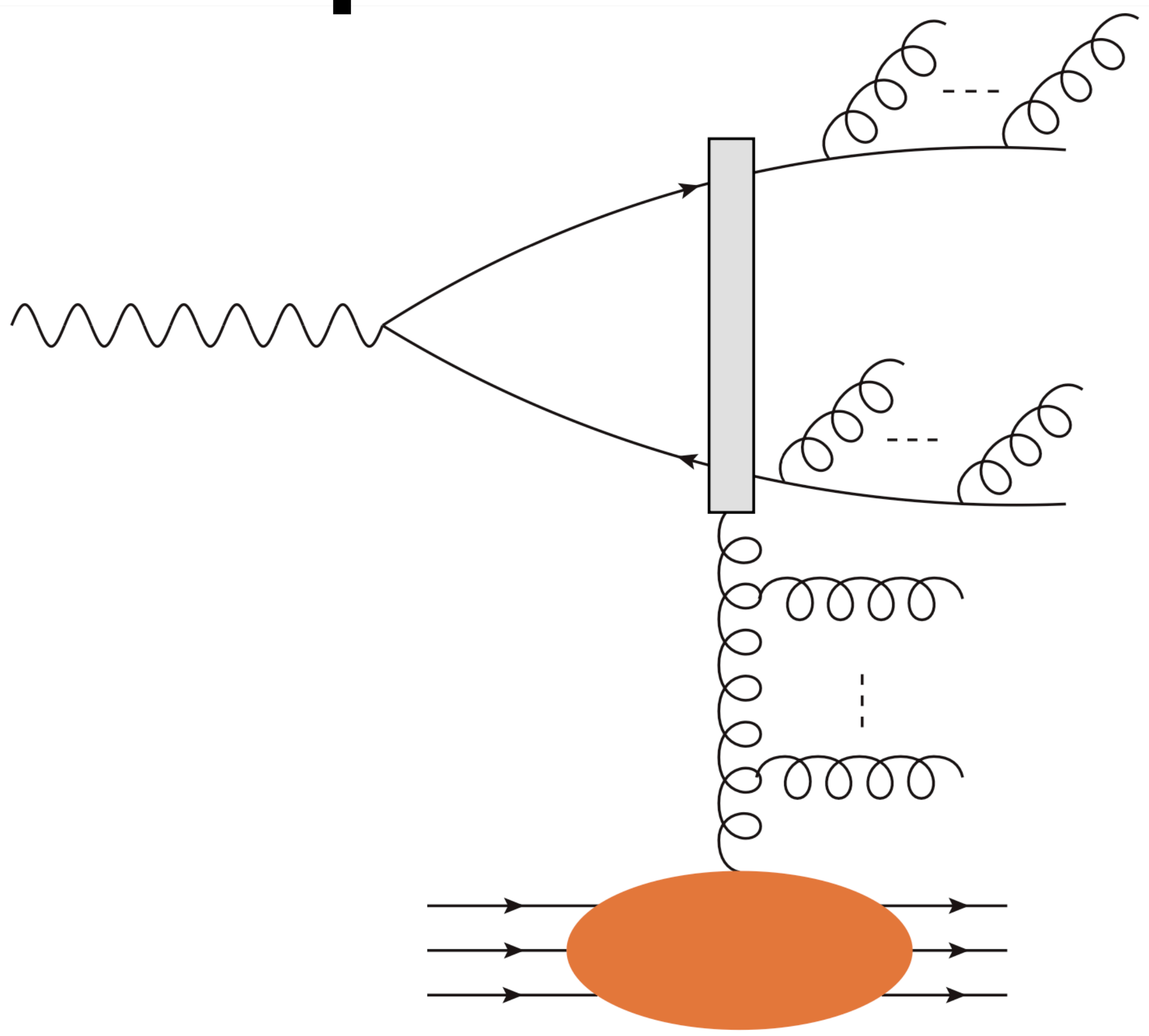
Soft gluon radiations and qt broadening

LO

azimuthal asymmetry

$$\frac{d\sigma^{LO}}{d^2\boldsymbol{q}} \propto xf(\boldsymbol{q}) + \cos(2\phi_{Pq})xh(\boldsymbol{q})$$

($\boldsymbol{q}$ ~ Saturation scale Q_s)



Final radiations from the quark pair

Initial radiation from the gluon

LO+ Soft-gluon radiations

$$\frac{d\sigma}{d^2\boldsymbol{q}} \propto \int \frac{d\sigma_{\mathrm{LO}}}{d^2\boldsymbol{q}'} \otimes \mathcal{S}_r(\boldsymbol{k}_g)\delta^{(2)}\left(\boldsymbol{q} - \boldsymbol{k}_g - \boldsymbol{q}'\right)$$

Soft gluon radiations

$$\mathcal{S}_r(\boldsymbol{k}_g) \propto \frac{\bar{\alpha}_s}{\boldsymbol{k}_g^2} \ln \frac{M_{in}^2}{\boldsymbol{k}_g^2} + \ldots \rightarrow \sum_{n=0}^{\infty} \frac{1}{n!}(\alpha_s L)^n = e^{\alpha_s L}$$

Soft-gluon radiations Saturation effect

[Muller, Xiao, Yuan, PRD88, 114010 (2013); Stasto, Wei, Xiao, Feng Yuan, Phys.Lett.B 784 (2018) 301-306; Hatta, Xiao, Yuan, Zhou, PRD104, 054037 (2021); Shao, YS, Zhang, Zhou, Zhou, JHEP 07 (2024) 189]

The Sudakov resummation in the heavy quark pair/di-lepton

$\gamma\gamma \to l^+l^-$

[Klein, Mueller, Xiao, Yuan, PRL.122, 132301 (2019); PRD 102, 094013 (2020); Hatta, Xiao, Yuan, Zhou, PRD, 21; YS, Chen, Wei, Xiao, PLB 862, 139317 (2025)]

$e^+e^- \to Q\bar{Q}$

[Aglietti, Ferrera, Eur.Phys.J.C 85 (2025) 3, 272]

$gg \to Q\bar{Q}$

[Catani, Grazzini, Torre, Nucl.Phys.B 890 (2014) 518-538; Catani, Grazzini , Sargsyan, JHEP 06 (2017) 017]

$\gamma g \to Q\bar{Q}$

[Muller, Xiao, Yuan, PRD 88, 114010 (2013)]

$P^2 \ll m_Q^2$

[Zhu, Sun, Yuan, PLB 727(2013) 474-479] Angular average

[Castillo, Echevarria, Makris, Scimemi, JHEP 01 (2021) 088] SCET

$P^2 \gg m_Q^2$

Our work focus on the Angular dependent…

The Sudakov resummation in the heavy quark pair photo-production

- we computed the Sudakov factor for heavy quark photo-production

- Following the method in [Hatta, Xiao, Yuan, Zhou, PRD, 21], we obtain

$$\frac{d\sigma}{dy_1 dy_2 d^2\boldsymbol{P} d^2\boldsymbol{q}} \propto \int \frac{d^2\boldsymbol{b}}{(2\pi)^2} e^{i\boldsymbol{q}\cdot\boldsymbol{b}} e^{-\mathrm{Sud}(\boldsymbol{b},M_{\mathrm{in}})} \left[1 + C_{2n}\cos(2n\phi_{bP})\right]$$
$$\times \int d^2\boldsymbol{q}' e^{-i\boldsymbol{q}'\cdot\boldsymbol{b}} \left[xf_{x_g}(\boldsymbol{q}') + \cos(2\phi_{Pq'}) xh_{x_g}(\boldsymbol{q}')\right]$$

