Fast or Furious surgery: New primitives for QLDPC computation

FTQT 2026 @ Benasque, Spain

Katie Chang

Yale Quantum Institute, Yale University

Joint works with:

[2603.02157] KC, Zhiyang He, Guanyu Zhu, Theodore Yoder, Tomas Jochym-O'Connor

[in prep.] KC, Anasuya Lyons, Yuanjie Ren, Harald Putterman, Victor Albert, Ben Brown, Nathaniel Tantivasadakarn, Dominic Williamson

QLDPC Codes

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QLDPC codes enable fault-tolerant memories at low space overhead

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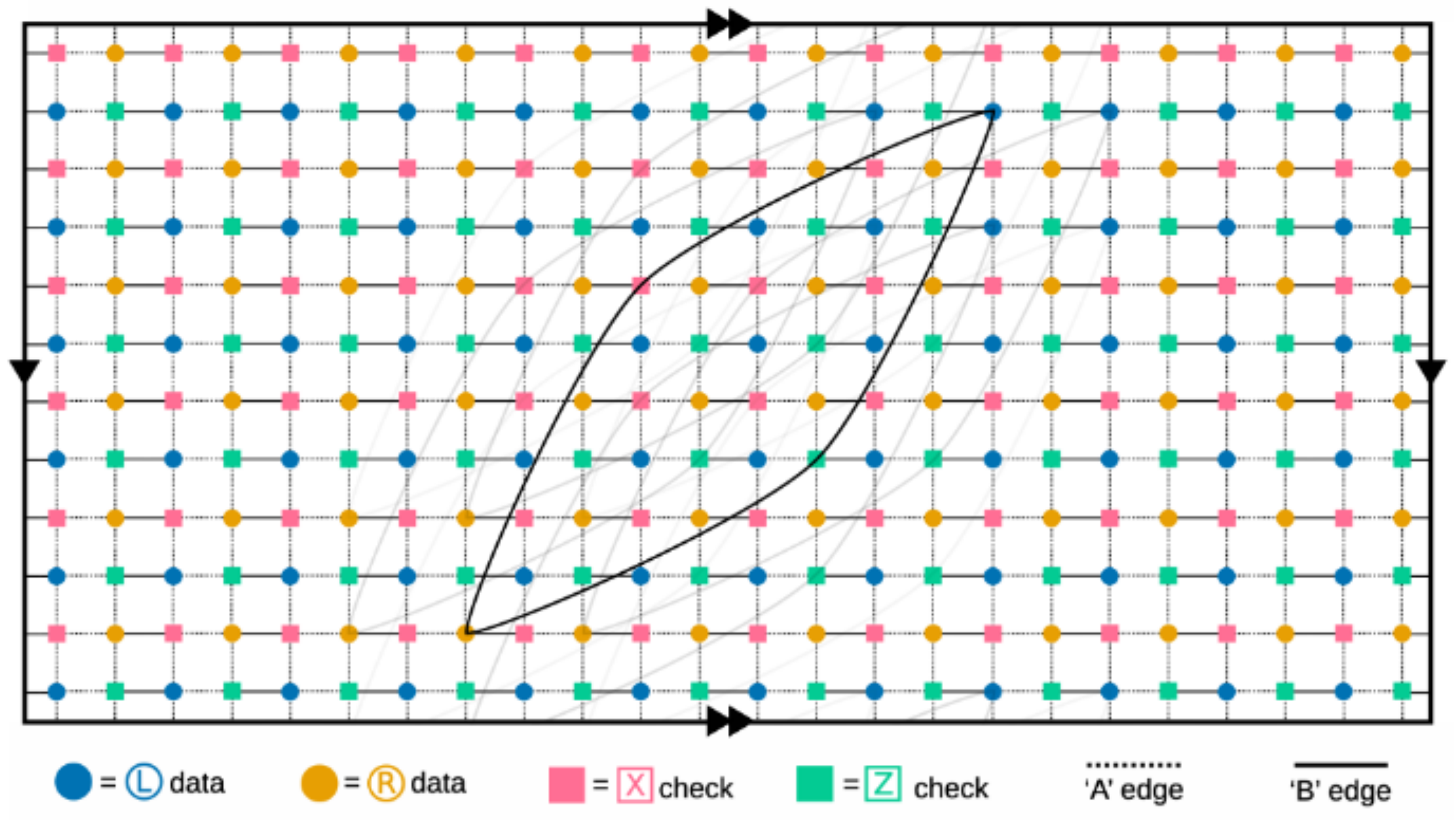

Bivariate Bicycle Code

QLDPC Codes

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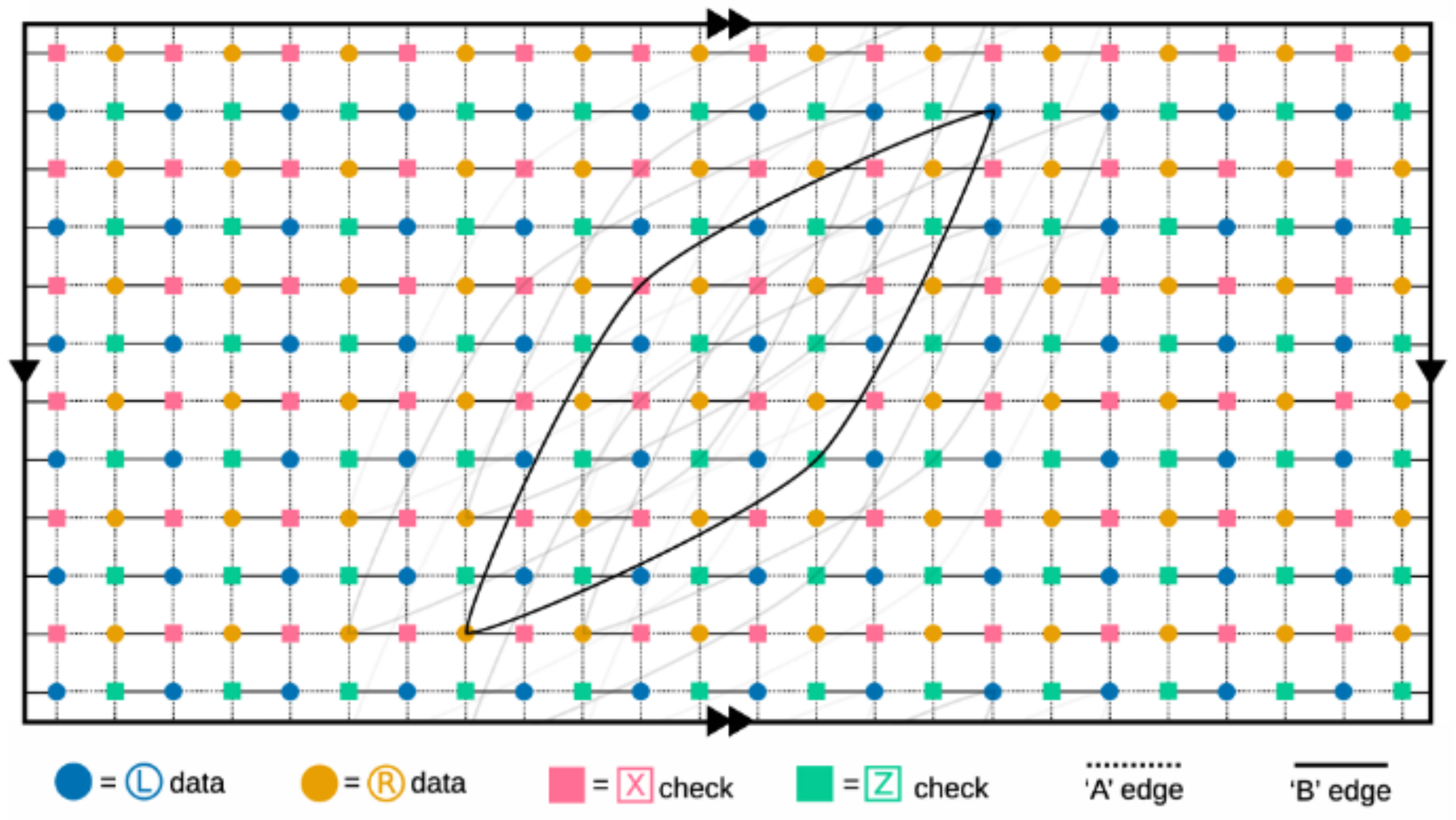

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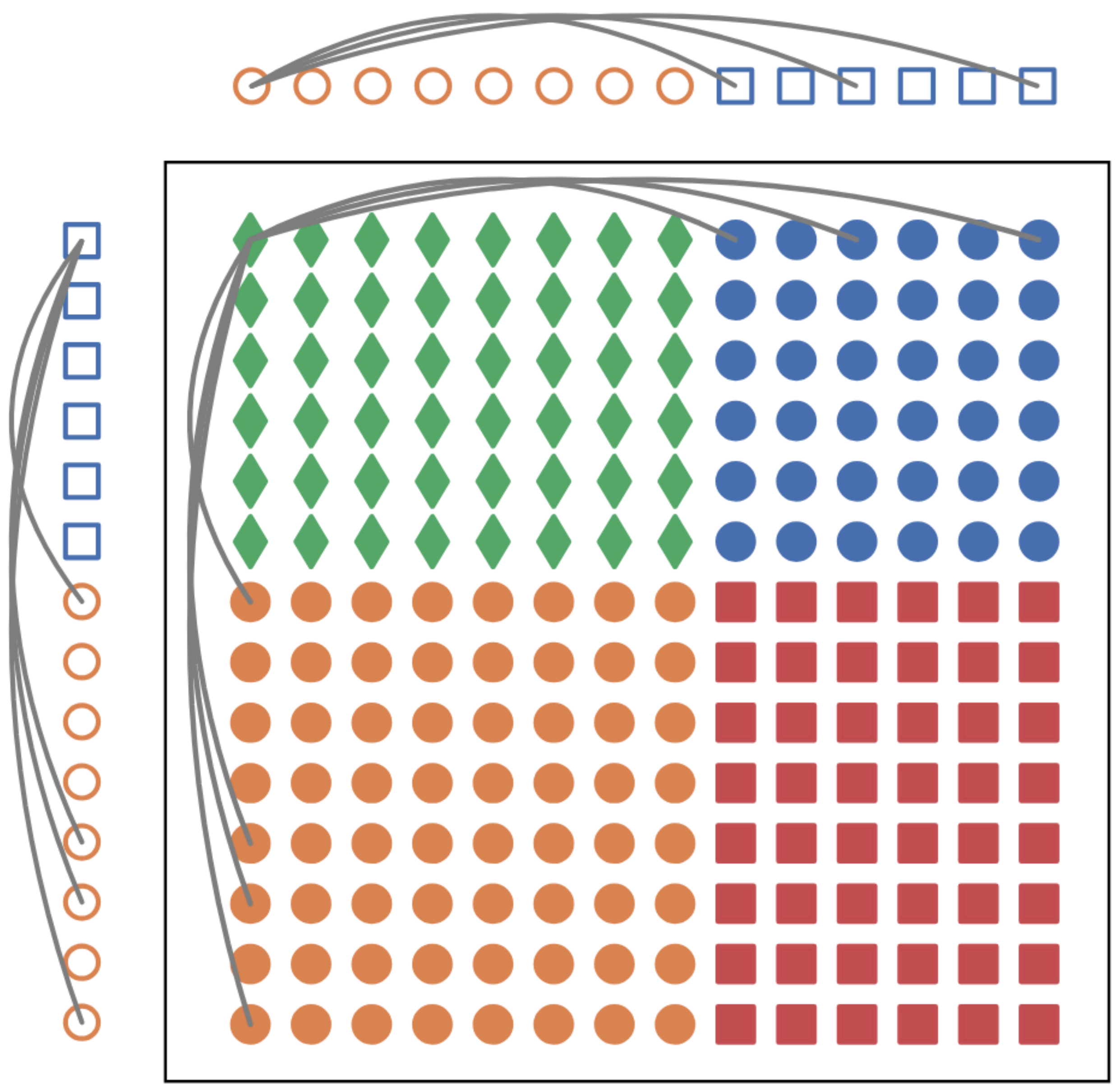

Hypergraph Product Code

QLDPC Codes

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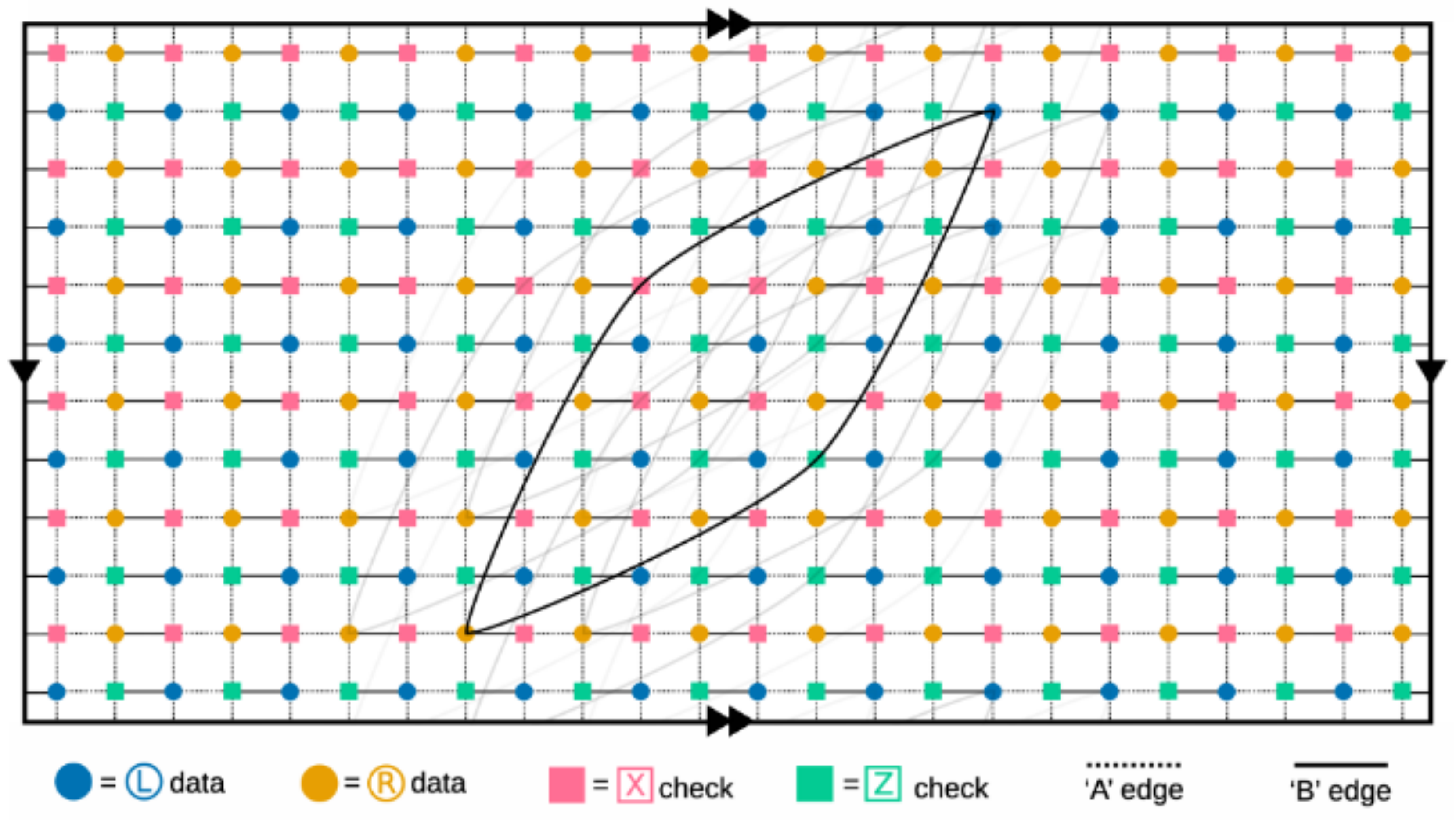

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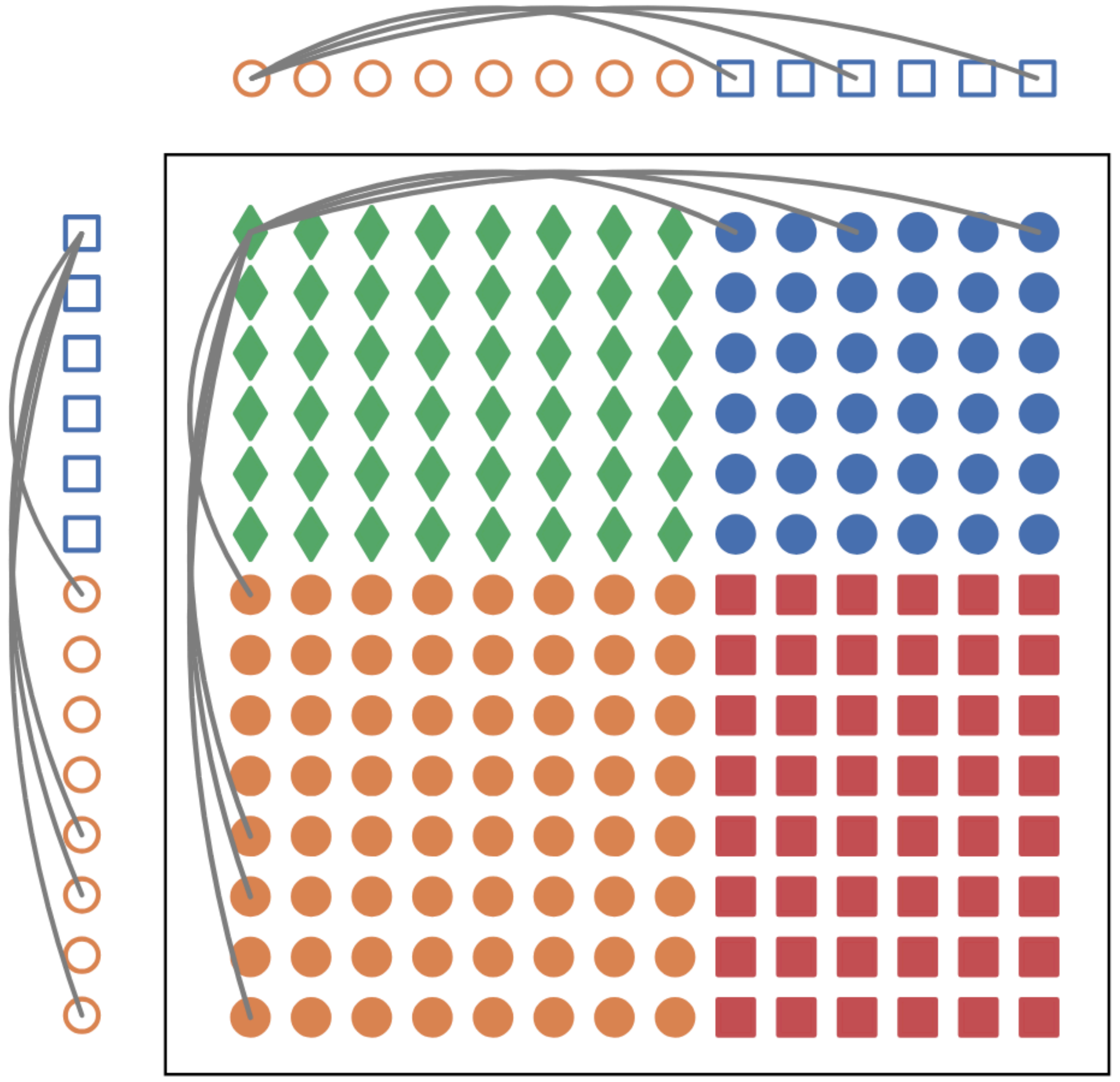

Hypergraph Product Code

$$H_X = \left(\begin{array}{cccccc|cccccc} F_0 & F_1 & F_2 & F_3 & F_4 & F_5 & G_0 & G_1 & G_2 & G_3 & G_4 & G_5 \\ F_5 & F_0 & F_1 & F_2 & F_3 & F_4 & G_5 & G_0 & G_1 & G_2 & G_3 & G_4 \\ F_4 & F_5 & F_0 & F_1 & F_2 & F_3 & G_4 & G_5 & G_0 & G_1 & G_2 & G_3 \end{array}\right)$$

$$H_Z = \left(\begin{array}{cccccc|cccccc} G_0^T & G_5^T & G_4^T & G_3^T & G_2^T & G_1^T & F_0^T & F_5^T & F_4^T & F_3^T & F_2^T & F_1^T \\ G_1^T & G_0^T & G_5^T & G_4^T & G_3^T & G_2^T & F_1^T & F_0^T & F_5^T & F_4^T & F_3^T & F_2^T \\ G_2^T & G_1^T & G_0^T & G_5^T & G_4^T & G_3^T & F_2^T & F_1^T & F_0^T & F_5^T & F_4^T & F_3^T \end{array}\right)$$

Affine Permutation Matrices (APM) Code

QLDPC Codes

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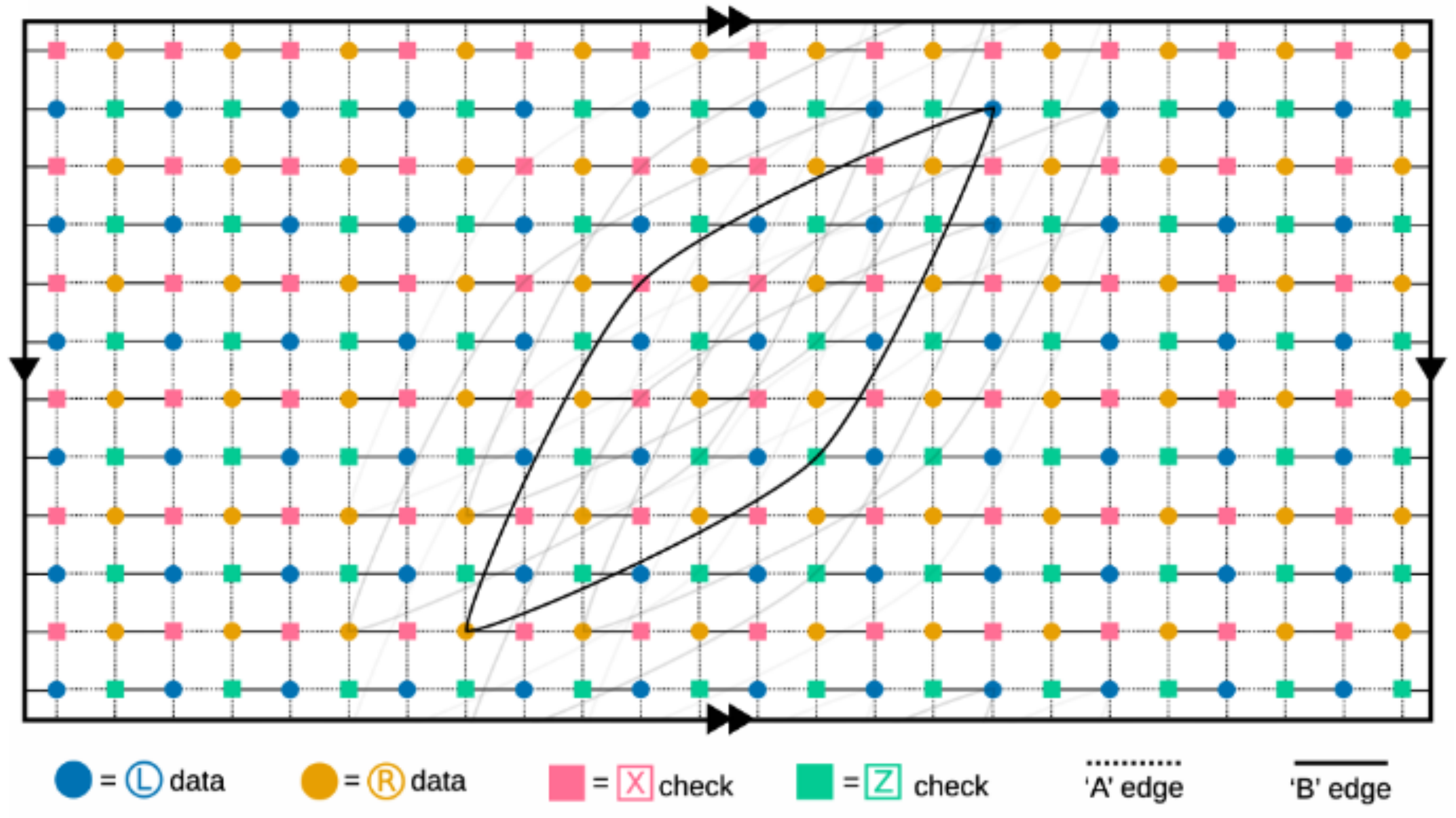

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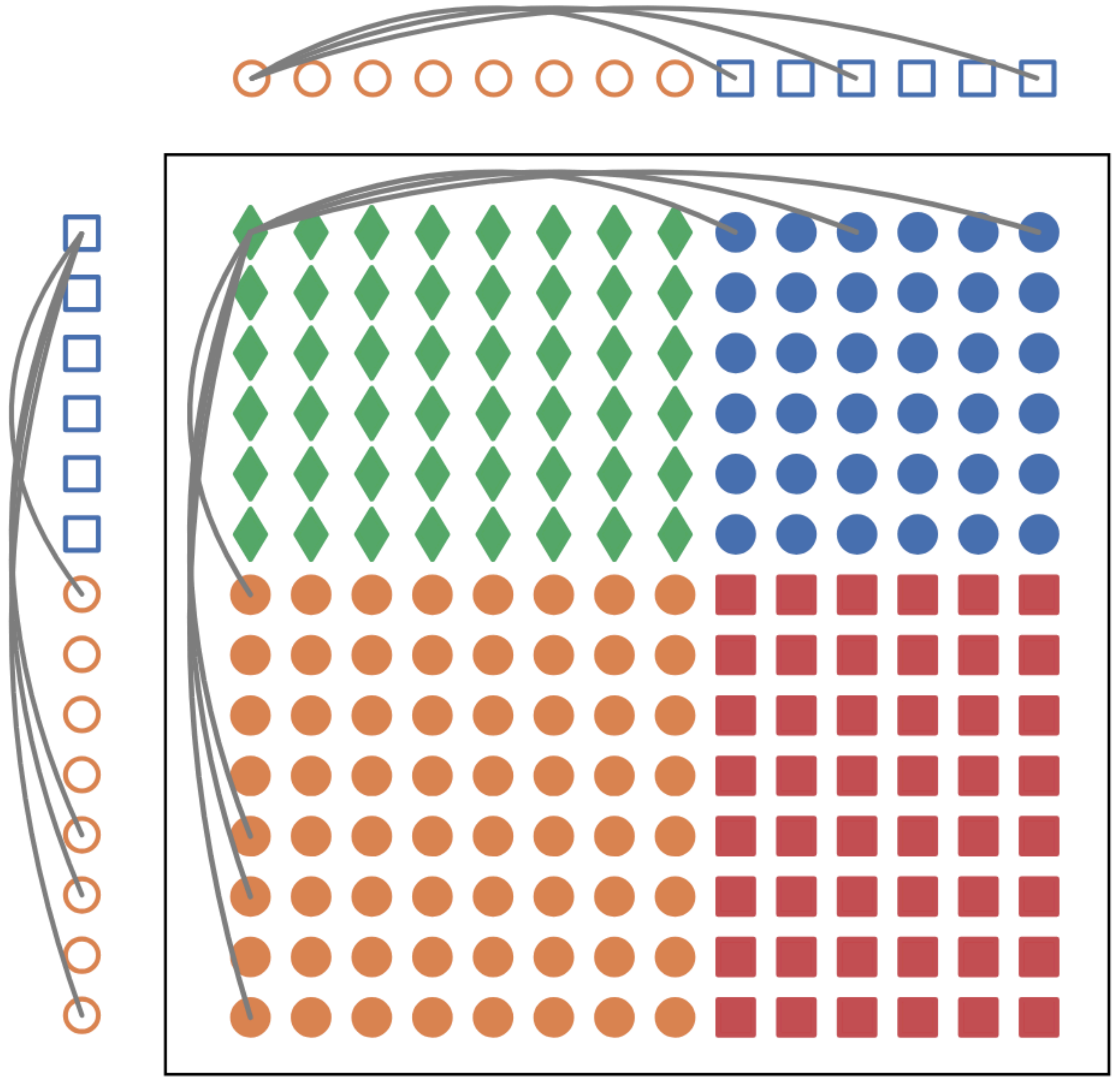

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$$H_Z = \left(\begin{array}{cccccc|cccccc} G_0^T & G_5^T & G_4^T & G_3^T & G_2^T & G_1^T & F_0^T & F_5^T & F_4^T & F_3^T & F_2^T & F_1^T \\ G_1^T & G_0^T & G_5^T & G_4^T & G_3^T & G_2^T & F_1^T & F_0^T & F_5^T & F_4^T & F_3^T & F_2^T \\ G_2^T & G_1^T & G_0^T & G_5^T & G_4^T & G_3^T & F_2^T & F_1^T & F_0^T & F_5^T & F_4^T & F_3^T \end{array}\right)$$

Affine Permutation Matrices (APM) Code

Lots of work in the past two years that study logical computation on QLDPC codes.

Computation on QLDPC codes

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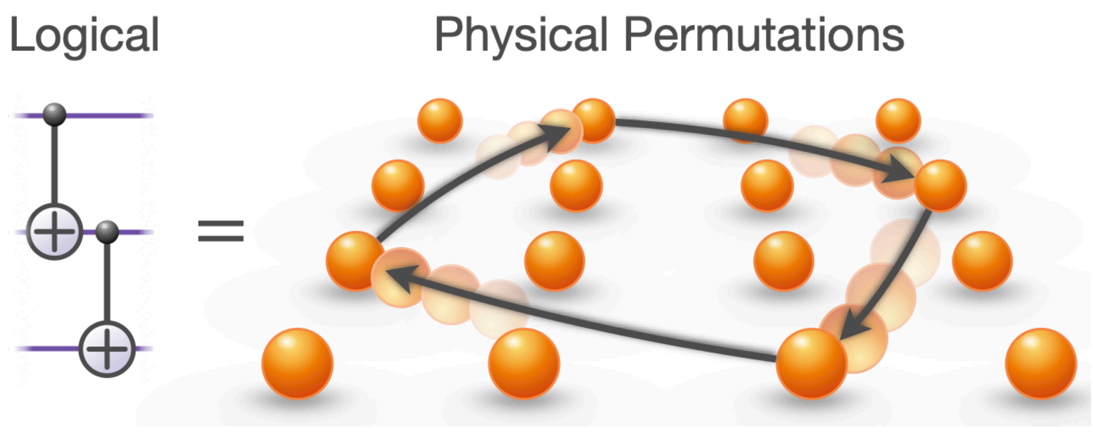
Clifford gates

Non-Clifford gates

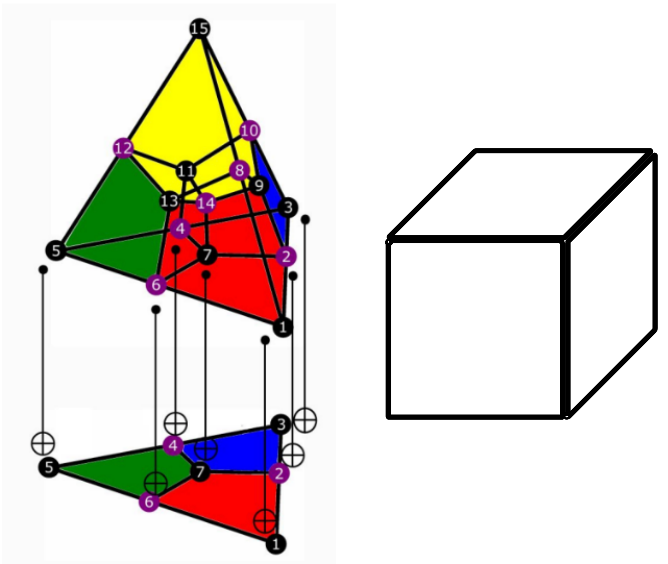
Computation on QLDPC codes

Clifford gates

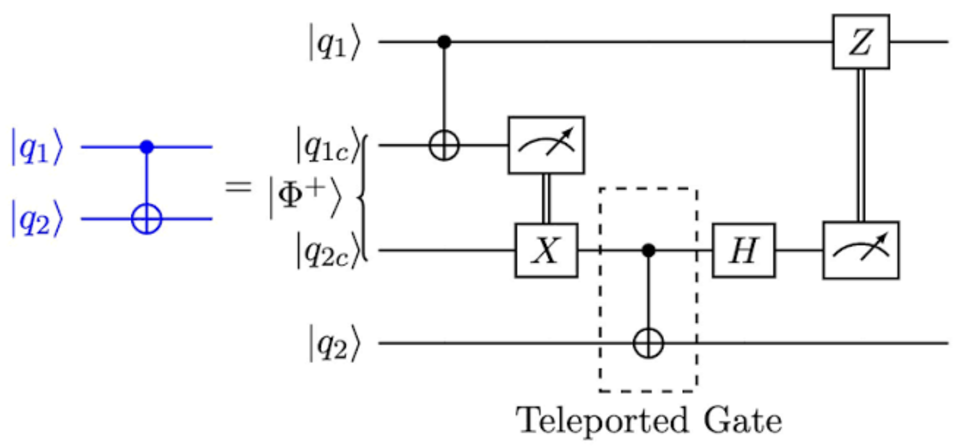
Non-Clifford gates



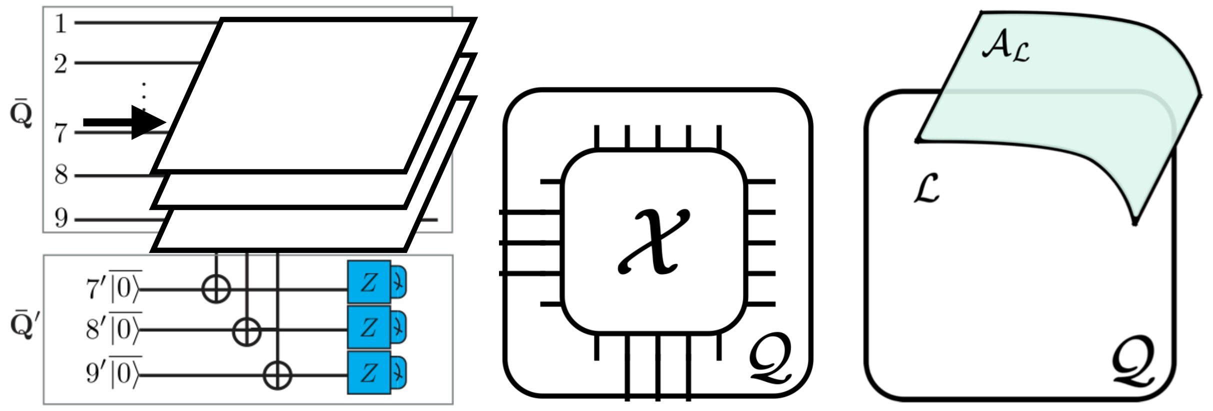
Automorphisms,
Low-depth unitaries



Code-switching



State prep. & gate teleportation



Measuring Pauli operators

Fig. Credits: Koh, *et al.* [2601.20927], Daguerre, Kim. PRR **7** (2025), Xu, *et al.* PRX **15** (2025), He, *et al.* [2503.10390],

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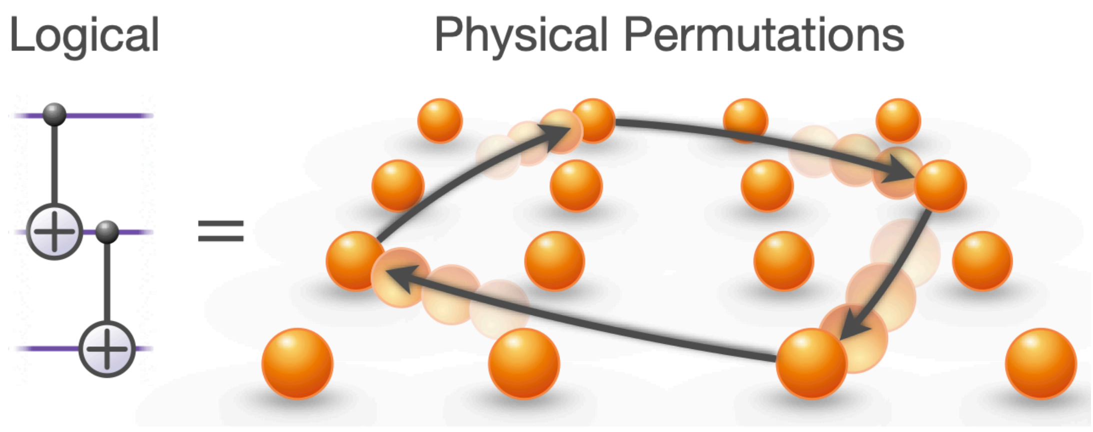

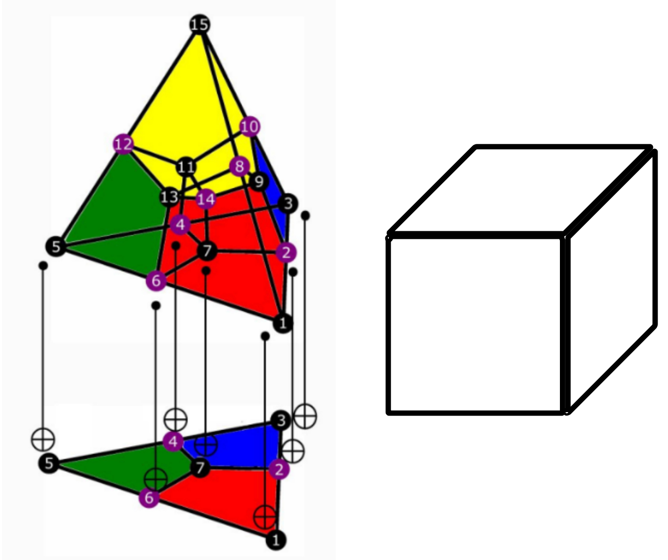

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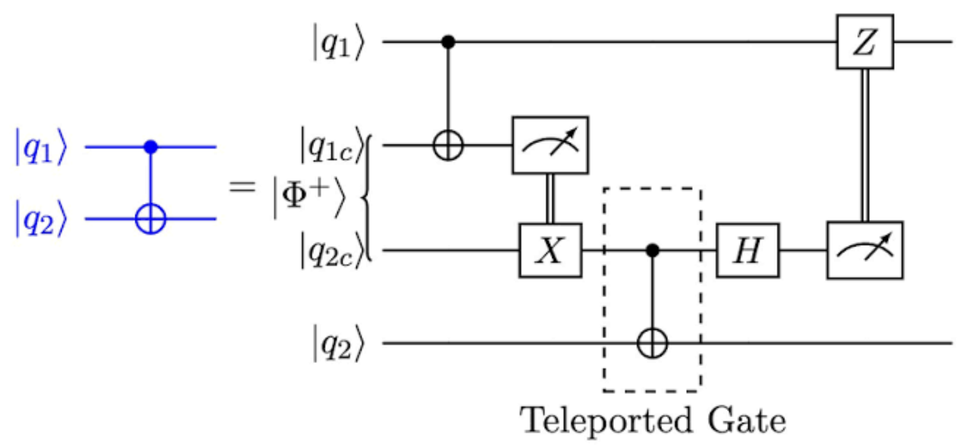

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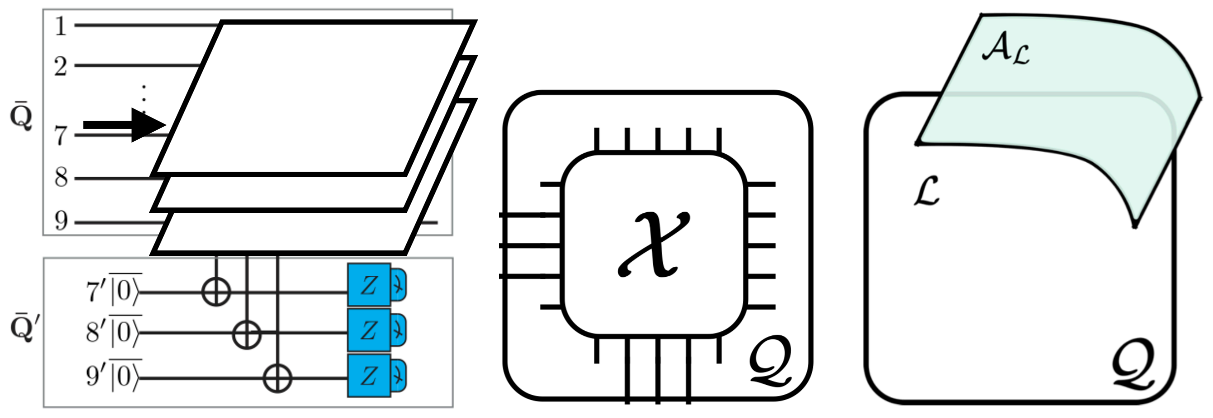
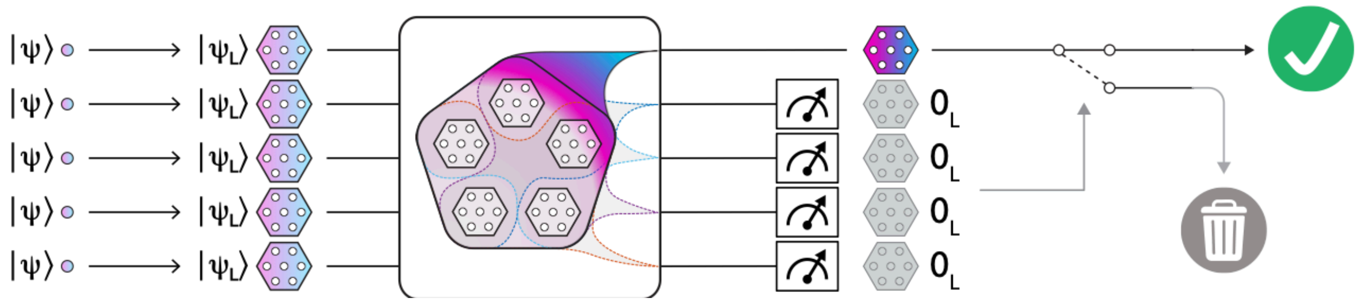
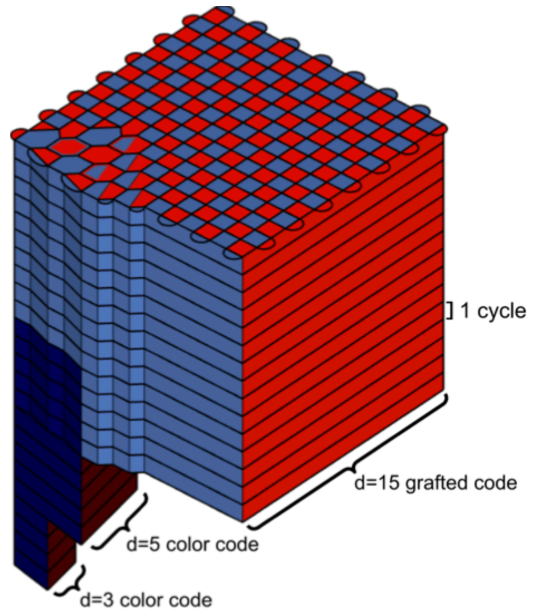


Measuring Pauli operators

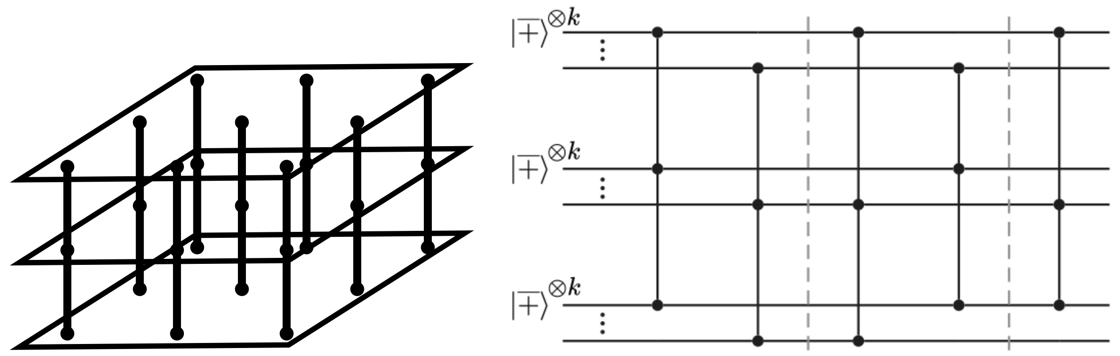
Non-Clifford gates



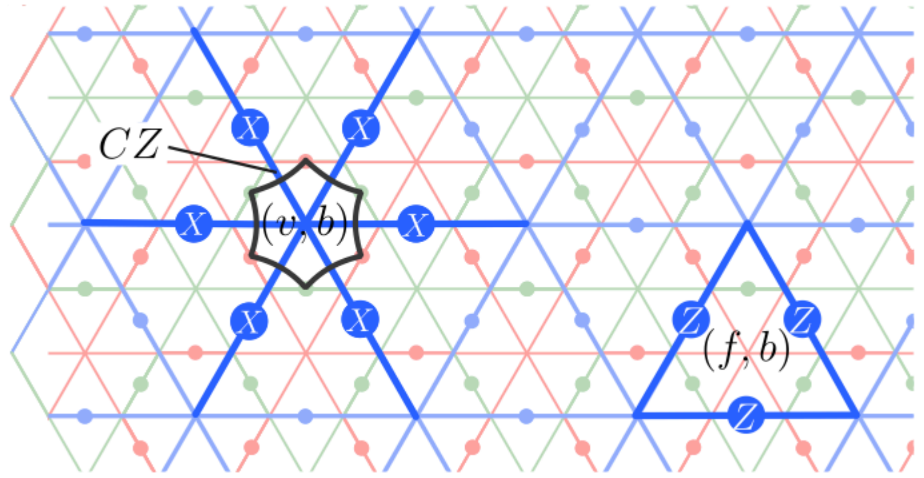
Magic state distillation



Magic state cultivation



Low-depth unitaries



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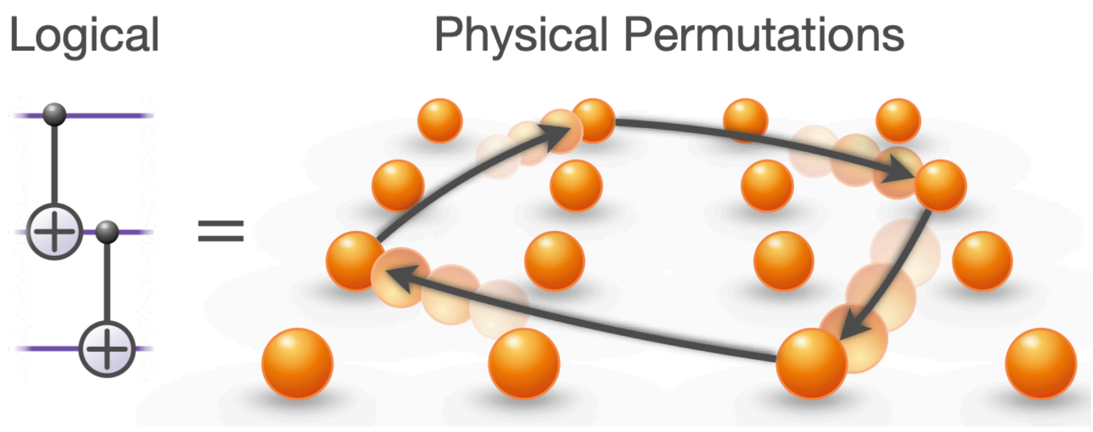
Computation on QLDPC codes

Clifford gates

Non-Clifford gates



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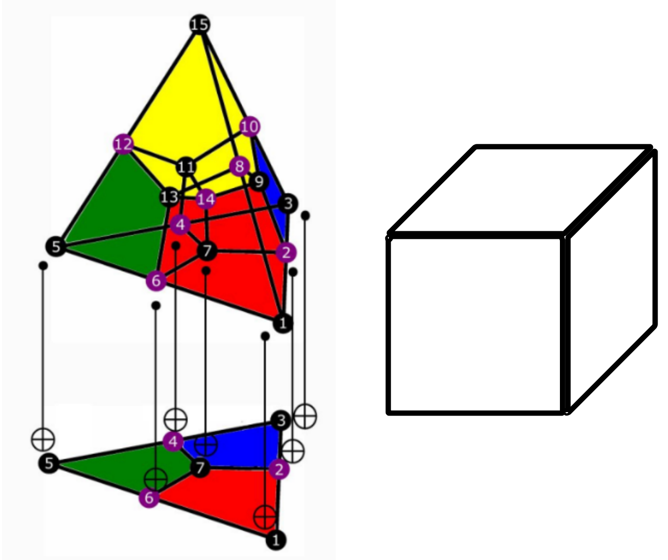

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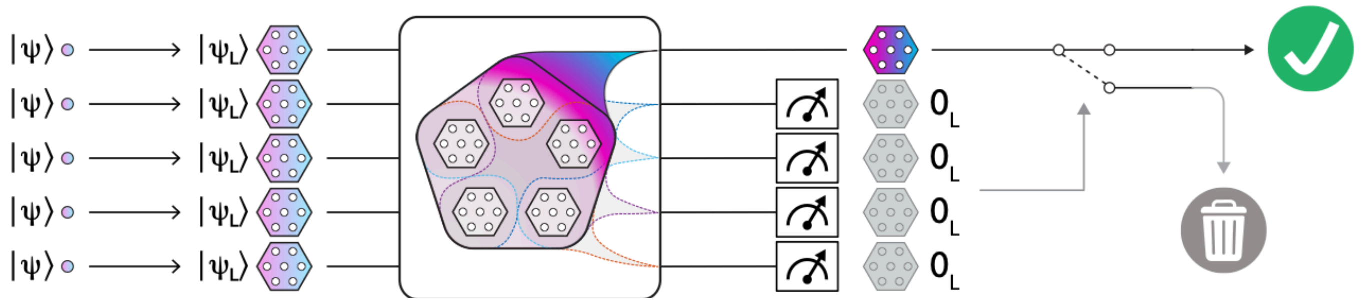

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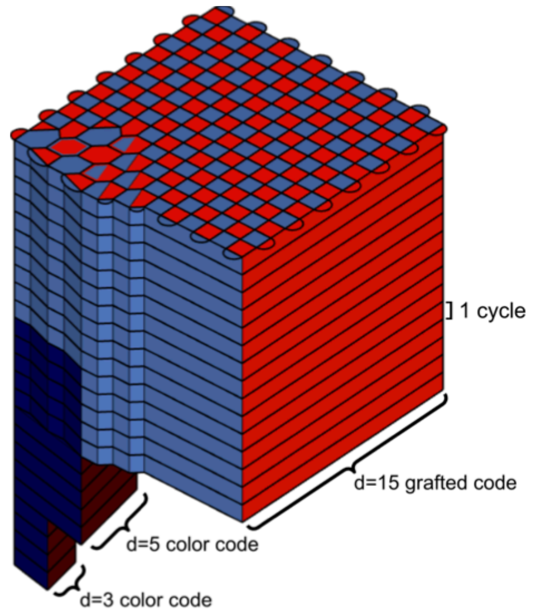

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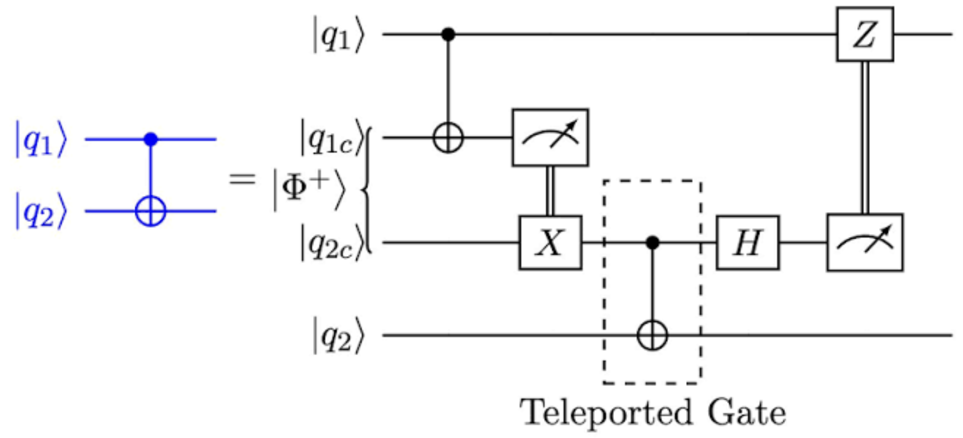
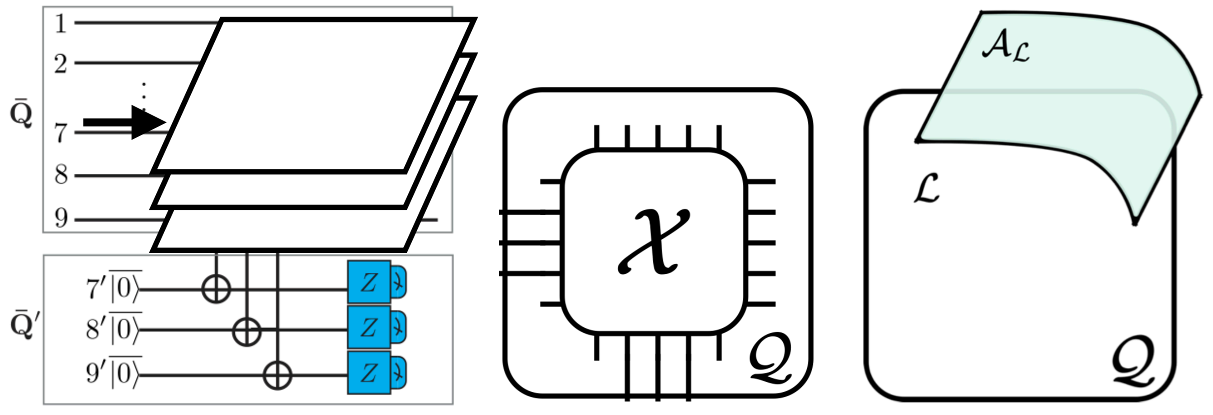


State prep. & gate teleportation



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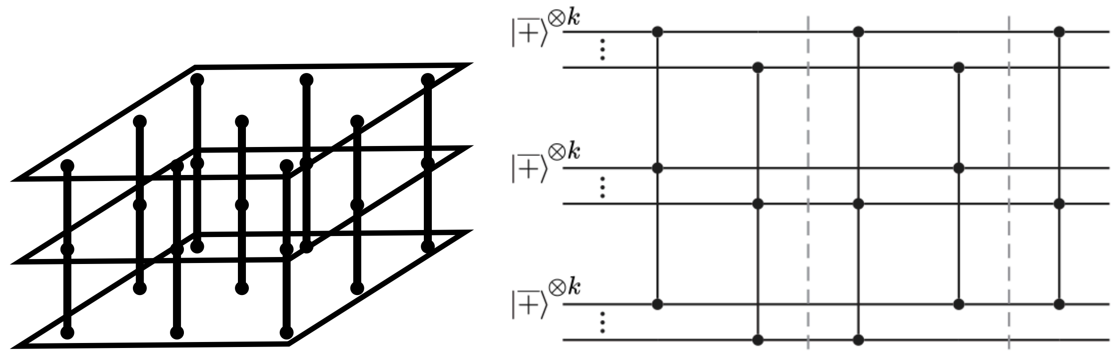
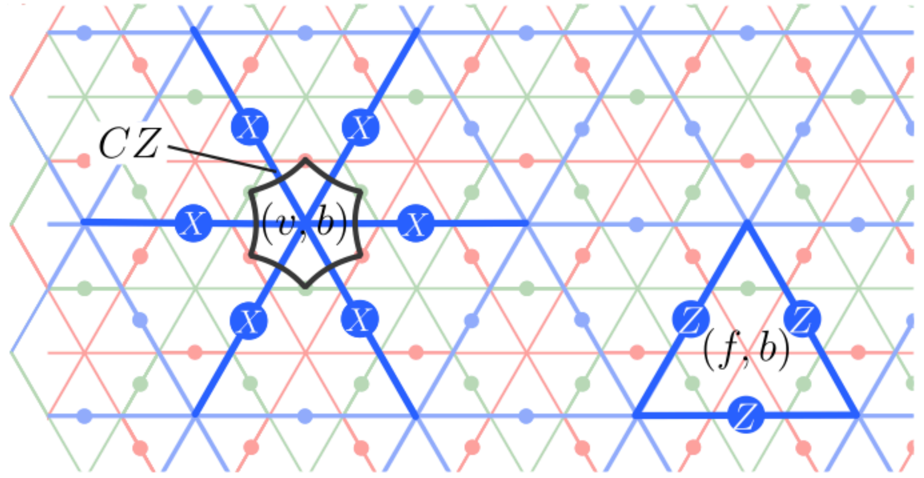


Low-depth unitaries



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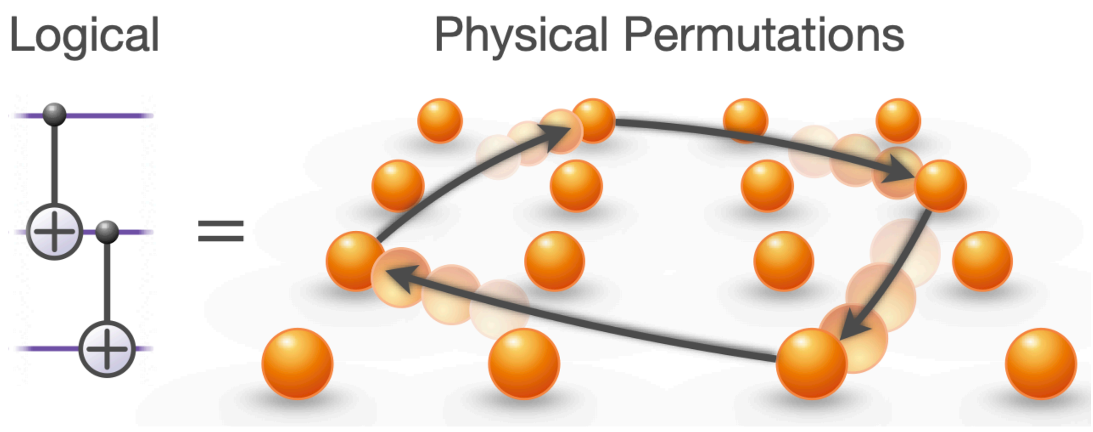

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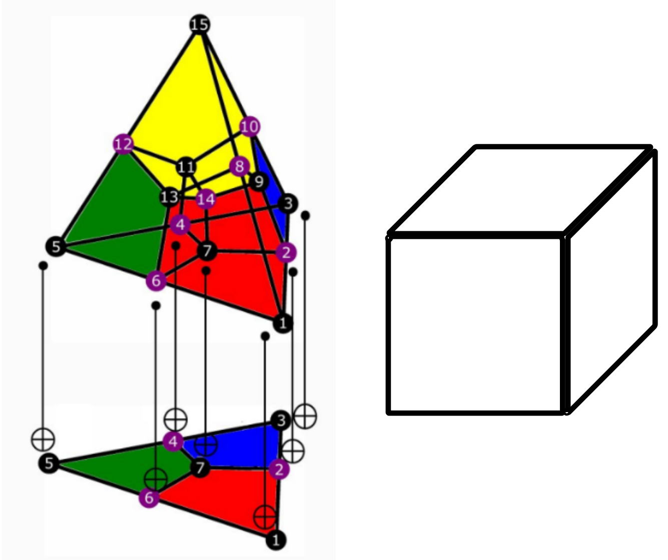
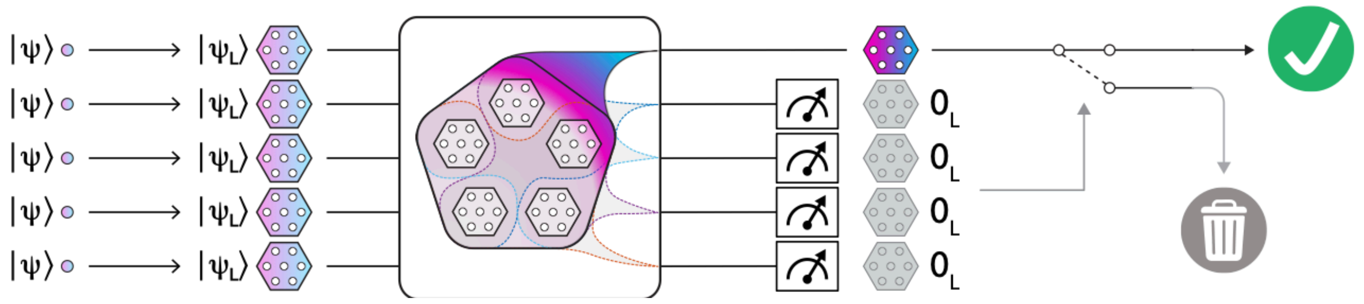


Code-switching

Non-Clifford gates



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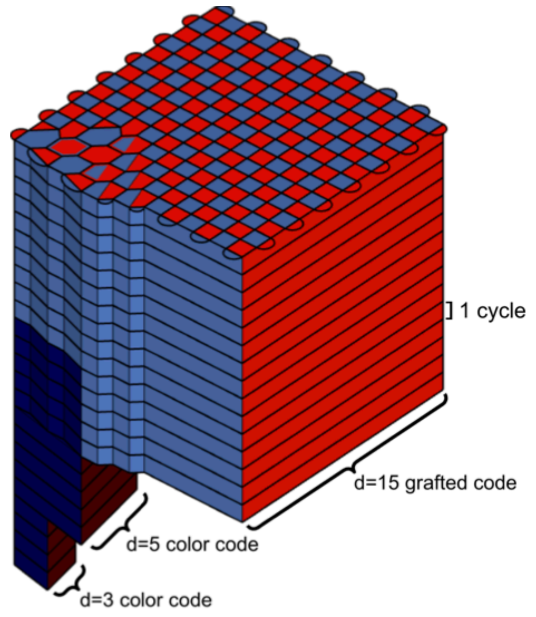
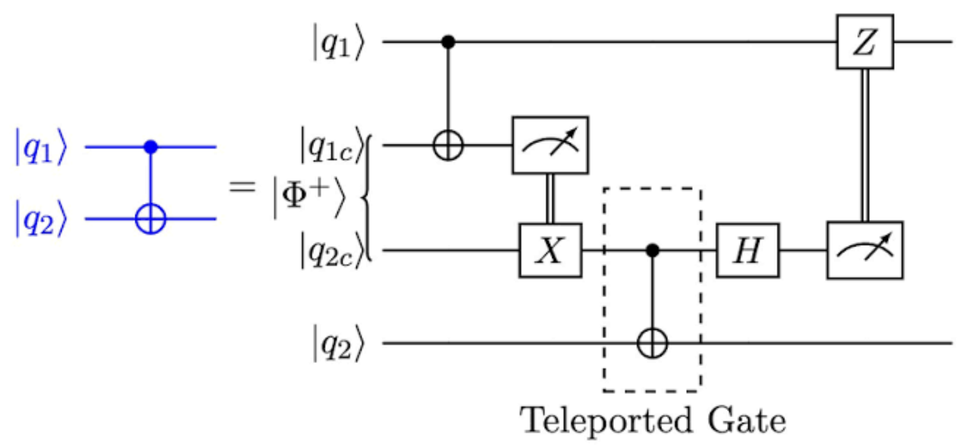
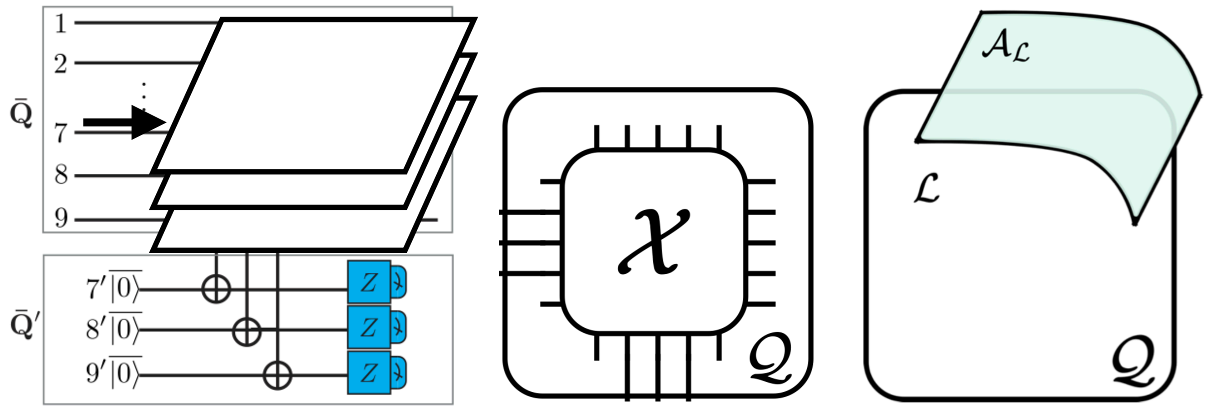


Magic state cultivation



State prep. & gate teleportation



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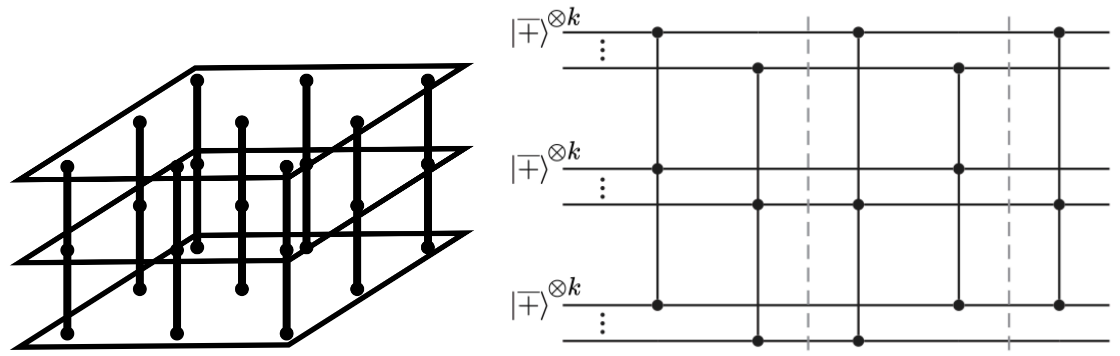
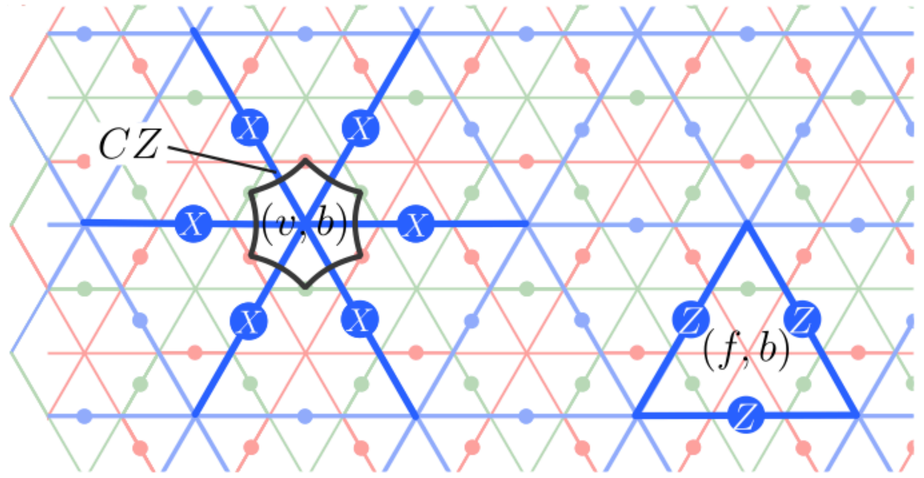


Low-depth unitaries



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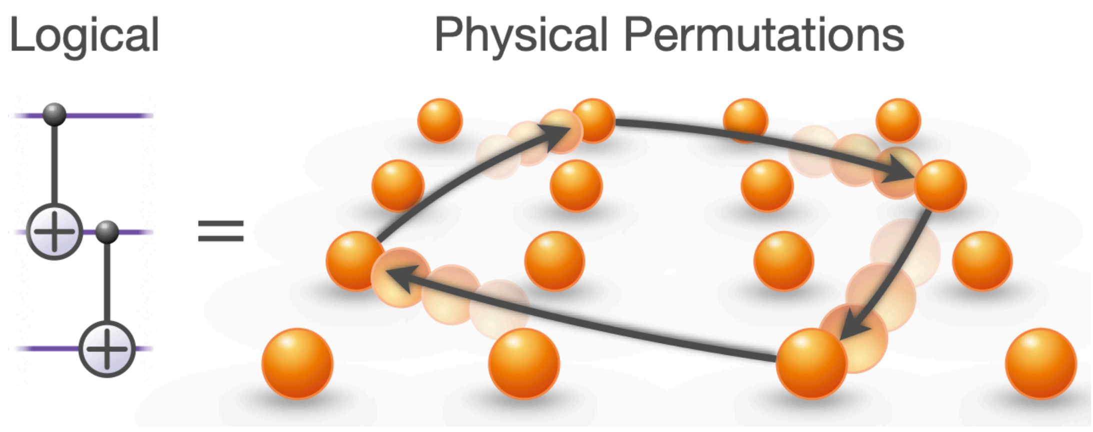
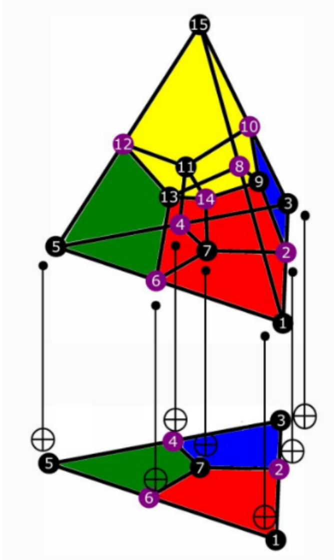
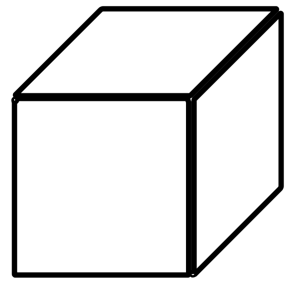
Computation on QLDPC codes

Clifford gates







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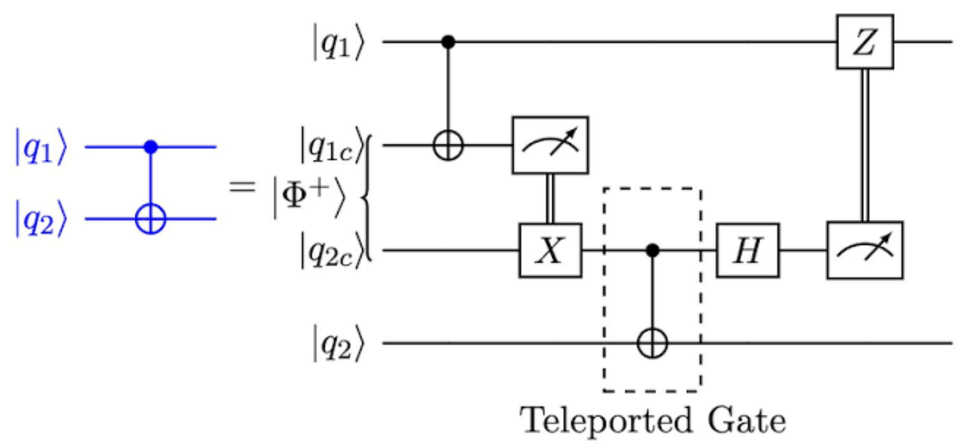

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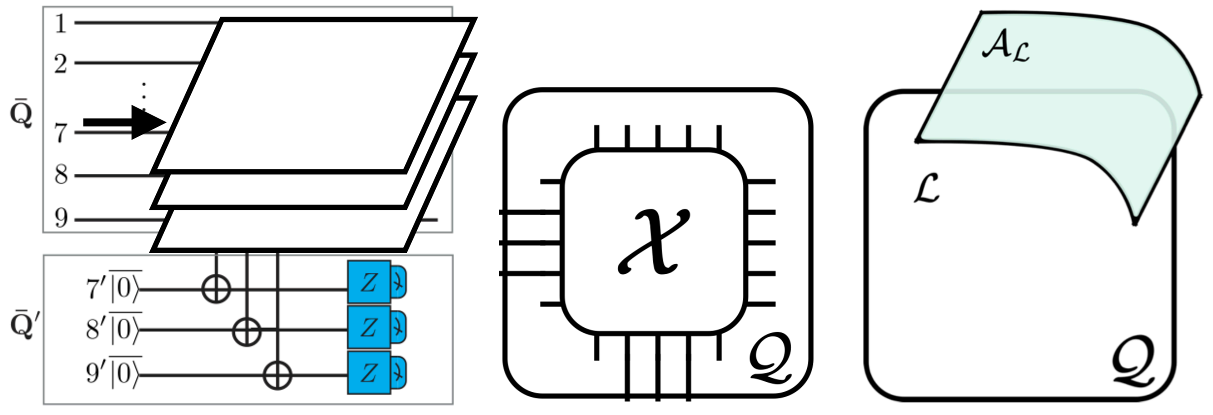
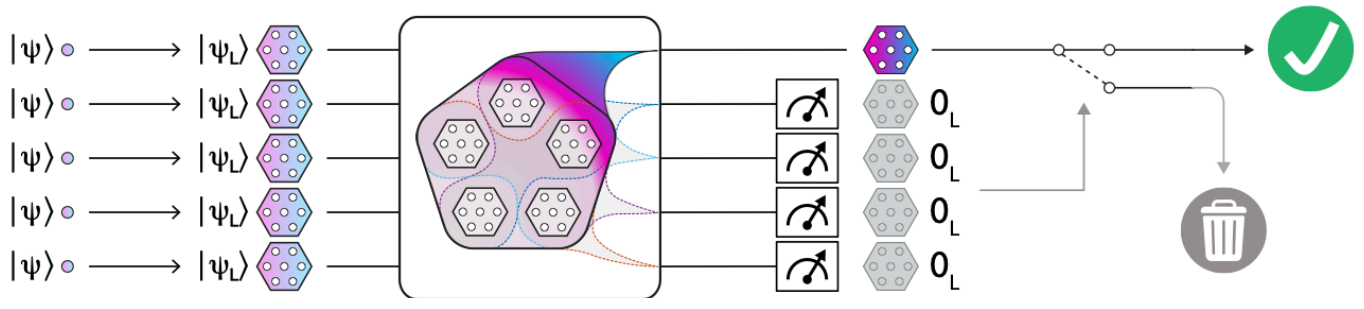
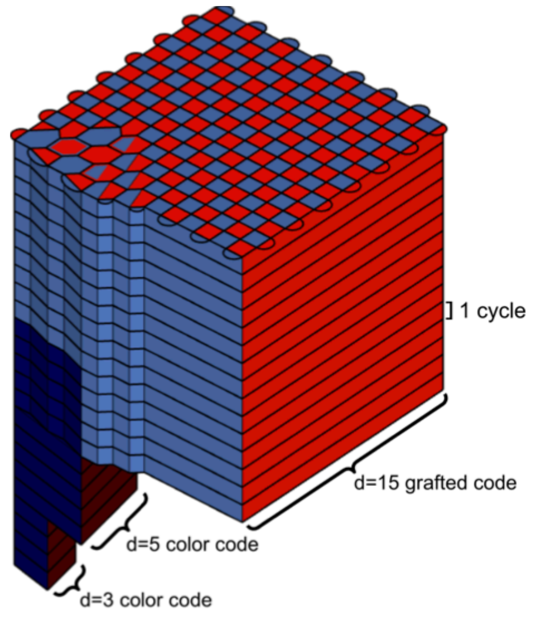


Measuring Pauli operators

Part I. *Fast* primitives for Pauli operator measurements

Non-Clifford gates





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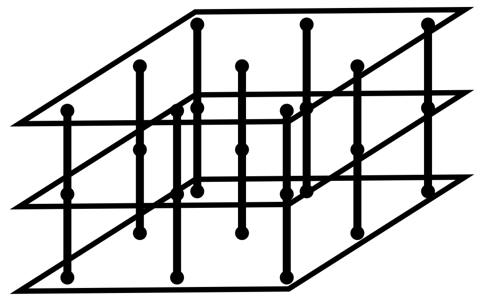
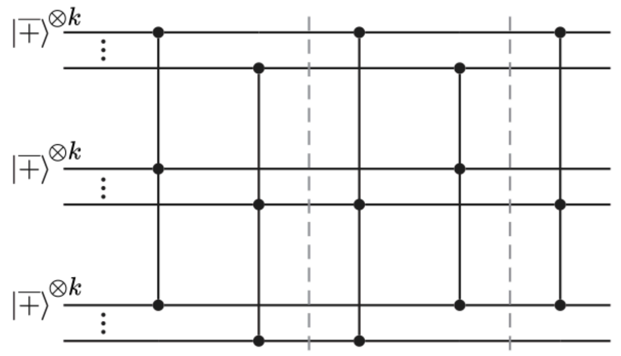
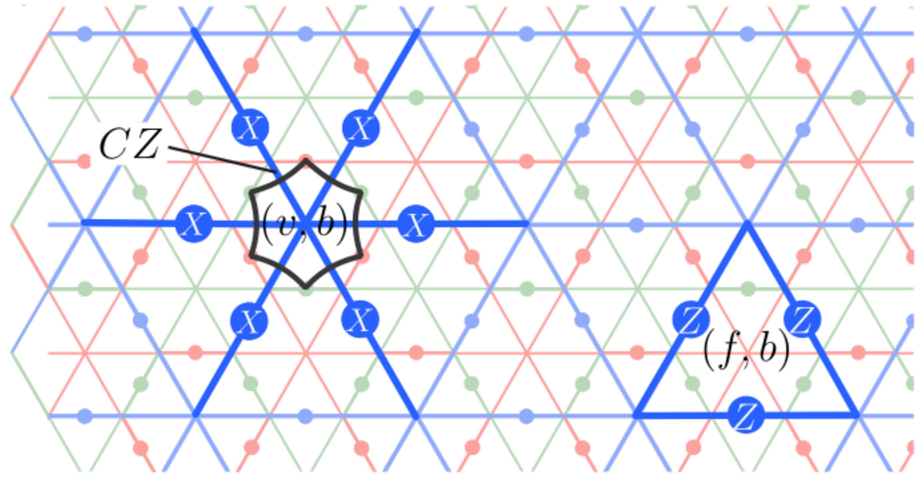




Low-depth unitaries



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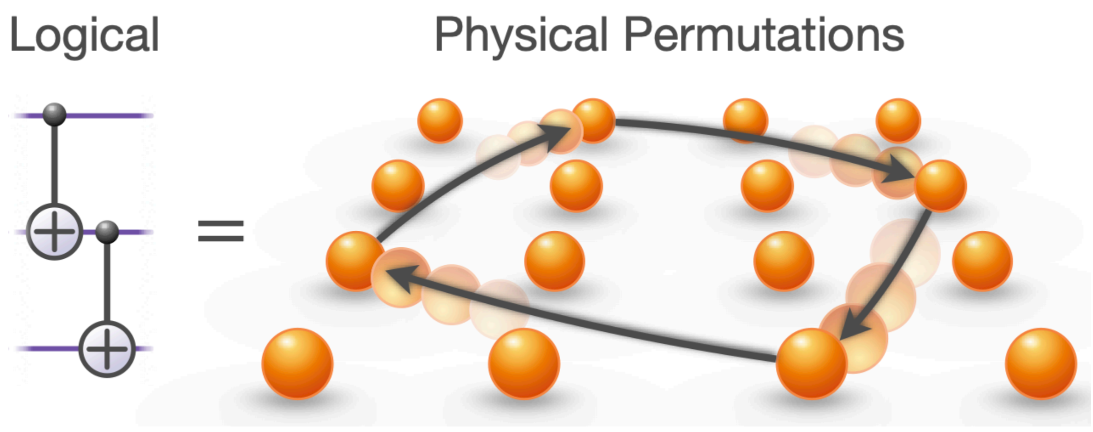
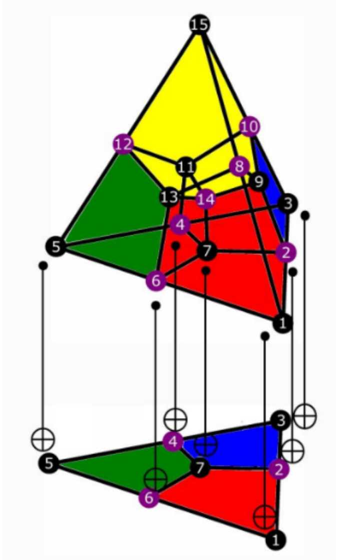
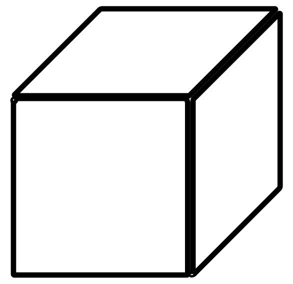
Computation on QLDPC codes

Clifford gates







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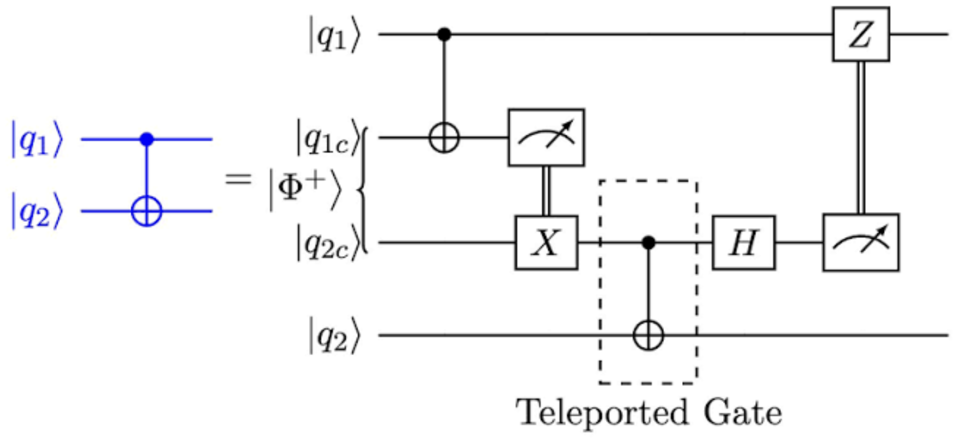

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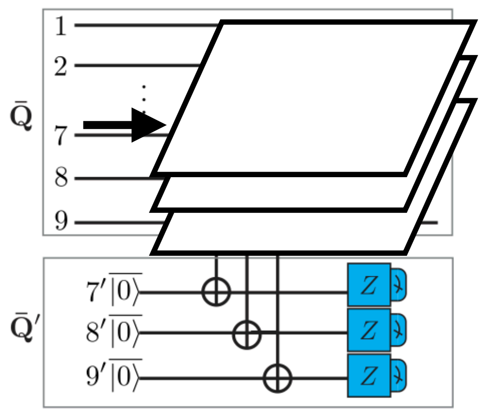
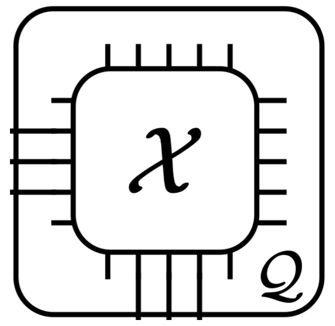
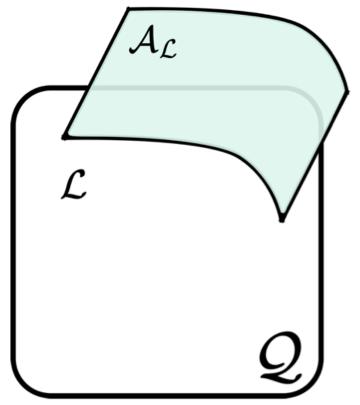
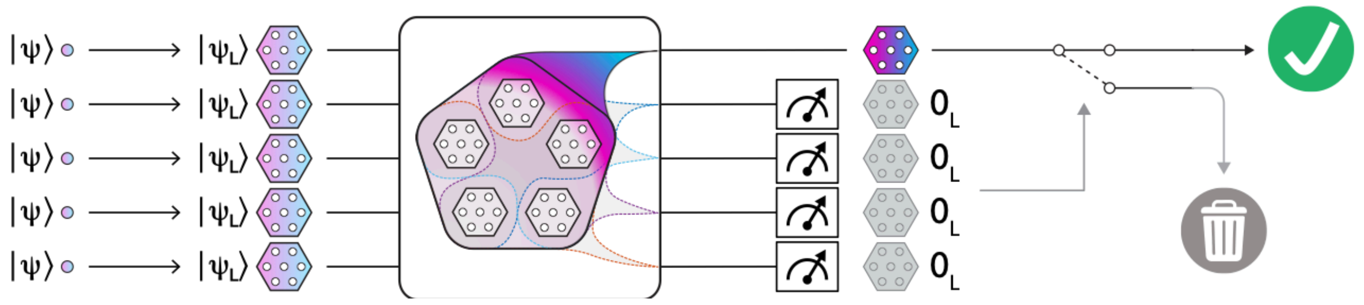


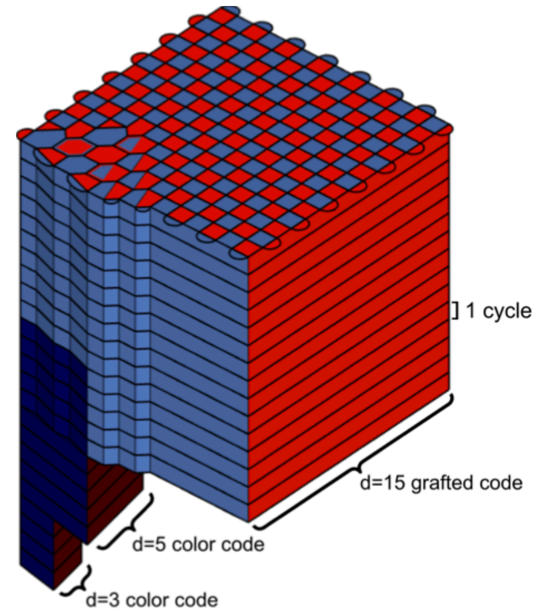




Measuring Pauli operators

Non-Clifford gates





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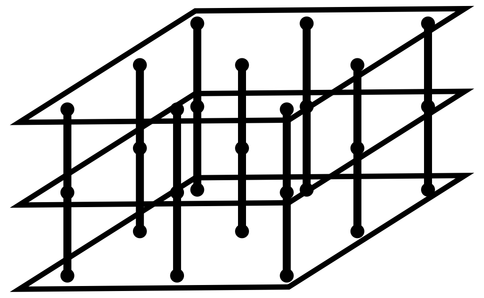

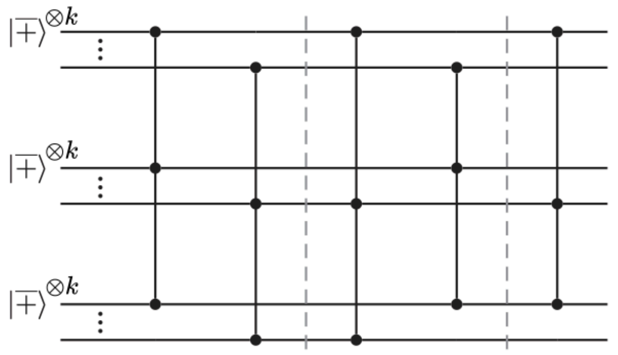

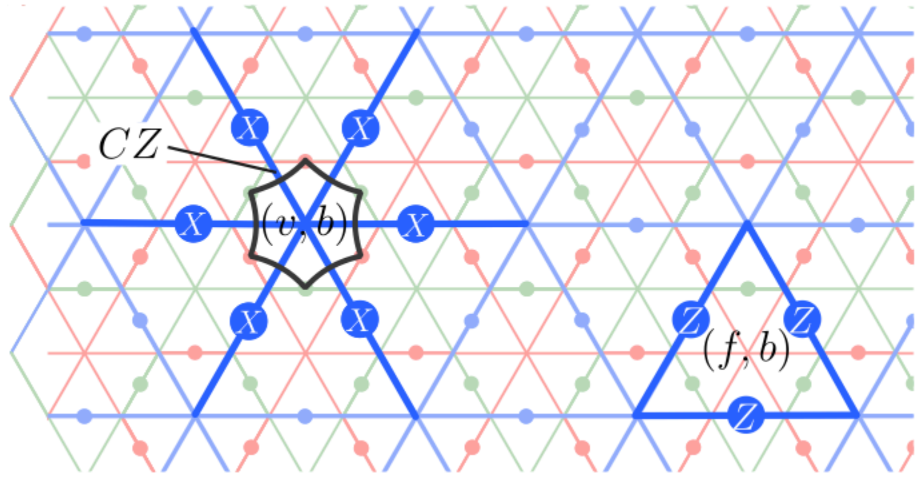

Low-depth unitaries

Measuring Clifford operators

Part I. *Fast* primitives for Pauli operator measurements

Part II. New (*Furious*?) techniques for measuring Clifford operators on *QLDPC* codes

Fig. Credits: Koh, *et al.* [2601.20927], Daguerre, Kim. PRR **7** (2025), Xu, *et al.* PRX **15** (2025), He, *et al.* [2503.10390], Sales Rodriguez, *et al.* Nature **645** (2025), Gidney, *et al.* [2409.17595], Menon, *et* al. PRX **16** (2026).Davydova, *et al.* PRX **16** (2026)

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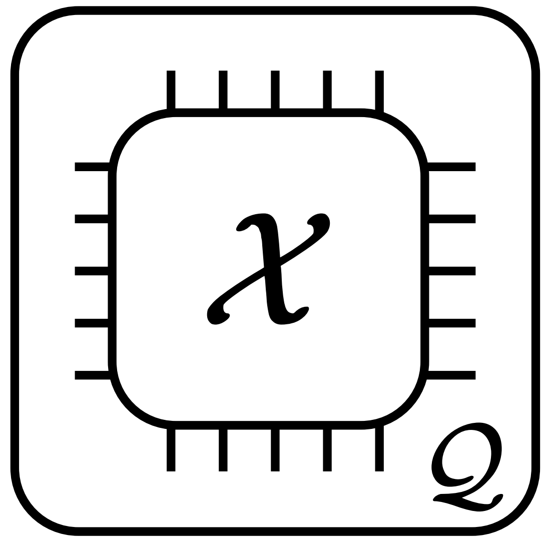

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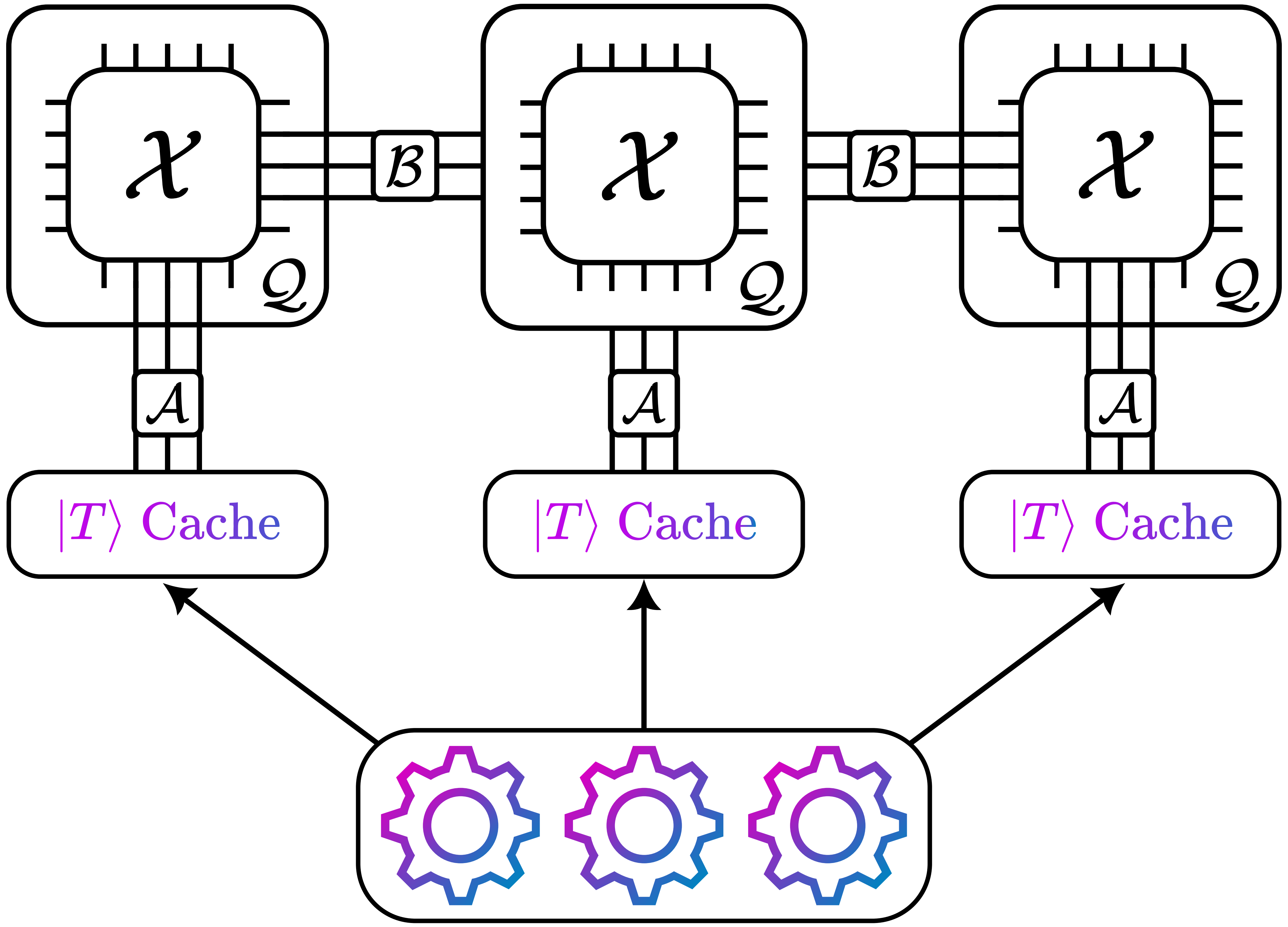
  - Or in parallel with different designs.[3,12,13]
- Can be adapted to many hardware platforms.[5,6,8,10,11]



Extractor Architectures,
a model for surgery architectures

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Next: why does surgery take O(d) time?

