The Renormalization Group for LSS

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(LMU/Cambridge)

In collaboration with Fabian Schmidt (MPA)

and also: Charalampos Nikolis, Mathias Garny, Thomas Bakx, Zvonimir Vlah

Benasque, July 2025

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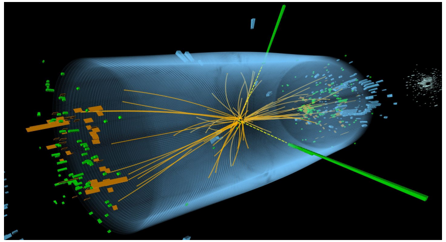
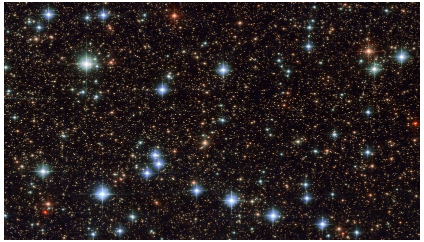
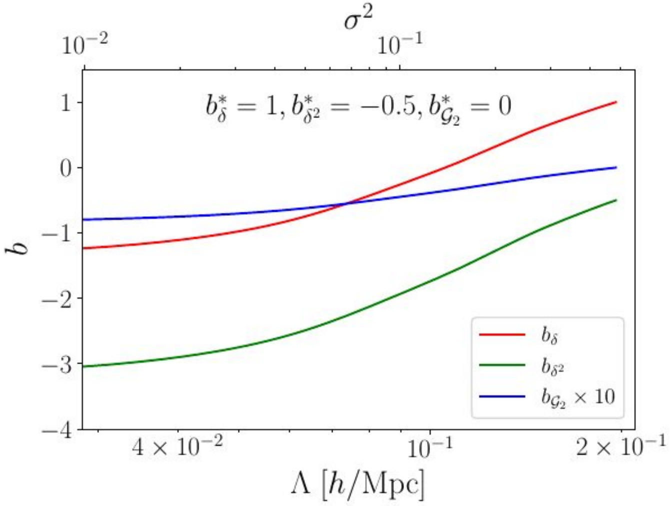
Based on:
2307.15031, 2404.16929,
2405.21002, 2507.13905

Message to take home

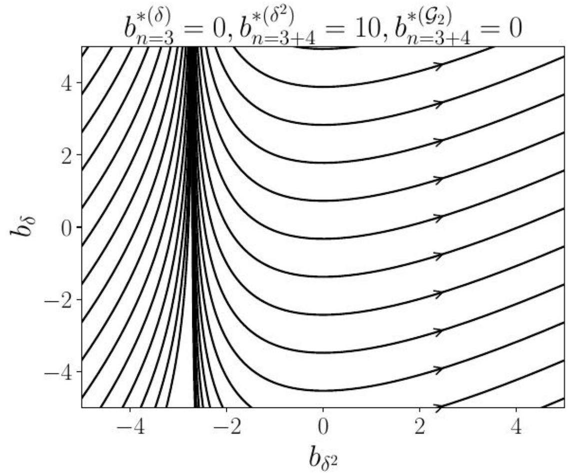


$\sim$





$$\frac{\partial g}{\partial \ln \mu} = \beta(g) \quad \sim \quad \begin{aligned} \frac{db_\delta}{d\Lambda} &= -\left[\frac{68}{21} b_{\delta^2} + 3 b^*_{\delta^3} - \frac{4}{3} b^*_{\mathcal{G}_2 \delta}\right] \frac{d\sigma^2_\Lambda}{d\Lambda}\,, \\ \frac{db_{\delta^2}}{d\Lambda} &= -\left[\frac{8126}{2205} b_{\delta^2} + \frac{17}{7} b^*_{\delta^3} - \frac{376}{105} b^*_{\mathcal{G}_2 \delta} + b^{*(\delta^2)}_{n=4}\right] \frac{d\sigma^2_\Lambda}{d\Lambda}\,, \\ \frac{db_{\mathcal{G}_2}}{d\Lambda} &= -\left[\frac{254}{2205} b_{\delta^2} + \frac{116}{105} b^*_{\mathcal{G}_2 \delta} + b^{*(\mathcal{G}_2)}_{n=4}\right] \frac{d\sigma^2_\Lambda}{d\Lambda}\,. \end{aligned}$$

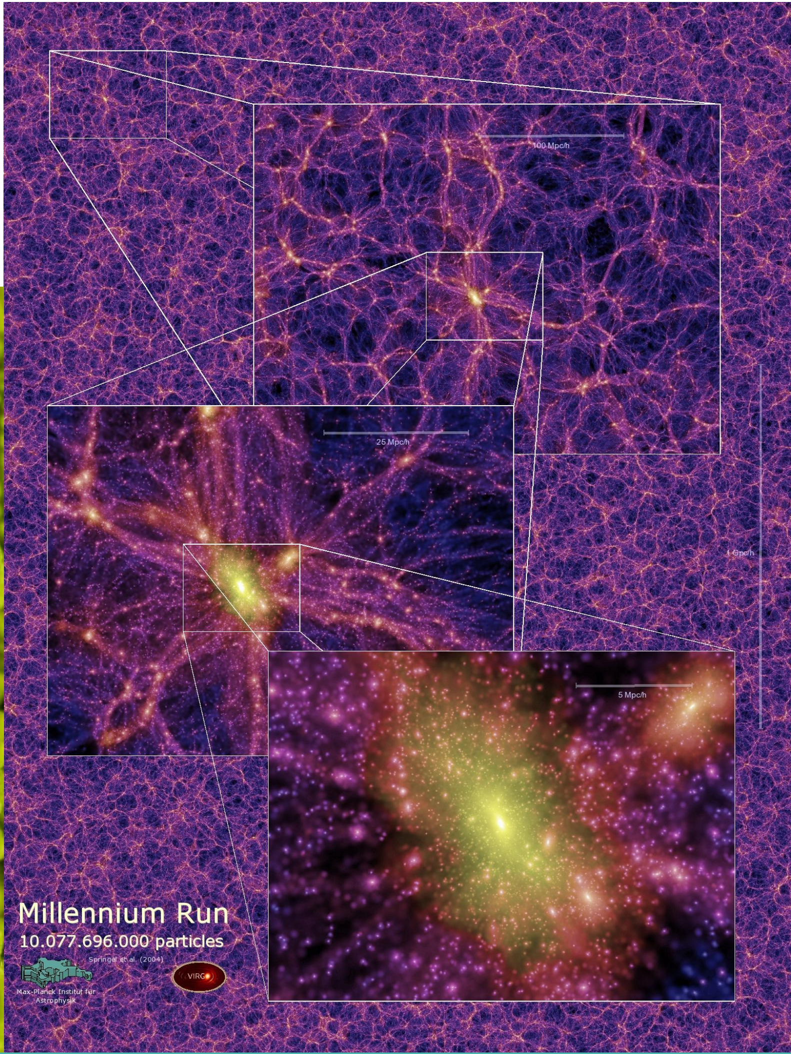


Many things to explore:

- Systematic construction of operator basis,
- Systematic renormalization,
- Cross-checks,
- More information from galaxy clustering (TBD)

Part I - Preamble

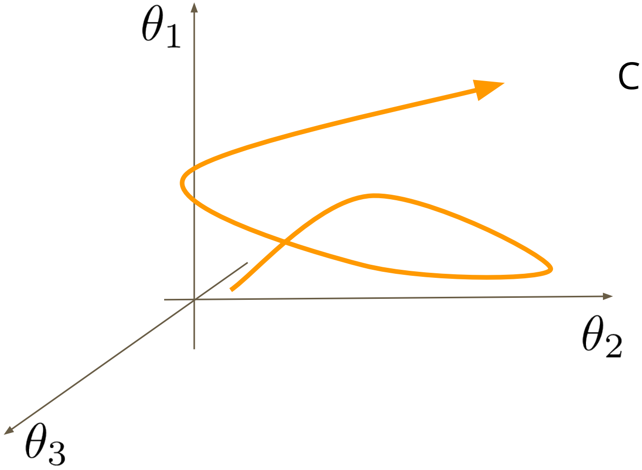
How things change with scale?
(from food to galaxies)



QFT101

Coupling constants evolve "flow" with the cutoff

Observables don't depend on the cutoff!



Callan-Symanzik eq:

$$\frac{\partial g}{\partial \ln \mu} = \beta(g)$$

For the fine-structure constant (QED):

$$\boxed{\frac{d\alpha}{d\ln\mu} = \beta_{1\mathrm{L}}\alpha^2 + \beta_{2\mathrm{L}}\alpha^3 + O(\alpha^4)}$$

$$\beta_{1\mathrm{L}} = 2/(3\pi)$$

$$\beta_{2\mathrm{L}} = 1/(4\pi^2)$$

Solution to the RG

$$\alpha(\mu)\big|_{\mathrm{LL}} = \frac{\alpha}{1 - \beta_{1\mathrm{L}}\alpha\ln(\mu/\mu_*)}$$

$$= \alpha\left[1 + \beta_{1\mathrm{L}}\alpha\ln(\mu/\mu_*) - \beta_{1\mathrm{L}}^2\alpha^2\ln^2(\mu/\mu_*) + \ldots\right]$$

QFT101

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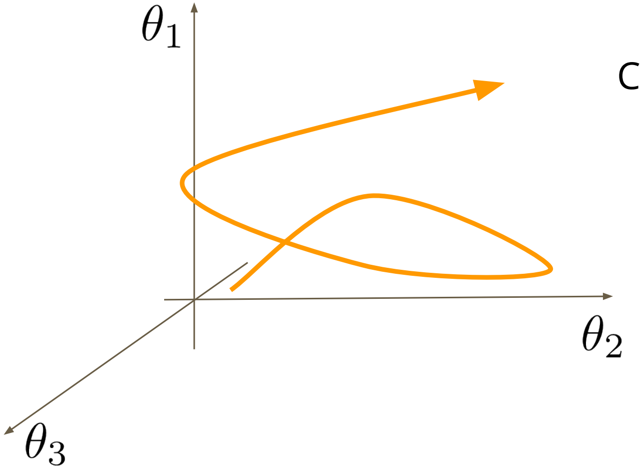

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Suppose you have an amplitude

$$\boxed{\frac{\sigma_{\ell\mathrm{L}}}{\sigma_{\mathrm{tree}}} = \alpha^\ell\left[c^{(\ell,\ell)}\ln^\ell(\mu/\mu_*) + c^{(\ell,\ell-1)}\ln^{\ell-1}(\mu/\mu_*) + \ldots\right]}$$

$$\frac{\sigma_{\mathrm{tree}}}{\sigma_{\mathrm{tree}}} = \alpha^0[c^{(0,0)}\ln^0]$$

$$\frac{\sigma_{1\mathrm{L}}}{\sigma_{\mathrm{tree}}} = \alpha^1[c^{(1,1)}\ln^1 + c^{(1,0)}\ln^0]$$

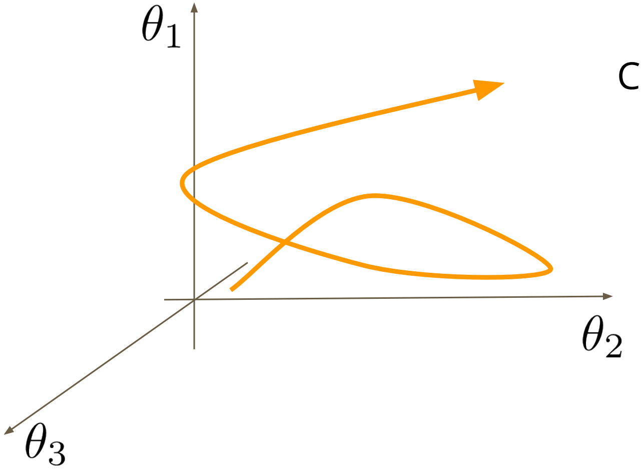
$$\frac{\sigma_{2\mathrm{L}}}{\sigma_{\mathrm{tree}}} = \alpha^2[c^{(2,2)}\ln^2 + c^{(2,1)}\ln^1 + c^{(2,0)}\ln^0]$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

QFT101

Coupling constants evolve "flow" with the cutoff

Observables don't depend on the cutoff!