- the Sudakov factor

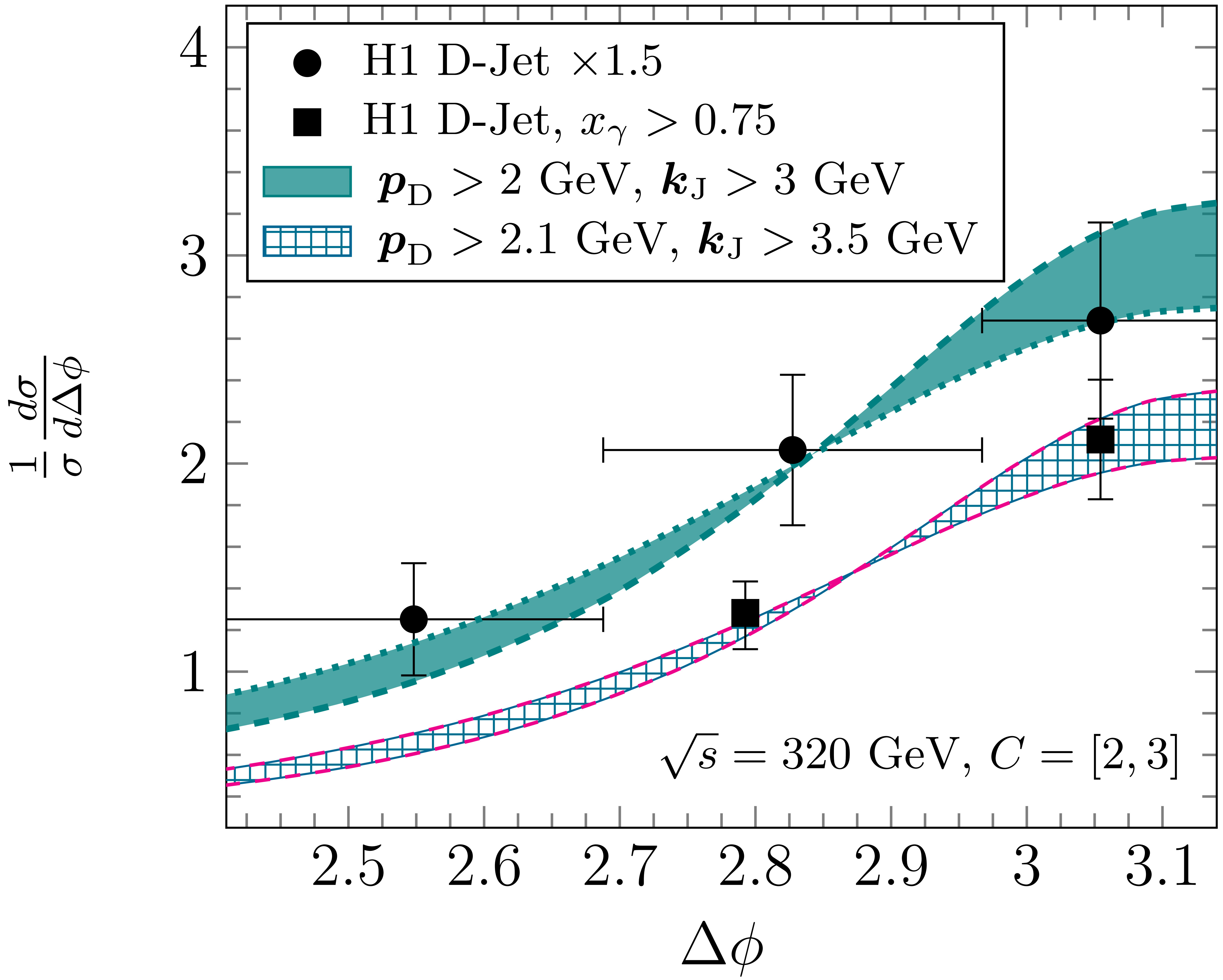
$$\mathrm{Sud}(\boldsymbol{b}, M_{\mathrm{in}}) = \mathrm{Sud}_i(\boldsymbol{b}, M_{\mathrm{in}}) + 2\mathrm{Sud}_f(\boldsymbol{b}, M_{\mathrm{in}}) + \mathrm{Sud}^i_{\mathrm{NP}}(\boldsymbol{b}, M_{\mathrm{in}})$$

Initial gluon radiation | Final heavy quarks radiation | Non-perturbative effects

[Marquet, YS, in preparation]

D-jet correlation in H1data

[Eur. Phys. J. C 50, 251 (2007), Eur. Phys. J.C 72 (2012)]



- H1 data can be explained by pQCD resummation for the first time.

the Sudakov resummation in heavy quark pair

- we computed the Sudakov factor for heavy quark production
- Following the method in [Hatta, Xiao, Yuan, Zhou, PRD, 21], we obtain

Soft gluon radiations also generate azimuthal asymmetry!!!

$$\frac{d\sigma}{dy_1 dy_2 d^2\boldsymbol{P} d^2\boldsymbol{q}} \propto \int \frac{d^2\boldsymbol{b}}{(2\pi)^2} e^{i\boldsymbol{q}\cdot\boldsymbol{b}} e^{-\mathrm{Sud}(\boldsymbol{b},M_{\mathrm{in}})} \left[1 + C_{2n}\cos(2n\phi_{bP})\right]$$
$$\times \int d^2\boldsymbol{q}' e^{-i\boldsymbol{q}'\cdot\boldsymbol{b}} \left[x f_{x_g}(\boldsymbol{q}') + \cos(2\phi_{Pq'}) x h_{x_g}(\boldsymbol{q}')\right]$$

Linearly-polarized gluon generate azimuthal asymmetry.