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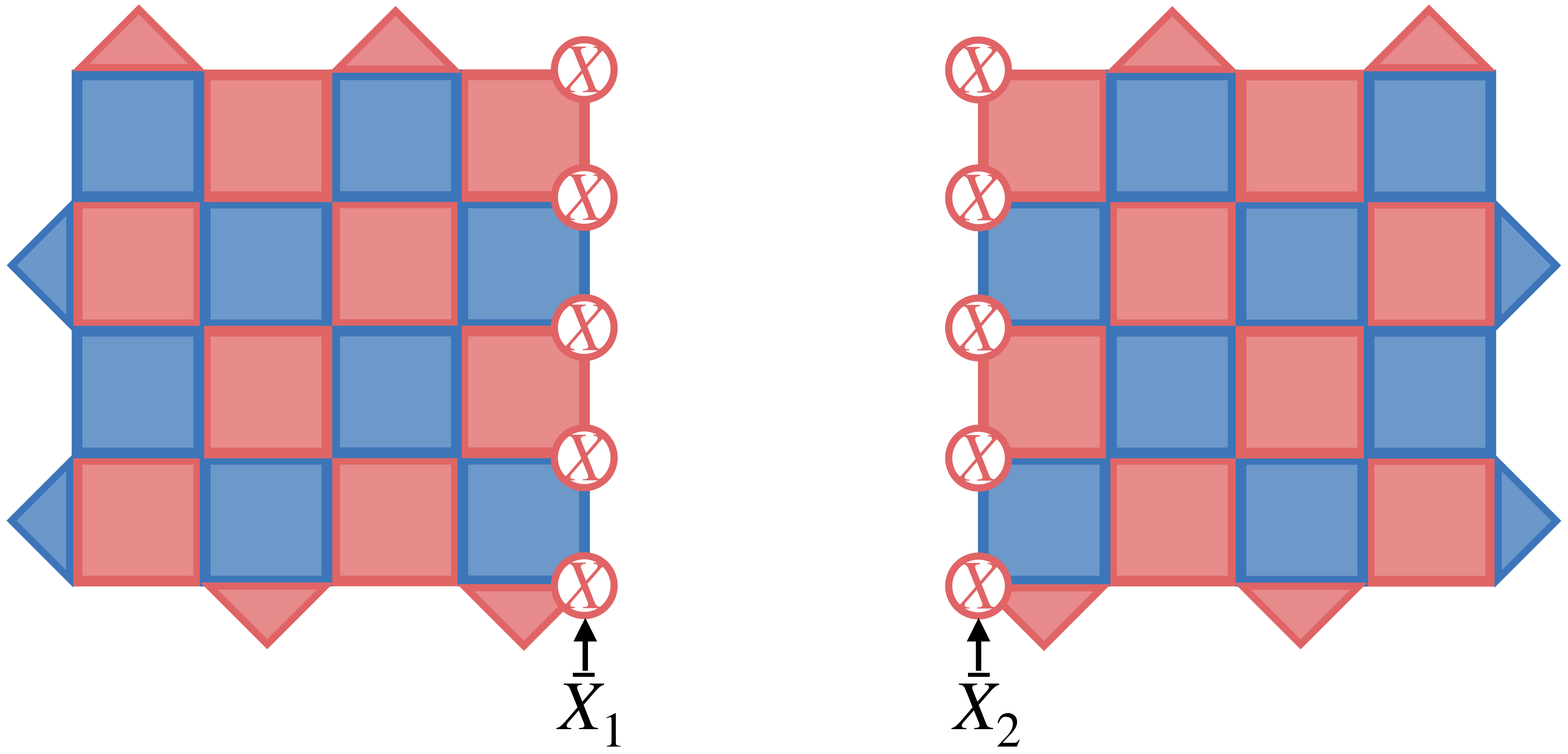
Lattice surgery on the surface code

Ex: Measuring $\bar{X}_1\bar{X}_2$



[1] Vuillot *et al.* NJP **21** (2019).

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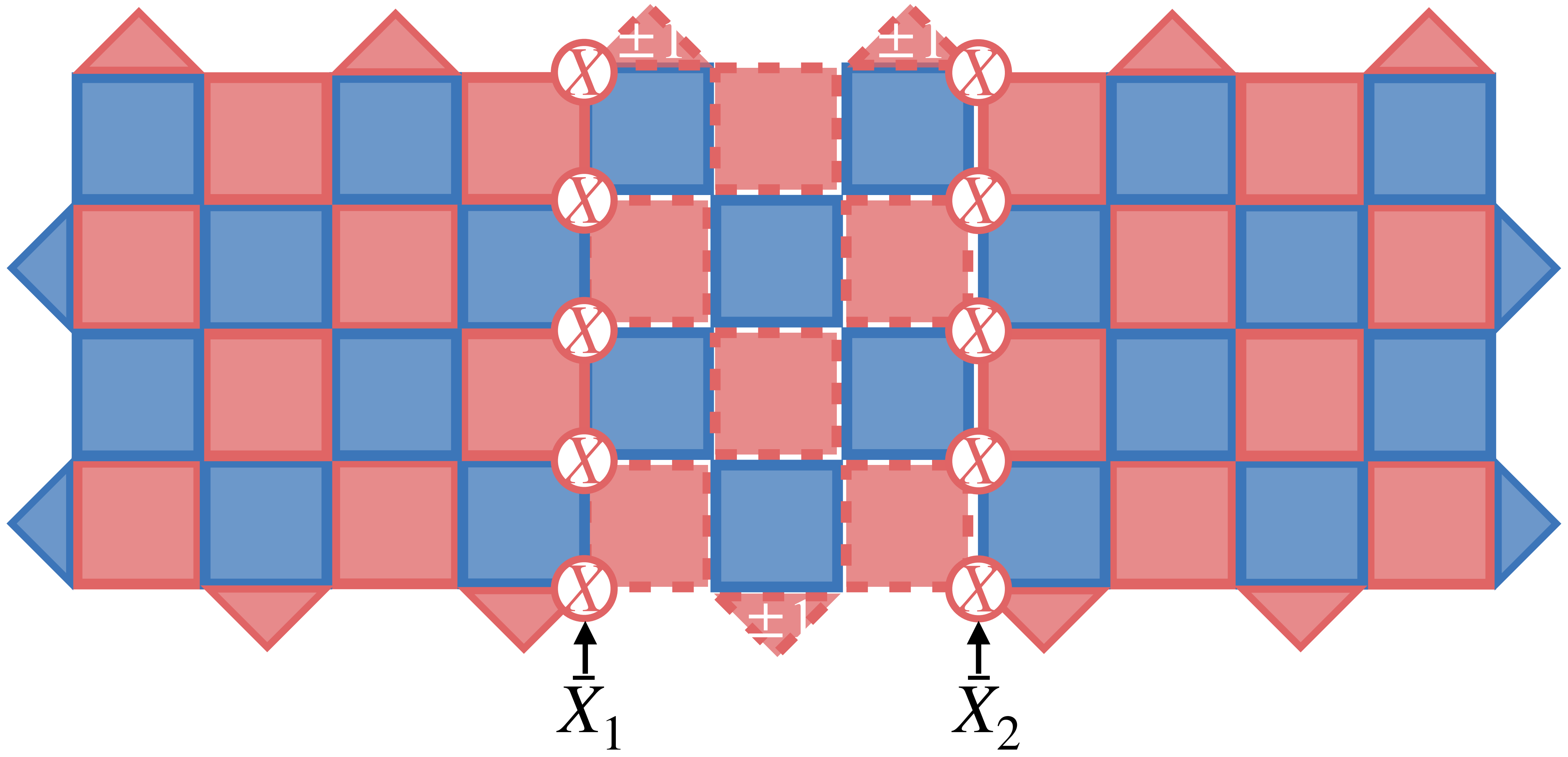
Ex: Measuring $\bar{X}_1\bar{X}_2$

$\bar{X}_1\bar{X}_2 = \prod$



- Switch to deformed/merged code where logical operator to be measured = products of stabilizers[1]

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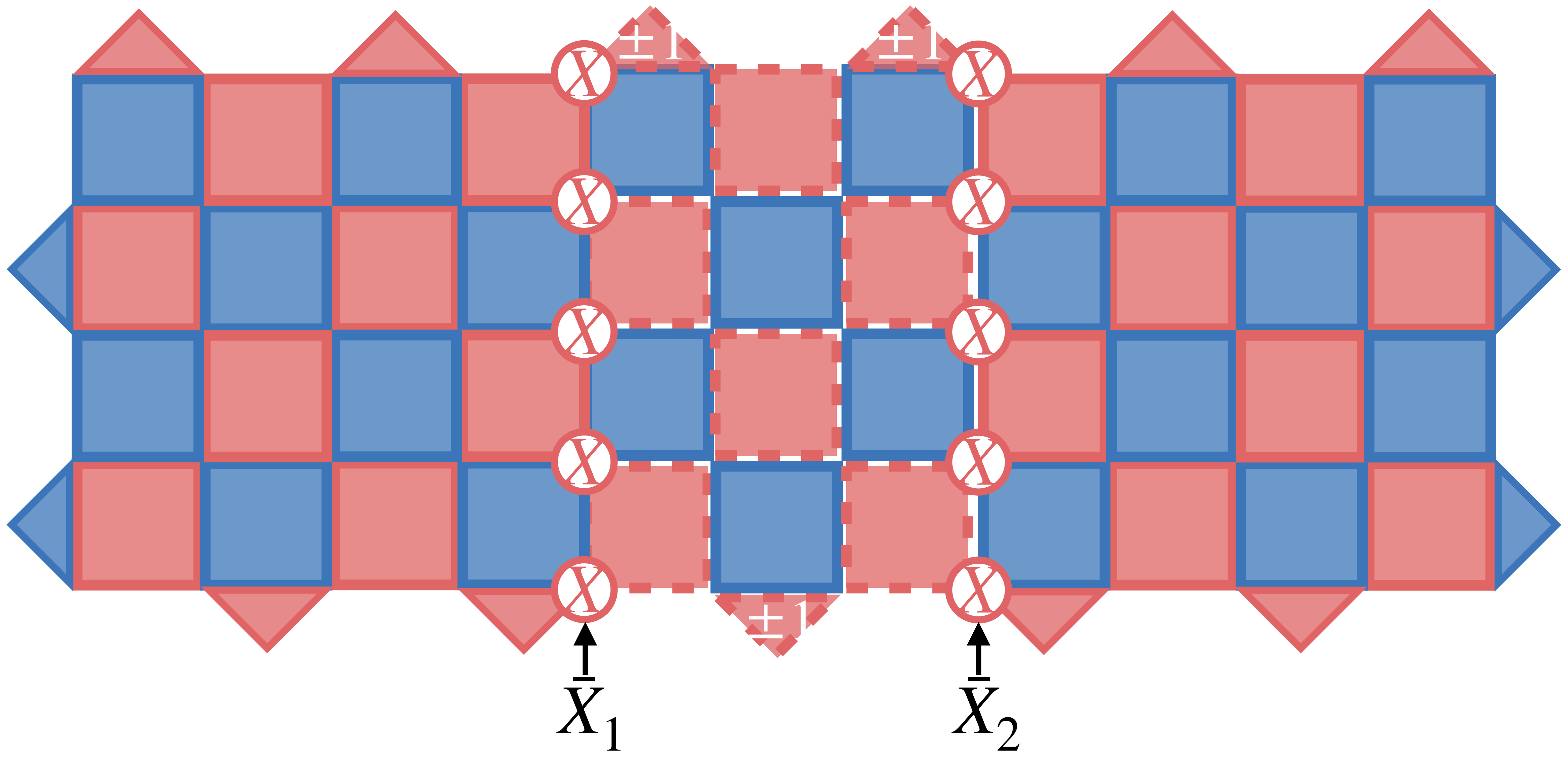

- Switch to deformed/merged code where logical operator to be measured = products of stabilizers[1]
- Measurement errors → need to repeat for $O(d)$ rounds.

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What is fast surgery?

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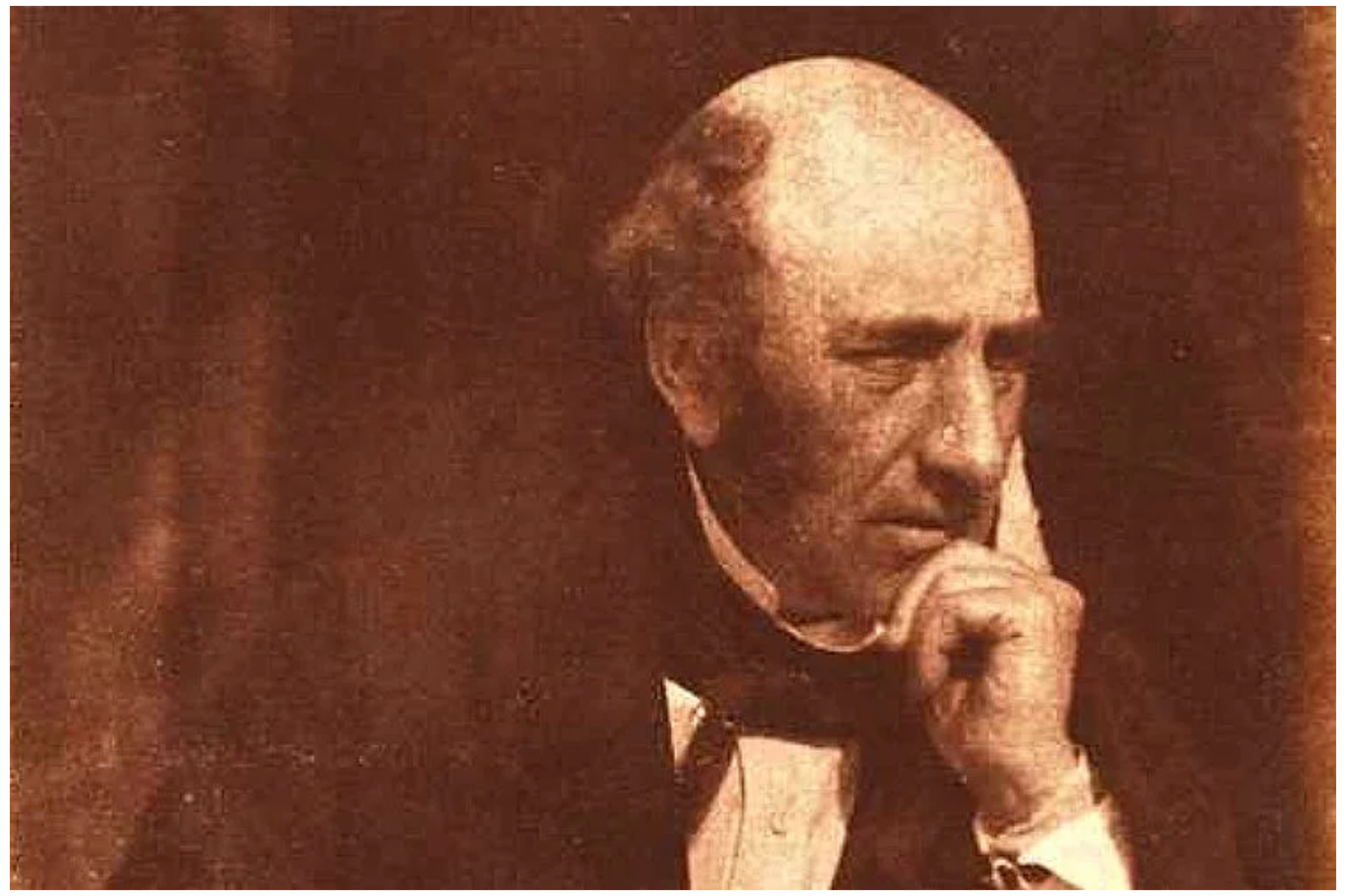

Robert Liston, infamous 17th century Scottish surgeon

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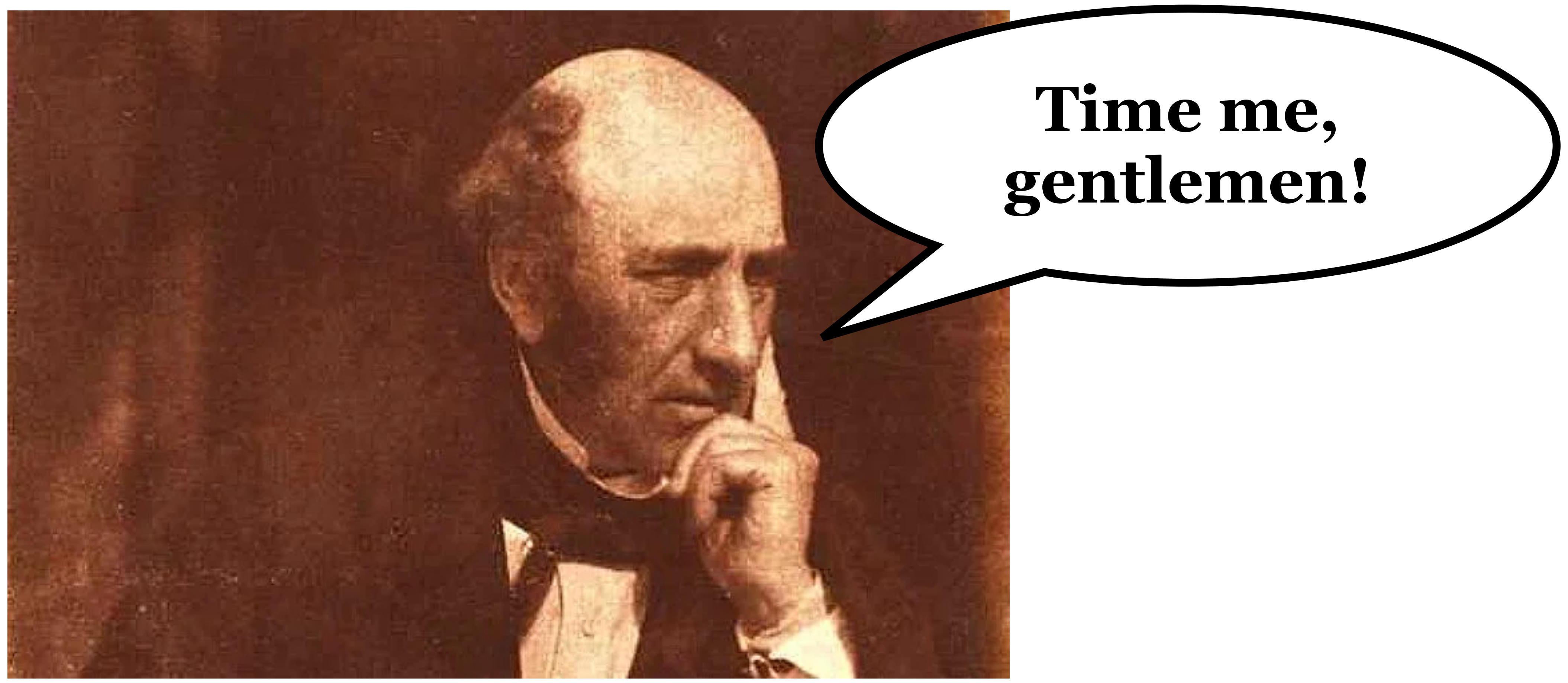

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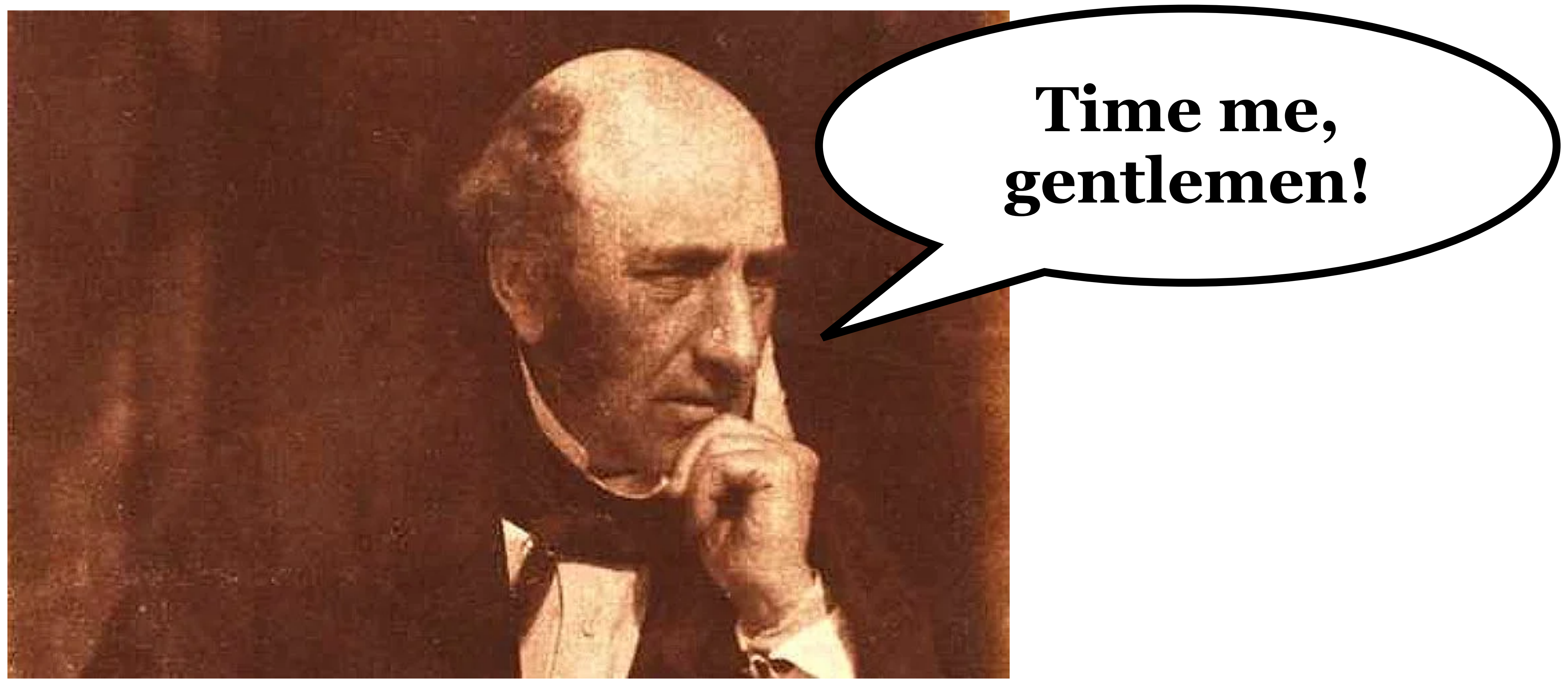

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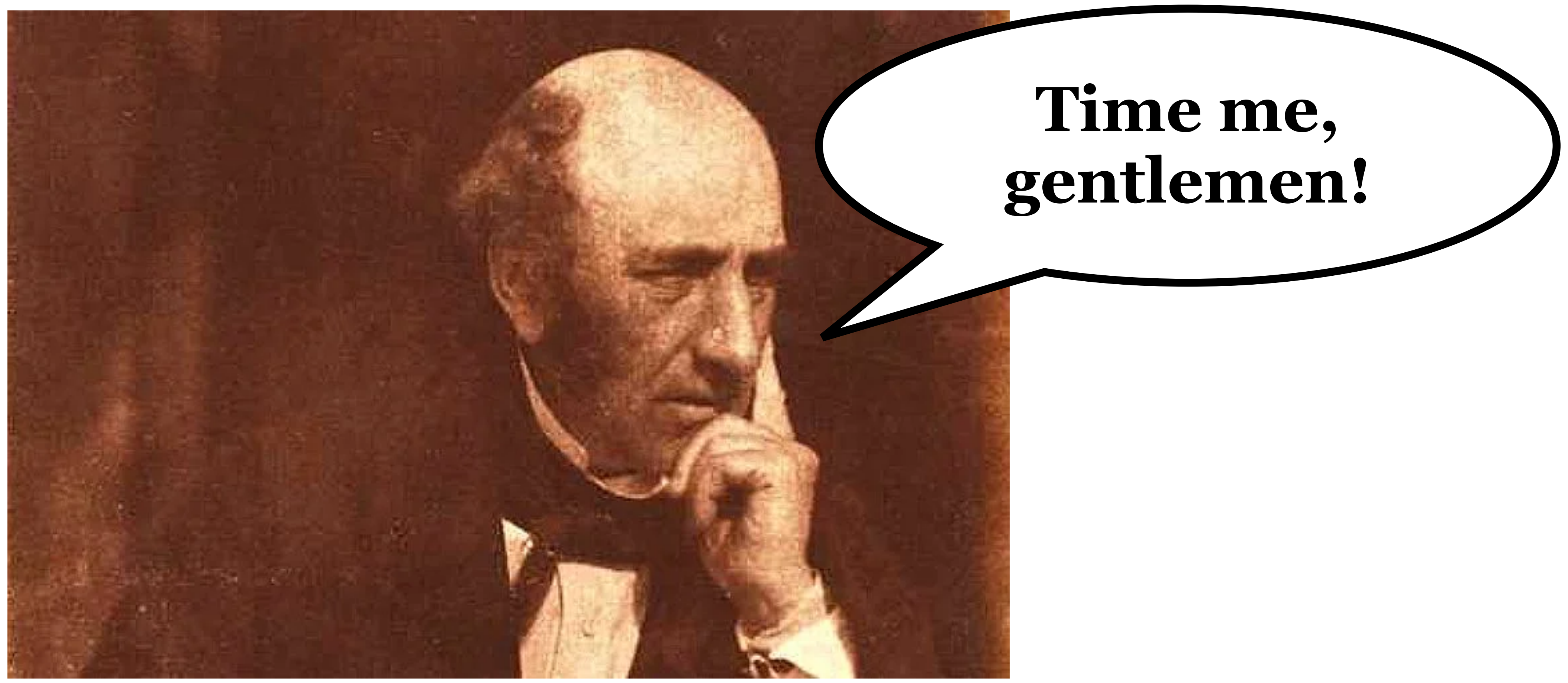

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300% mortality rate

Regular surgery → Fast surgery

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3-term chain complex for **regular code surgery, Z measurement**

Regular surgery $\rightarrow$ Fast surgery

3-term chain complex for **regular code surgery, Z measurement**

	Z checks	Qubits	X checks

$$
\begin{tikzcd}
A_\bullet = & A_1 \arrow[r] \arrow[dr, "f_1"] & A_0 \arrow[r] \arrow[dr, "f_0"] & A_{-1} \\
C_\bullet = & C_2 \arrow[r] & C_1 \arrow[r] & C_0
\end{tikzcd}
$$

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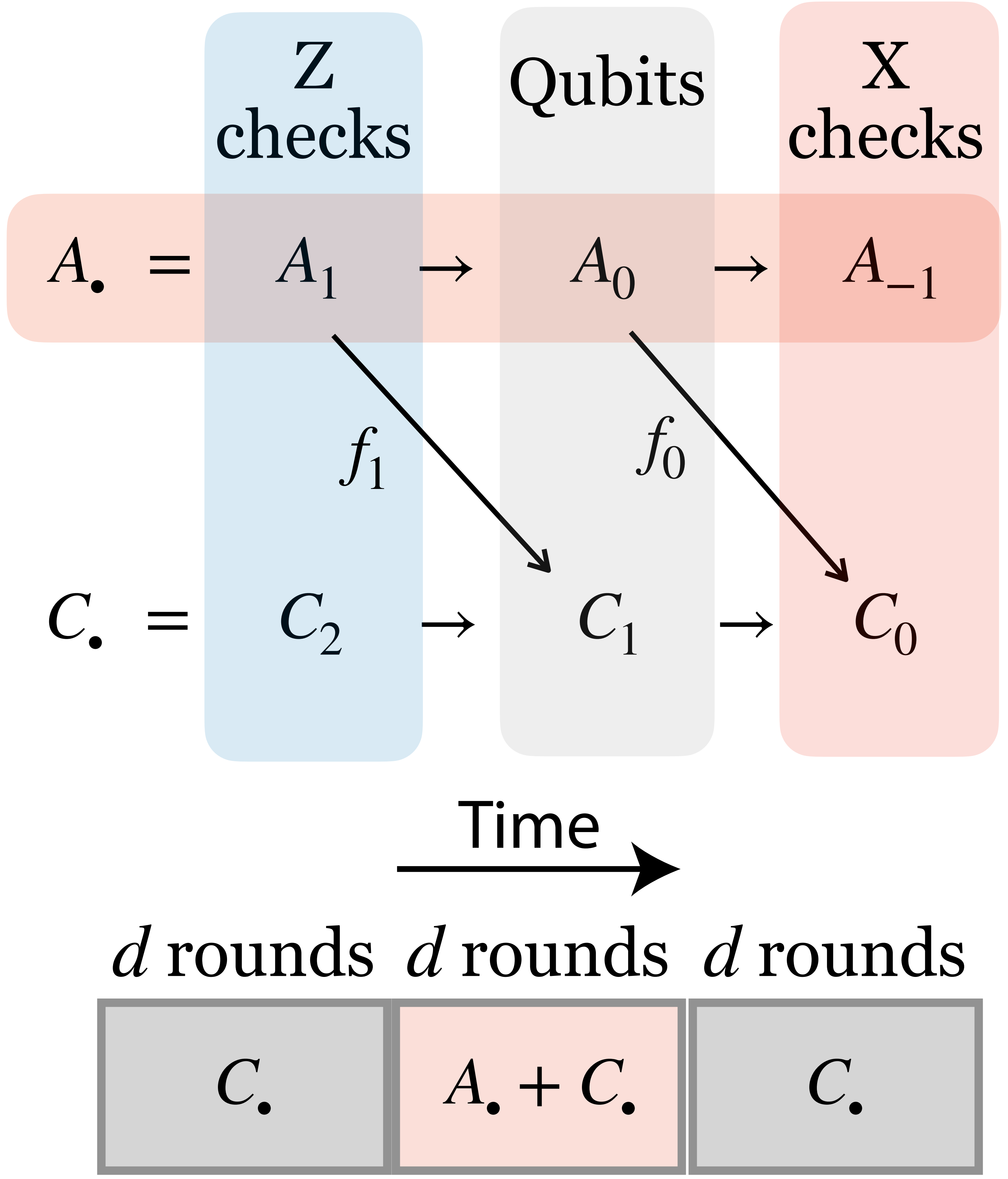

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Time

d rounds	d rounds	d rounds
$C_\bullet$	$A_\bullet + C_\bullet$	$C_\bullet$

4-term chain complex for **fast surgery**

Take a ***4-term*** chain ancilla complex, $A_\bullet$, $f_\bullet : A_\bullet \to C_\bullet$, st:

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Detects measurement errors

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Detects measurement errors

What is the surgery procedure now?

Want fault-distance d for data qubit & measurement errors

Is this fault-tolerant?

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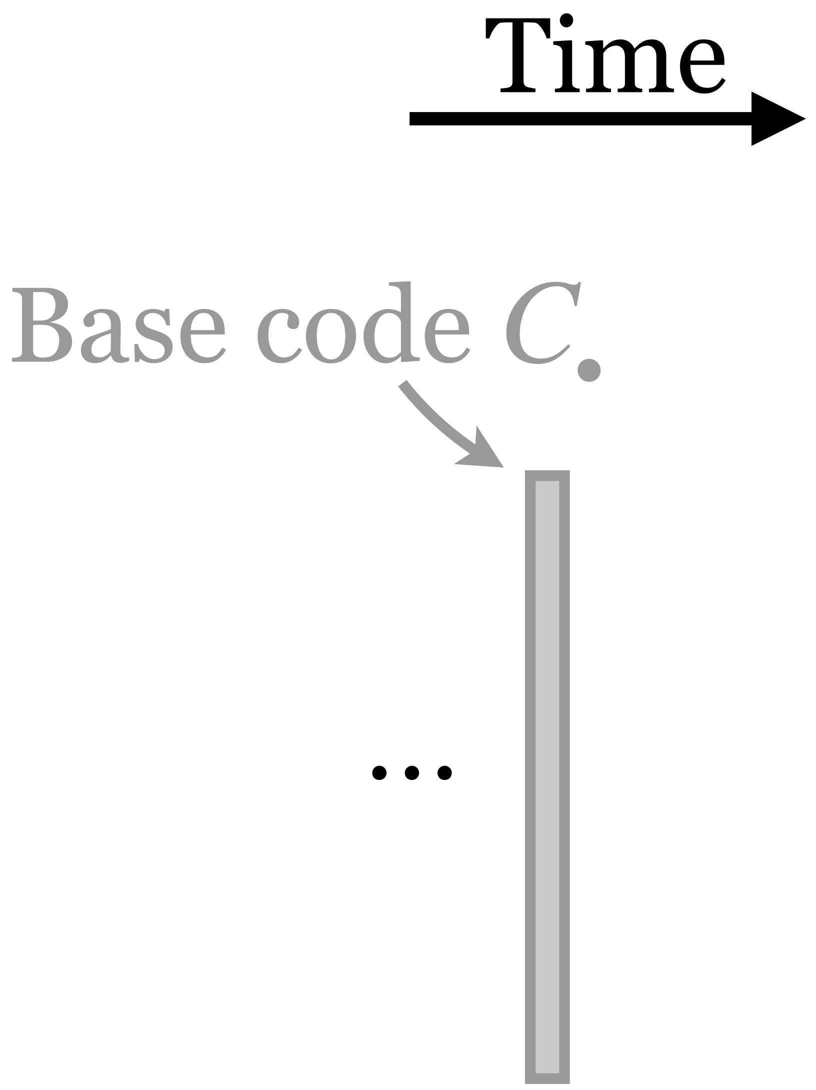
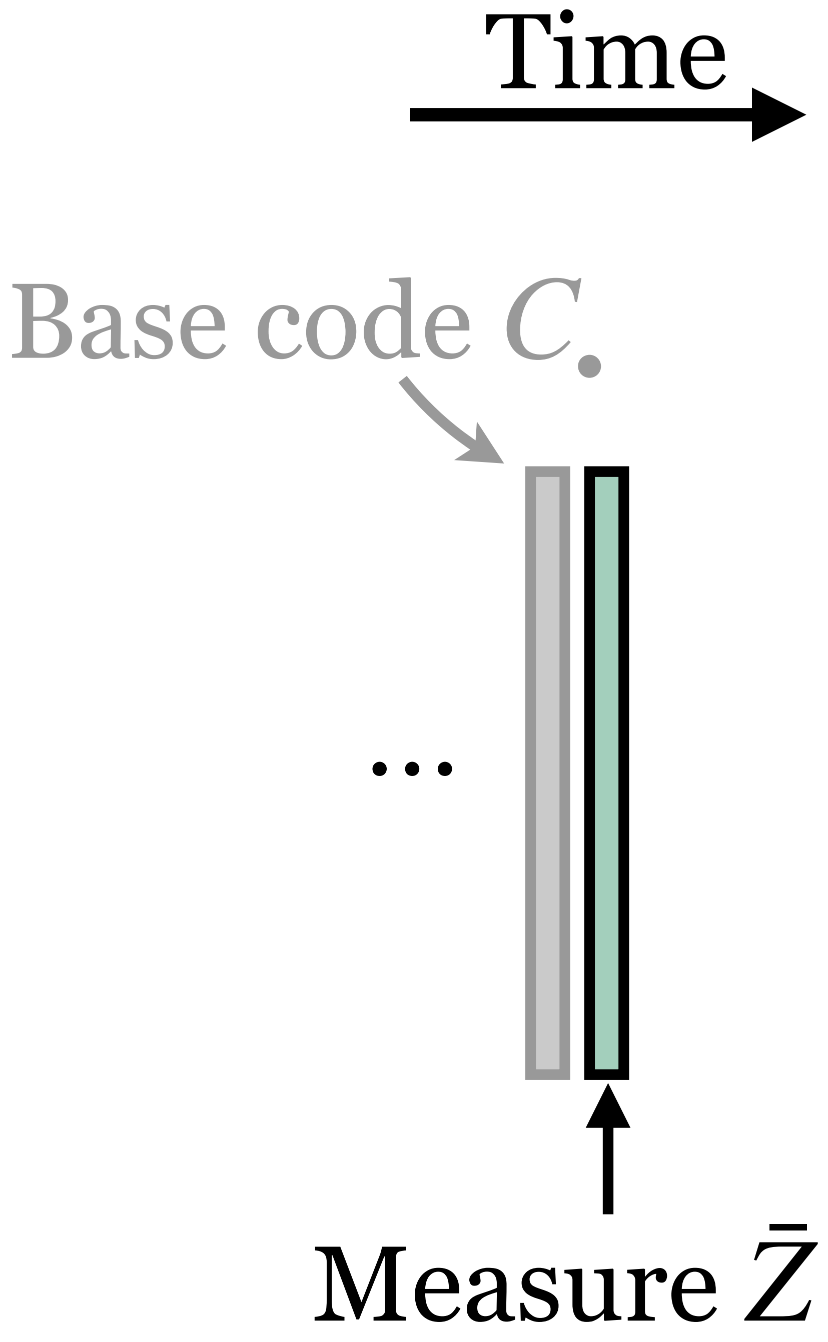
1. 1 round in the base code $C_\bullet$



Is this fault-tolerant?

1. 1 round in the base code $C_\bullet$
2. 1 round in the deformed code (measure $\bar{Z}$)



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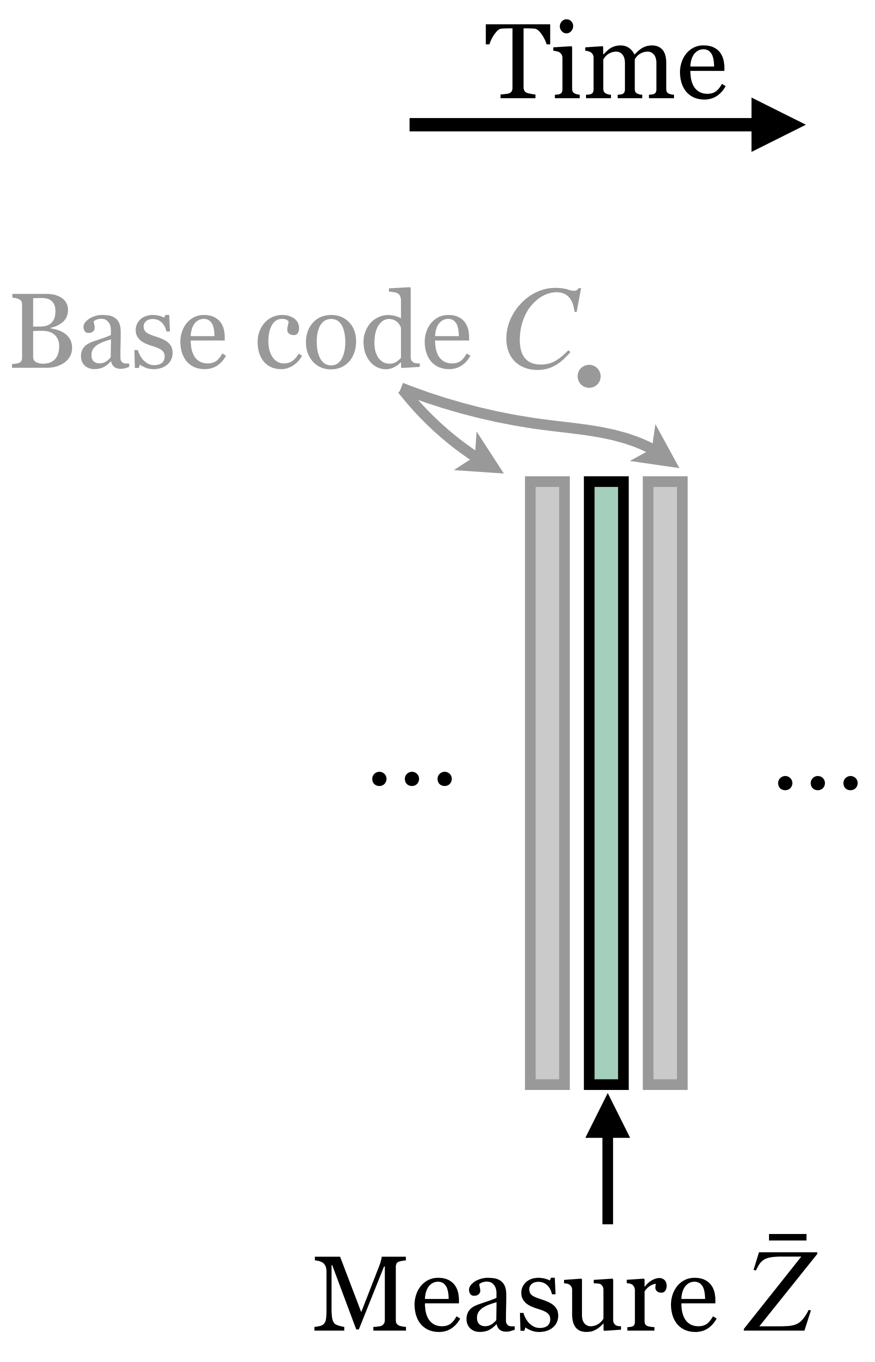

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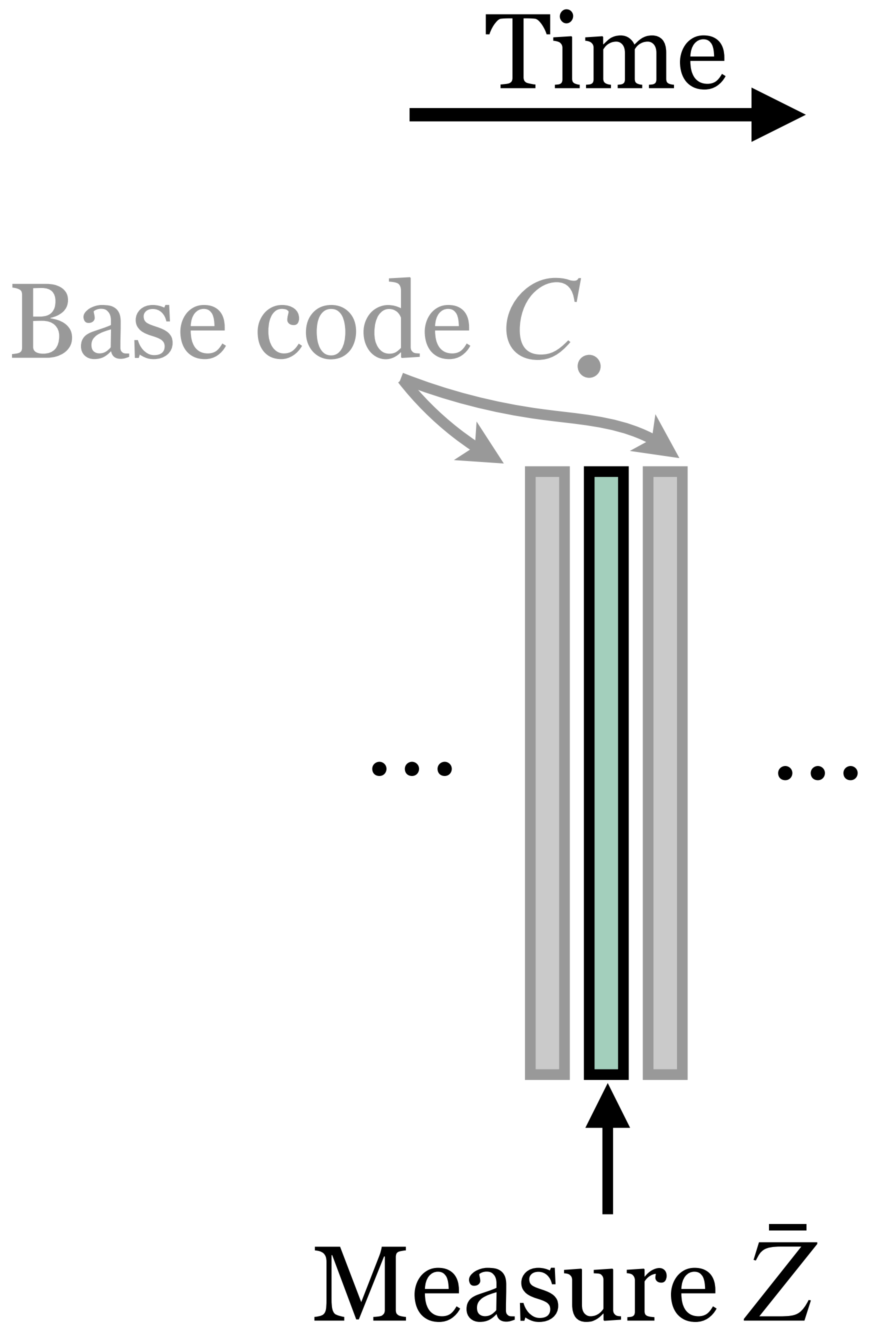

Measure $\bar{Z}$

Meta-checks | Z checks | Qubits | X checks

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\begin{tikzcd}
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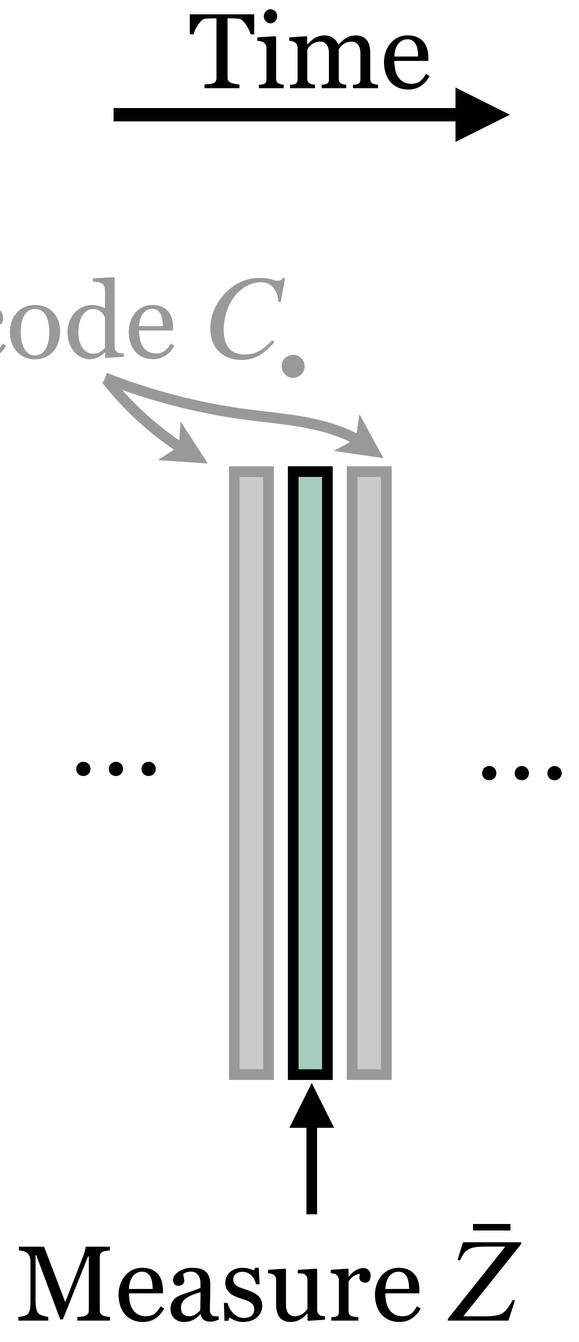

Start: init. A_0 qubits in $|+\rangle$

Measure $\bar{Z}$

Meta-checks	Z checks	Qubits	X checks

$$A_\bullet = A_2 \to A_1 \to A_0 \to A_{-1}$$

$$f_2,\ f_1,\ f_0$$

$$C_\bullet = C_2 \to C_1 \to C_0$$

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1. 1 round in the base code $C_\bullet$
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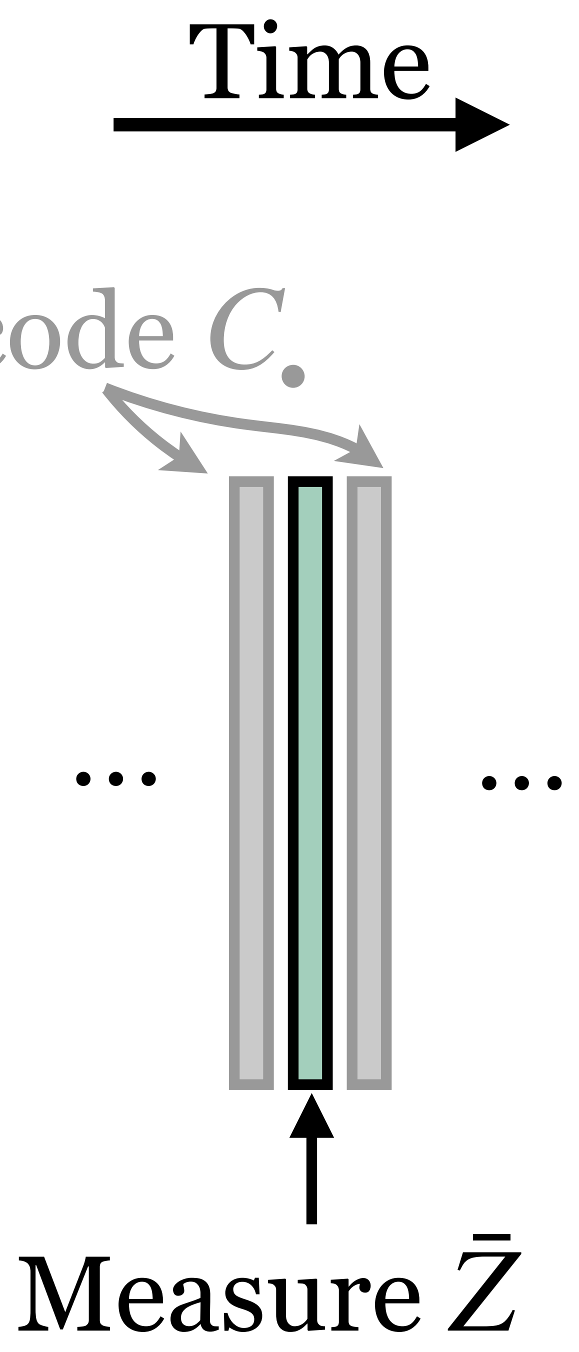

Start: init. A_0 qubits in $|+\rangle$

Finish: measure A_0 qubits in X

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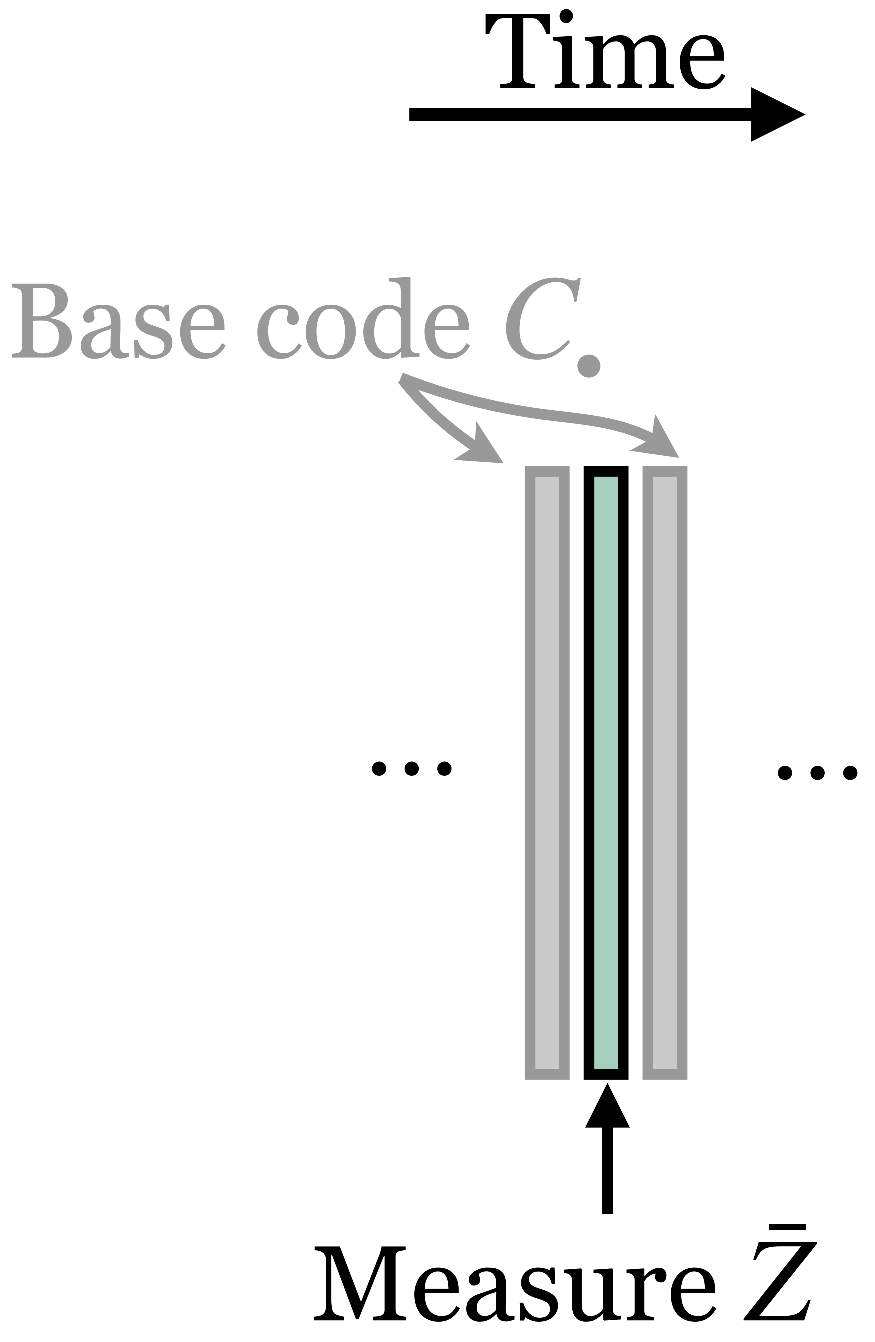

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Updated after X measurement, *random*.

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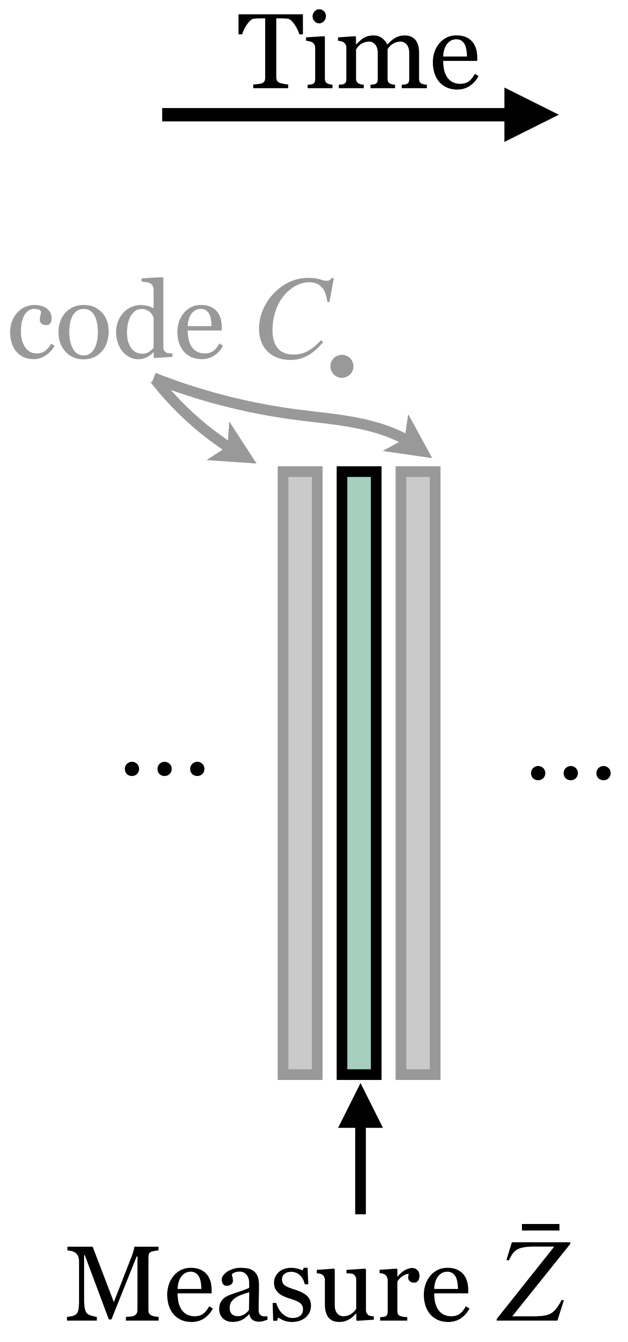
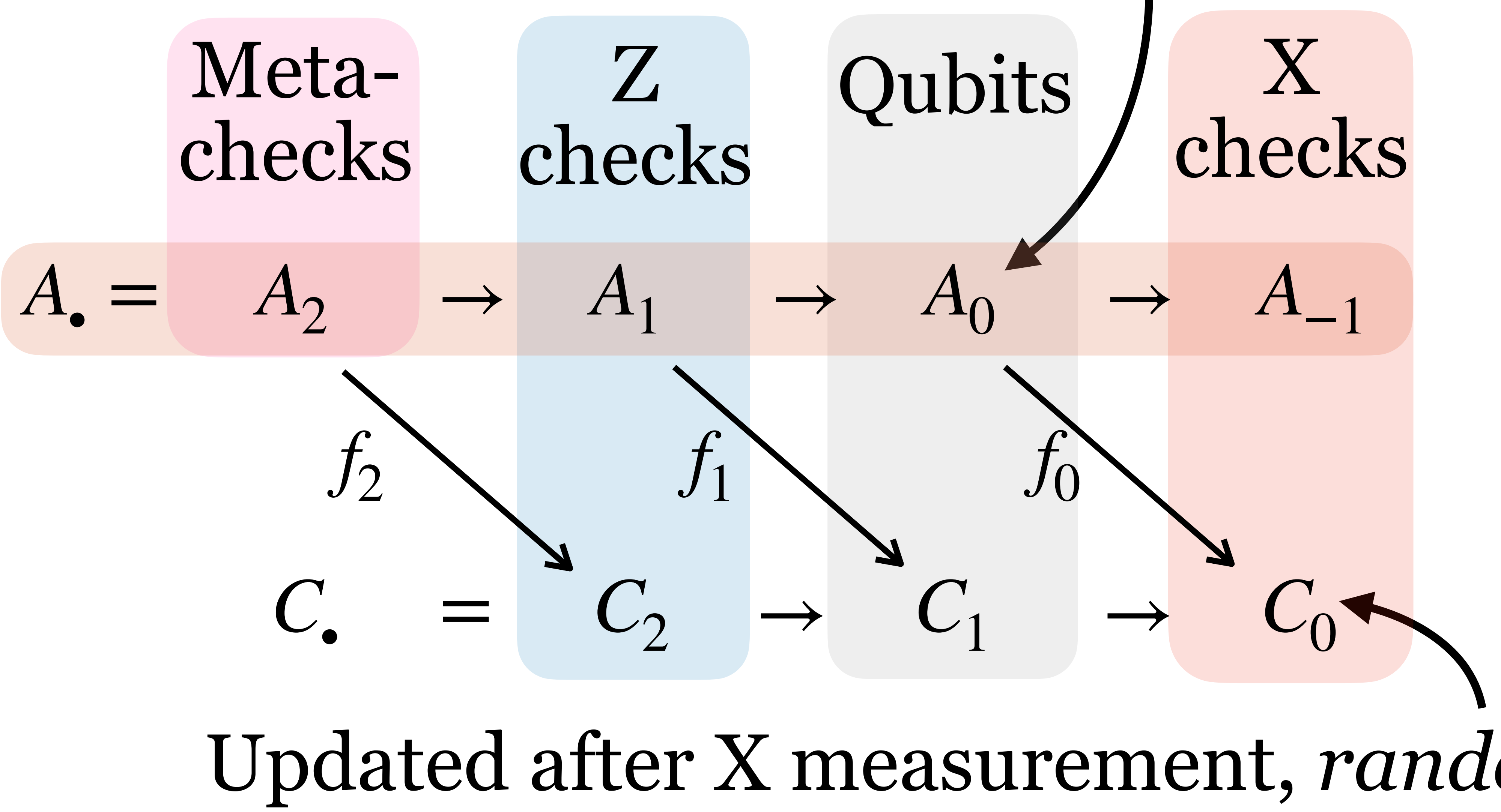


Start: init. A_0 qubits in $|+\rangle$

Finish: measure A_0 qubits in X

Measure $\bar{Z}$



- State preparation-like error. Pauli frame is random after surgery.

Is this fault-tolerant?

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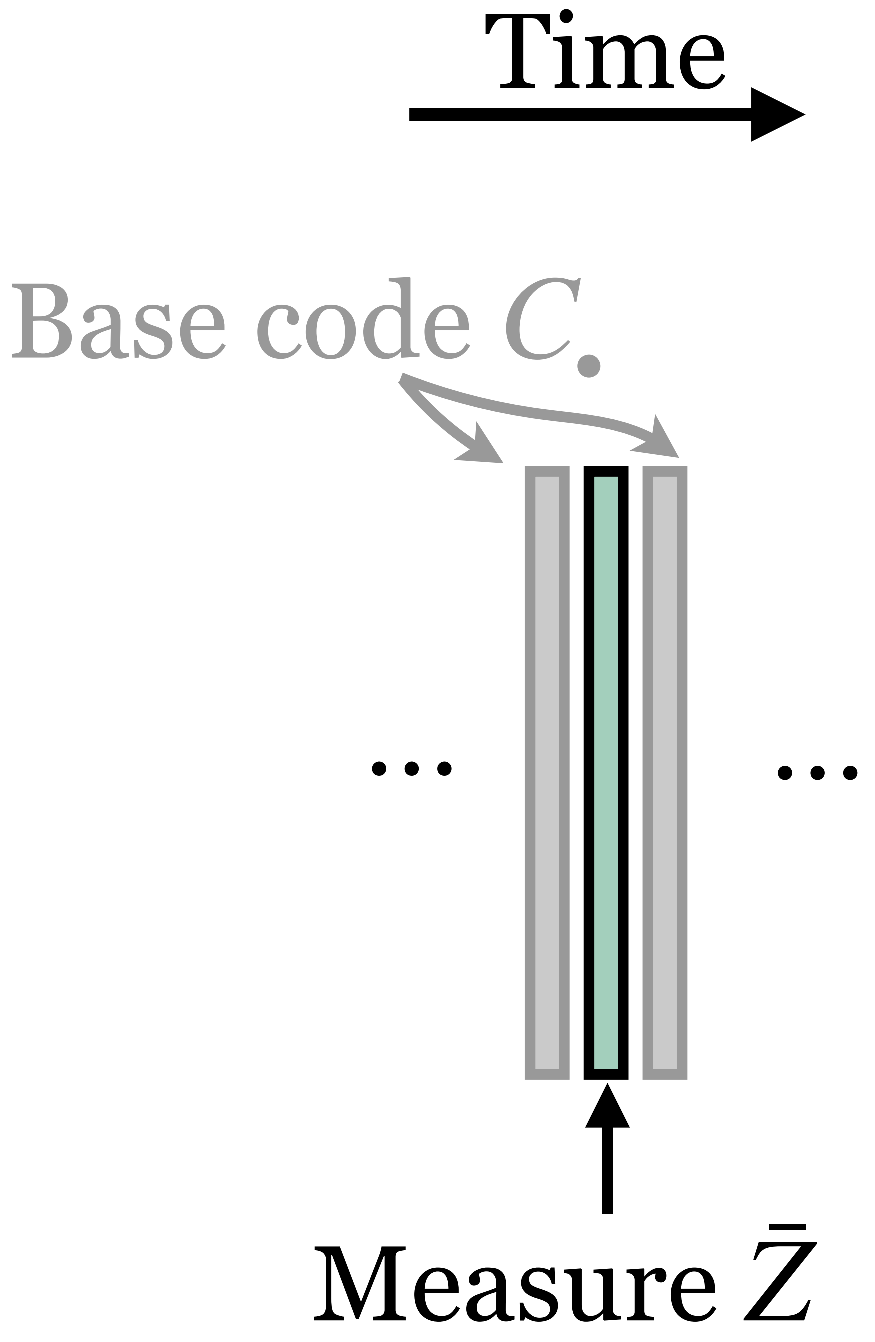

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Measure $\bar{Z}$

	Meta-checks		Z checks		Qubits		X checks
$A_\bullet =$	A_2	$\rightarrow$	A_1	$\rightarrow$	A_0	$\rightarrow$	A_{-1}

$$\begin{tikzcd} A_2 \arrow[r] \arrow[dr, "f_2"] & A_1 \arrow[r] \arrow[dr, "f_1"] & A_0 \arrow[r] \arrow[dr, "f_0"] & A_{-1} \\ C_\bullet = & C_2 \arrow[r] & C_1 \arrow[r] & C_0 \end{tikzcd}$$

Updated after X measurement, *random*.

- State preparation-like error. Pauli frame is random after surgery.
- Only 1 round in $C_\bullet$ to learn the new stabilizer values $\rightarrow$ incorrectly inferred Pauli frame

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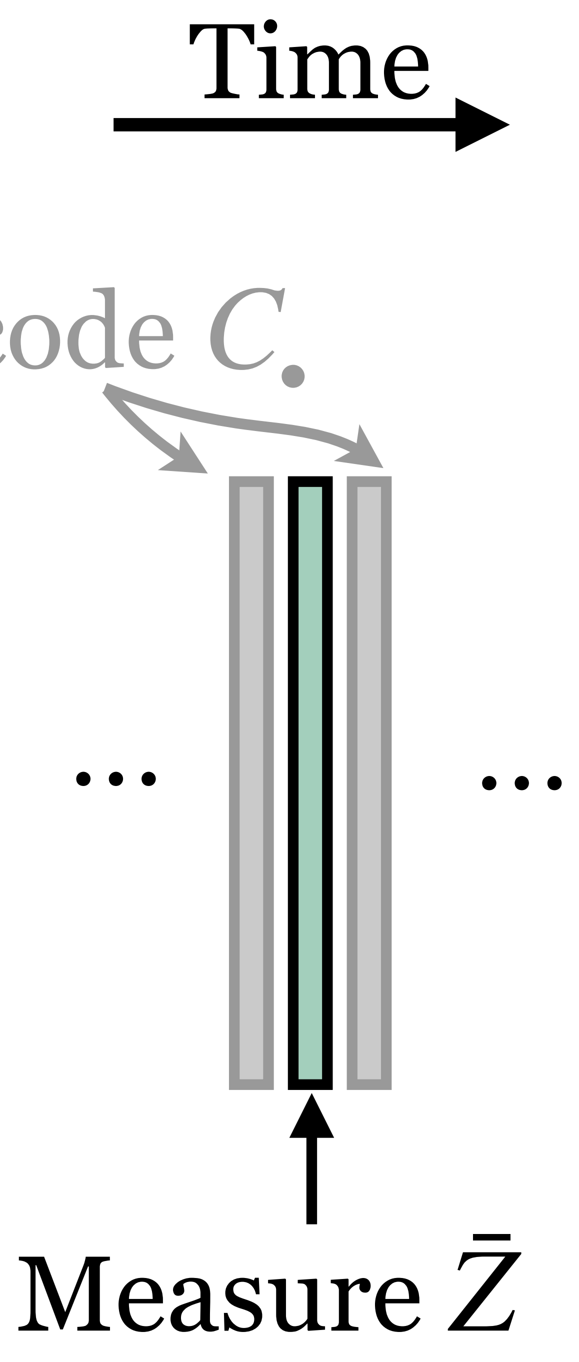

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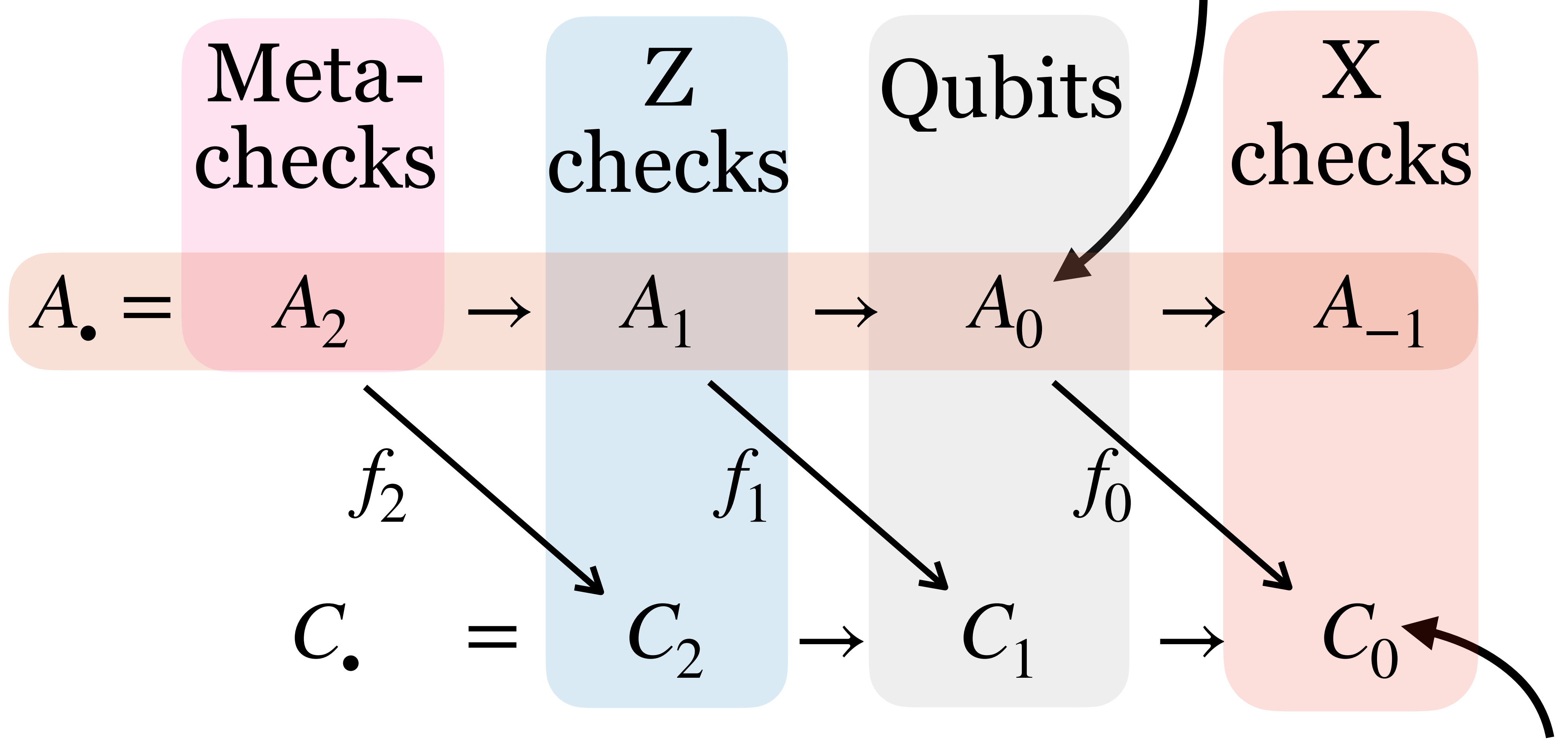

Updated after X measurement, *random*.

- State preparation-like error. Pauli frame is random after surgery.
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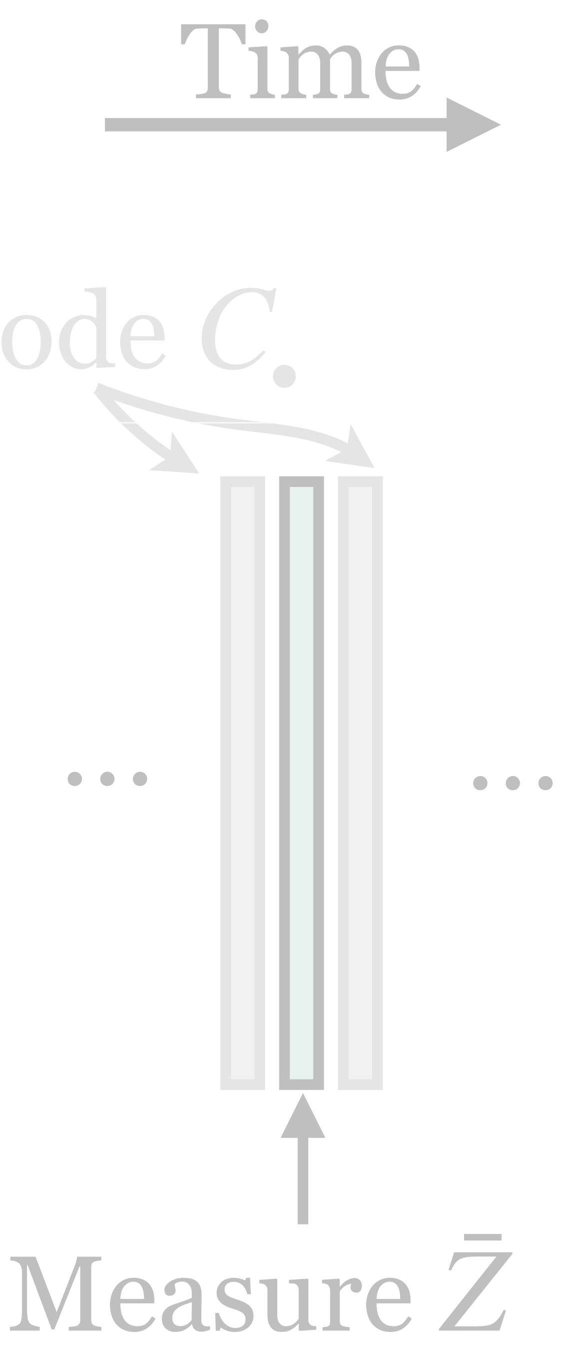

Base code $C_\bullet$ must infer syndromes w/
1 round of syndrome extraction

Satisfied if single-shot state preparation

Start: init. A_0 qubits

Finish: measure A_0 qu

Measure $\bar{Z}$

Meta-checks

Z checks

$A_\bullet = A_2 \rightarrow A_1 \rightarrow A_0 \rightarrow A_{-1}$

f_2 f_1 f_0

$C_\bullet = C_2 \rightarrow C_1 \rightarrow C_0$

Updated after X measurement, *random*.

error. Pauli frame is random after surgery.