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Solution to the RG

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$$= \alpha\left[1 + \beta_{1\mathrm{L}}\alpha\ln(\mu/\mu_*) - \beta_{1\mathrm{L}}^2\alpha^2\ln^2(\mu/\mu_*) + \ldots\right]$$

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LL (1loop RG)

NLL (2loop RG)

$\mathrm{N}^2\mathrm{LL}$ (3loop RG)

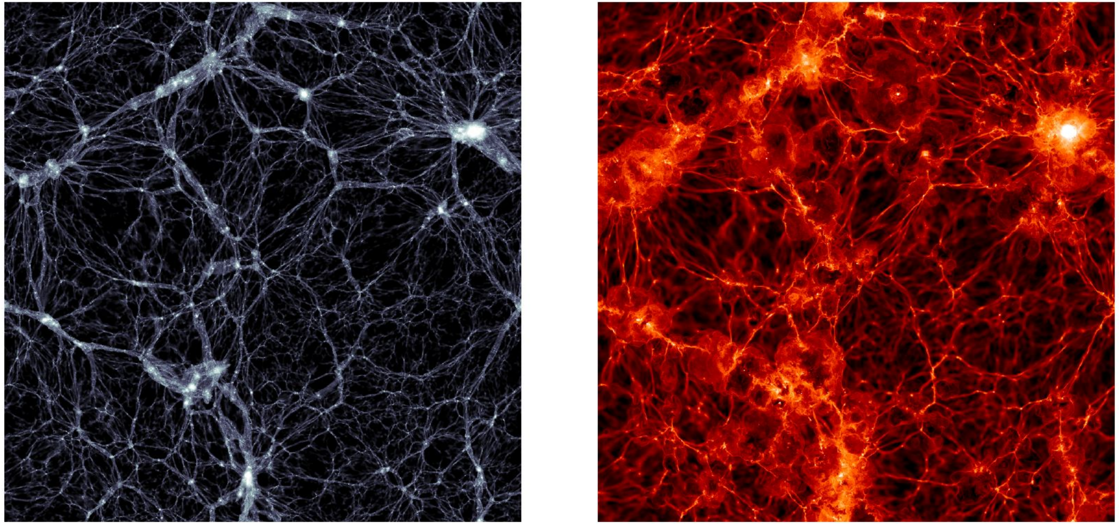
$$\frac{\sigma_{\mathrm{tree}}}{\sigma_{\mathrm{tree}}} = \alpha^0\left[c^{(0,0)}\ln^0\right]$$

$$\frac{\sigma_{1\mathrm{L}}}{\sigma_{\mathrm{tree}}} = \alpha^1\left[c^{(1,1)}\ln^1 + c^{(1,0)}\ln^0\right]$$

$$\frac{\sigma_{2\mathrm{L}}}{\sigma_{\mathrm{tree}}} = \alpha^2\left[c^{(2,2)}\ln^2 + c^{(2,1)}\ln^1 + c^{(2,0)}\ln^0\right]$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

The galaxy bias expansion



(a) dark matter (b) baryons

From Illustris simulation,
Haiden, Steinhauser, Vogelsberger,
Genel, Springel, Torrey, Hernquist, 15

Stochastic field

$$\delta_g(\boldsymbol{x},\tau) \equiv \frac{n_g(\boldsymbol{x},\tau)}{\bar{n}_g(\tau)} - 1 = \sum_O \left[\boxed{b_O}(\tau) + c_{\epsilon,O}(\tau)\boxed{\epsilon(\boldsymbol{x},\tau)}\right] O(\boldsymbol{x},\tau) + \boxed{\epsilon(\boldsymbol{x},\tau)}$$

Bias

Bias review: Desjacques, Jeong, Schmidt

Renormalizing the bias parameters

Important: those are the same parameters for all n-pt functions

In a nutshell, it is an **Operator Product Expansion (OPE)**

$$\delta_g(\boldsymbol{x},\tau) \equiv \frac{n_g(\boldsymbol{x},\tau)}{\bar{n}_g(\tau)} - 1 = \sum_O \left[b_O(\tau) + c_{\epsilon,O}(\tau)\epsilon(\boldsymbol{x},\tau)\right] O(\boldsymbol{x},\tau) + \epsilon(\boldsymbol{x},\tau)$$

$$O[\delta](\boldsymbol{k}) = \int_{\boldsymbol{p}_1,\ldots,\boldsymbol{p}_n} \delta_{\mathrm{D}}(\boldsymbol{k}-\boldsymbol{p}_{1\ldots n}) S_O(\boldsymbol{p}_1,\ldots \boldsymbol{p}_n)\delta(\boldsymbol{p}_1)\cdots\delta(\boldsymbol{p}_n)$$

$$\begin{aligned} \text{First order:}\quad & \delta\,; \\ \text{Second order:}\quad & \delta^2,\ \mathcal{G}_2\,; \\ \text{Third order:}\quad & \delta^3,\ \delta\,\mathcal{G}_2,\ \Gamma_3,\ \mathcal{G}_3\,; \end{aligned}$$

Contribution from arbitrarily small scales!

Renormalizing the bias parameters

Important: those are the same parameters for all n-pt functions

In a nutshell, it is an **Operator Product Expansion (OPE)**

$$\delta_g(\boldsymbol{x},\tau) \equiv \frac{n_g(\boldsymbol{x},\tau)}{\bar{n}_g(\tau)} - 1 = \sum_O \left[\overset{\Lambda}{b}_O(\tau) + \overset{\Lambda}{c}_{\epsilon,O}(\tau)\overset{\Lambda}{\epsilon}(\boldsymbol{x},\tau)\right] \overset{\Lambda}{O}(\boldsymbol{x},\tau) + \overset{\Lambda}{\epsilon}(\boldsymbol{x},\tau)$$

$+$ counter-terms (Λ)

$$O[\delta](\boldsymbol{k}) = \int_{\boldsymbol{p}_1,\ldots,\boldsymbol{p}_n}^{\Lambda} \delta_{\rm D}(\boldsymbol{k} - \boldsymbol{p}_{1\ldots n}) S_O(\boldsymbol{p}_1,\ldots \boldsymbol{p}_n)\delta(\boldsymbol{p}_1)\cdots\delta(\boldsymbol{p}_n)$$

Notation:

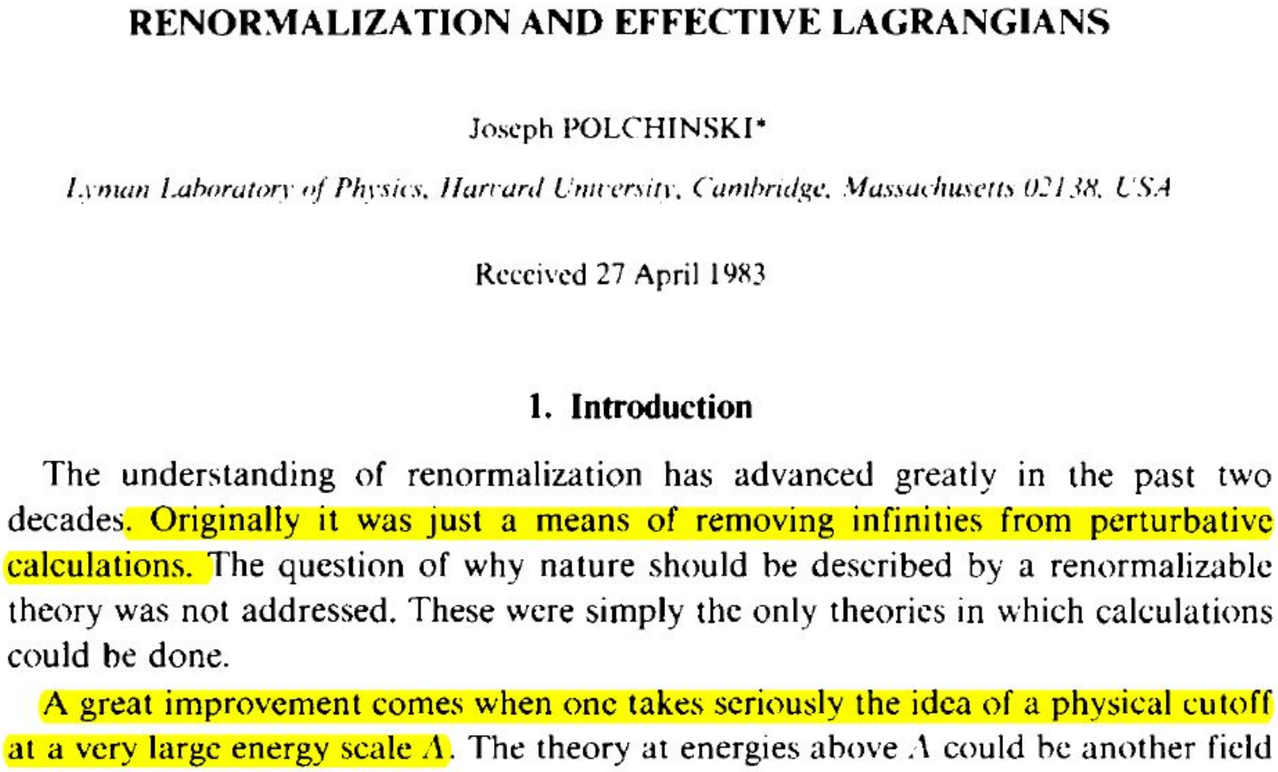
$$[\![O]\!] = O^{\Lambda}$$ +counter-terms (Λ)

Mc. Donald 09
Assassi+ 14

First order: $\delta\,;$
Second order: $\delta^2,\ \mathcal{G}_2\,;$
Third order: $\delta^3,\ \delta\,\mathcal{G}_2,\ \Gamma_3,\ \mathcal{G}_3\,;$

Contribution from arbitrarily small scales!

Motivation



RENORMALIZATION AND EFFECTIVE LAGRANGIANS

Joseph POLCHINSKI*

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Received 27 April 1983

1. Introduction

The understanding of renormalization has advanced greatly in the past two decades. Originally it was just a means of removing infinities from perturbative calculations. The question of why nature should be described by a renormalizable theory was not addressed. These were simply the only theories in which calculations could be done.

A great improvement comes when one takes seriously the idea of a physical cutoff at a very large energy scale Λ. The theory at energies above Λ could be another field

In a nutshell: instead of simply removing the cutoff dependence, allow for the operators to depend on the cutoff

Motivation (for different tastes)

Lattice person: "At field level you smooth out over your cutoff and those bias parameters have to be defined at a fixed scale!"