- the Sudakov factor

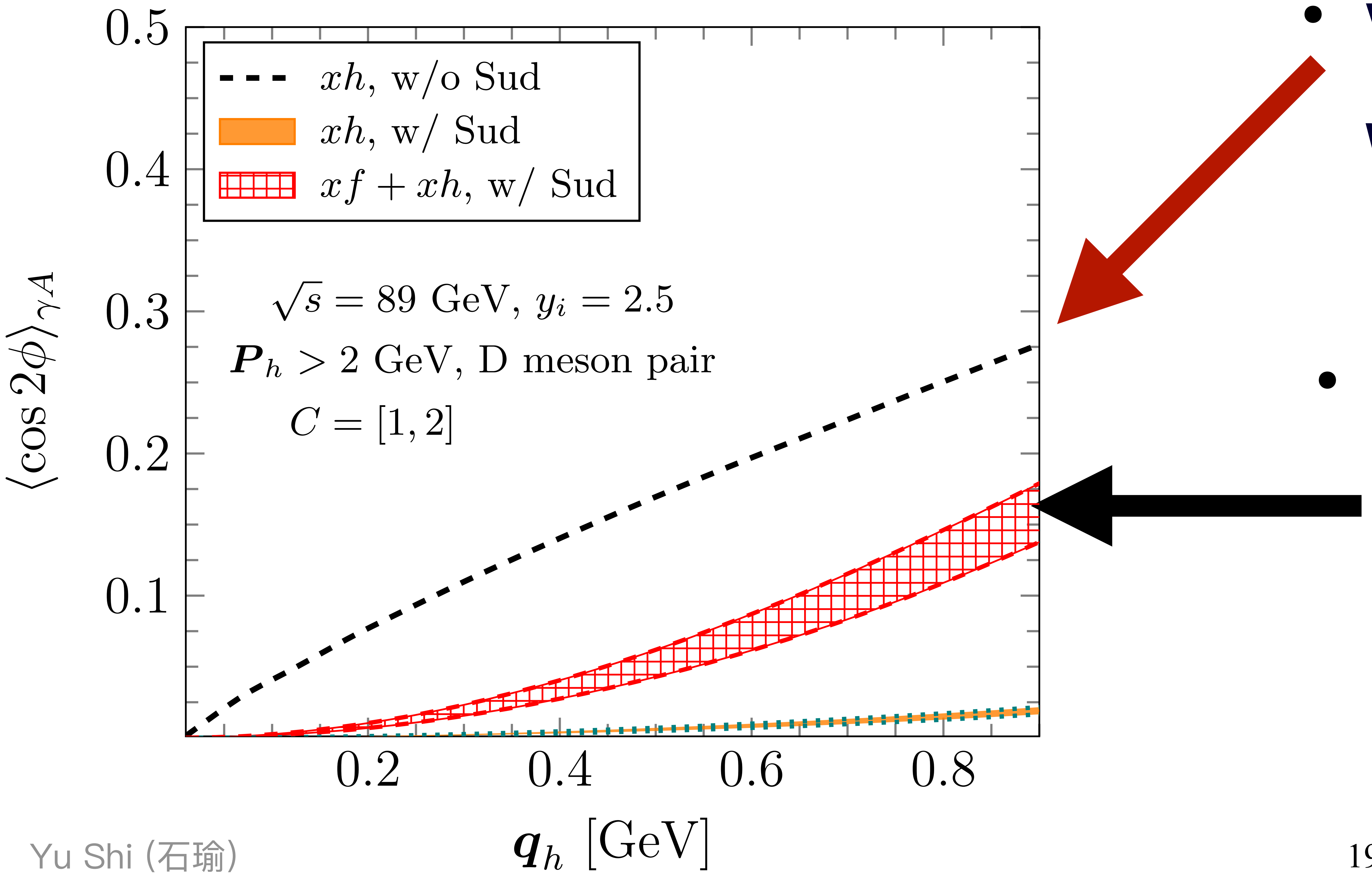
$$\mathrm{Sud}(\boldsymbol{b}, M_{\mathrm{in}}) = \mathrm{Sud}_i(\boldsymbol{b}, M_{\mathrm{in}}) + 2\mathrm{Sud}_f(\boldsymbol{b}, M_{\mathrm{in}}) + \mathrm{Sud}^i_{\mathrm{NP}}(\boldsymbol{b}, M_{\mathrm{in}})$$

Initial gluon radiation | Final heavy quarks radiation | Non-perturbative effects

[Marquet, YS, in preparation]

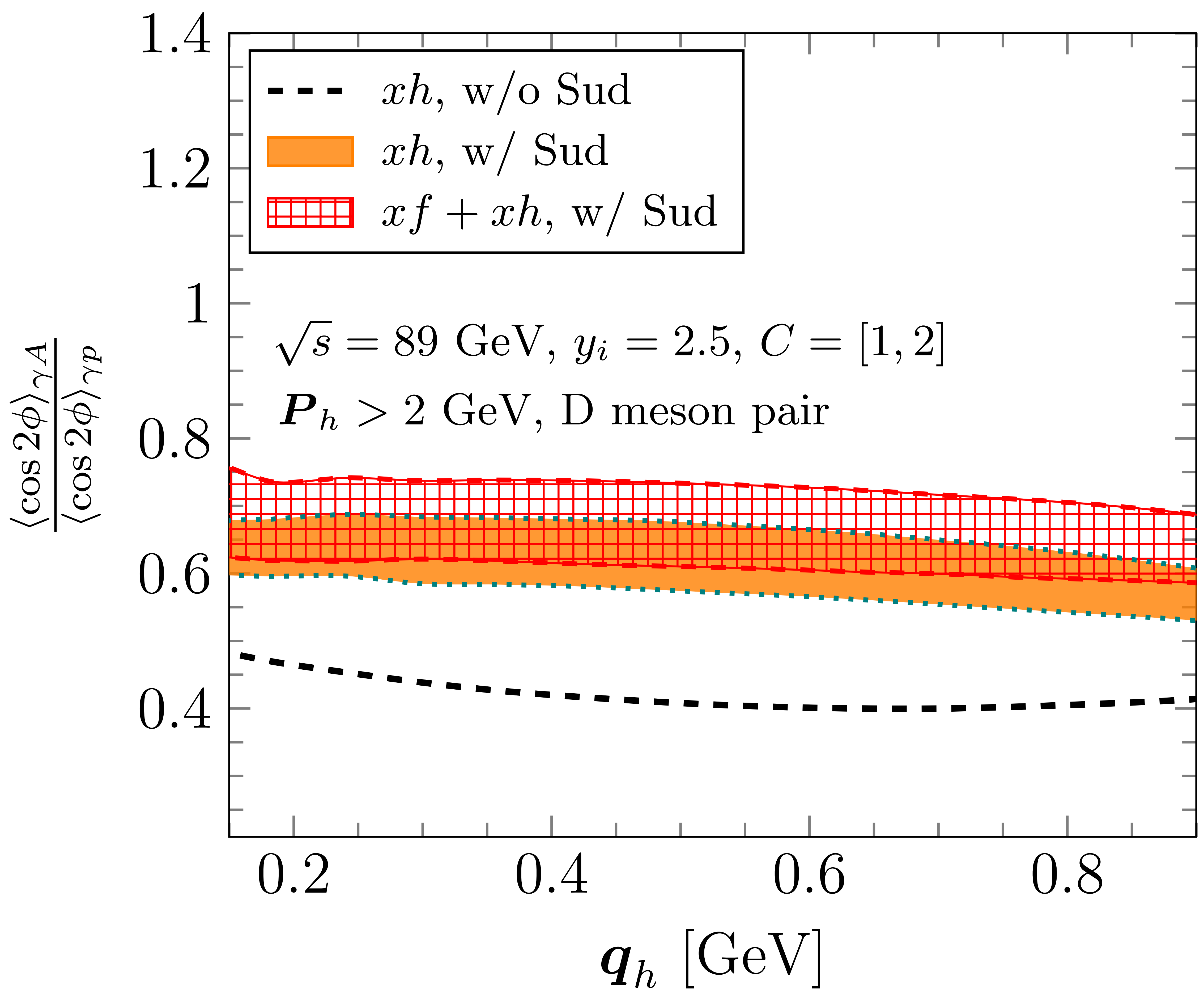
Preliminary results for heavy quark pair photo-production at EIC

$$\langle \cos 2\phi \rangle = \frac{\int d\mathscr{P}\,.\,\mathscr{S}\,.\cos(2\phi)\frac{d\sigma}{d\mathscr{P}\,.\,\mathscr{S}\,.}}{\int d\mathscr{P}\,.\,\mathscr{S}\,.\frac{d\sigma}{d\mathscr{P}\,.\,\mathscr{S}\,.}}$$



- without Sudakov effect, only linearly polarised WW gluon xh generates $\langle \cos 2\phi \rangle$.
- With Sudakov effect, the linearly polarised WW has been suppressed, and the soft-gluon radiation with unpolarised gluon xf dominates.