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Single-shot surgery [1-3]

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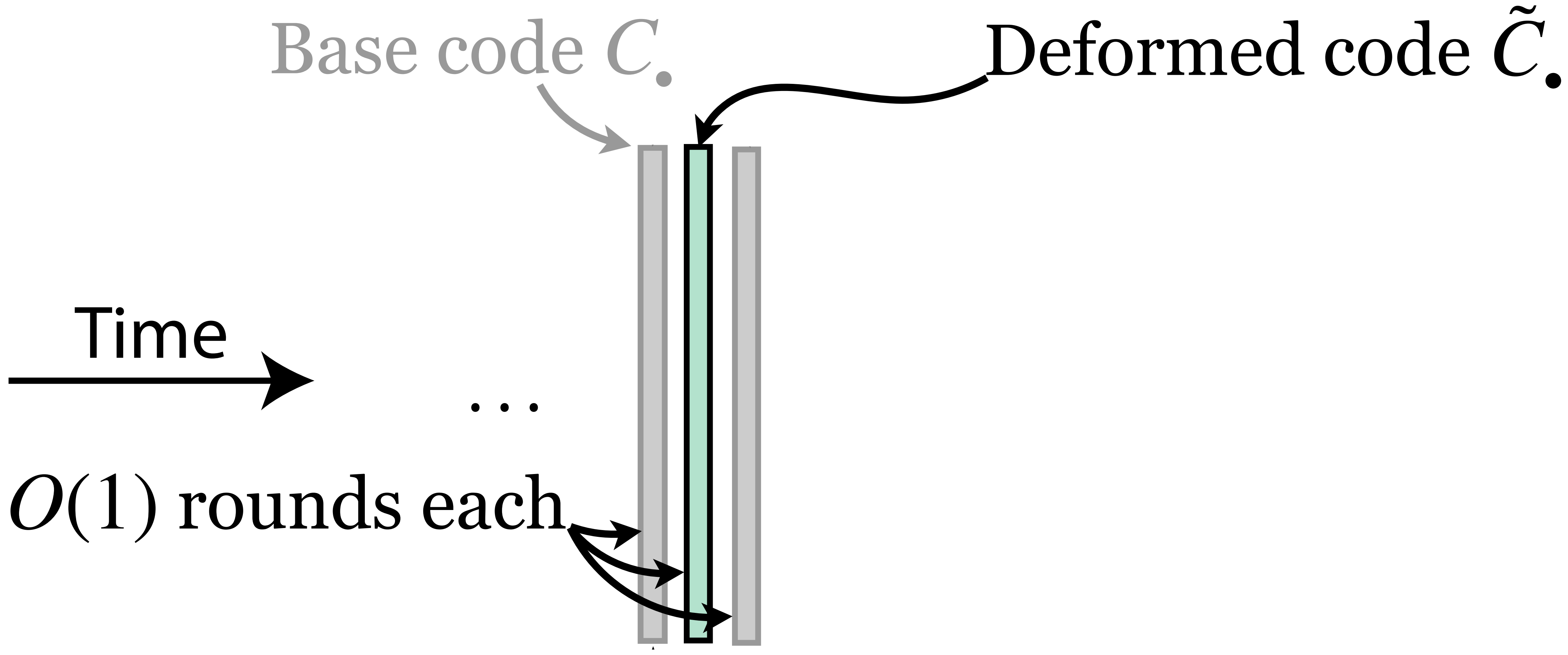

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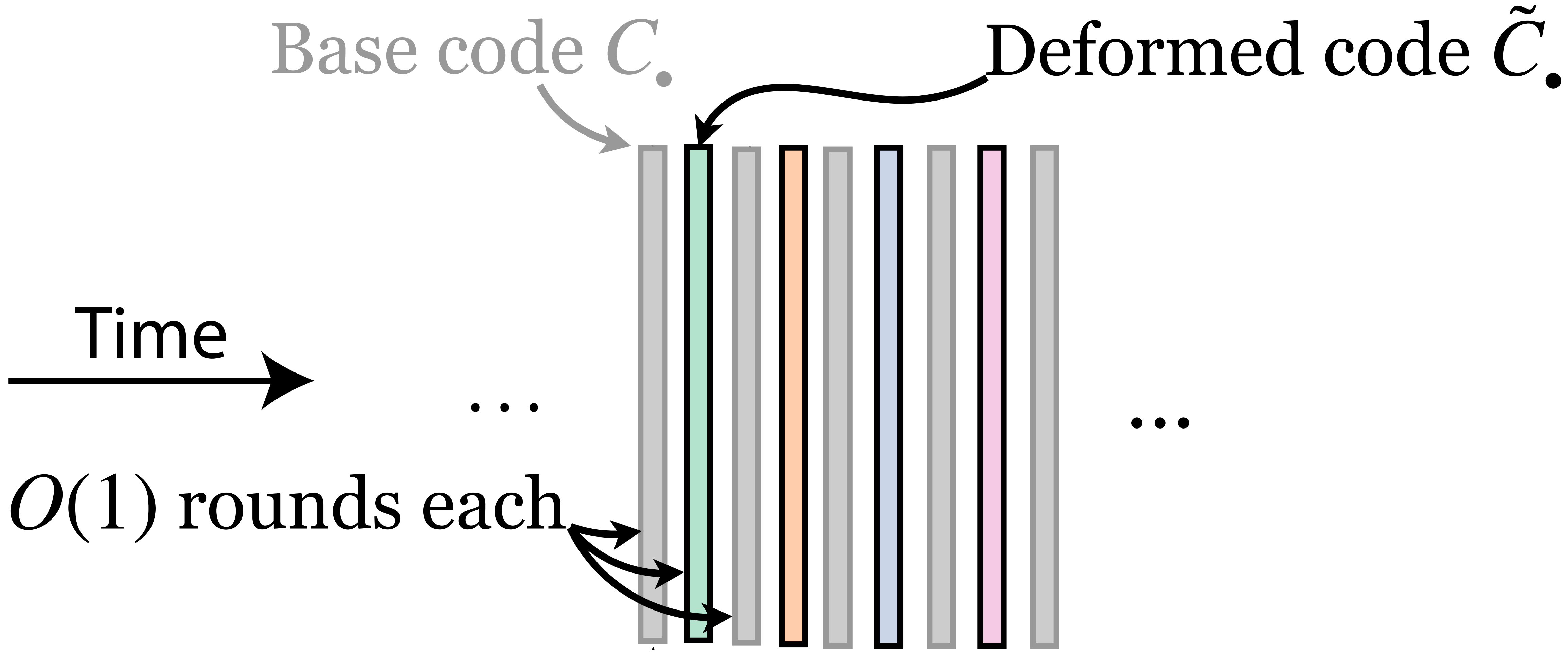

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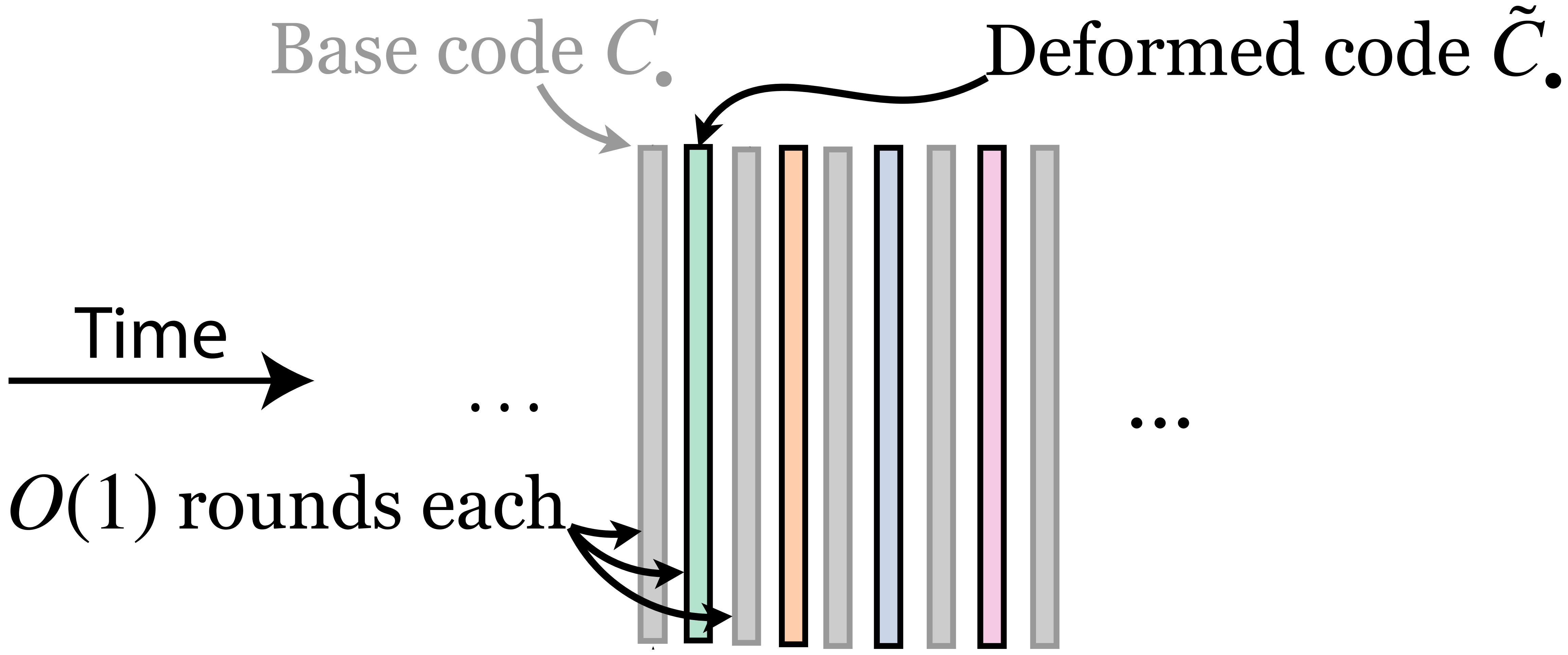

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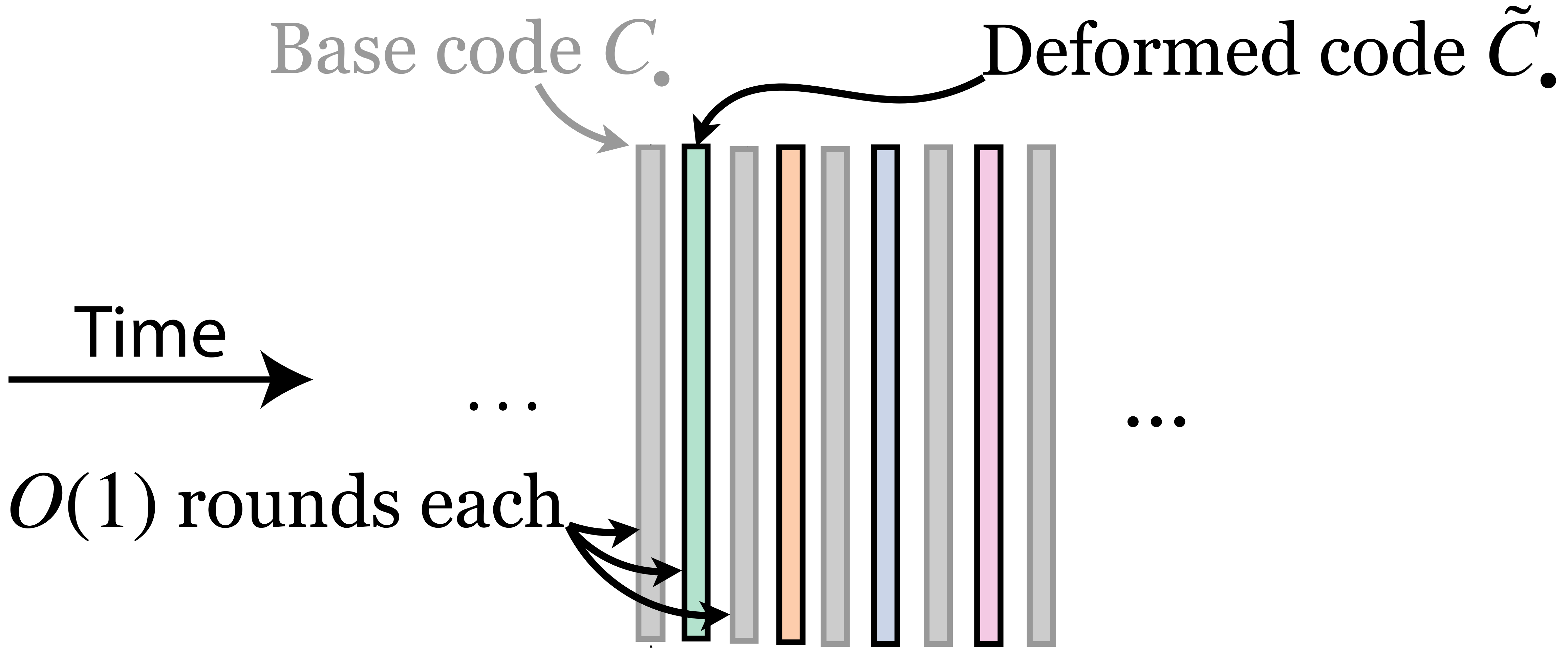

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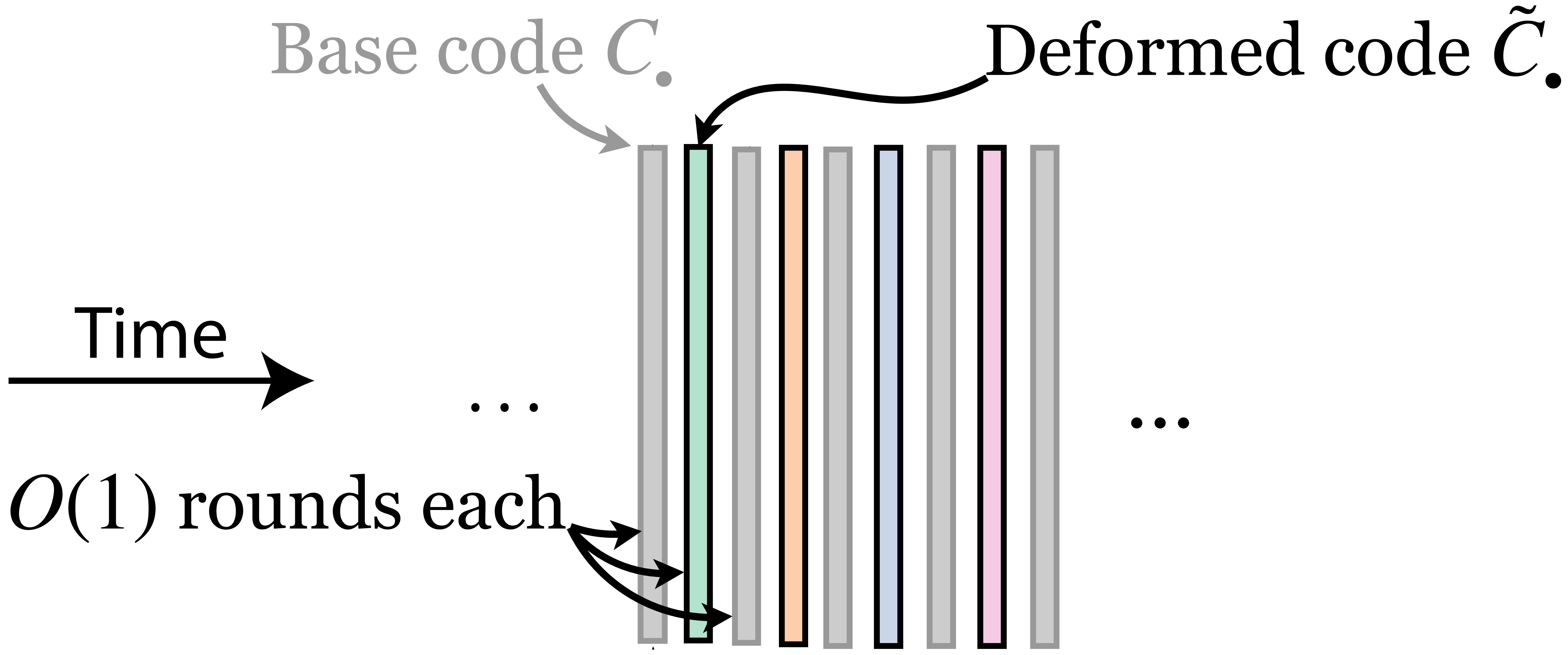

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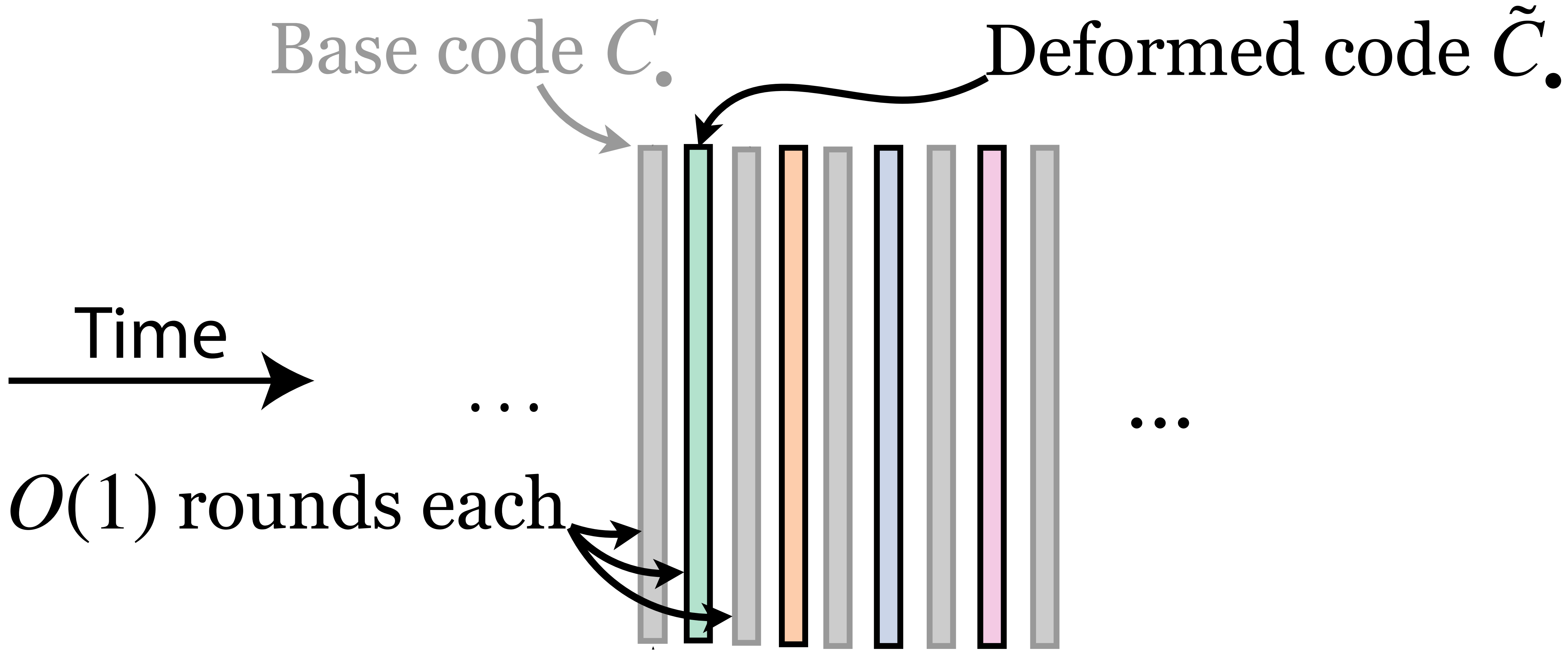

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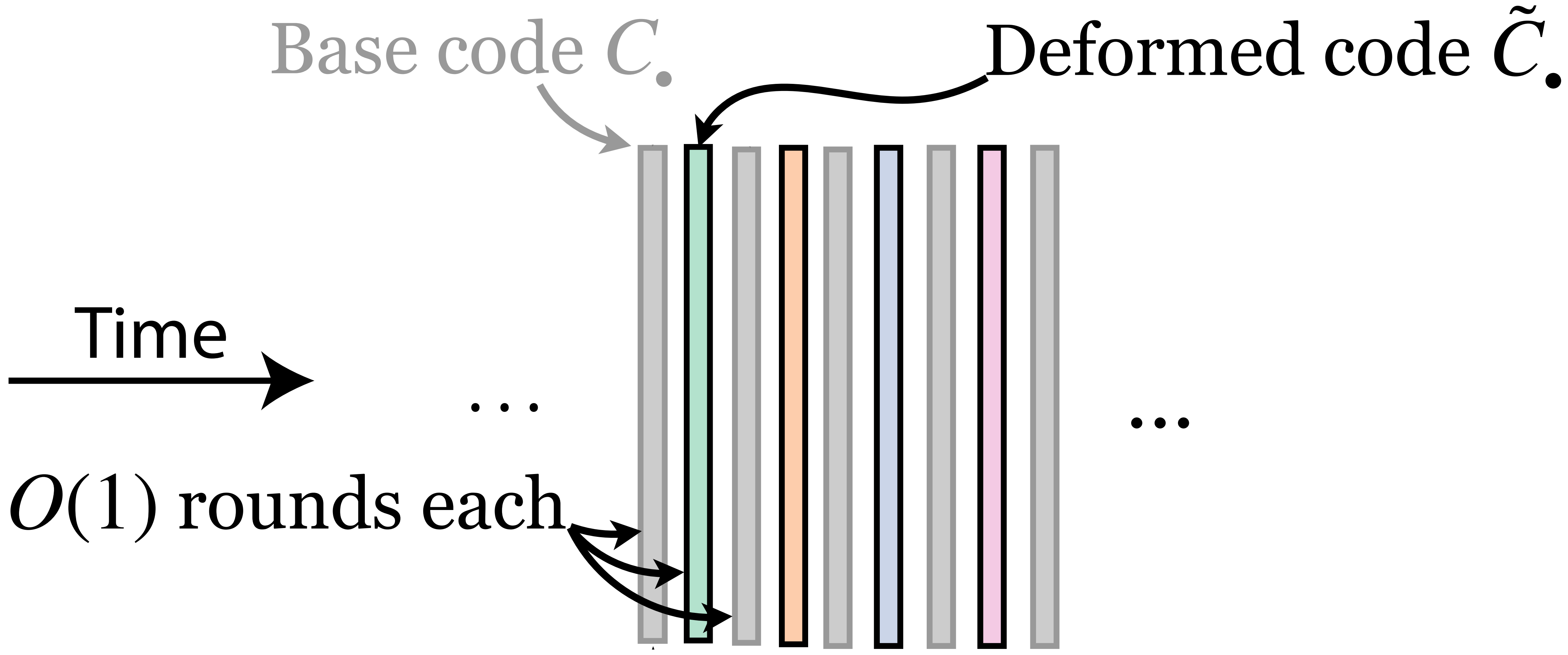

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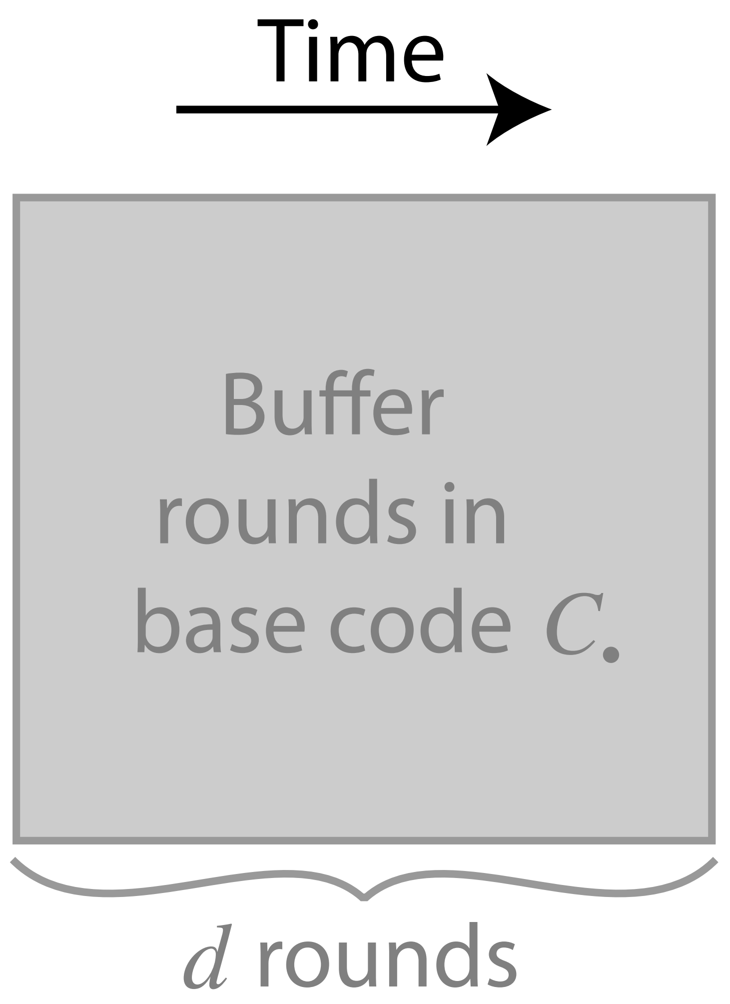

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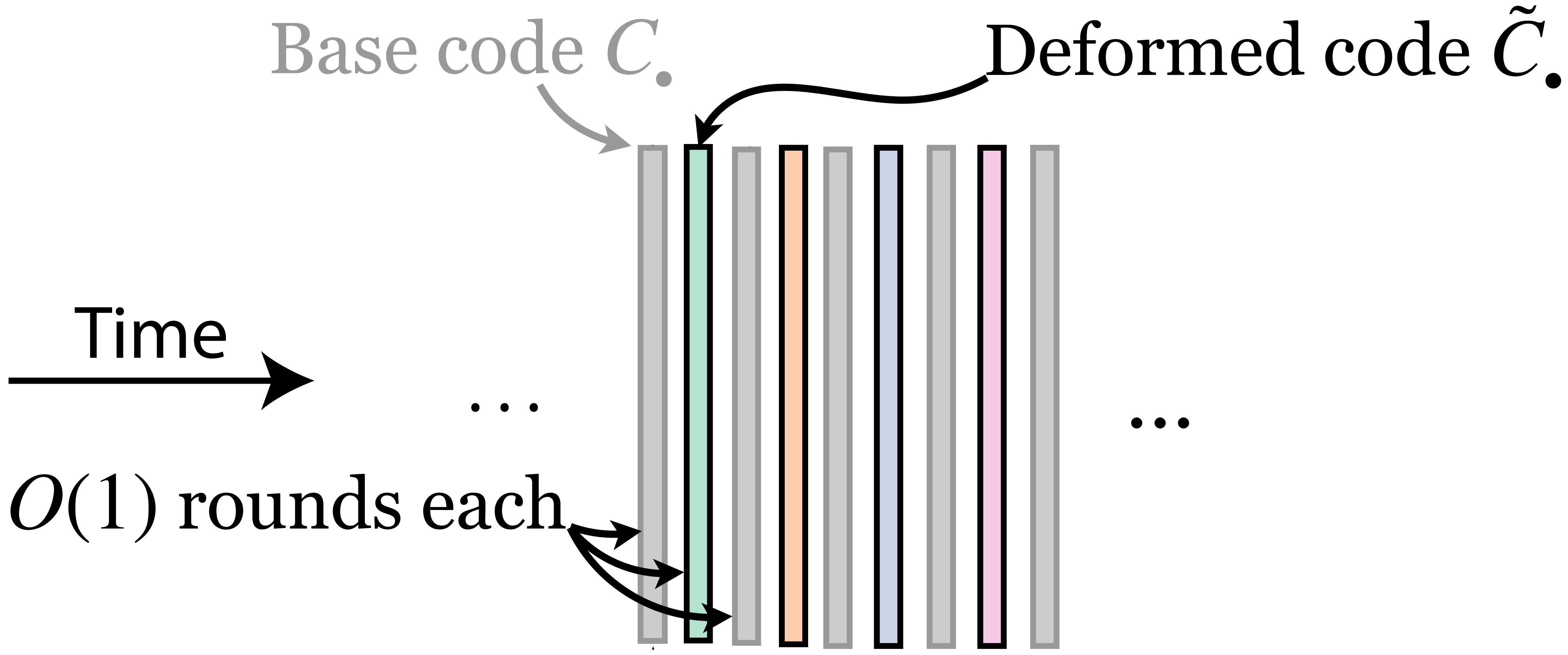

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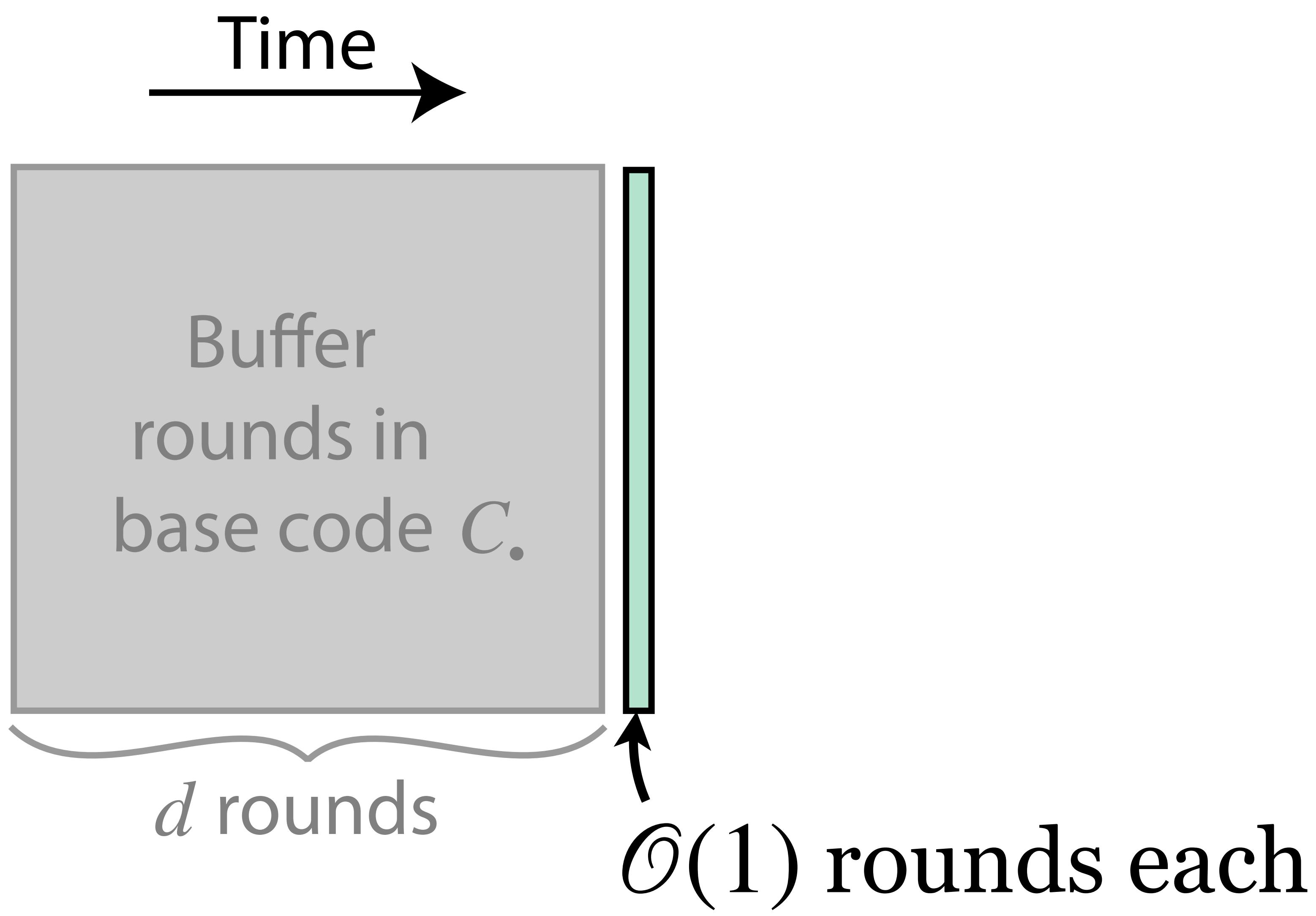

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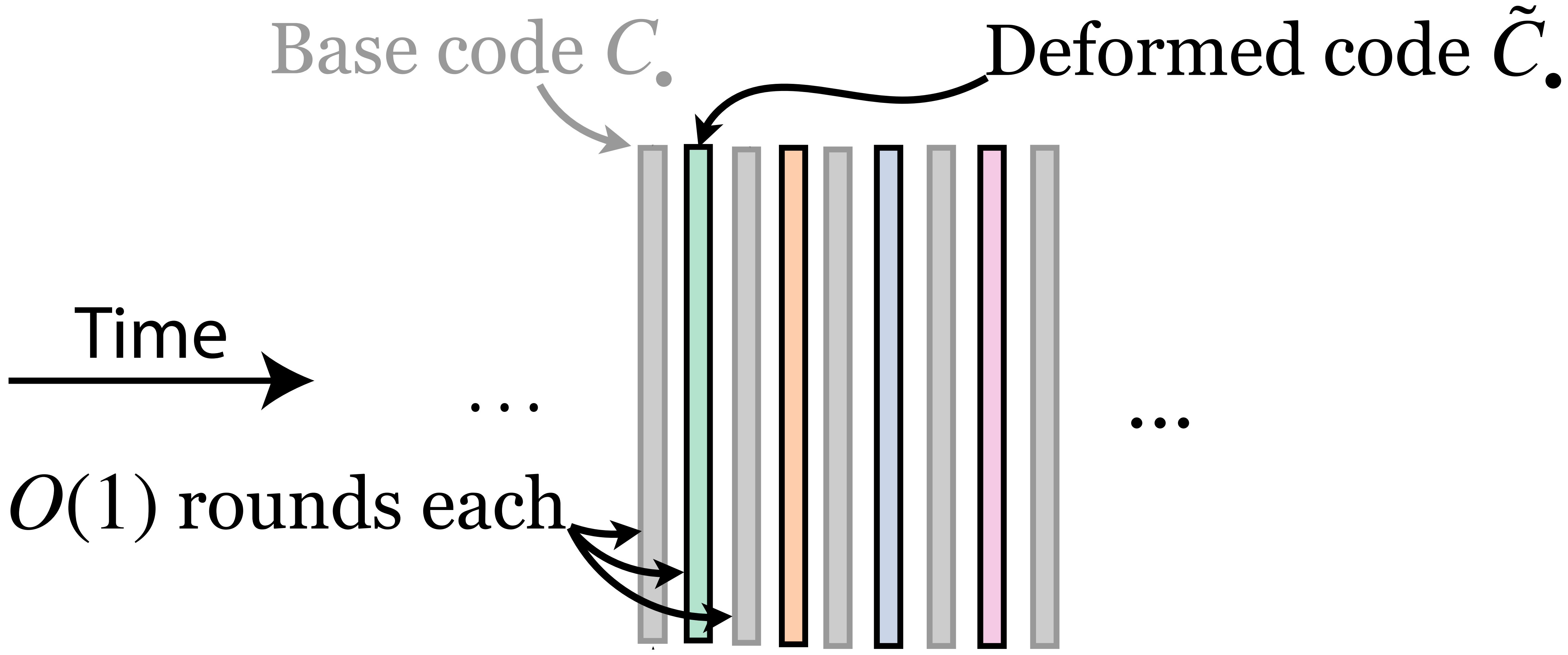

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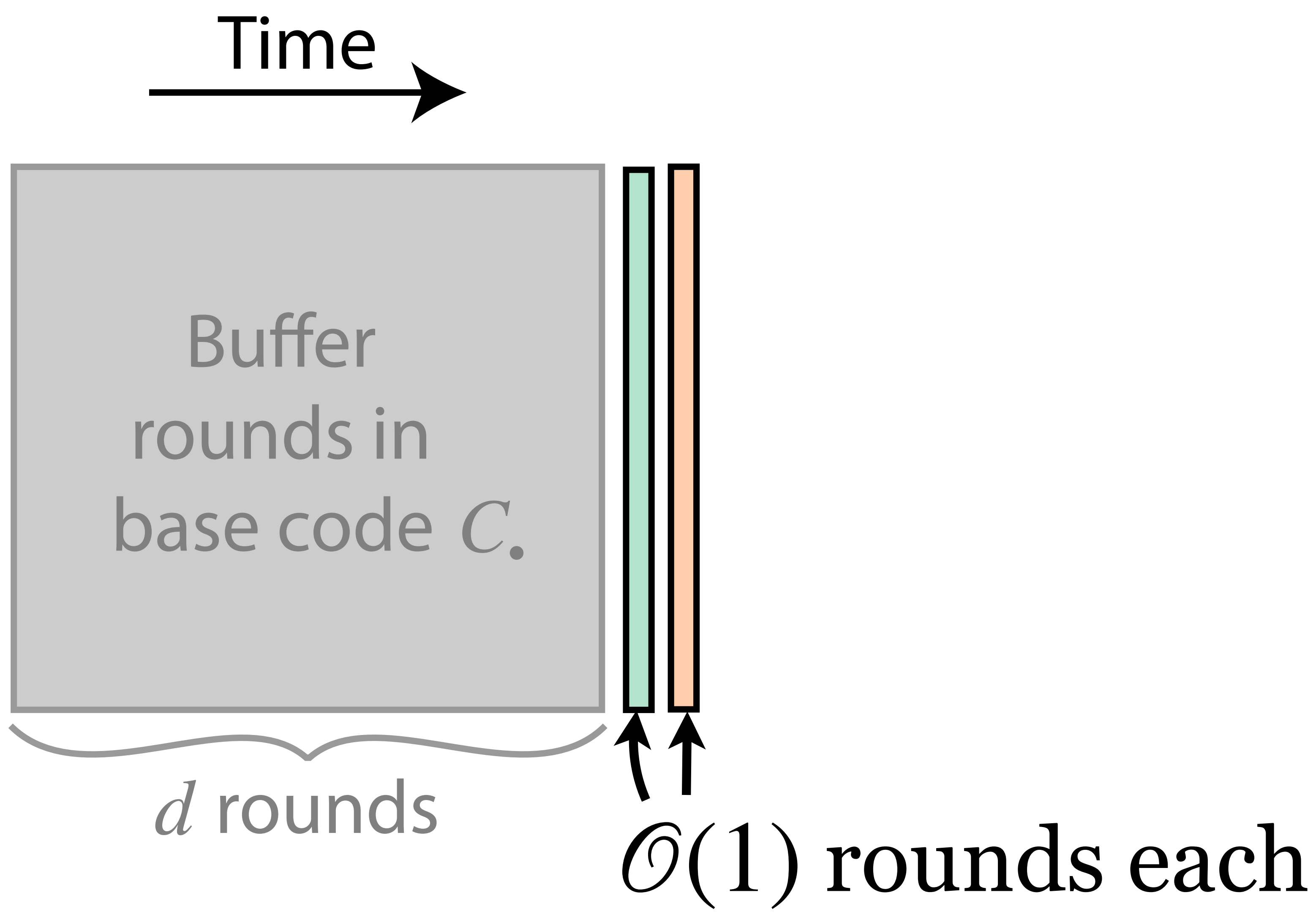

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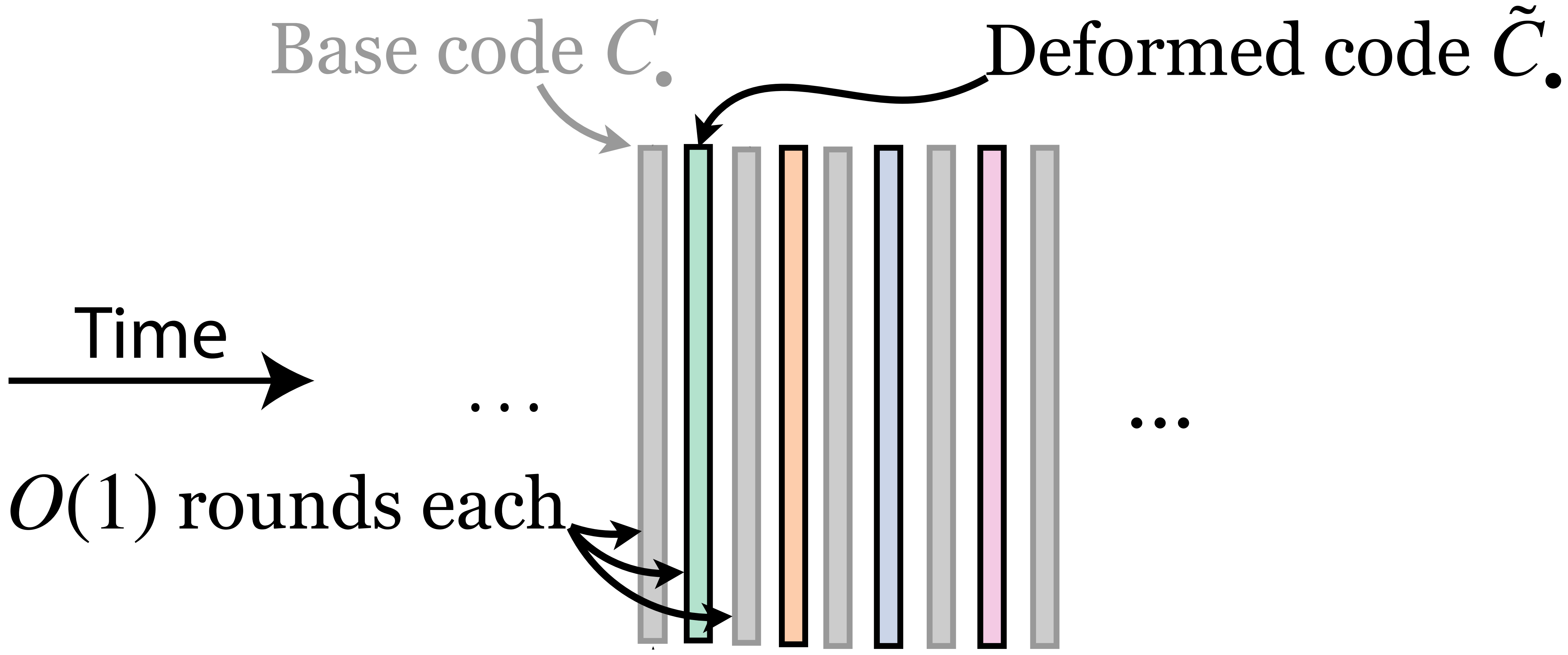

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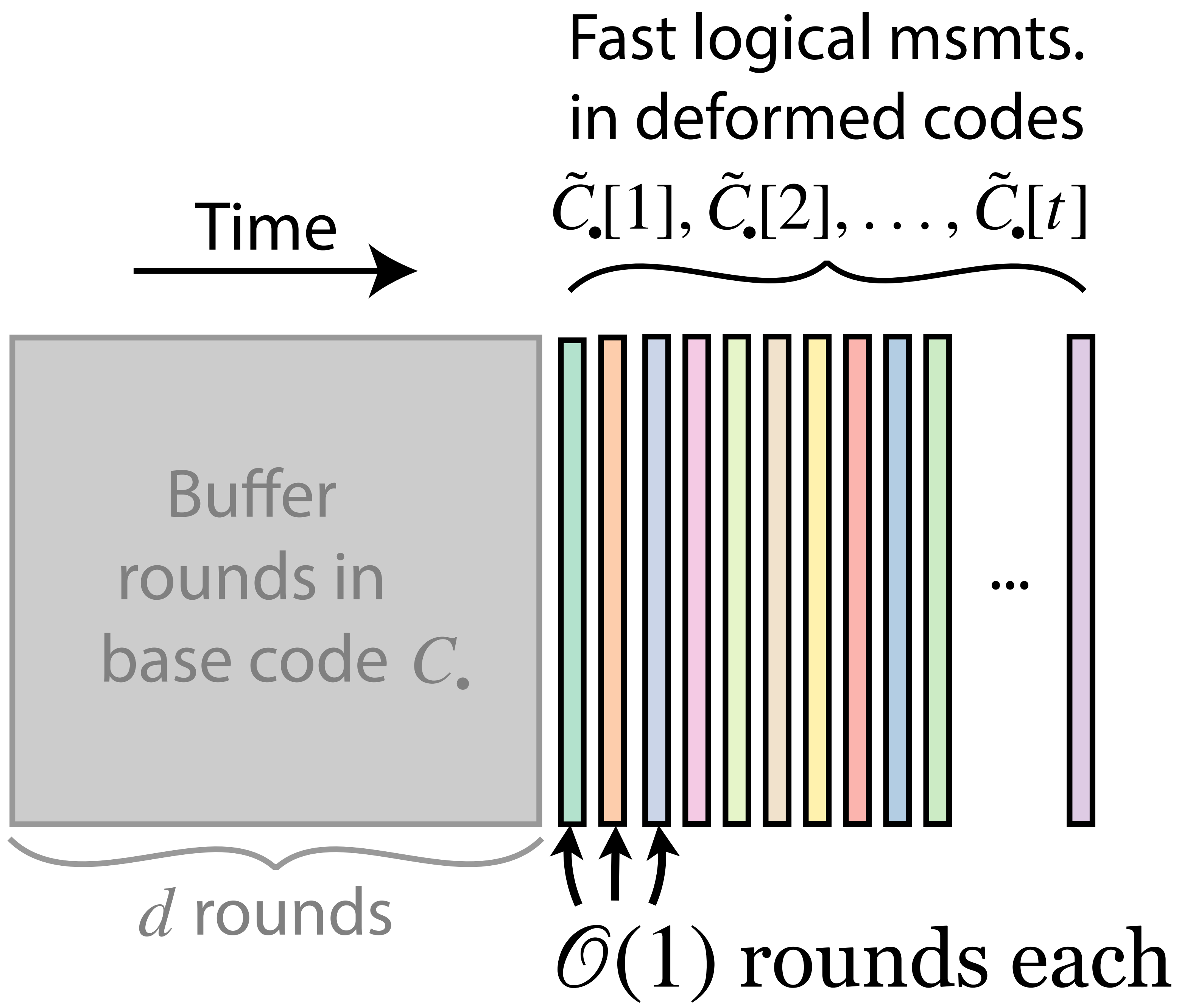

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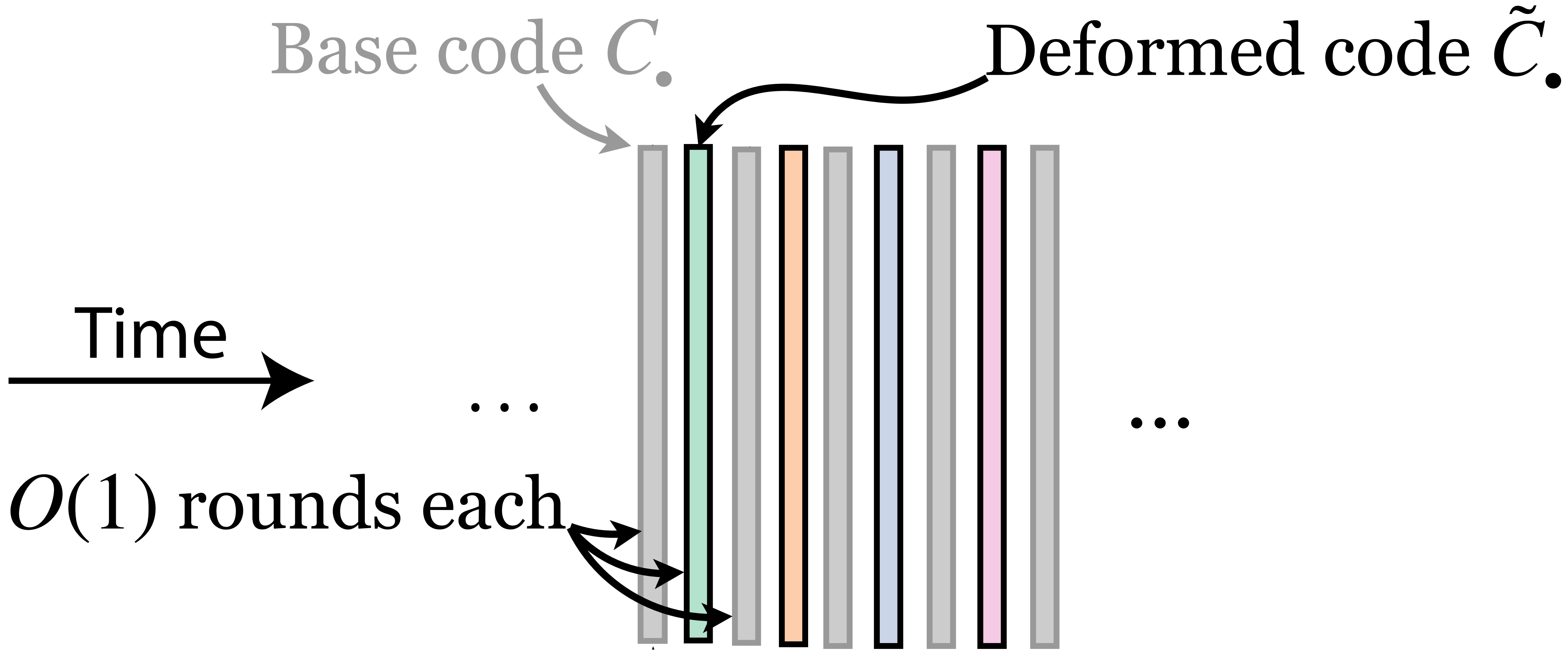

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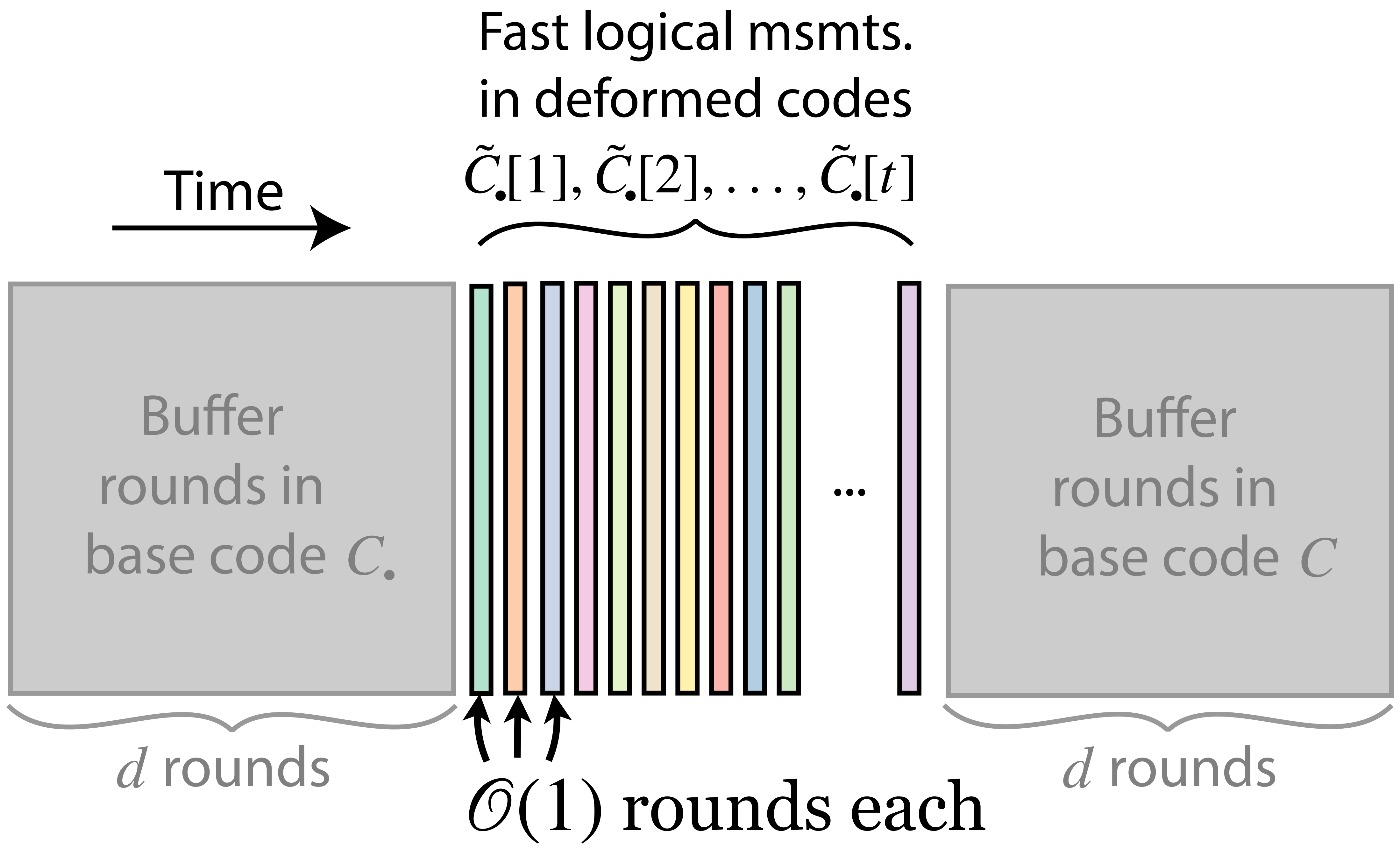

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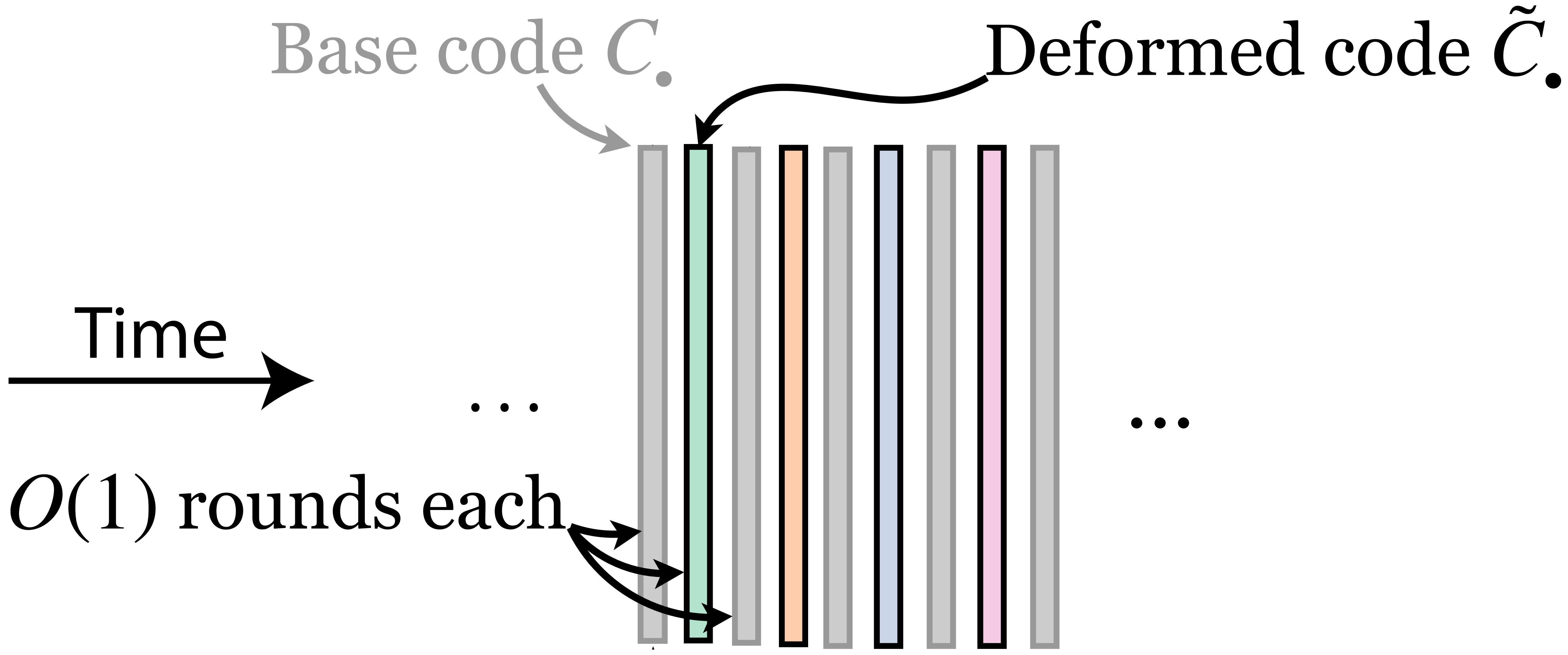

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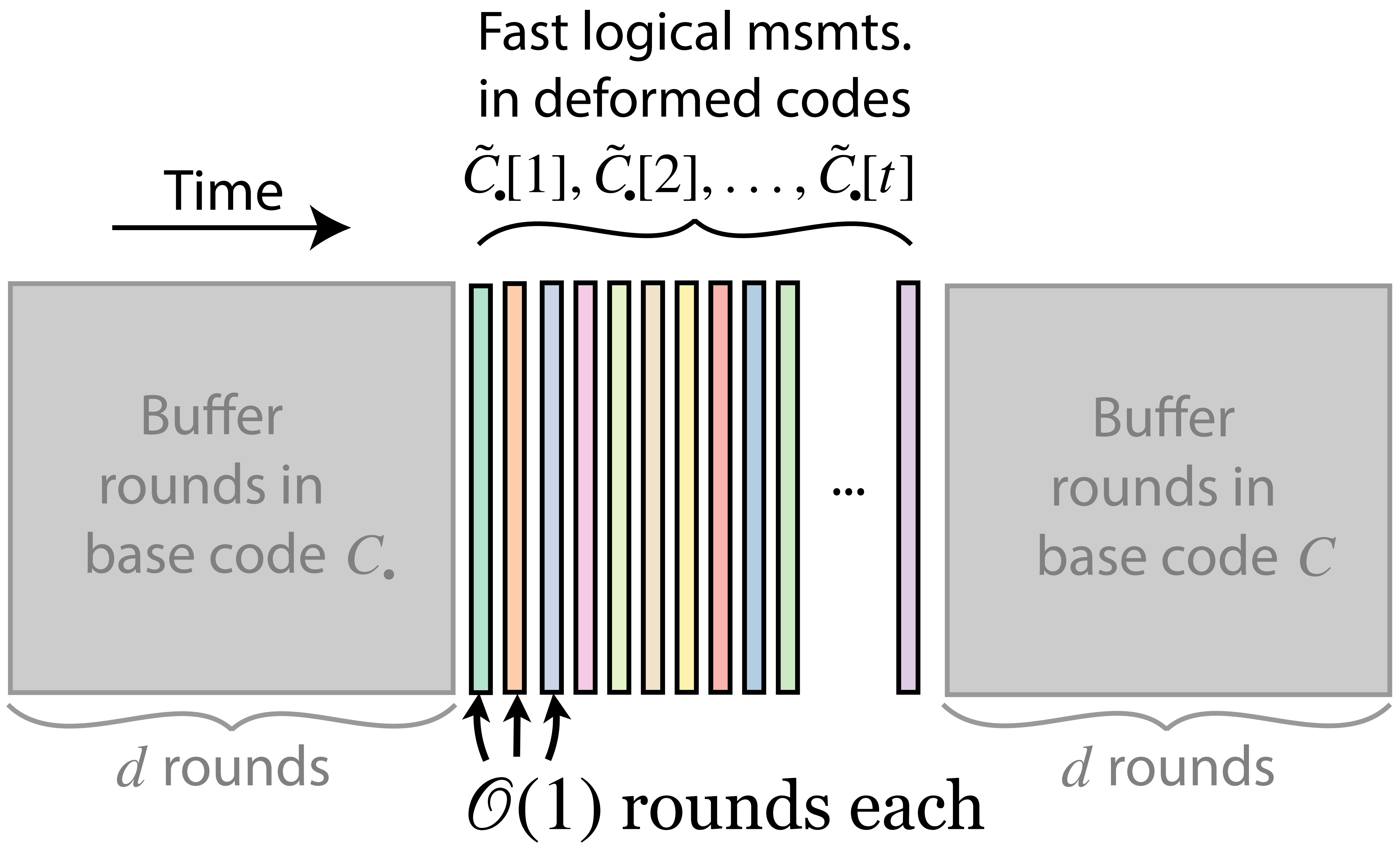

- C need not be single-shot state preparable

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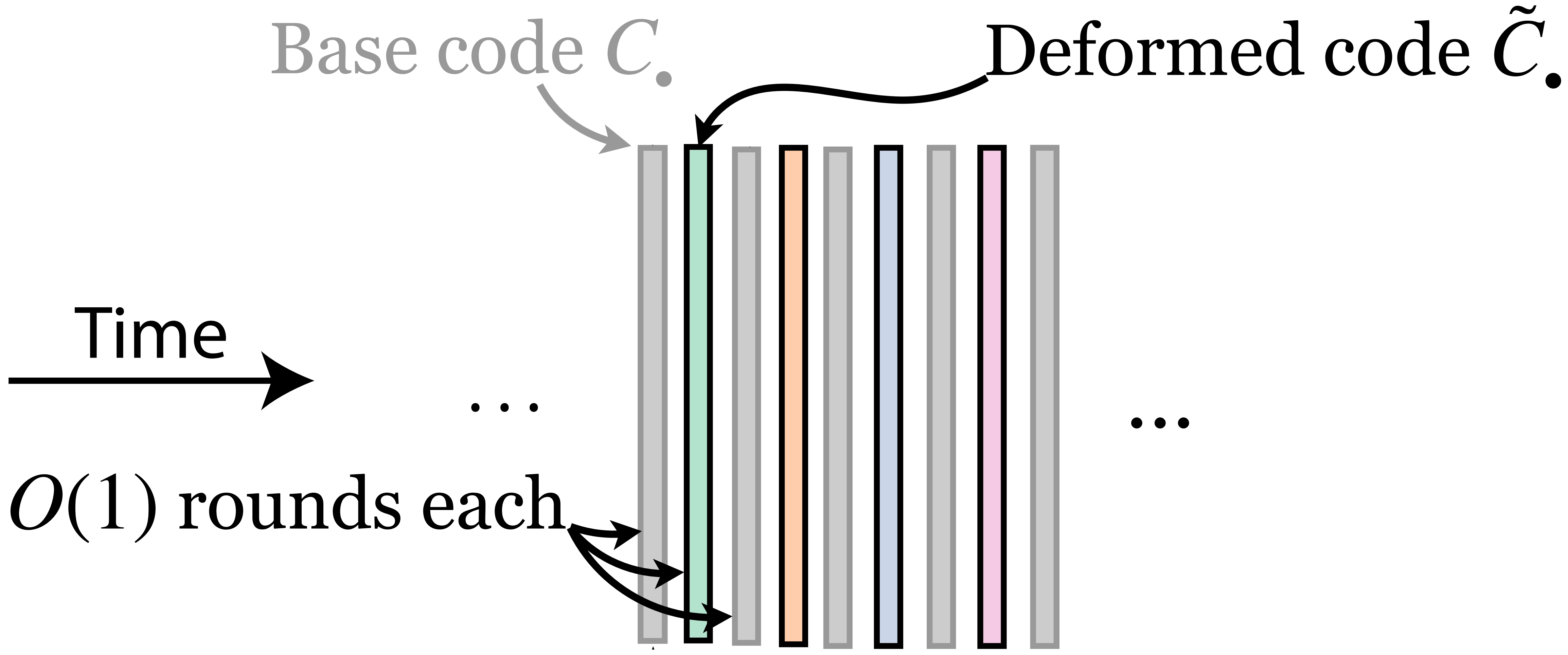

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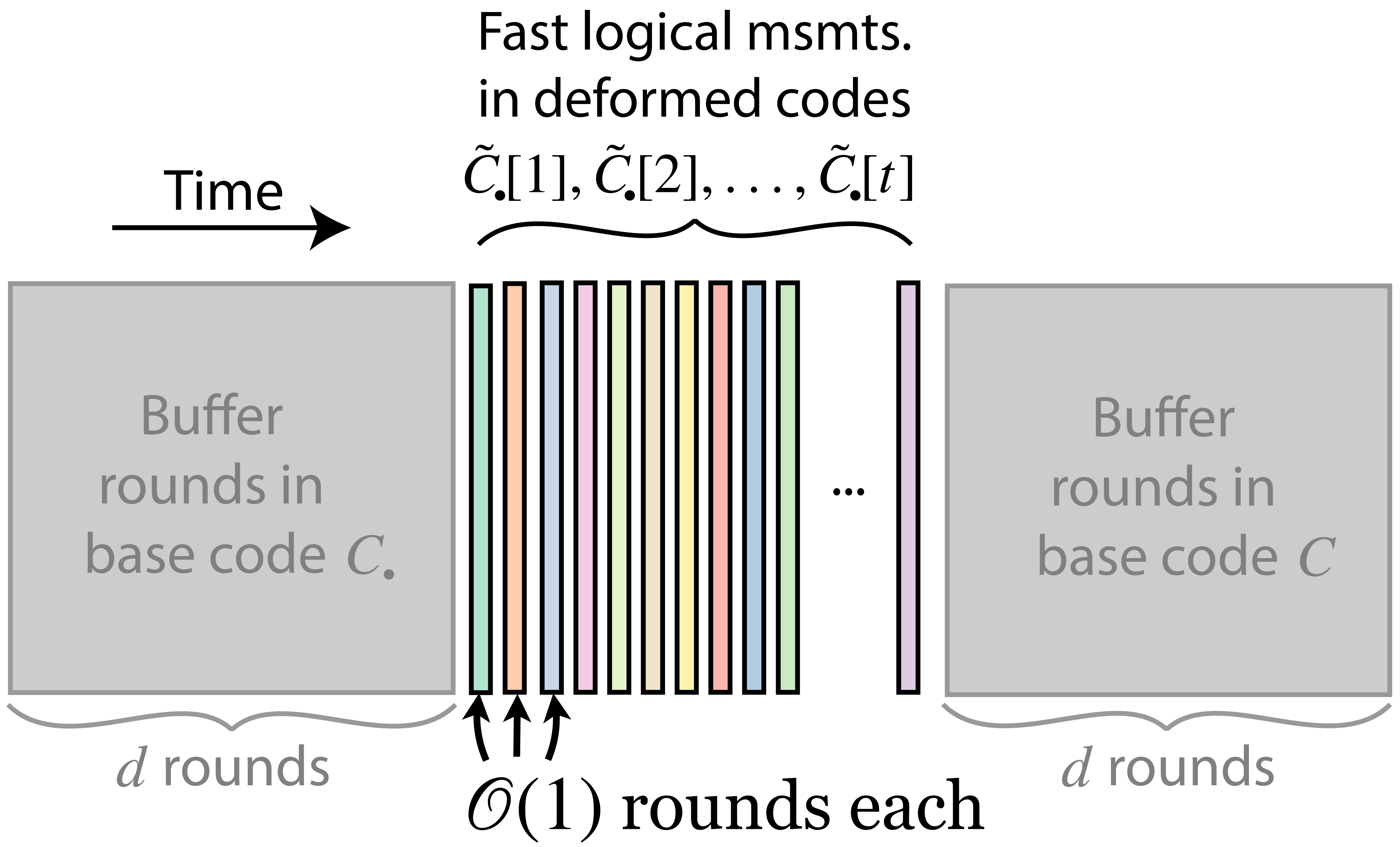

- C need not be single-shot state preparable
- Sufficient detectors to infer the Pauli frame after each msmt

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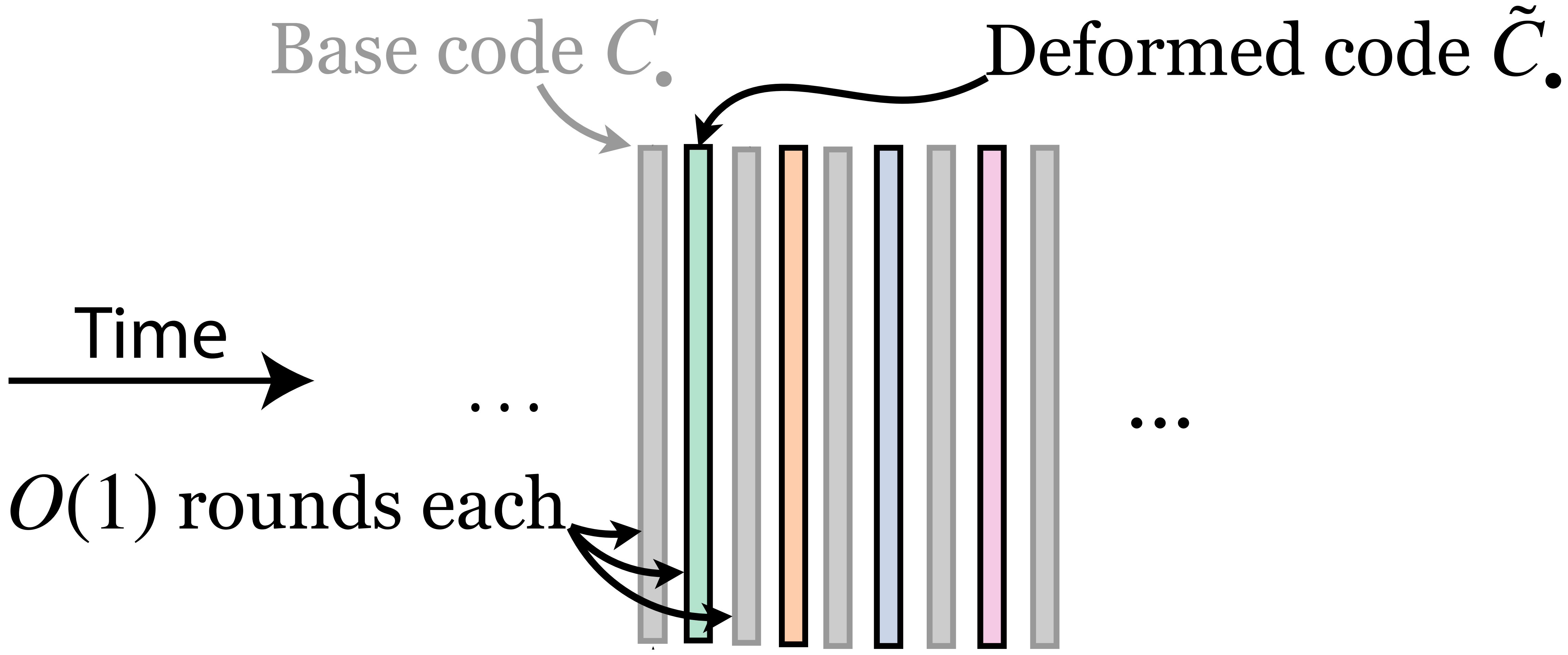

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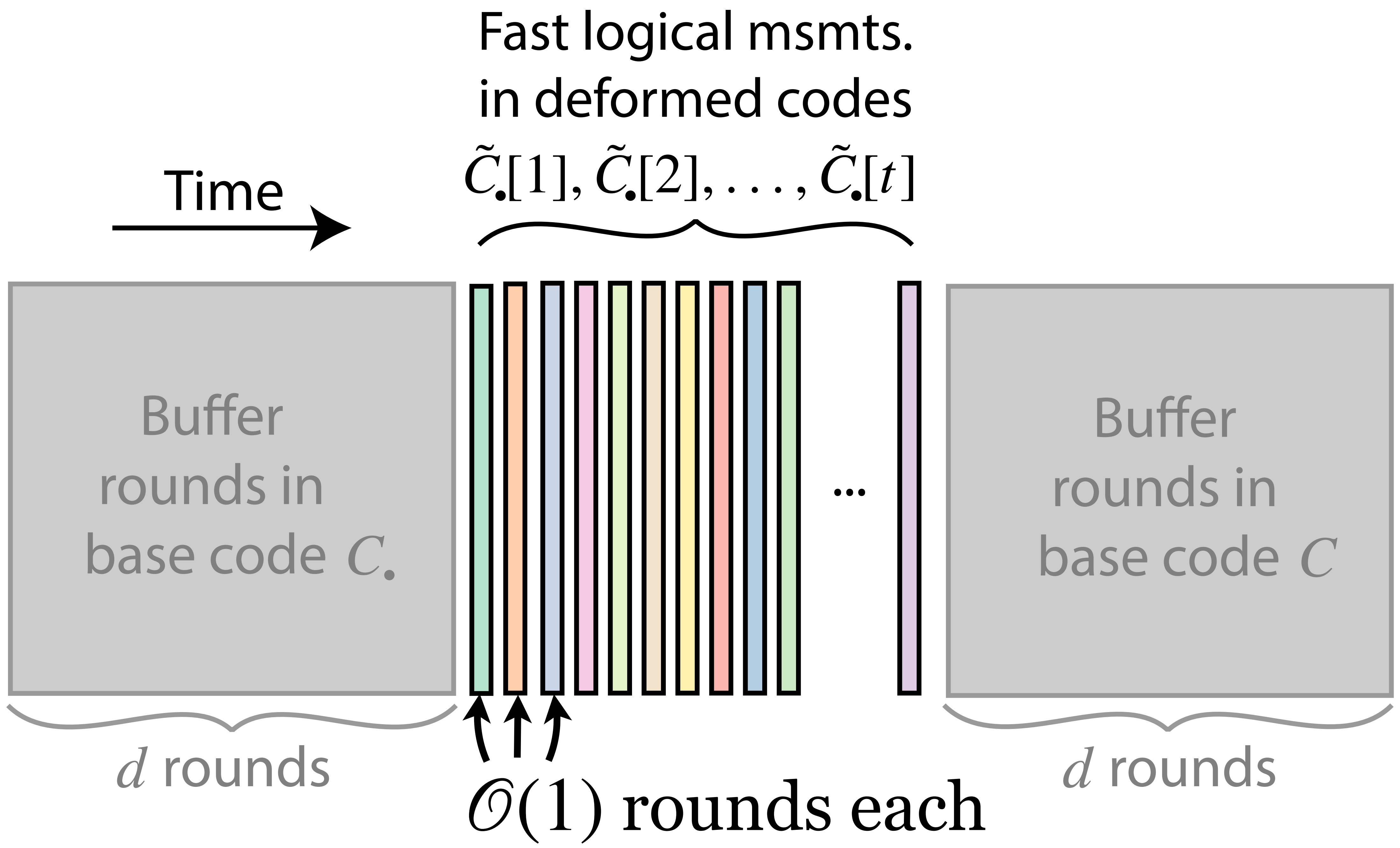

- C need not be single-shot state preparable
- Sufficient detectors to infer the Pauli frame after each msmt
- Each $\tilde{C}[i]$ needs to satisfy certain properties to ensure fault-tolerance

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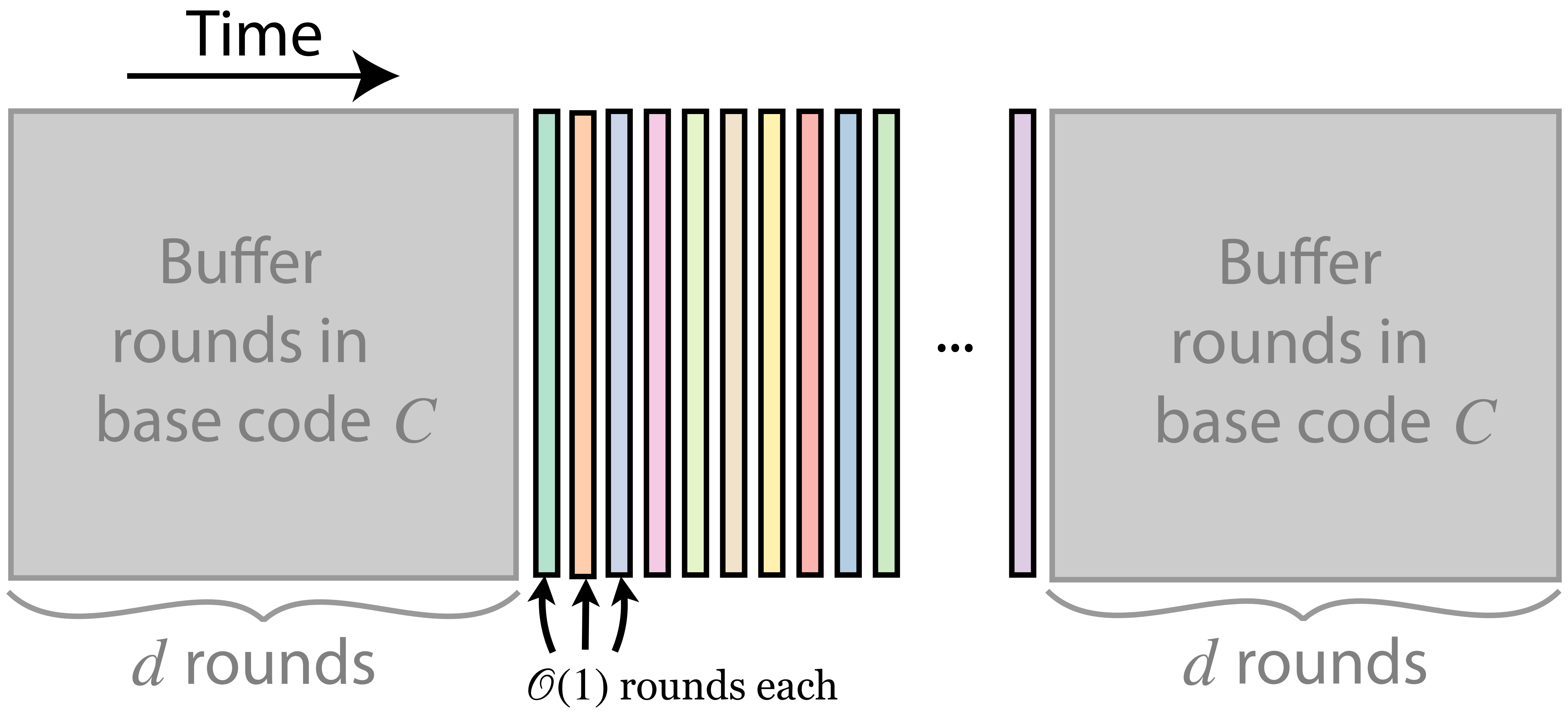

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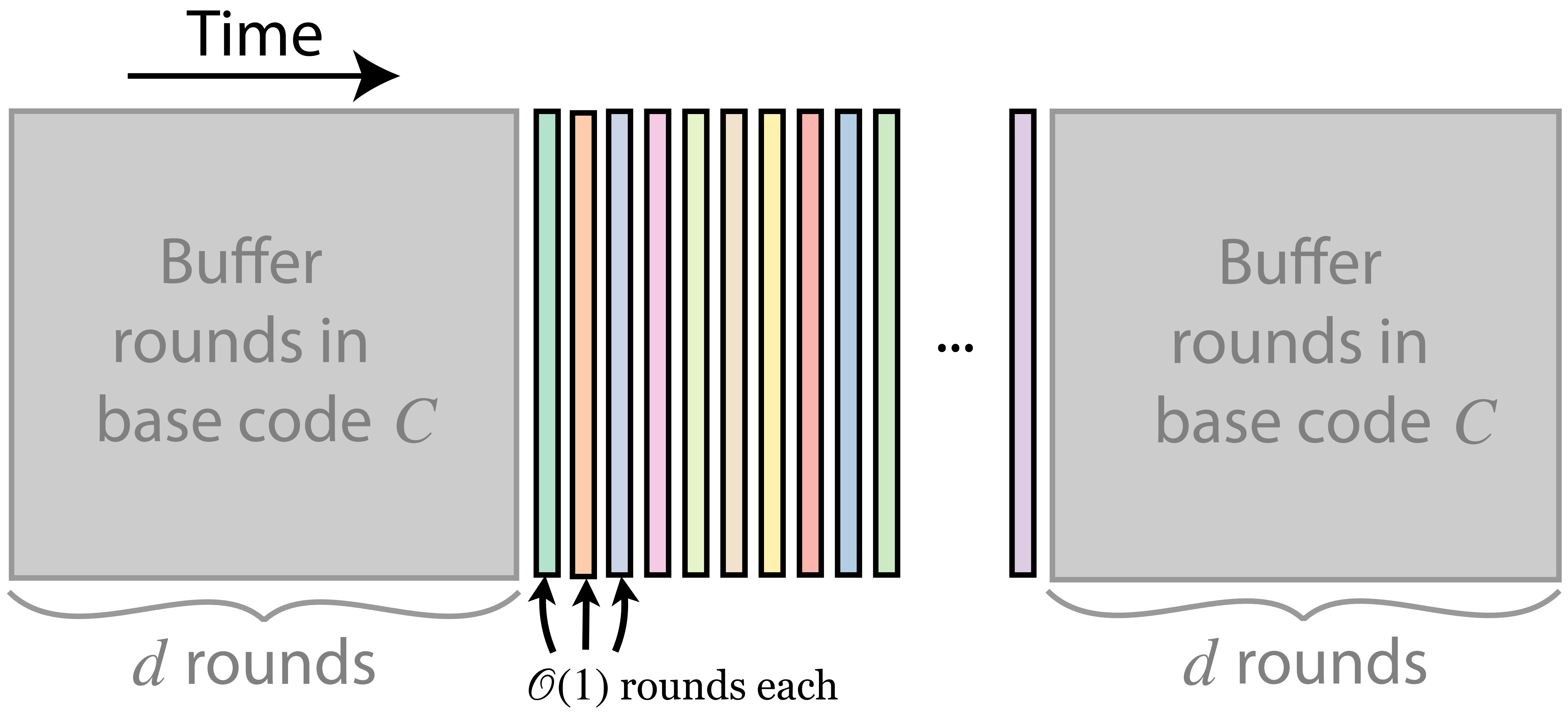

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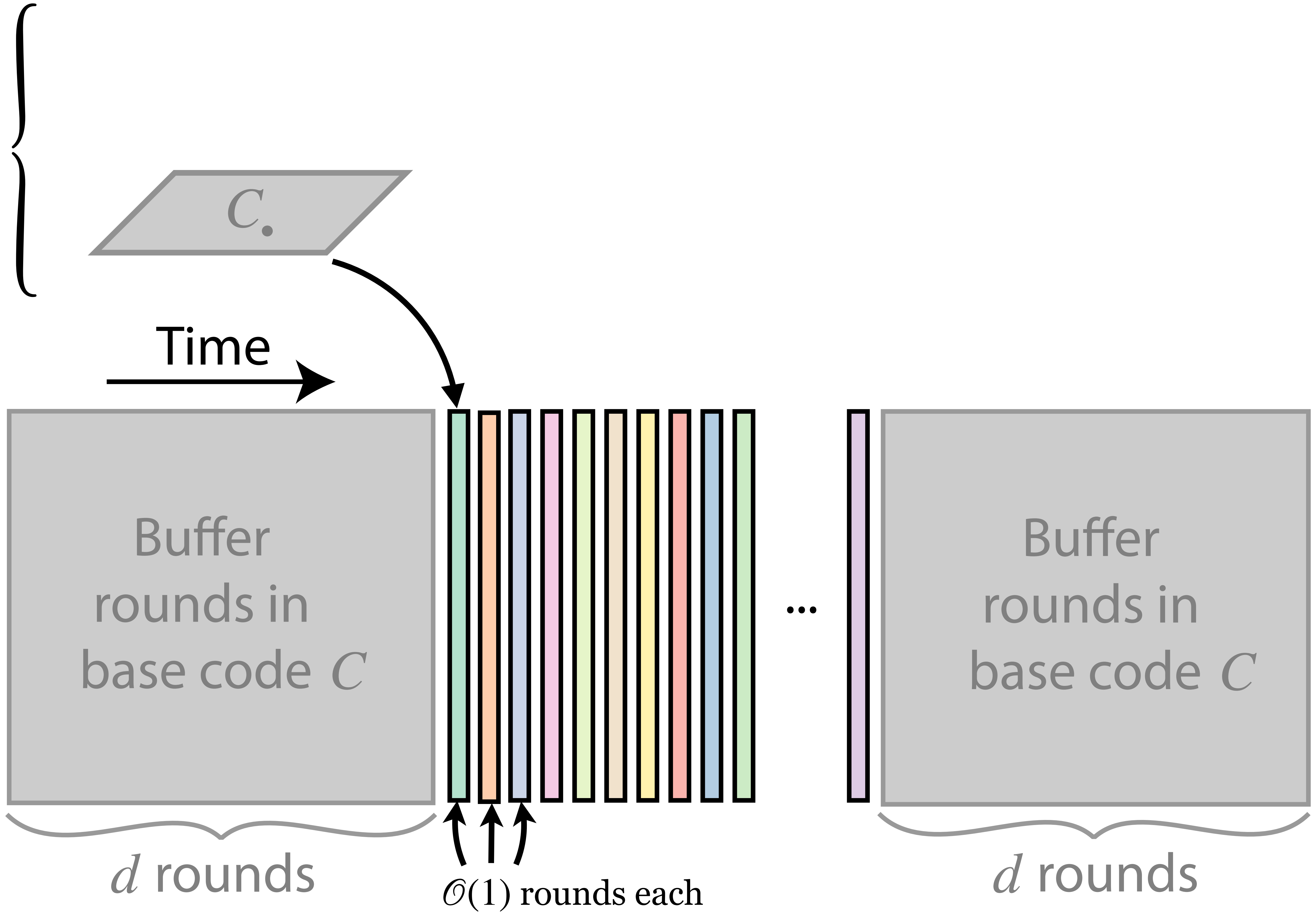

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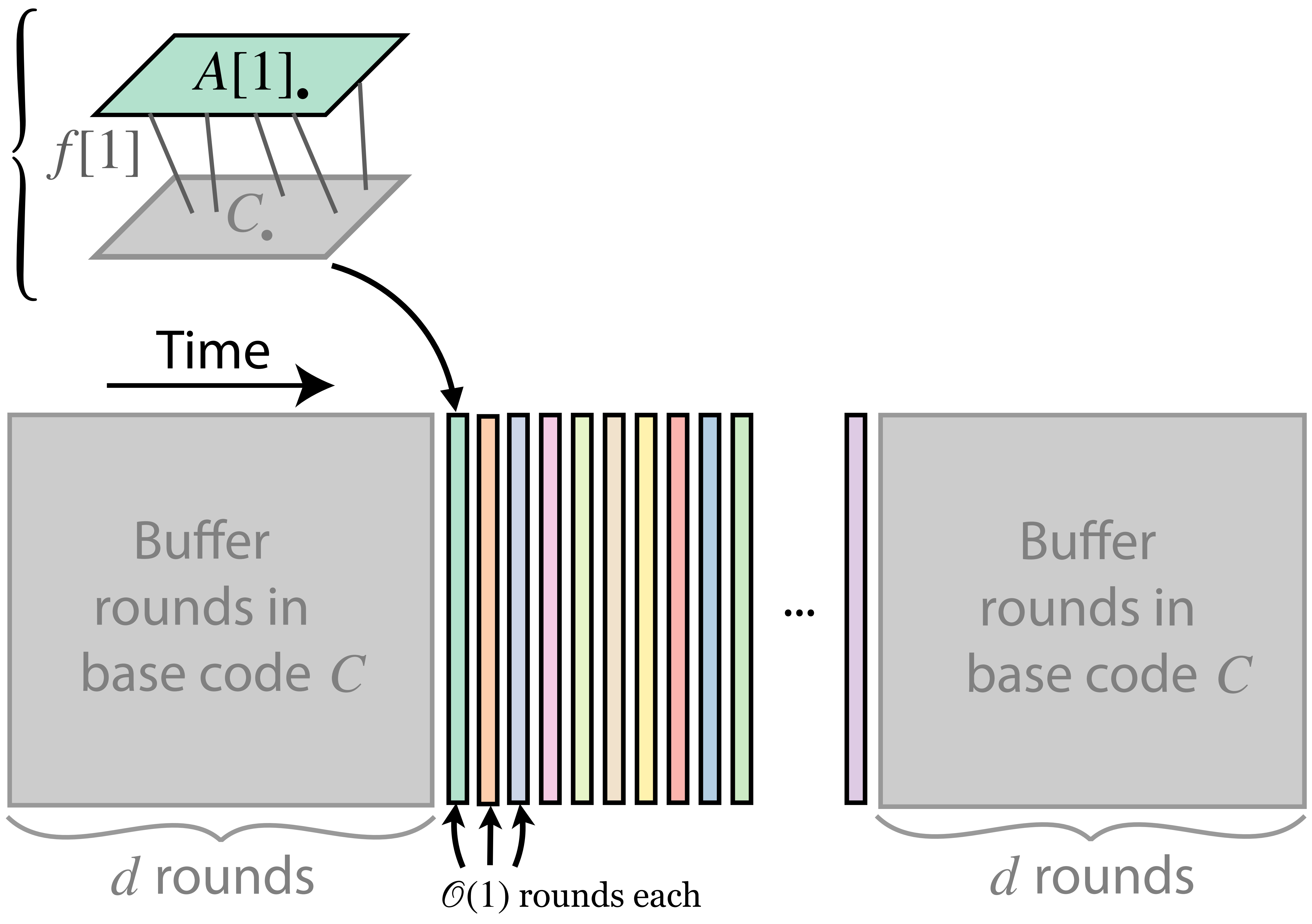

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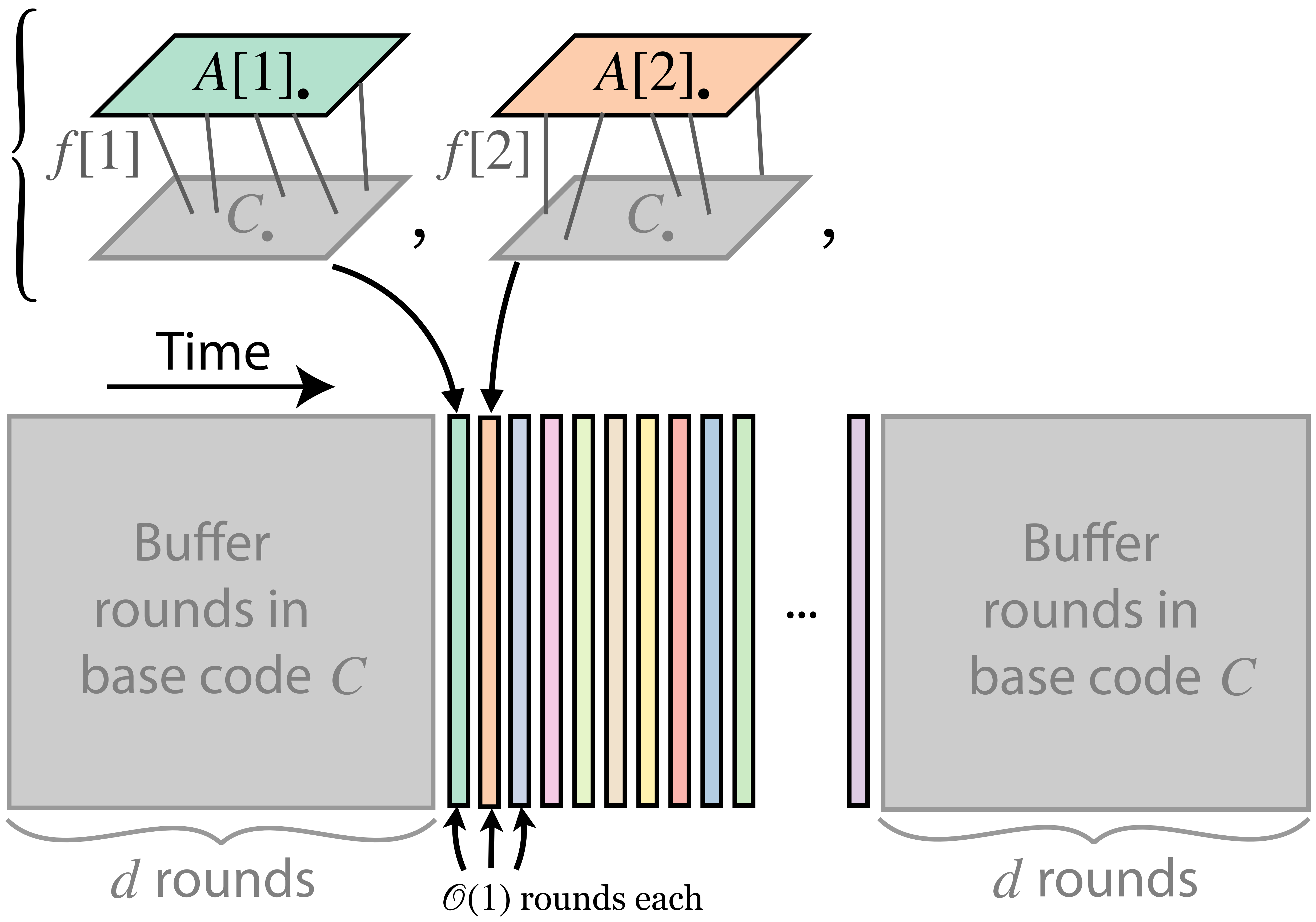

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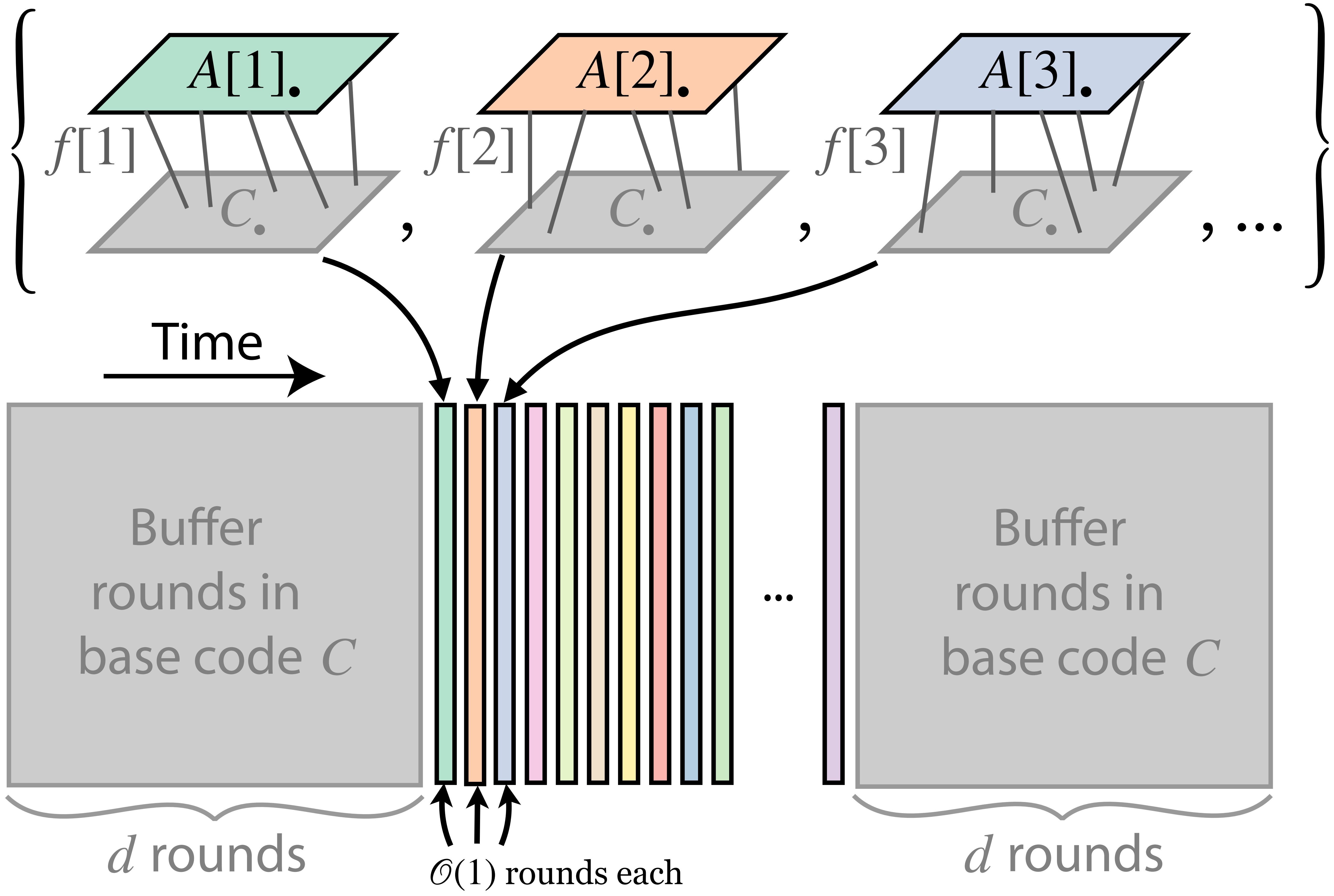

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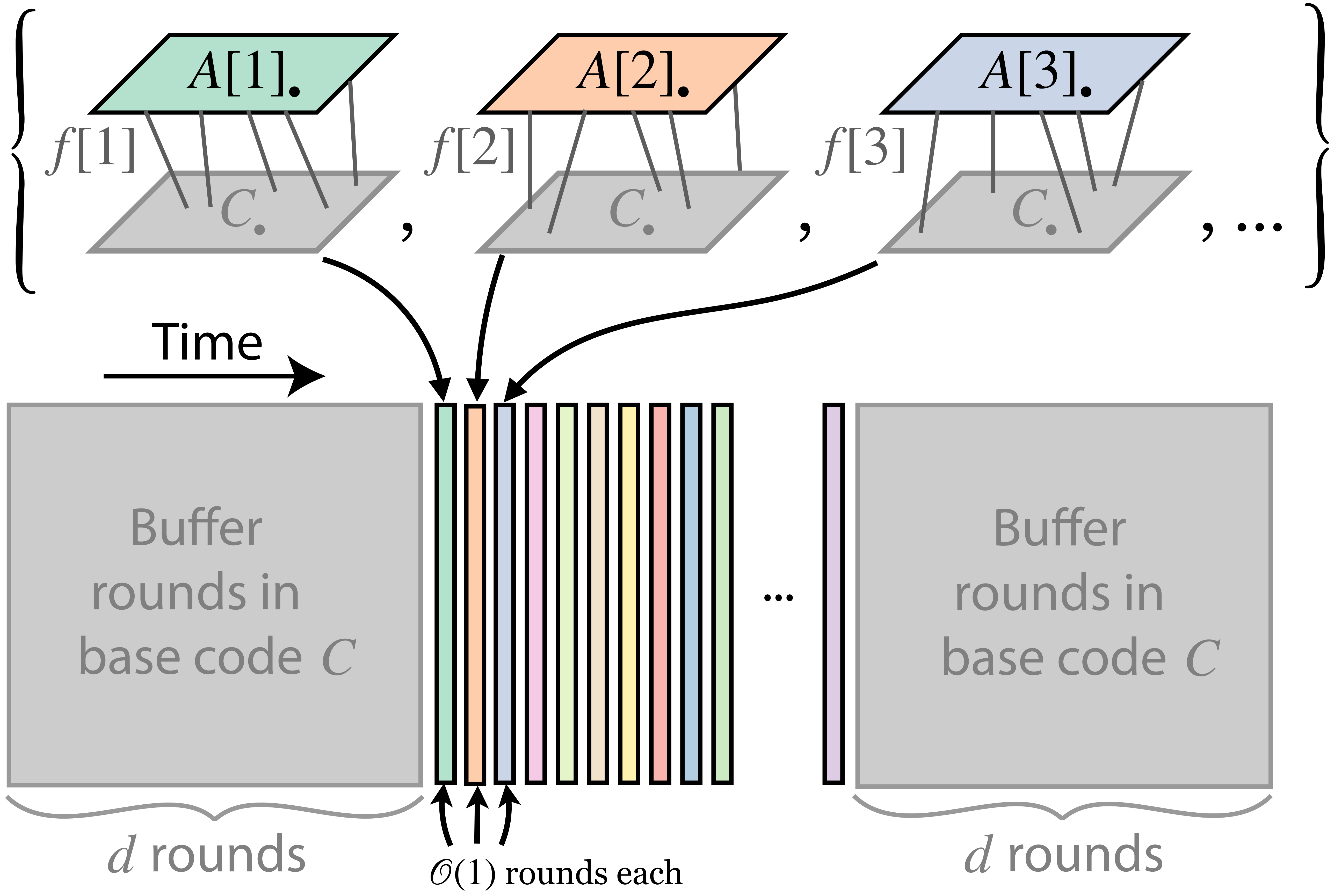

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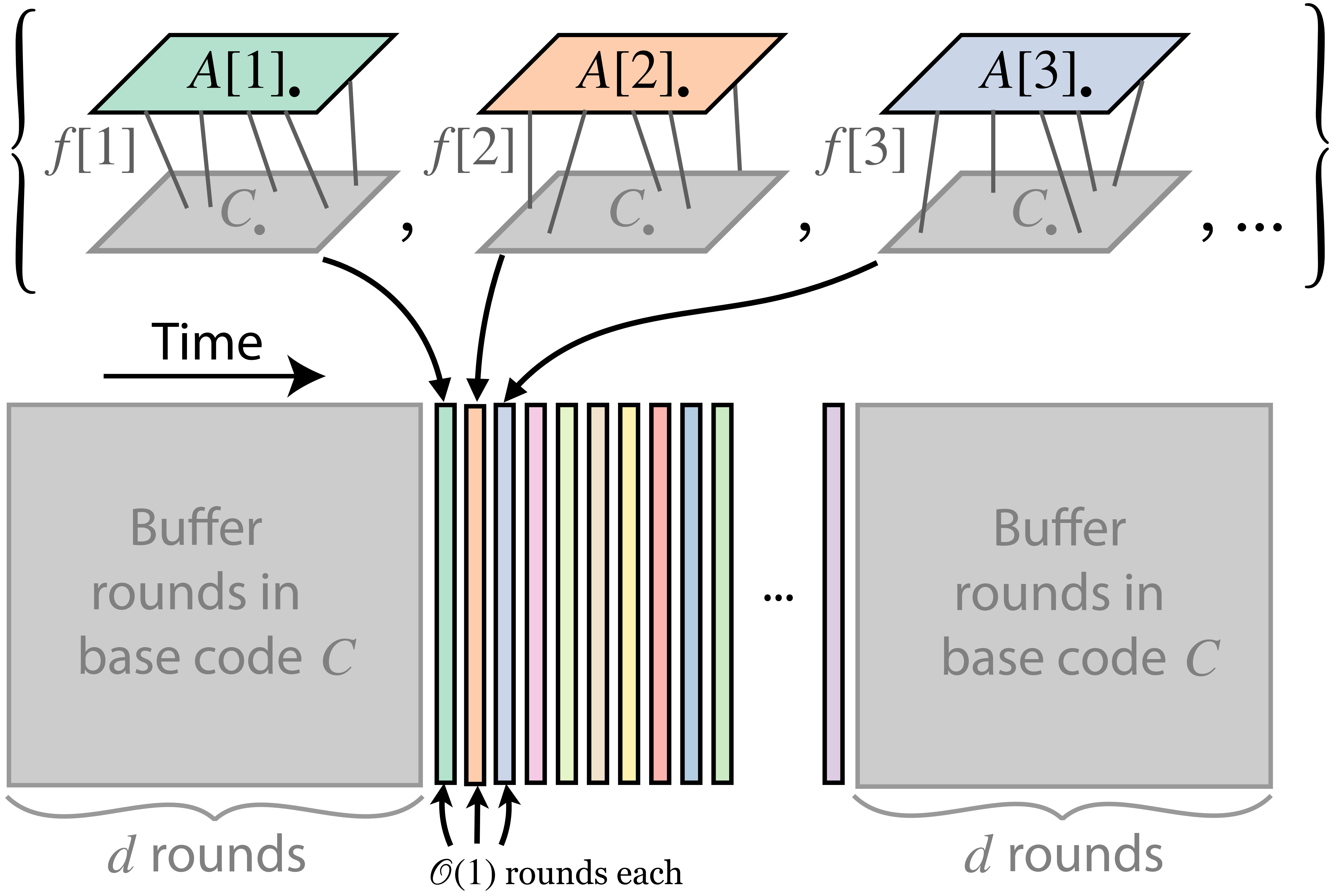
- Each ancilla complex $A[i]_\bullet$ has meta-check distance at least d.