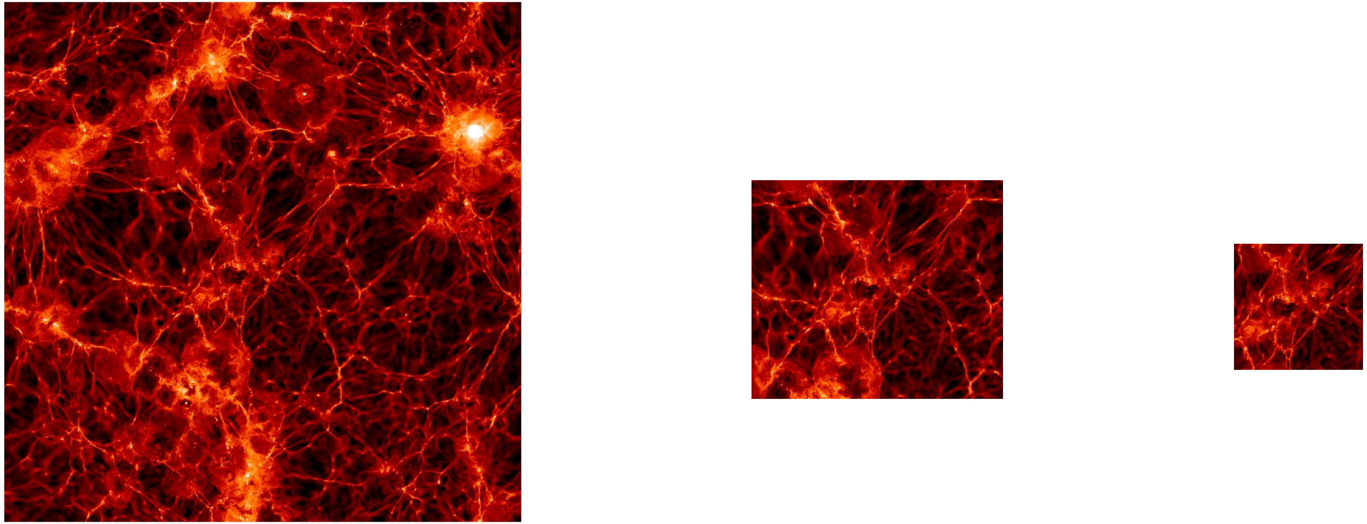
HEP person: "Everything is an EFTs and RG-flow is the next thing to do. "

Cosmo-MCMC person: "How can we be sure we are not messing up with the priors in my EFT analysis? Maybe extract more information..."

EFT-complainer: "You have a bunch of free parameters. How can you trust them?"

Intuition time

Smooth simulations (initial conditions) at different Λ and measure b_O



Part II - The RG equations

Warning (and apologies in advance):
next 3 slides will be technical

From Λ -independence to bias running

$$\boxed{0 = \frac{d}{d\Lambda}\delta_g(\boldsymbol{x}) = \frac{db_a}{d\Lambda}\mathcal{O}_a(\boldsymbol{x}) + b_a\frac{d\mathcal{O}_a(\boldsymbol{x})}{d\Lambda}}$$

Then we expand…

$$\frac{db_a}{d\Lambda} = \left.\frac{db_a}{d\Lambda}\right|_{1\mathrm{L}} + \left.\frac{db_a}{d\Lambda}\right|_{2\mathrm{L}} + \ldots$$

From Λ -independence to bias running

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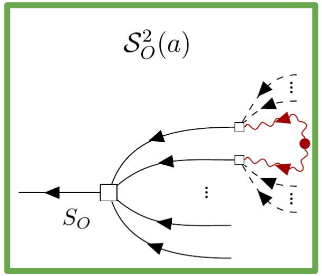
Then we expand...

$$\frac{db_a}{d\Lambda} = \frac{db_a}{d\Lambda}\Big|_{1\mathrm{L}} + \frac{db_a}{d\Lambda}\Big|_{2\mathrm{L}} + \dots$$

and calculate introducing correlations:

$$0 = \frac{db_a}{d\Lambda}\Big|_{1\mathrm{L}}\langle\mathcal{O}_a\delta_L(\boldsymbol{k}_1)\cdots\delta_L(\boldsymbol{k}_n)\rangle_{\mathrm{tree}} + b_a\frac{d}{d\Lambda}\langle\mathcal{O}_a\delta_L(\boldsymbol{k}_1)\cdots\delta_L(\boldsymbol{k}_n)\rangle_{1\mathrm{L}}$$



one-loop: $$\boxed{\frac{db_a}{d\Lambda}\Big|_{1\mathrm{L}} = -b_b s^{1\mathrm{L}}_{ba}\frac{d\sigma^2_\Lambda}{d\Lambda}}$$ **HR**, Schmidt, 23

$s^O_{O'}$	δ	δ^2	$\mathcal{G}_2$	δ^3	$\mathcal{G}_3$	Γ_3	$\delta\mathcal{G}_2$
$\mathbb{1}$	-	-	-	-	-	-	-
δ	-	$68/21$	-	3	-	-	$-4/3$
δ^2	-	$8126/2205$	-	$68/7$	-	-	$-376/105$
$\mathcal{G}_2$	-	$254/2205$	-	-	-	-	$116/105$

From Λ -independence to bias running

$$\boxed{0 = \frac{d}{d\Lambda}\delta_g(\boldsymbol{x}) = \frac{db_a}{d\Lambda}\mathcal{O}_a(\boldsymbol{x}) + b_a\frac{d\mathcal{O}_a(\boldsymbol{x})}{d\Lambda}}$$

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one-loop:

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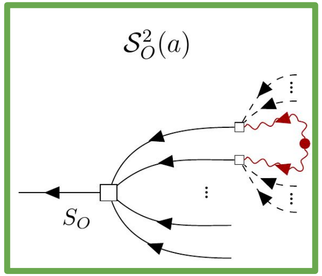
HR, Schmidt, 23

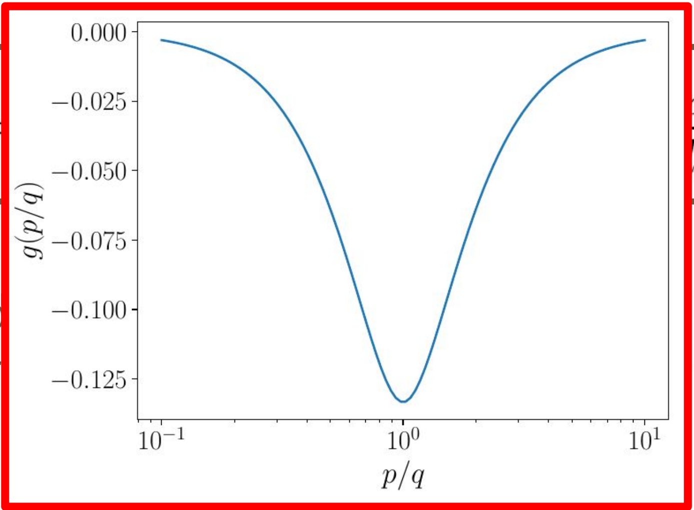


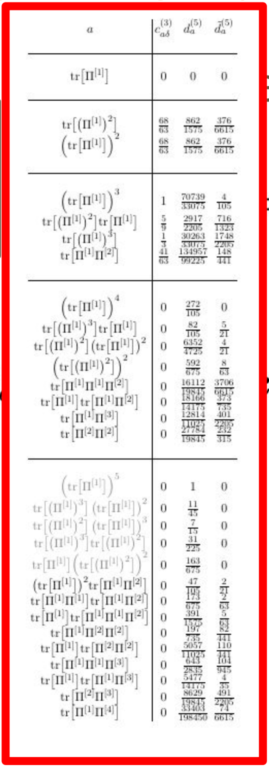
two-loop:

$$\boxed{\frac{db_\delta}{d\Lambda}\Big|_{2\mathrm{L}} = -30 b_b \tilde{d}^{(5)}_b \frac{d\sigma^2_\Lambda}{d\Lambda}\int_0^\Lambda dq \frac{q^2 P^{\mathrm{lin}}(q)}{2\pi^2} g(q/\Lambda)\,,}$$

Bakx, Garny, **HR**, Vlah

From Λ -independence to bias running





a	$c^{(3)}_{a\delta}$	$d^{(5)}_a$	$\tilde{d}^{(5)}_a$
$\mathrm{tr}[\Pi^{[1]}]$	0	0	0
$\mathrm{tr}[(\Pi^{[1]})^2]$	$\frac{68}{63}$	$\frac{862}{1575}$	$\frac{376}{6615}$
$(\mathrm{tr}[\Pi^{[1]}])^2$	$\frac{68}{63}$	$\frac{862}{1575}$	$\frac{376}{6615}$
$(\mathrm{tr}[\Pi^{[1]}])^3$	1	$\frac{70739}{33075}$	$\frac{4}{105}$
$\mathrm{tr}[(\Pi^{[1]})^2]\mathrm{tr}[\Pi^{[1]}]$	$\frac{5}{9}$	$\frac{2917}{2205}$	$\frac{716}{1323}$
$\mathrm{tr}[(\Pi^{[1]})^3]$	$\frac{1}{3}$	$\frac{30263}{33075}$	$\frac{1748}{2205}$
$\mathrm{tr}[\Pi^{[1]}\Pi^{[2]}]$	$\frac{41}{63}$	$\frac{134957}{99225}$	$\frac{148}{441}$
$(\mathrm{tr}[\Pi^{[1]}])^4$	0	$\frac{272}{105}$	0
$\mathrm{tr}[(\Pi^{[1]})^3]\mathrm{tr}[\Pi^{[1]}]$	0	$\frac{82}{105}$	$\frac{5}{21}$
$\mathrm{tr}[(\Pi^{[1]})^2](\mathrm{tr}[\Pi^{[1]}])^2$	0	$\frac{6352}{4725}$	$\frac{4}{21}$
$(\mathrm{tr}[(\Pi^{[1]})^2])^2$	0	$\frac{592}{675}$	$\frac{8}{63}$
$\mathrm{tr}[\Pi^{[1]}\Pi^{[1]}\Pi^{[2]}]$	0	$\frac{16112}{19845}$	$\frac{3706}{6615}$
$\mathrm{tr}[\Pi^{[1]}]\mathrm{tr}[\Pi^{[1]}\Pi^{[2]}]$	0	$\frac{18166}{14175}$	$\frac{373}{735}$
$\mathrm{tr}[\Pi^{[1]}\Pi^{[3]}]$	0	$\frac{12814}{11025}$	$\frac{401}{2205}$
$\mathrm{tr}[\Pi^{[2]}\Pi^{[2]}]$	0	$\frac{27784}{19845}$	$\frac{232}{315}$
$(\mathrm{tr}[\Pi^{[1]}])^5$	0	1	0
$\mathrm{tr}[(\Pi^{[1]})^3](\mathrm{tr}[\Pi^{[1]}])^2$	0	$\frac{11}{45}$	0
$\mathrm{tr}[(\Pi^{[1]})^2](\mathrm{tr}[\Pi^{[1]}])^3$	0	$\frac{7}{15}$	0
$\mathrm{tr}[(\Pi^{[1]})^3]\mathrm{tr}[(\Pi^{[1]})^2]$	0	$\frac{31}{225}$	0
$\mathrm{tr}[\Pi^{[1]}](\mathrm{tr}[(\Pi^{[1]})^2])^2$	0	$\frac{163}{675}$	0
$(\mathrm{tr}[\Pi^{[1]}])^2\mathrm{tr}[\Pi^{[1]}\Pi^{[2]}]$	0	$\frac{47}{105}$	$\frac{2}{21}$
$\mathrm{tr}[\Pi^{[1]}\Pi^{[1]}]\mathrm{tr}[\Pi^{[1]}\Pi^{[2]}]$	0	$\frac{173}{675}$	$\frac{2}{63}$
$\mathrm{tr}[\Pi^{[1]}]\mathrm{tr}[\Pi^{[1]}\Pi^{[1]}\Pi^{[2]}]$	0	$\frac{391}{1575}$	$\frac{5}{63}$
$\mathrm{tr}[\Pi^{[1]}\Pi^{[2]}\Pi^{[2]}]$	0	$\frac{197}{735}$	$\frac{82}{441}$
$\mathrm{tr}[\Pi^{[1]}]\mathrm{tr}[\Pi^{[2]}\Pi^{[2]}]$	0	$\frac{5057}{11025}$	$\frac{110}{441}$
$\mathrm{tr}[\Pi^{[1]}\Pi^{[1]}\Pi^{[3]}]$	0	$\frac{643}{2835}$	$\frac{104}{945}$
$\mathrm{tr}[\Pi^{[1]}]\mathrm{tr}[\Pi^{[1]}\Pi^{[3]}]$	0	$\frac{5477}{14175}$	$\frac{4}{35}$
$\mathrm{tr}[\Pi^{[2]}\Pi^{[3]}]$	0	$\frac{8629}{19845}$	$\frac{491}{2205}$
$\mathrm{tr}[\Pi^{[1]}\Pi^{[4]}]$	0	$\frac{33403}{198450}$	$\frac{74}{6615}$

en we expand...