Preliminary results for heavy quark pair photo-production at EIC



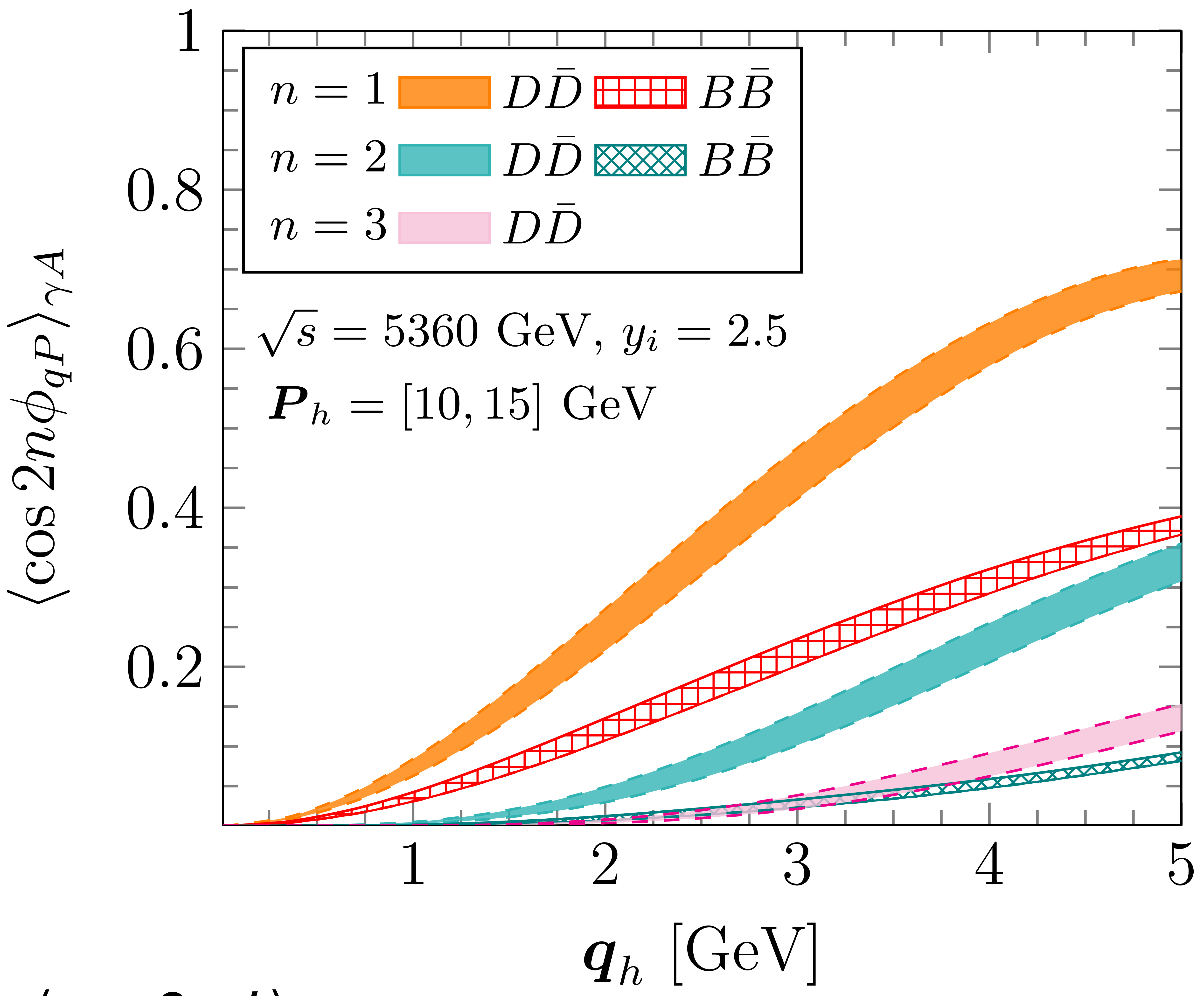
$$\frac{\langle \cos 2\phi \rangle_{\gamma A}}{\langle \cos 2\phi \rangle_{\gamma p}} = 1$$ → No saturation effect

$$\frac{\langle \cos 2\phi \rangle_{\gamma A}}{\langle \cos 2\phi \rangle_{\gamma p}} < 1$$ → saturation effect emerges

strong saturation effect!!!

$\frac{\langle \cos 2\phi \rangle_{\gamma A}}{\langle \cos 2\phi \rangle_{\gamma p}}$ suppression survives after the Sudakov effect is included.