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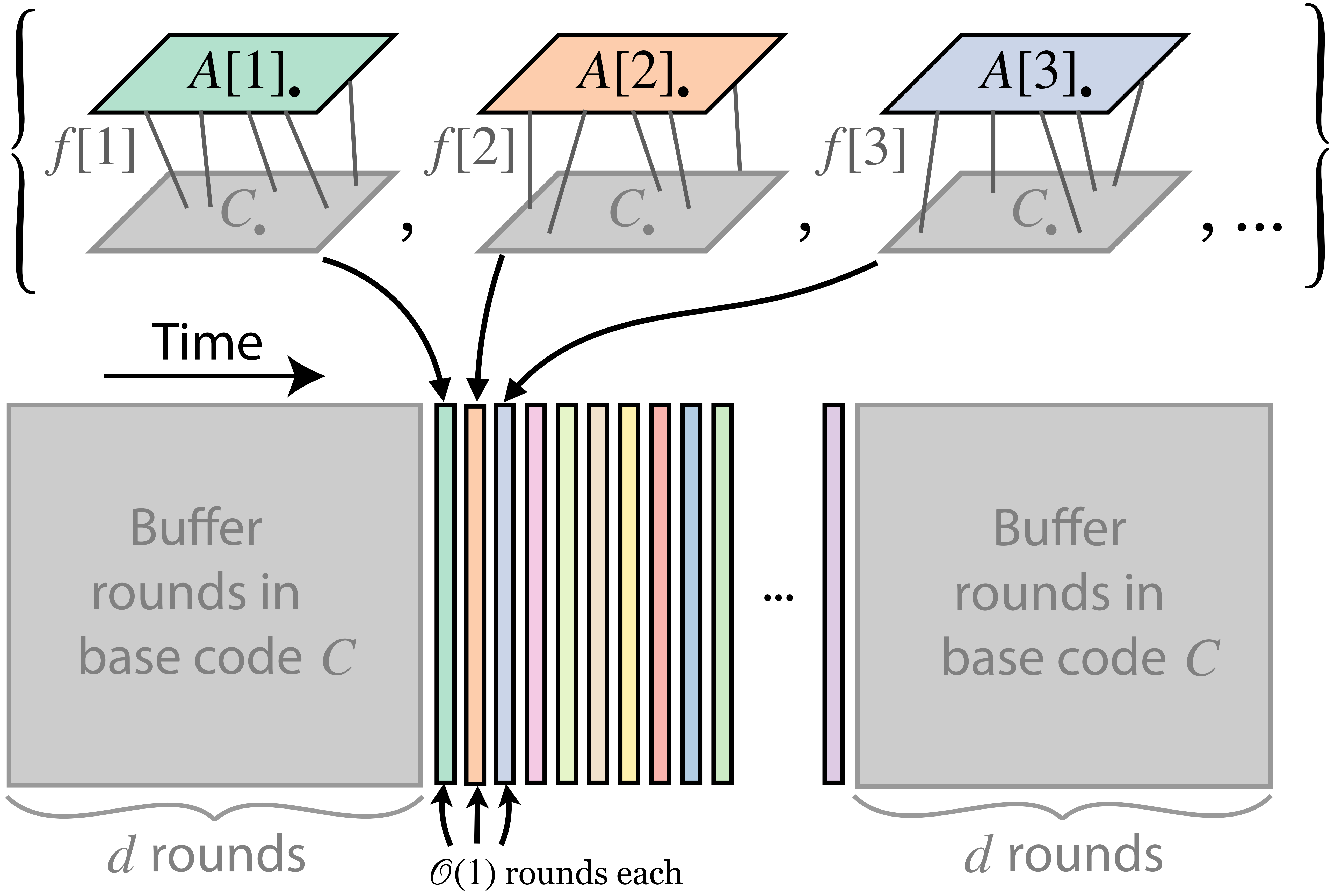
Can protect up to d ancilla measurement errors



Fault-tolerance conditions for constant time surgery: Cowtan, *et al.* [2510.14895]

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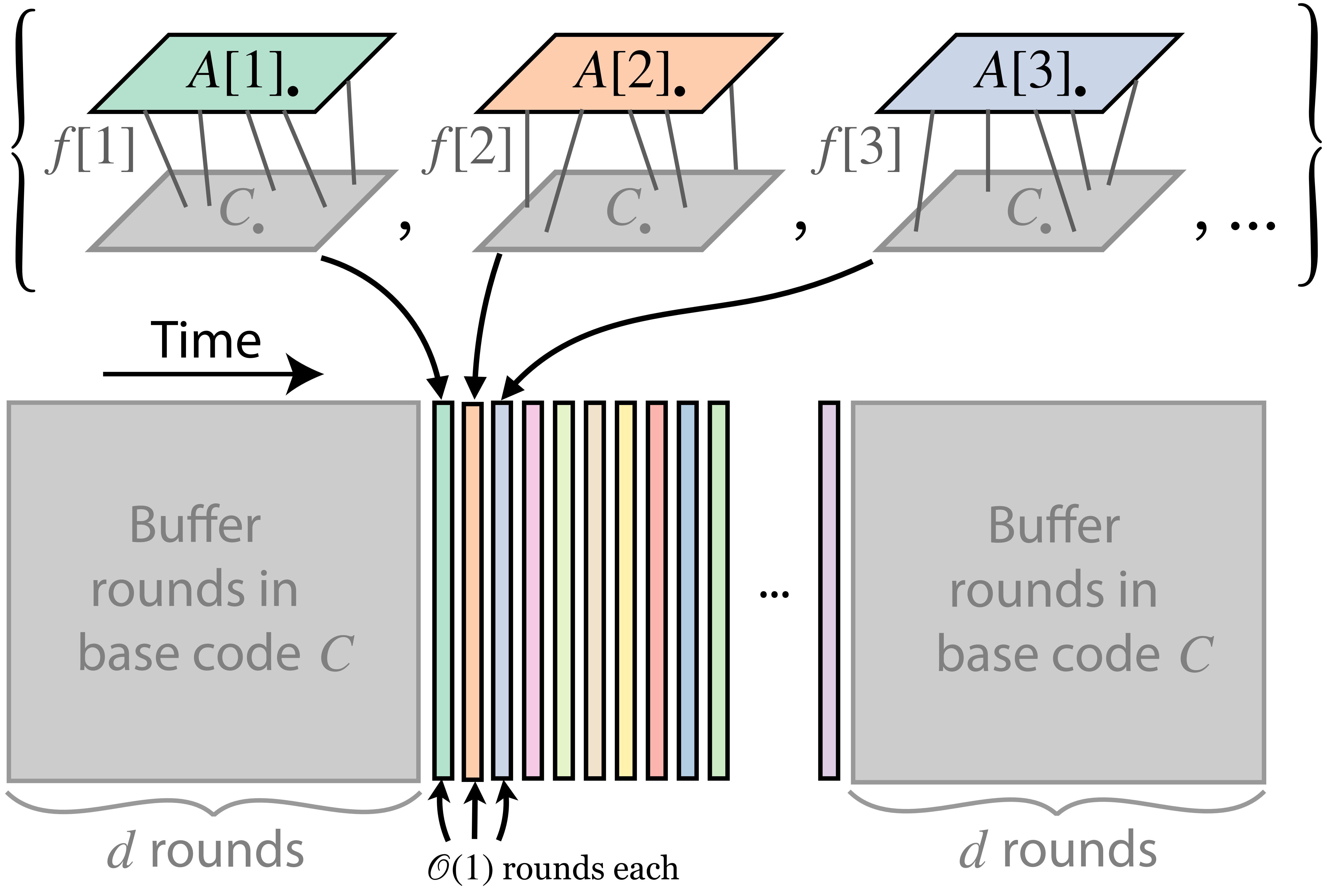

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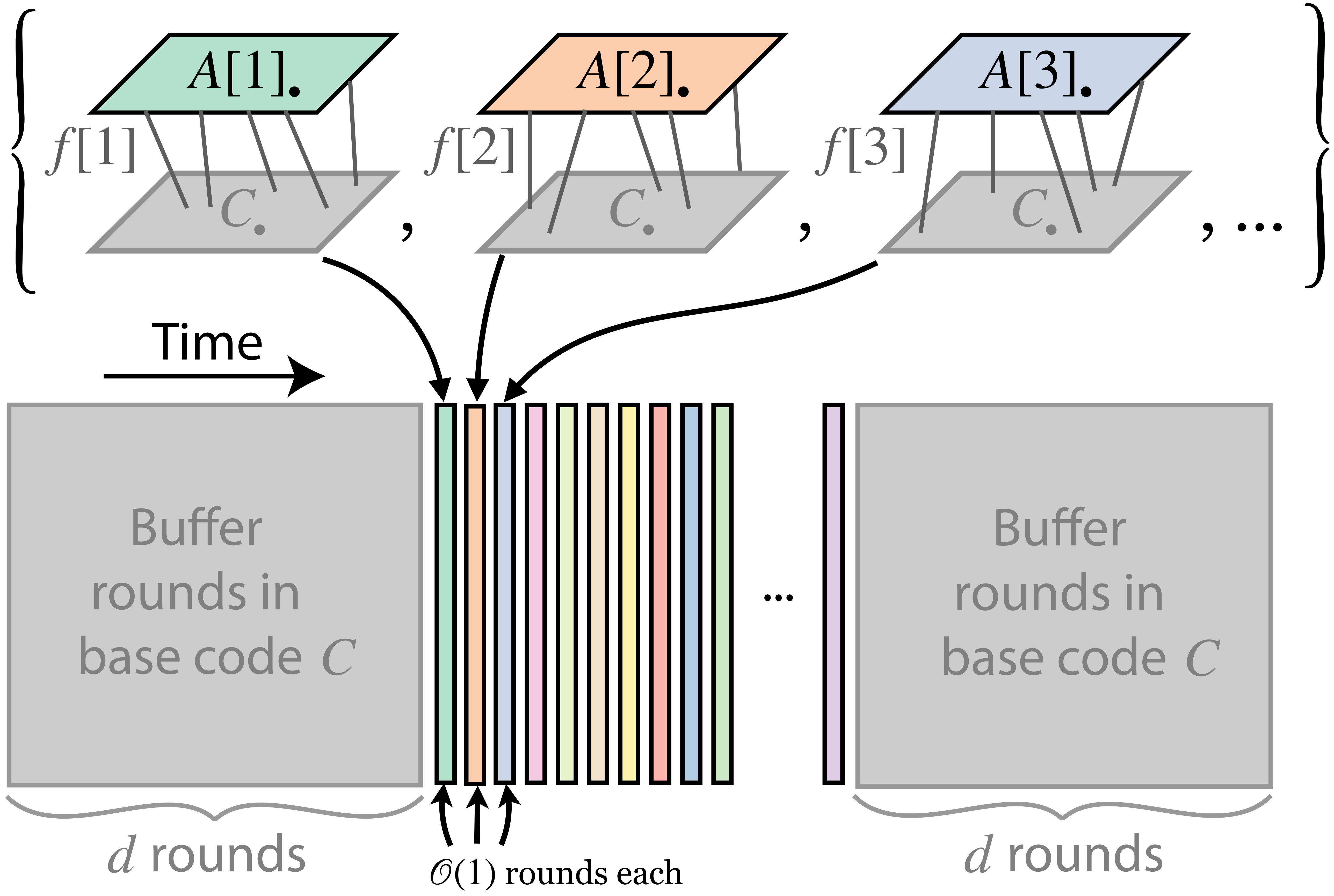

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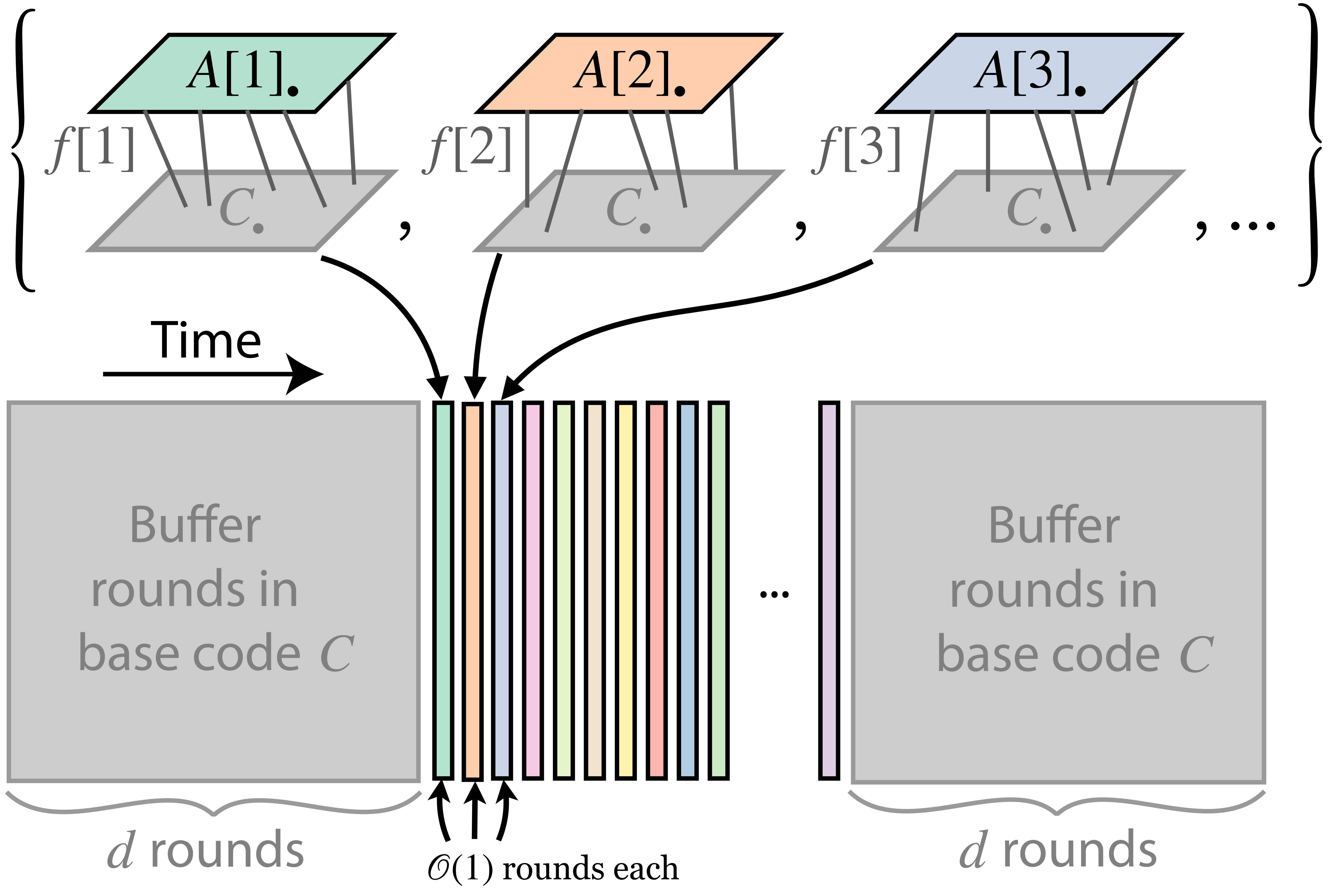

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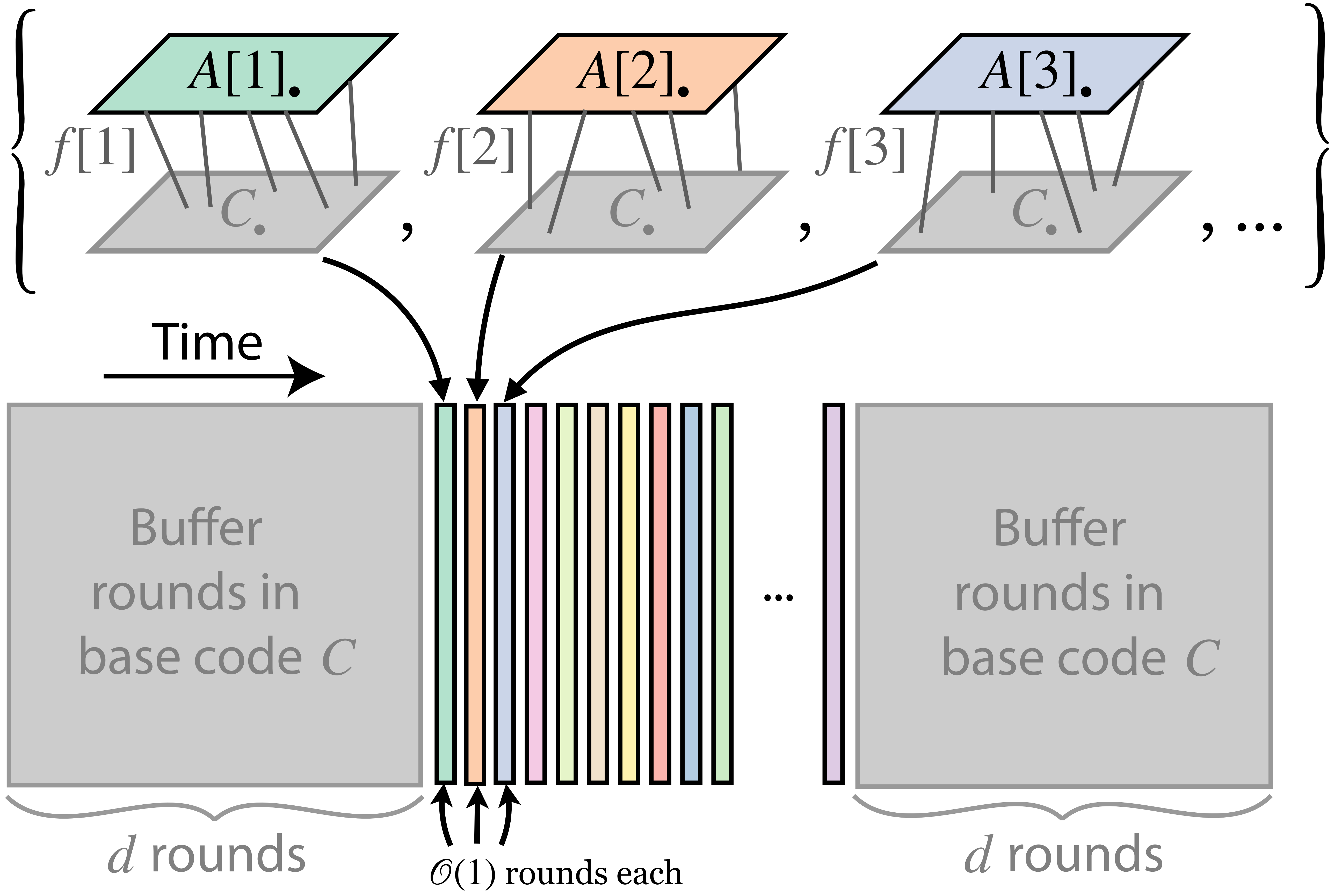

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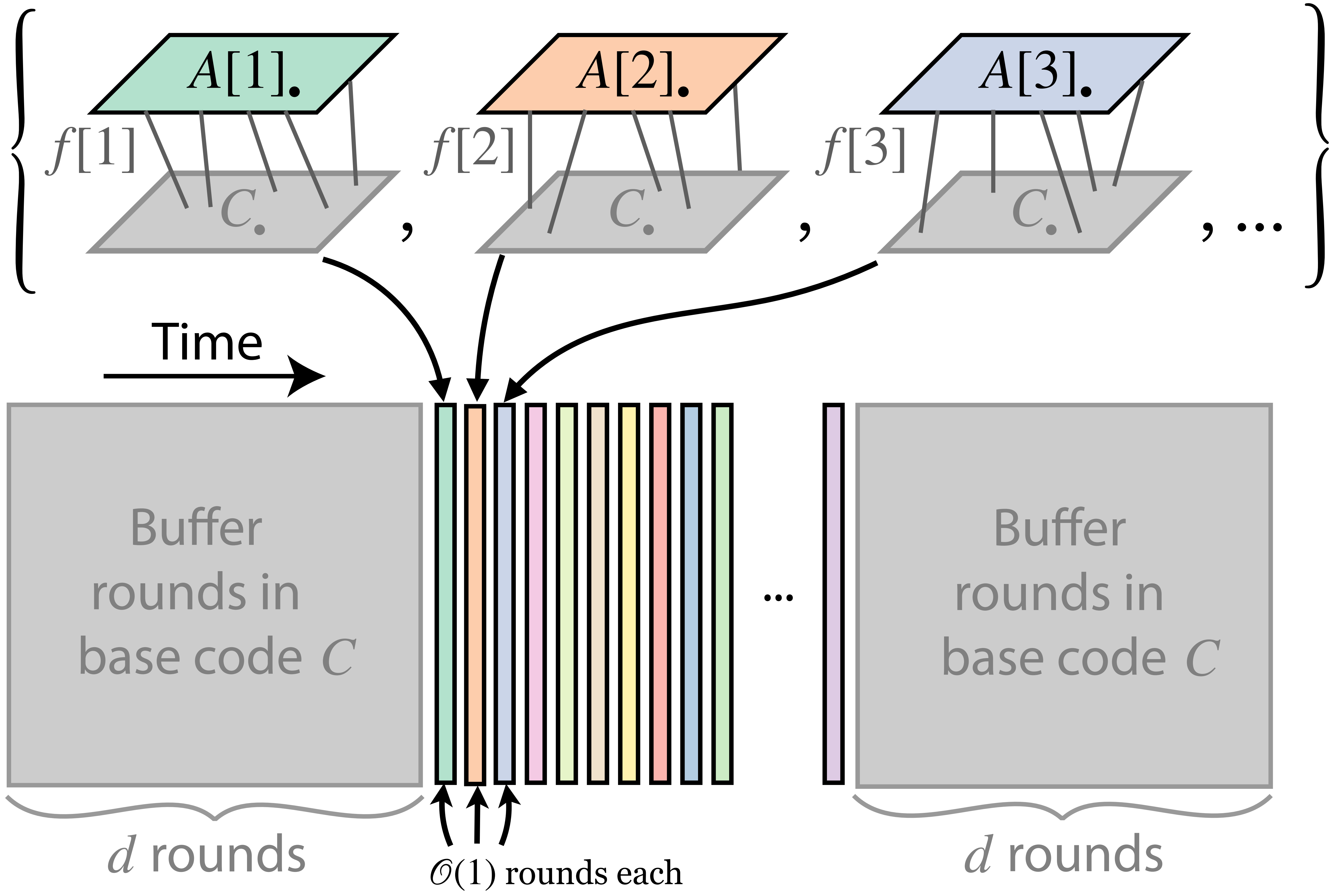

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- Need to consider all surgery systems together — logicals may be weight reduced across different surgery systems.

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- **Constant time per measurement:** Each surgery operation takes $\mathcal{O}(1)$ **time** in amortization.
- **Low space overhead:** Each complex **uses** $\mathcal{O}(n\log(n))$ **physical ancilla qubits**.

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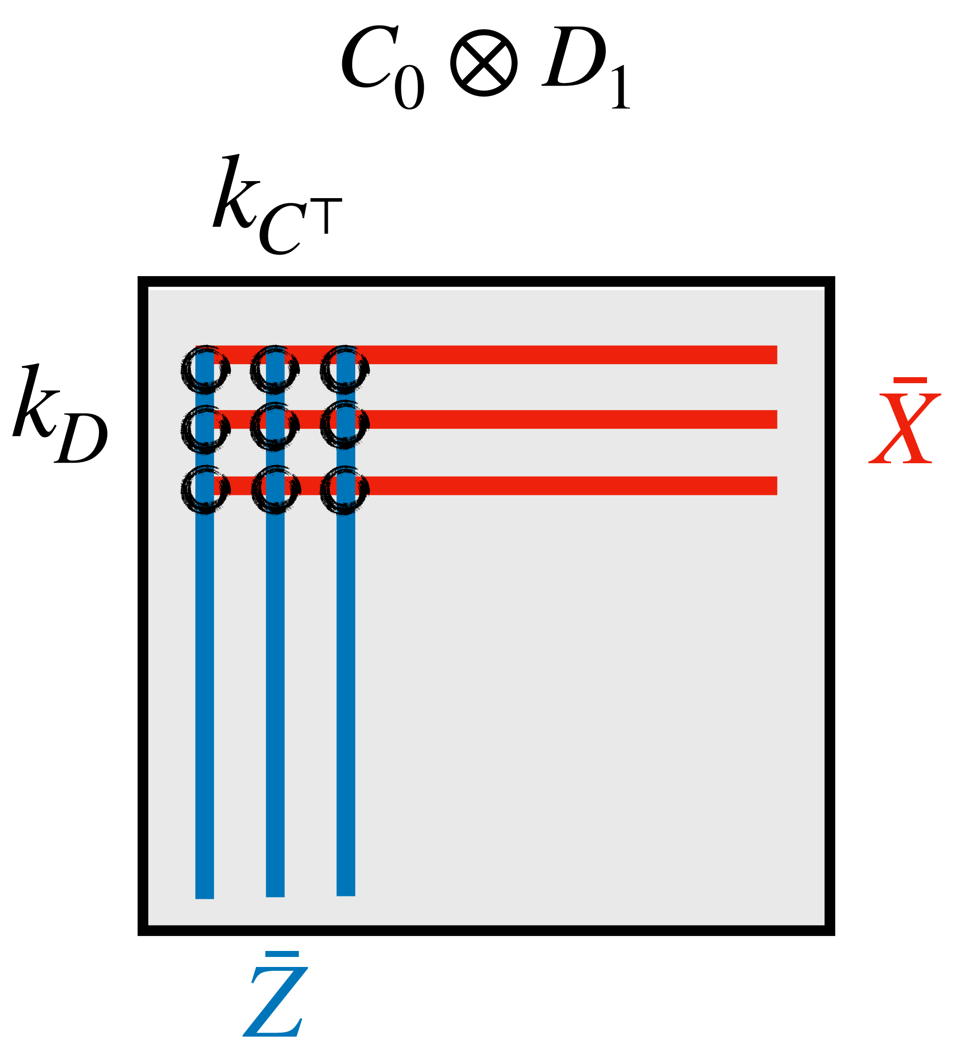

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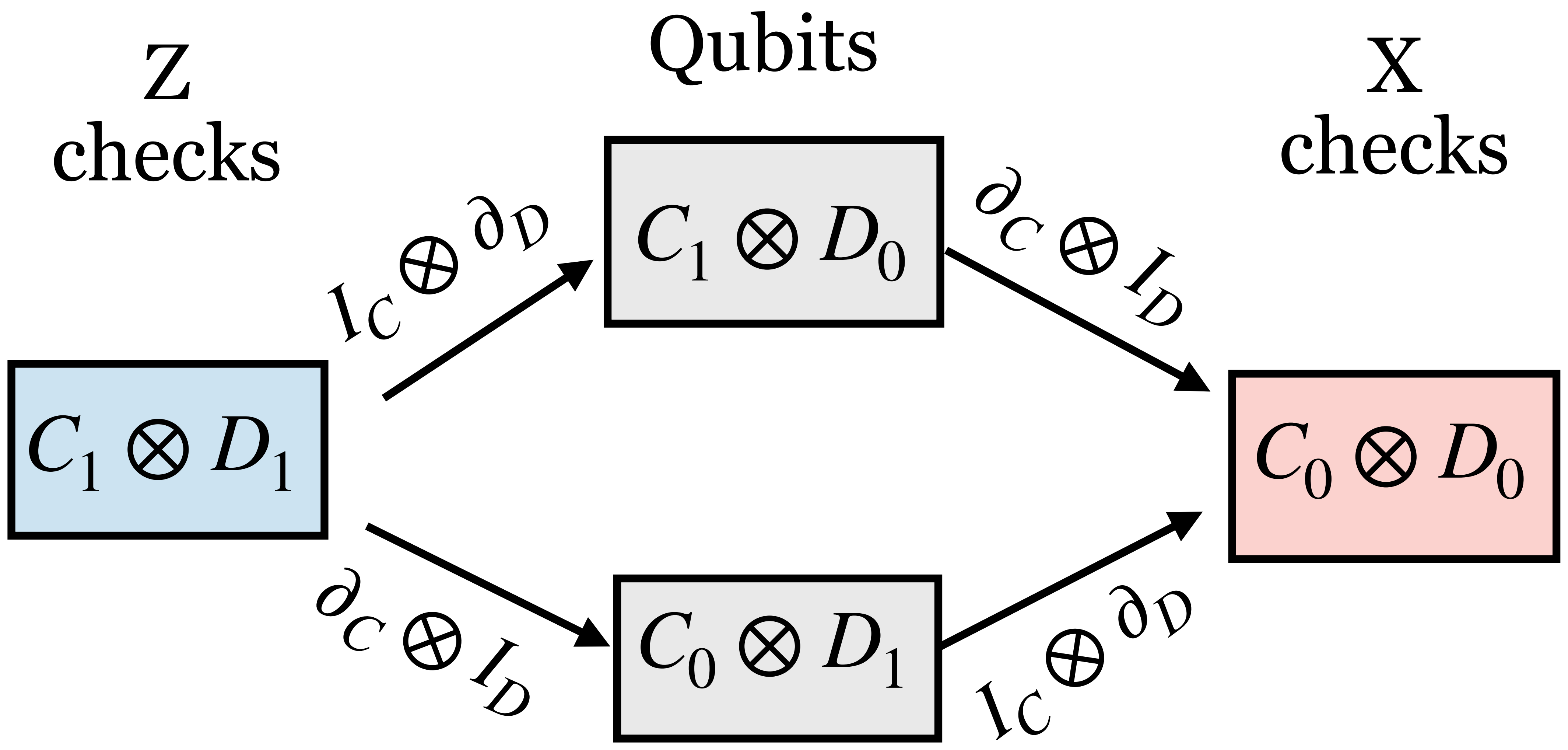

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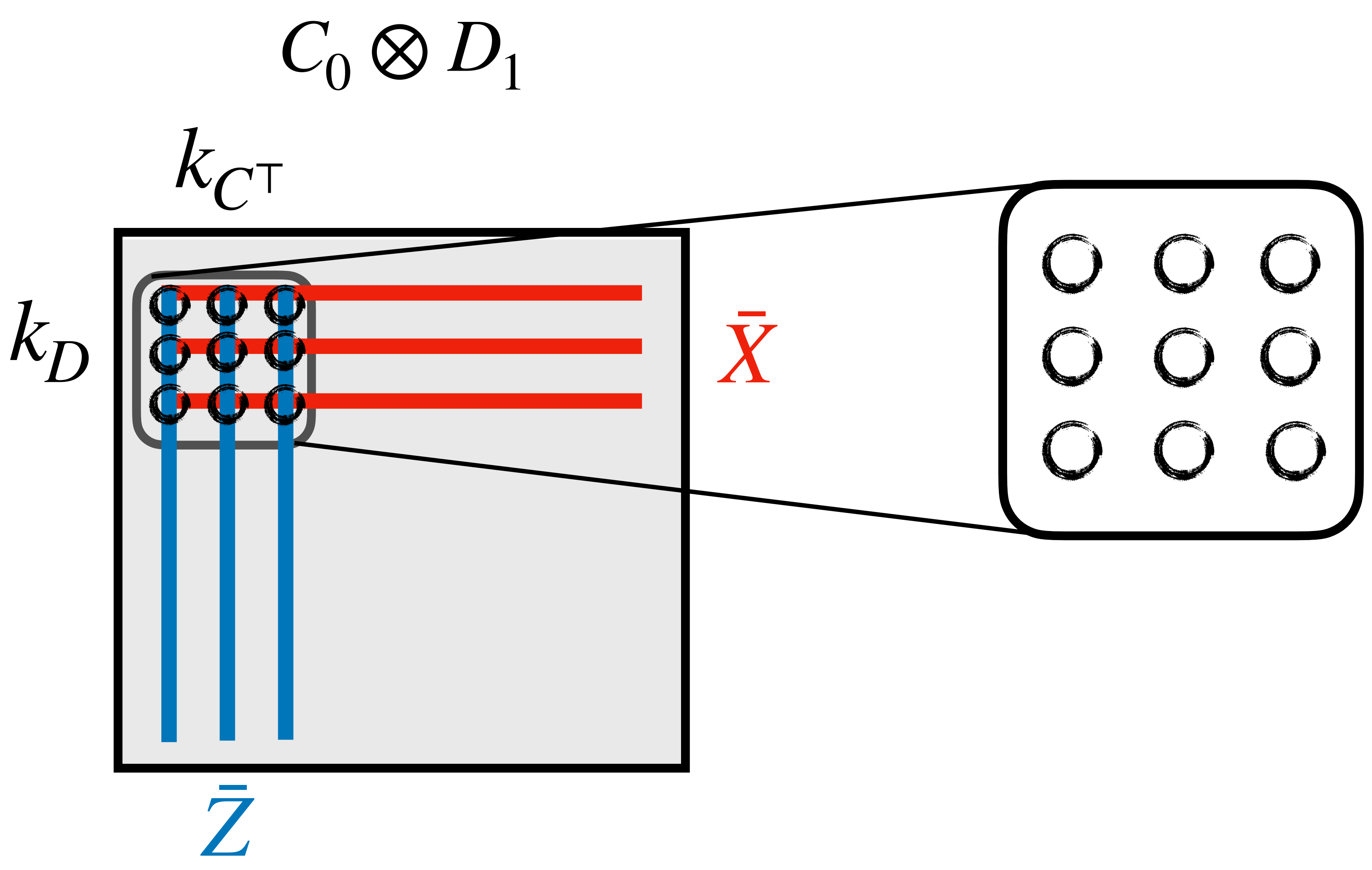

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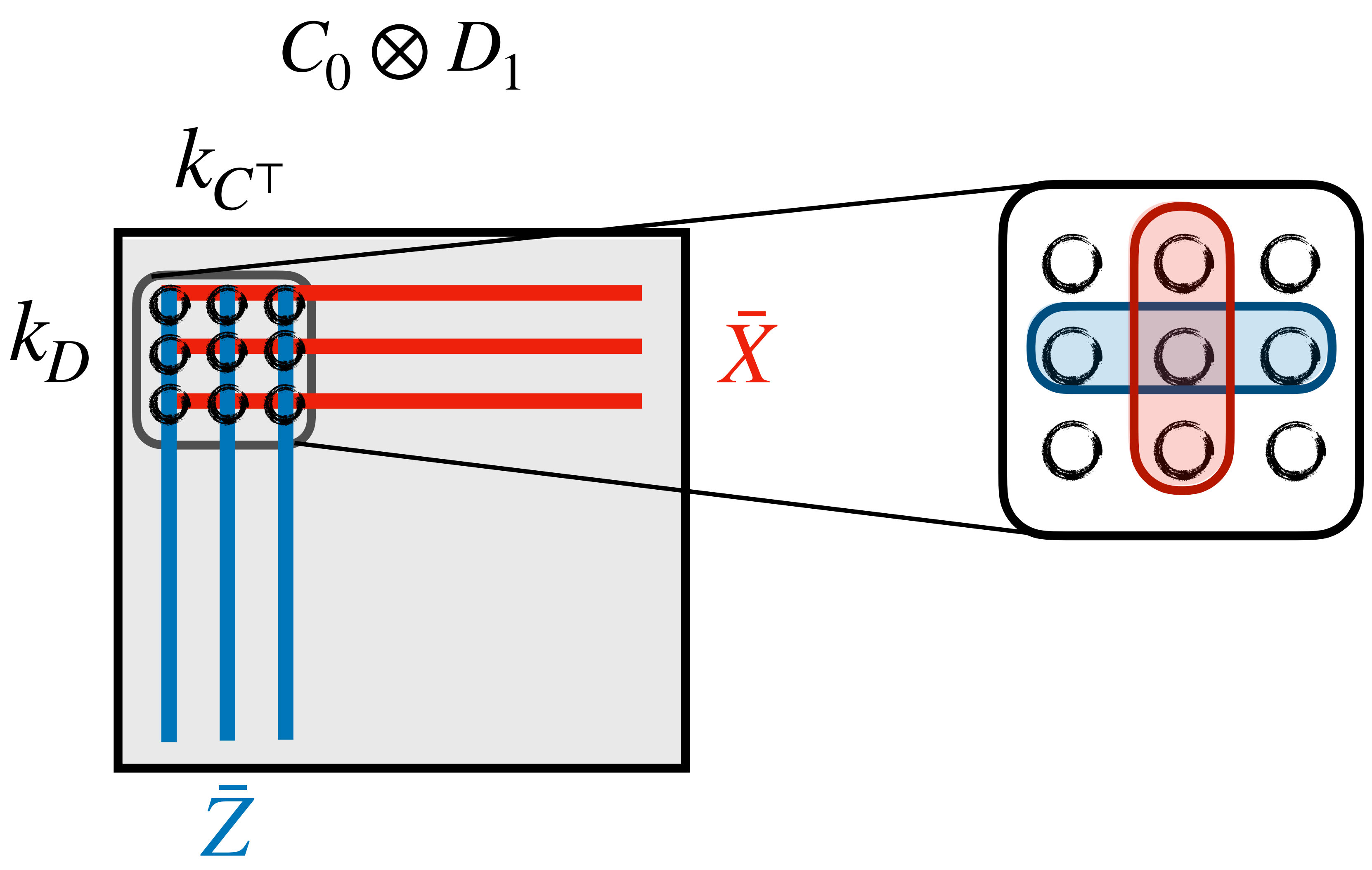

Each row (col) corresponds to a codeword of $D_\bullet$ ($C_\bullet^\perp$)

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Limited addressability, parallel measurements

We can measure rows of $\bar{Z}$ logicals (or columns of $\bar{X}$)

Constructing constant-time surgery gadgets

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1. Apply surgery gadget
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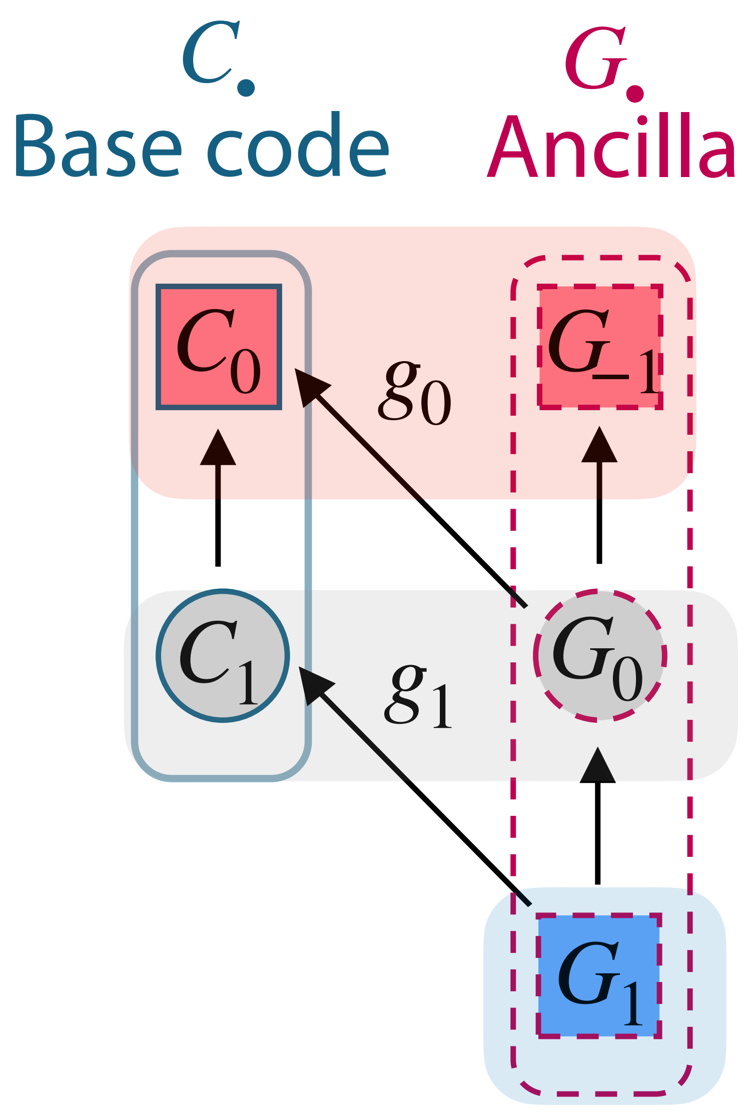
Surgery on classical code
(3-term chain complex)



Constructing constant-time surgery gadgets

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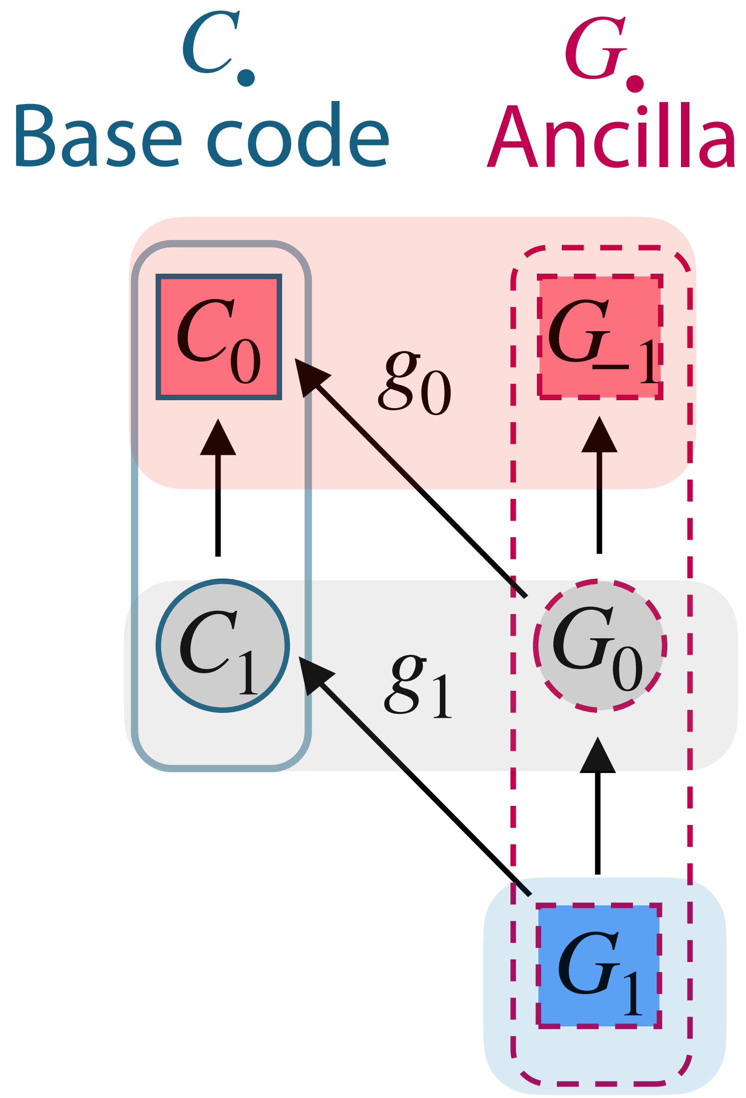

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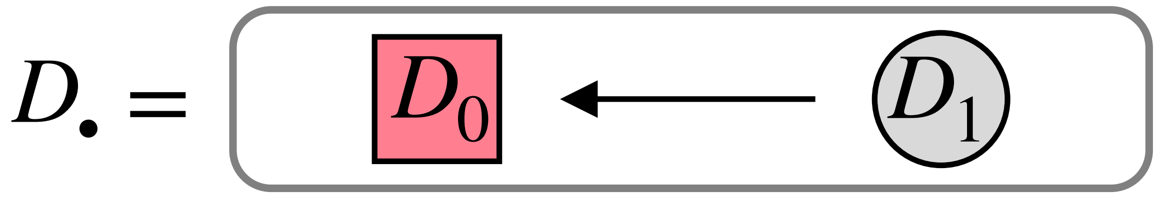

Constructing constant-time surgery gadgets

1. Apply surgery gadget on the classical code $C_\bullet$. The ancilla = $G_\bullet$

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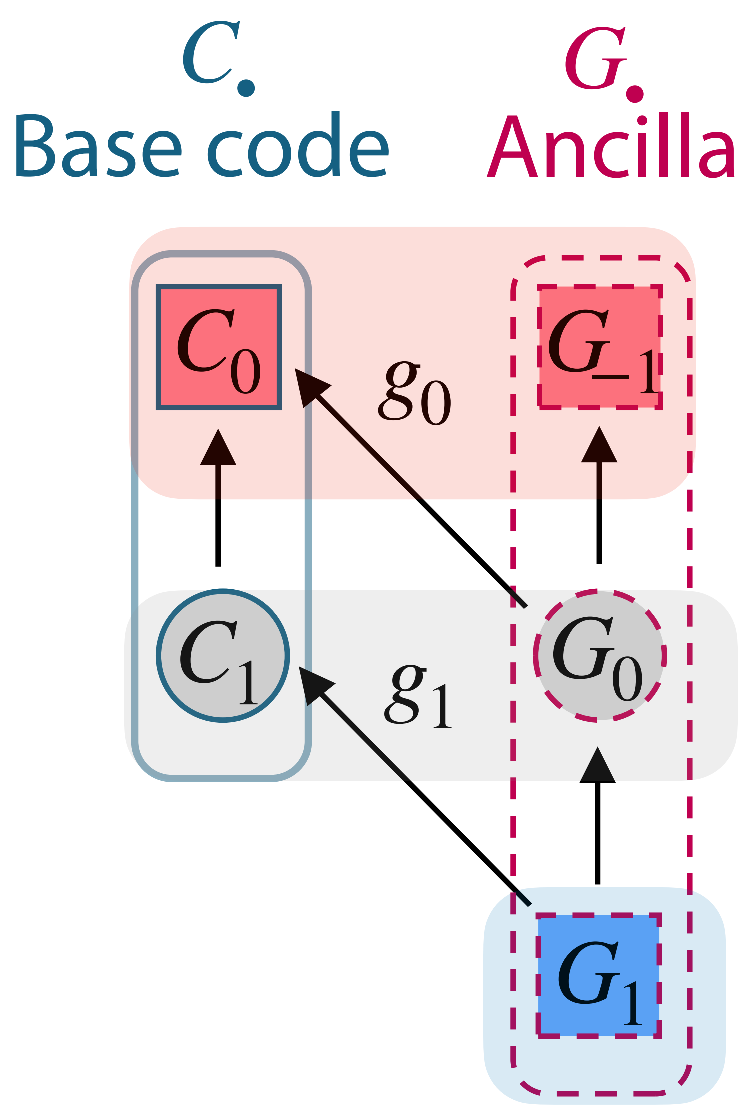

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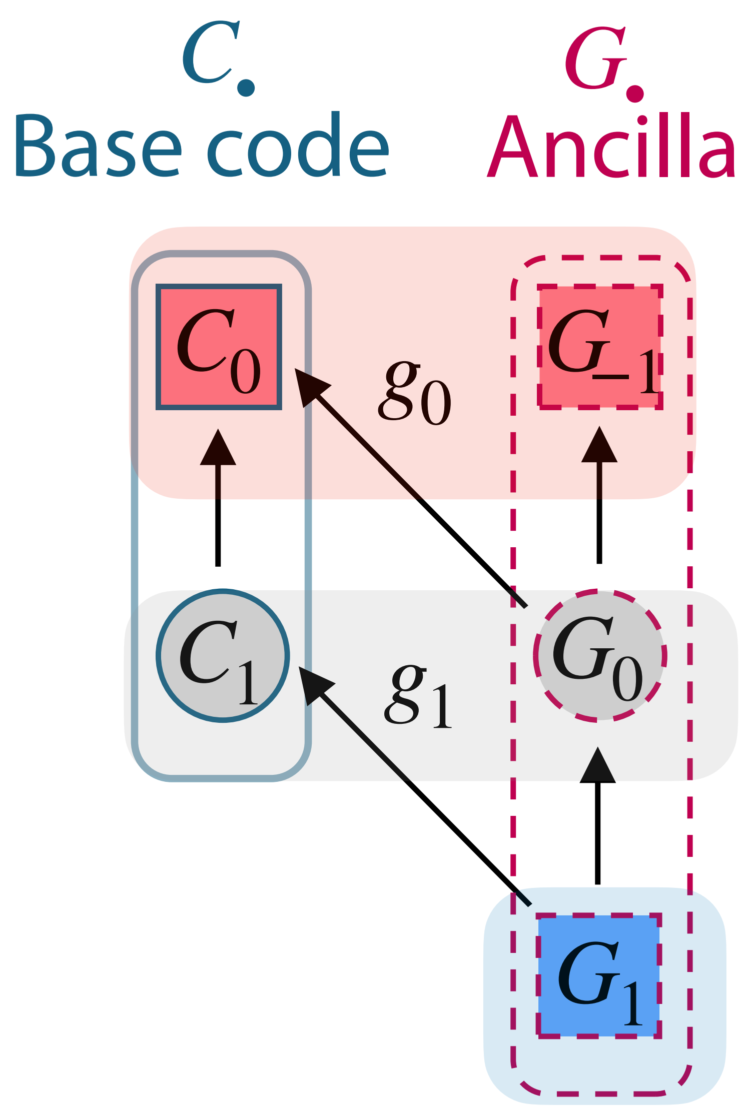

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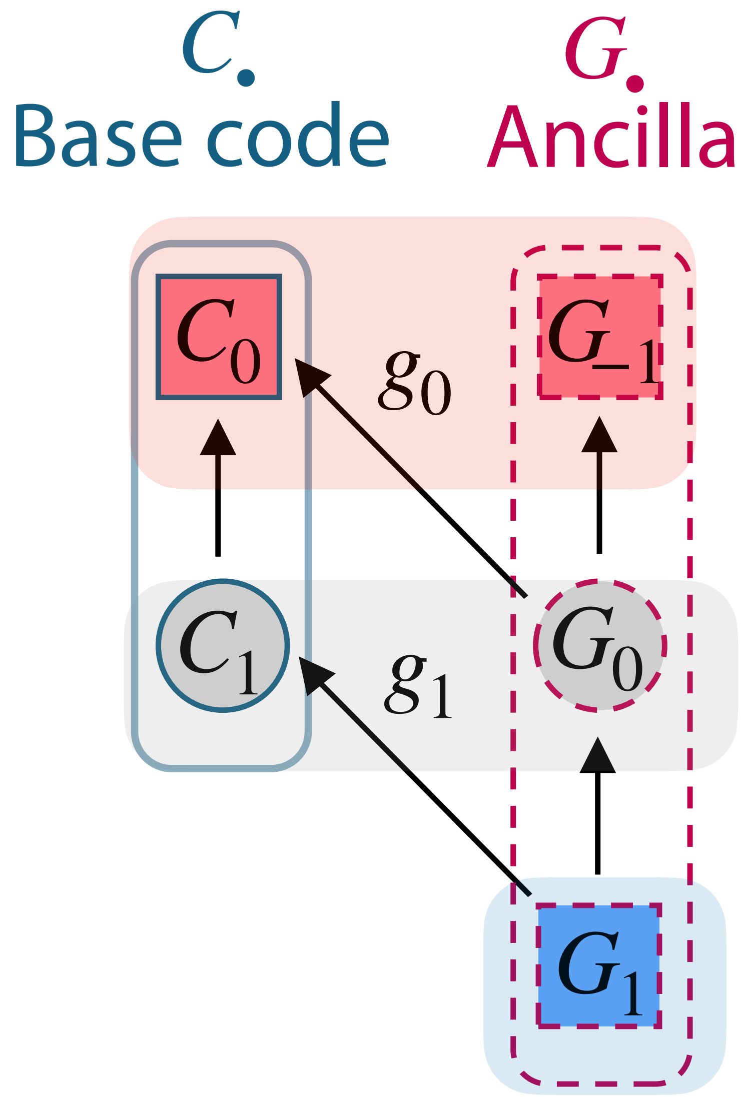

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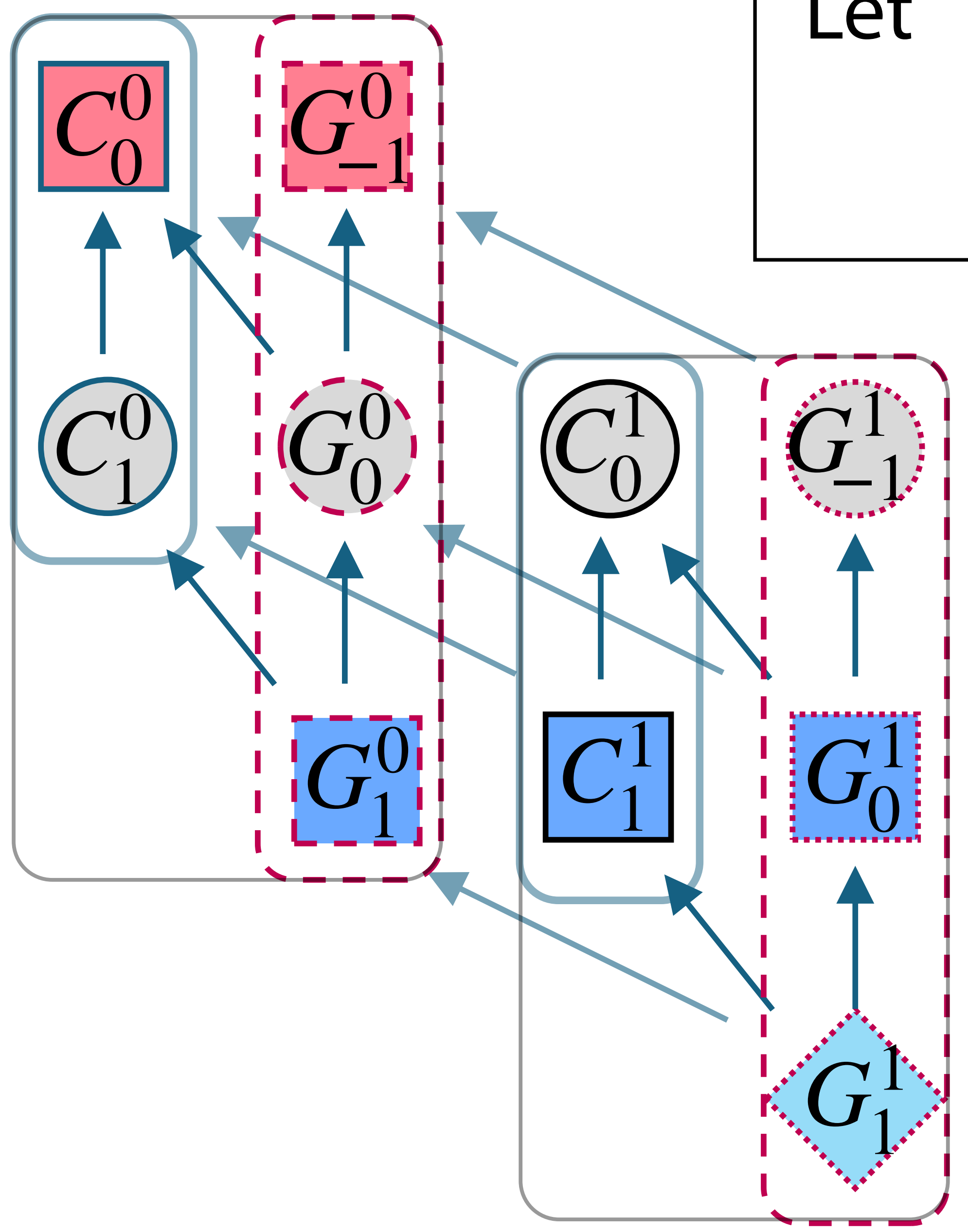

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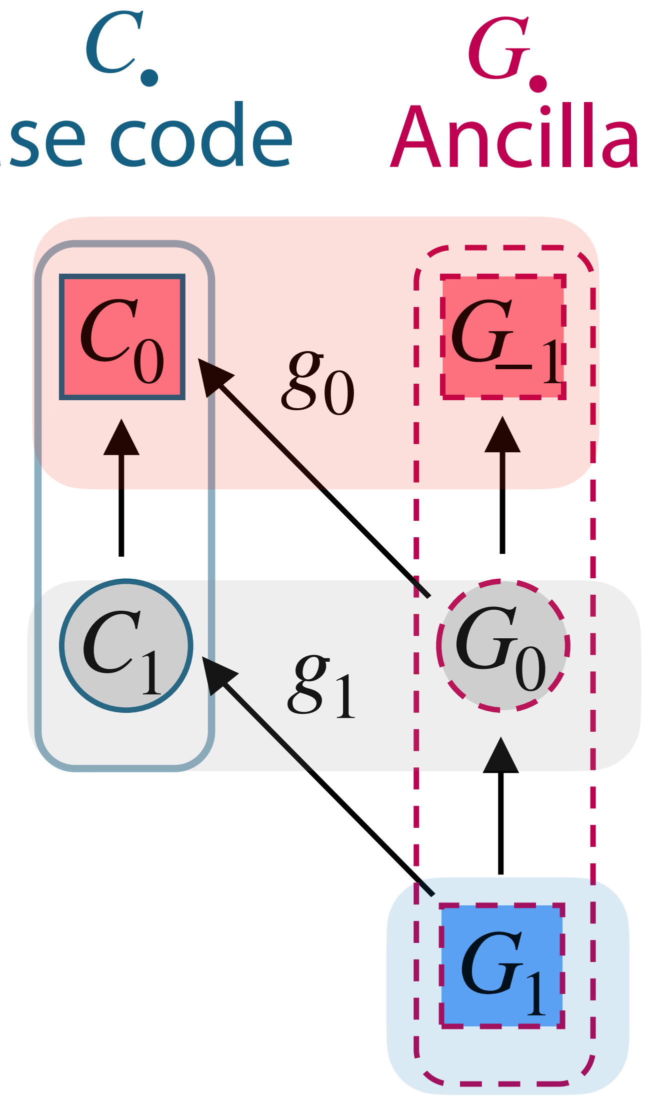

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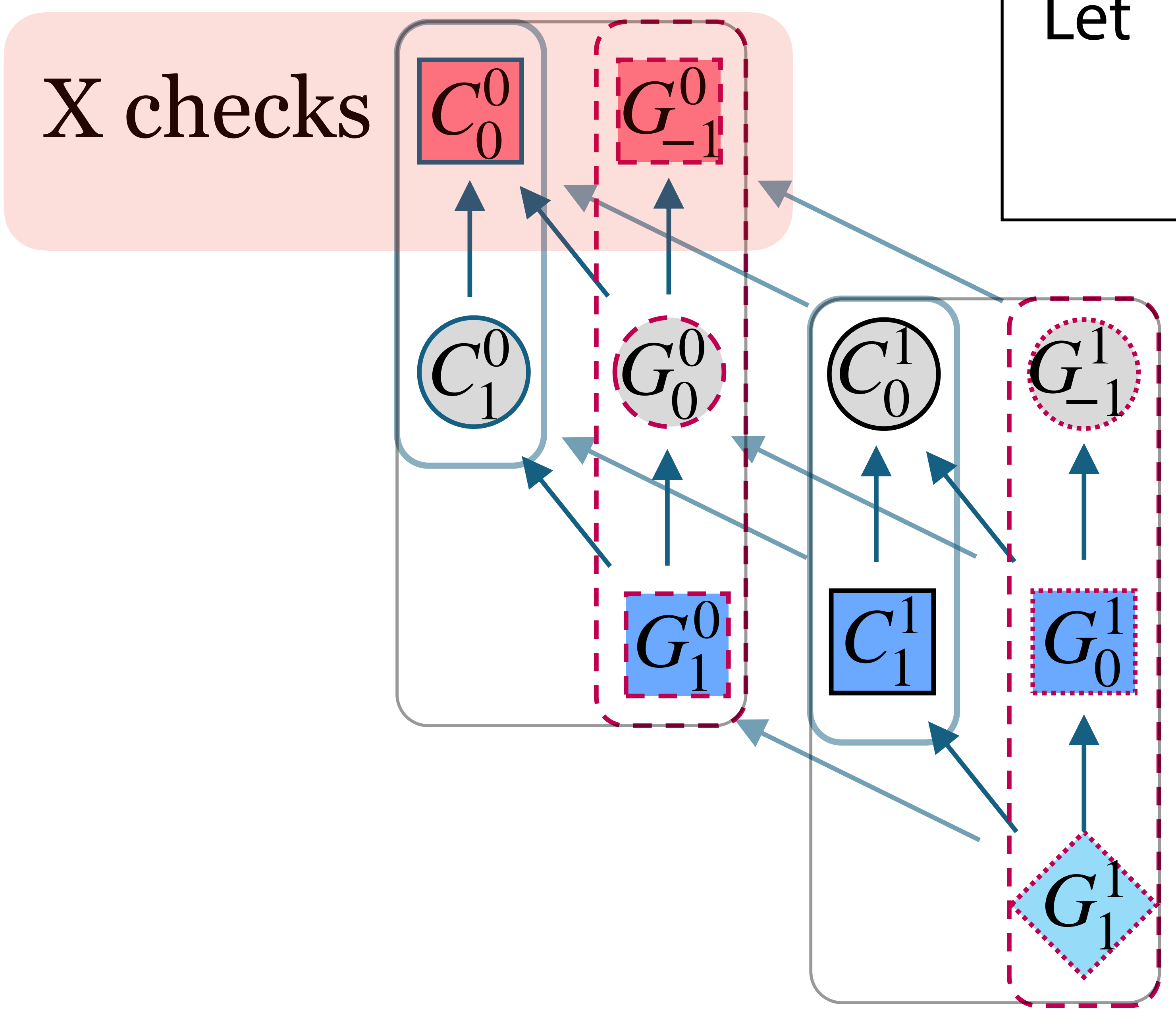

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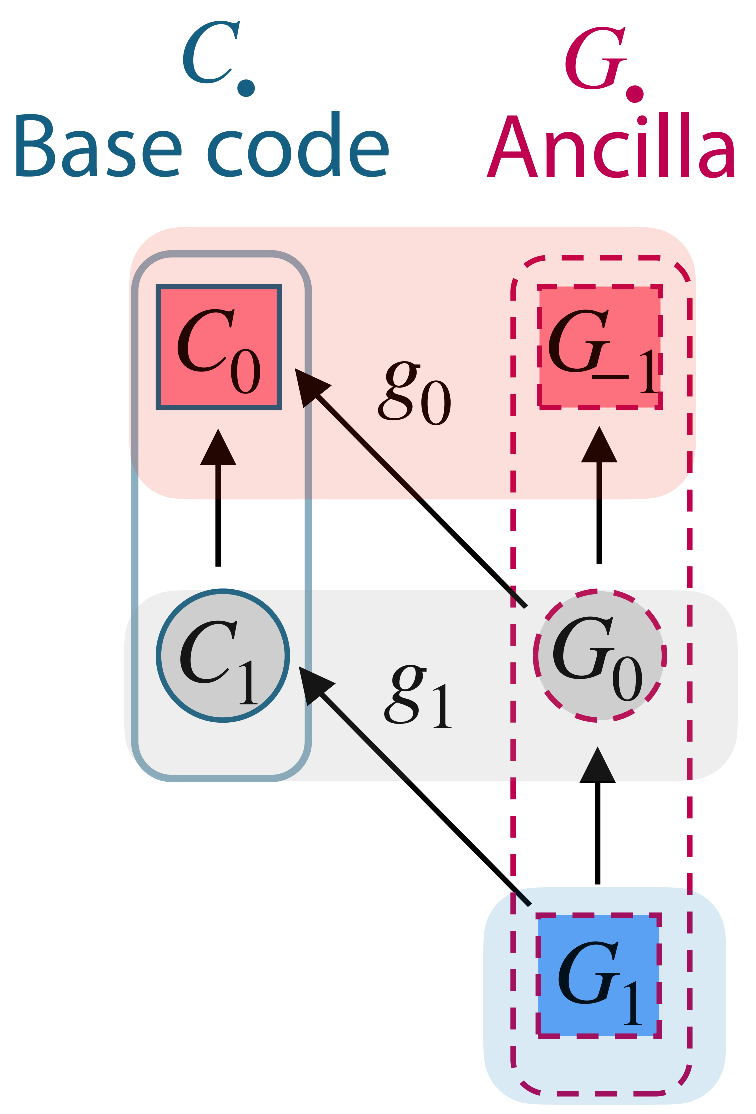

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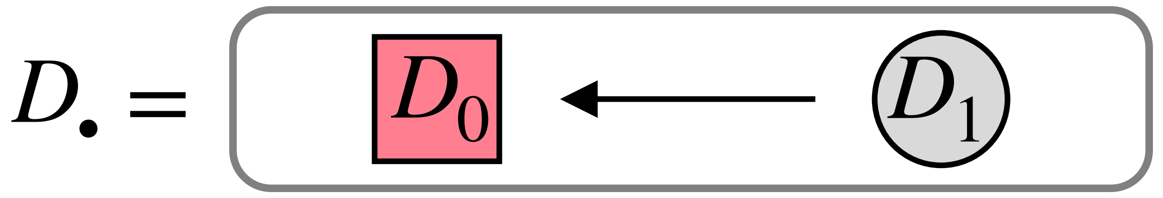

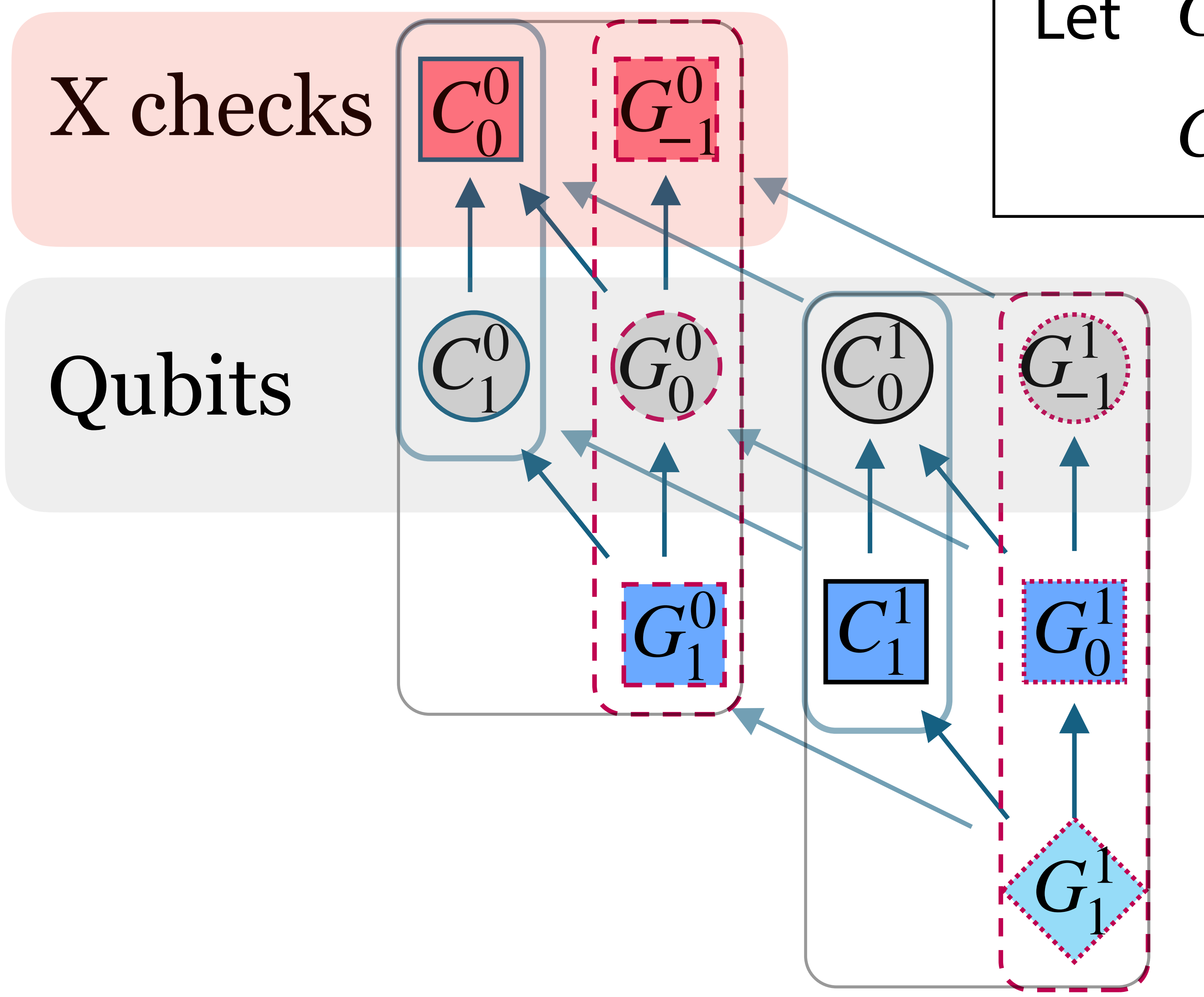

Let $G_i^j = G_i \otimes D_j$

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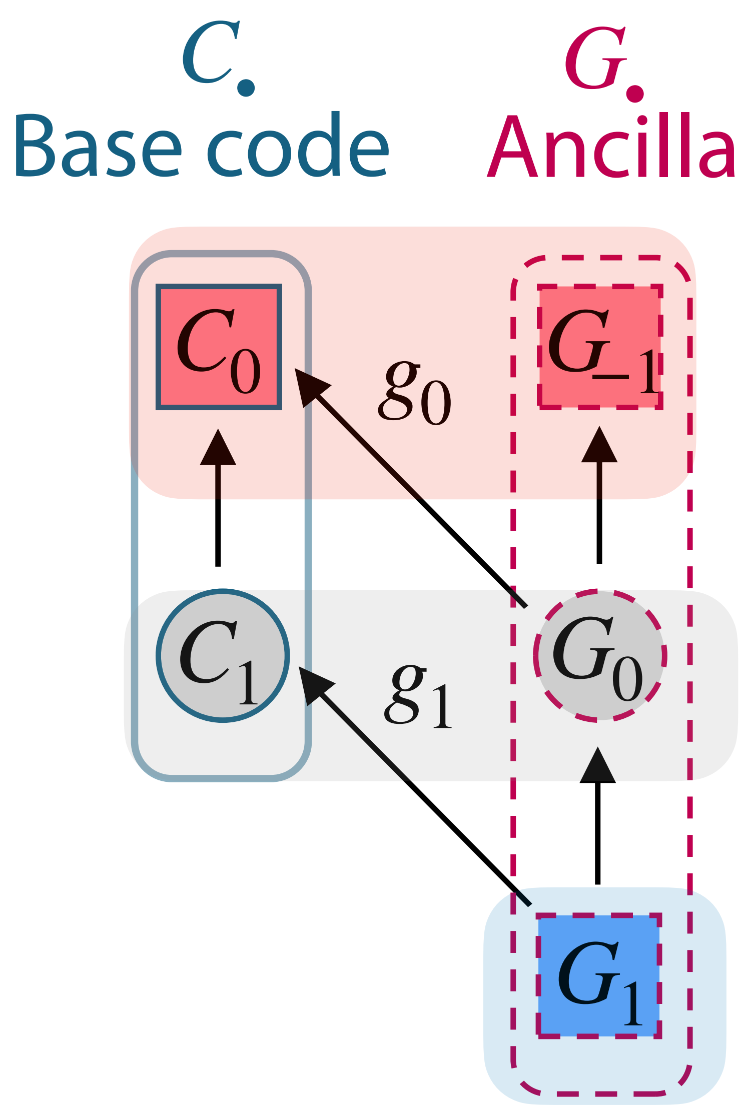
Constructing constant-time surgery gadgets

1. Apply surgery gadget on the classical code $C_\bullet$. The ancilla = $G_\bullet$
2. Take the tensor product with $D_\bullet$
3. Arrive at deformed code, "$(G + C) \otimes D$"

Surgery on classical code *(3-term chain complex)* $\otimes$ Classical code *(2-term chain complex)* $=$ Quantum code *(4-term. chain complex)*



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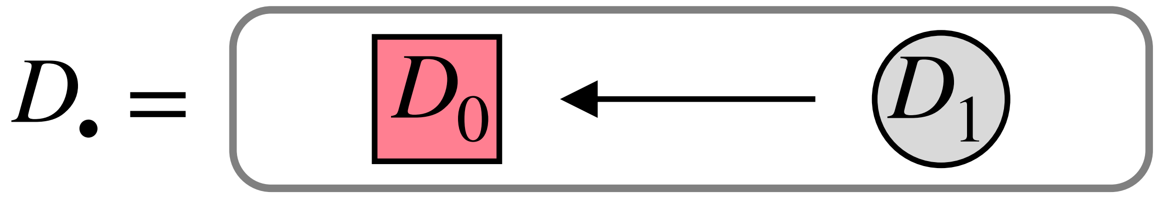