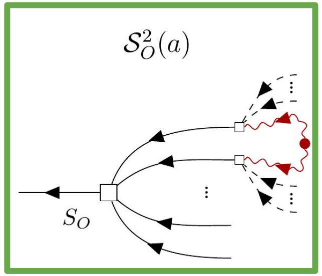
$$0 \qquad a(\boldsymbol{x}) \qquad d\Lambda$$

$$= \frac{db_a}{d\Lambda}\Big|_{1\mathrm{L}} + \frac{db_a}{d\Lambda}\Big|_{2\mathrm{L}} + \ldots$$

and

intr

$$\Big|_{1\mathrm{L}}\langle\mathcal{O} \qquad n))\rangle_{\mathrm{tree}} + b_a\frac{d}{d\Lambda}\langle\mathcal{O}_a\delta_L(\boldsymbol{k}_1)\cdots\delta_L(\boldsymbol{k}_n)\rangle_{1\mathrm{L}}$$



one-loop: $$\frac{db_a}{d\Lambda}\Big|_{1\mathrm{L}} = -b_b s^{1\mathrm{L}}_{ba}\frac{d\sigma^2_\Lambda}{d\Lambda}$$

HR,

two-loop: $$\frac{db_\delta}{d\Lambda}\Big|_{2\mathrm{L}} = -30 b_b \tilde{d}^{(5)}_b\frac{d\sigma^2_\Lambda}{d\Lambda}\int_0^\Lambda dq\frac{q^2P^{\mathrm{lin}}(q)}{2\pi^2}g(q/\Lambda)\,,$$

Bakx, Garny,
HR, Vlah

Part IV - The One-loop RG results

HR, Schmidt 23

Solutions

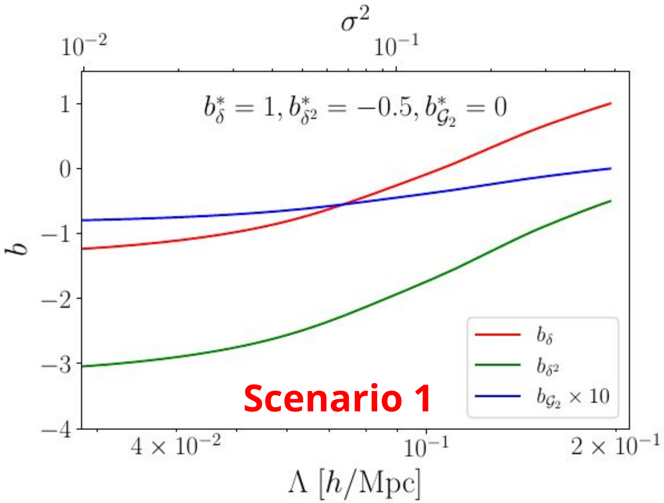
Wilson-Polchinski RG-equations

$$\frac{db_\delta}{d\Lambda} = -\left[\frac{68}{21}b_{\delta^2} + 3b^*_{\delta^3} - \frac{4}{3}b^*_{\mathcal{G}_2\delta}\right]\frac{d\sigma^2_\Lambda}{d\Lambda}\,,$$

$$\frac{db_{\delta^2}}{d\Lambda} = -\left[\frac{8126}{2205}b_{\delta^2} + \frac{17}{7}b^*_{\delta^3} - \frac{376}{105}b^*_{\mathcal{G}_2\delta} + b^{*(\delta^2)}_{n=4}\right]\frac{d\sigma^2_\Lambda}{d\Lambda}\,,$$

$$\frac{db_{\mathcal{G}_2}}{d\Lambda} = -\left[\frac{254}{2205}b_{\delta^2} + \frac{116}{105}b^*_{\mathcal{G}_2\delta} + b^{*(\mathcal{G}_2)}_{n=4}\right]\frac{d\sigma^2_\Lambda}{d\Lambda}\,.$$

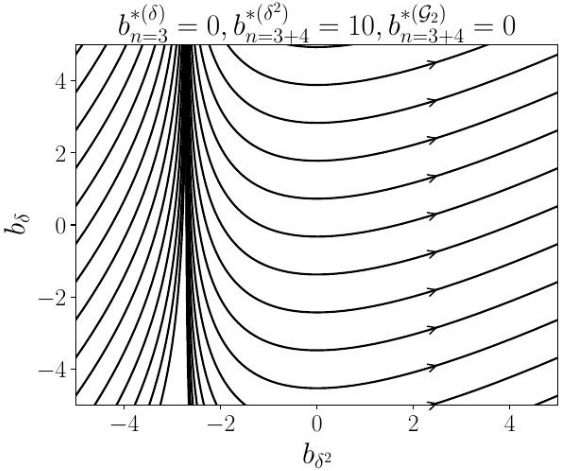
HR, Schmidt 23



Scenario 1

Notice that:

- **Bias parameter that are zero, may be sourced;**
- **Bias parameters may change sign!**

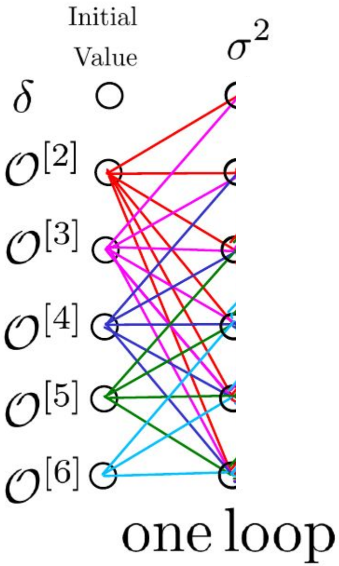


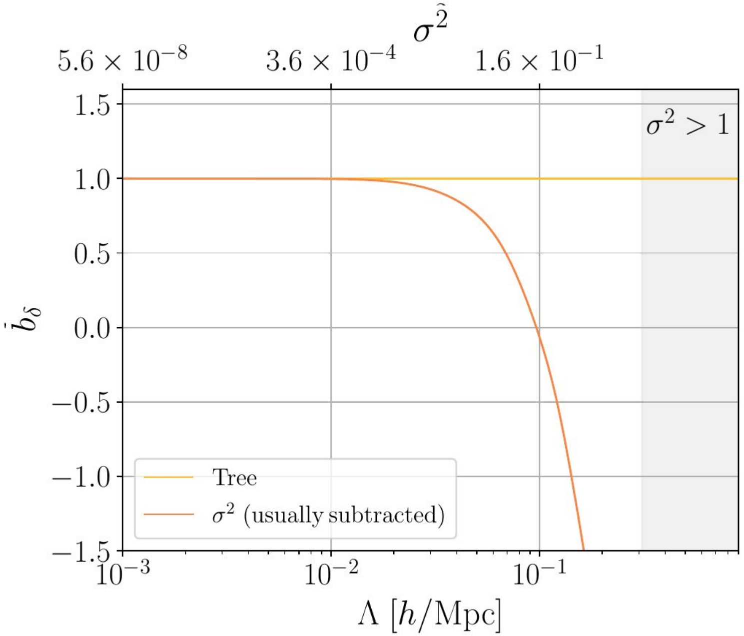
Resumming terms with the RG equations

Resumming terms with the RG equations

1Loop RG eq.

$$\frac{db_a}{d\sigma^2} = -\bar{s}^{1\mathrm{L}}_{ac} b_c$$





Solution

$$b_a(\sigma^2)$$

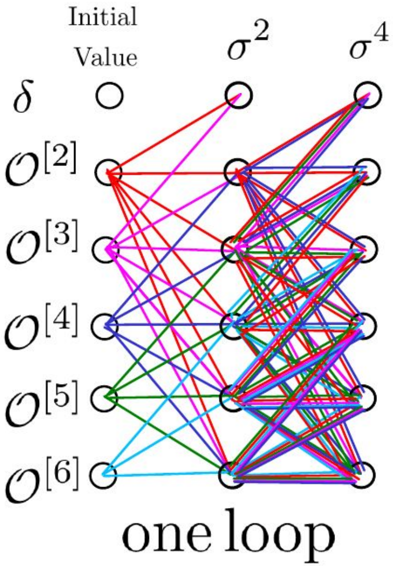
$$= b_a^* - (\sigma^2 - \sigma_*^2)\bar{s}^{1\mathrm{L}}_{ac} b_c^*$$

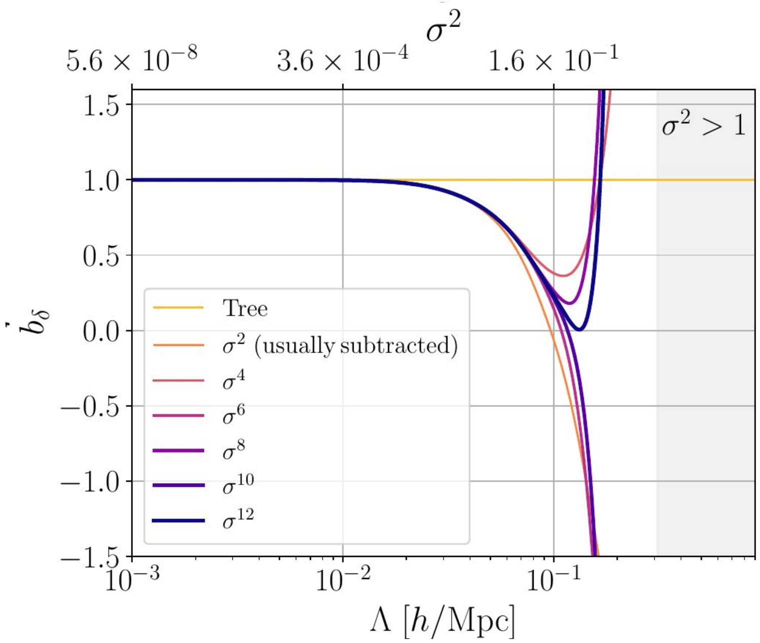
Assassi et al, 14

Resumming terms with the RG equations

1Loop RG eq.