Preliminary results for D&B meson pair photo-production at LHC regime



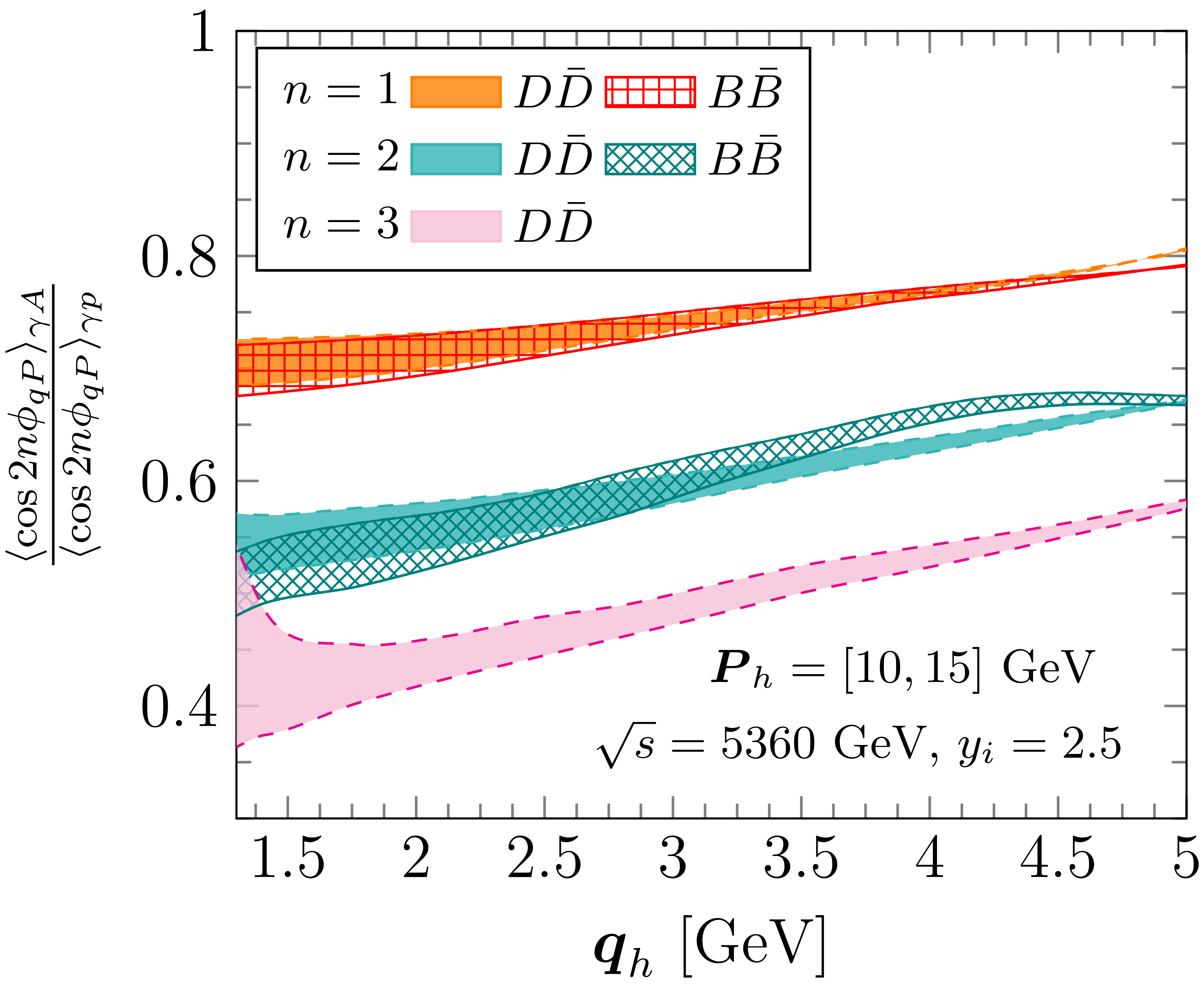
$$\langle \cos 6\phi_{qP} \rangle < \langle \cos 4\phi_{qP} \rangle < \langle \cos 2\phi_{qP} \rangle$$

$$\langle \cos 2n\phi_{qP} \rangle \propto c_{2n} \int \ldots \propto \ln \frac{M_{\rm in}^2}{m_Q^2} \int \ldots$$

$\langle \cos 2n\phi \rangle$ show a clear mass hierarchy

$$\langle \cos 2n\phi \rangle_{m_c} > \langle \cos 2n\phi \rangle_{m_b}$$

Preliminary results for D&B meson pair photo-production at LHC regime



$$\frac{\langle \cos 6\phi_{qP}\rangle_{\gamma A}}{\langle \cos 6\phi_{qP}\rangle_{\gamma p}} < \frac{\langle \cos 4\phi_{qP}\rangle_{\gamma A}}{\langle \cos 4\phi_{qP}\rangle_{\gamma p}} < \frac{\langle \cos 2\phi_{qP}\rangle_{\gamma A}}{\langle \cos 2\phi_{qP}\rangle_{\gamma p}} < 1$$

$$\langle \cos 2n\phi_{qP}\rangle \propto c_{2n} \int \ldots \propto \ln \frac{M_{\rm in}^2}{m_Q^2} \int \ldots$$

It reveal increasingly strong suppression at higher harmonic order.

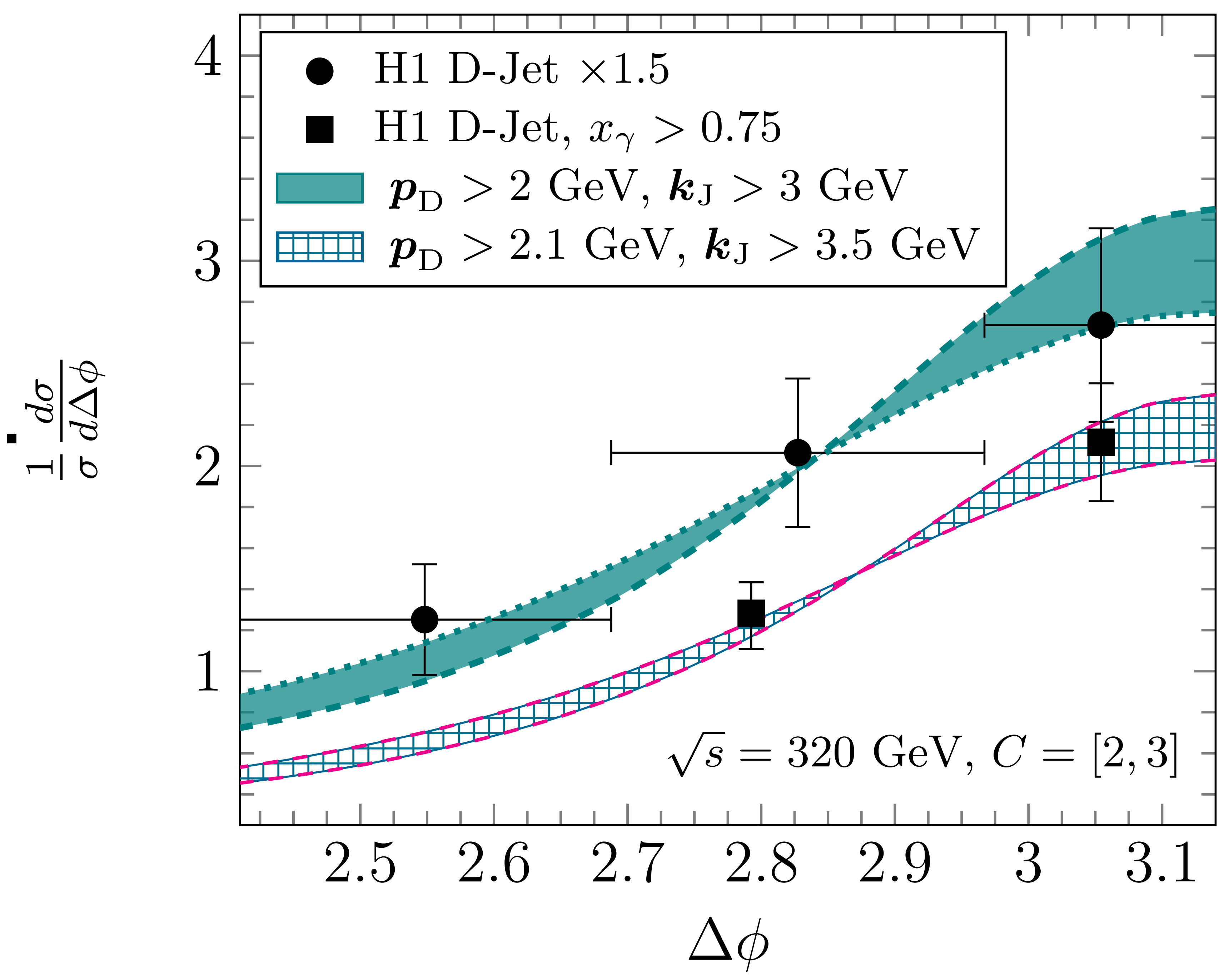
The $\cos(2n\phi)$ ratios may serve as a good process to detect the saturation