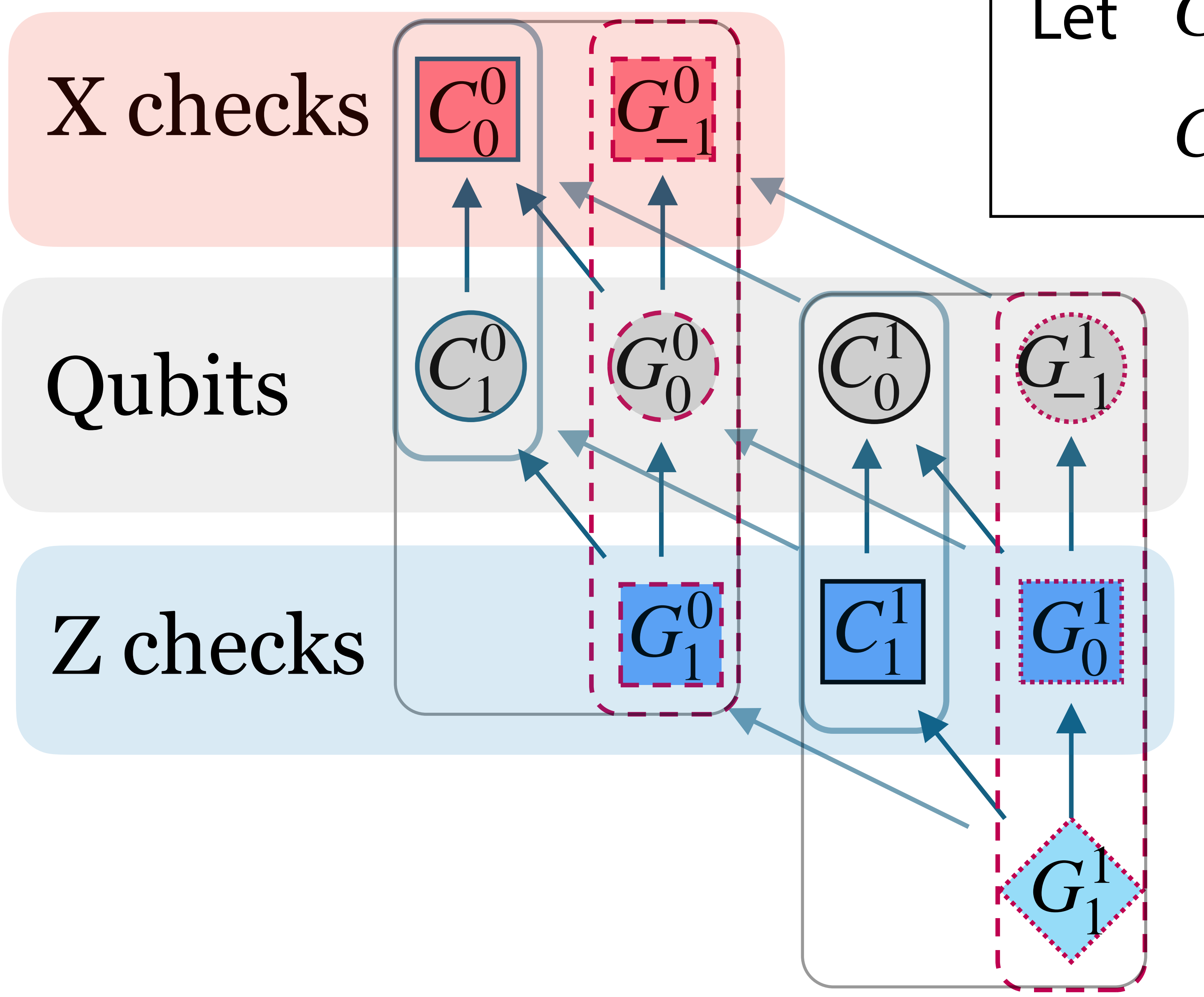

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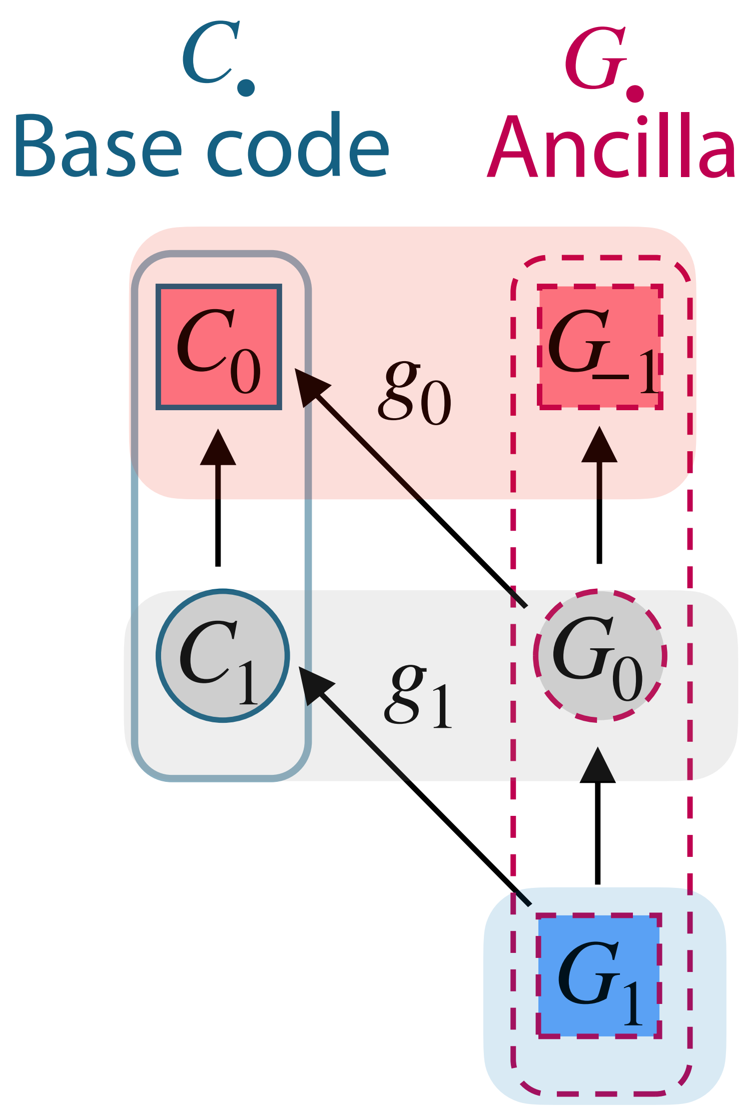
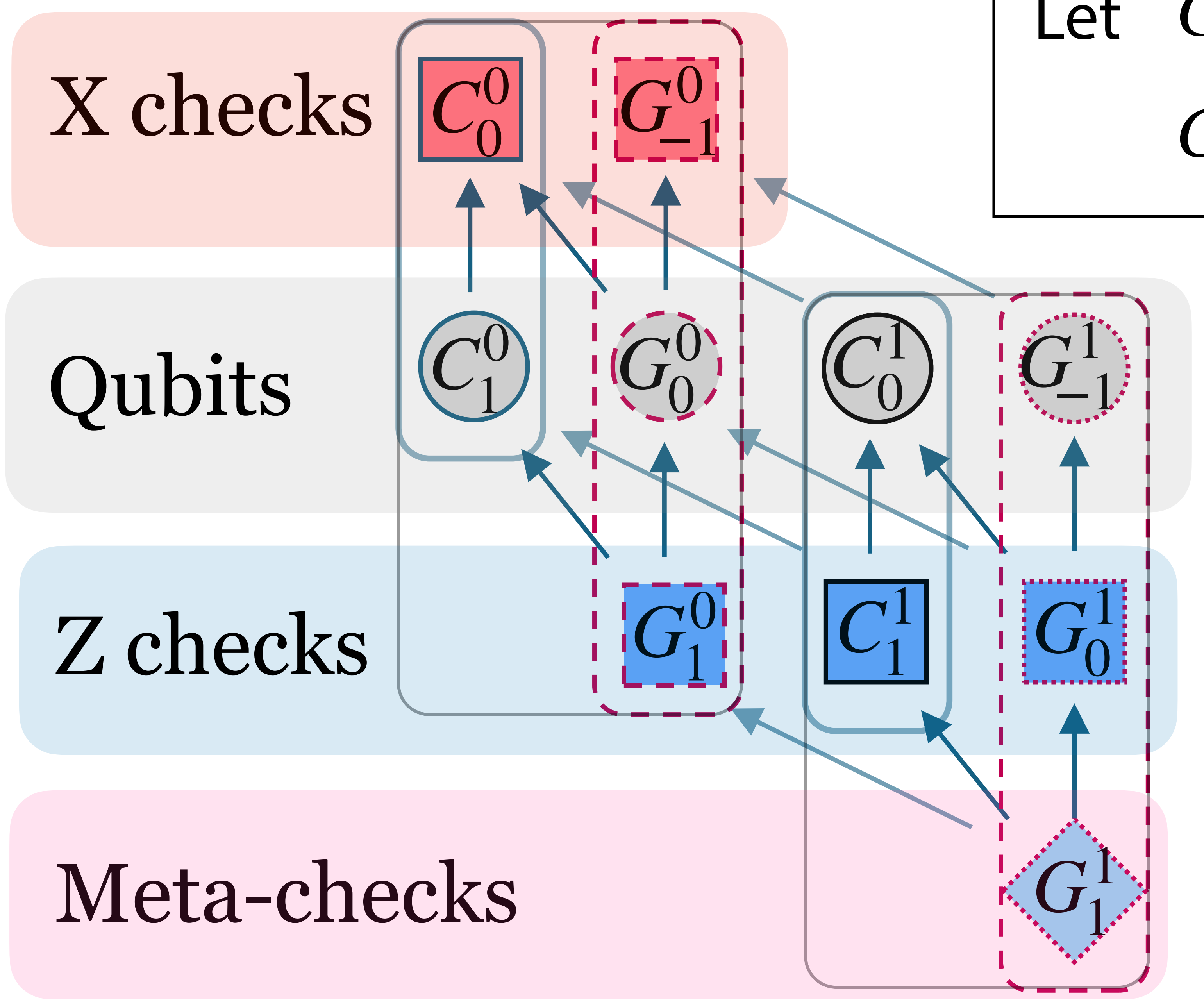


$\otimes$

$$D_\bullet = \boxed{D_0 \longleftarrow D_1}$$



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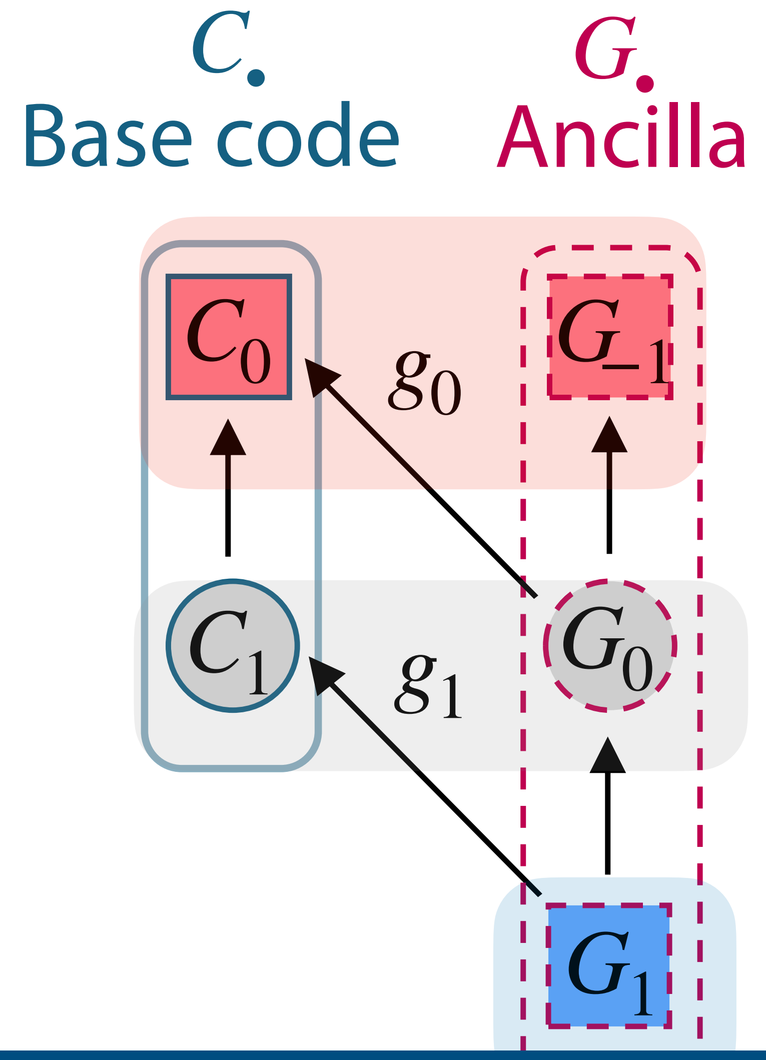

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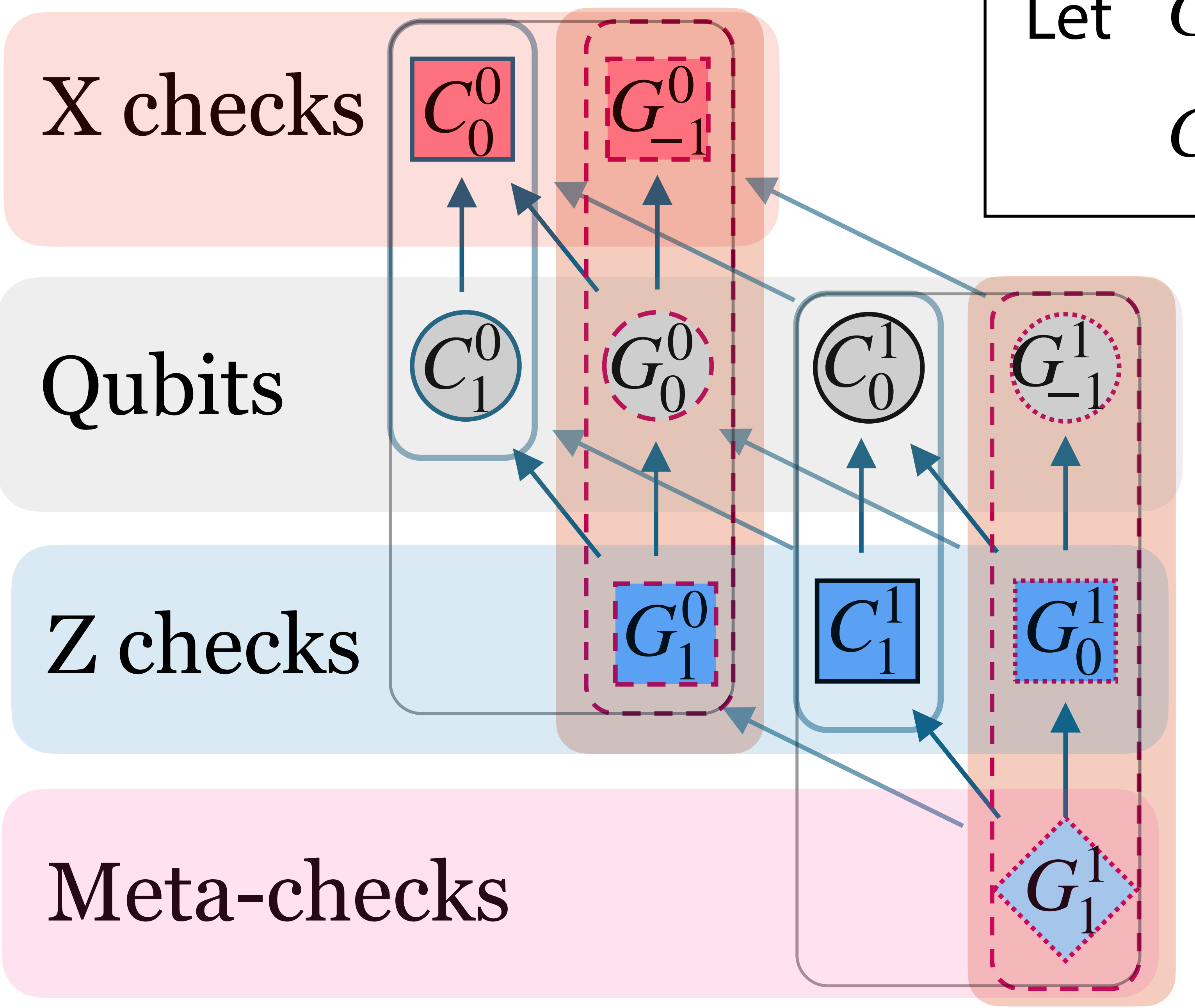

Let $G_i^j = G_i \otimes D_j$

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The ancilla complex $A_\bullet$ is $(G \otimes D)_\bullet$

The chain map is $f : (G \otimes D)_\bullet \xrightarrow{g \otimes \mathrm{Id}_D} (C \otimes D)_\bullet$

Ex. Constructing the deformed code

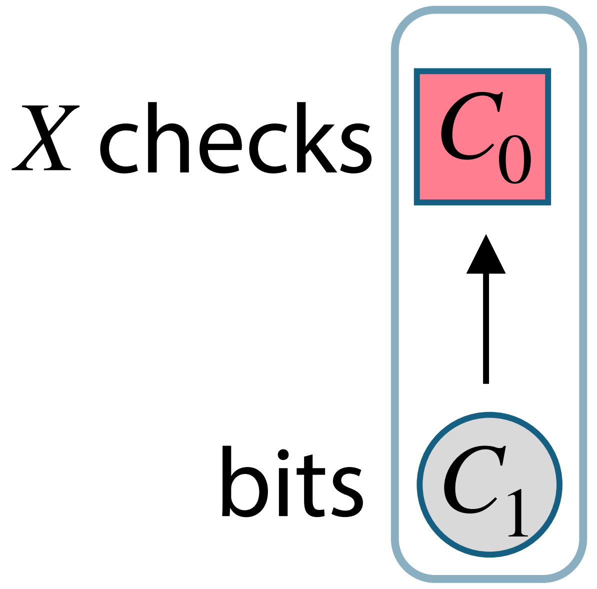
Ex. Constructing the deformed code

1. Take the base classical code C.

Ex. Constructing the deformed code

1. Take the base classical code $C_\bullet$

Chain complex



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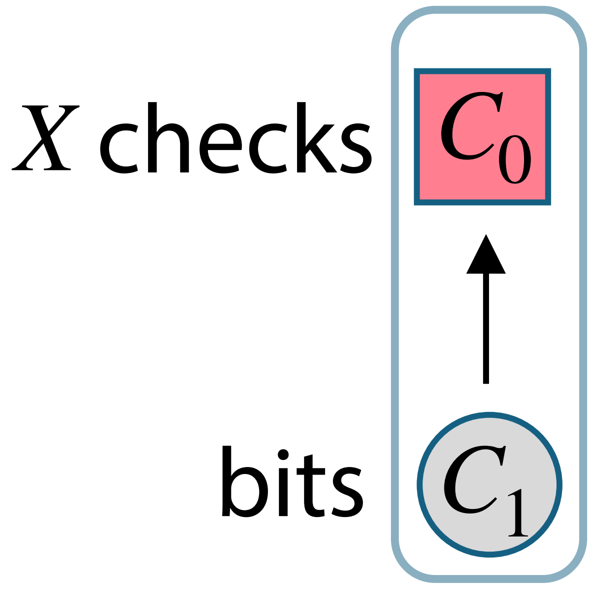

Tanner graph Ex.

[7,4,3]

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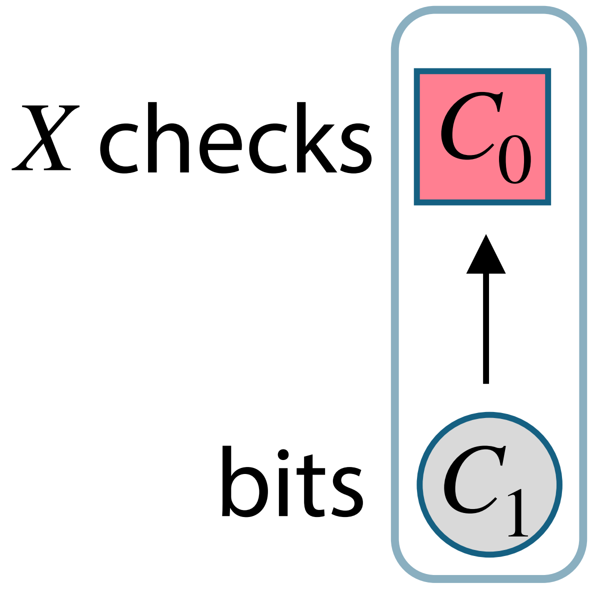

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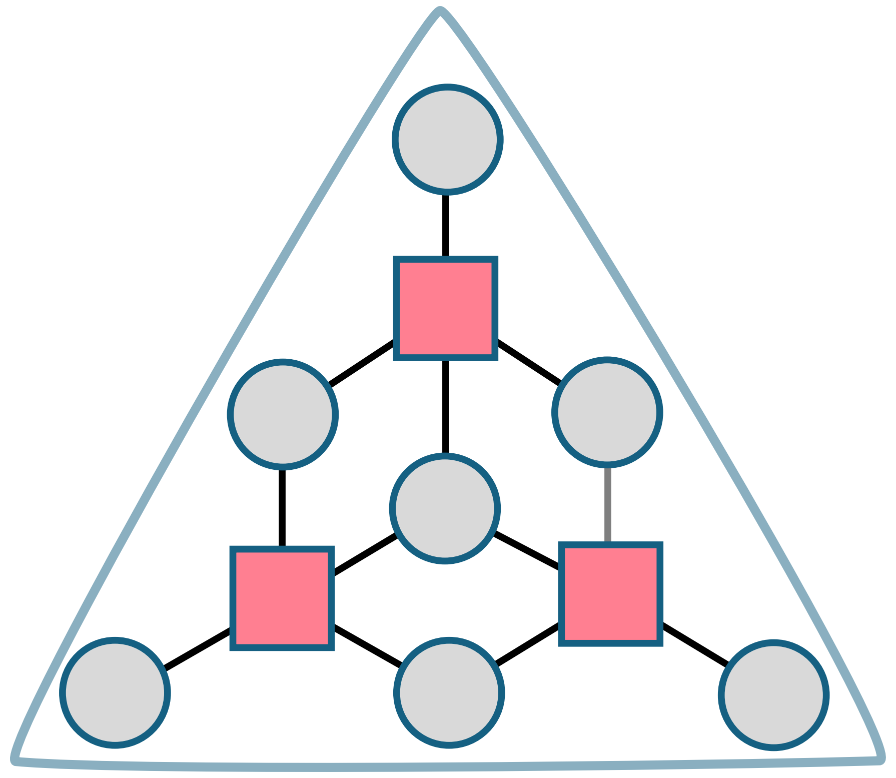

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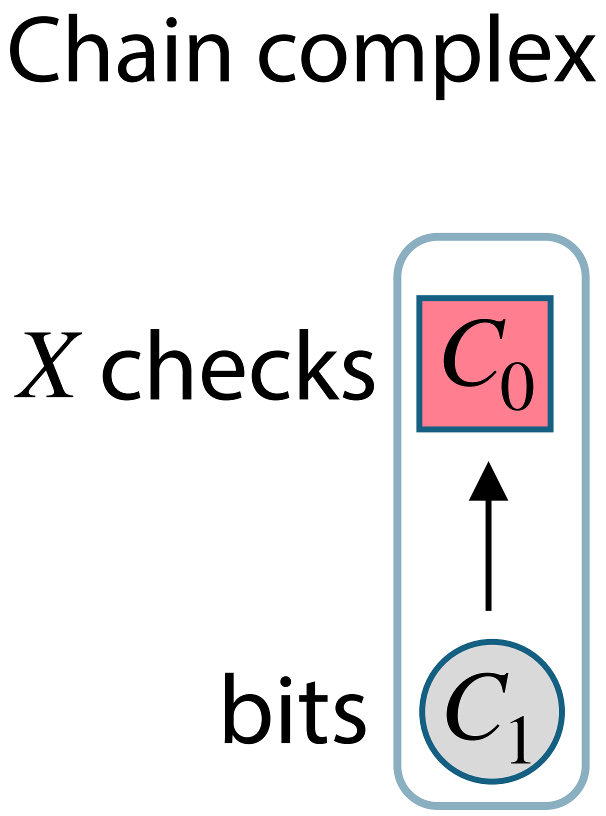

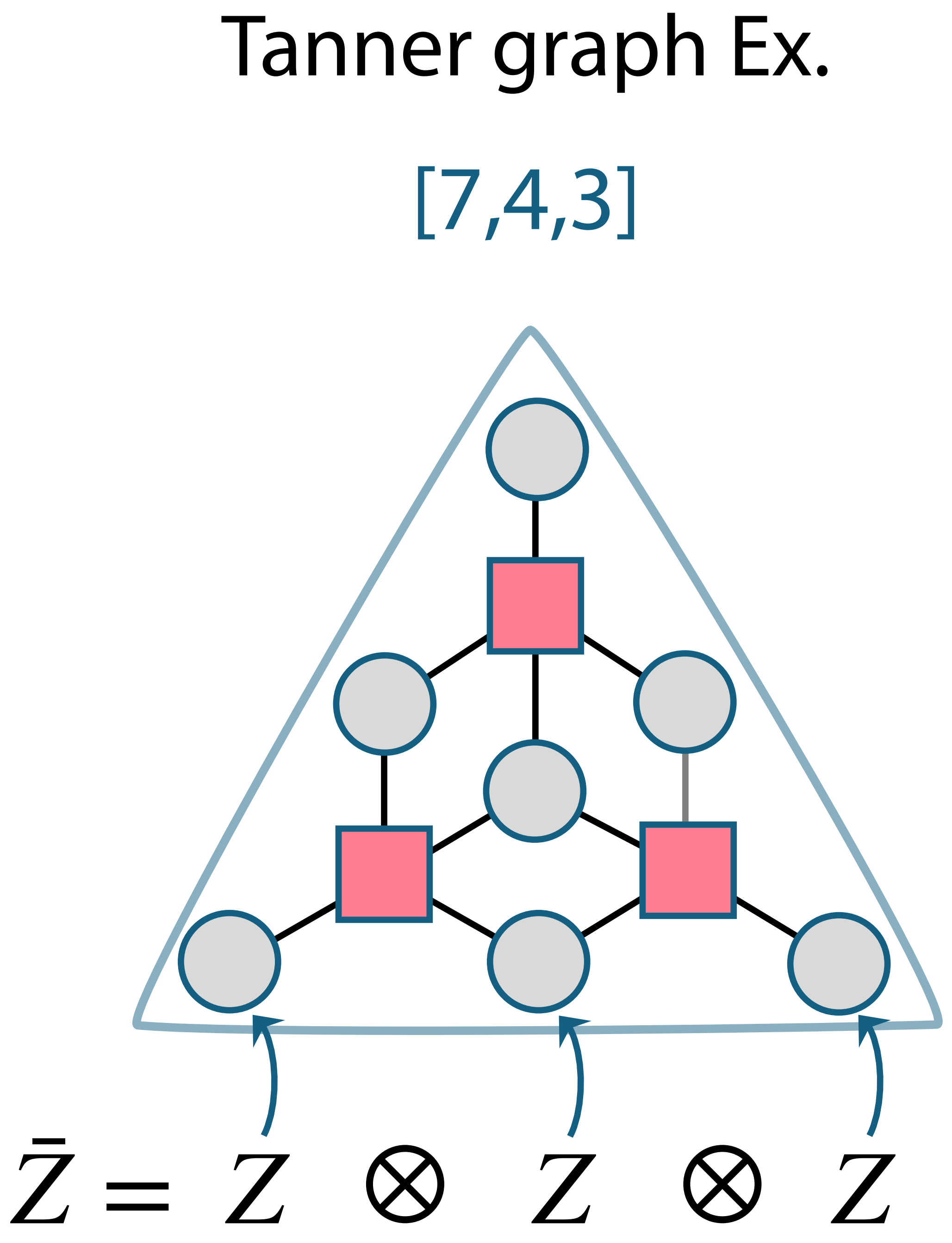

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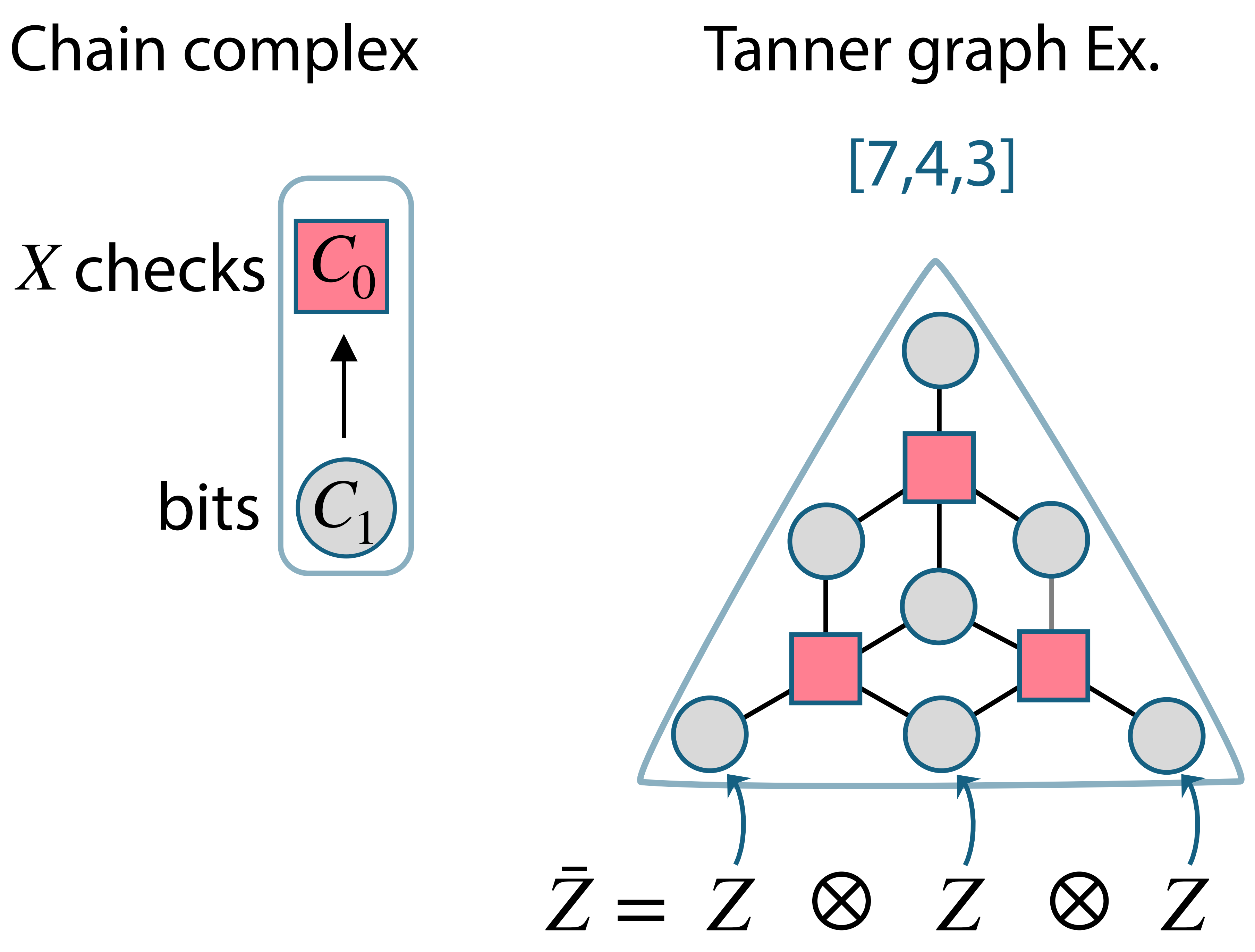

2. Augment the classical code with known surgery techniques[1,2], measure the classical codeword.

[1] Williamson, Yoder. [2410.02213] [2] Ide *et al.* PRX **15** (2025).

Ex. Constructing the deformed code

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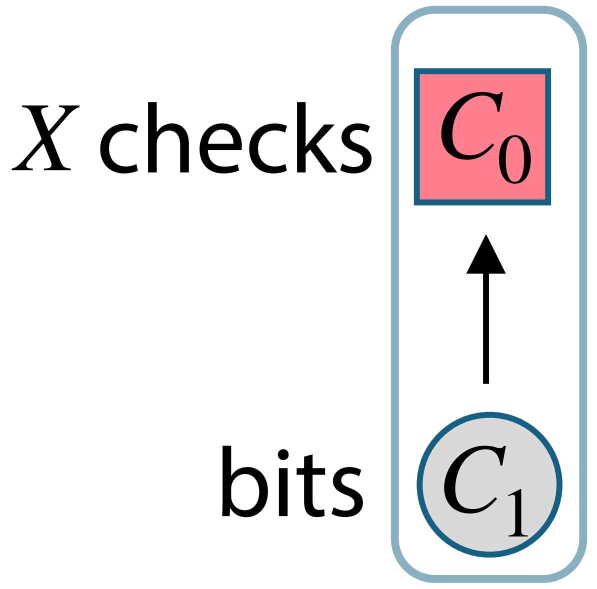

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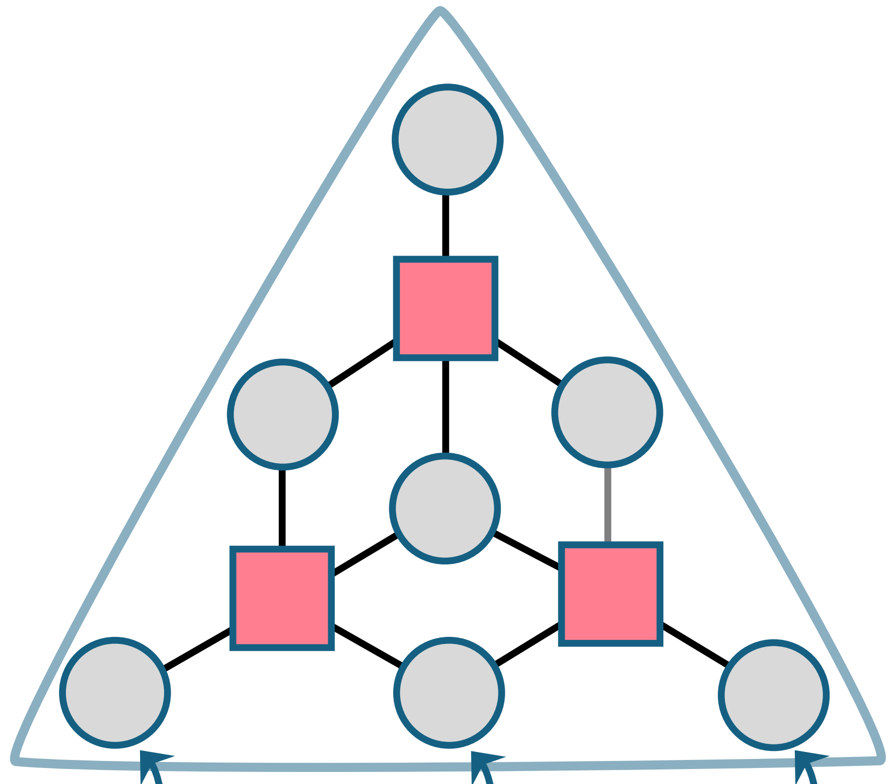

$$\bar{Z} = Z \otimes Z \otimes Z$$

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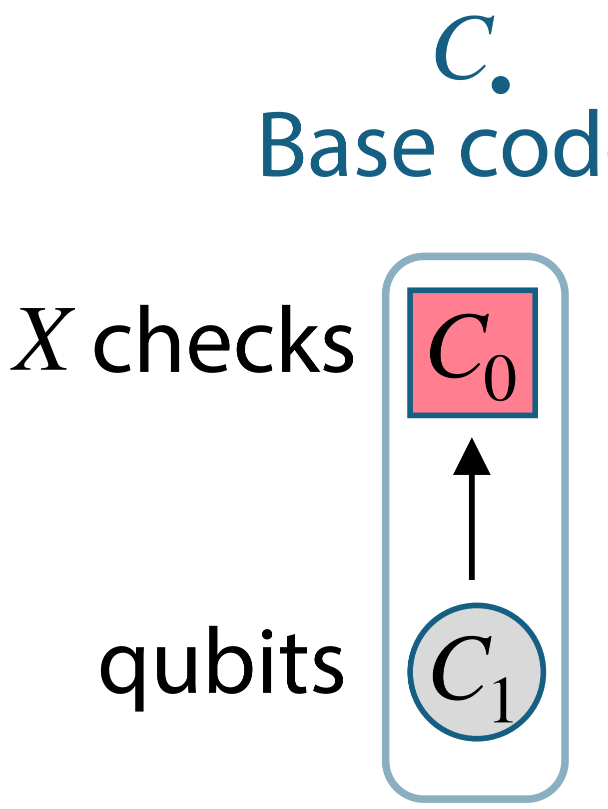
Chain complex



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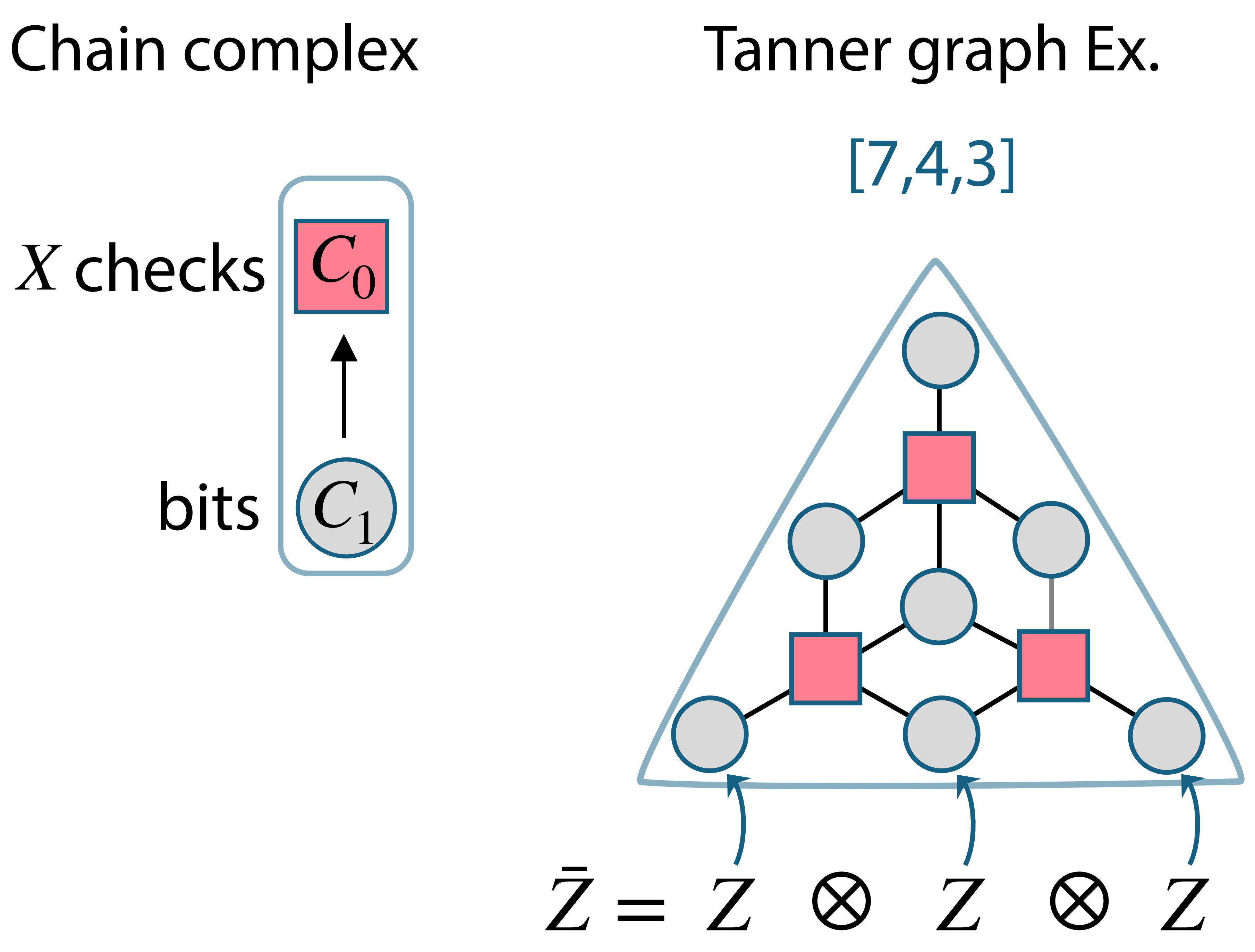

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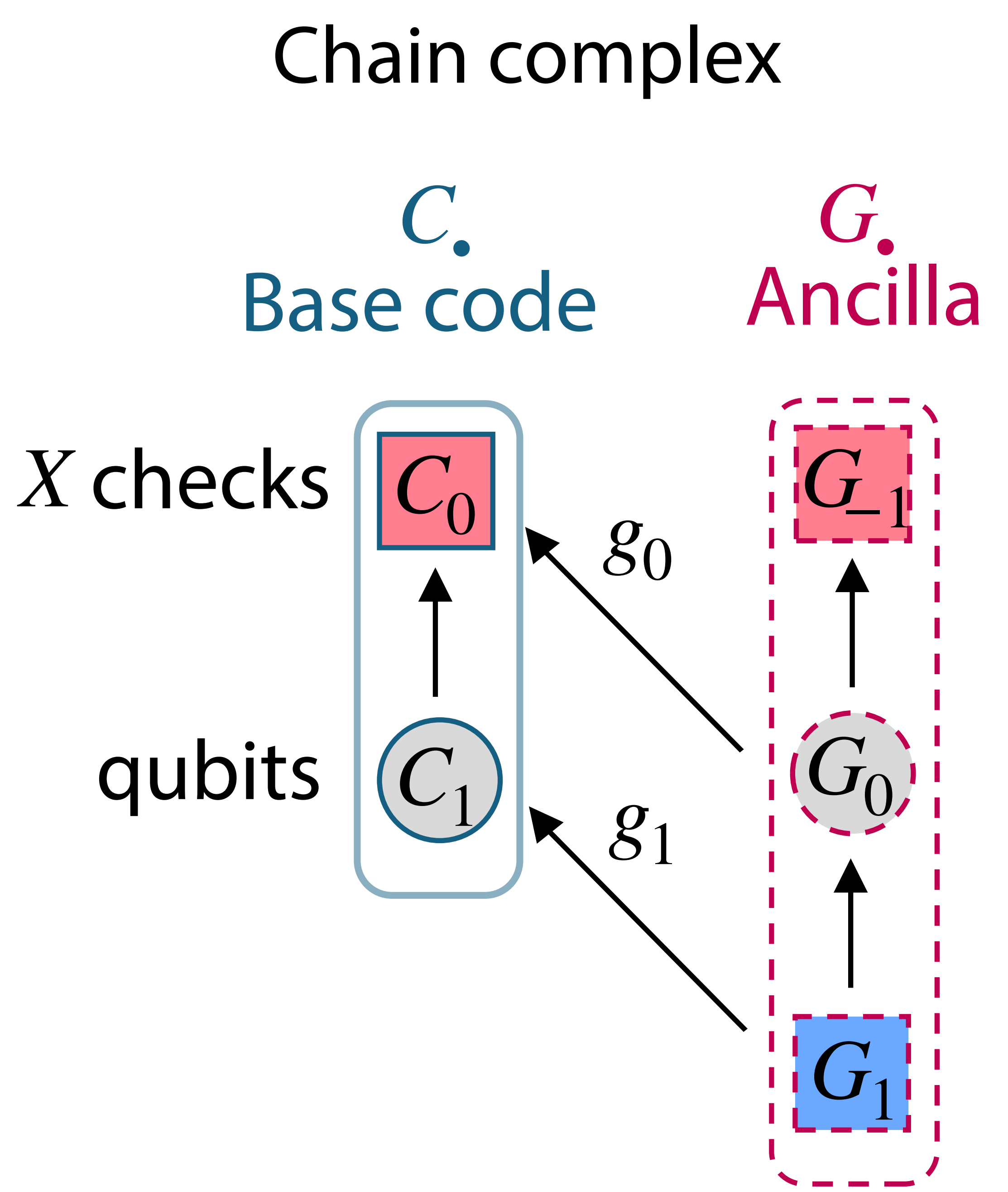

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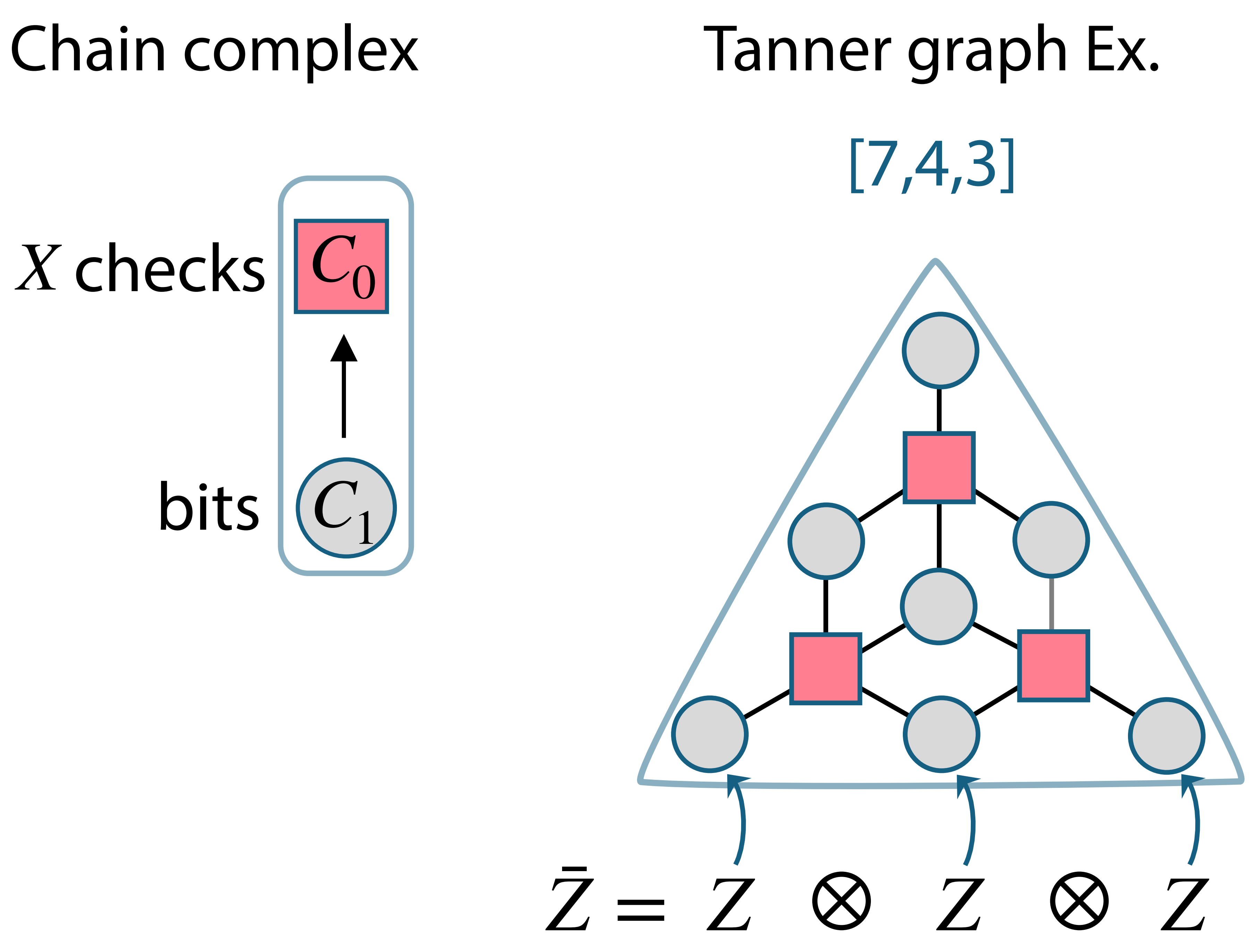

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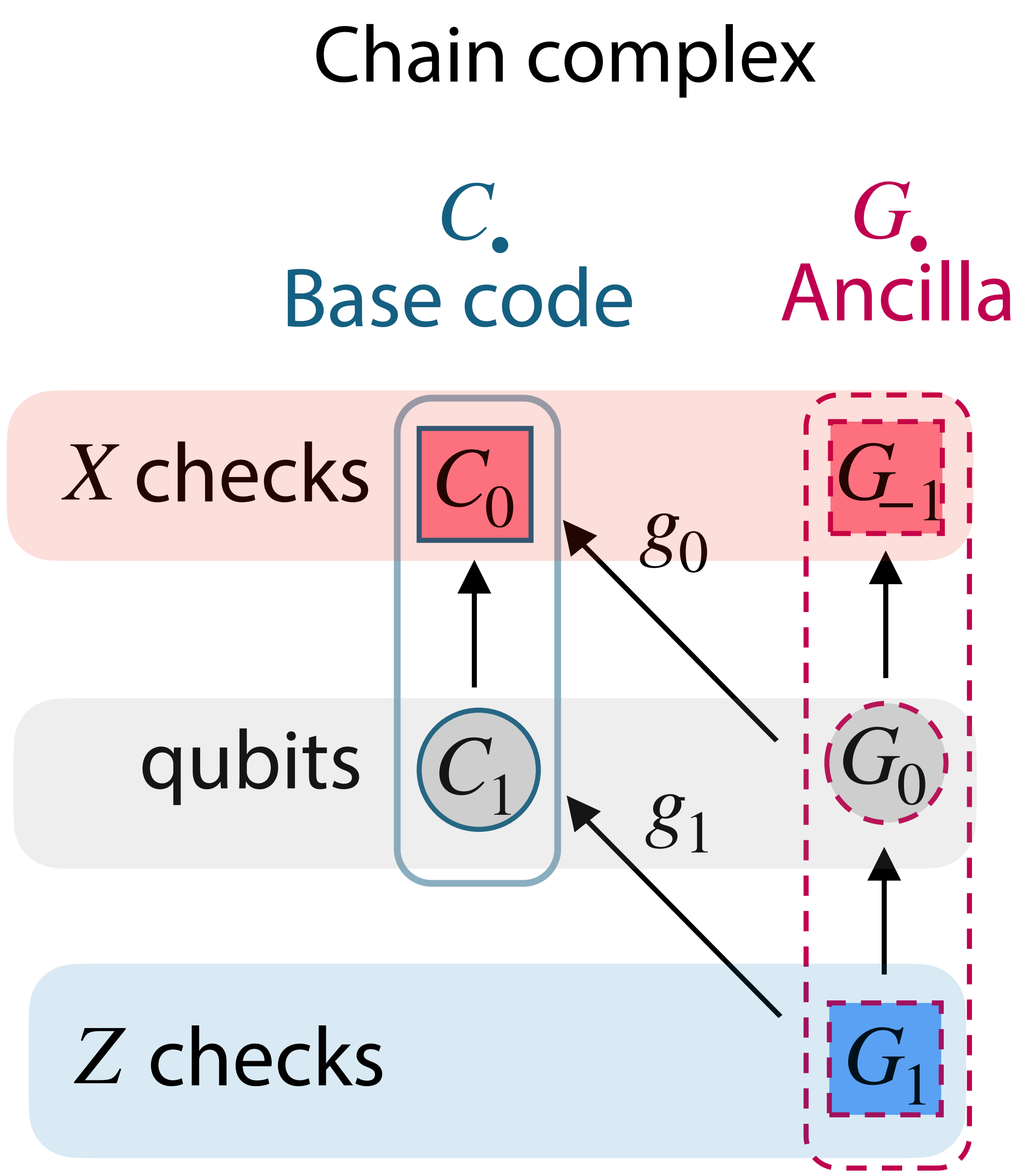

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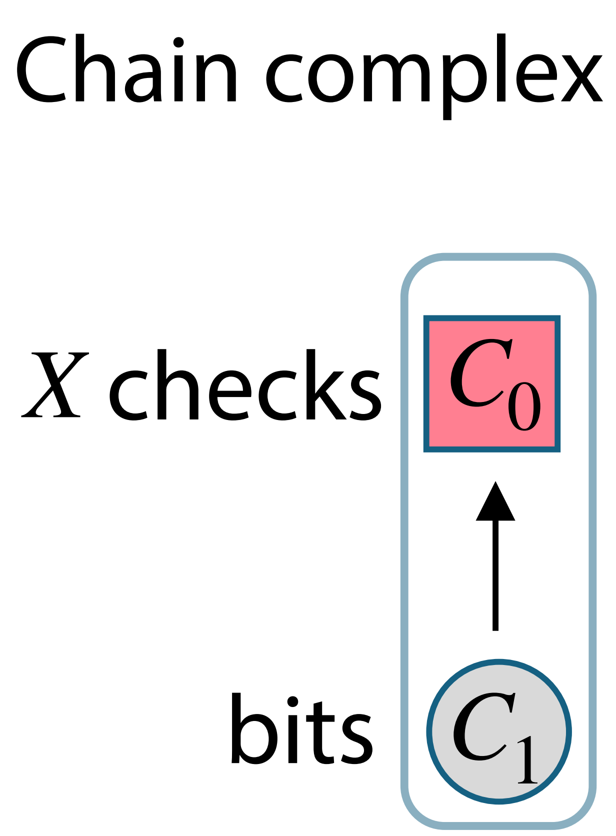

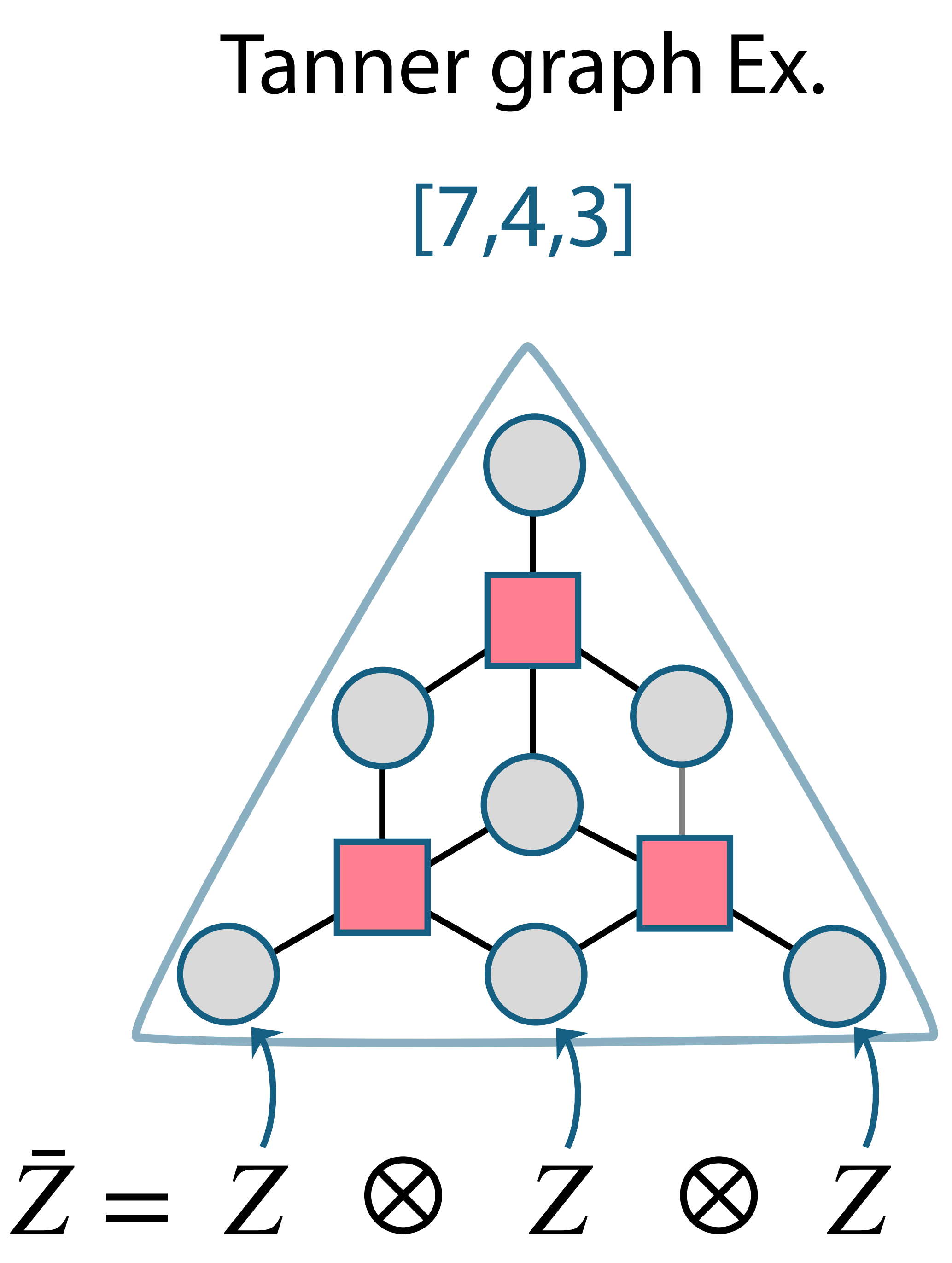

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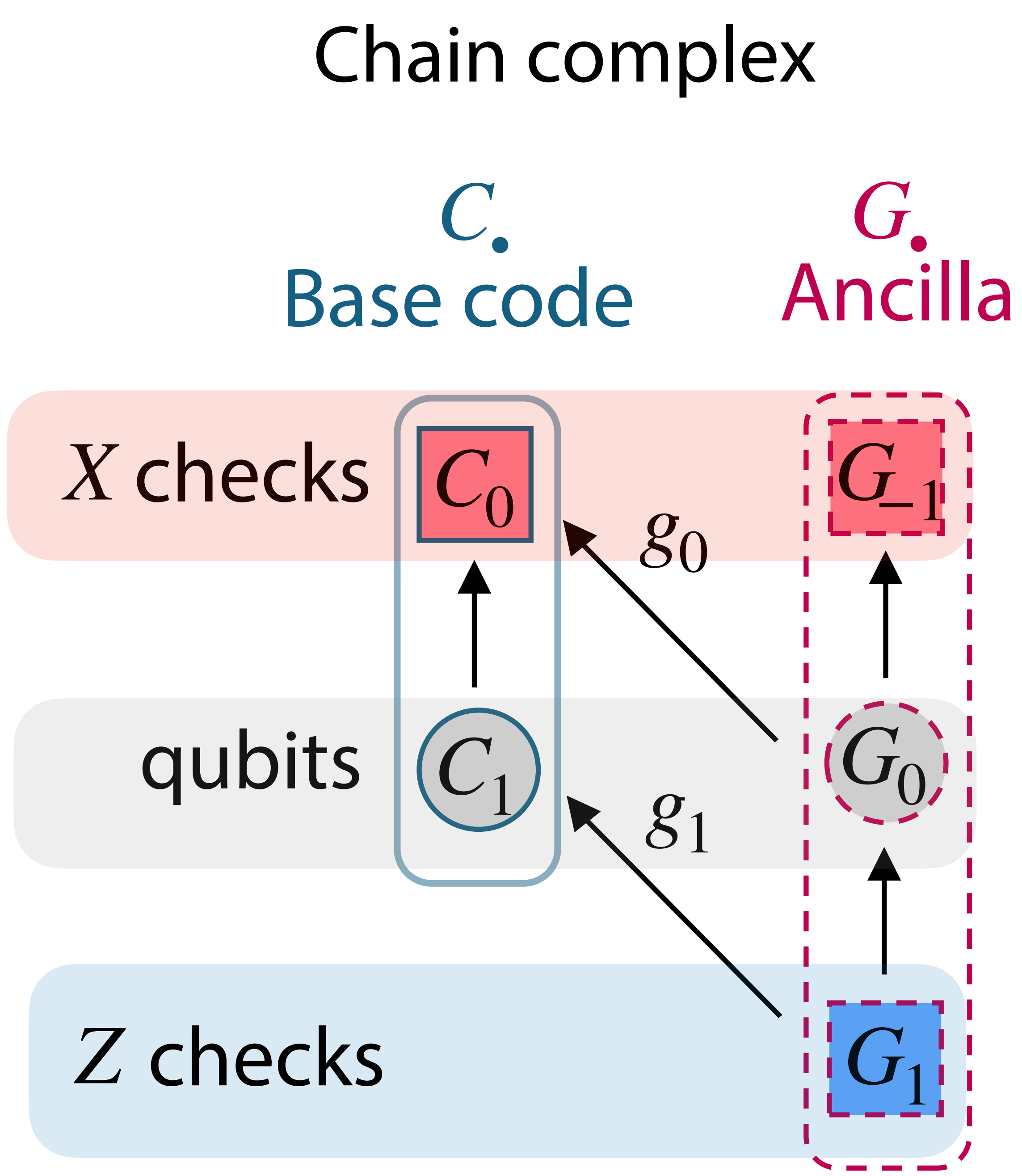

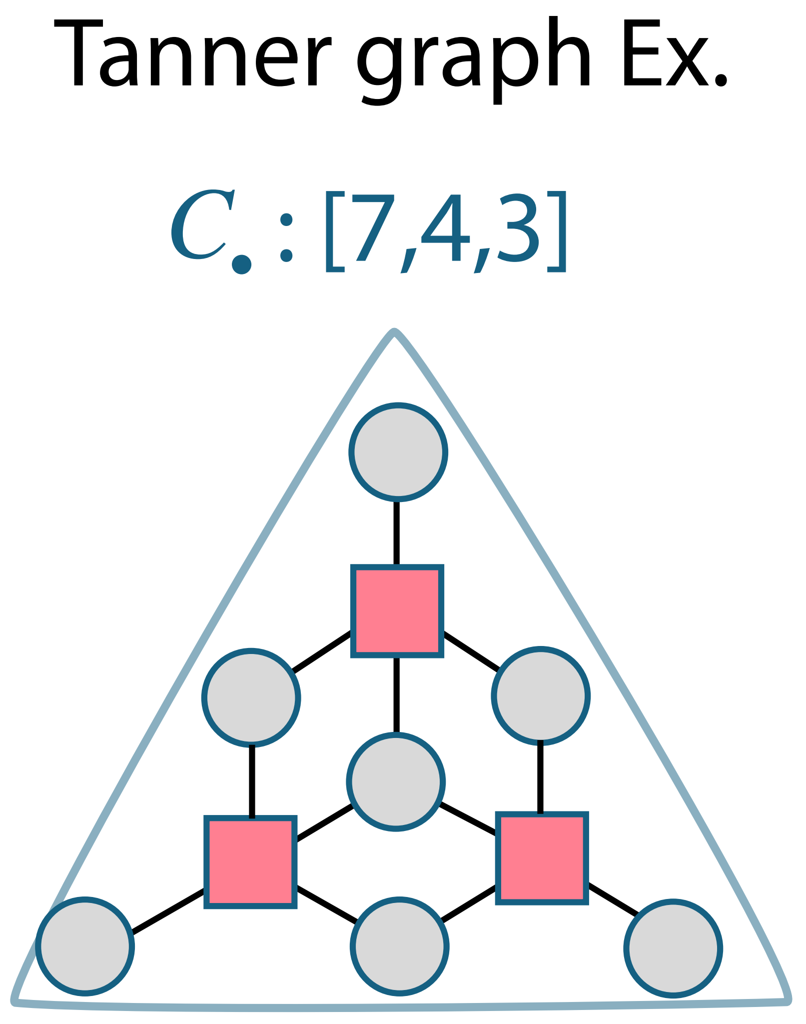

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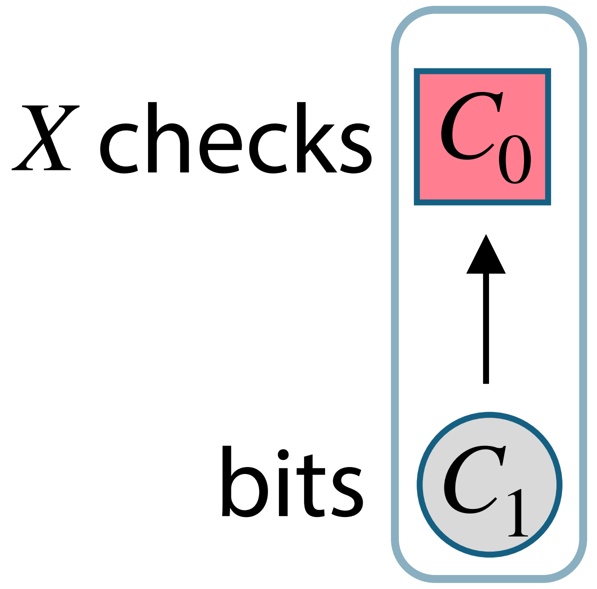
Ex. Constructing the deformed code

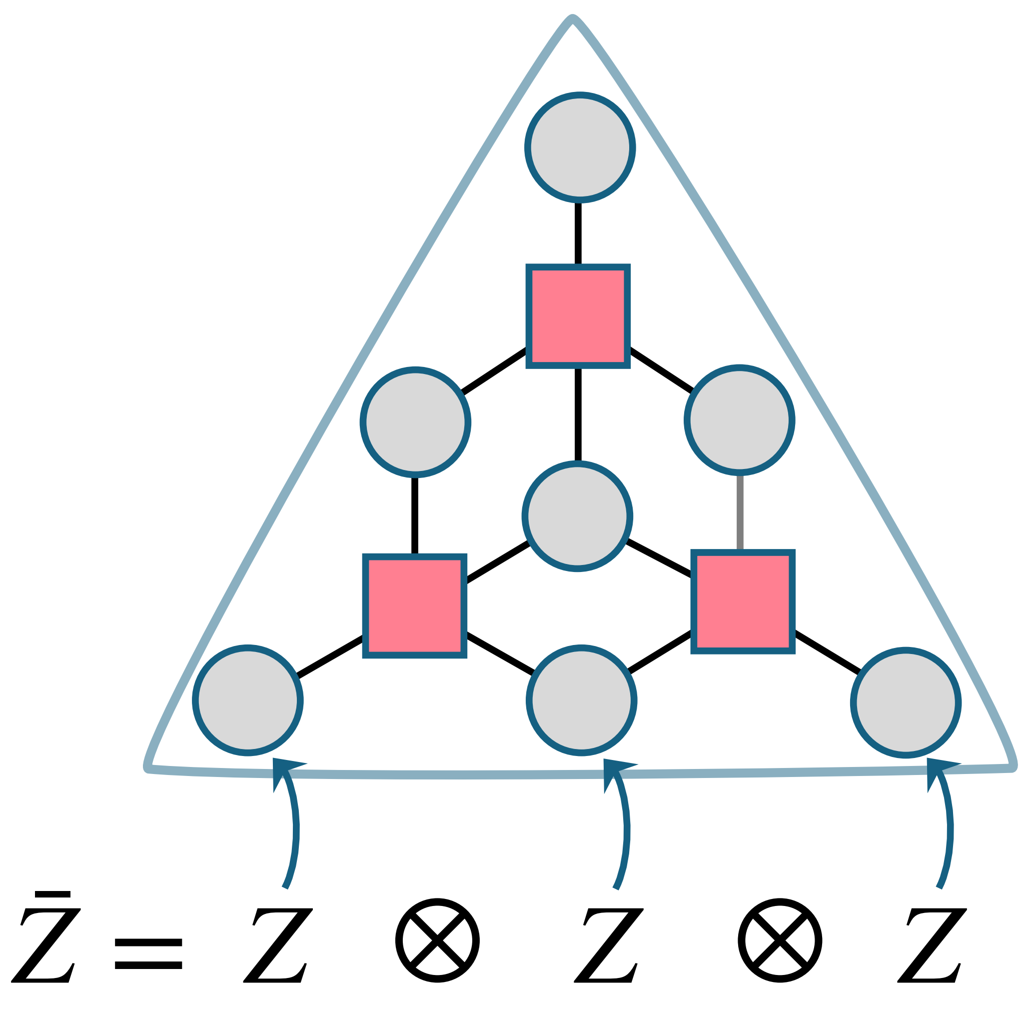
1. Take the base classical code $C_\bullet$

Chain complex



Tanner graph Ex.

[7,4,3]



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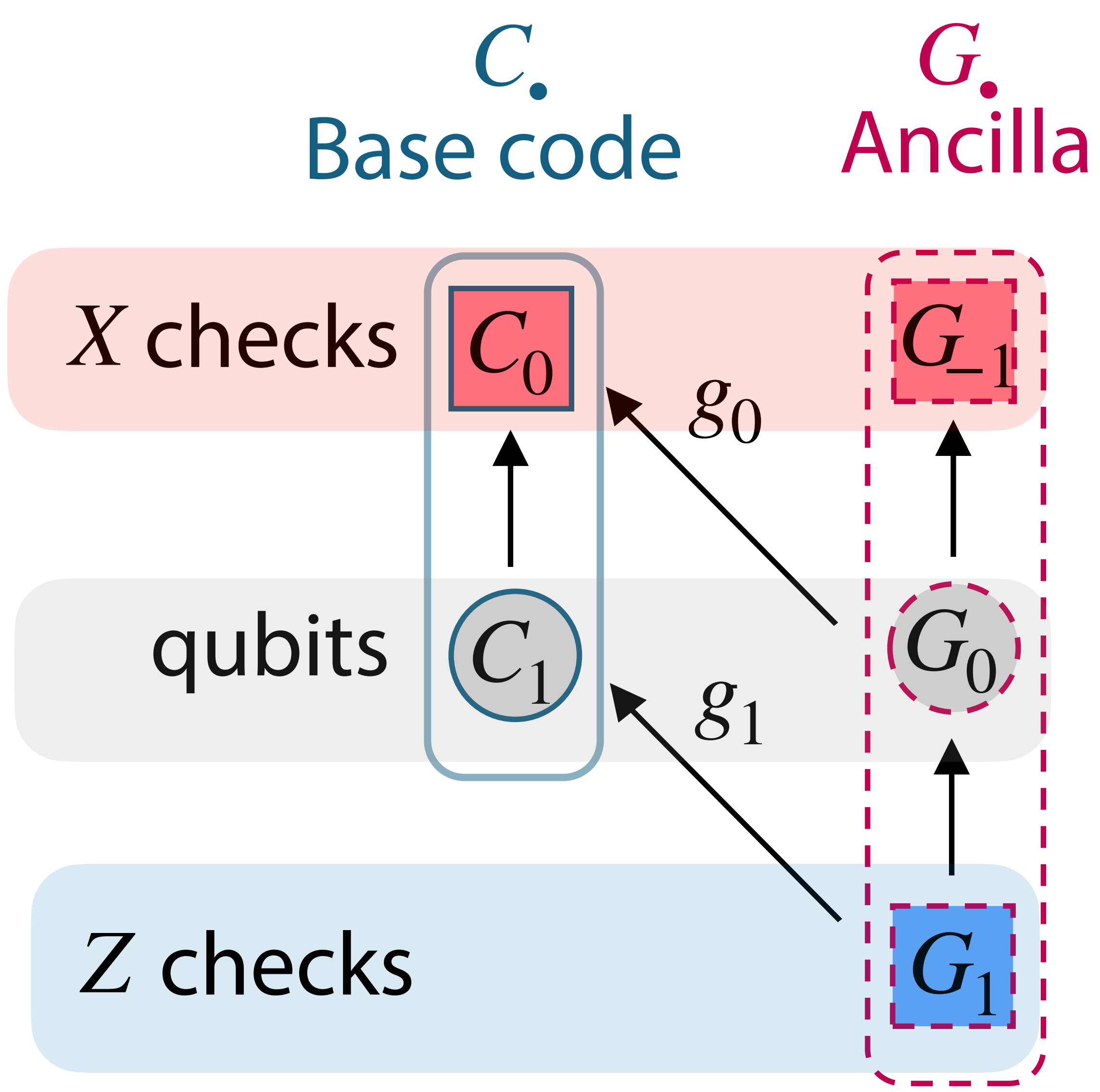
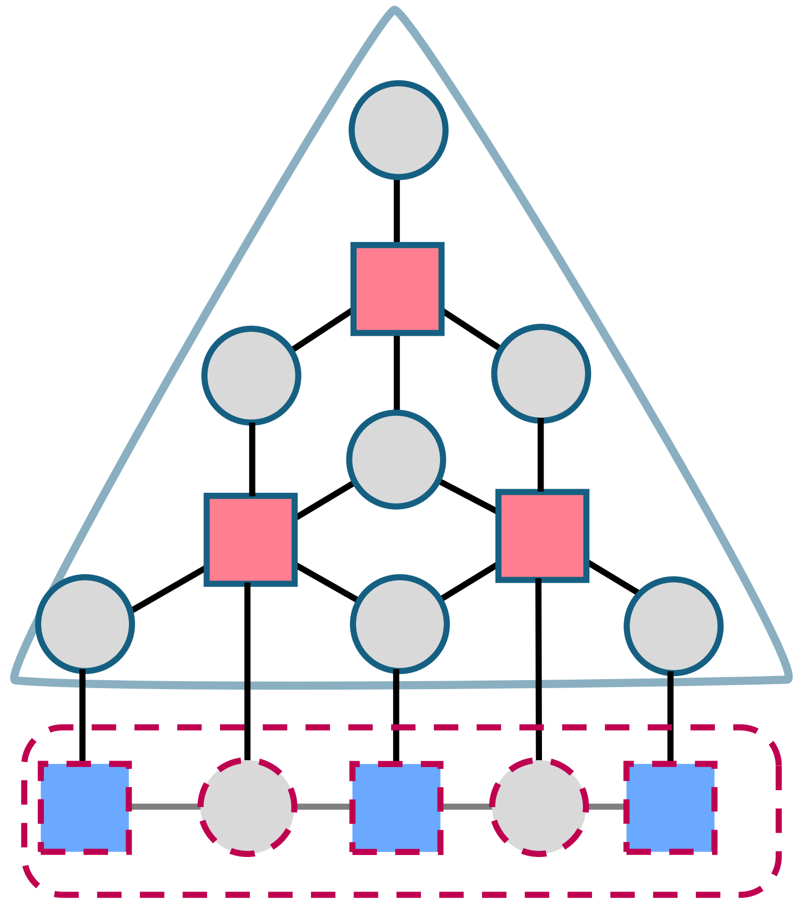


Tanner graph Ex.

$C_\bullet$: [7,4,3]



$G_\bullet$: $[3,1,3]^\top$

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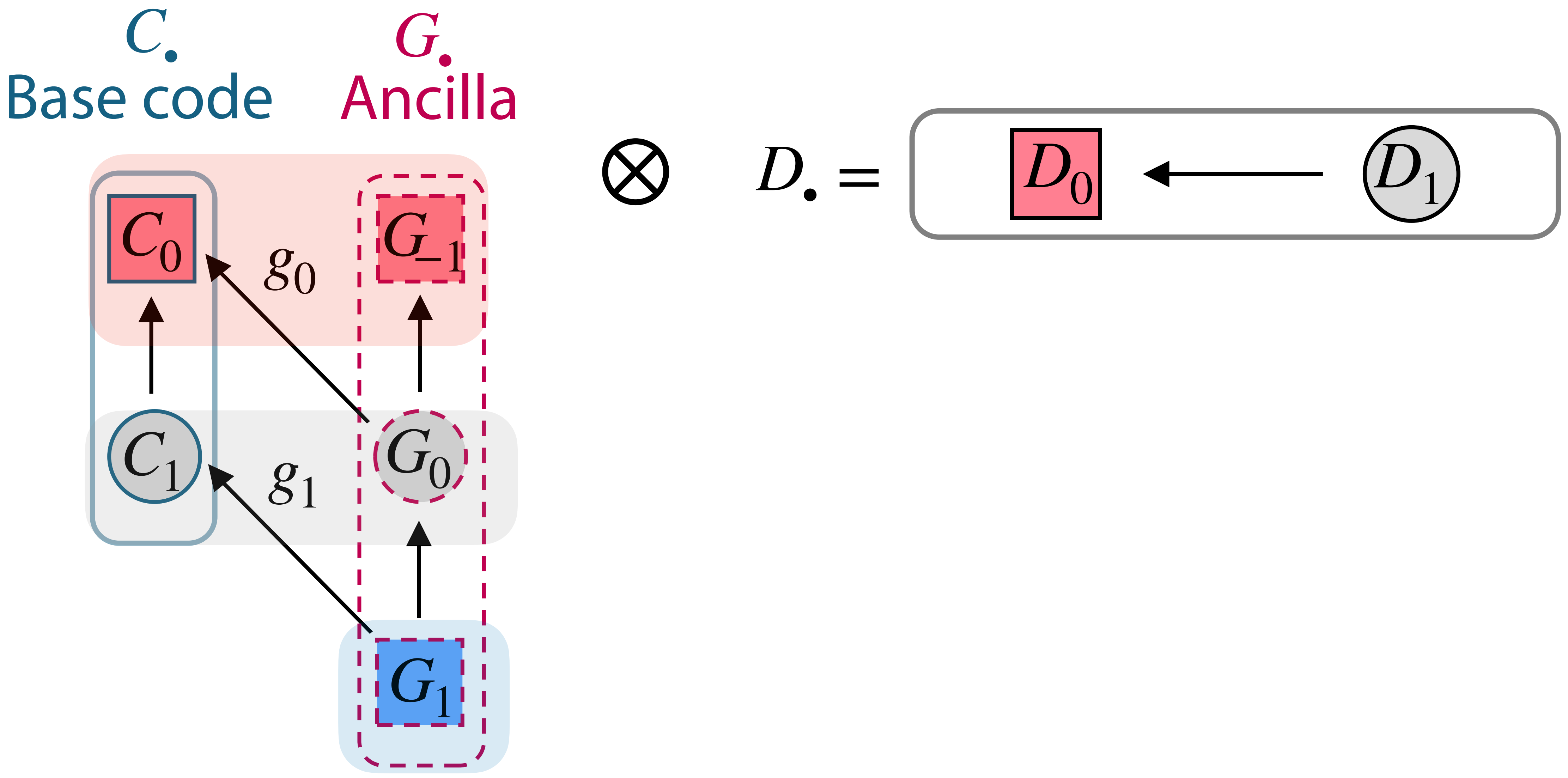
Ex. Constructing the deformed code

3. Take the hypergraph product with the other base classical code, $D_\bullet$.



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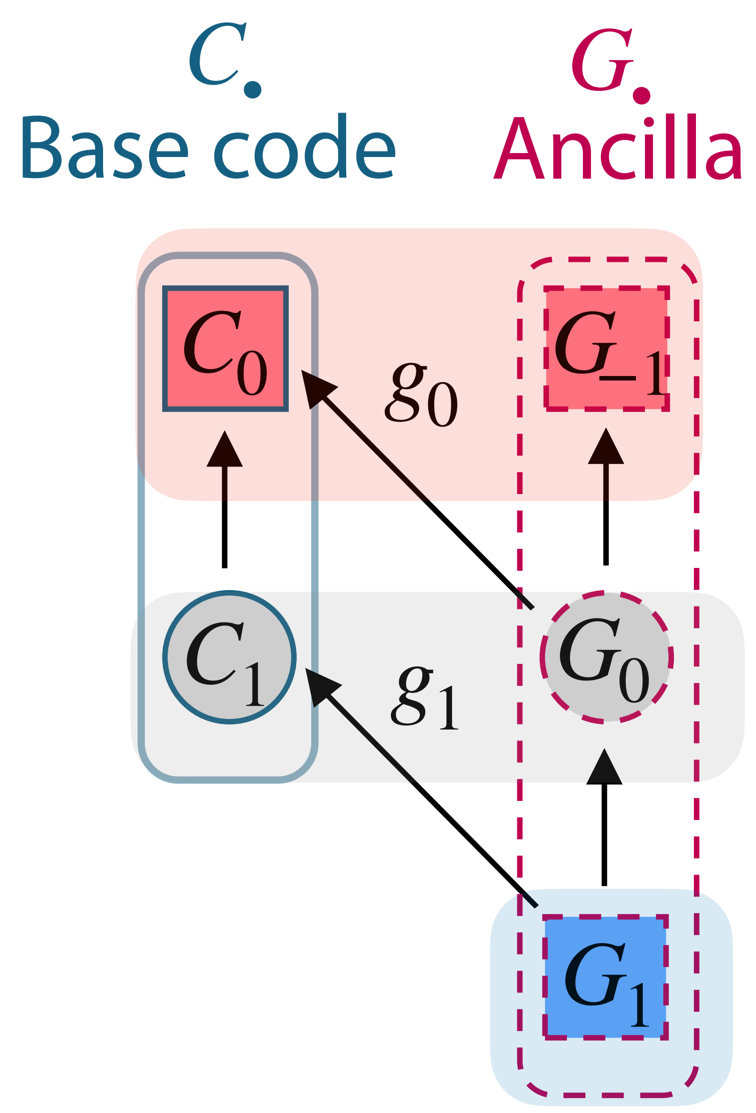
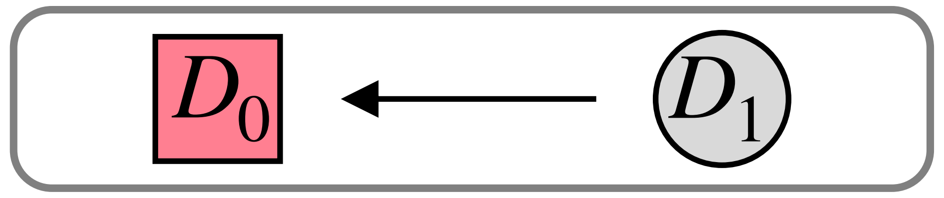


$\otimes$ $D_\bullet =$



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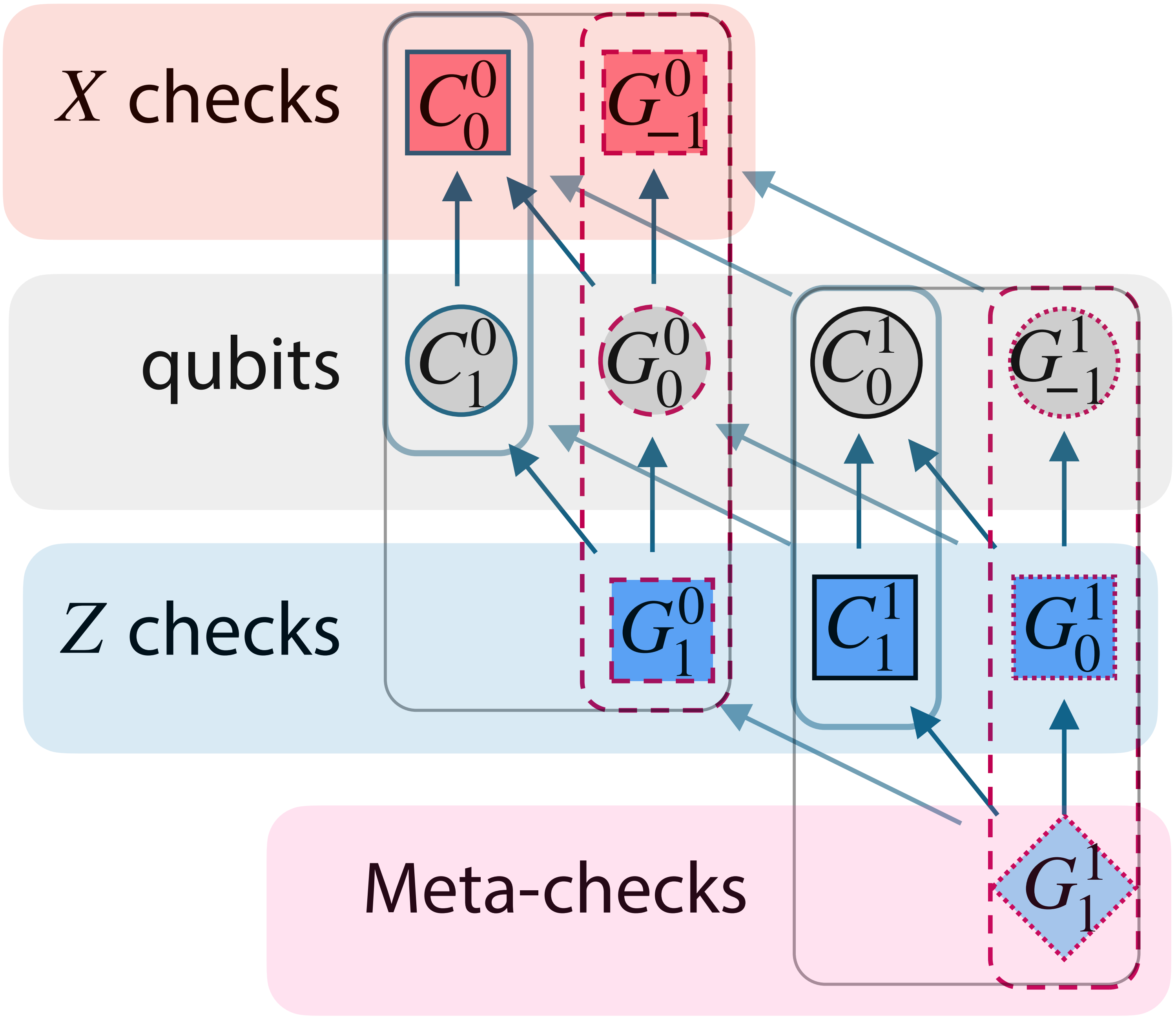

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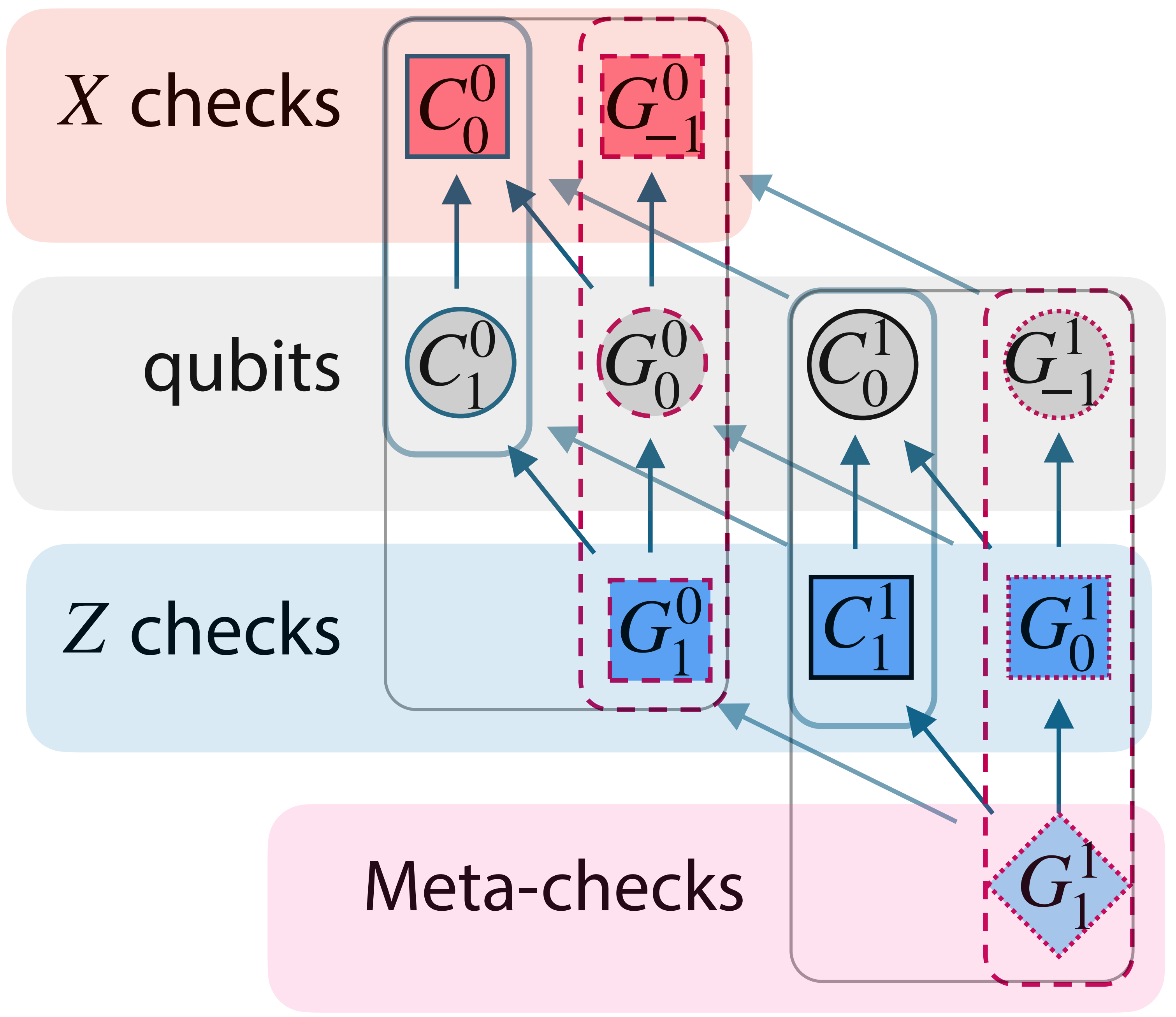

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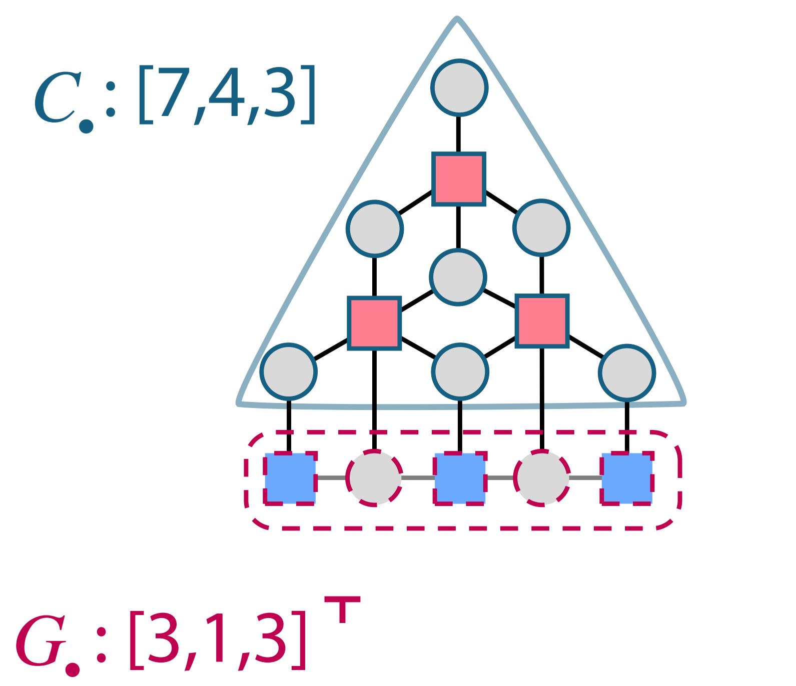