$$\frac{db_a}{d\sigma^2} = -\bar{s}^{1\mathrm{L}}_{ac} b_c$$





Solution

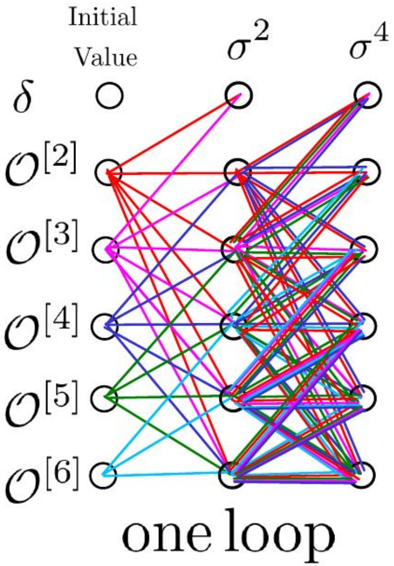
$b_a(\sigma^2)$

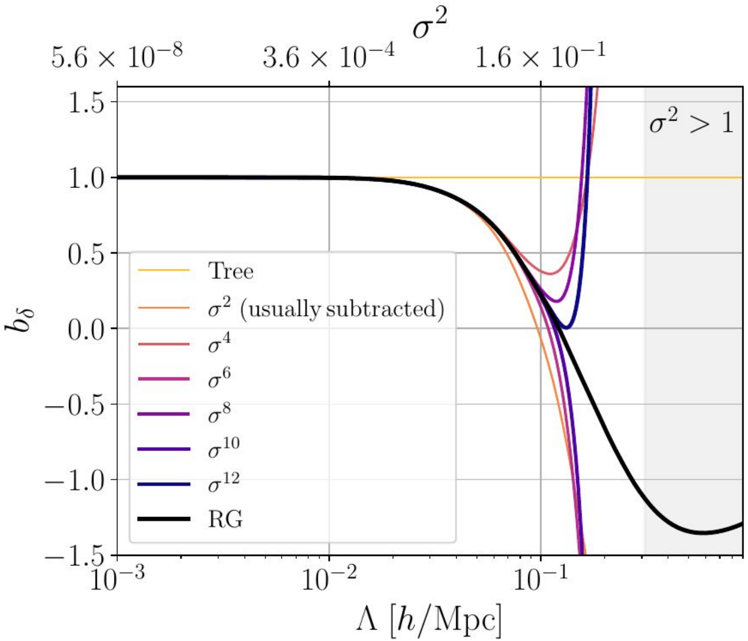
$$= b^*_a - (\sigma^2 - \sigma^2_*)\bar{s}^{1\mathrm{L}}_{ac} b^*_c + \frac{1}{2}(\sigma^2 - \sigma^2_*)^2 \bar{s}^{1\mathrm{L}}_{ab}\bar{s}^{1\mathrm{L}}_{bc} b^*_c - \frac{1}{6}(\sigma^2 - \sigma^2_*)^3 \bar{s}^{1\mathrm{L}}_{ab}\bar{s}^{1\mathrm{L}}_{bd}\bar{s}^{1\mathrm{L}}_{dc} b^*_c + \dots$$

Resumming terms with the RG equations

1Loop RG eq.

$$\frac{db_a}{d\sigma^2} = -\bar{s}^{1\mathrm{L}}_{ac} b_c$$





Solution

$$\boxed{b_a(\sigma^2) = \left[e^{-\bar{s}^{1\mathrm{L}}\times(\sigma^2-\sigma_*^2)}\right]_{ac} b_c^*}$$

$$= b_a^* - (\sigma^2 - \sigma_*^2)\bar{s}^{1\mathrm{L}}_{ac} b_c^* + \frac{1}{2}(\sigma^2 - \sigma_*^2)^2 \bar{s}^{1\mathrm{L}}_{ab}\bar{s}^{1\mathrm{L}}_{bc} b_c^* - \frac{1}{6}(\sigma^2 - \sigma_*^2)^3 \bar{s}^{1\mathrm{L}}_{ab}\bar{s}^{1\mathrm{L}}_{bd}\bar{s}^{1\mathrm{L}}_{dc} b_c^* + \ldots$$

RG resums the series!

What do the solutions of the RG tell us?

What do the solutions of the RG tell us?

We can always diagonalize the bias basis

$$\frac{db_i^{\text{diag}}}{d\sigma^2} = \lambda_i b_i^{\text{diag}}$$

$$b_a(\sigma^2) = p_{ai} e^{\lambda_i(\sigma^2 - \sigma_*^2)} c_i$$

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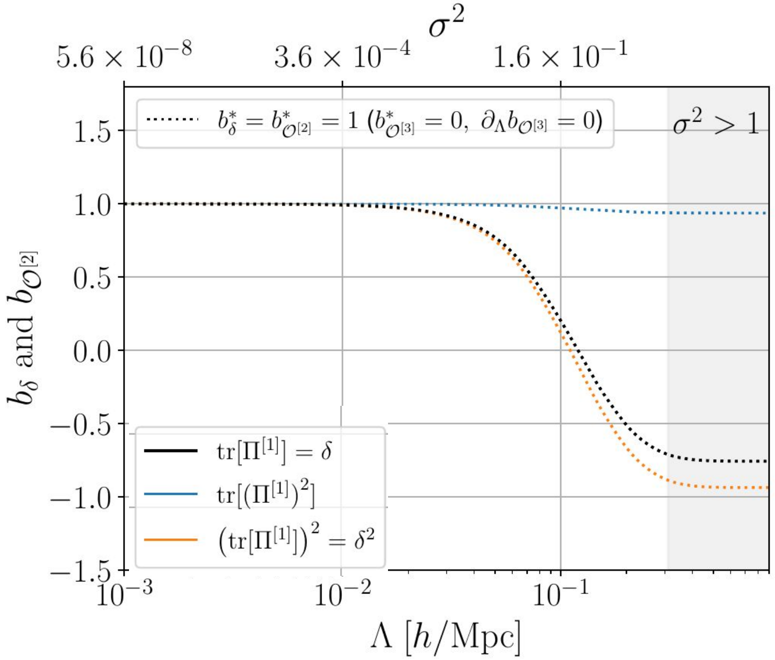
If we stop at second-order, we find:

$$\{\lambda_1, \lambda_2, \lambda_3\} \simeq \boxed{\{0, 0,} \boxed{-3.69\}}$$

Marginal *Relevant*



What do the solutions of the RG tell us?

Bakx, Garny, **HR**, Vlah

We can always diagonalize the bias basis

$$\frac{db_i^{\mathrm{diag}}}{d\sigma^2} = \lambda_i b_i^{\mathrm{diag}}$$

$$b_a(\sigma^2) = p_{ai} e^{\lambda_i(\sigma^2 - \sigma_*^2)} c_i$$

If we stop at second-order, we find:

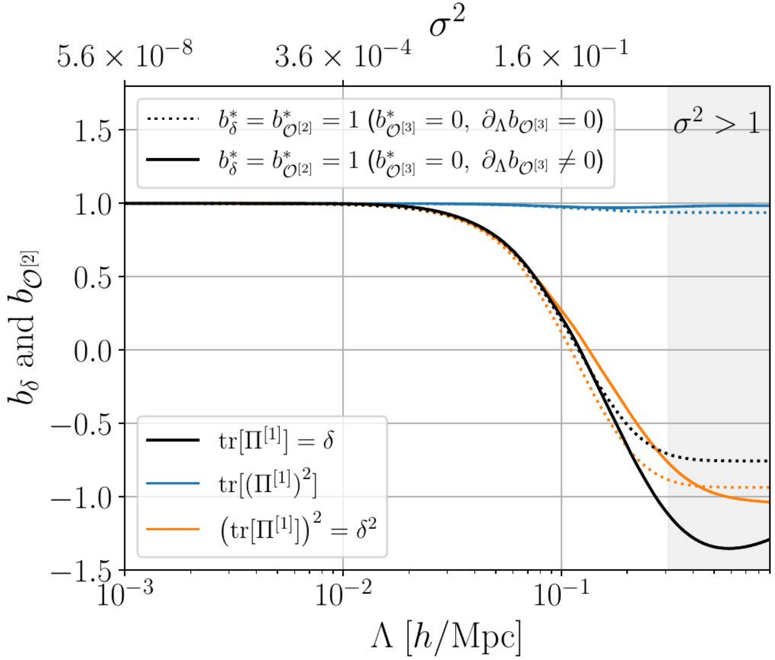
$$\{\lambda_1, \lambda_2, \lambda_3\} \simeq \boxed{\{0, 0,} \boxed{-3.69\}}$$

Marginal *Relevant*

Extending to third-order:

Irrelevant

$$\boxed{\{0, 0, 0,} \boxed{-12.6, -3.44, -2.01}, \boxed{0.220}\}$$

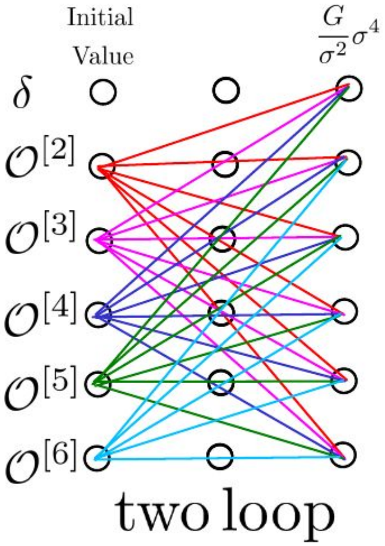


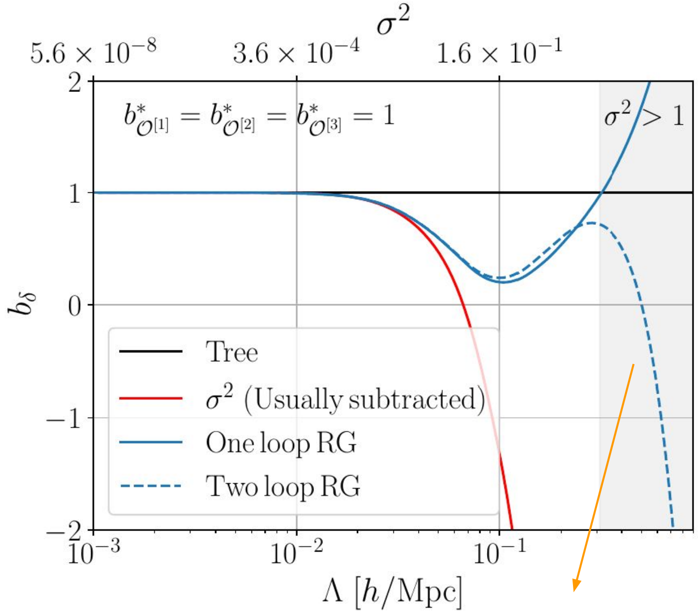
Caution to interpret: what happens if we go to higher order? TBD

Part III - The Two-loop RG

Bakx, Garny, **HR**, Vlah

Two-loop RG





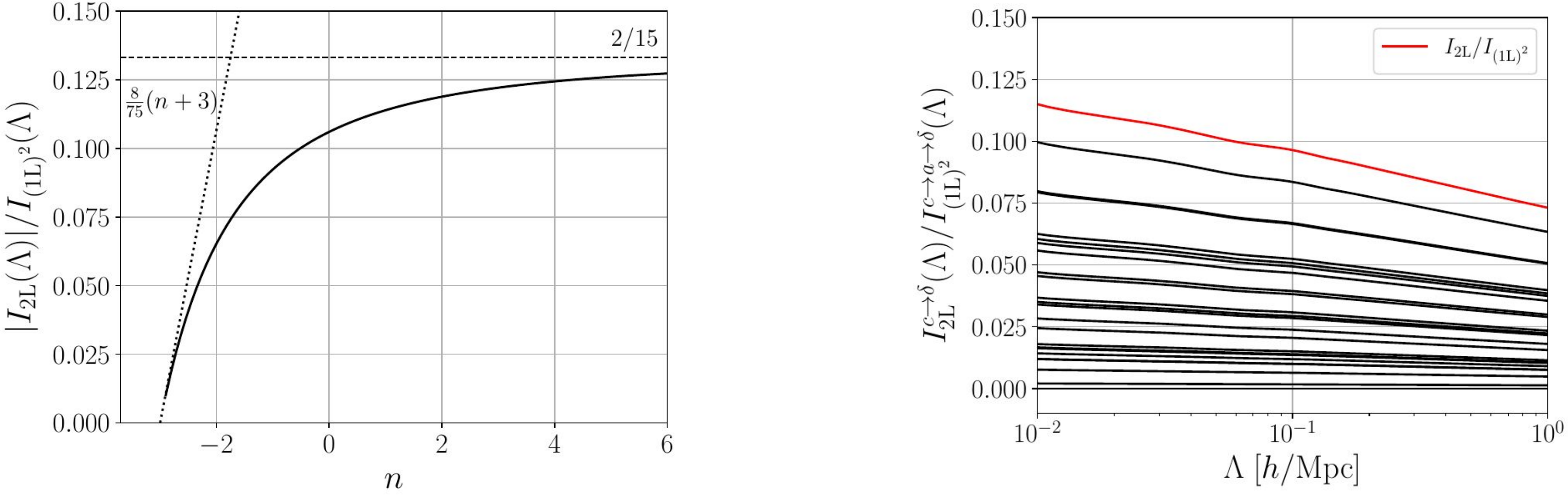
Small corrections compared to the one-loop

$$\frac{db_\delta}{d\ln\Lambda} = -\left[\sum_{c\in\mathcal{O}^{[2]}} s^{1\mathrm{L}}_{c\delta} b_c + \sum_{c\in\mathcal{O}^{[3]}} s^{1\mathrm{L}}_{c\delta} b_c\right]\frac{d\sigma^2_\Lambda}{d\ln\Lambda} - \sum_{c\in\mathrm{all}} \frac{d\sigma^2_\Lambda}{d\ln\Lambda}\overset{G}{\boxed{\int_0^\Lambda \frac{d\Lambda'}{\Lambda'}\frac{d\sigma^2_{\Lambda'}}{d\ln\Lambda'}\left[s^{2\mathrm{L}}_{c\delta}(\Lambda'/\Lambda) - s^{2\mathrm{L}}_{c\delta}(0)\right] b_c}}$$

So... The 2Loop is small. Why should you care?