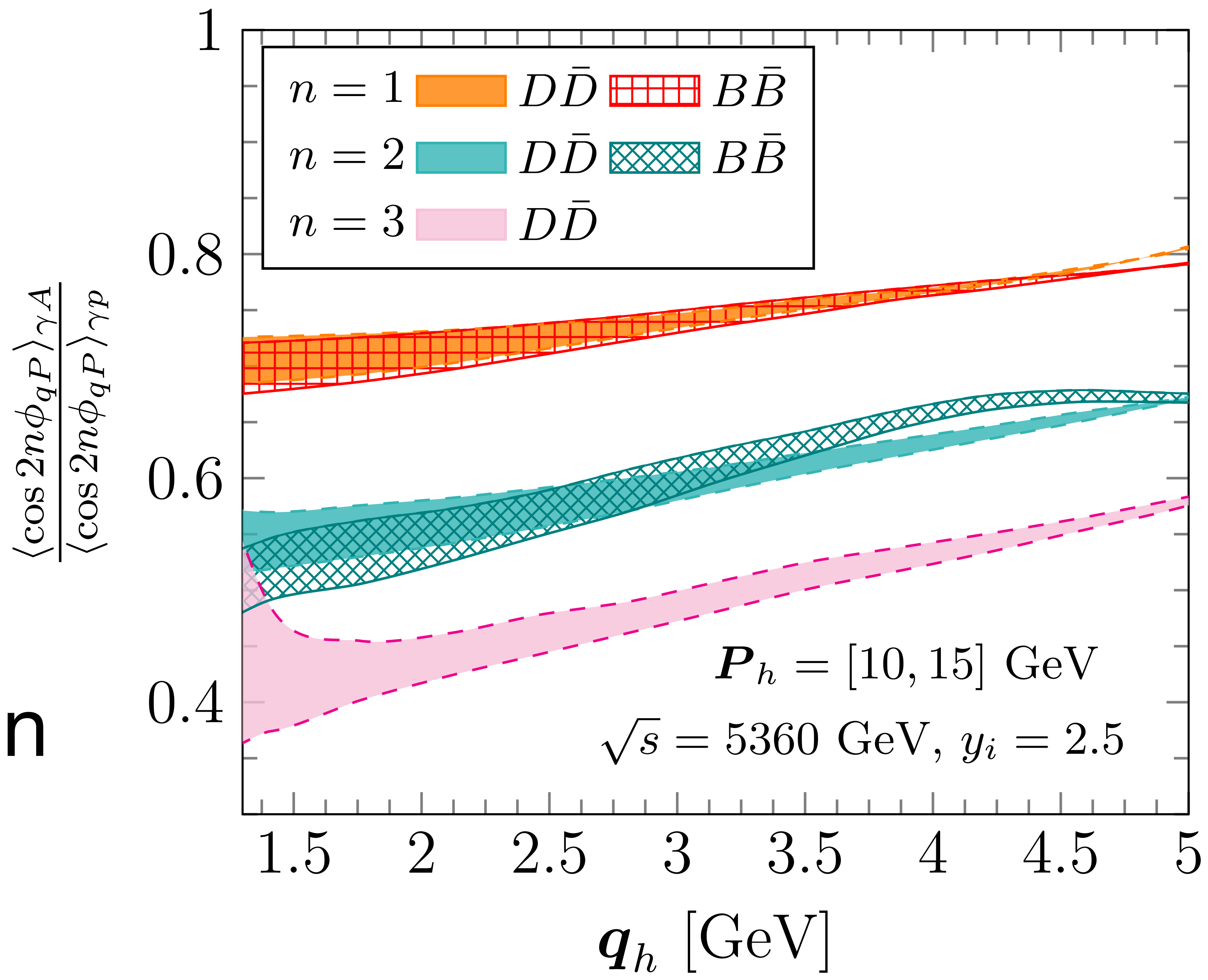
Summary

- We investigate the impact of the multi-soft-gluon radiation on heavy quark/meson pair photo-production (small x + Sudakov).
- Two observables: angle correlation and azimuthal Asymmetry.
- H1 data can be explained by pQCD resummation for the first time.
- Linearly-polarized WW gluon has been suppressed by the Sudakov effect.

Question: how to detect the linearly-polarized WW gluon?

- Sensitivity of the $\dfrac{\langle \cos 2n\phi \rangle_A}{\langle \cos 2n\phi \rangle_p}$ ratio to saturation effect.
  - Great potential in searching for compelling evidences of gluon saturation in the near future.