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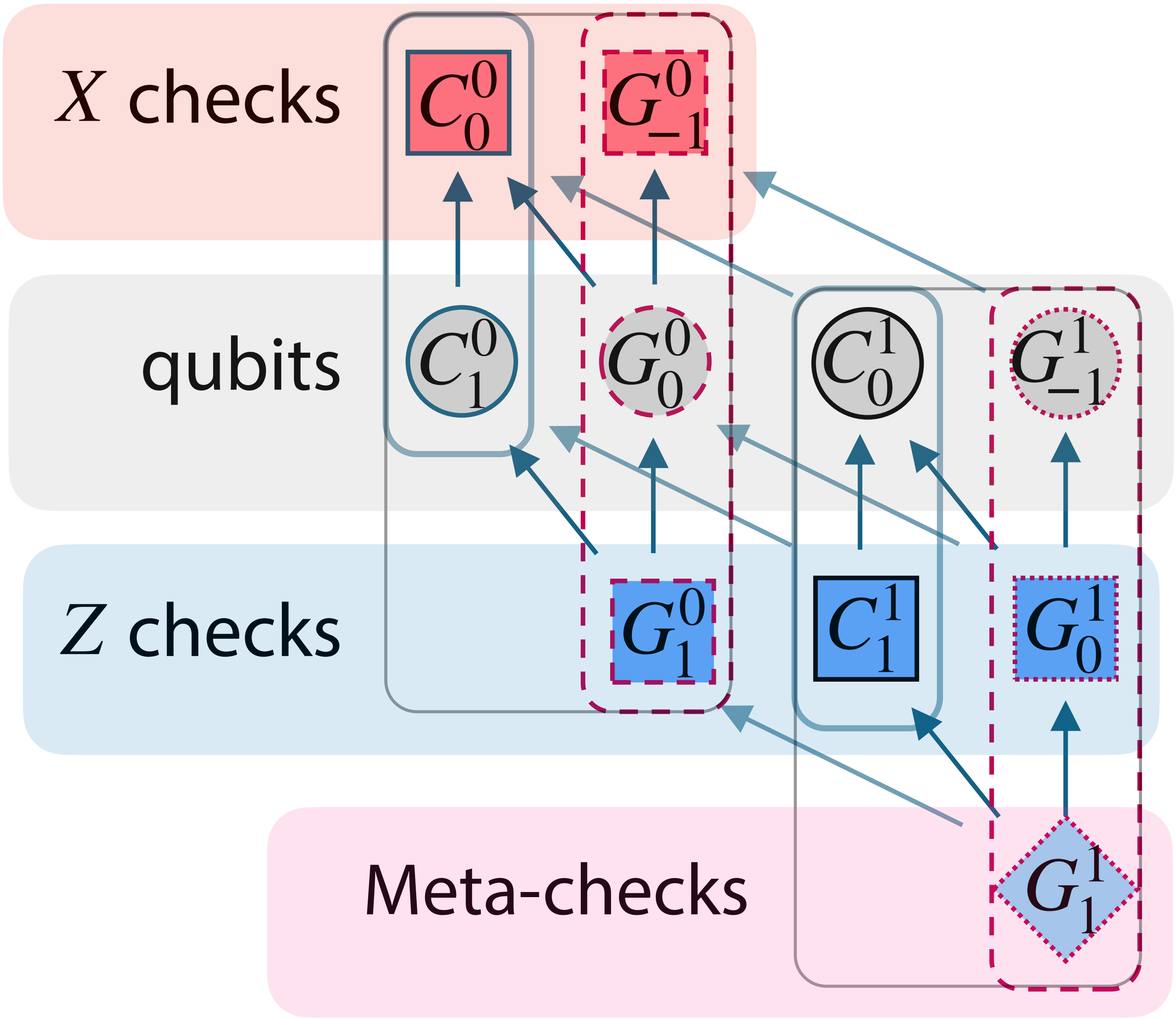

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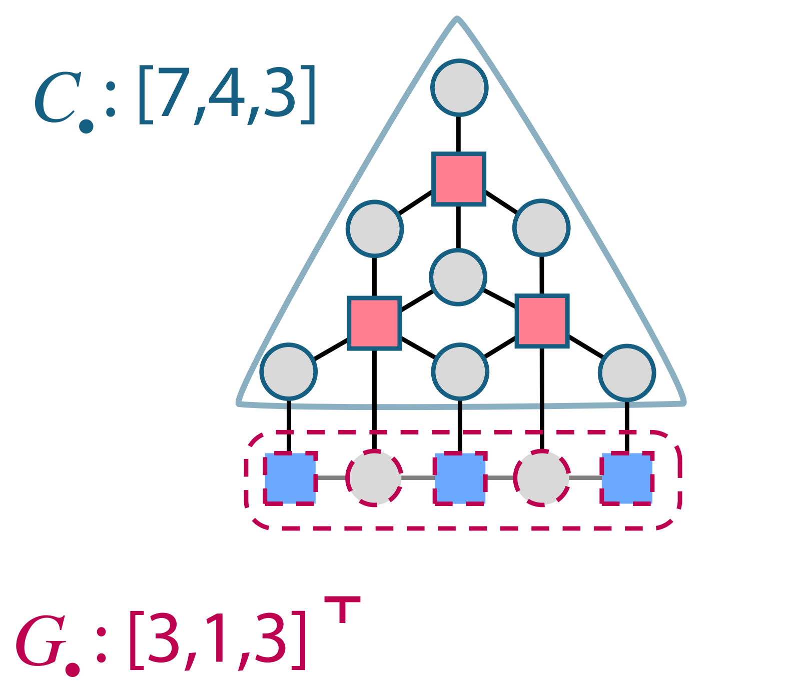

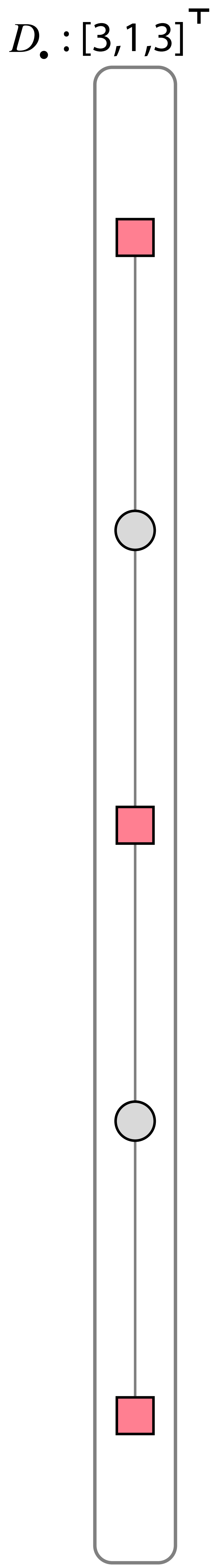

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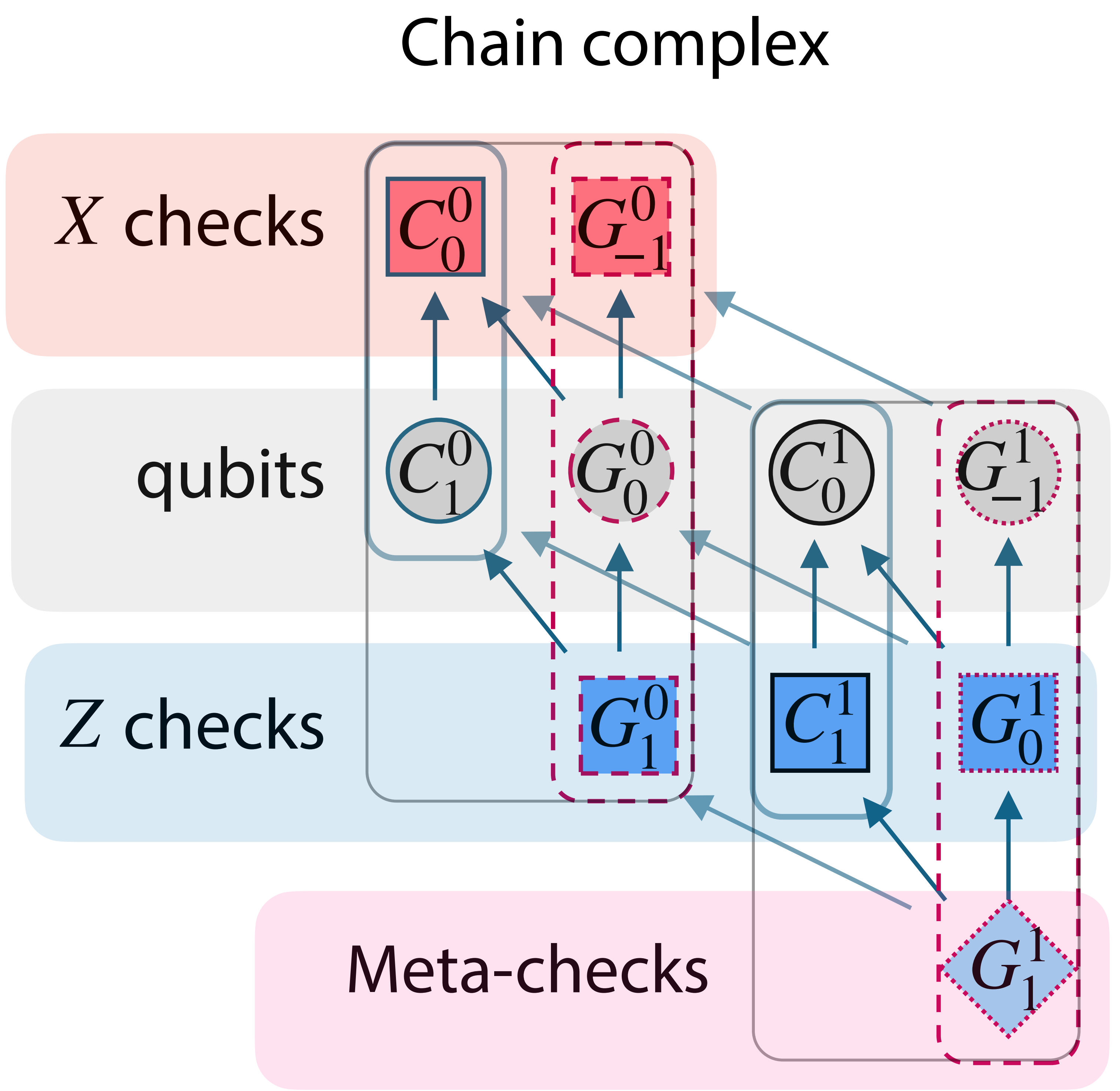

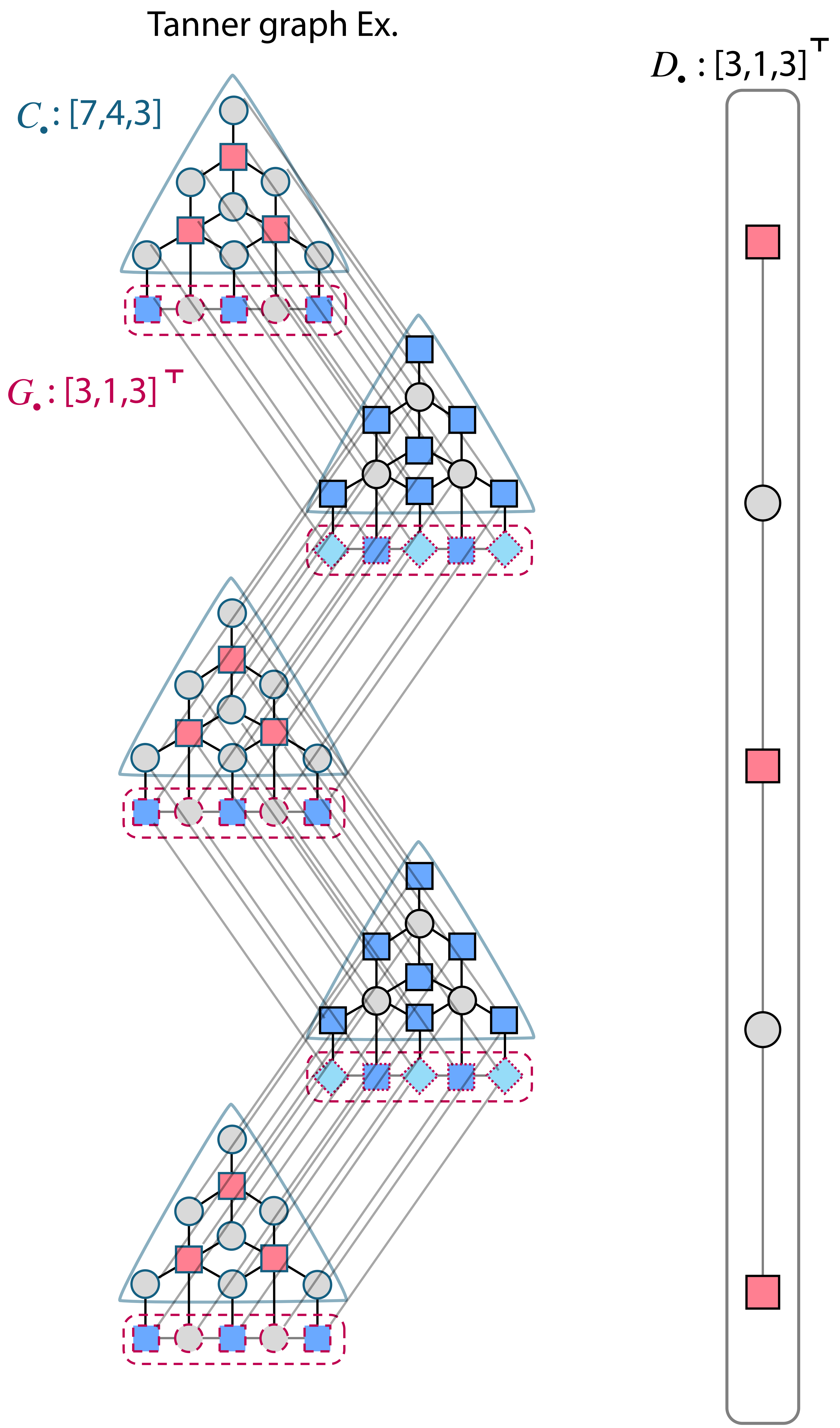

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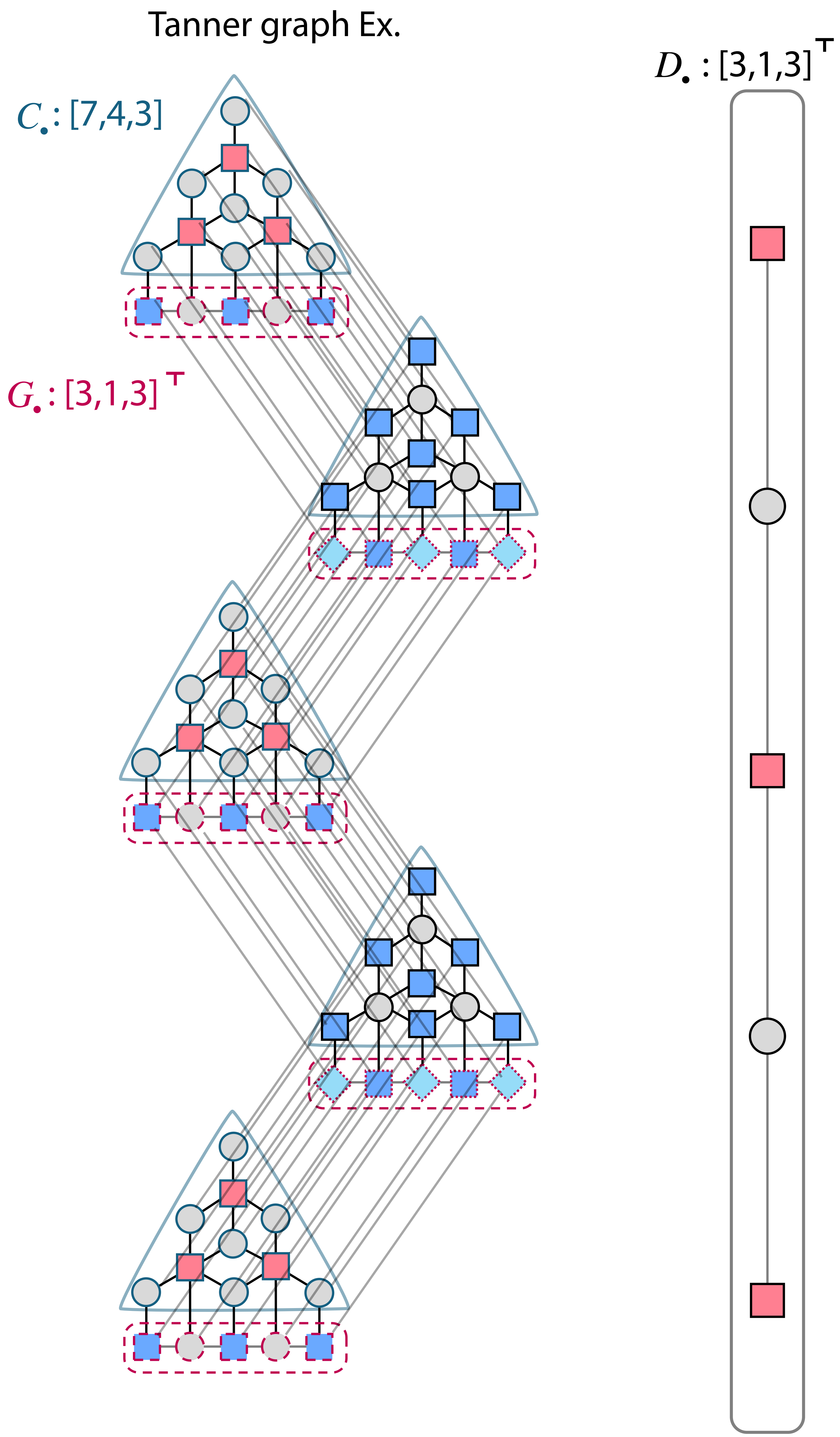
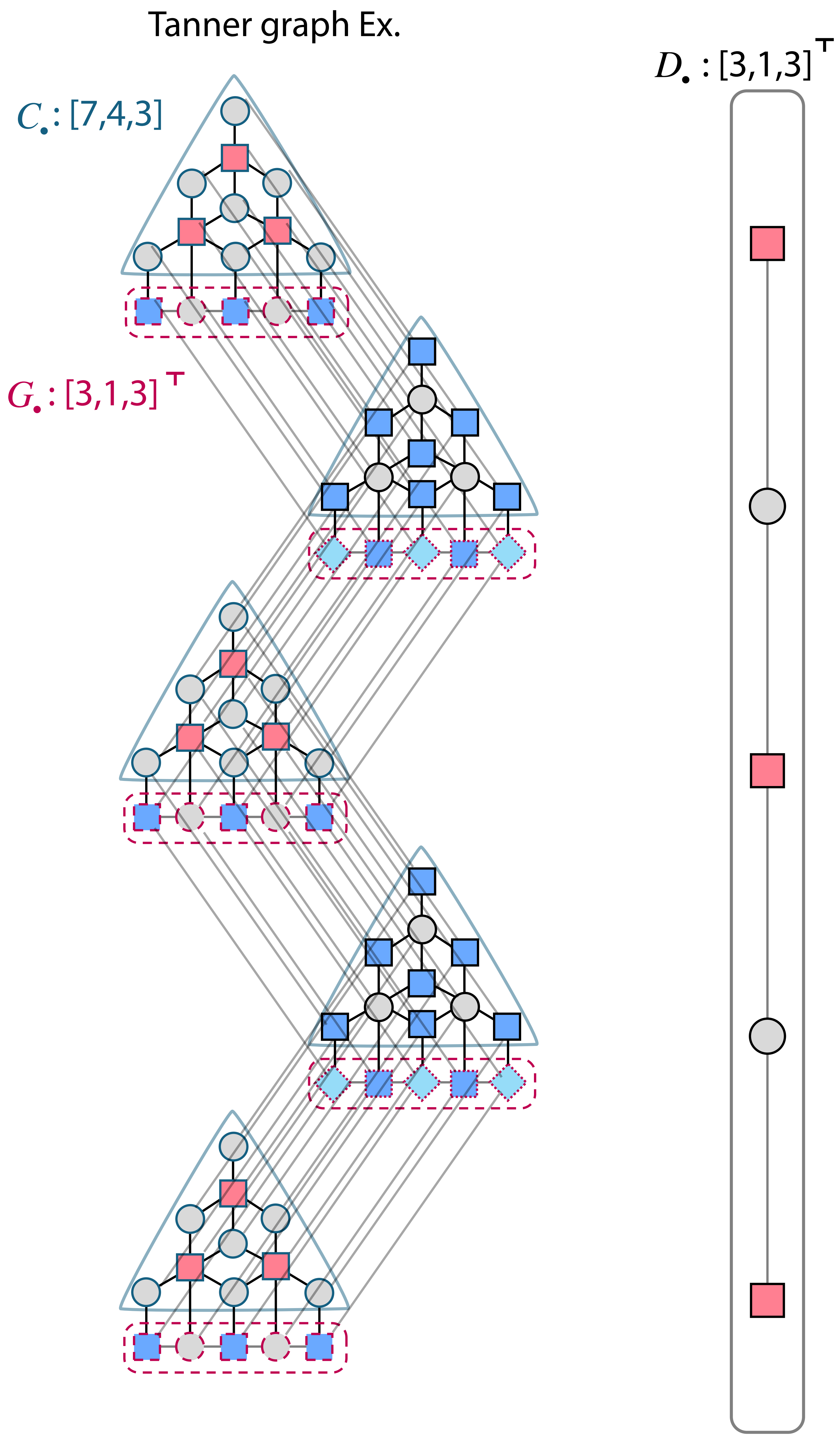


Ex. Constructing the deformed code

What does measuring the stabilizers of this deformed code accomplish?



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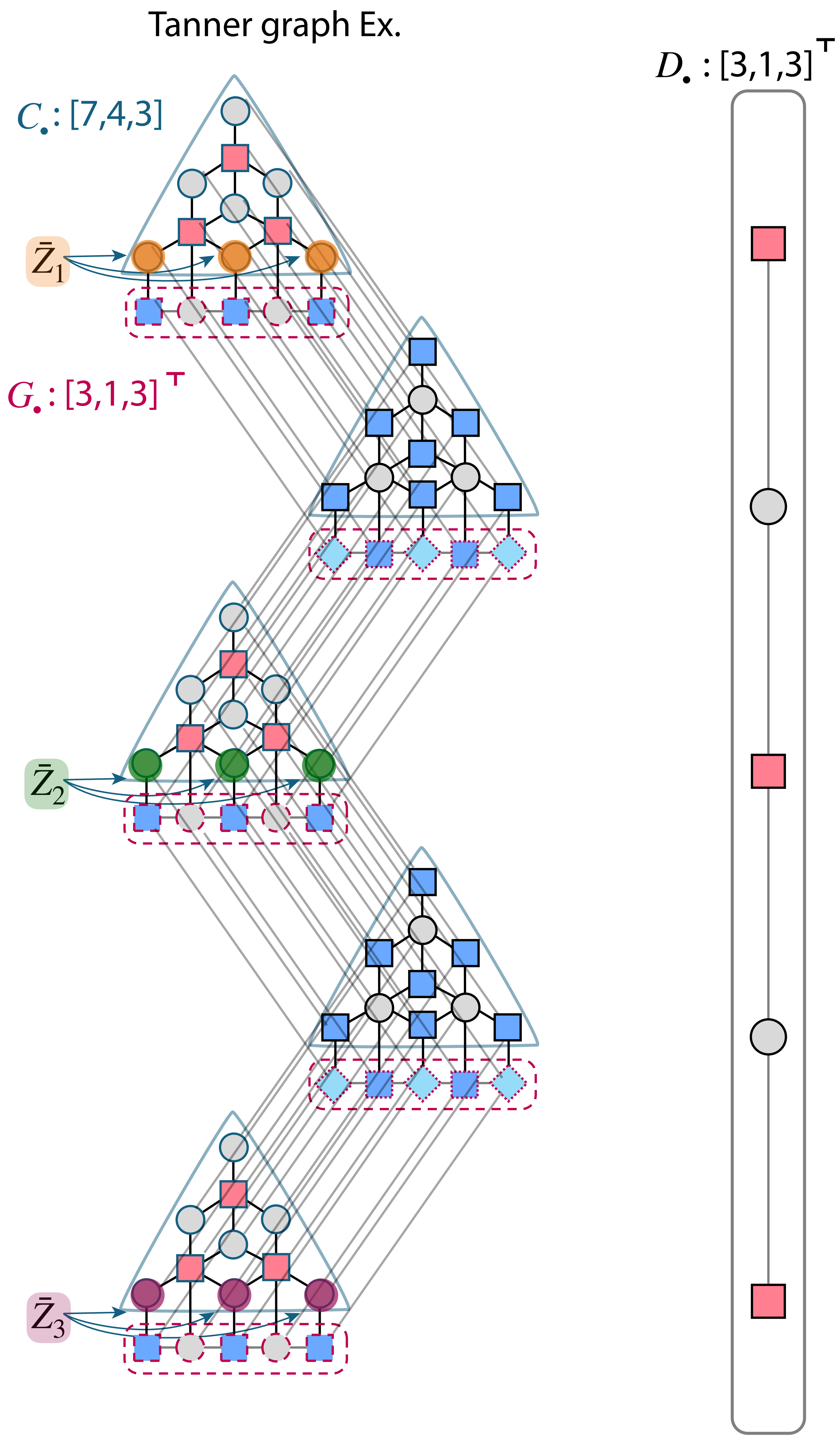

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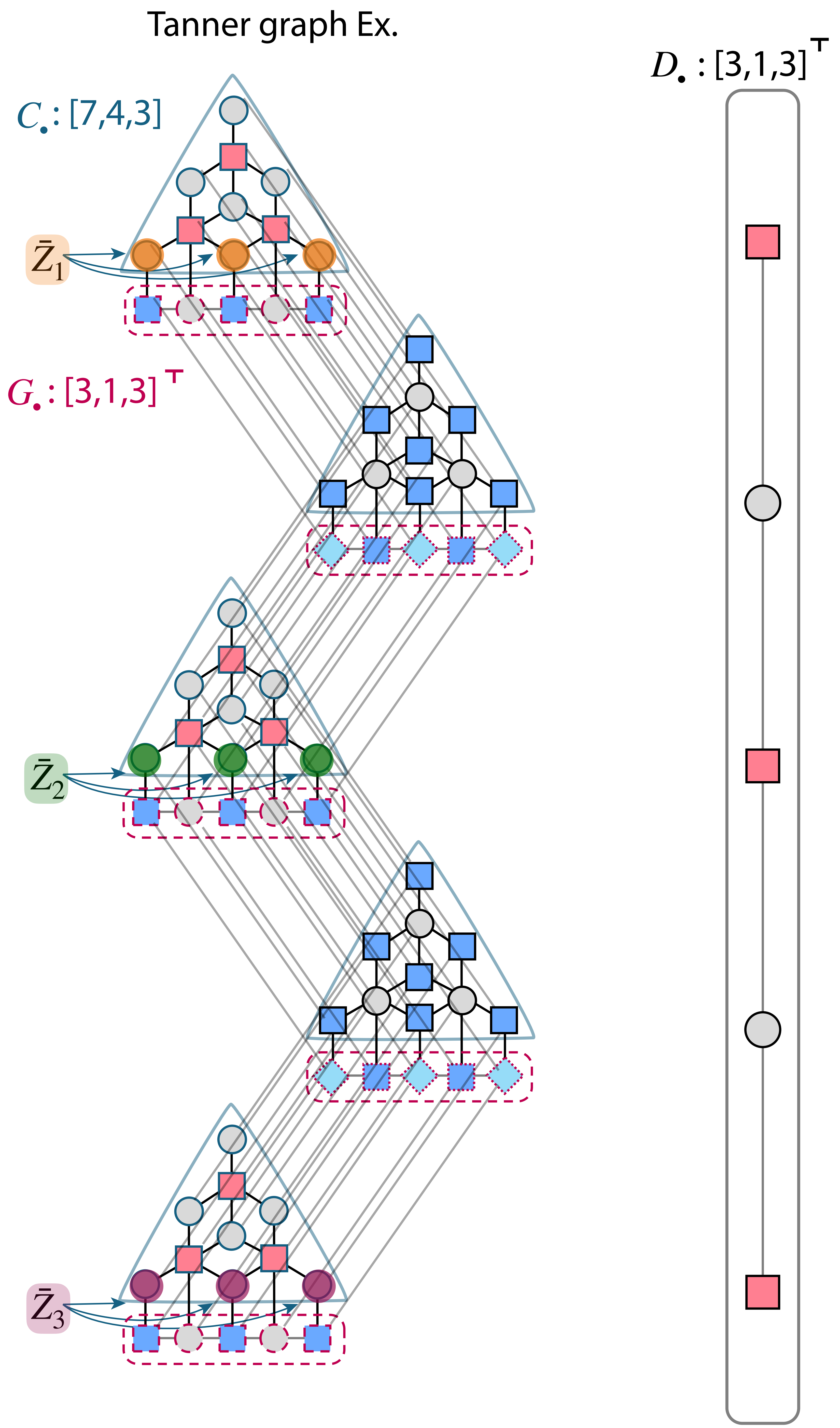

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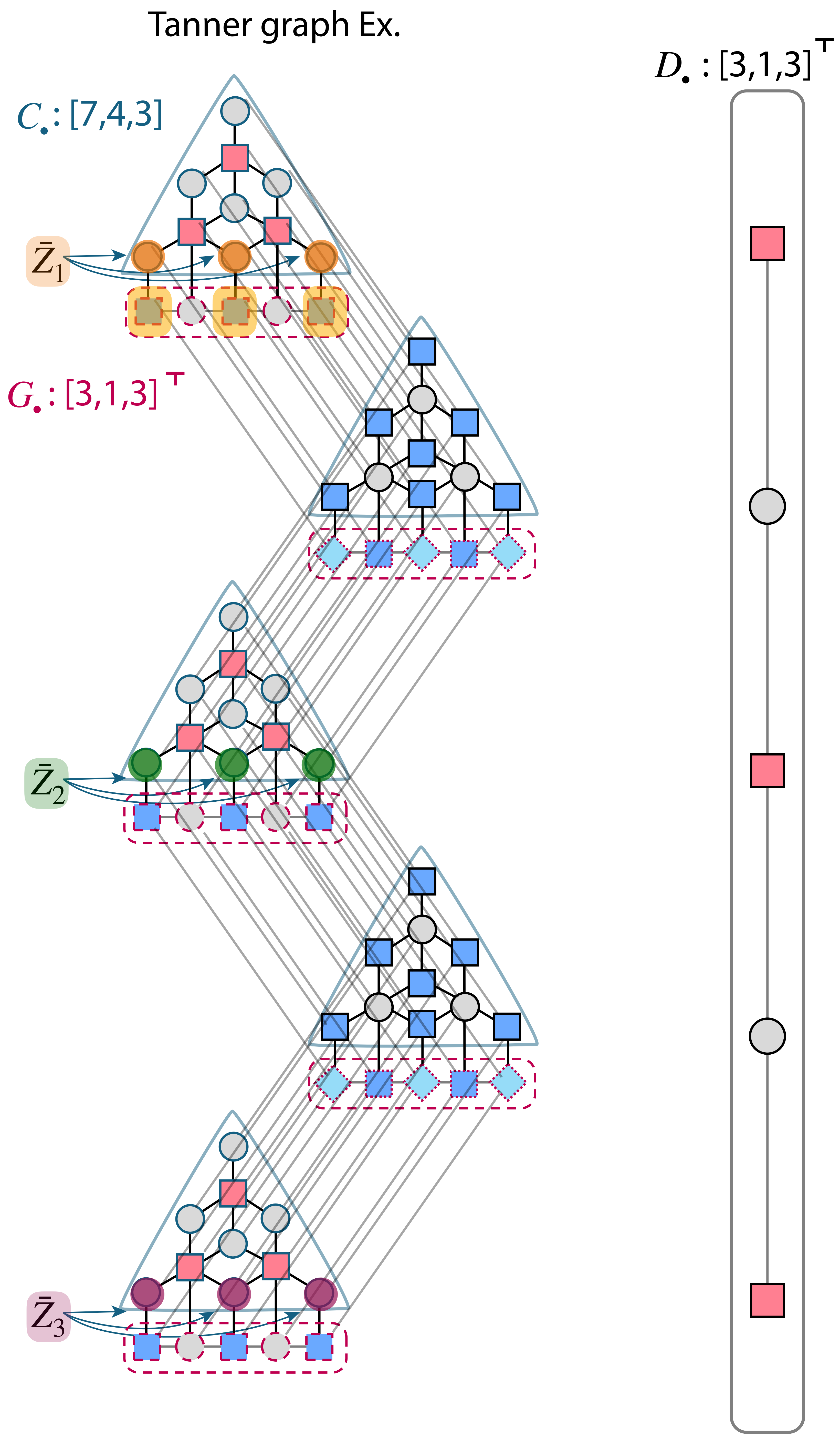

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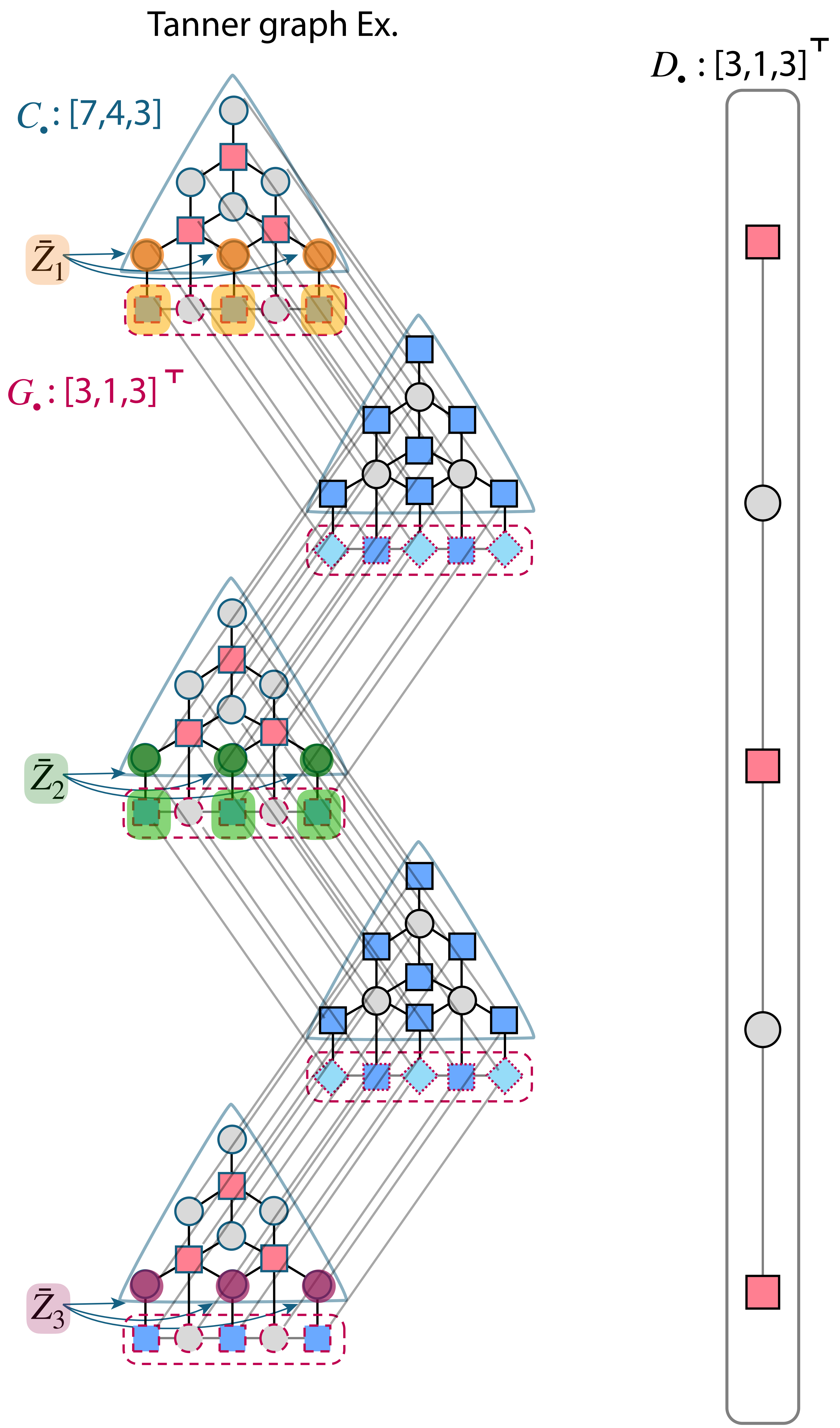

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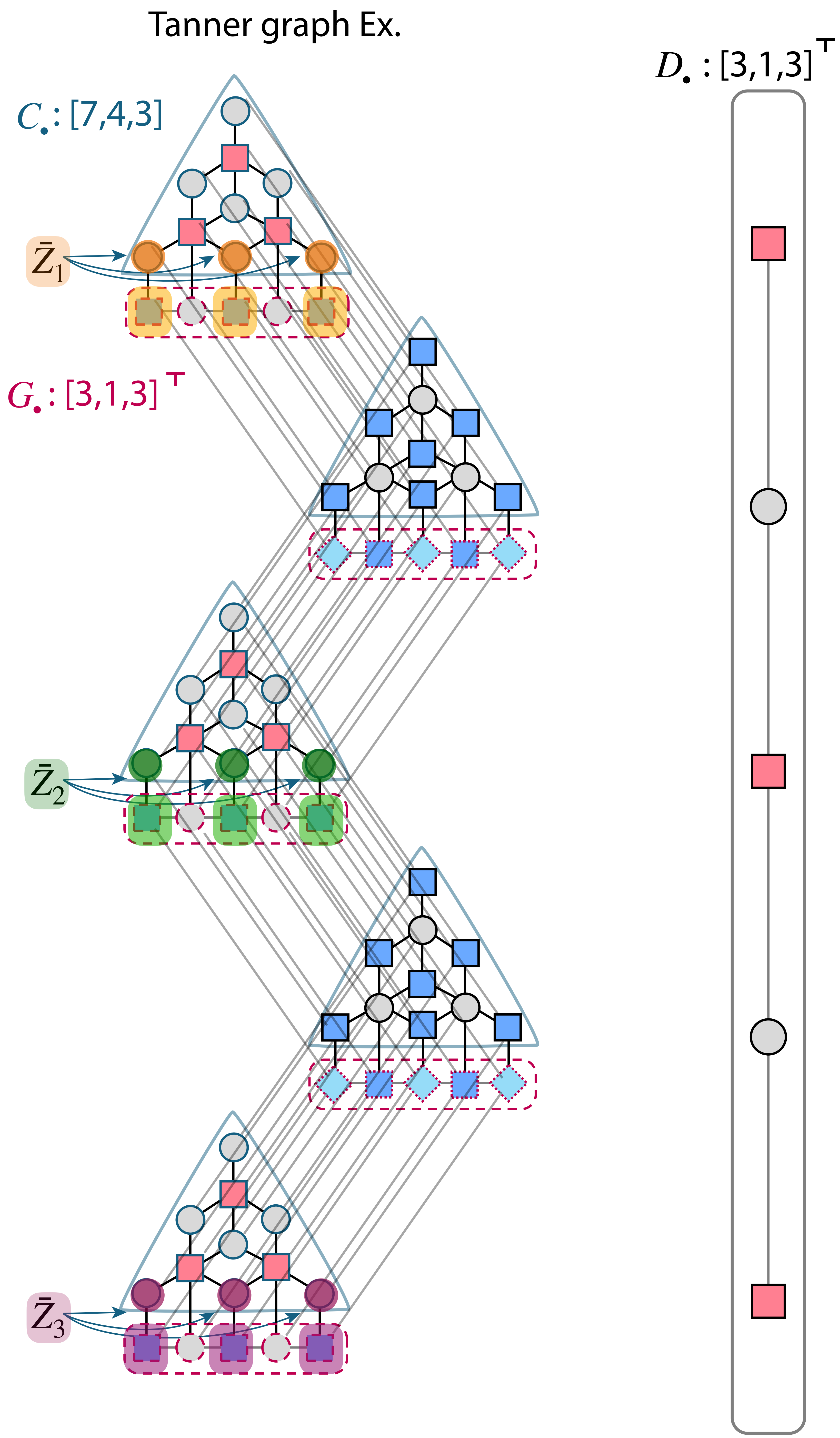

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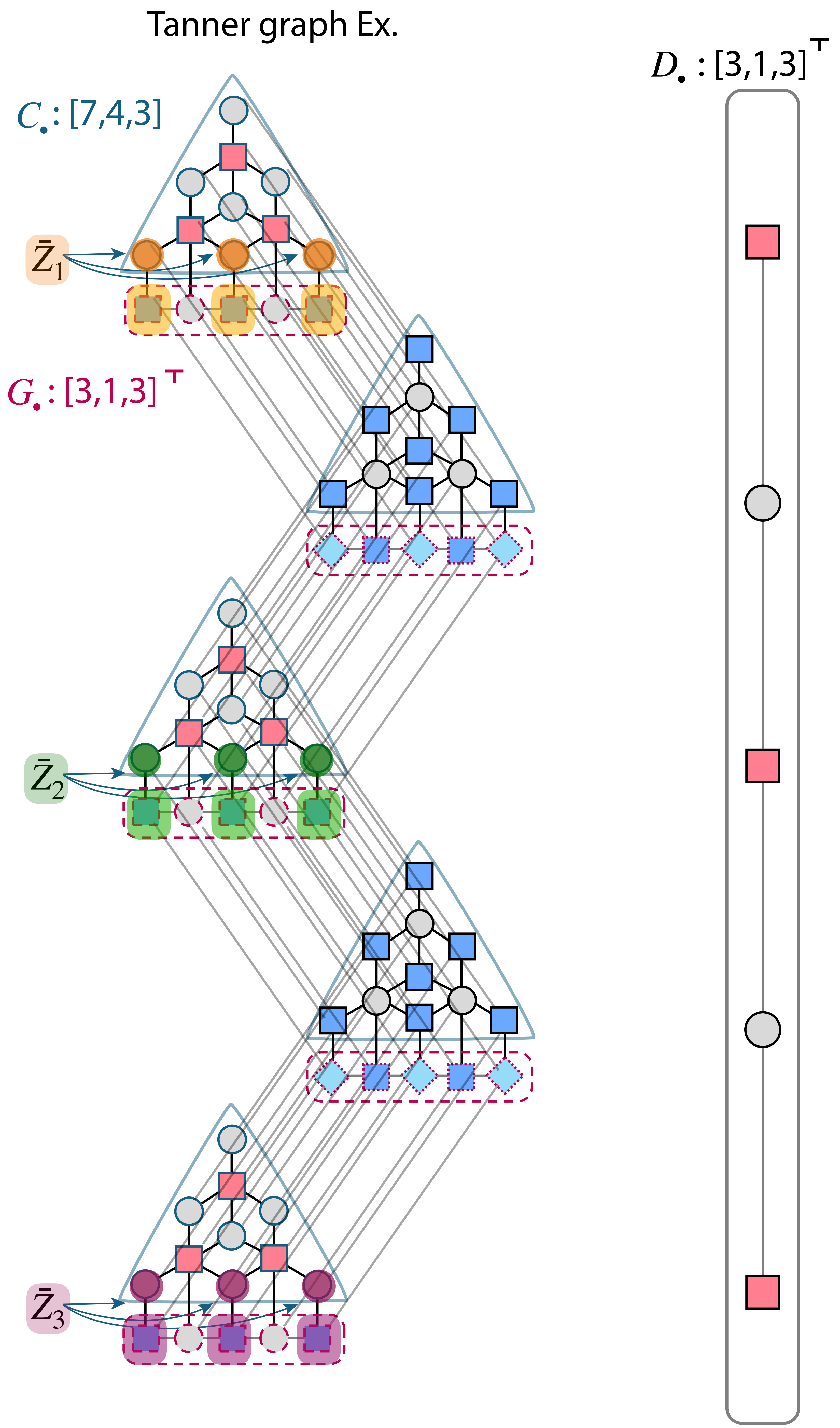
Redundancy **by measuring different** ***representatives*** **of the same logical operator.**

Ensure fault tolerance against measurement errors.



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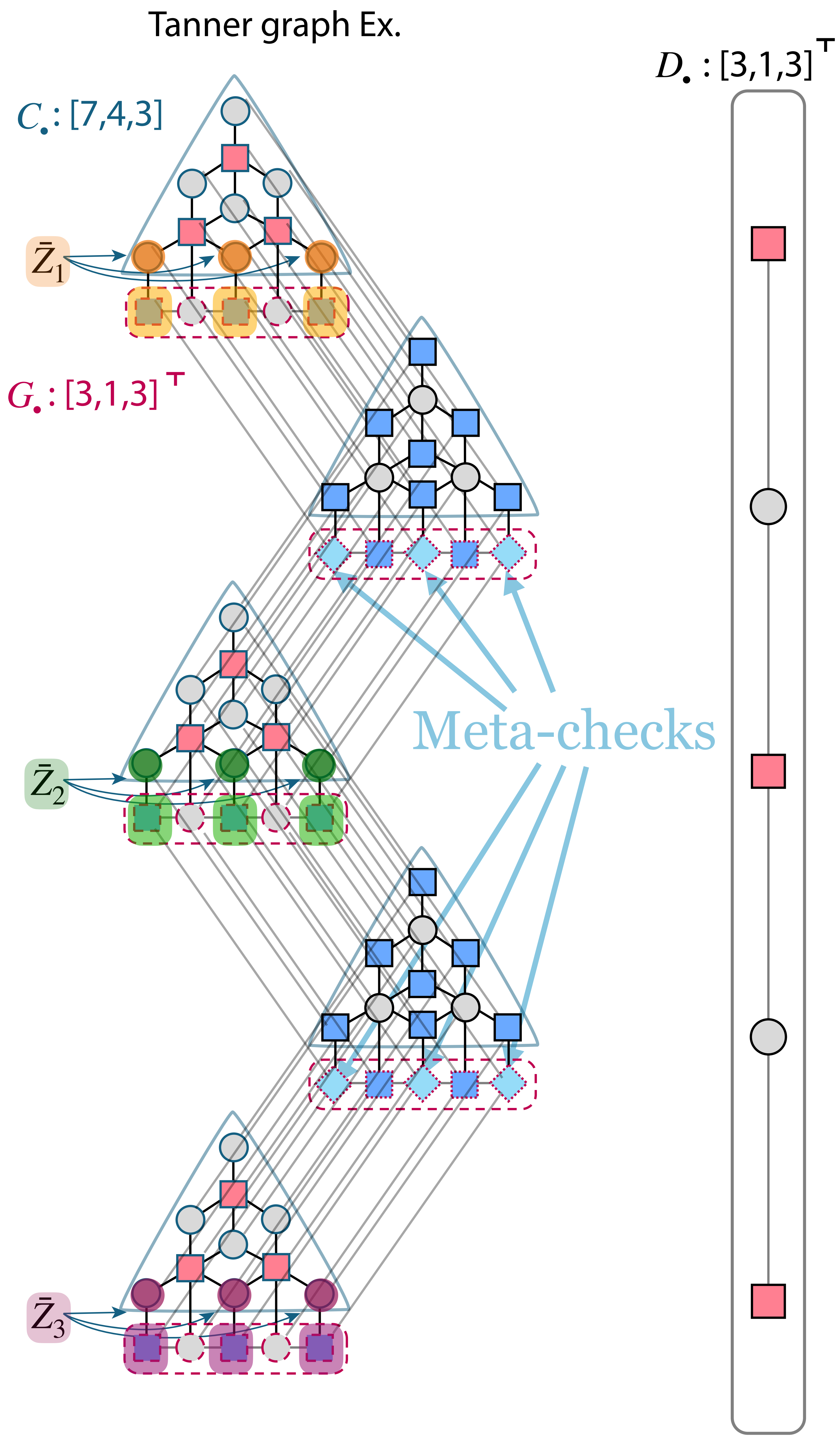

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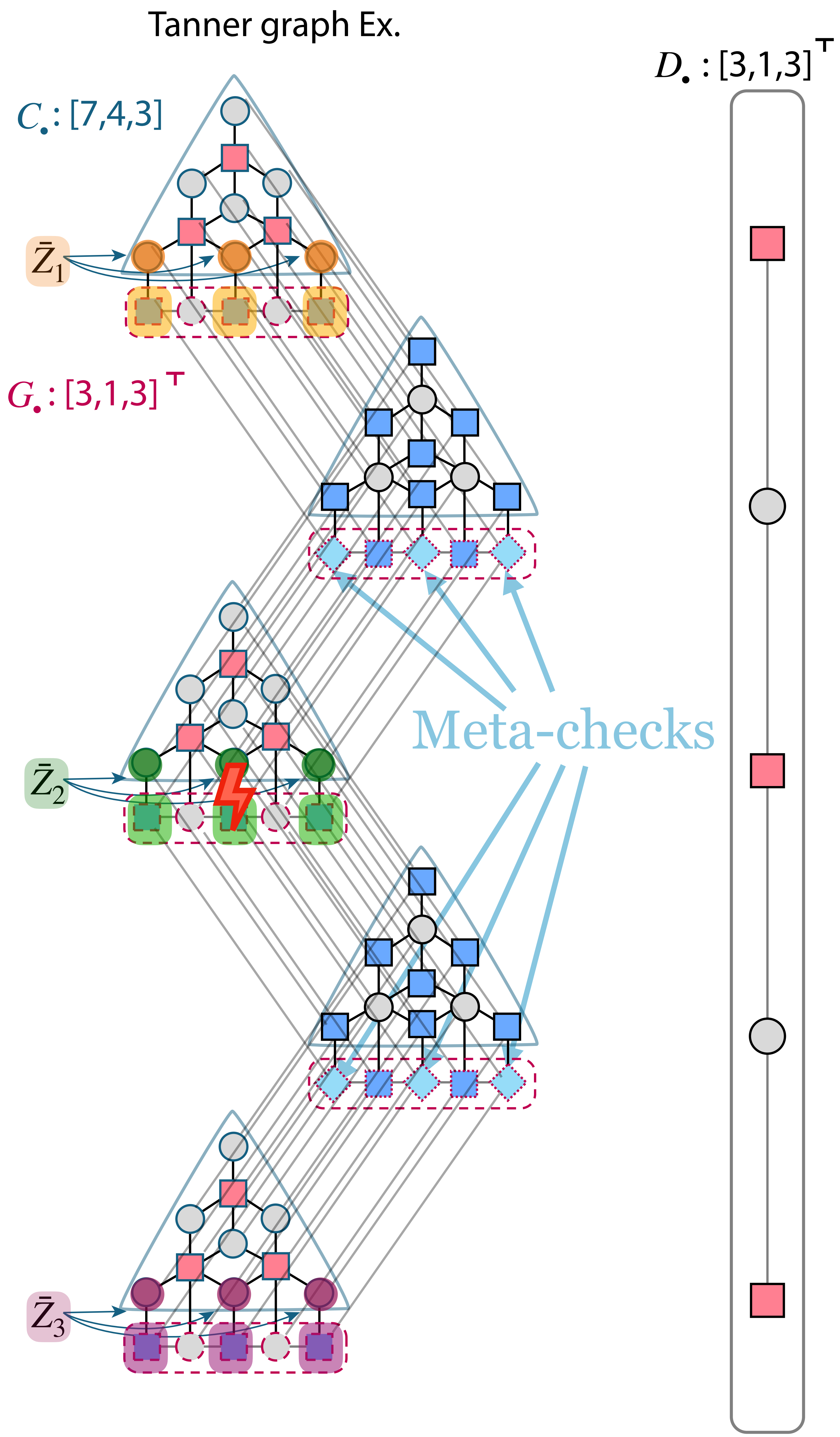

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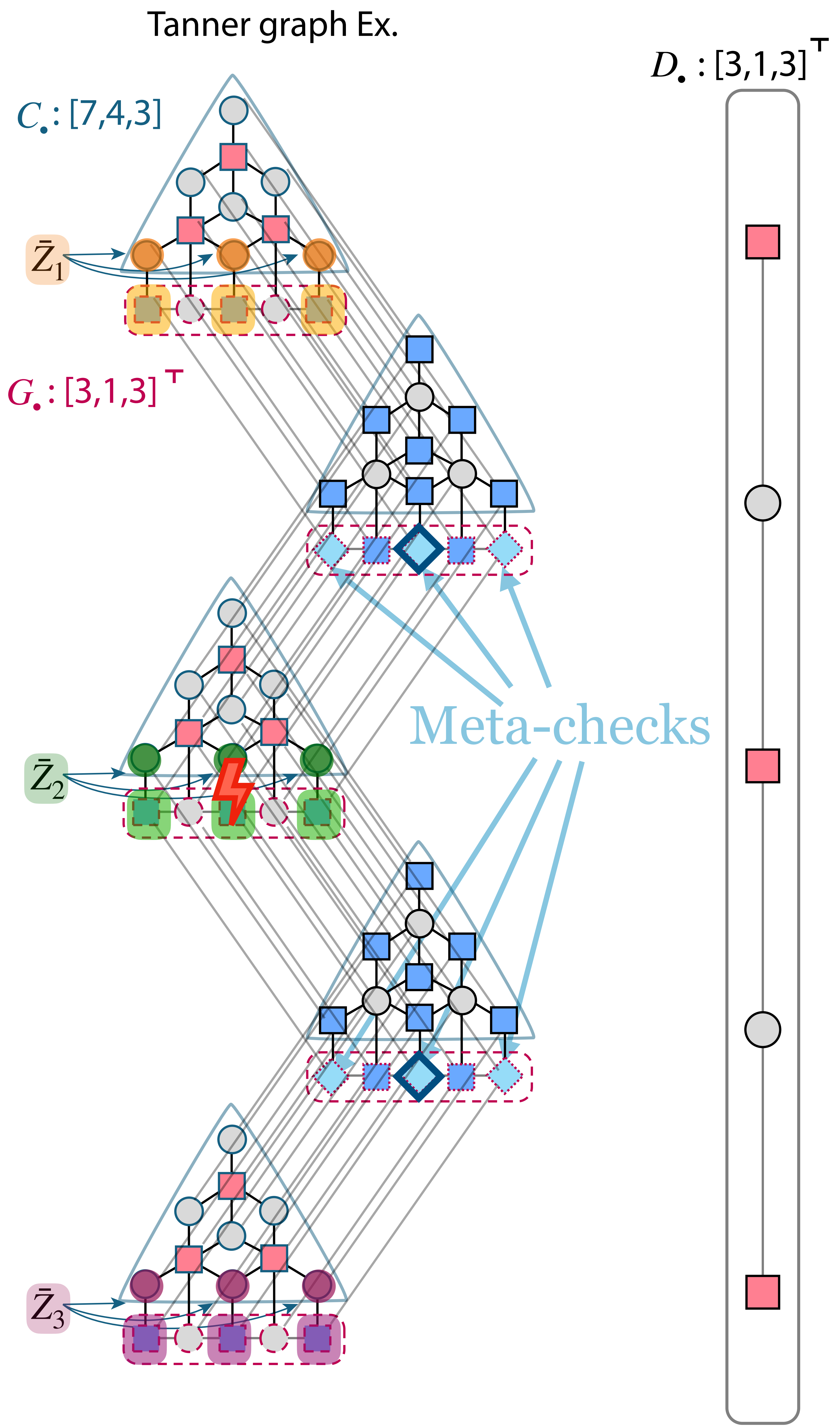
- $\bar{Z}_1$, $\bar{Z}_2$, $\bar{Z}_3$ are logically equivalent representatives of the same logical operator.
- $\bar{Z}_1$, $\bar{Z}_2$, $\bar{Z}_3$ logical representatives are measured simultaneously — each equal a product of ancilla Z checks

***Redundancy* by measuring different *representatives* of the same logical operator.**

Ensure fault tolerance against measurement errors.



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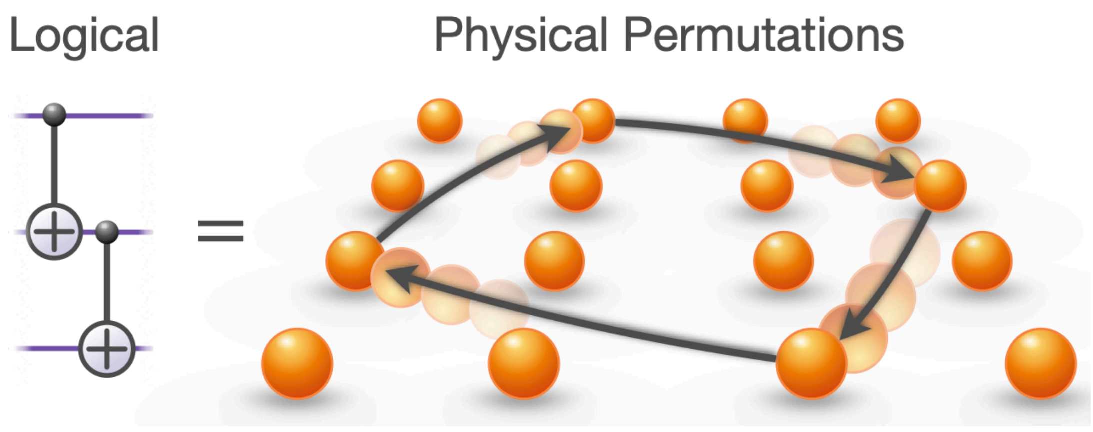
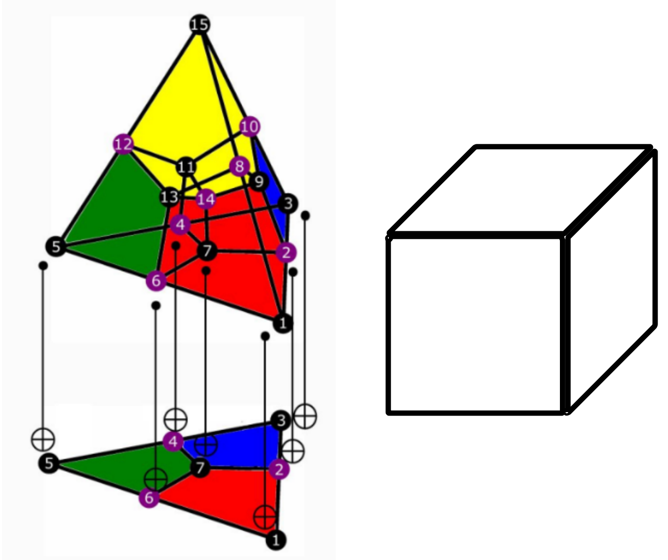
Computation on QLDPC codes

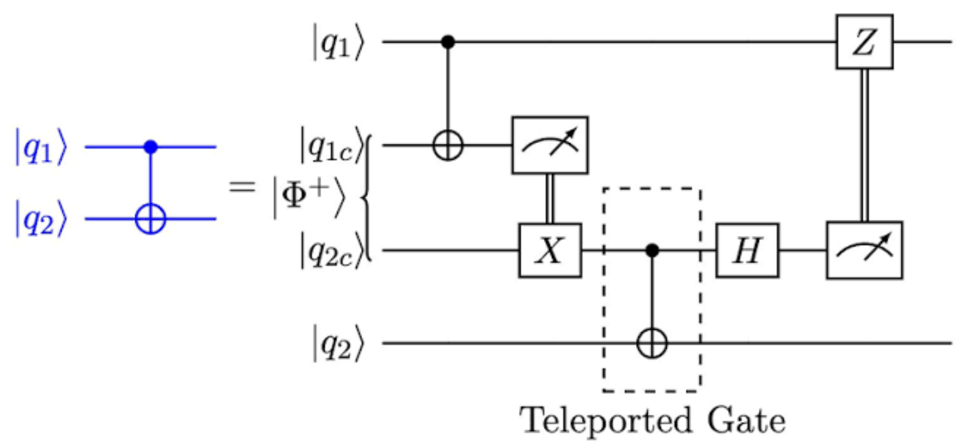
Clifford gates



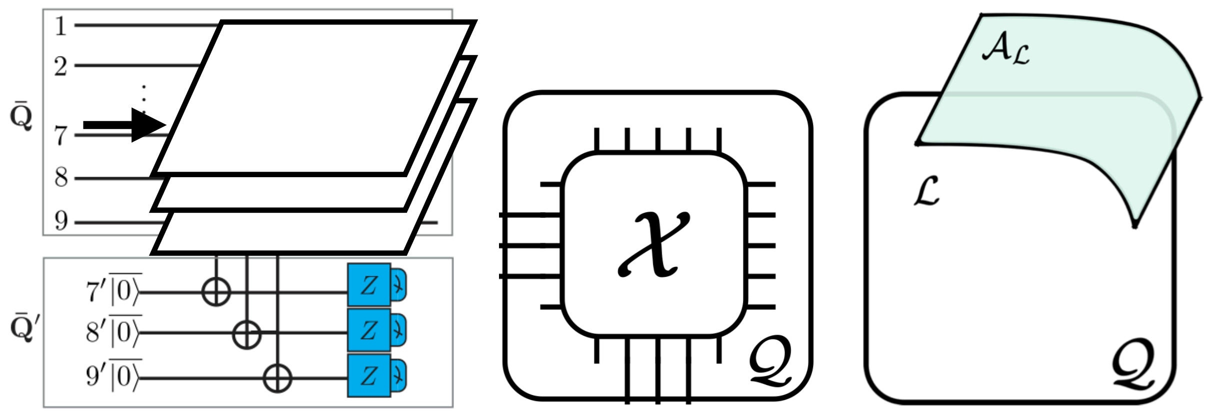


Automorphisms,
Low-depth unitaries

Code-switching



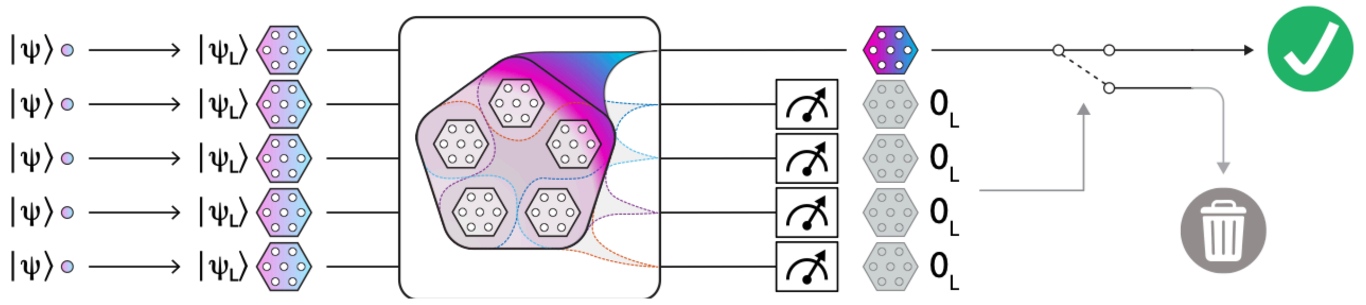
State prep. & gate teleportation

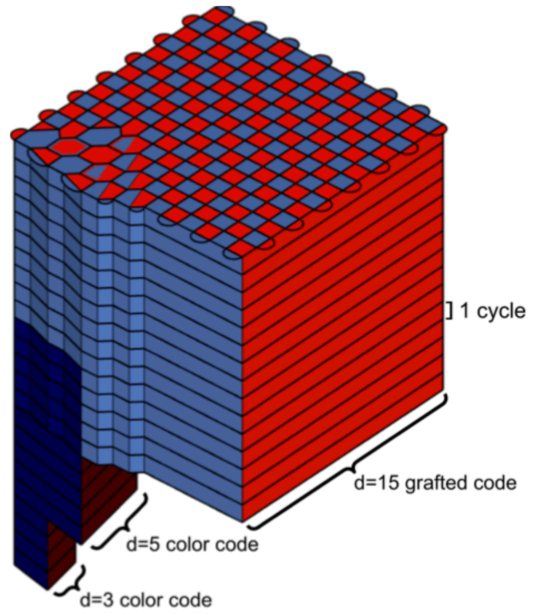


Measuring Pauli operators

Part I. *Fast* primitives for Pauli operator measurements

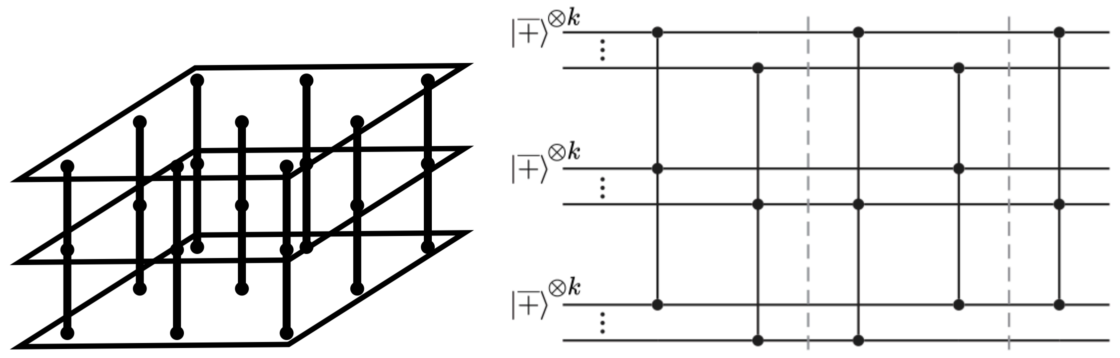
Non-Clifford gates





Magic state distillation

Magic state cultivation



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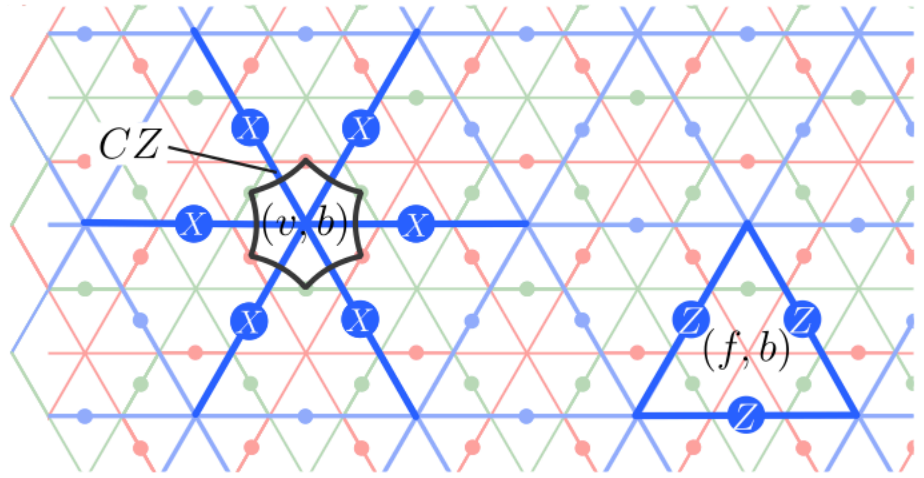

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Fig. Credits: Koh, *et al.* [2601.20927], Daguerre, Kim. PRR **7** (2025), Xu, *et al.* PRX **15** (2025), He, *et al.* [2503.10390], Sales Rodriguez, *et al.* Nature **645** (2025), Gidney, *et al.* [2409.17595], Menon, *et* al. PRX **16** (2026).Davydova, *et al.* PRX **16** (2026)

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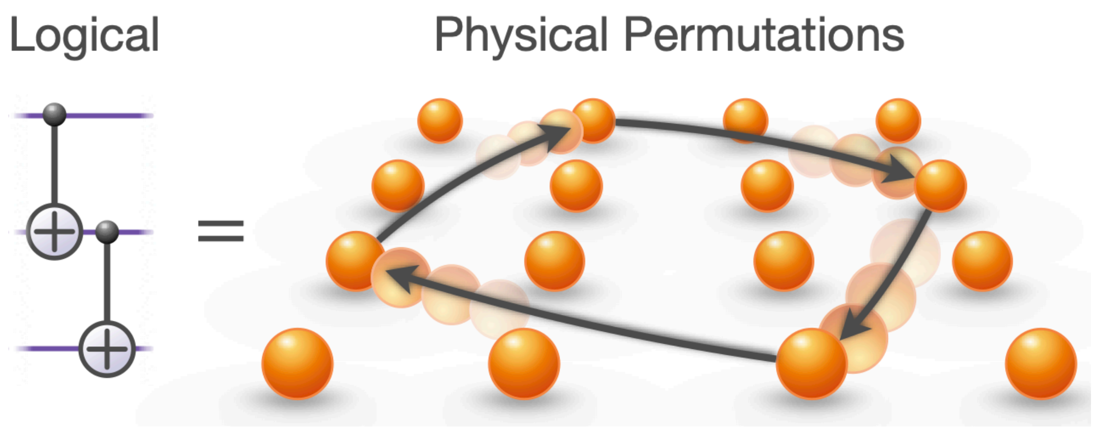

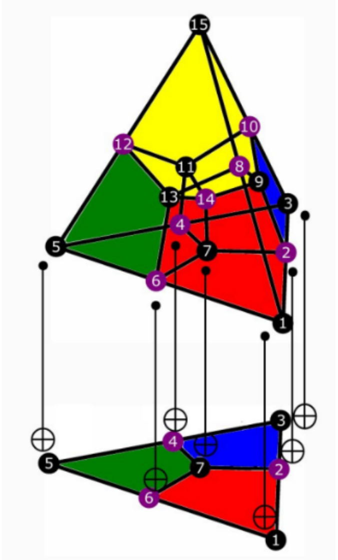

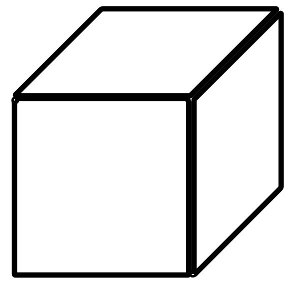

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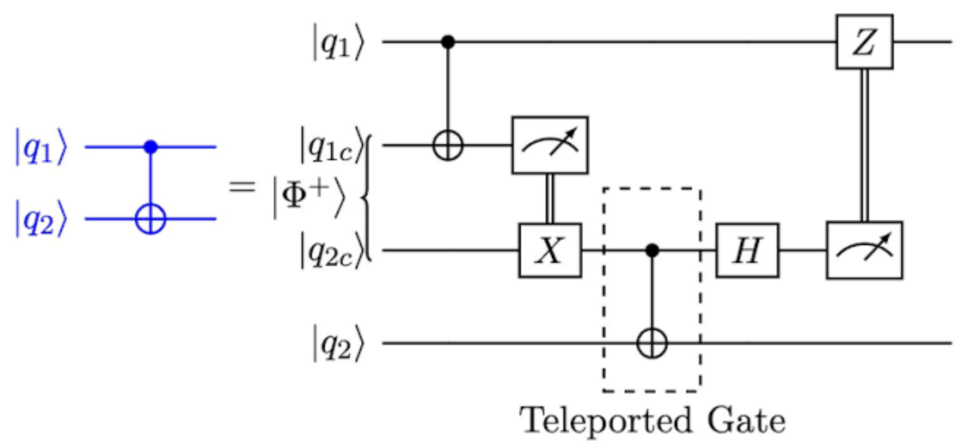

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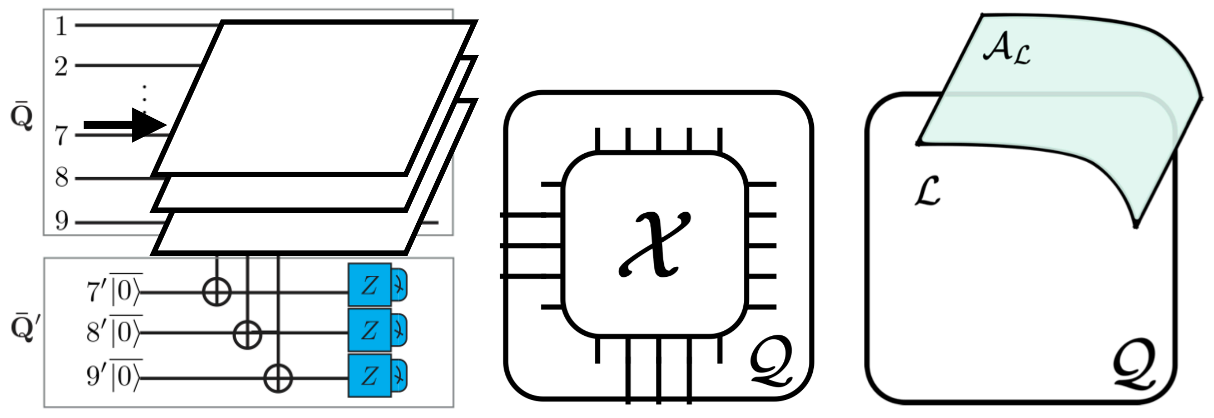

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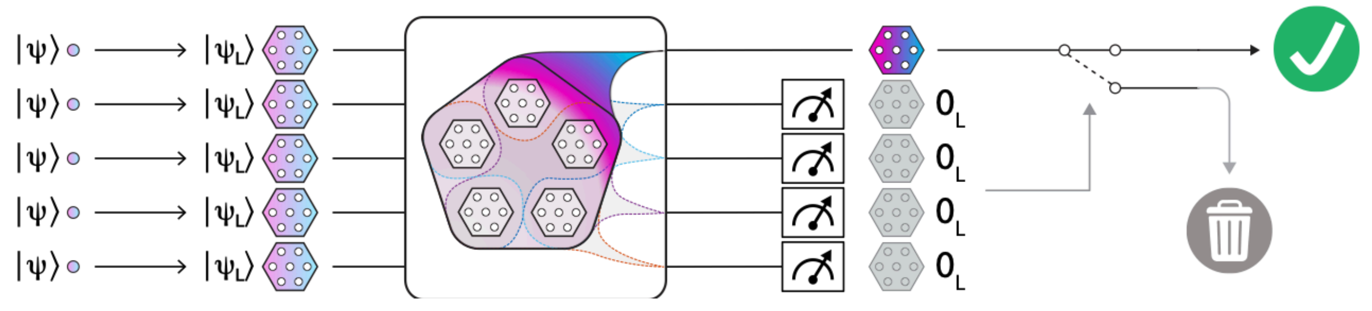

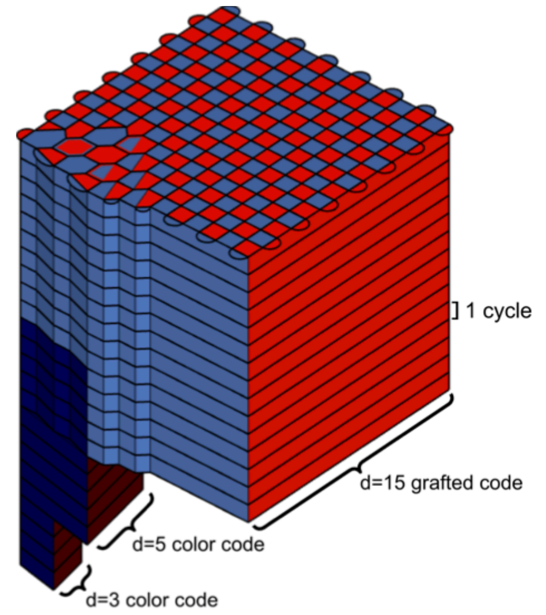

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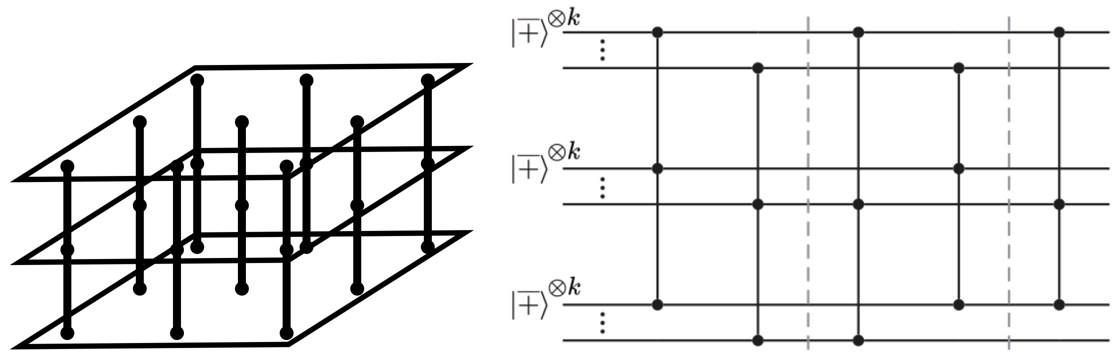

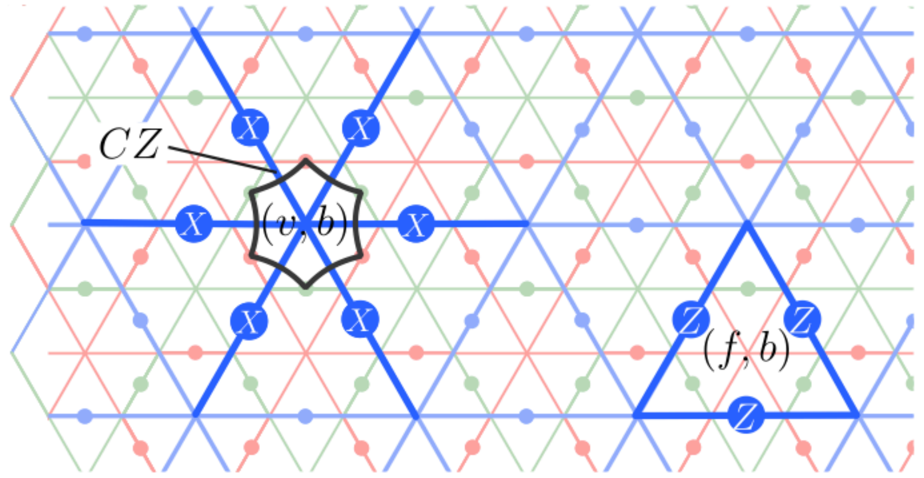

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Part I. *Fast* primitives for Pauli operator measurements

Part II. New (*Furious*?) techniques for measuring Clifford operators on *QLDPC* codes

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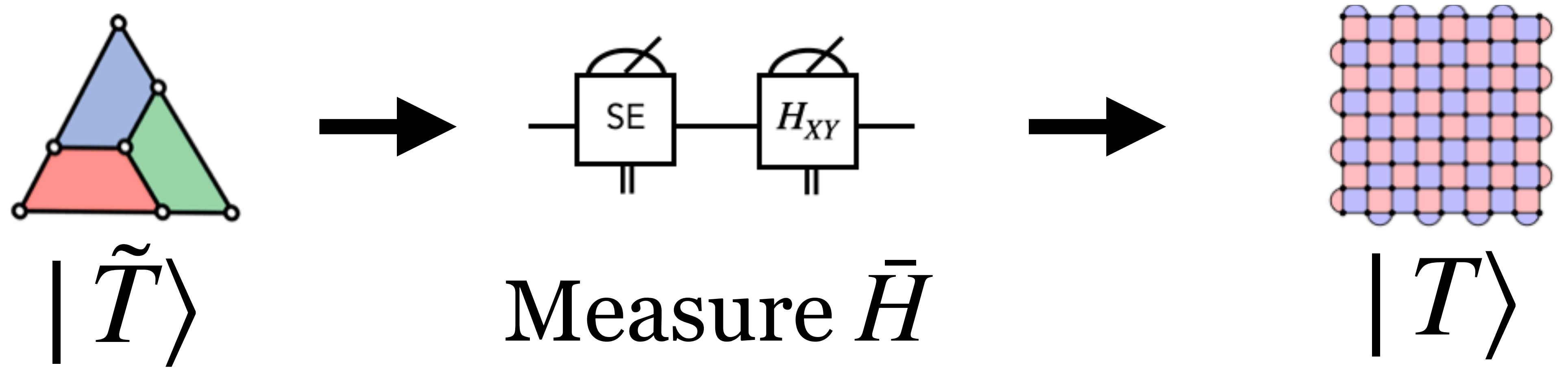

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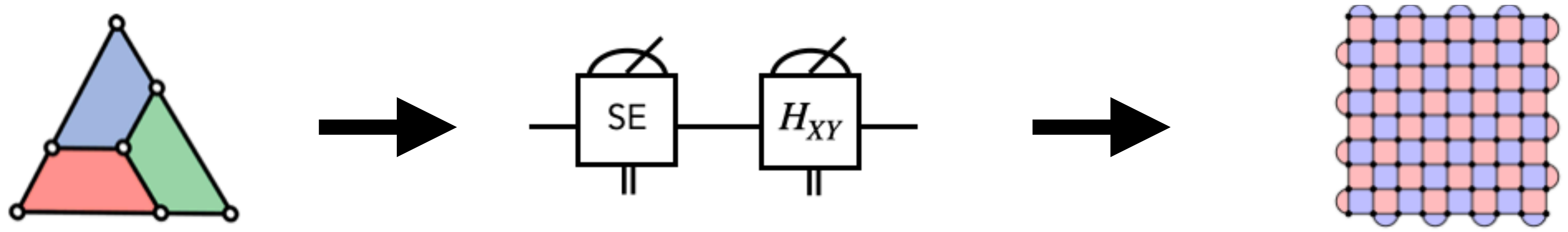

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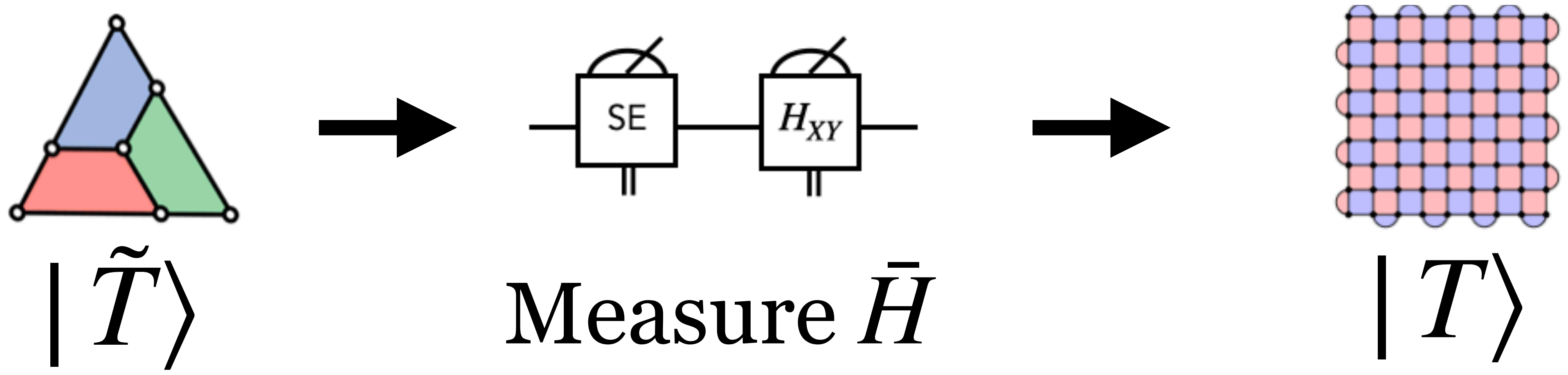

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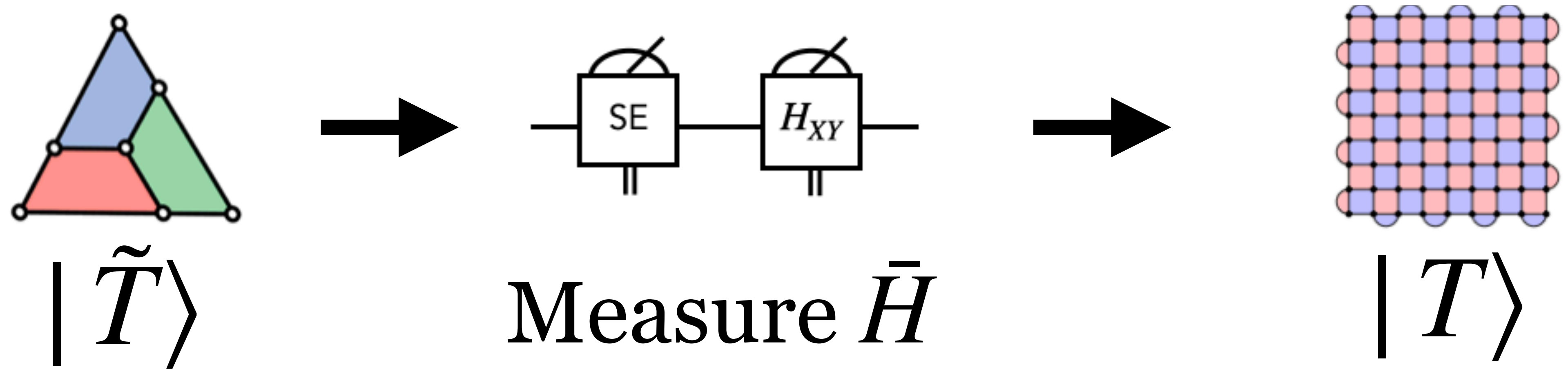

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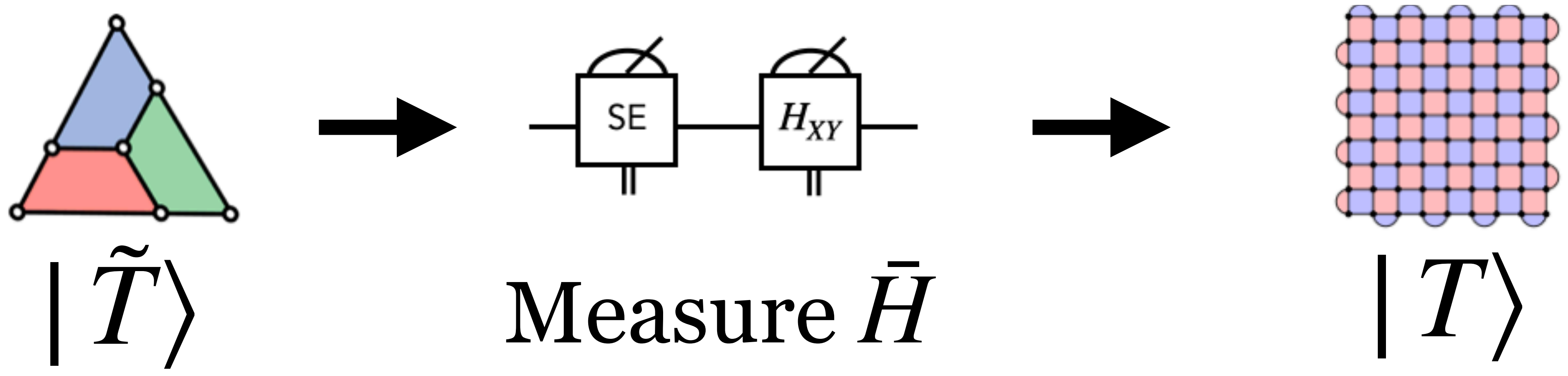

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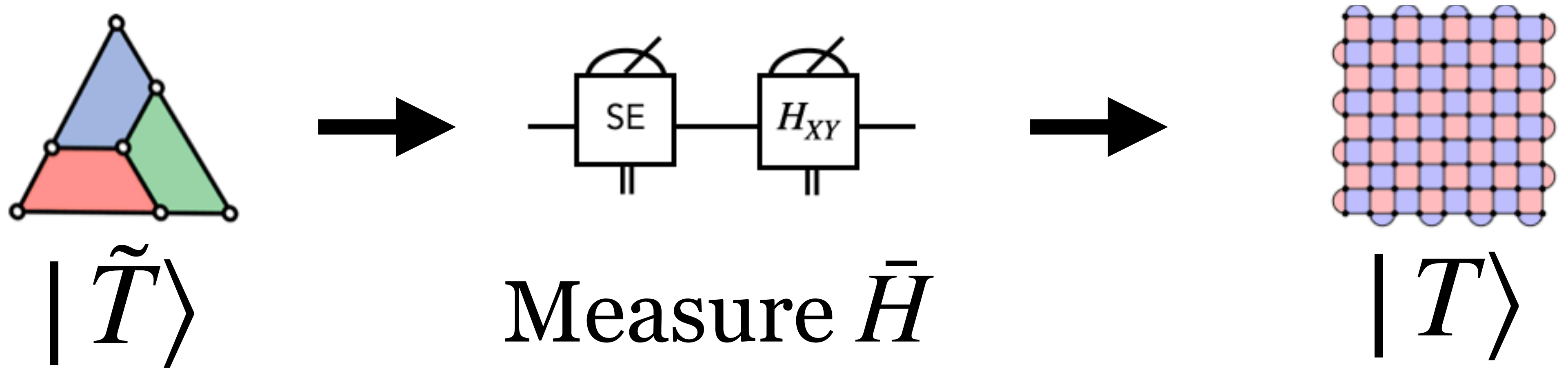

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On topological codes, see related work by Brown. Sci. Adv. **6** (2024), Davydova, et al. PRX **16.3** (2026), Bauer, Magdalena de la Fuente PRXQ **7.2** (2026).
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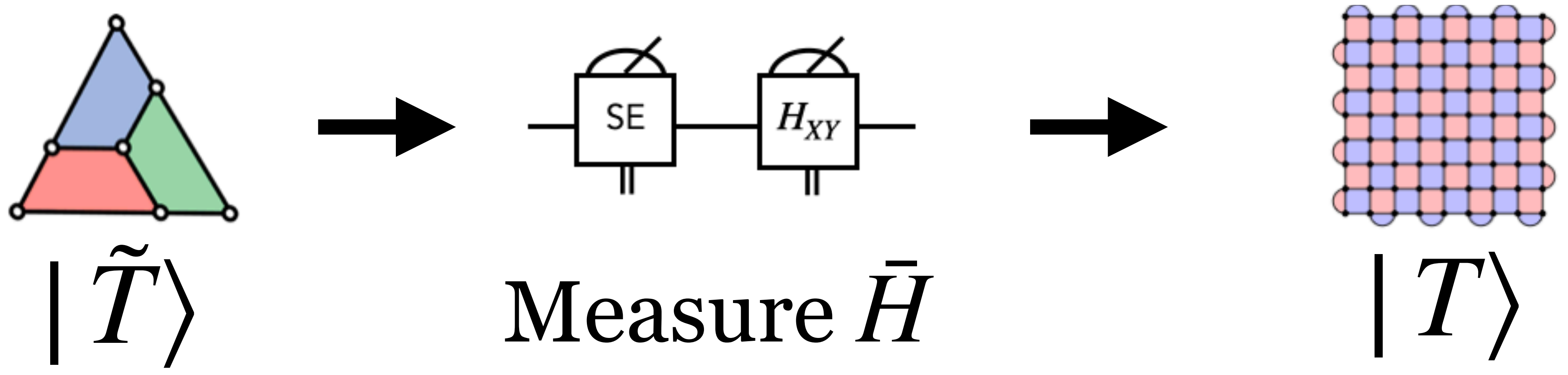

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See also, Christos et al. [2602.12228]. Williamson [2601.22939]. Zhu, et al. [2601.06736].