- Good news: 1Loop RG takes care of most of the information
- It is not just small, it is **PARAMETRICALLY** small as n→ -3

Power law **Ratio 2Loop over 1Loop squared** LCDM



So the 2Loop is small. Why should you care?

We can write EFT loops as:

$$\left.\frac{P_{ab}^{\ell\mathrm{L}}(k)}{P^{\mathrm{lin}}(k)}\right|_{k\ll\Lambda} = (\Delta_\Lambda^2)^\ell \times \left[c_{ab}^{(\ell,\ell)}\left(\frac{1}{n+3}\right)^\ell + c_{ab}^{(\ell,\ell-1)}\left(\frac{1}{n+3}\right)^{\ell-1} + \dots\right]$$

$$\begin{aligned}
\frac{P_{ab}^{\mathrm{lin}}}{P^{\mathrm{lin}}} &= (\Delta_\Lambda^2)^0\left[c_{ab}^{(0,0)}\left(\frac{1}{n+3}\right)^0\right] \\
\frac{P_{ab}^{1\mathrm{L}}}{P^{\mathrm{lin}}} &= (\Delta_\Lambda^2)^1\left[c_{ab}^{(1,1)}\left(\frac{1}{n+3}\right)^1 + c_{ab}^{(1,0)}\left(\frac{1}{n+3}\right)^0\right] \\
\frac{P_{ab}^{2\mathrm{L}}}{P^{\mathrm{lin}}} &= (\Delta_\Lambda^2)^2\left[c_{ab}^{(2,2)}\left(\frac{1}{n+3}\right)^2 + c_{ab}^{(2,1)}\left(\frac{1}{n+3}\right)^1 + c_{ab}^{(2,0)}\left(\frac{1}{n+3}\right)^0\right] \\
&\vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots
\end{aligned}$$

So the 2Loop is small. Why should you care?

We can write EFT loops as:

$$\left.\frac{P^{\ell\mathrm{L}}_{ab}(k)}{P^{\mathrm{lin}}(k)}\right|_{k\ll\Lambda} = (\Delta^2_\Lambda)^\ell \times \left[c^{(\ell,\ell)}_{ab}\left(\frac{1}{n+3}\right)^\ell + c^{(\ell,\ell-1)}_{ab}\left(\frac{1}{n+3}\right)^{\ell-1} + \dots\right]$$

$$\begin{array}{lccc}
 & \mathrm{LL} & \mathrm{NLL} & \mathrm{N}^2\mathrm{LL}\\
 & (1\mathrm{loop\,RG}) & (2\mathrm{loop\,RG}) & (3\mathrm{loop\,RG})\\
\frac{P^{\mathrm{lin}}_{ab}}{P^{\mathrm{lin}}} = (\Delta^2_\Lambda)^0 & \Big[c^{(0,0)}_{ab}\left(\frac{1}{n+3}\right)^0\Big] & & \\
\frac{P^{1\mathrm{L}}_{ab}}{P^{\mathrm{lin}}} = (\Delta^2_\Lambda)^1 & \Big[c^{(1,1)}_{ab}\left(\frac{1}{n+3}\right)^1 & + c^{(1,0)}_{ab}\left(\frac{1}{n+3}\right)^0\Big] & \\
\frac{P^{2\mathrm{L}}_{ab}}{P^{\mathrm{lin}}} = (\Delta^2_\Lambda)^2 & \Big[c^{(2,2)}_{ab}\left(\frac{1}{n+3}\right)^2 & + c^{(2,1)}_{ab}\left(\frac{1}{n+3}\right)^1 & + c^{(2,0)}_{ab}\left(\frac{1}{n+3}\right)^0\Big]\\
\vdots & \vdots & \vdots & \vdots
\end{array}$$

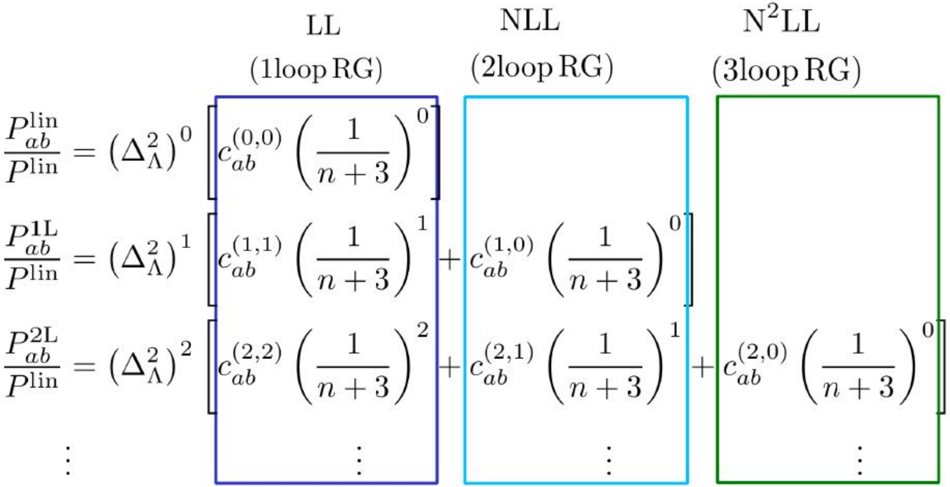
$$\begin{array}{lccc}
 & \mathrm{LL} & \mathrm{NLL} & \mathrm{N}^2\mathrm{LL}\\
 & (1\mathrm{loop\,RG}) & (2\mathrm{loop\,RG}) & (3\mathrm{loop\,RG})\\
\frac{\sigma_{\mathrm{tree}}}{\sigma_{\mathrm{tree}}} = \alpha^0 & [c^{(0,0)}\ln^0] & & \\
\frac{\sigma_{1\mathrm{L}}}{\sigma_{\mathrm{tree}}} = \alpha^1 & [c^{(1,1)}\ln^1 & + c^{(1,0)}\ln^0] & \\
\frac{\sigma_{2\mathrm{L}}}{\sigma_{\mathrm{tree}}} = \alpha^2 & [c^{(2,2)}\ln^2 & + c^{(2,1)}\ln^1 & + c^{(2,0)}\ln^0]\\
\vdots & \vdots & \vdots & \vdots
\end{array}$$

So the 2Loop is small. Why should you care?

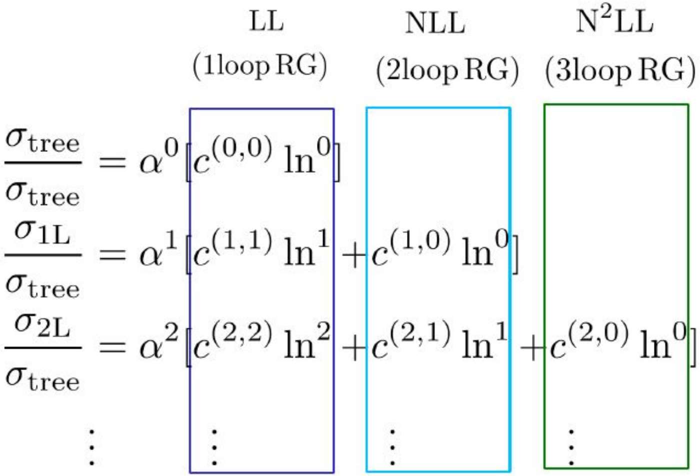
We can write EFT loops as:

$$\left.\frac{P^{\ell\mathrm{L}}_{ab}(k)}{P^{\mathrm{lin}}(k)}\right|_{k\ll\Lambda} = (\Delta^2_\Lambda)^\ell \times \left[c^{(\ell,\ell)}_{ab}\left(\frac{1}{n+3}\right)^{\ell} + c^{(\ell,\ell-1)}_{ab}\left(\frac{1}{n+3}\right)^{\ell-1} + \dots\right]$$

*Caution to interpret: scales in between we have to resum the integrals still tbd



$$\begin{array}{llll}
 & \text{LL} & \text{NLL} & \text{N}^2\text{LL} \\
 & (1\text{loop RG}) & (2\text{loop RG}) & (3\text{loop RG}) \\
\frac{P^{\mathrm{lin}}_{ab}}{P^{\mathrm{lin}}} = (\Delta^2_\Lambda)^0 & \left[c^{(0,0)}_{ab}\left(\frac{1}{n+3}\right)^0\right] & & \\
\frac{P^{1\mathrm{L}}_{ab}}{P^{\mathrm{lin}}} = (\Delta^2_\Lambda)^1 & \left[c^{(1,1)}_{ab}\left(\frac{1}{n+3}\right)^1\right. & \left.+c^{(1,0)}_{ab}\left(\frac{1}{n+3}\right)^0\right] & \\
\frac{P^{2\mathrm{L}}_{ab}}{P^{\mathrm{lin}}} = (\Delta^2_\Lambda)^2 & \left[c^{(2,2)}_{ab}\left(\frac{1}{n+3}\right)^2\right. & +c^{(2,1)}_{ab}\left(\frac{1}{n+3}\right)^1 & \left.+c^{(2,0)}_{ab}\left(\frac{1}{n+3}\right)^0\right] \\
\vdots & \vdots & \vdots & \vdots
\end{array}$$



$$\begin{array}{llll}
 & \text{LL} & \text{NLL} & \text{N}^2\text{LL} \\
 & (1\text{loop RG}) & (2\text{loop RG}) & (3\text{loop RG}) \\
\frac{\sigma_{\mathrm{tree}}}{\sigma_{\mathrm{tree}}} = \alpha^0 & \left[c^{(0,0)}\ln^0\right] & & \\
\frac{\sigma_{1\mathrm{L}}}{\sigma_{\mathrm{tree}}} = \alpha^1 & \left[c^{(1,1)}\ln^1\right. & \left.+c^{(1,0)}\ln^0\right] & \\
\frac{\sigma_{2\mathrm{L}}}{\sigma_{\mathrm{tree}}} = \alpha^2 & \left[c^{(2,2)}\ln^2\right. & +c^{(2,1)}\ln^1 & \left.+c^{(2,0)}\ln^0\right] \\
\vdots & \vdots & \vdots & \vdots
\end{array}$$