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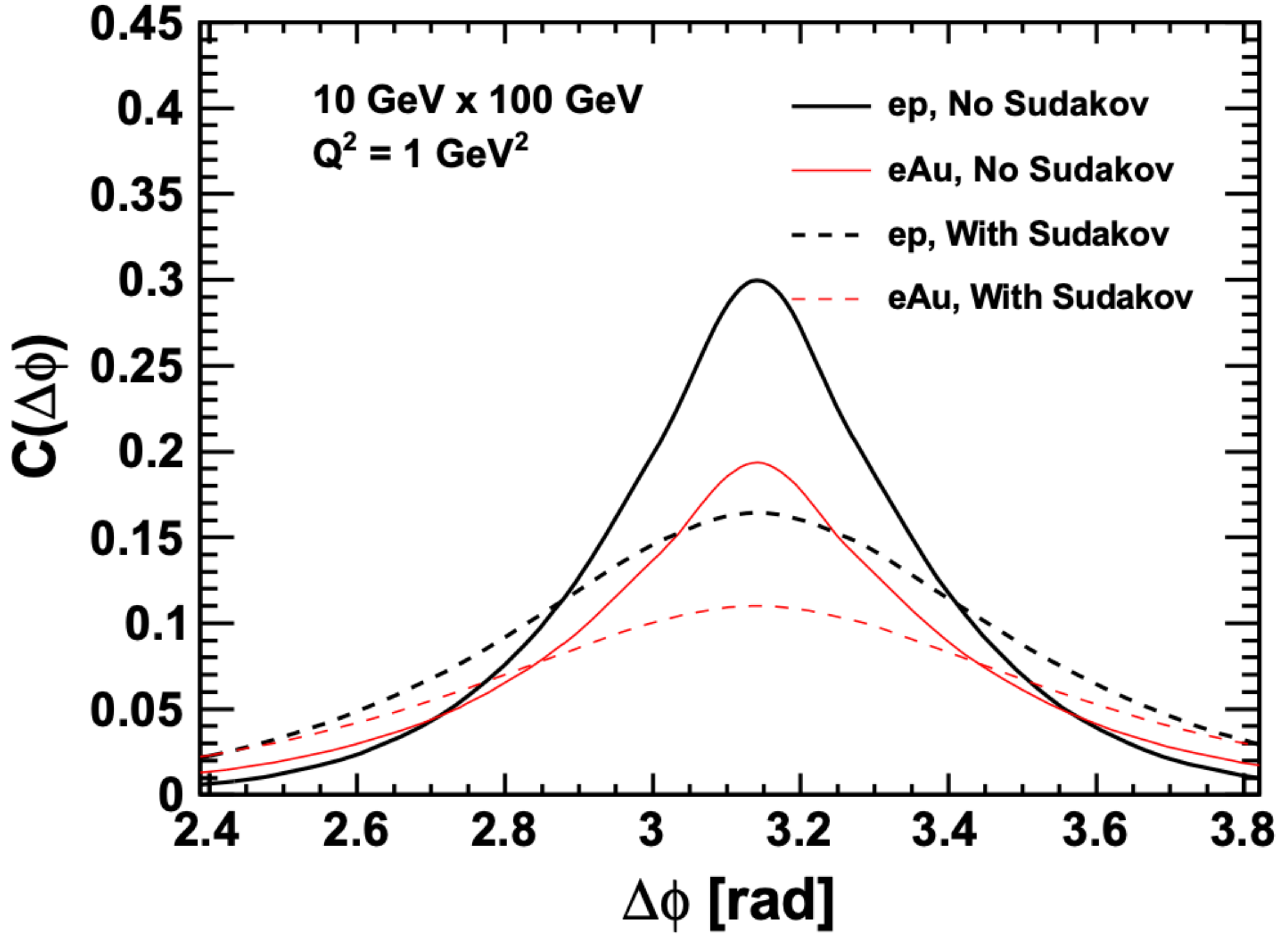
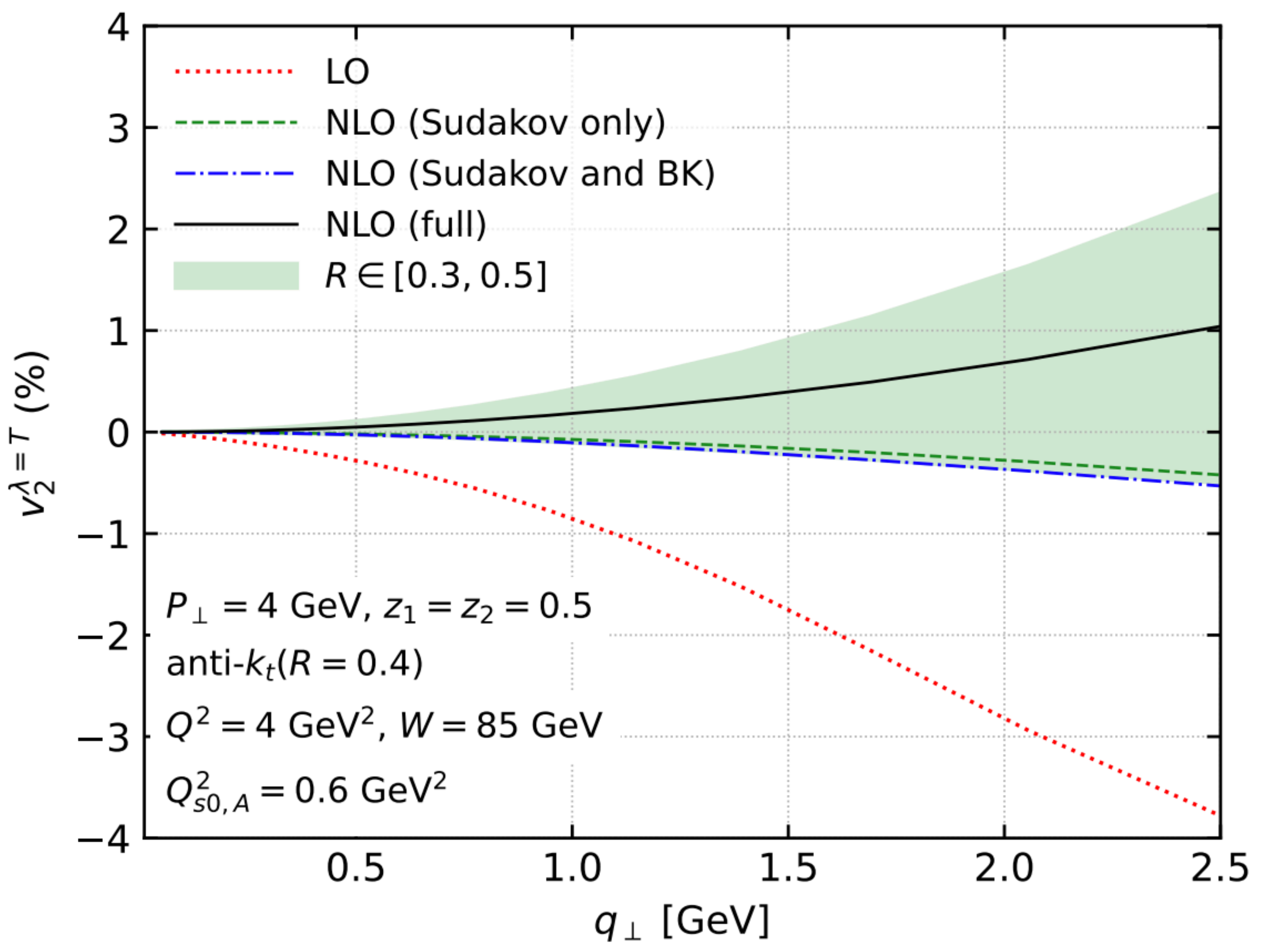
Examples: Sudakov resummation in the dijet/dihadron case

- Without Sudakov resummation, the CGC predicts sizable angular correlations and asymmetries.

[Zheng, Aschenauer, Lee, Xiao, PRD, 14]



[Caucal, Salazar, Schenke, Stebel and Venugopalan, PRL, 24]



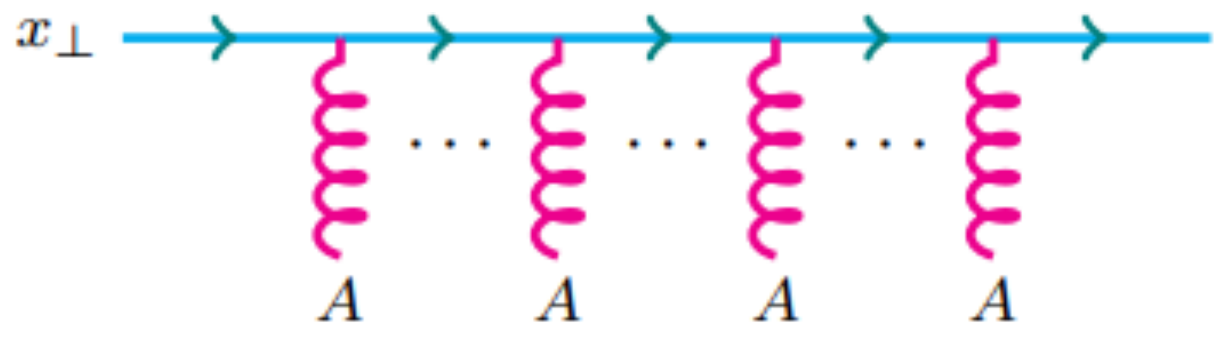
$$\langle \cos 2\phi \rangle \propto \frac{x h_{x_g}(\boldsymbol{q})}{x f_{x_g}(\boldsymbol{q})}$$