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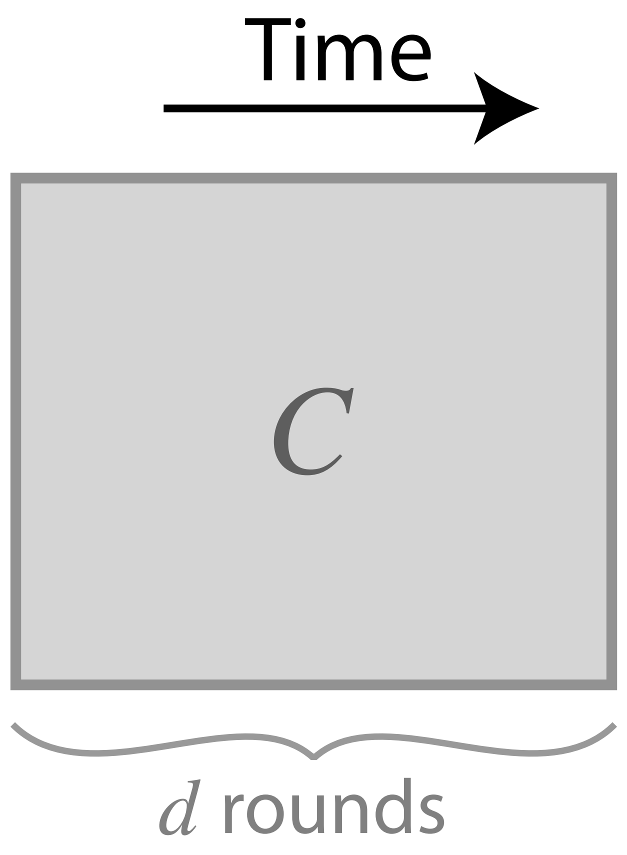

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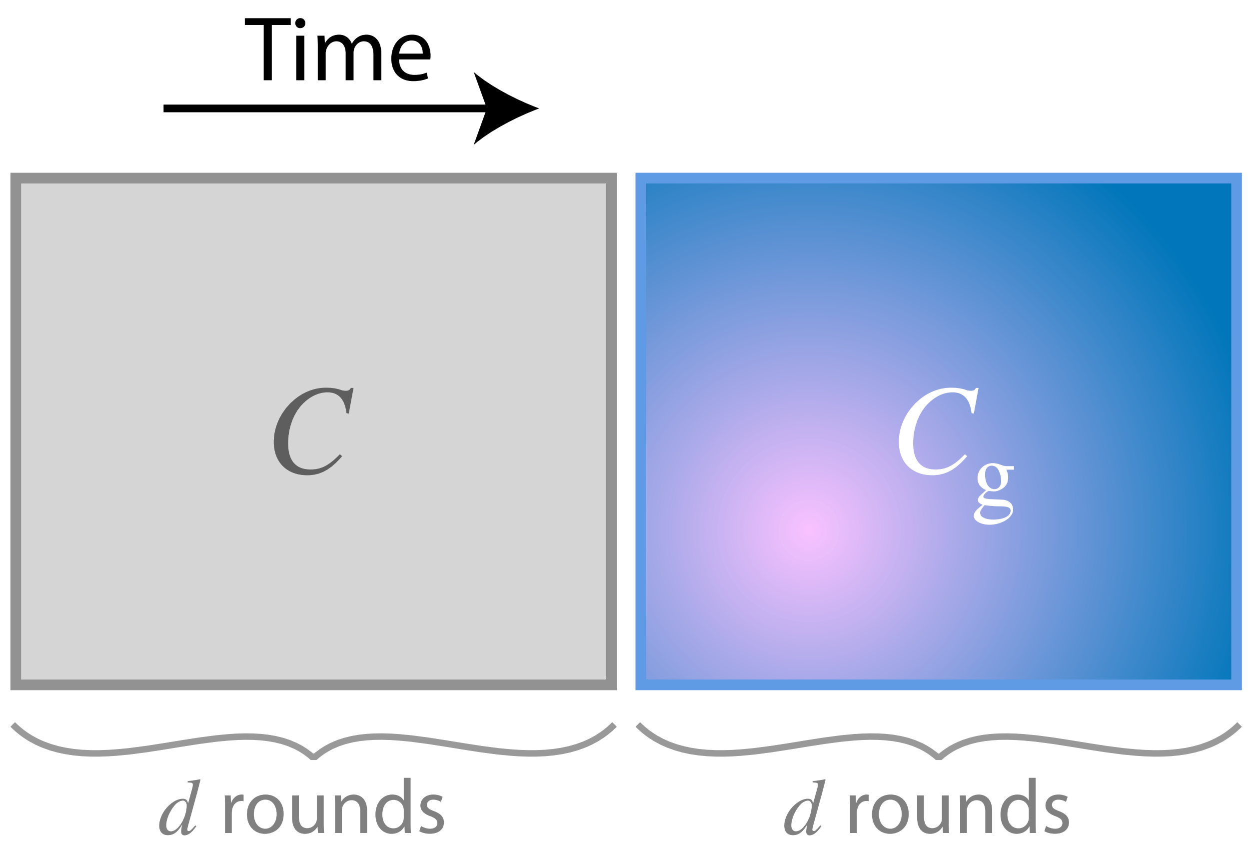

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2. Add ancilla qubits, measure stabilizers of C_{g}

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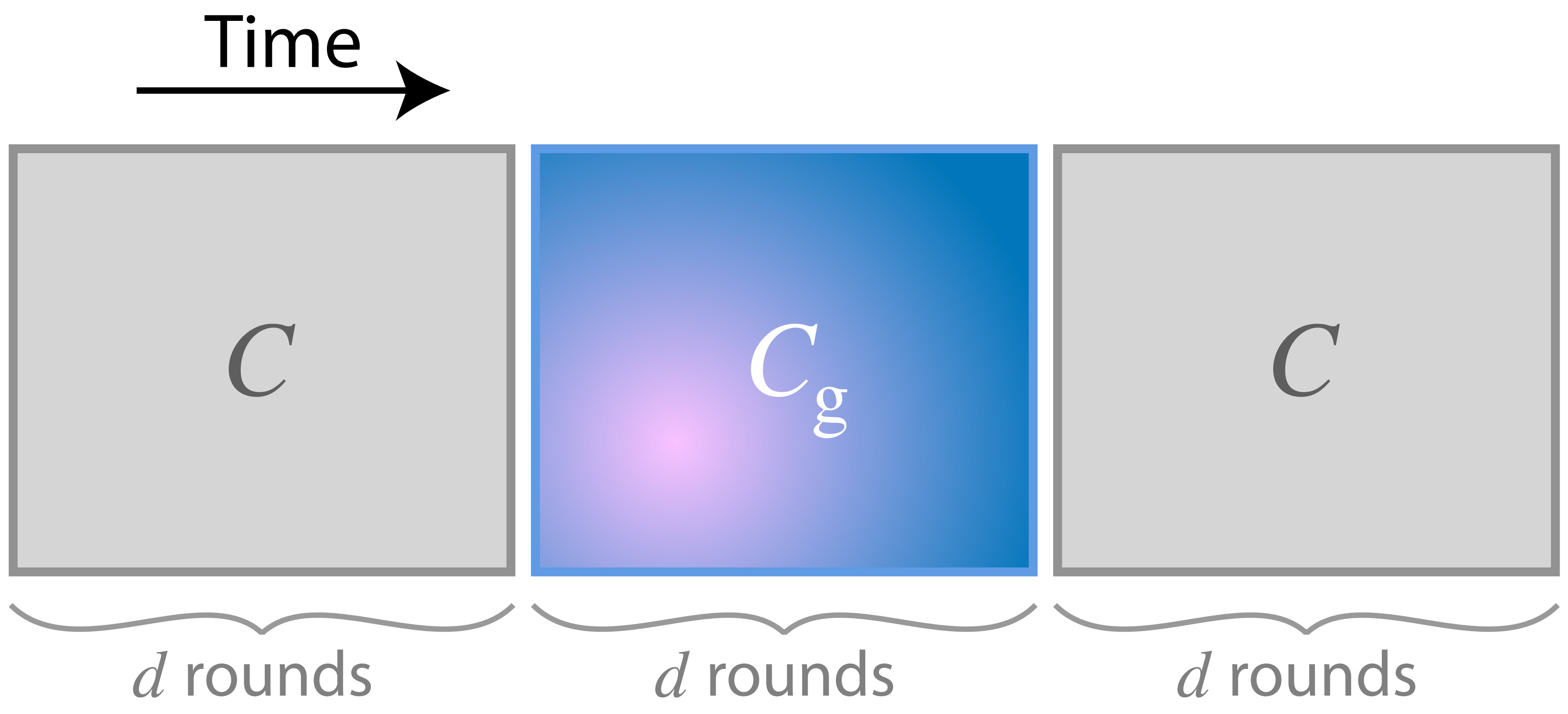
Let T be decomposable into a tensor product of constant-sized operators $\{\mathsf{T}_v\}_v$

$$\mathsf{T} = \otimes_v \mathsf{T}_v \qquad\qquad |\mathsf{T}_v| = O(1)$$

Then there is a protocol that switches to a gauged code, C_g, that measures T.
The whole protocol has an adversarial fault-distance to data qubit and measurement errors that is

$$d_F \geq d/\gamma \qquad\qquad \gamma \geq 1, \text{ and } \gamma = O(1).$$



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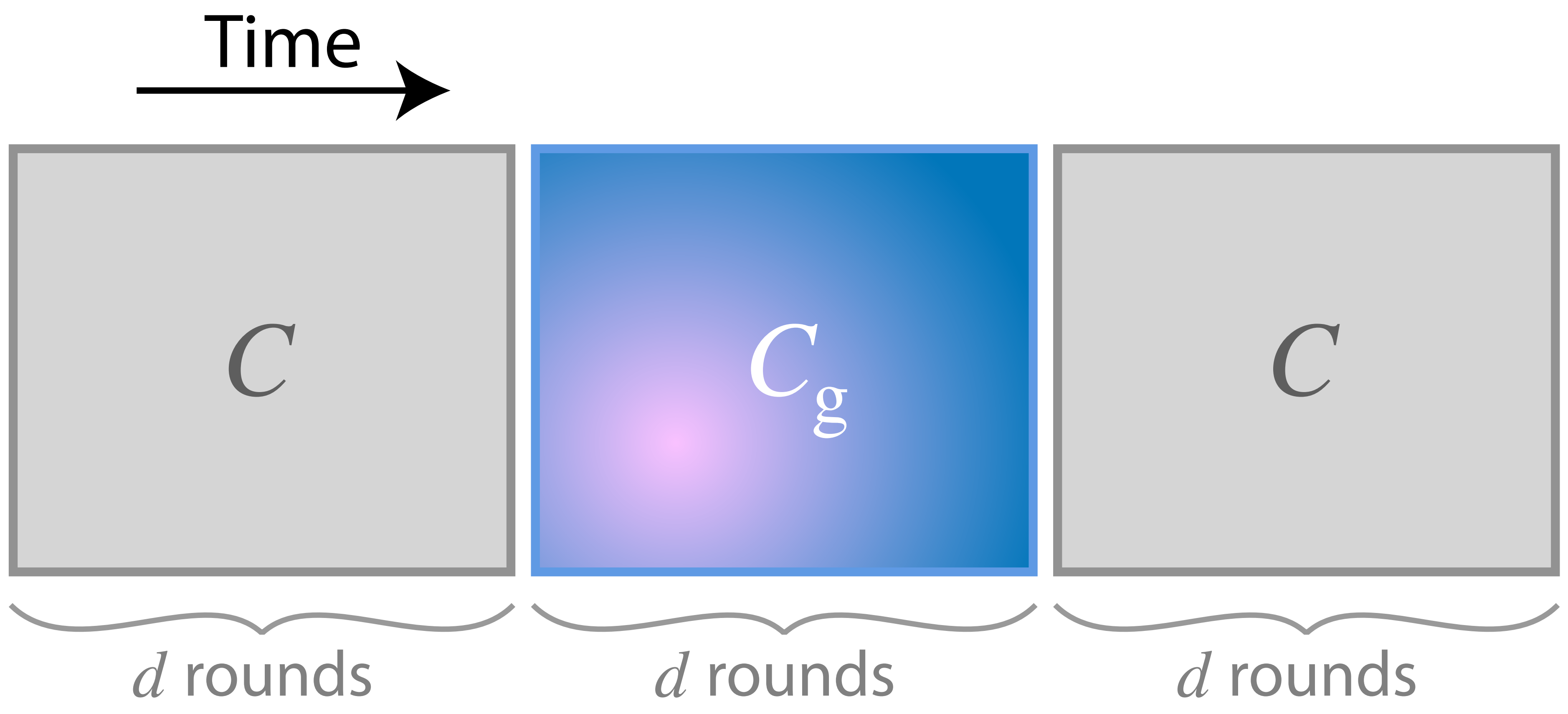

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Next: what is C_g?

Constructing the gauged code

Tantivasadakarn, et al. PRX **14** (2024)
Christos, et al. [2602.12228]

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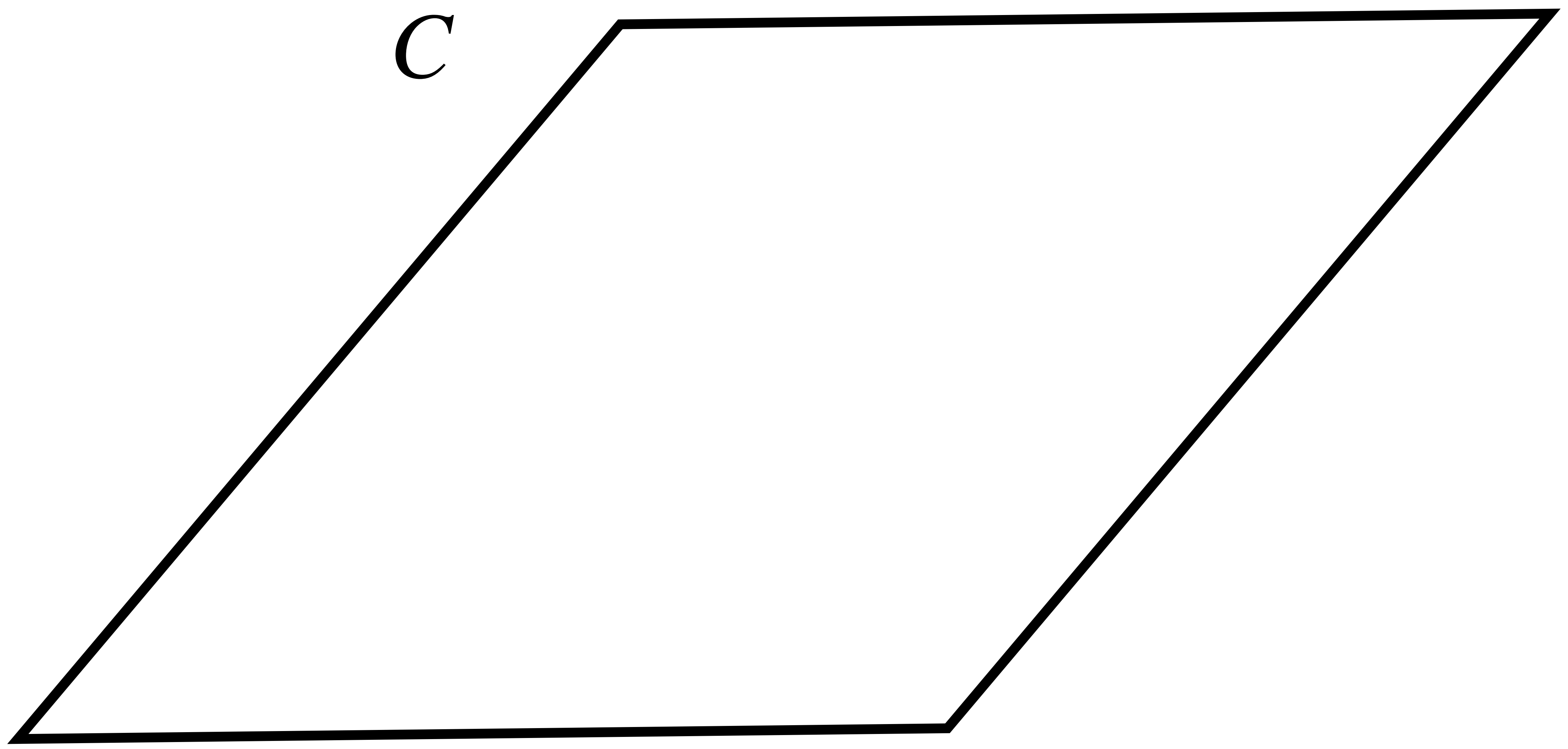

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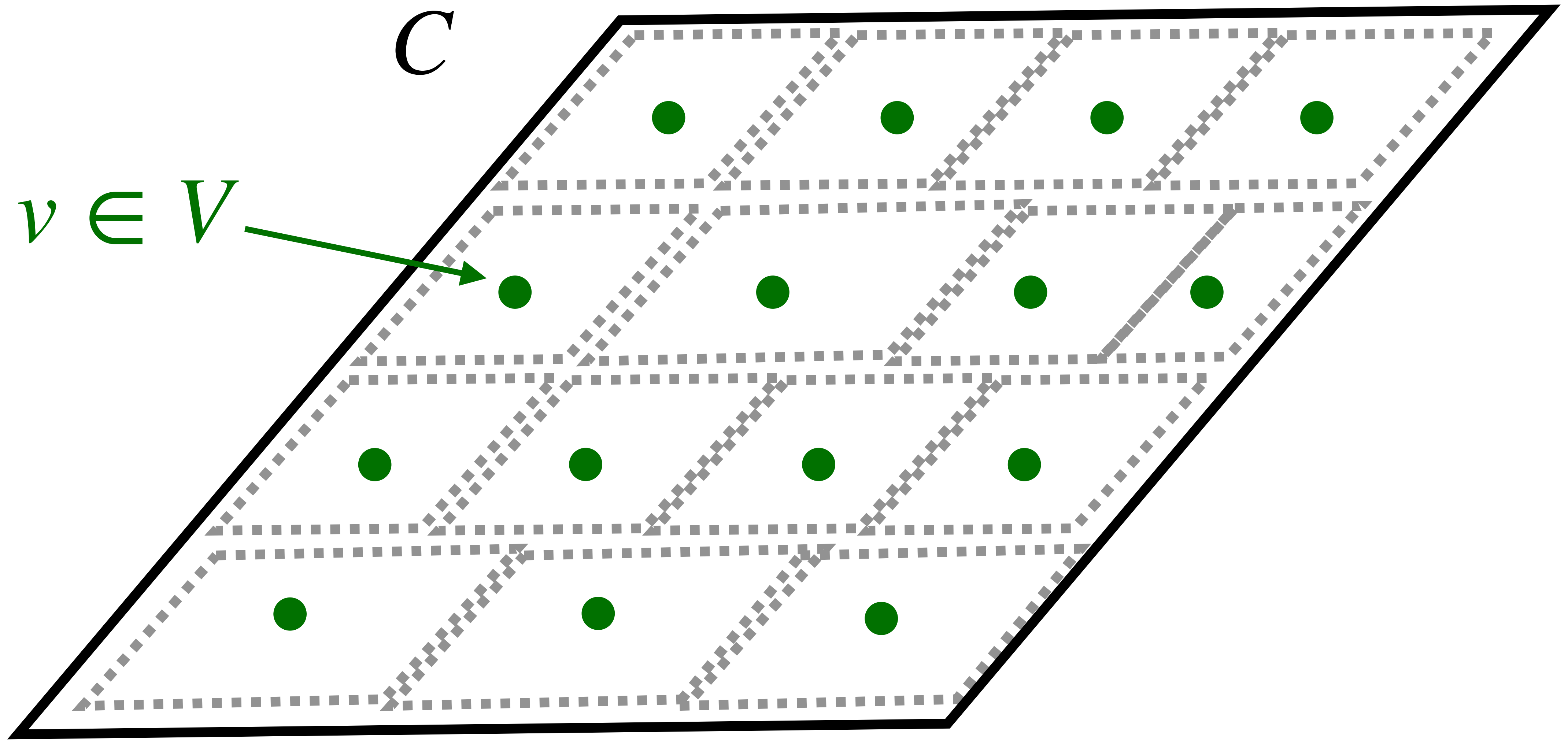

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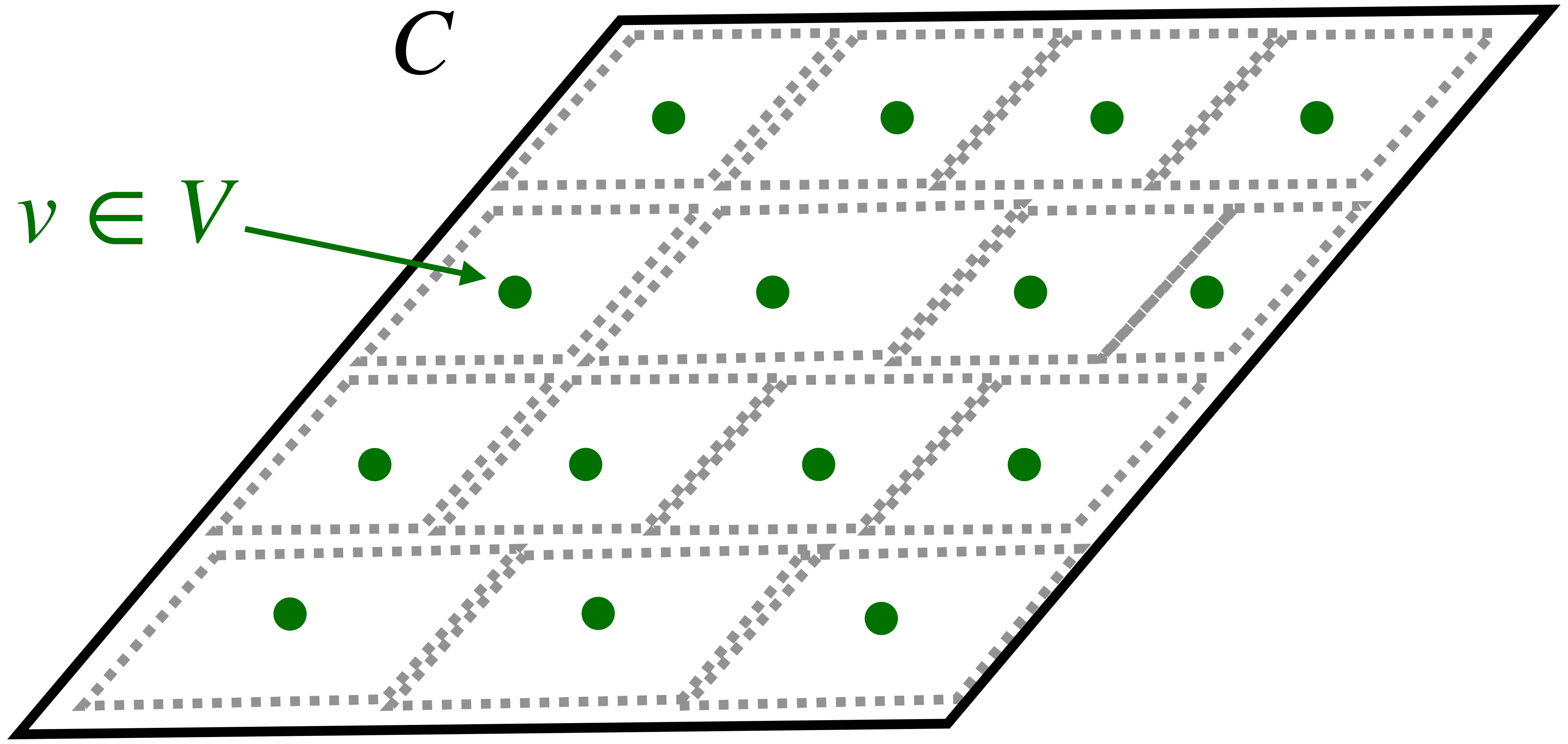

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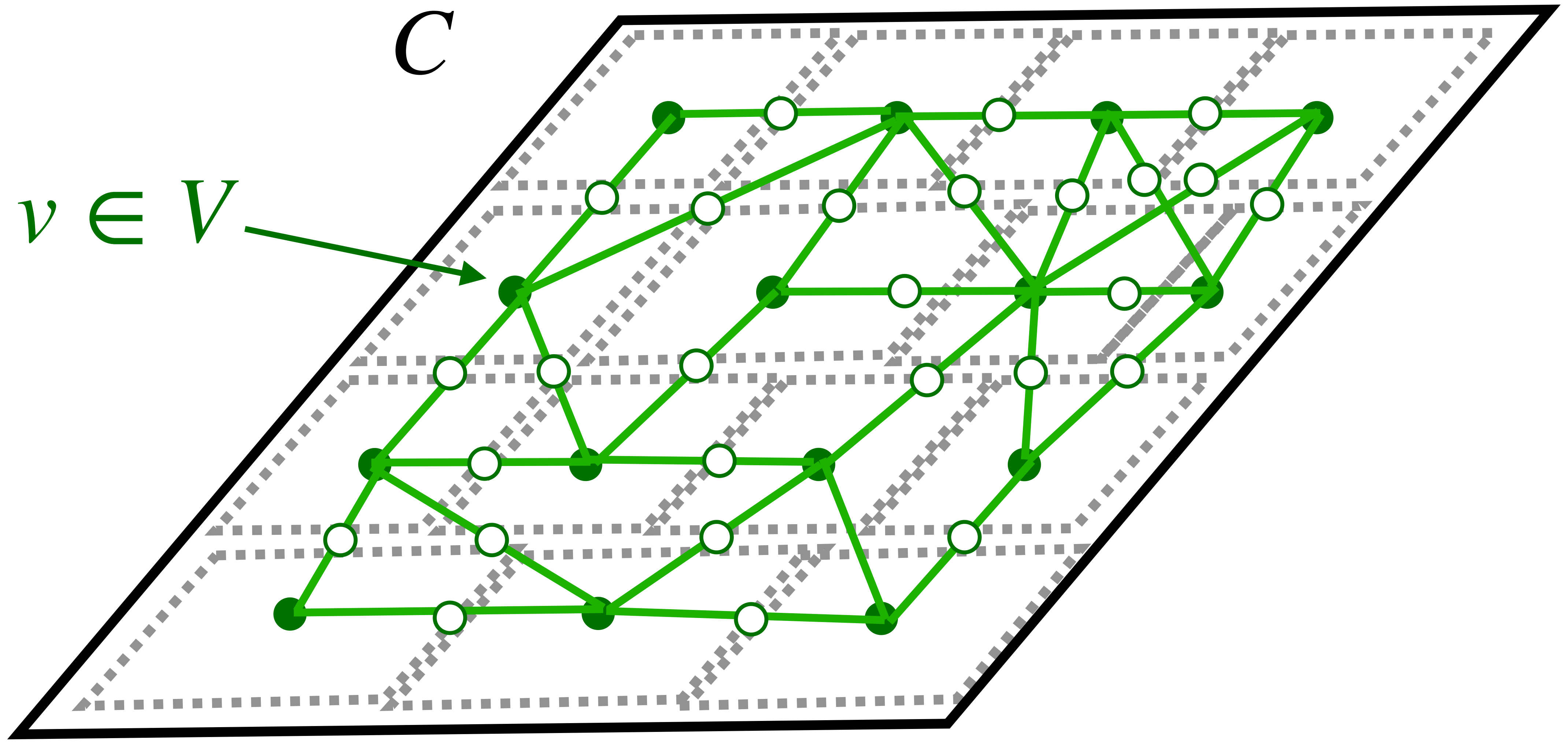

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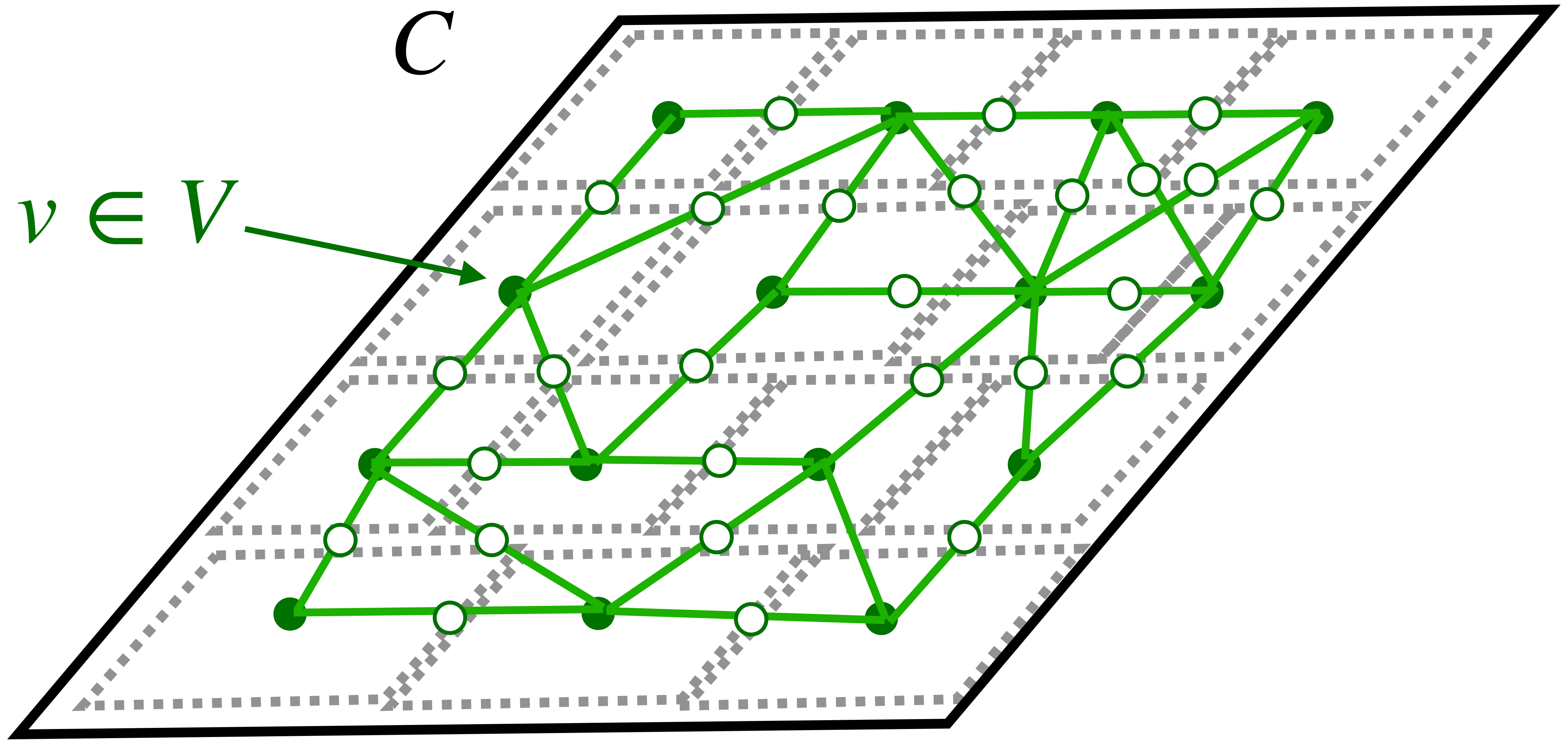

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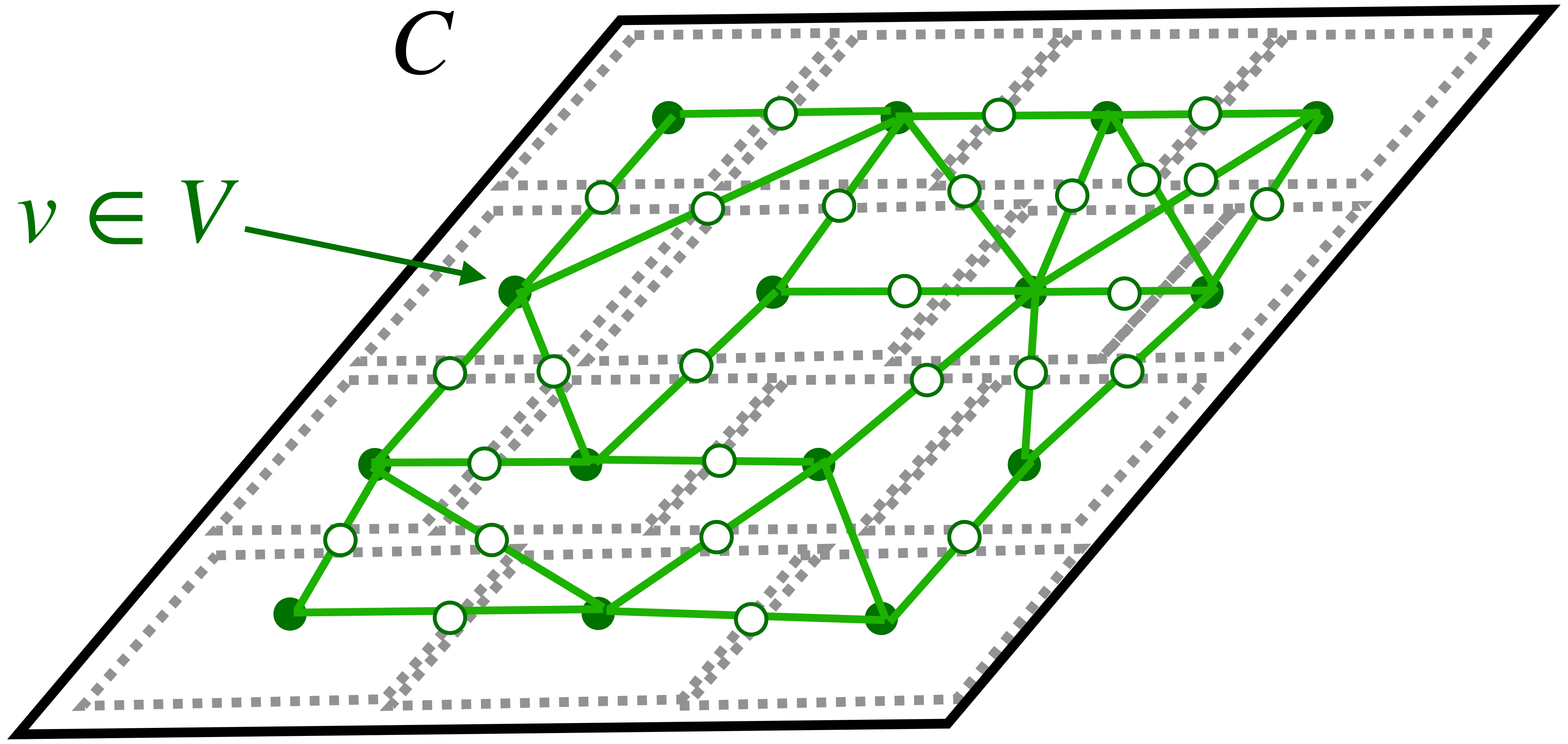

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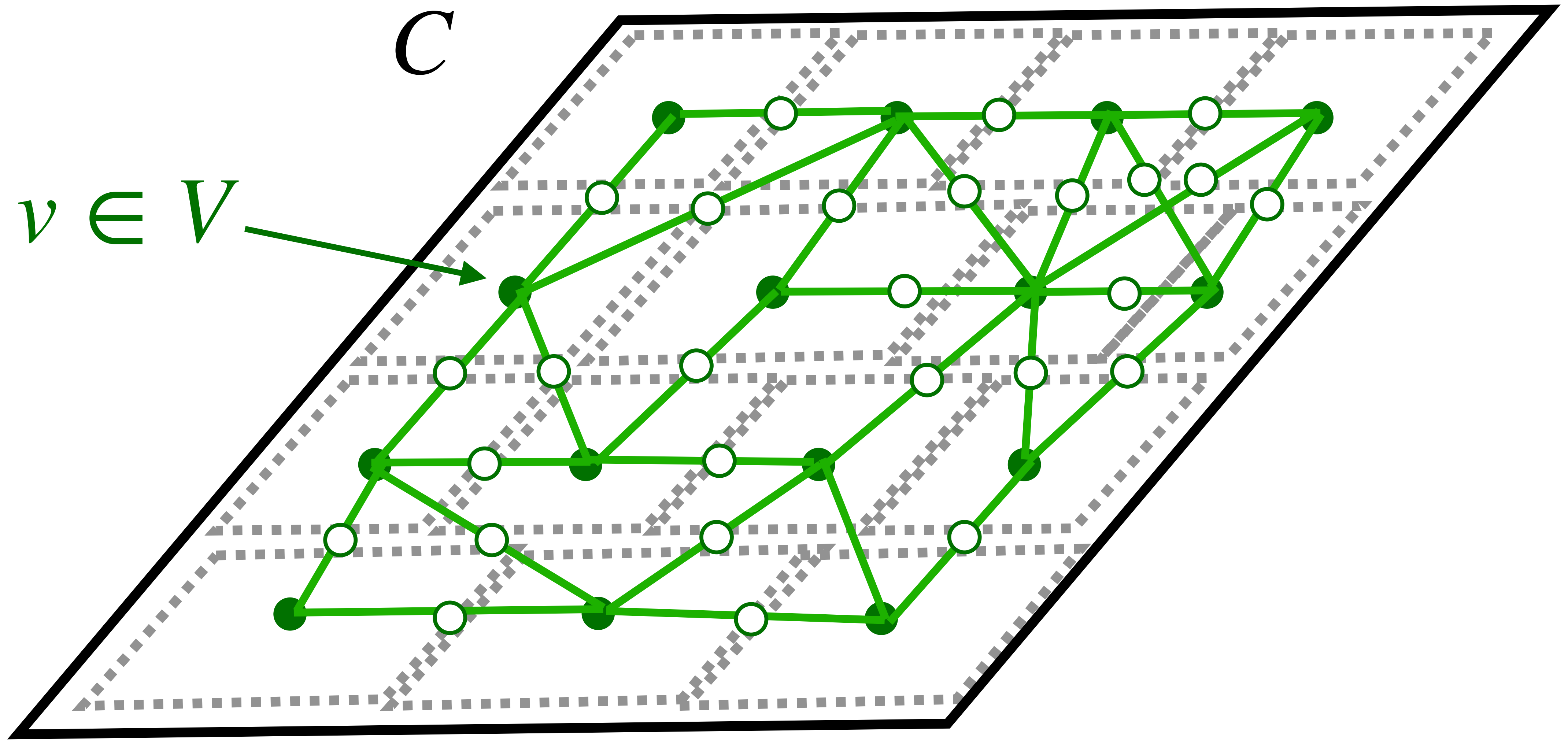

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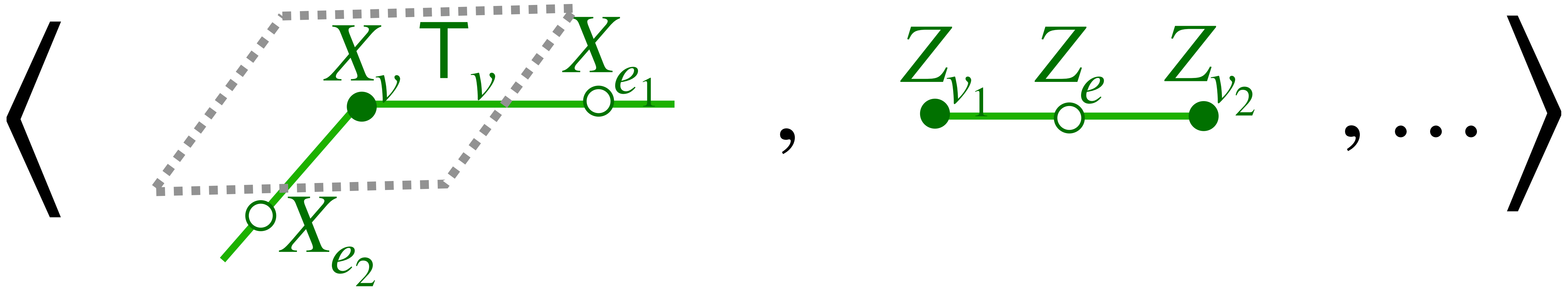

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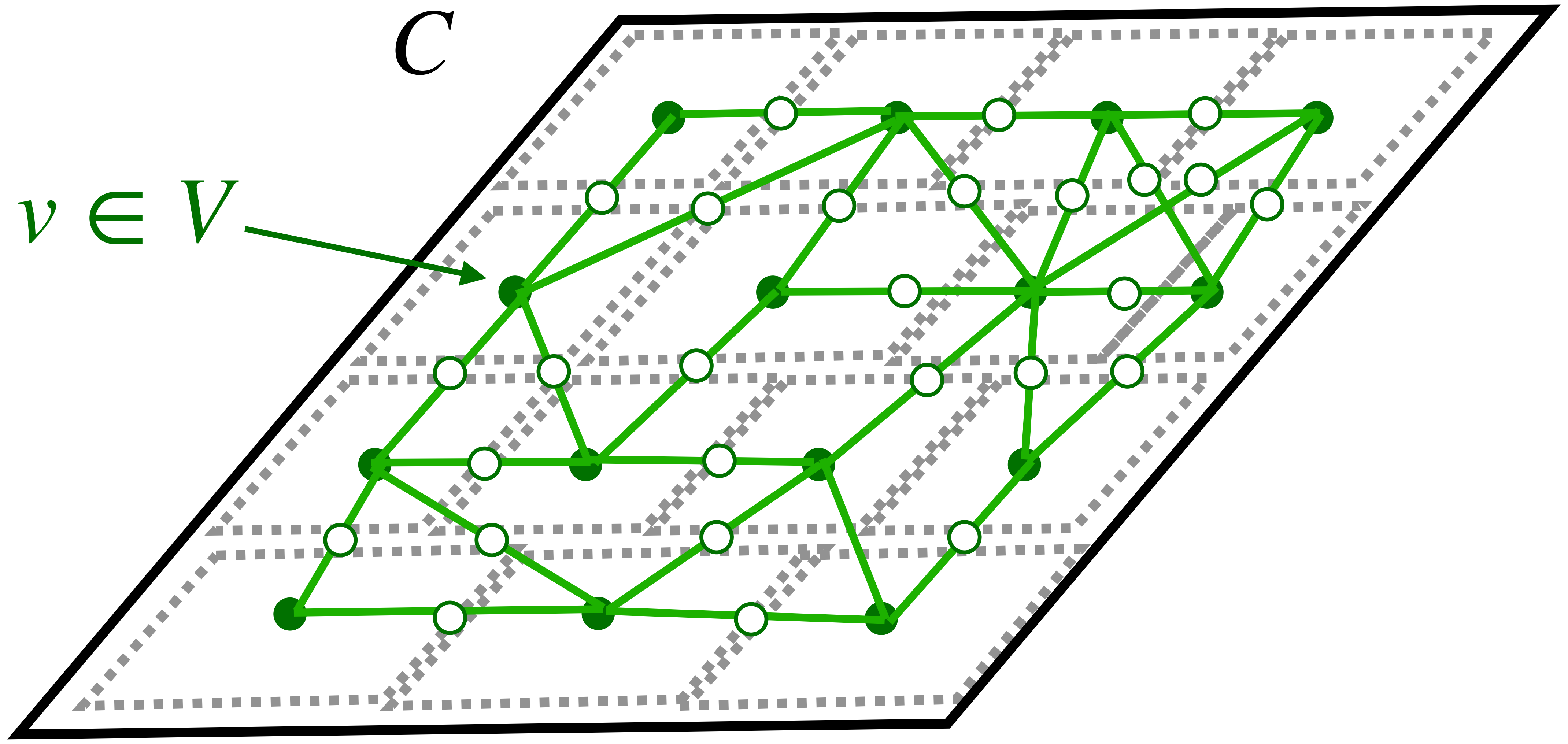

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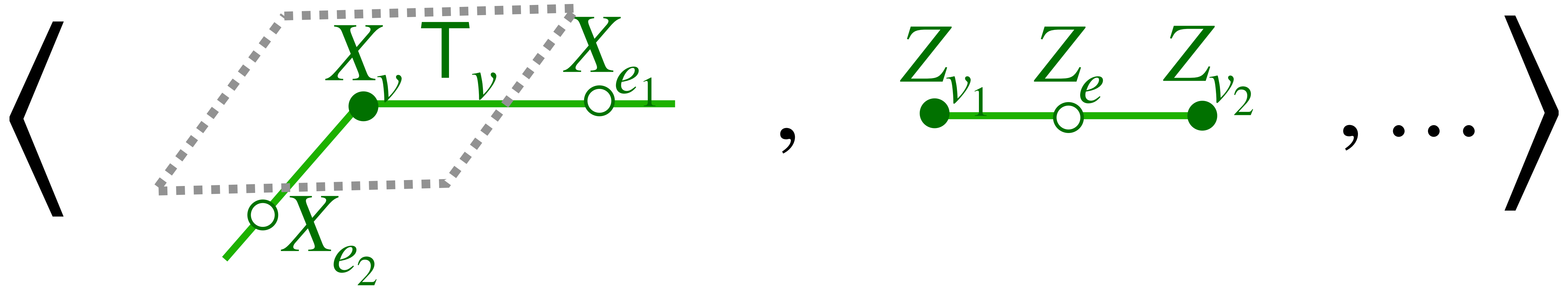

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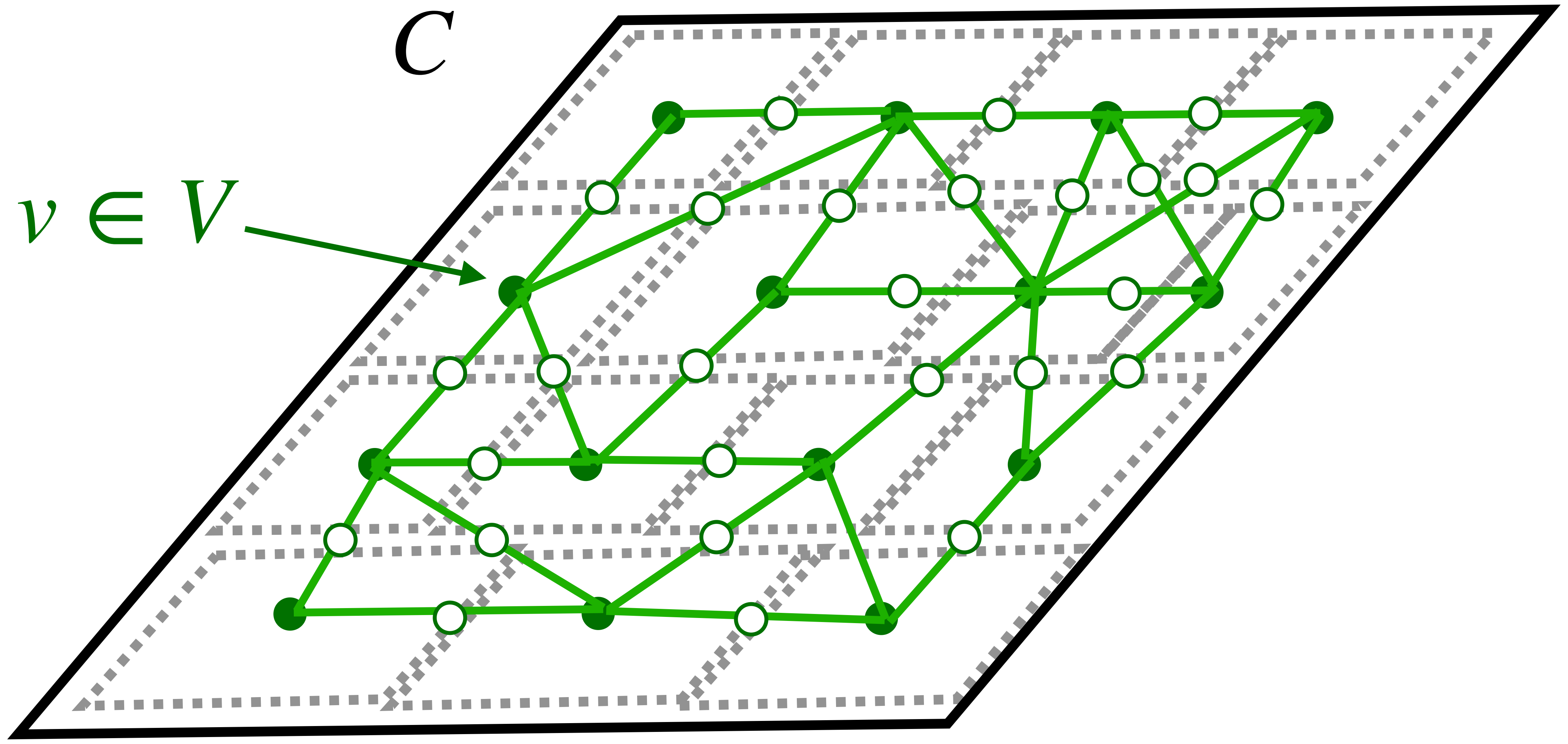

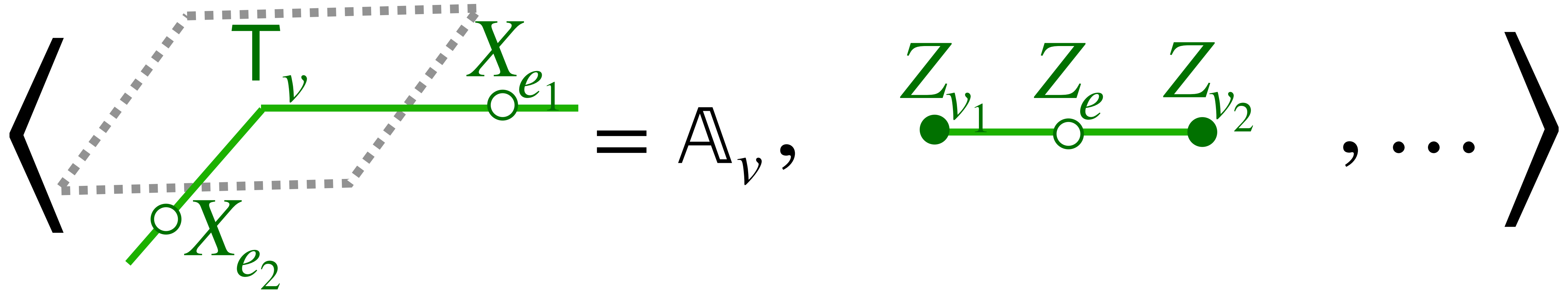

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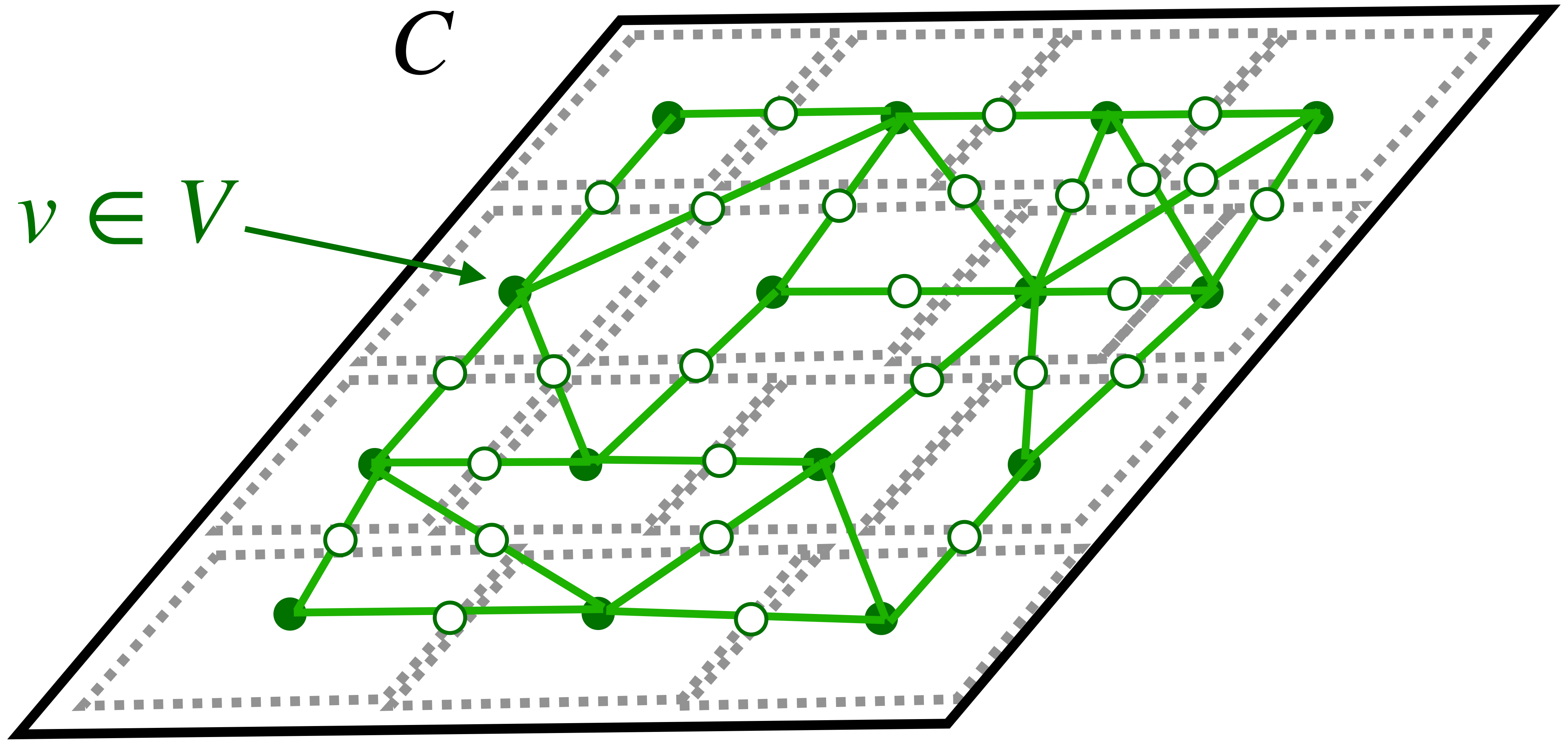

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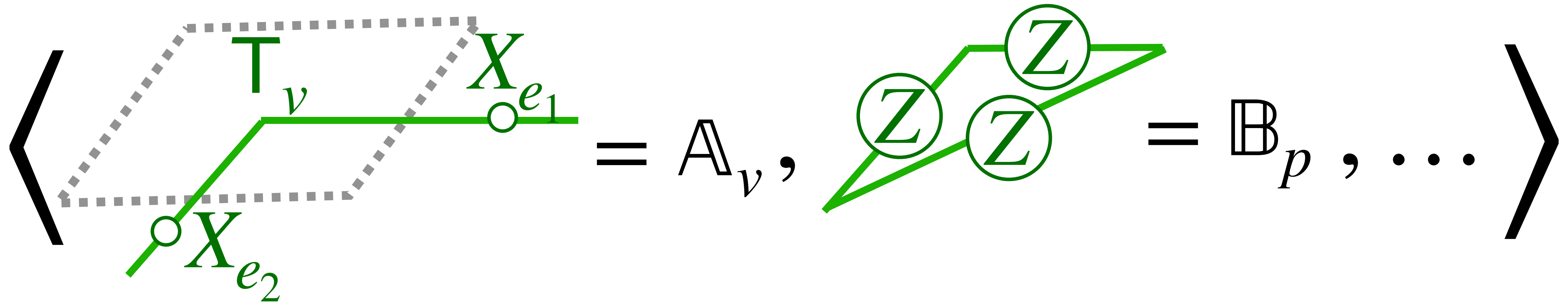

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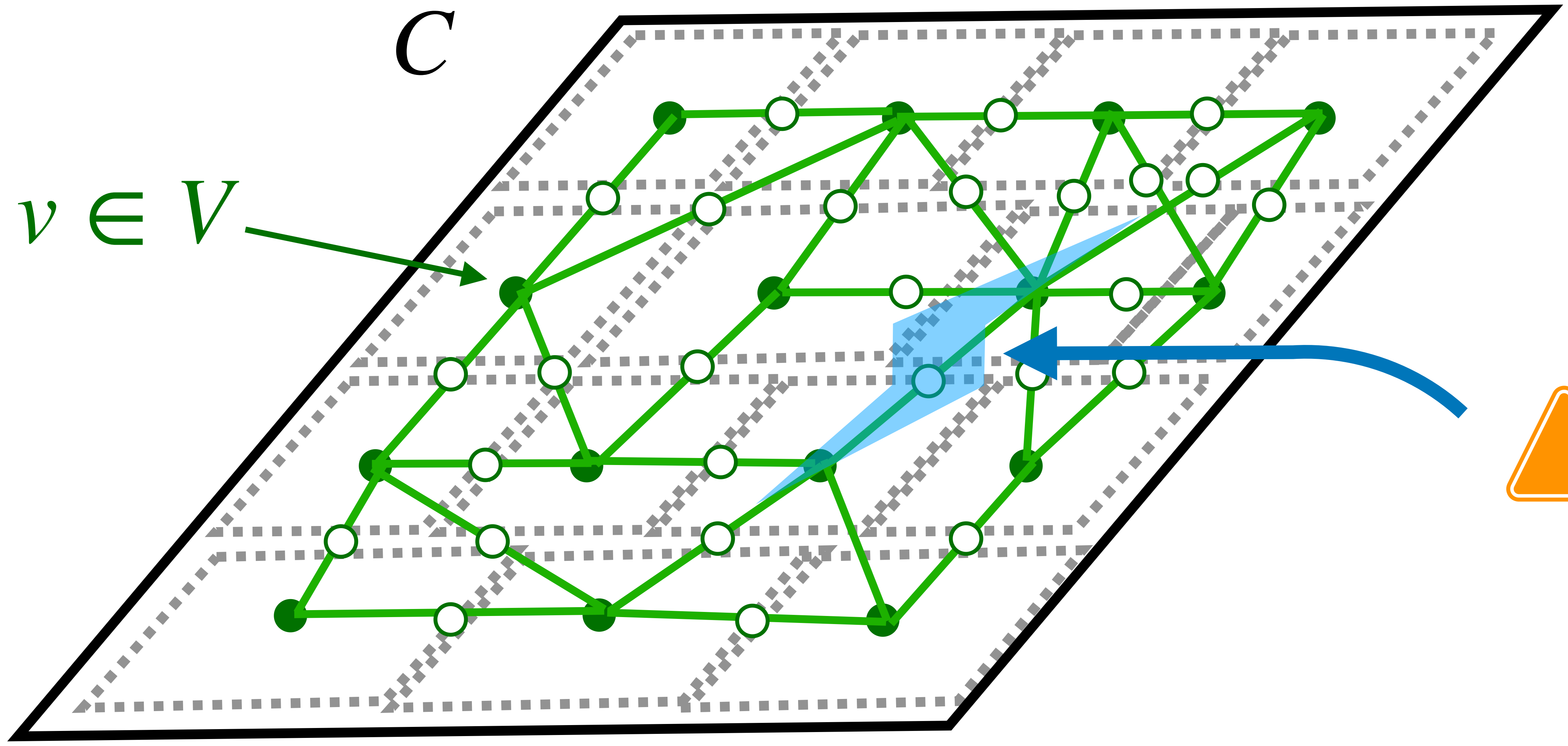

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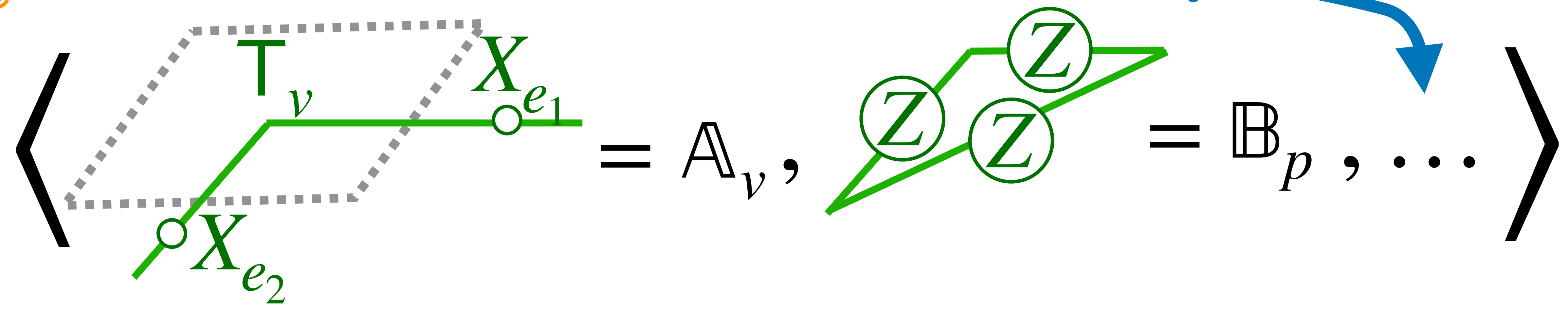

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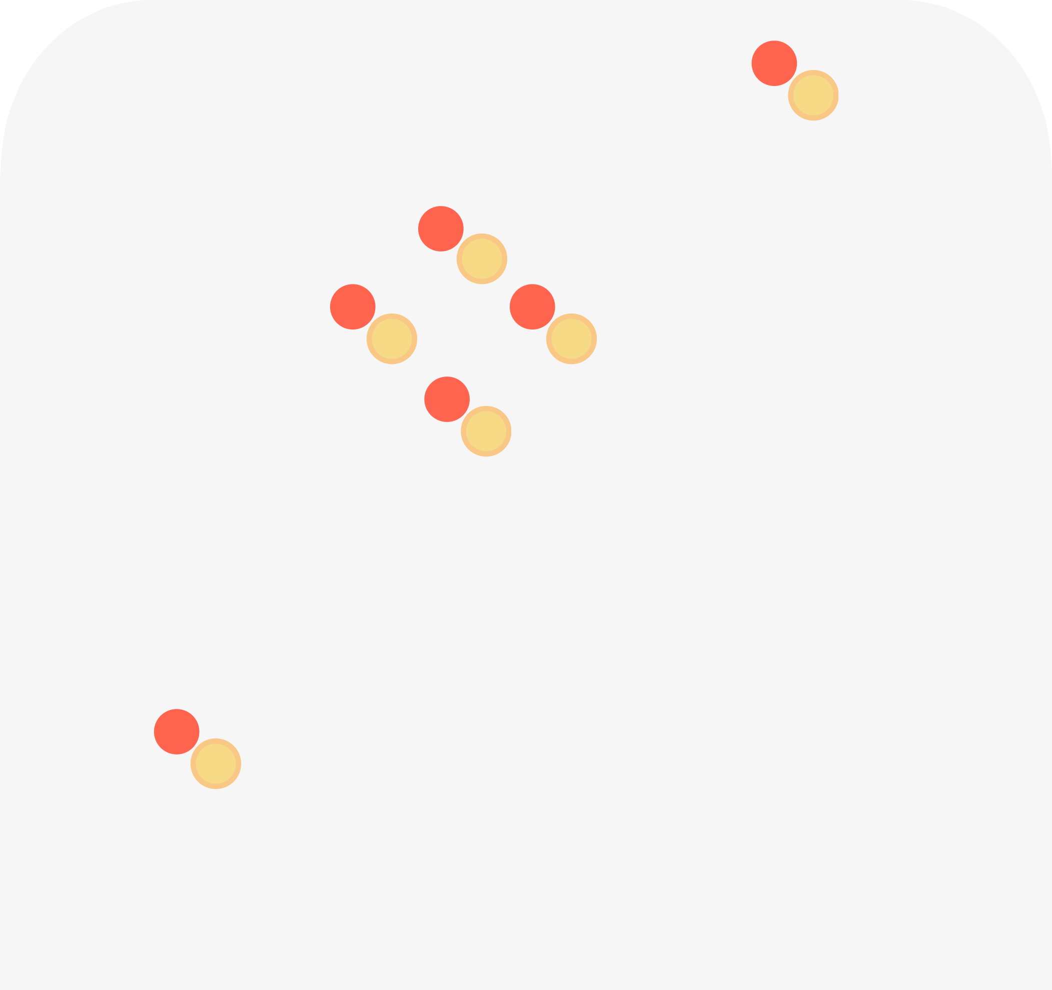

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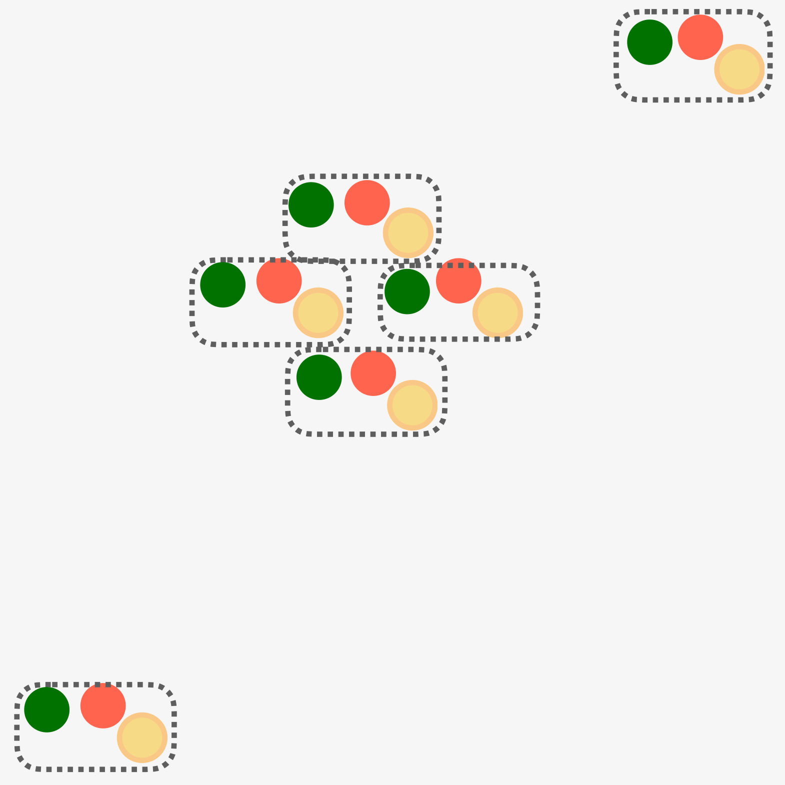

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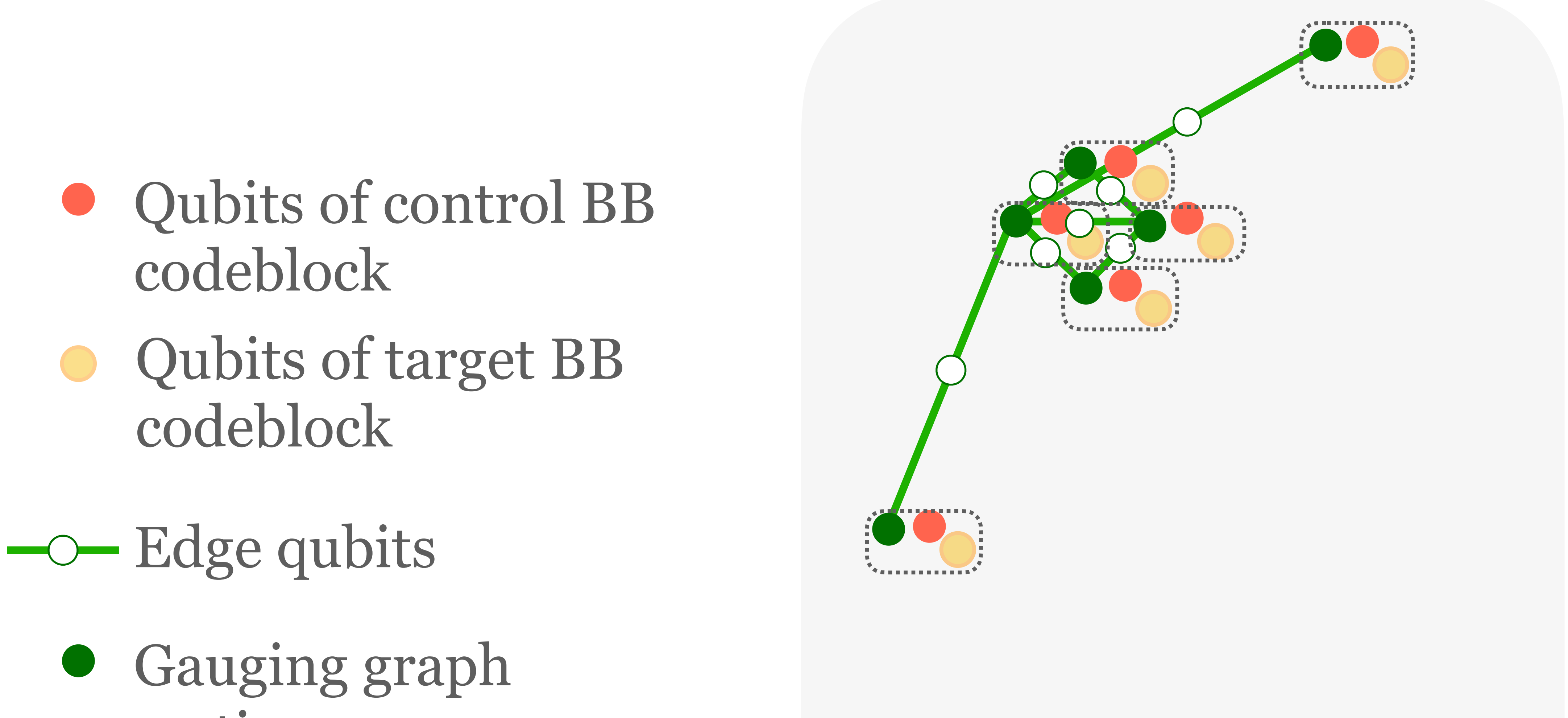

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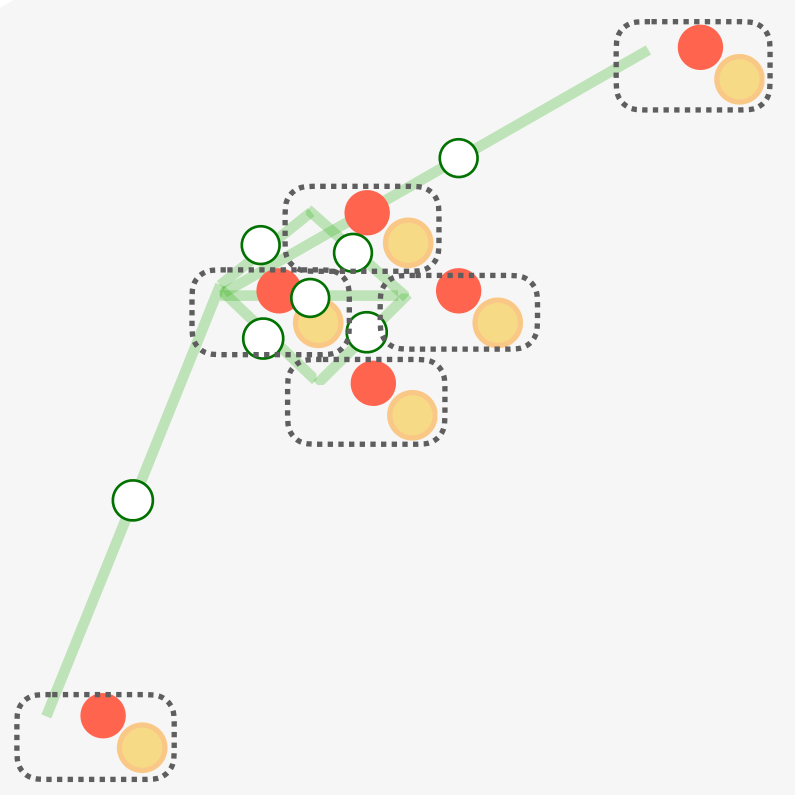

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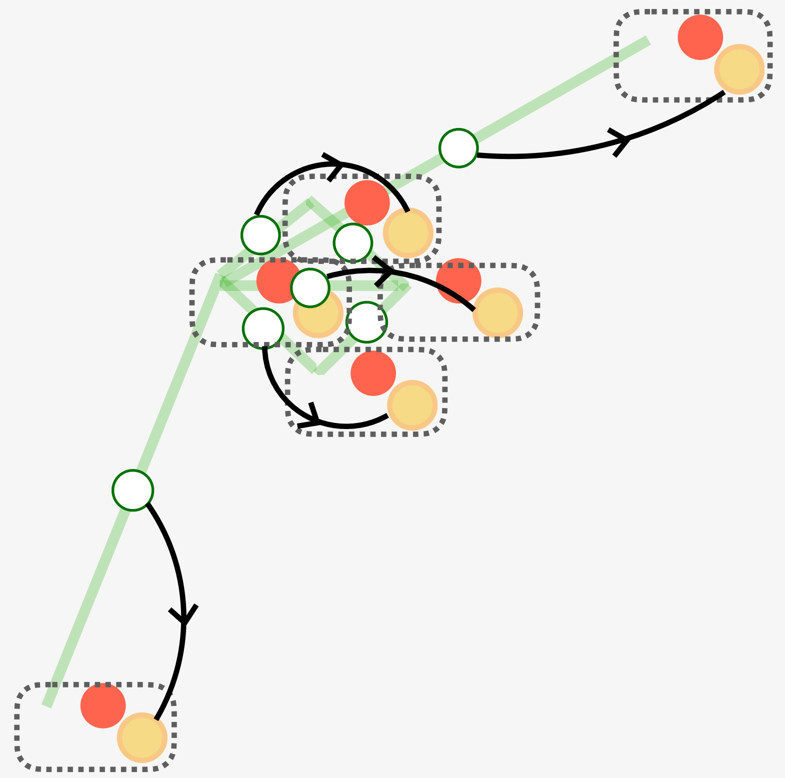

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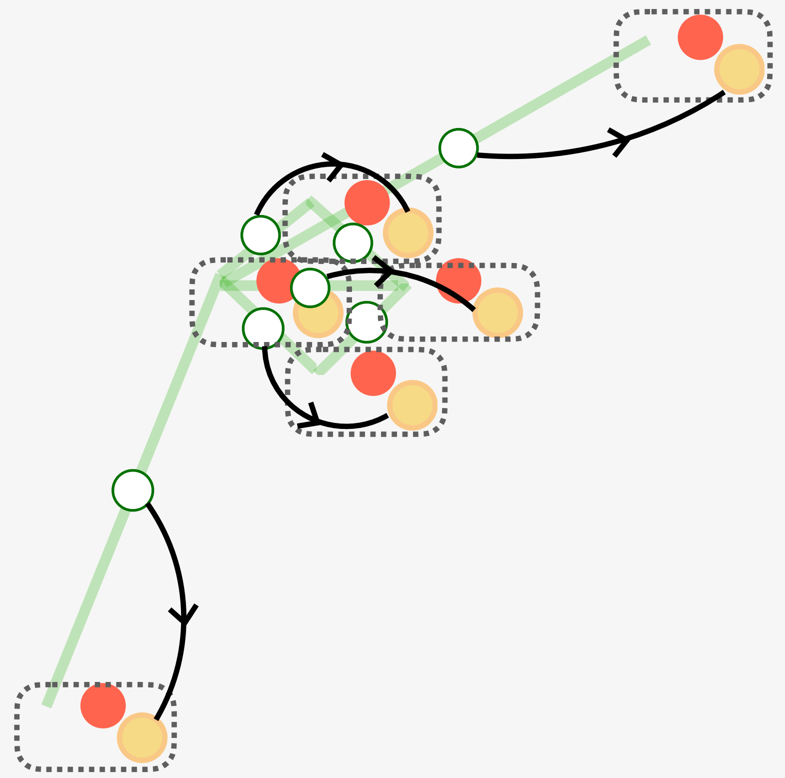

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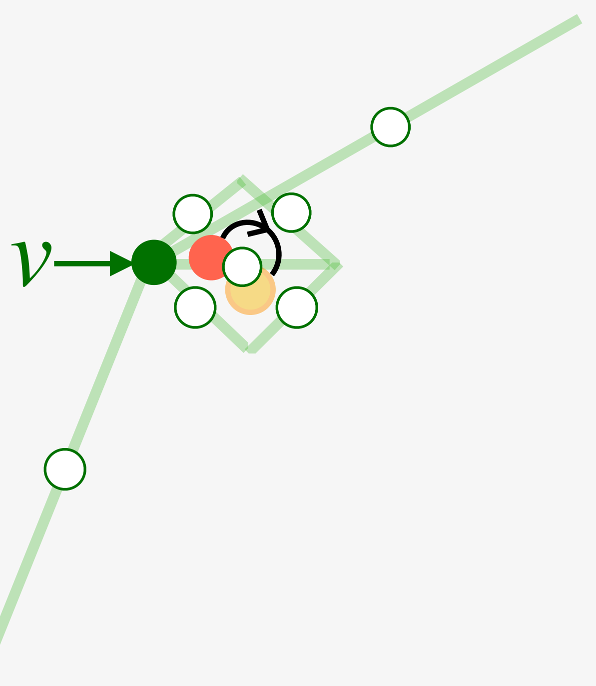

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A vertex stabilizer to measure logical CX

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$$[\![A, B]\!] := ABA^{-1}B^{-1} = I \text{ if } A, B \text{ commute}$$

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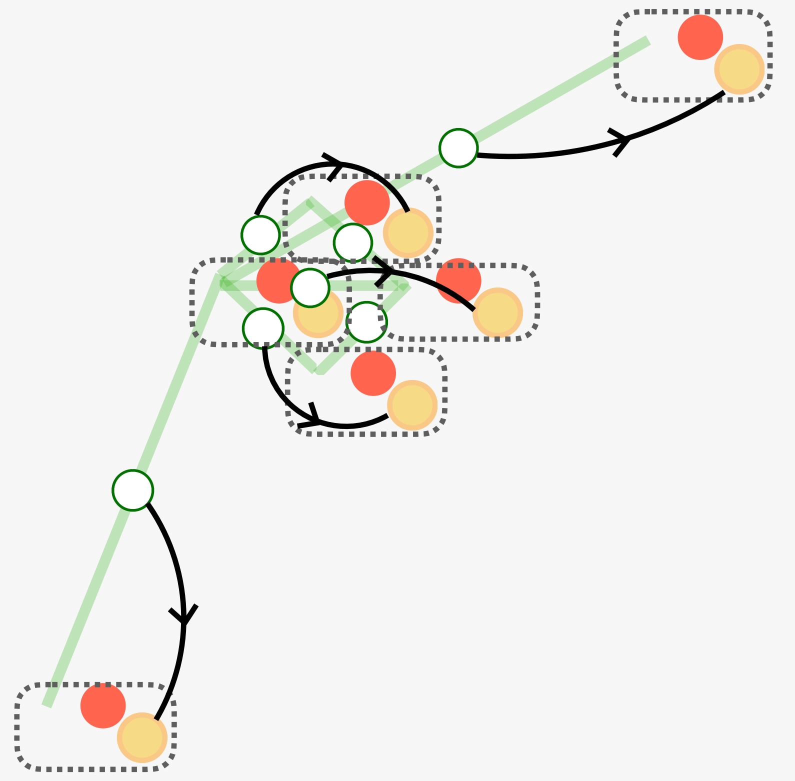

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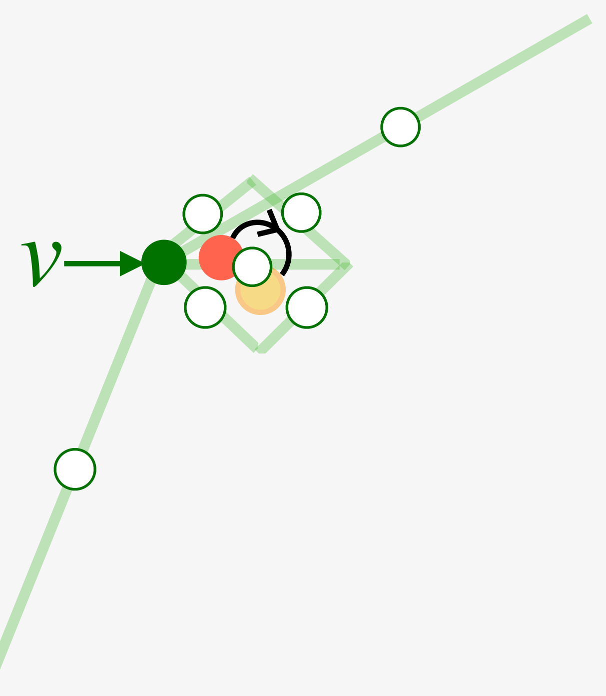

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- ☑ Some product of stabilizers of C_g should equal the Clifford logical, T. $(\prod_v \mathbb{A}_v = \mathsf{T})$
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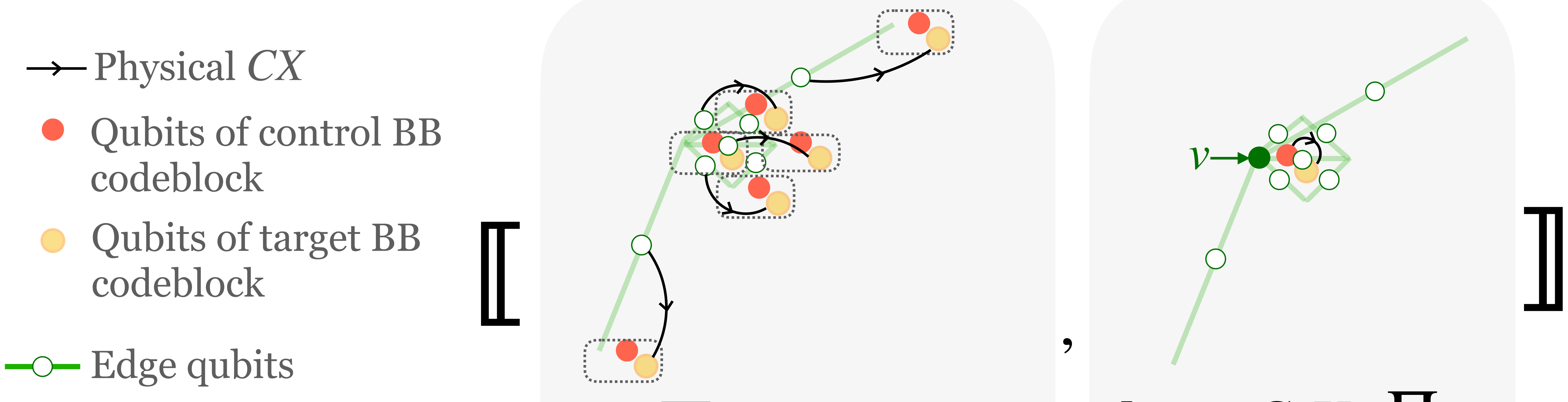
Ex. Stabilizers of BB code when gauging transversal $\overline{CX}$ $\quad [\![A, B]\!] := ABA^{-1}B^{-1} = I$ if A, B commute



$$[\![\ \mathbb{S} = \prod_{\color{red}\bullet} X_{\color{red}\bullet} \prod_{\to} C_{\circ} X_{\color{orange}\bullet}\ ,\ \mathbb{A}_v = C_{\color{red}\bullet} X_{\color{orange}\bullet} \prod_{\circ \in \delta(v)} X_{\circ}\ ]\!]$$

A decorated X-stabilizer of the control BB code

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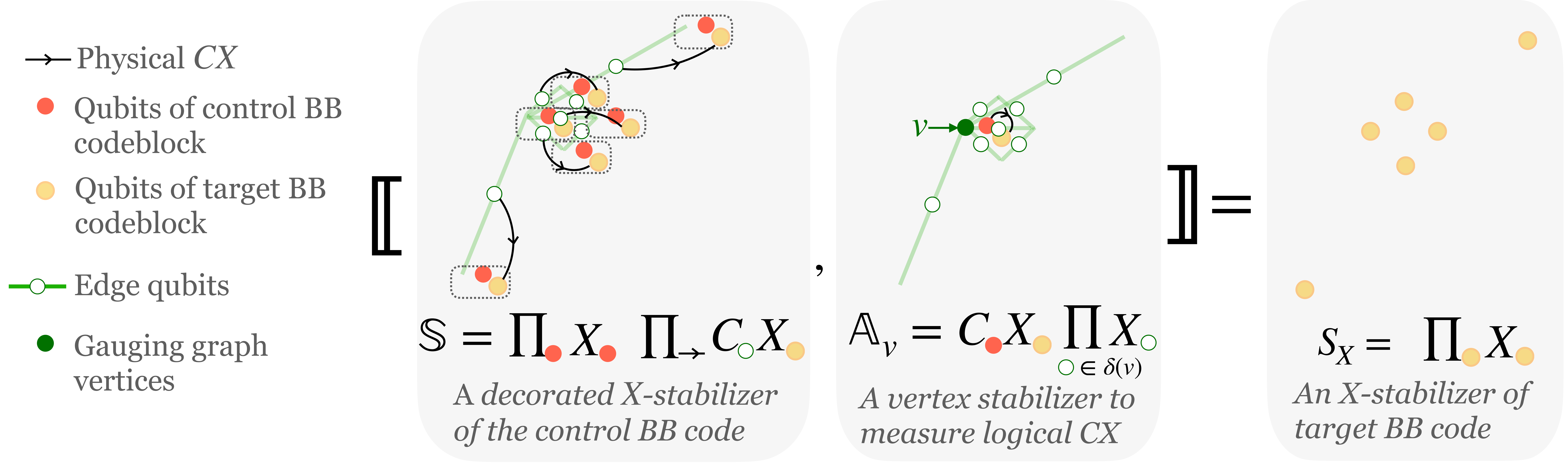