Part IV - PNG and Stochasticity

PNGs

Free term

$$\frac{db_\delta}{d\Lambda} = -\left[\frac{68}{21} b_{\delta^2}(\Lambda) + b^{*\{\delta\}\mathrm{G}}_{n=3}\right]\frac{d\sigma^2_\Lambda}{d\Lambda}$$

New interaction

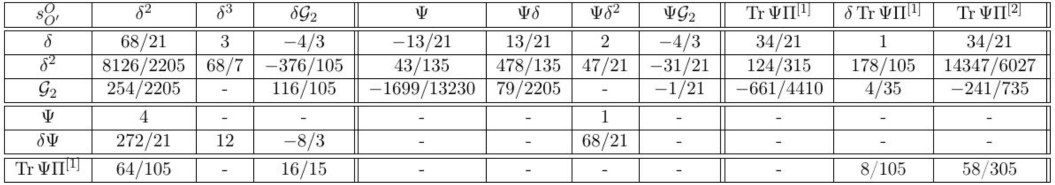
$$- a_0 f_{\rm NL}\left[-\frac{13}{21} b_\Psi + \frac{13}{21} b_{\Psi\delta} + b^{*\{\delta\}\mathrm{NG}}_{n=3}\right]\left(\frac{H_0}{\Lambda}\right)^2 \frac{3\,\Omega_m}{2\,T(\Lambda)}\frac{d\sigma^2_\Lambda}{d\Lambda}\,;$$

Now a coupled set of ODEs

$$\frac{db_\Psi}{d\Lambda} = -a_0 f_{\rm NL} b^{*\{\Psi\}\mathrm{NG}}_{n=3}\frac{d\sigma^2_\Lambda}{d\Lambda} - 4 a_0 f_{\rm NL} b_{\delta^2}\frac{d\sigma^2_\Lambda}{d\Lambda}\,,$$

$$\frac{db_{\Psi\delta}}{d\Lambda} = -a_0 f_{\rm NL}\left[\frac{272}{21} b_{\delta^2} + b^{*\{\Psi\delta\}\mathrm{G}}_{n=3+4} + b^{*\{\Psi\delta\}\mathrm{NG}}_{n=3+4}\right]\frac{d\sigma^2_\Lambda}{d\Lambda}\,,$$

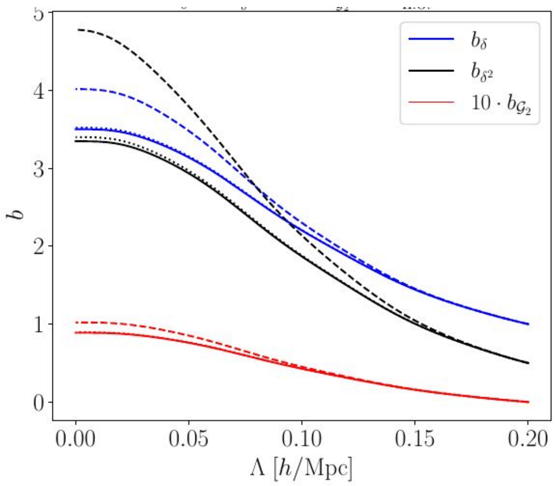
Rederivation of Dalal+ 07 (in an elegant way)



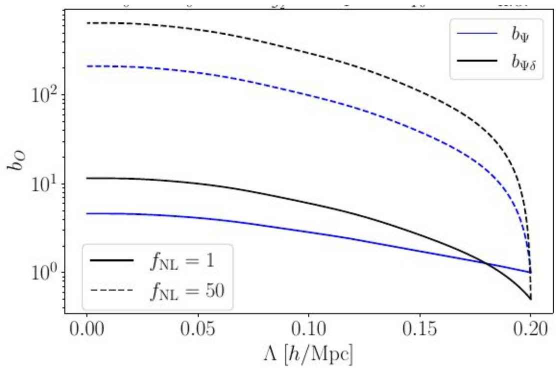
$s^{O}_{O'}$	δ^2	δ^3	$\delta\mathcal{G}_2$	Ψ	$\Psi\delta$	$\Psi\delta^2$	$\Psi\mathcal{G}_2$	$\mathrm{Tr}\,\Psi\Pi^{[1]}$	$\delta\,\mathrm{Tr}\,\Psi\Pi^{[1]}$	$\mathrm{Tr}\,\Psi\Pi^{[2]}$
δ	68/21	3	−4/3	−13/21	13/21	2	−4/3	34/21	1	34/21
δ^2	8126/2205	68/7	−376/105	43/135	478/135	47/21	−31/21	124/315	178/105	14347/6027
$\mathcal{G}_2$	254/2205	-	116/105	−1699/13230	79/2205	-	−1/21	−661/4410	4/35	−241/735
Ψ	4	-	-	-	-	1	-	-	-	-
$\delta\Psi$	272/21	12	−8/3	-	-	68/21	-	-	-	-
$\mathrm{Tr}\,\Psi\Pi^{[1]}$	64/105	-	16/15	-	-	-	-	-	8/105	58/305

Nikolis, **HR**, Schmidt

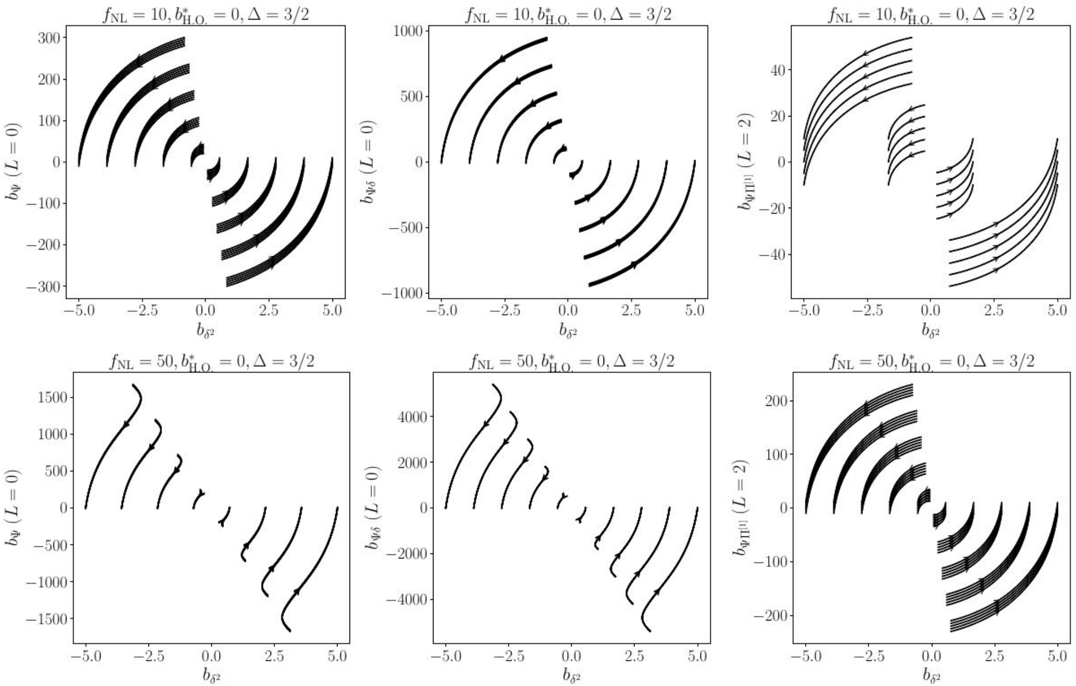
PNGs



b_δ
b_{δ^2}
$10 \cdot b_{\mathcal{G}_2}$
b
0
1
2
3
4
5
0.00
0.05
0.10
0.15
0.20
$\Lambda\ [h/\mathrm{Mpc}]$



b_Ψ
$b_{\Psi\delta}$
$f_{\rm NL} = 1$
$f_{\rm NL} = 50$
b_O
10^0
10^1
10^2
0.00
0.05
0.10
0.15
0.20
$\Lambda\ [h/\mathrm{Mpc}]$



$f_{\rm NL} = 10, b^*_{\rm H.O.} = 0, \Delta = 3/2$
$f_{\rm NL} = 10, b^*_{\rm H.O.} = 0, \Delta = 3/2$
$f_{\rm NL} = 10, b^*_{\rm H.O.} = 0, \Delta = 3/2$
$f_{\rm NL} = 50, b^*_{\rm H.O.} = 0, \Delta = 3/2$
$f_{\rm NL} = 50, b^*_{\rm H.O.} = 0, \Delta = 3/2$
$f_{\rm NL} = 50, b^*_{\rm H.O.} = 0, \Delta = 3/2$
$b_\Psi\ (L=0)$
$b_{\Psi\delta}\ (L=0)$
$b_{\Psi\Pi^{[1]}}\ (L=2)$
b_{δ^2}
−5.0
−2.5
0.0
2.5
5.0
300
200
100
0
−100
−200
−300
1000
500
−500
−1000
40
20
−20
−40
1500
1000
500
−500
−1000
−1500
4000
2000
−2000
−4000
200
100
−100
−200

Stochasticity

$$\delta_g(\boldsymbol{x},\tau) \equiv \frac{n_g(\boldsymbol{x},\tau)}{\bar{n}_g(\tau)} - 1 = \sum_O \left[b_O(\tau) + c_{\epsilon,O}(\tau)\boxed{\epsilon(\boldsymbol{x},\tau)}\right] O(\boldsymbol{x},\tau) \boxed{+\, \epsilon(\boldsymbol{x},\tau)}$$

$$\langle \epsilon(\boldsymbol{k}_1)\dots\epsilon(\boldsymbol{k}_m) O(\boldsymbol{k}_{m+1})\rangle = \hat{\delta}_{\mathrm{D}}(\boldsymbol{k}_{1\dots m})\boxed{C^{(m)}_{\epsilon,O}} O(\boldsymbol{k}_{m+1})$$

HR, Schmidt, 24

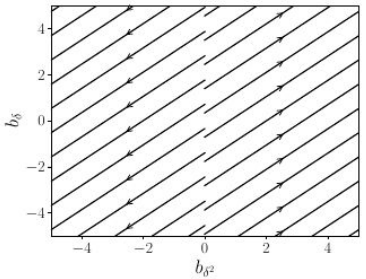
Stochasticity

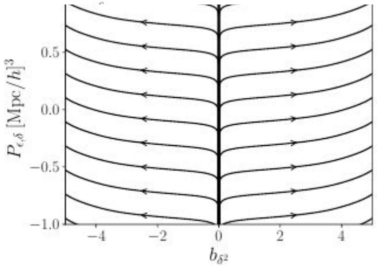
$$\delta_g(\boldsymbol{x},\tau) \equiv \frac{n_g(\boldsymbol{x},\tau)}{\bar{n}_g(\tau)} - 1 = \sum_O \left[b_O(\tau) + c_{\epsilon,O}(\tau)\boxed{\epsilon(\boldsymbol{x},\tau)]}\right] O(\boldsymbol{x},\tau) \boxed{+\,\epsilon(\boldsymbol{x},\tau)}$$

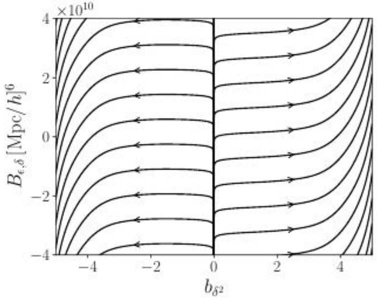
$$\langle \epsilon(\boldsymbol{k}_1)\ldots\epsilon(\boldsymbol{k}_m) O(\boldsymbol{k}_{m+1})\rangle = \hat{\delta}_{\rm D}(\boldsymbol{k}_{1\ldots m})\boxed{C^{(m)}_{\epsilon,O}}O(\boldsymbol{k}_{m+1})$$