- Soft-gluon emission (Sudakov effect) have a strong impact on the back-to-back regime.
- TMD (CSS) evolution suppresses angular asymmetry. [Boer, Mulders, Zhou and Zhou, 17]

Wilson lines in CGC

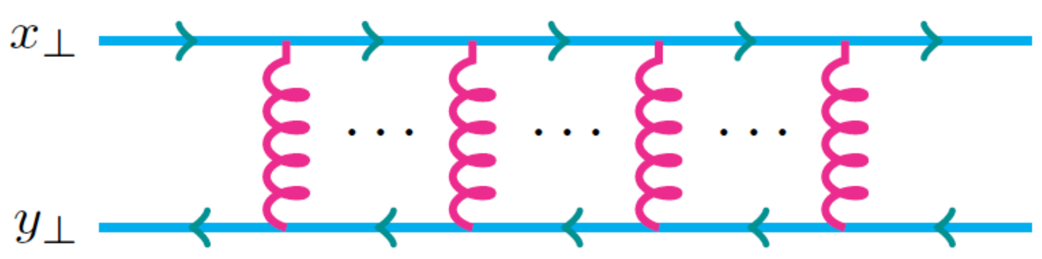
- Consider the multiple scattering between a fast quark and target background gluon fields.

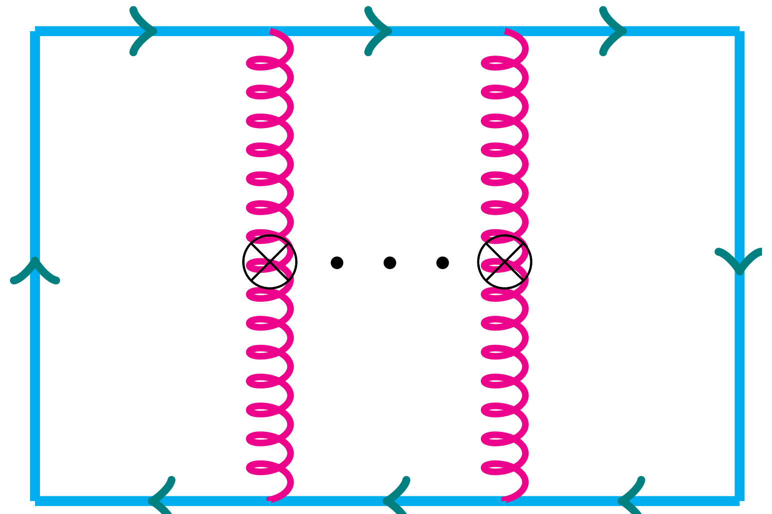
$$U(x_\perp) = \mathscr{P} \exp\left(-ig \int dz^+ A^-(x_\perp, z^+)\right)$$



- The Wilson loop (color singlet dipole)

$$\frac{1}{N_c}\langle \mathrm{Tr}\left[U_x U_y^\dagger\right]\rangle$$





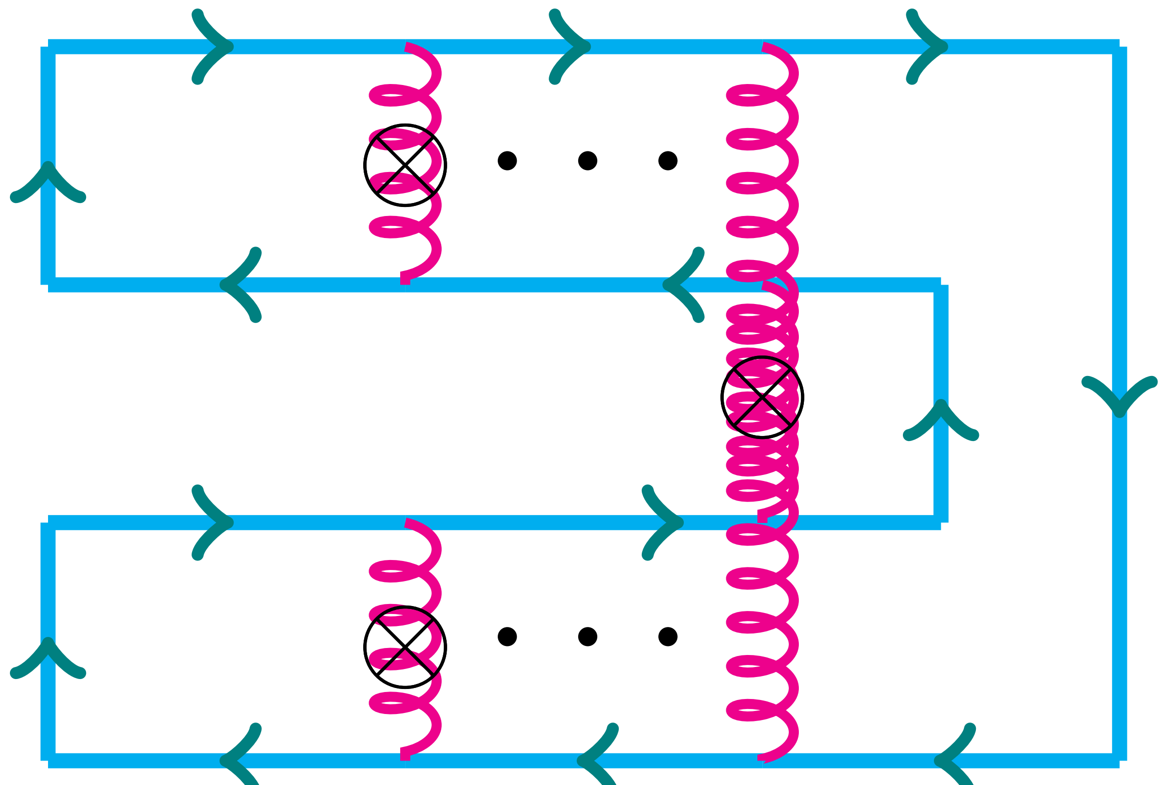
Dipole (DP gluon)

- Weizsacker-Williams (WW) gluon → Quadrupole

$$xG^{ij}(x_g, \boldsymbol{q}) = -\frac{4}{g_s^2}\int \frac{d^2\boldsymbol{x} d^2\boldsymbol{y}}{(2\pi)^3} e^{-\boldsymbol{q}\cdot(\boldsymbol{x}-\boldsymbol{y})}\langle \mathrm{Tr}\left[(\partial_i U_x)U_y^\dagger(\partial_i U_y)U_x^\dagger\right]\rangle$$

$$= \frac{\delta^{ij}}{2} xf(x_g, \boldsymbol{q}) + \left(\frac{\boldsymbol{q}_i \boldsymbol{q}_j}{\boldsymbol{q}^2} - \frac{\delta^{ij}}{2}\right) xh(x_g, \boldsymbol{q})$$

un-polarized linearly-polarized



Quadrupole (WW gluon)

[Dominguez, Marquet, Xiao, Yuan, Phys.Rev.D 83 (2011) 105005; Metz, Zhou, Phys.Rev.D 84 (2011) 051503 ; Dominguez, Qiu, Xiao, Yuan, Phys.Rev.D 85 (2012) 045003]