Simple expression for how stochastic terms talk to each other

$$\frac{d}{d\Lambda}C^{(m)}_O(\Lambda) \propto -[P_{\rm L}(\Lambda)]^{p-1}\frac{d\sigma^2_\Lambda}{d\Lambda}\sum_{O_1,O_2,\ldots O_m} s^O_{O_1 O_2\ldots O_m} C^{(i_1)}_{O_1}(\Lambda)\ldots C^{(i_p)}_{O_p}(\Lambda)$$







HR, Schmidt, 24

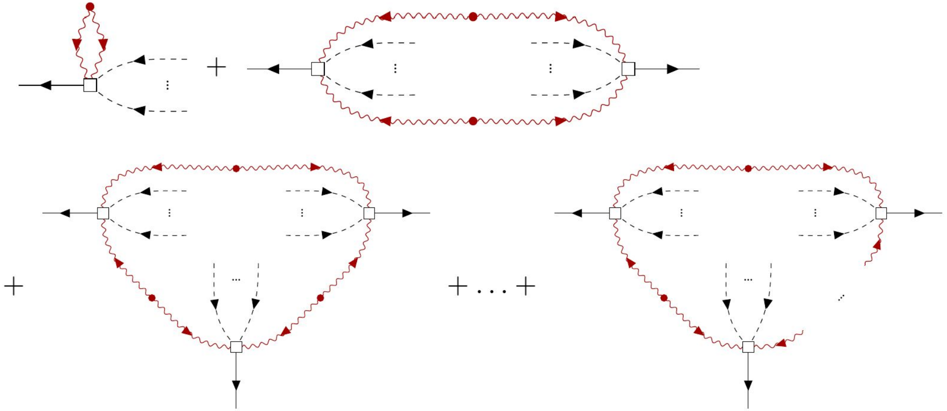
Stochasticity

$$\delta_g(\boldsymbol{x},\tau) \equiv \frac{n_g(\boldsymbol{x},\tau)}{\bar{n}_g(\tau)} - 1 = \sum_O \left[b_O(\tau) + c_{\epsilon,O}(\tau)\boxed{\epsilon(\boldsymbol{x},\tau)]}\,O(\boldsymbol{x},\tau)\right) \boxed{+\,\epsilon(\boldsymbol{x},\tau)}$$

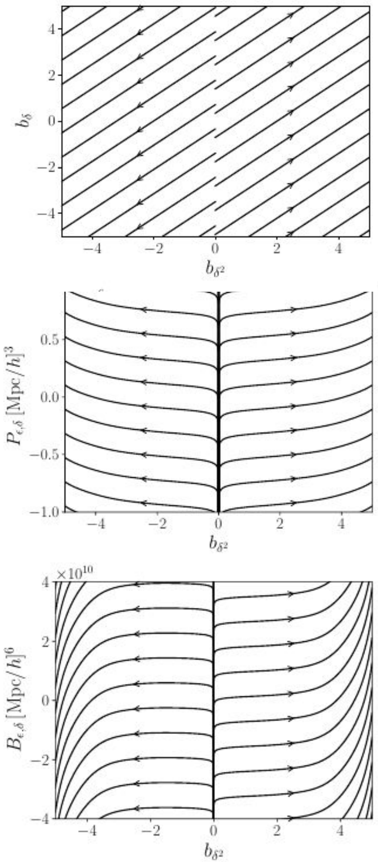
$$\langle \epsilon(\boldsymbol{k}_1)\ldots\epsilon(\boldsymbol{k}_m)O(\boldsymbol{k}_{m+1})\rangle = \hat{\delta}_{\rm D}(\boldsymbol{k}_{1\ldots m})\boxed{C^{(m)}_{\epsilon,O}}\,O(\boldsymbol{k}_{m+1})$$

Simple expression for how stochastic terms talk to each other

$$\frac{d}{d\Lambda}C^{(m)}_O(\Lambda) \propto -[P_{\rm L}(\Lambda)]^{p-1}\frac{d\sigma^2_\Lambda}{d\Lambda}\sum_{O_1,O_2,\ldots O_m} s^O_{O_1 O_2\ldots O_m}\,C^{(i_1)}_{O_1}(\Lambda)\ldots C^{(i_p)}_{O_p}(\Lambda)$$



Simple diagrammatic interpretation



HR, Schmidt, 24

Part IV - Final remarks

How to relate the renormalization schemes?

N-point function renormalized bias
(Assassi, Baumann, Green, Zaldarriaga)

Finite cutoff bias
(This work)

$$[\![O']\!](\boldsymbol{k}') \xleftrightarrow{\text{How to connect both?}} O'[\delta_\Lambda^{(1)}](\boldsymbol{k}')$$

How to relate the renormalization schemes?

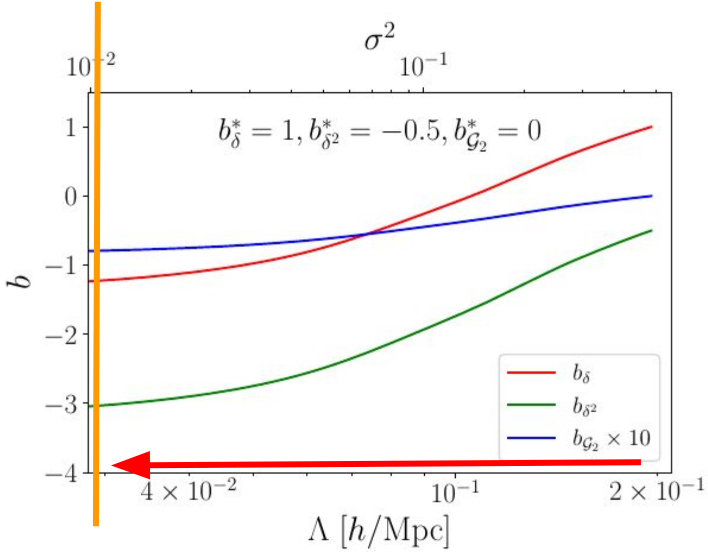
N-point function renormalized bias
(Assassi, Baumann, Green, Zaldarriaga)

Finite cutoff bias
(This work)

How to connect both?

$$[\![O']\!](\boldsymbol{k}') \longleftrightarrow O'[\delta_\Lambda^{(1)}](\boldsymbol{k}')$$

Separate Universe



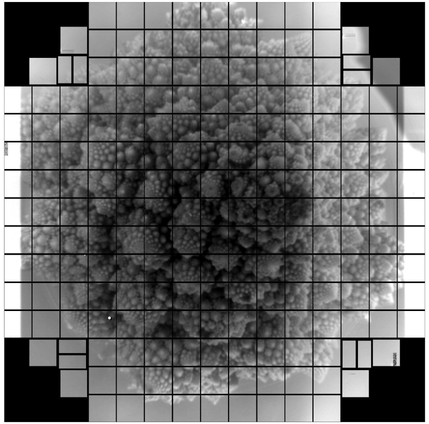
Solution: Run the bias towards

$$\Lambda \to 0$$

HR, Schmidt 23

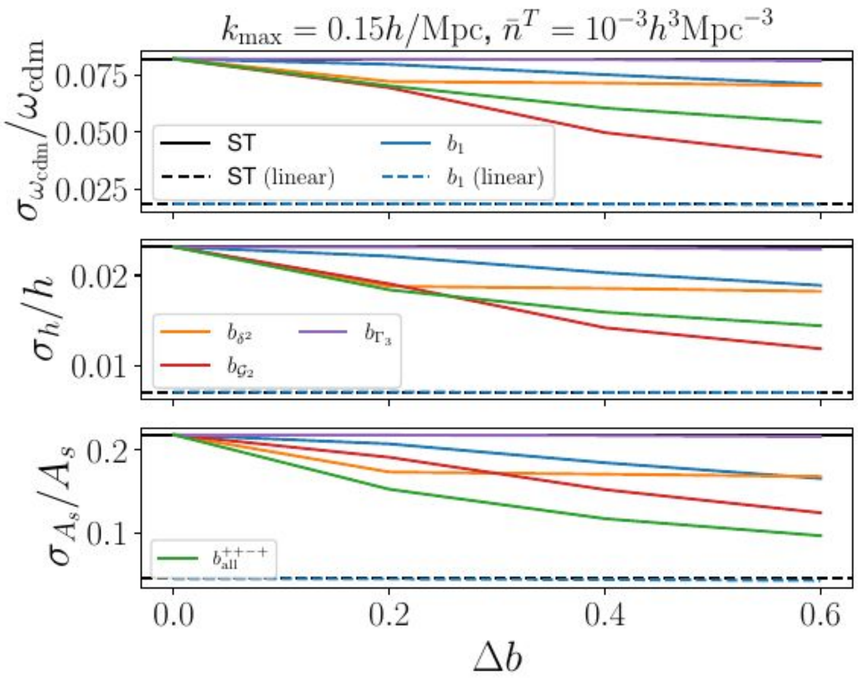
Why you should care

- Cross-check for EFT inference;
- Systematic renormalization (+ stochastic +PNG);
- Systematic renormalization of n-point functions. Self-consistent renormalization for P(k), B(k1,k2,k3), ...
- (Unambiguously) Define Priors for EFT analysis in $\Lambda \to 0$
- More information from resummation? TBD!

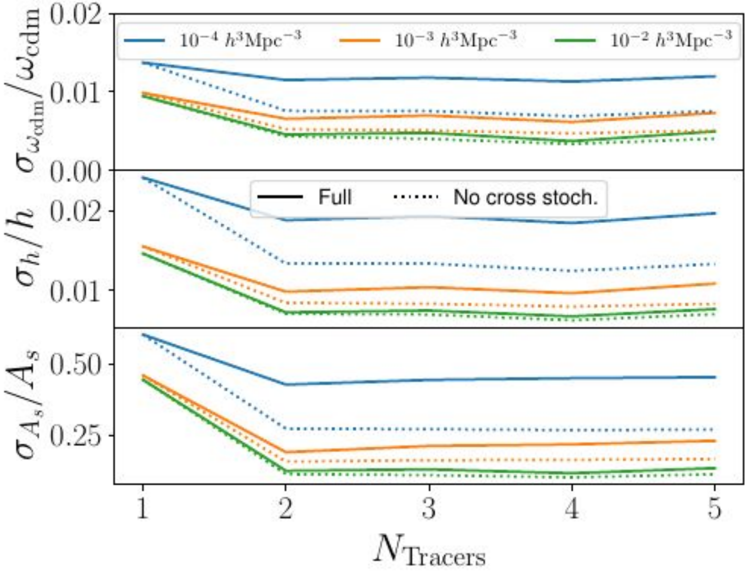


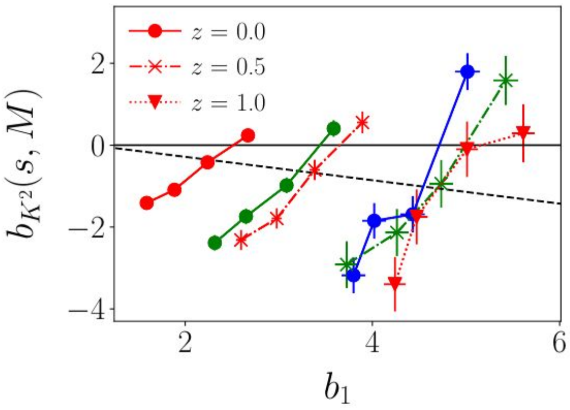
First images of Rubin

Multi-tracer



HR, Conteddu, see also:
Mergulhão, HR, Voivodic 23
Mergulhão, HR, Voivodic, Abramo





Lazeyras,
Barreira,
Schmidt

Thanks a lot!