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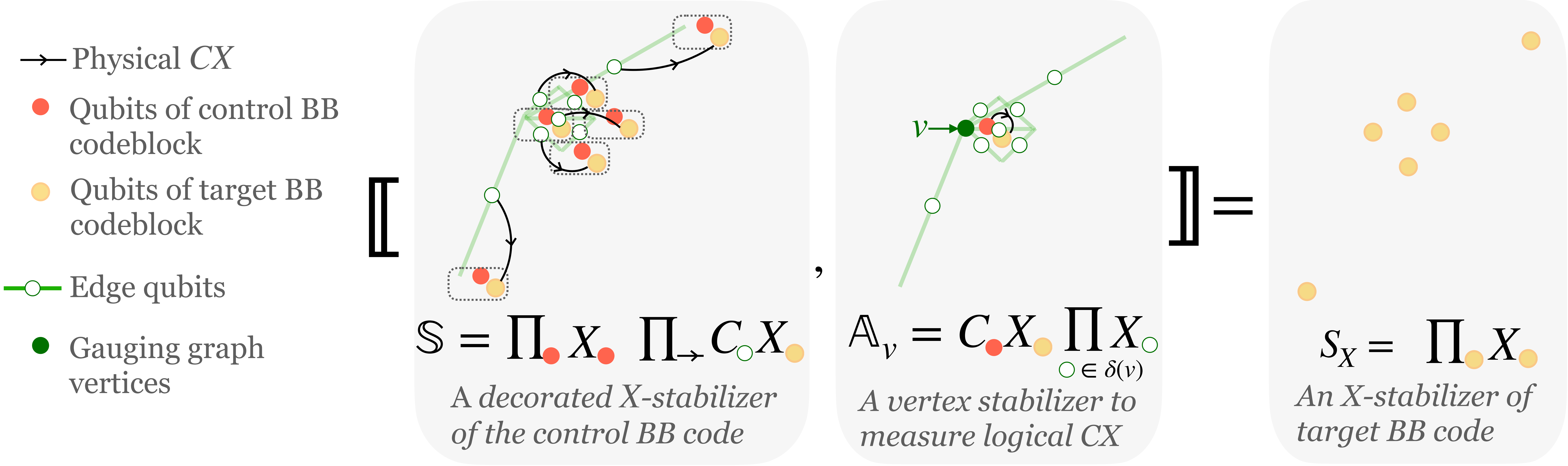

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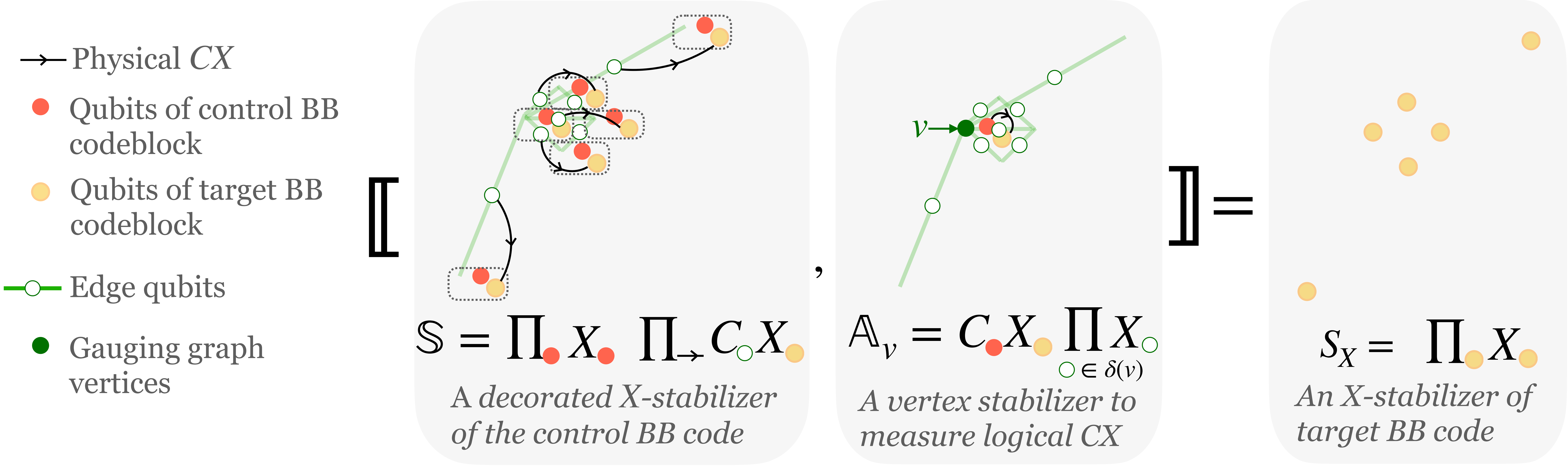

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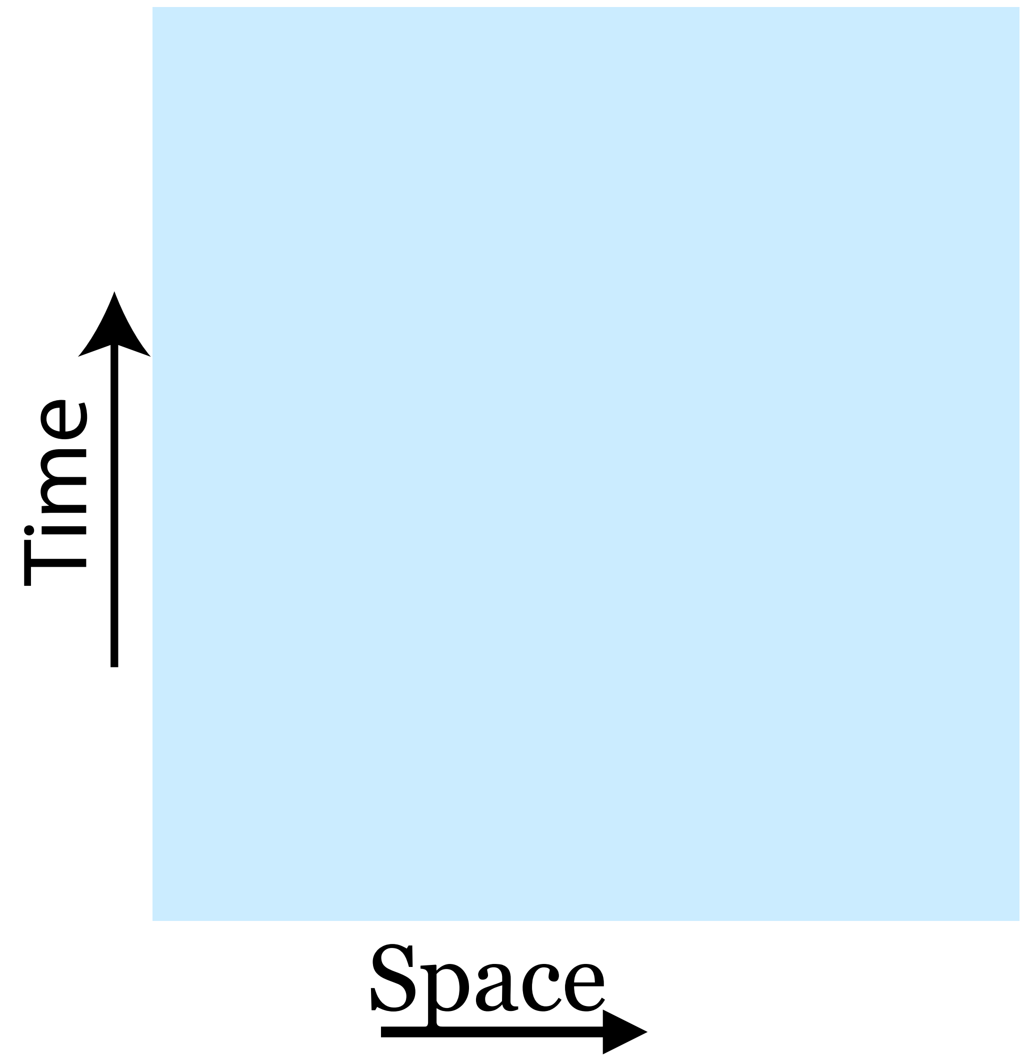

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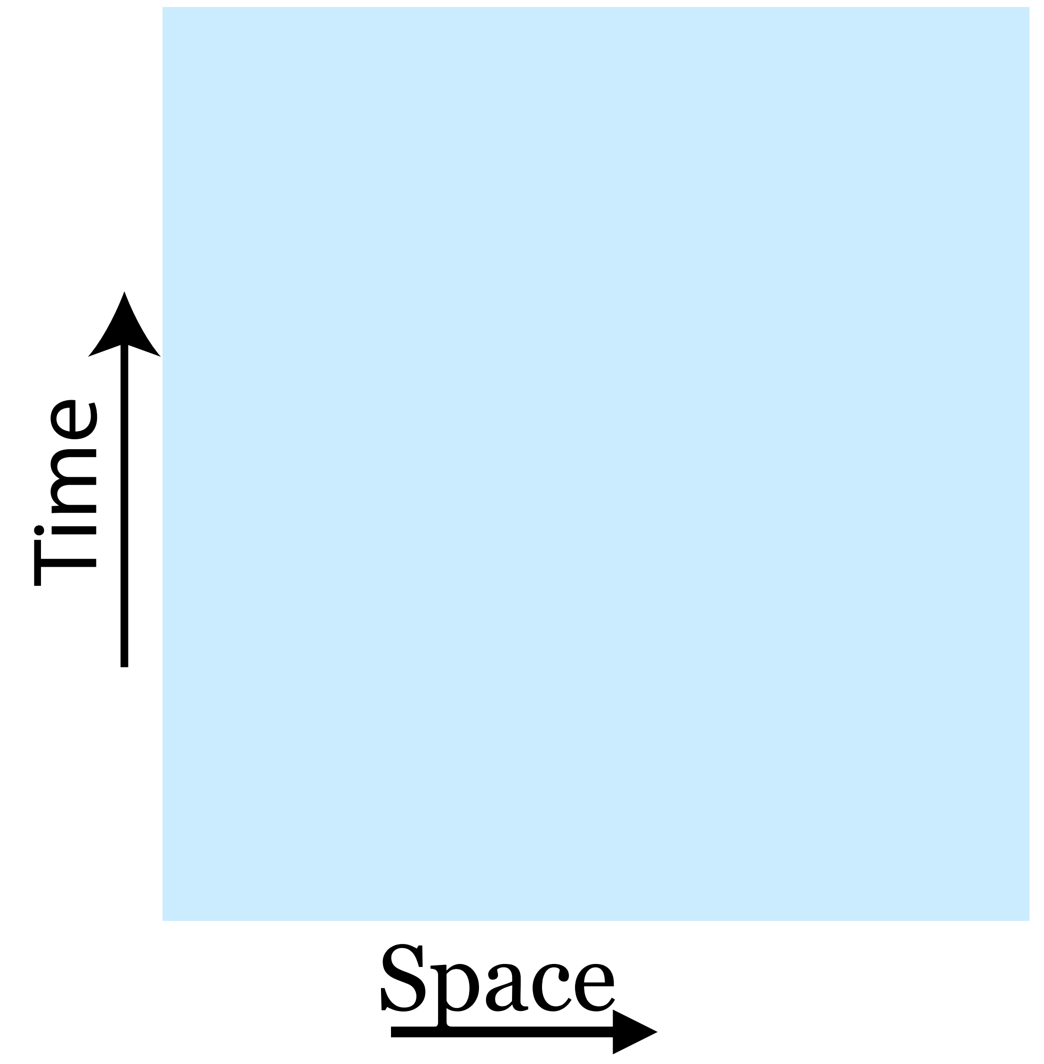

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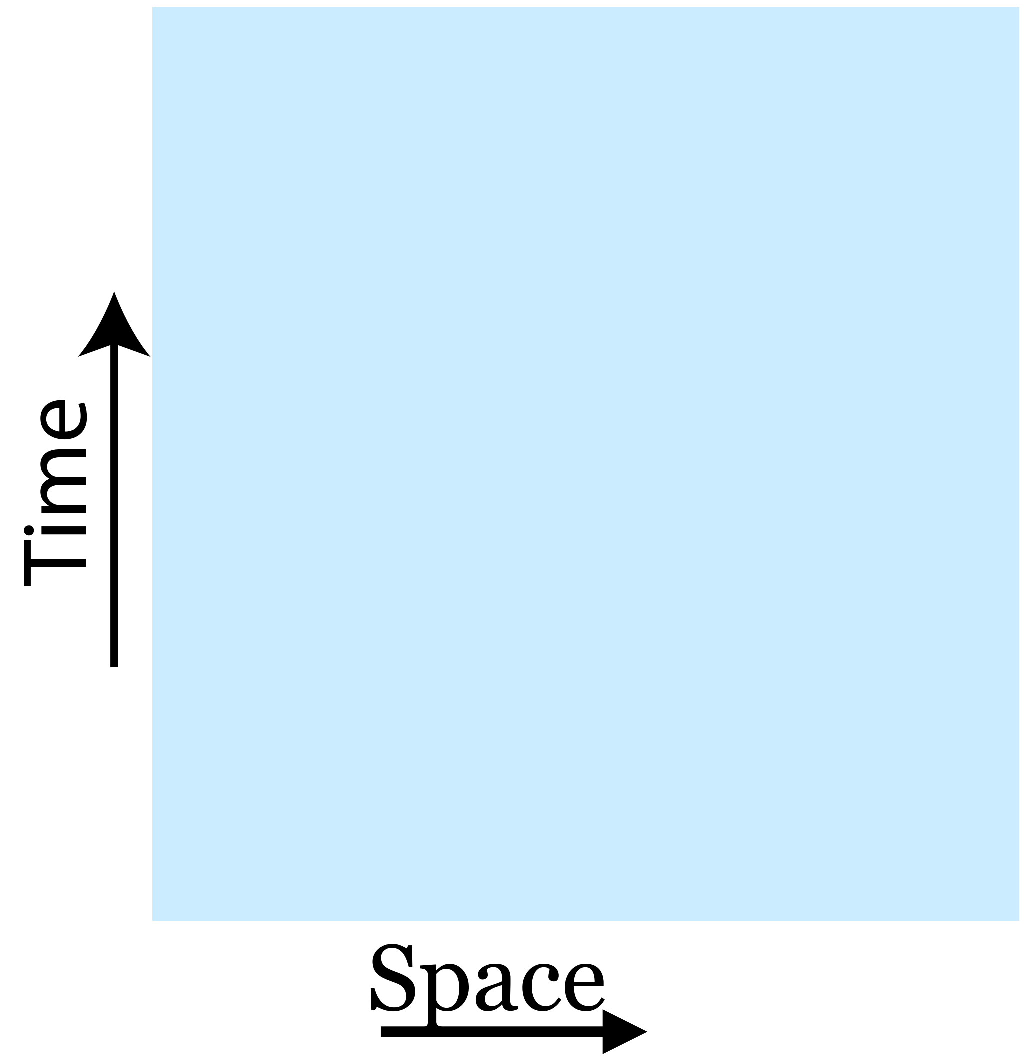

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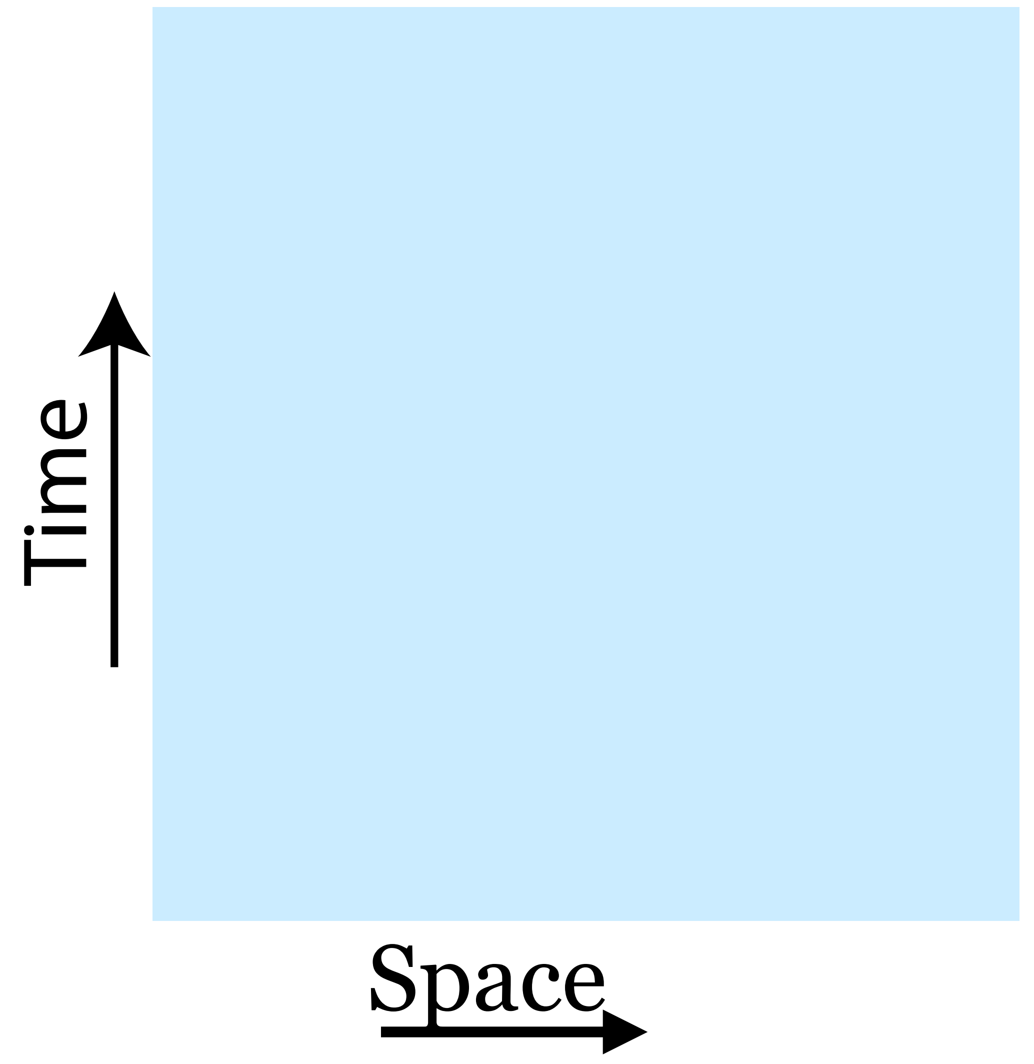

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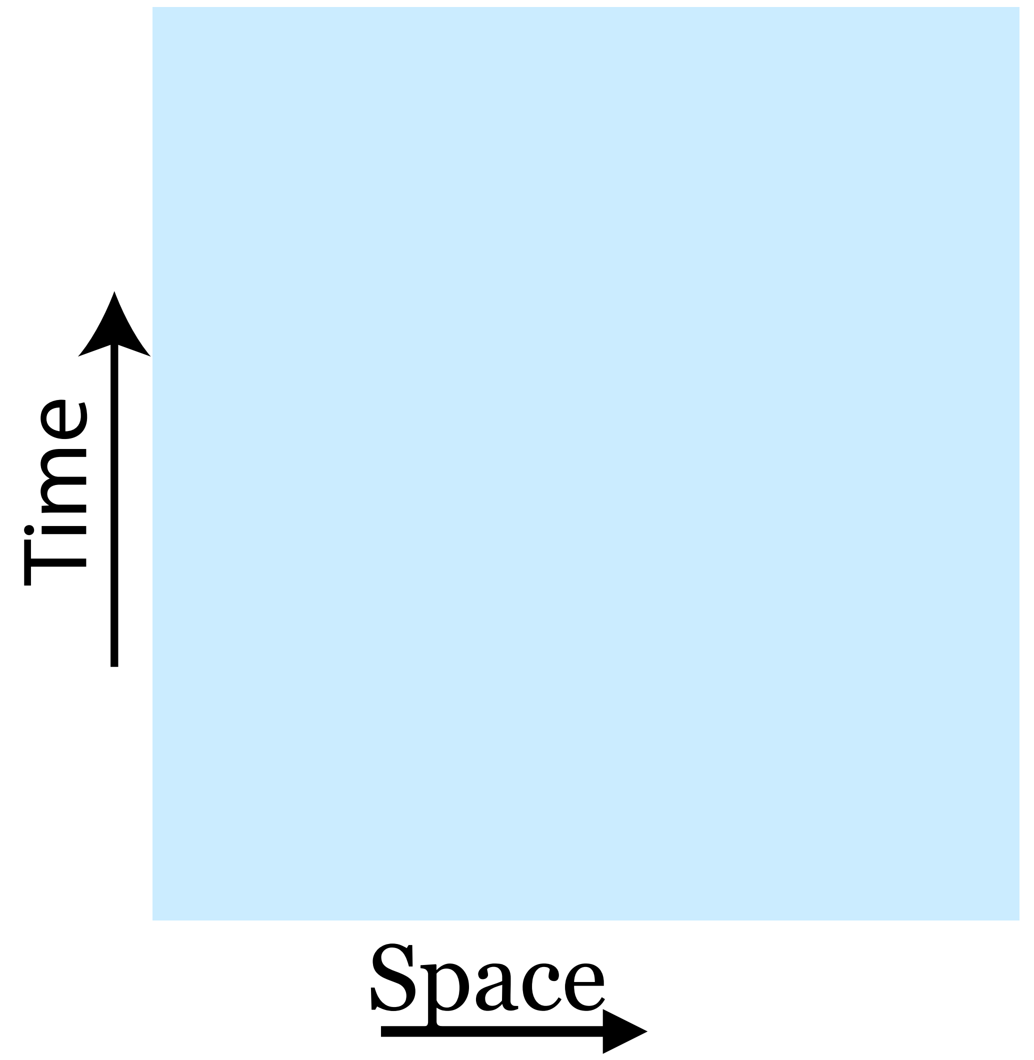

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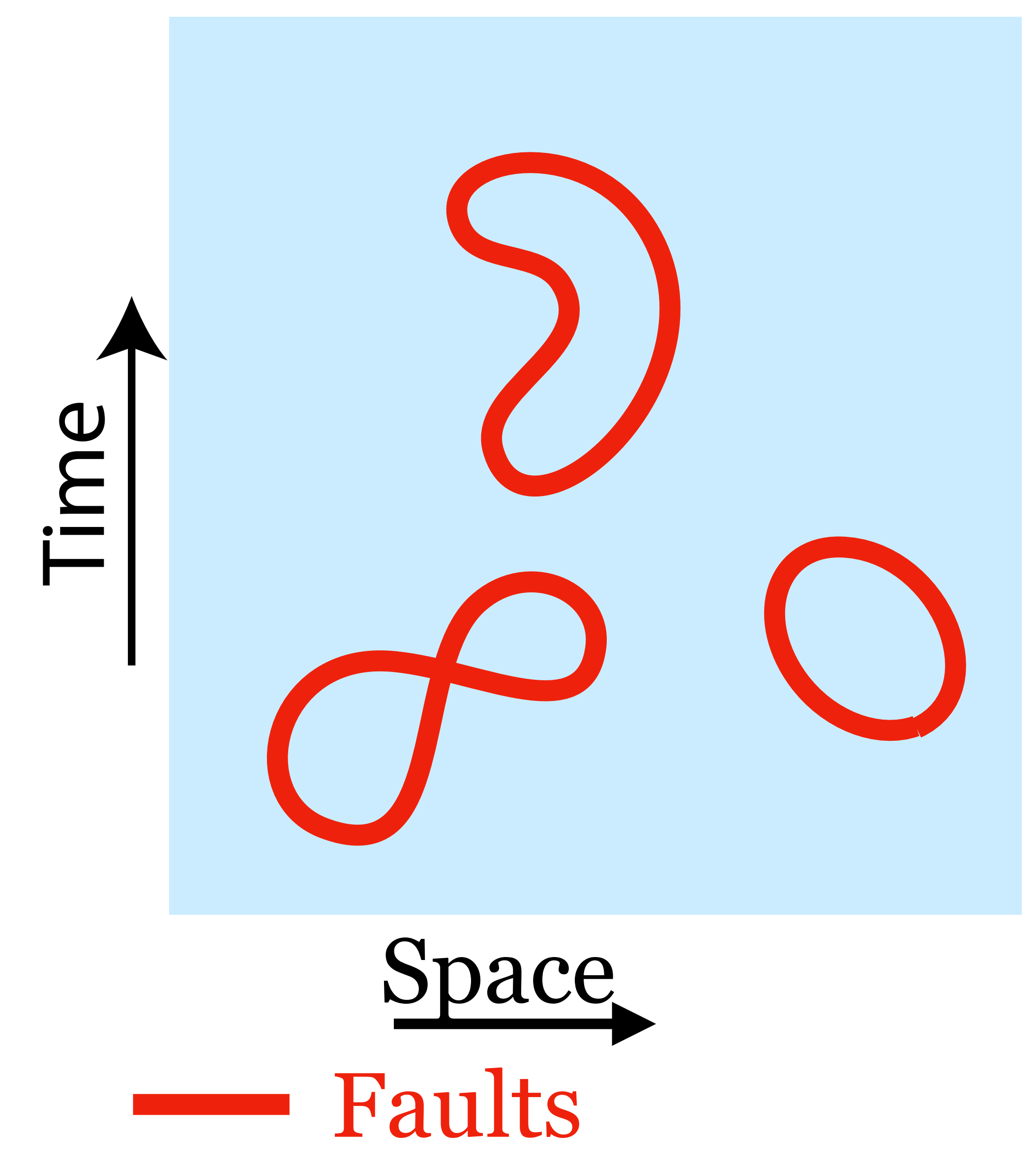

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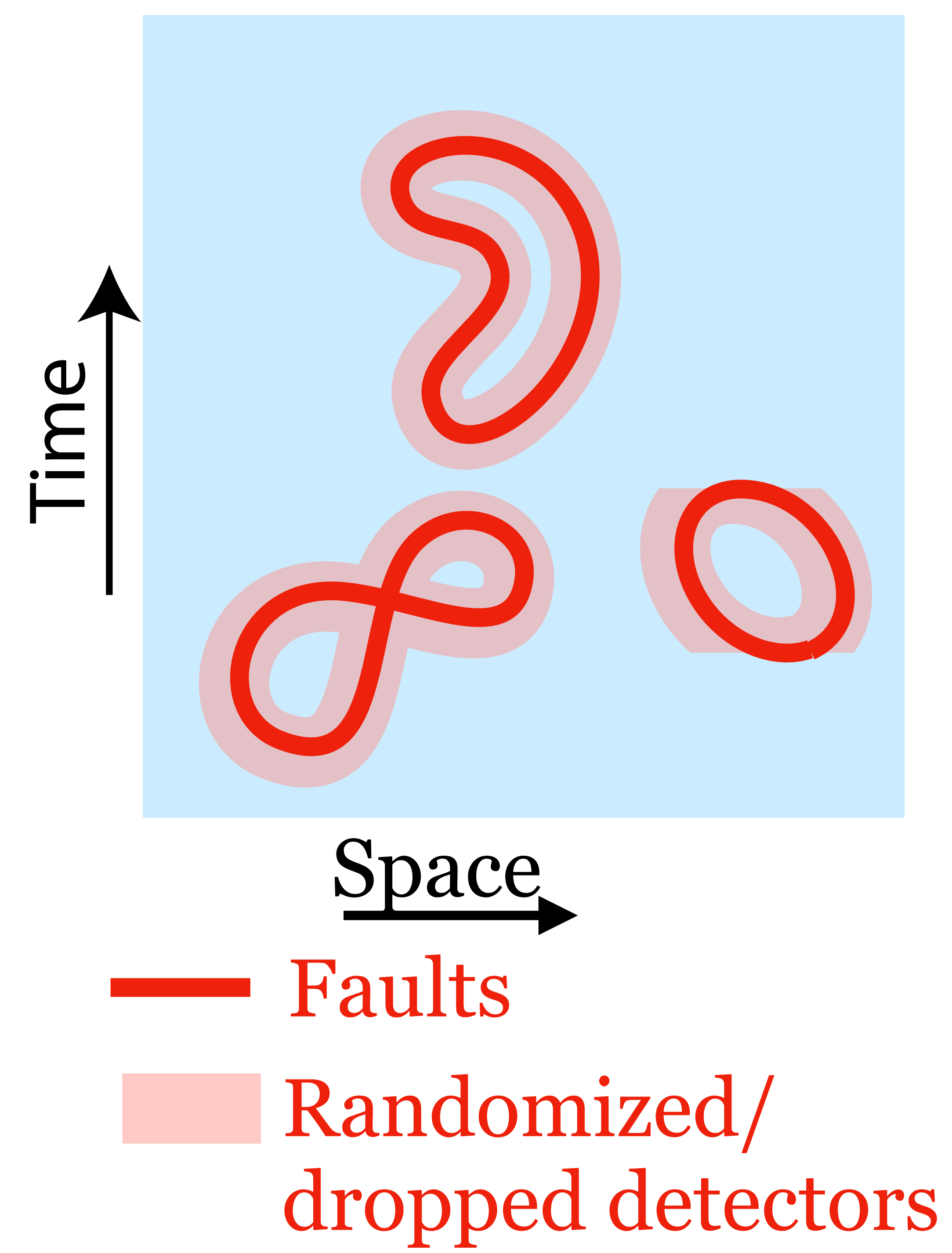

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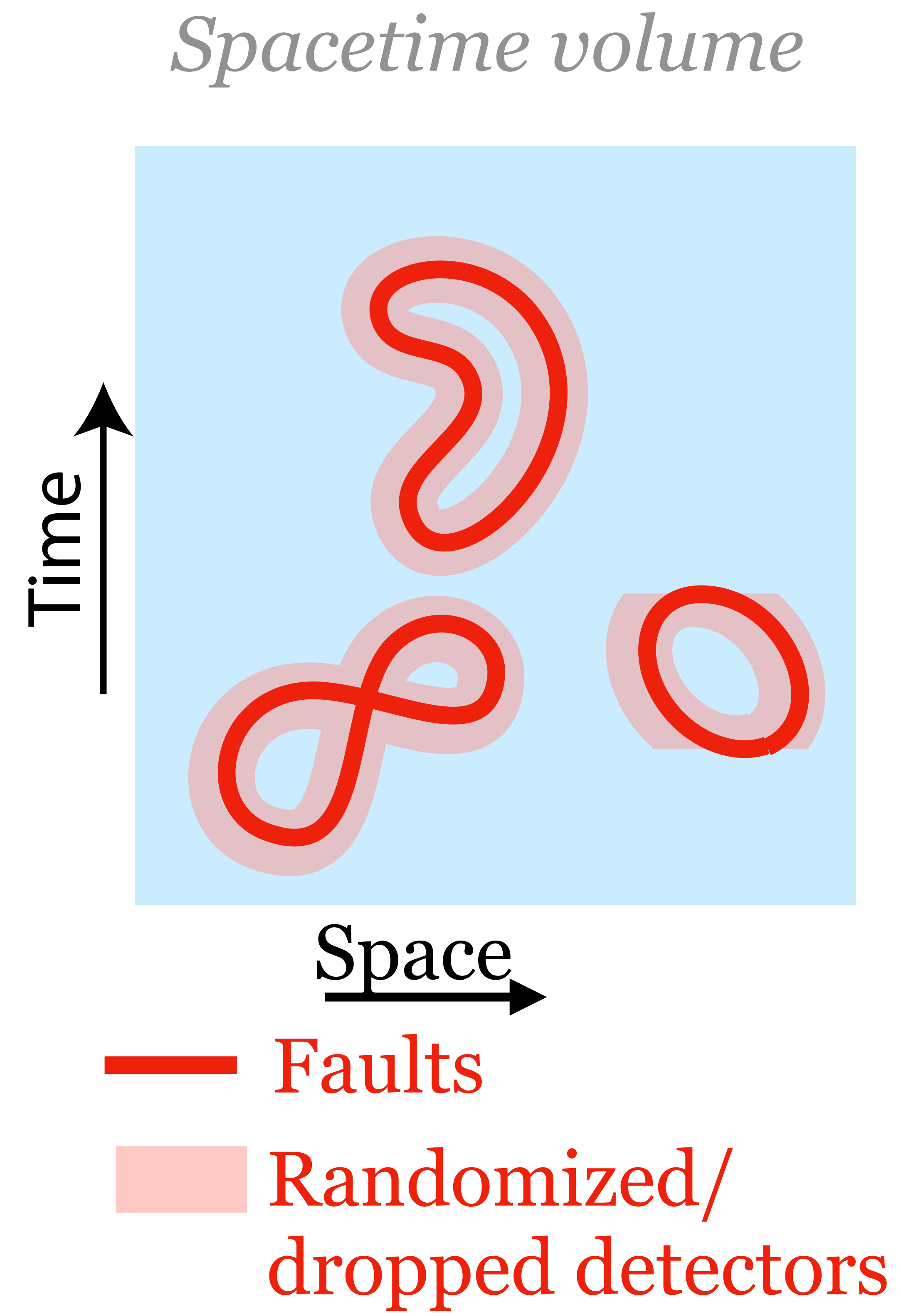

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The fault distance to Pauli and measurement errors is d/γ
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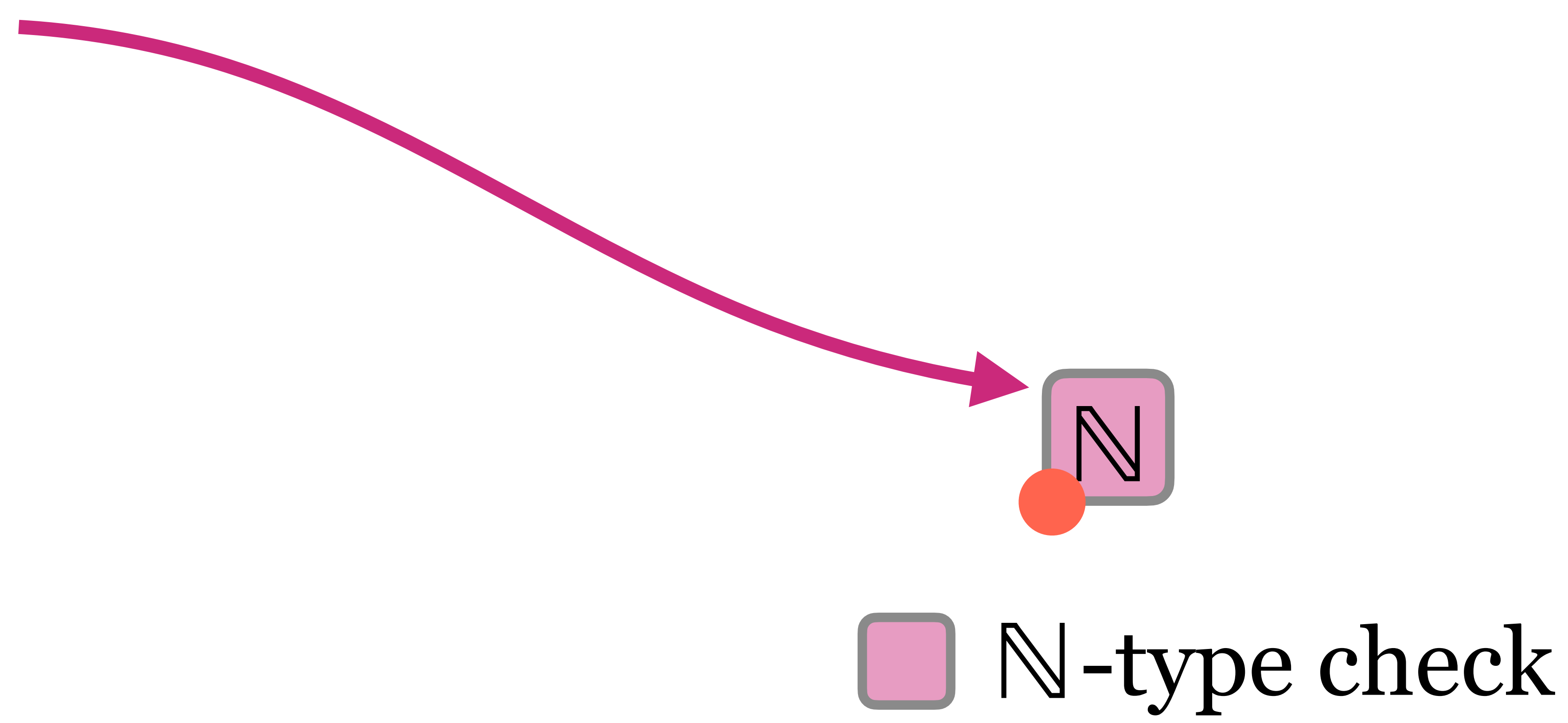

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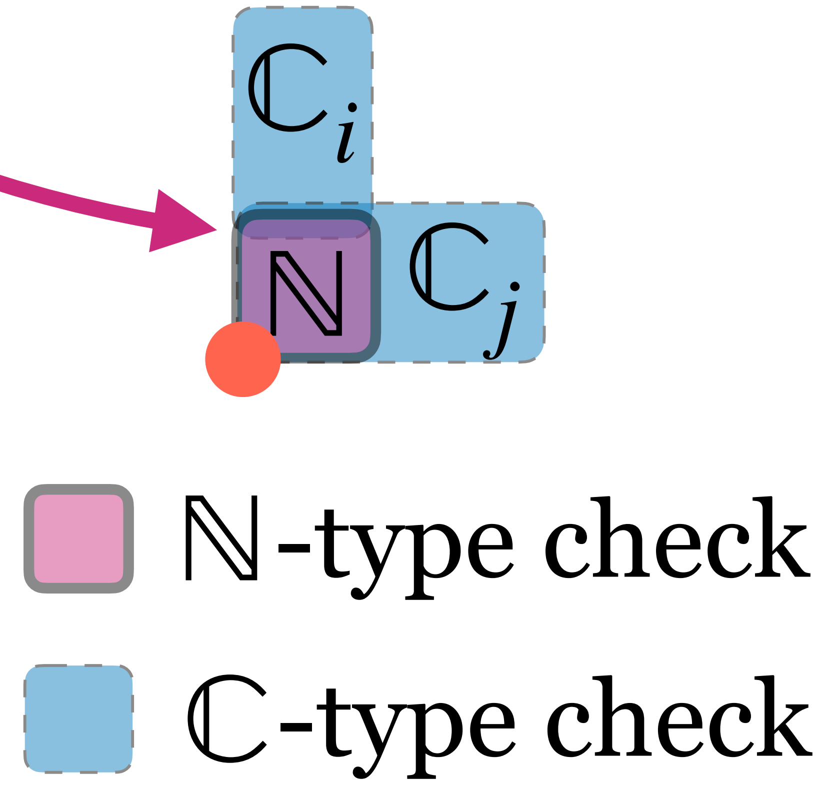

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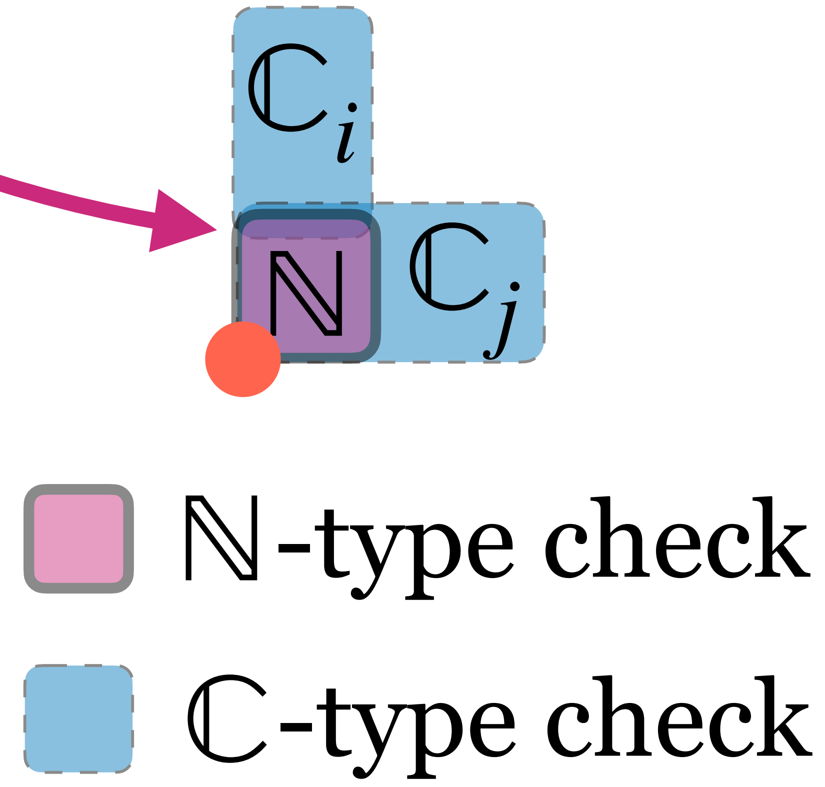

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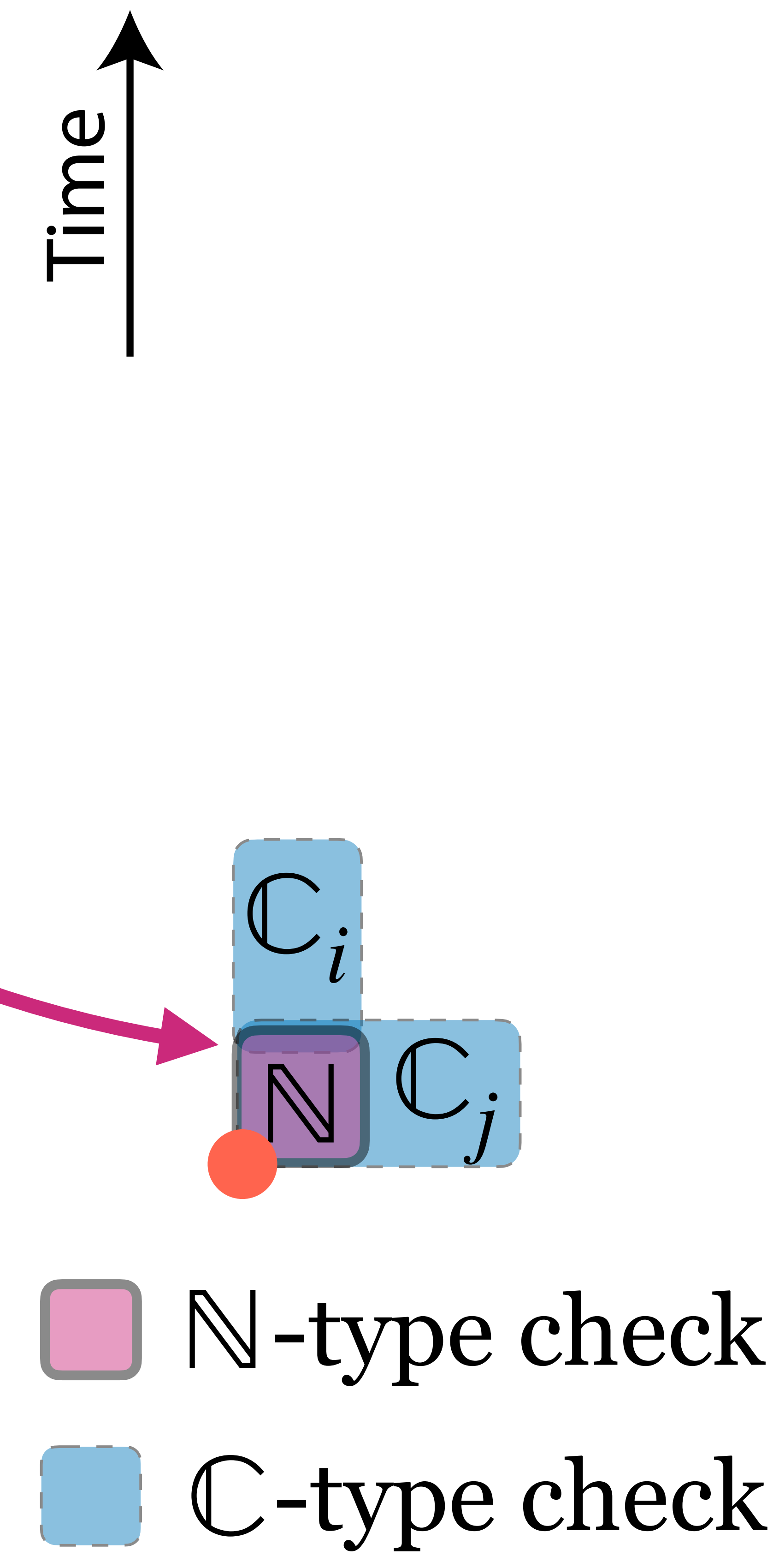

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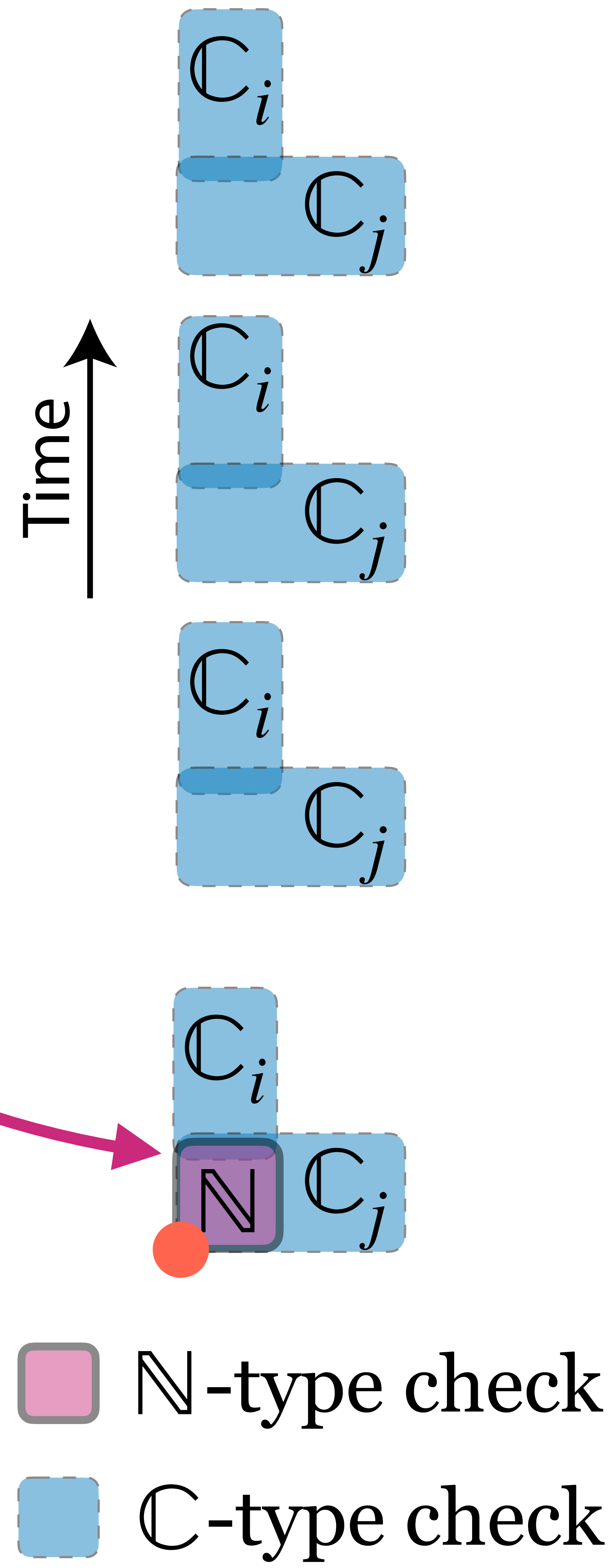

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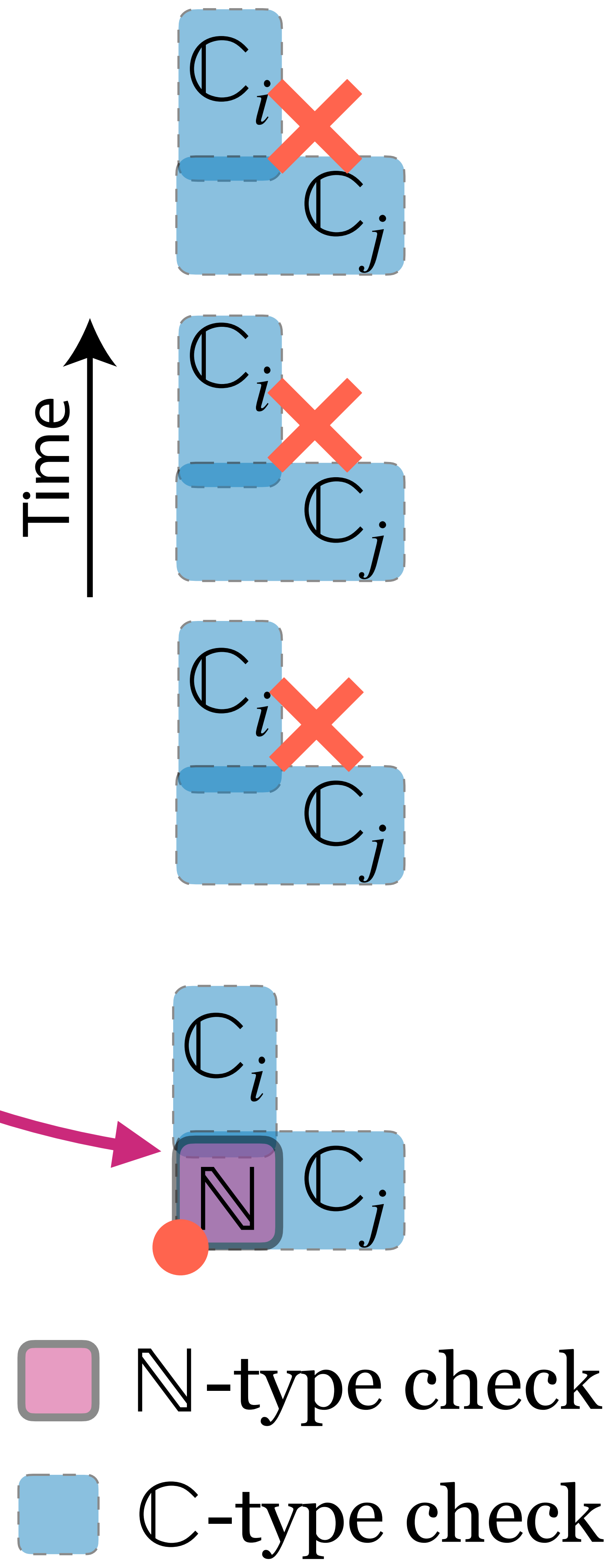

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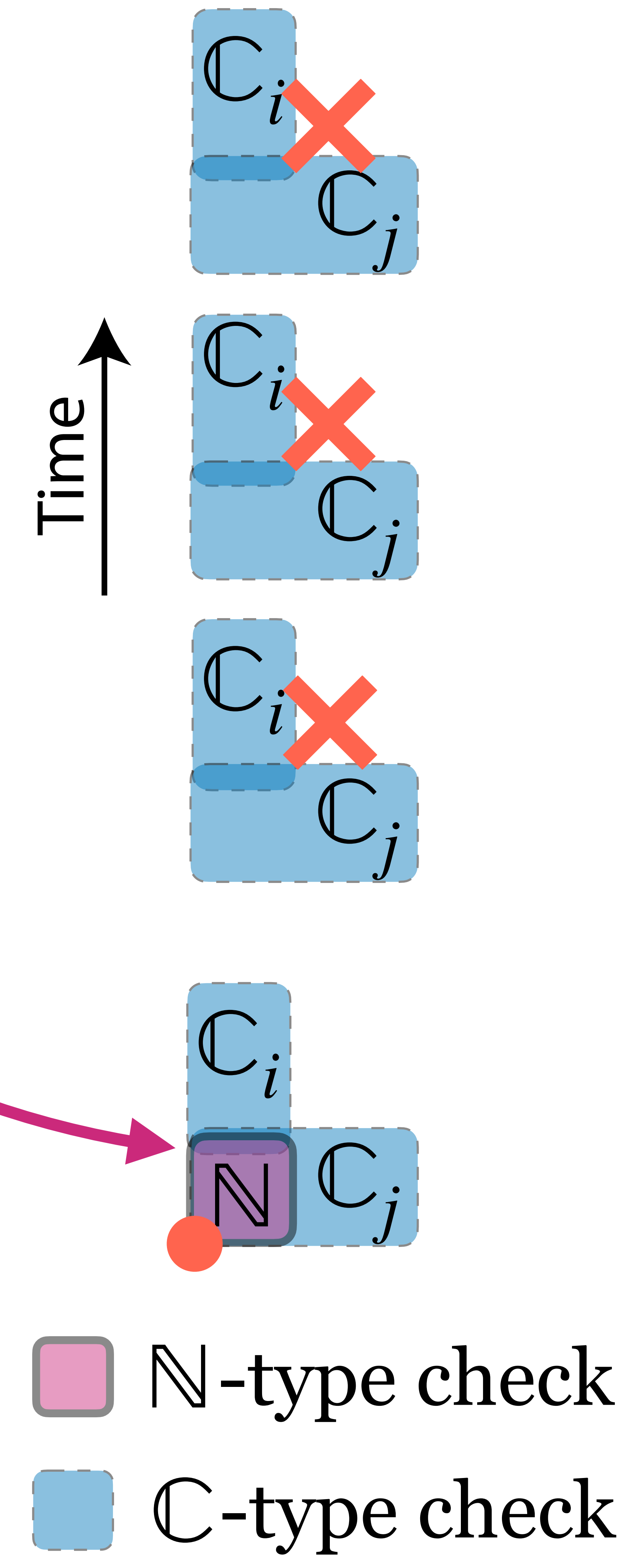

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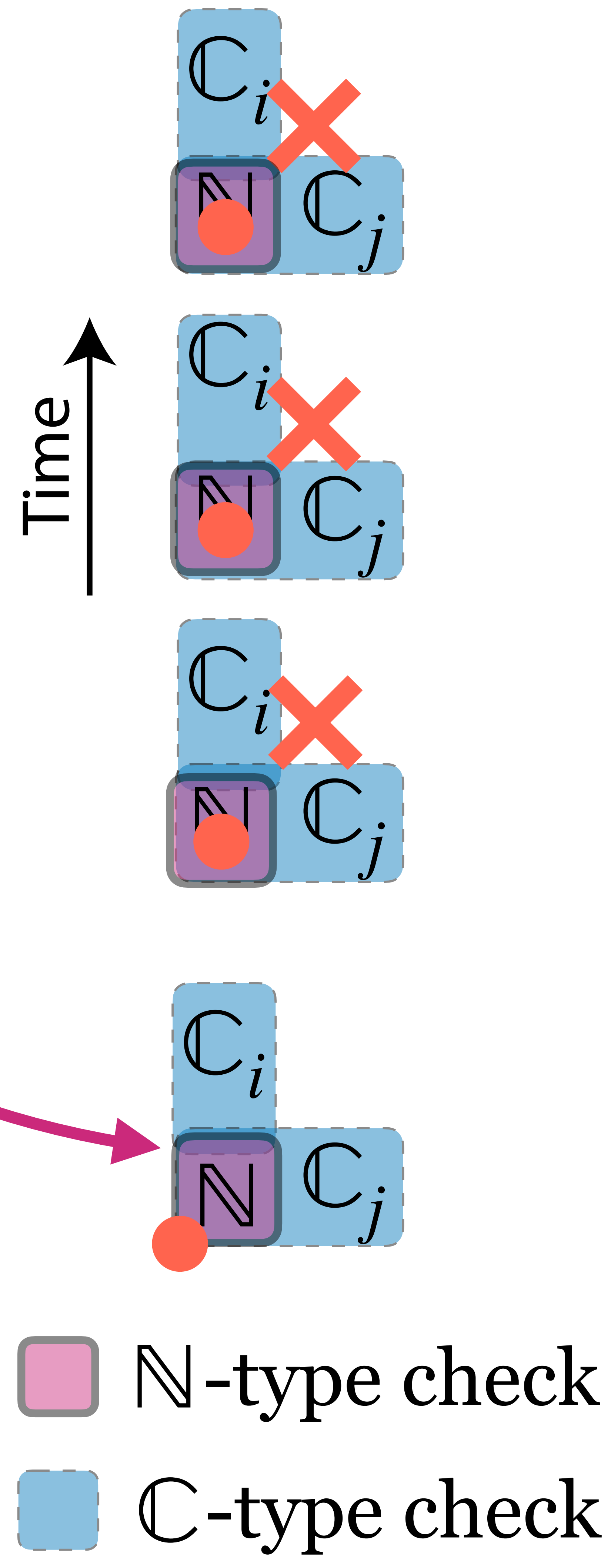
- For this space error to be undetected, a measurement error needed to have occurred on $\mathbb{N}$ for each round that $\mathbb{C}_i$ and $\mathbb{C}_j$ do not commute



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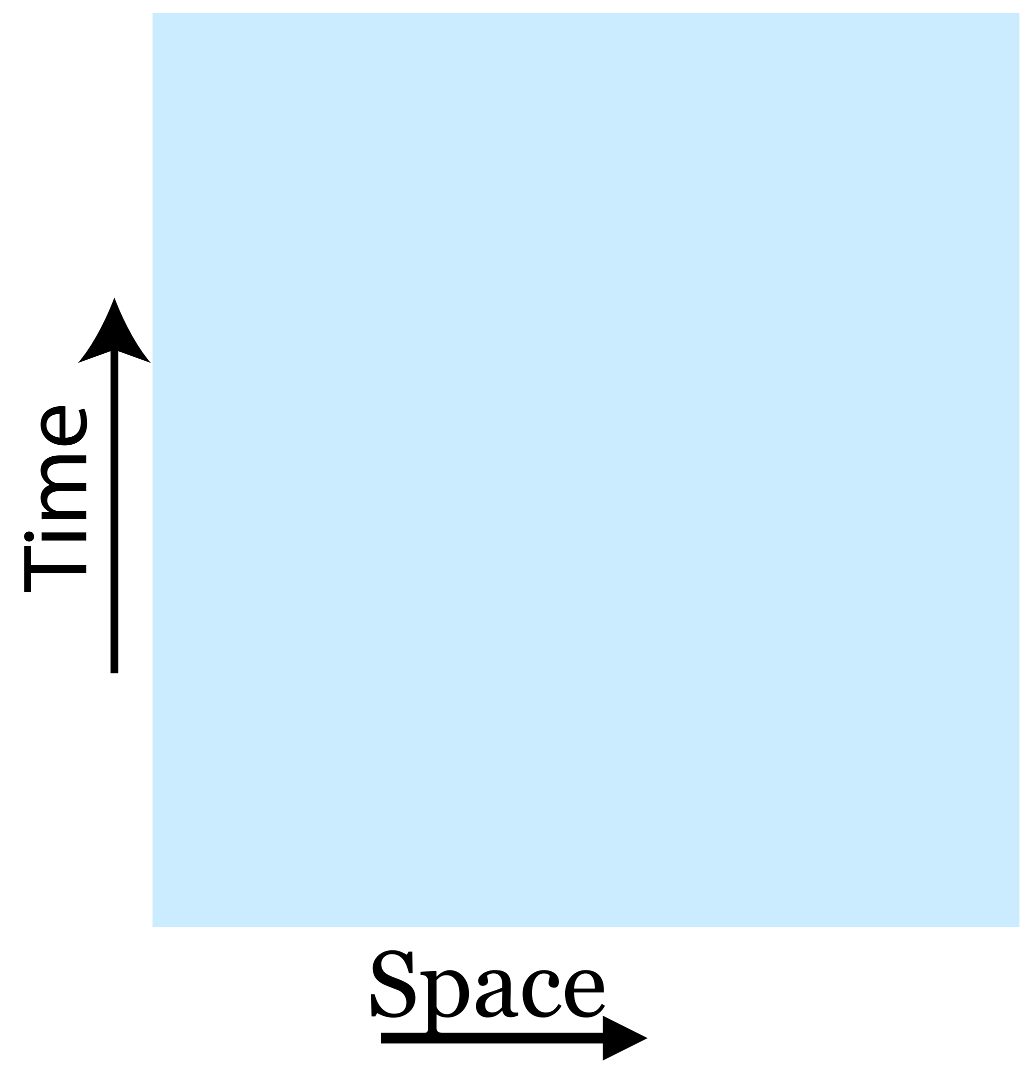

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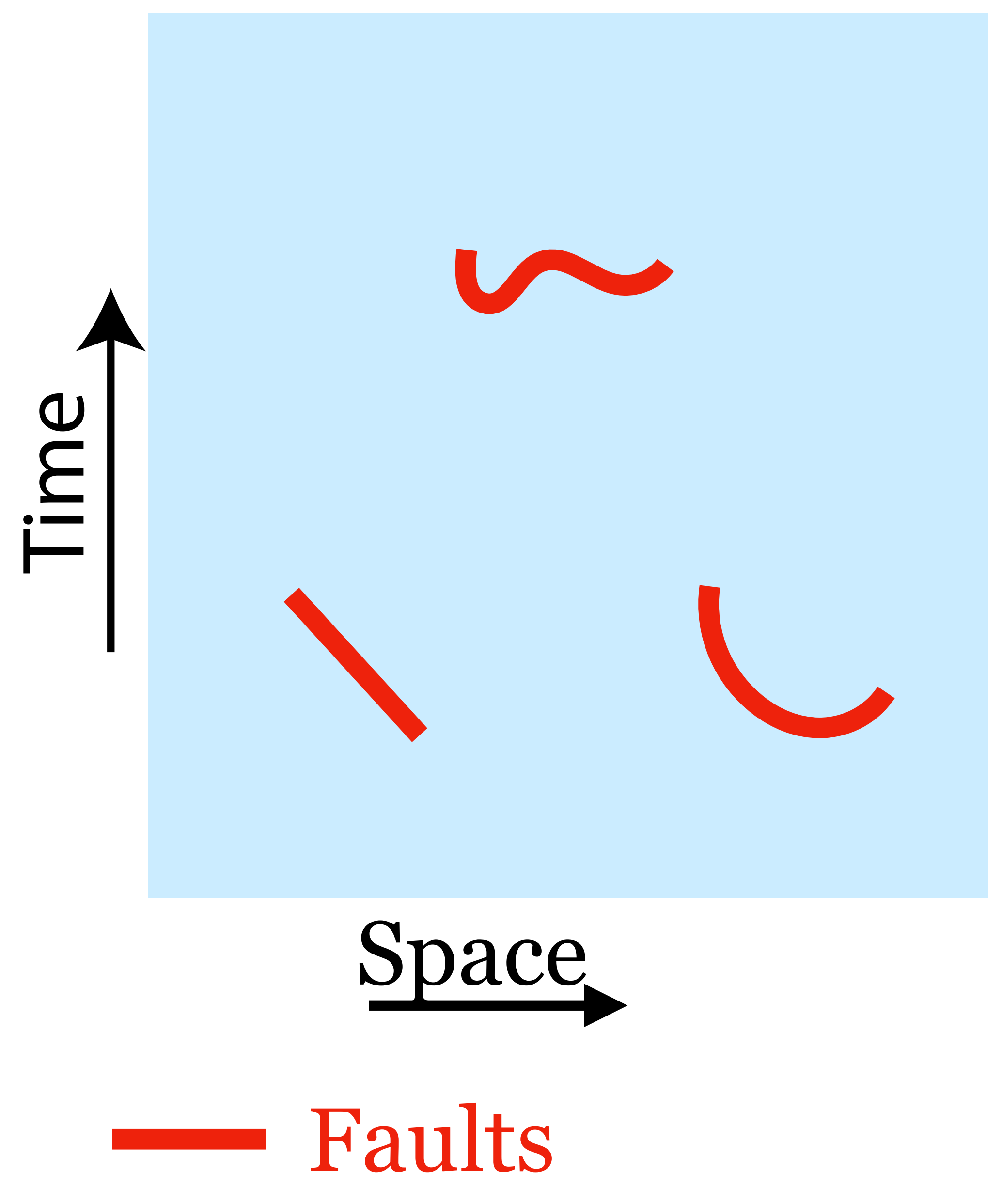

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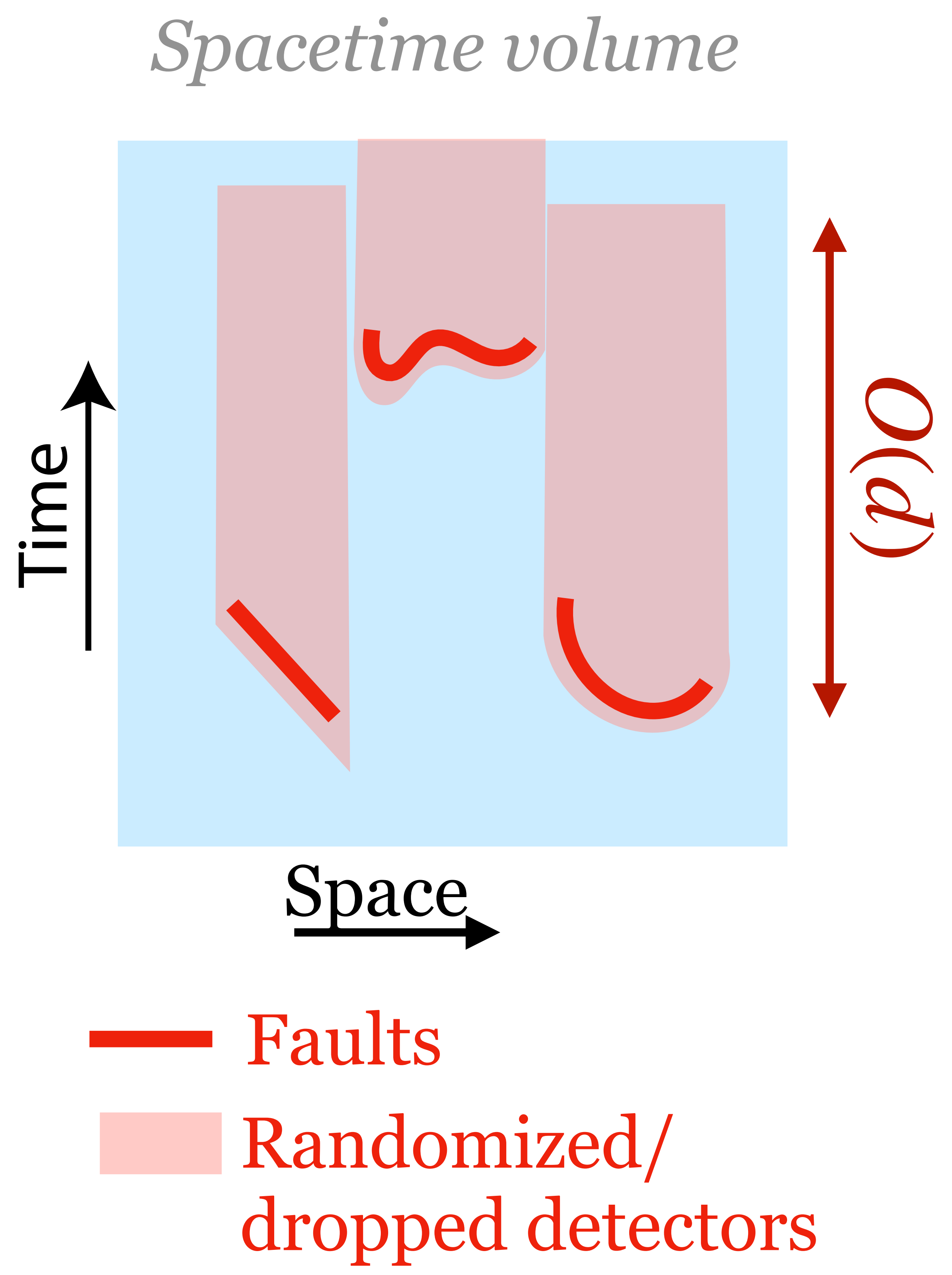

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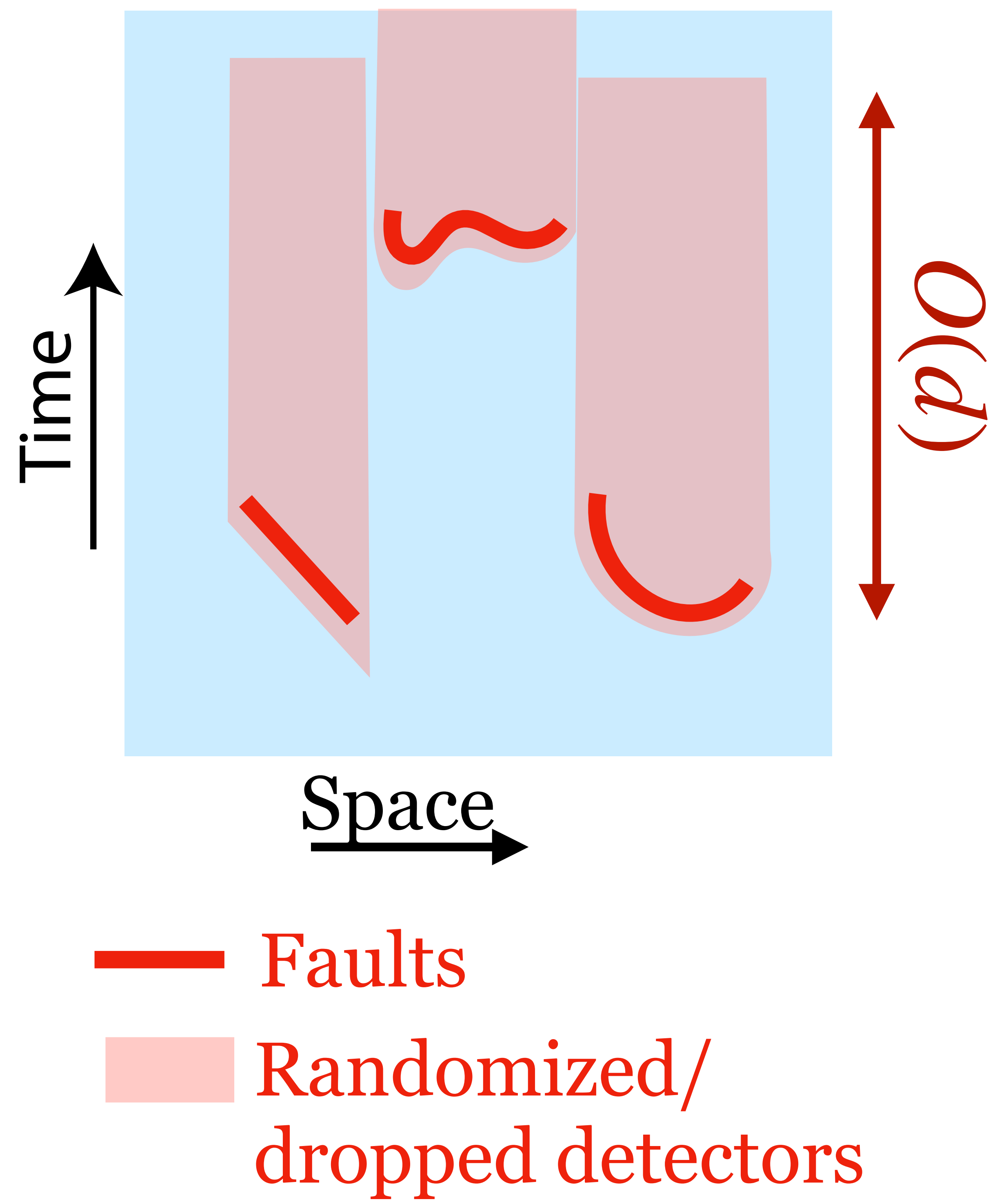

[1] Bombín. [1810.09571] [2] Brown. Sci. Adv. **6** (2024)

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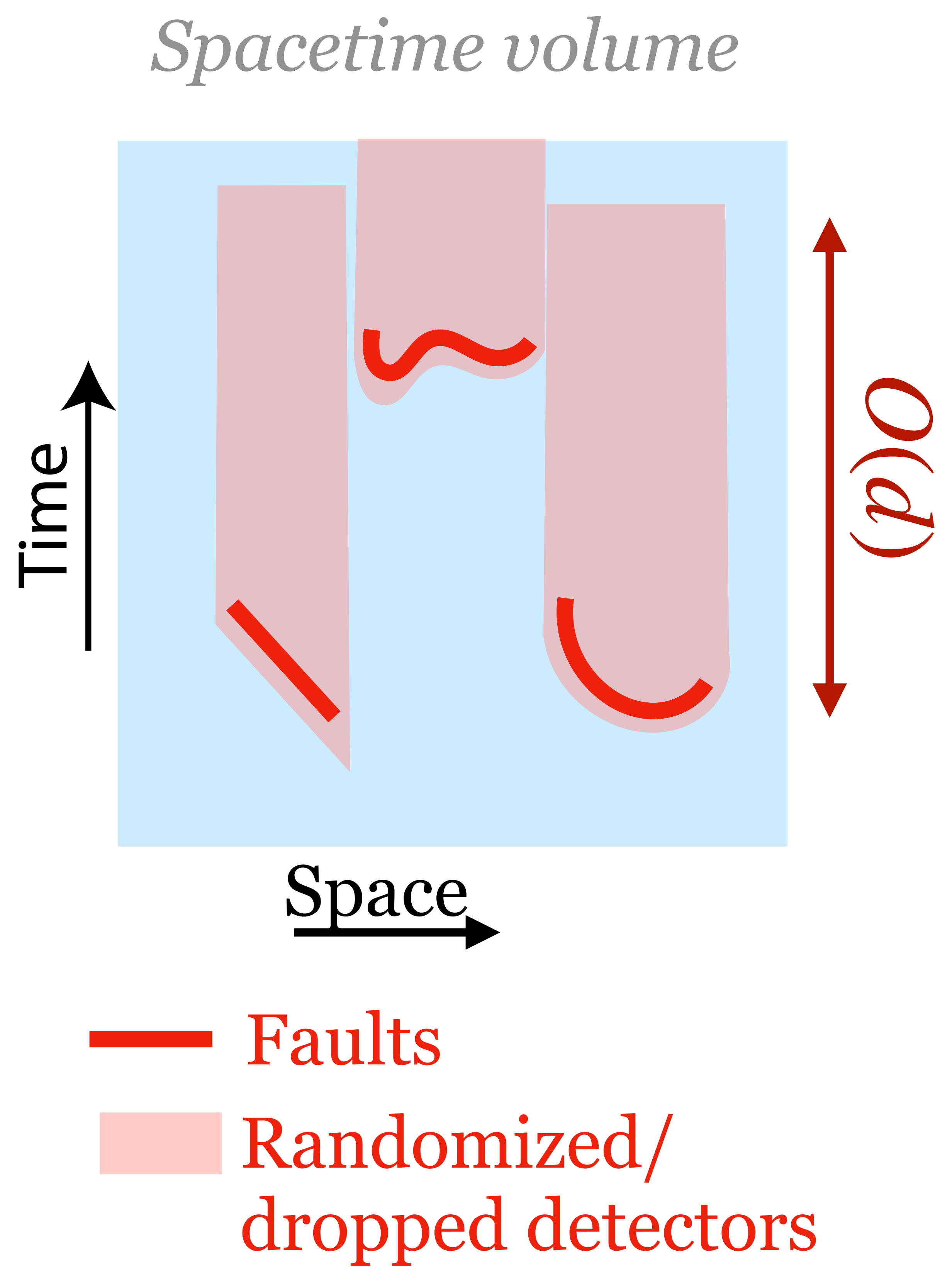
(WIP for QLDPC codes)

Heuristic decoders?[3,4]



[1] Bombín. [1810.09571] [2] Brown. Sci. Adv. **6** (2024) [3] Scruby, et al. Quantum **6** (2022). [4] Magdalena de la Fuente, et al. [2604.02033]

Toffoli gate resource states on any CSS code

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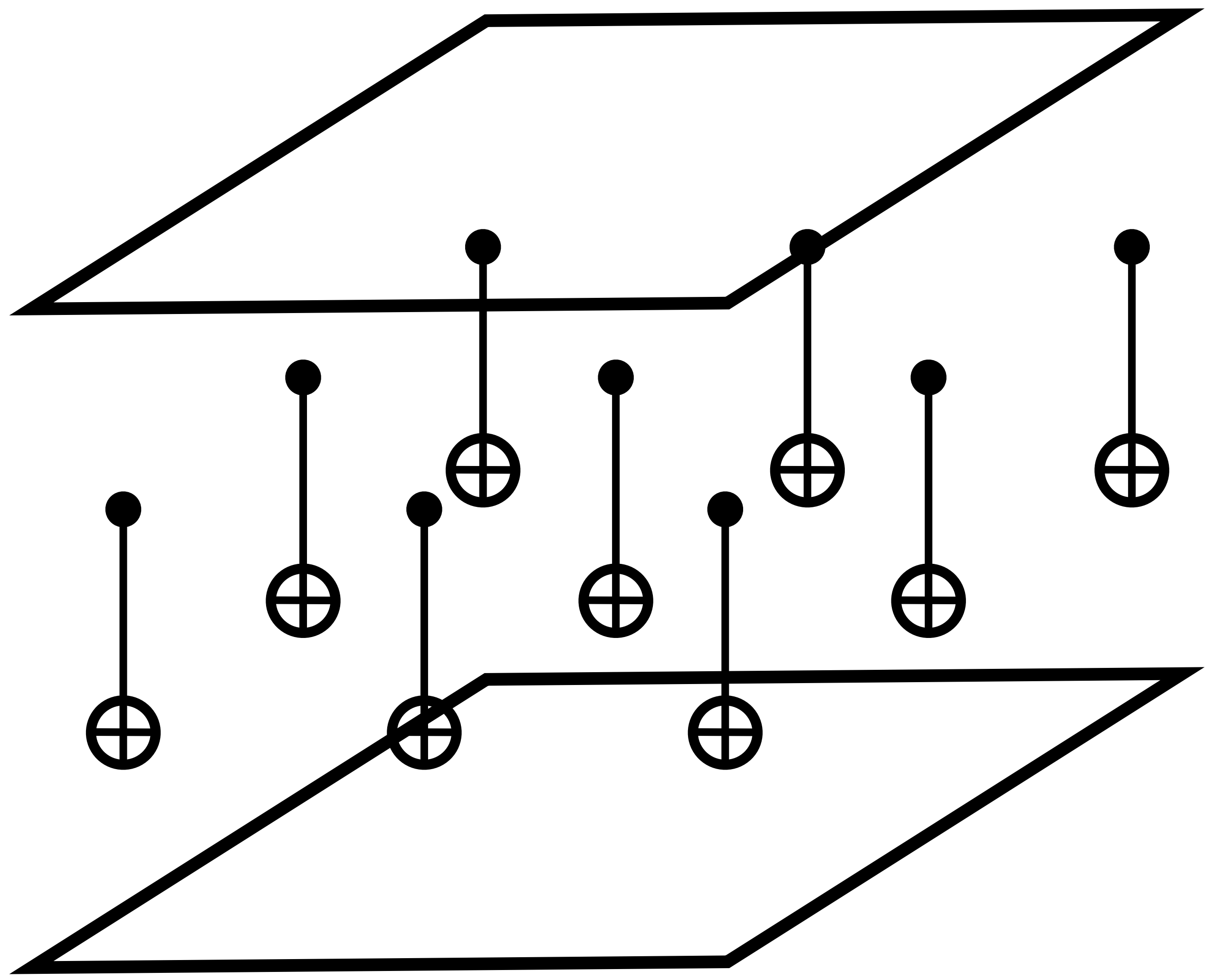

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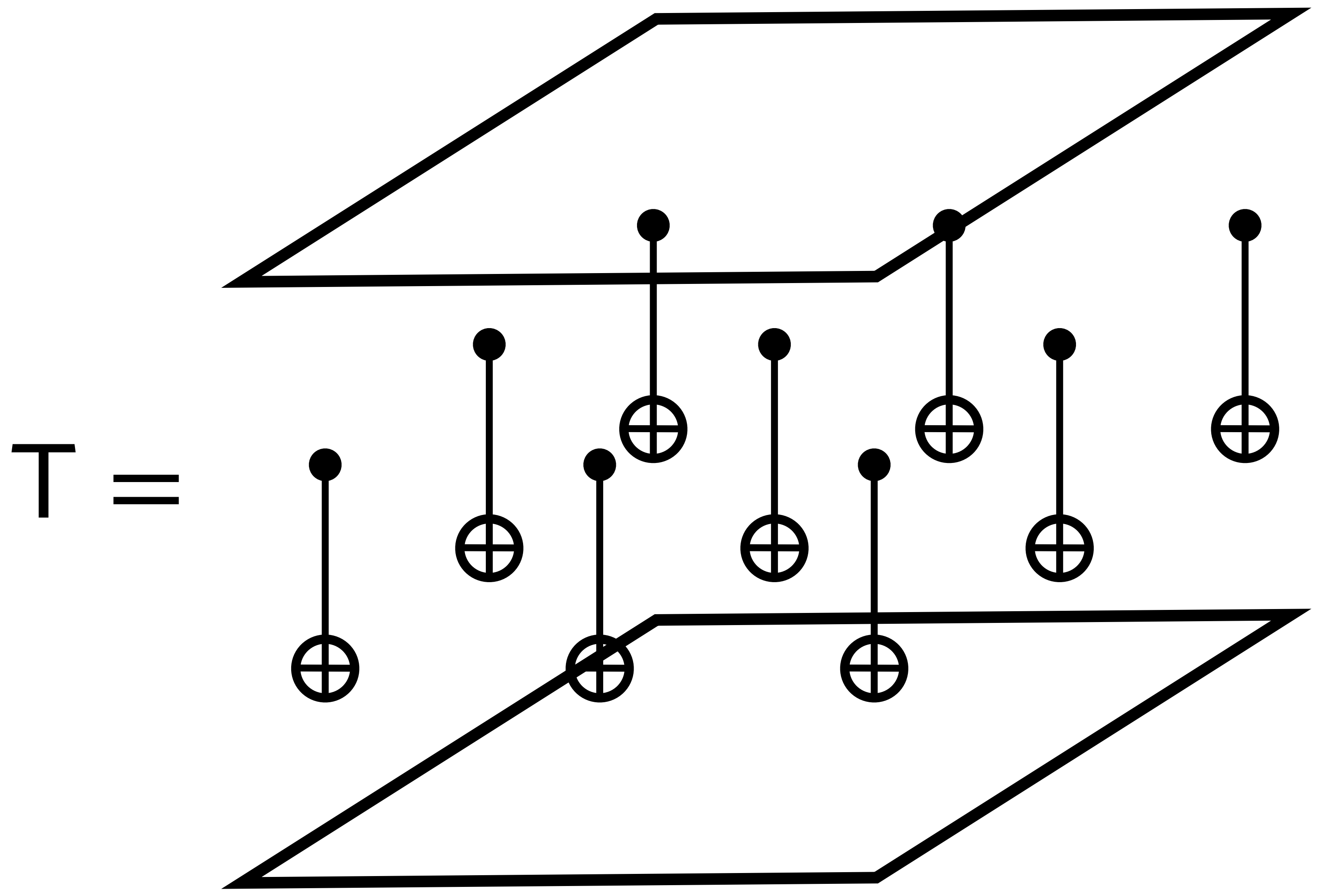

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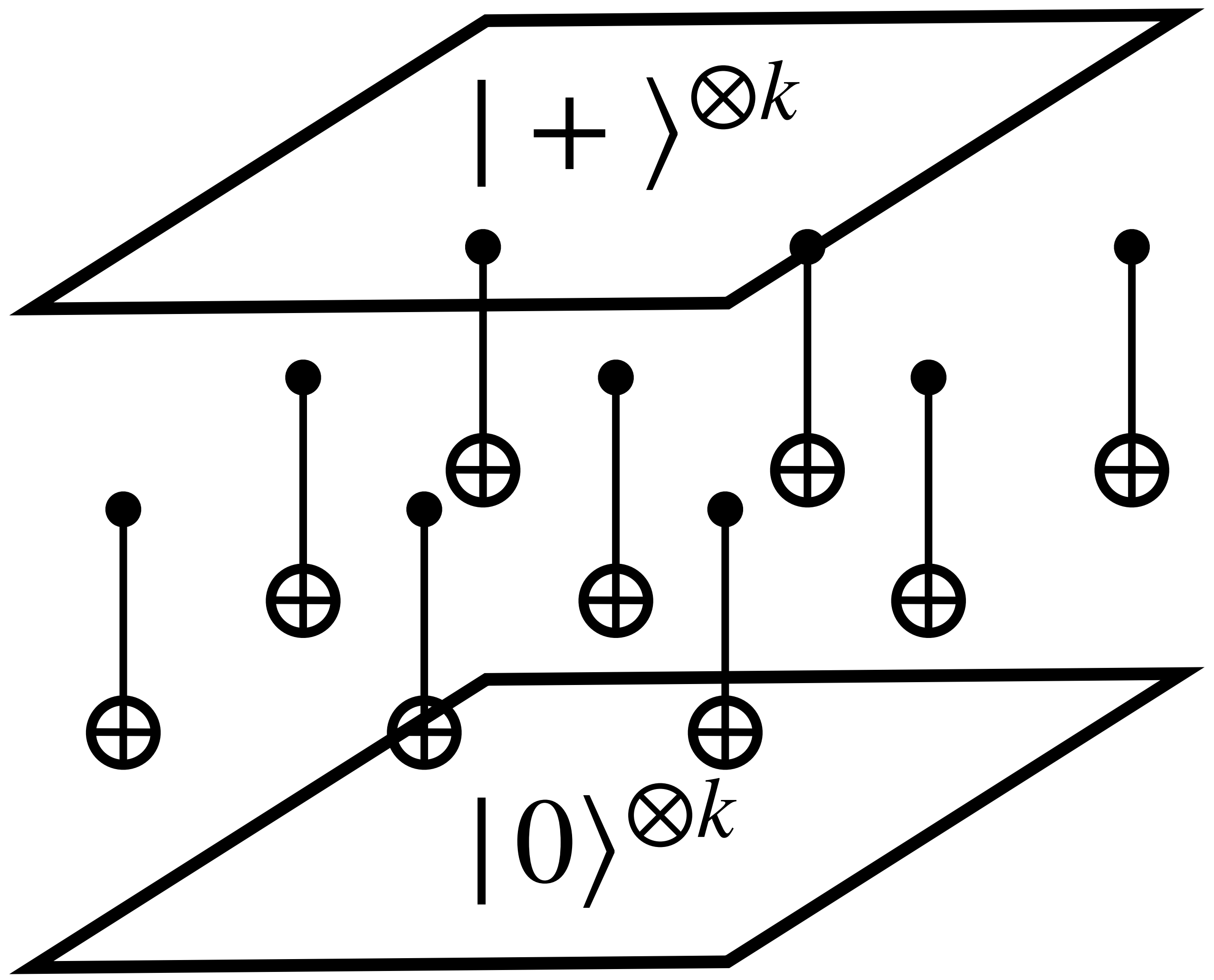

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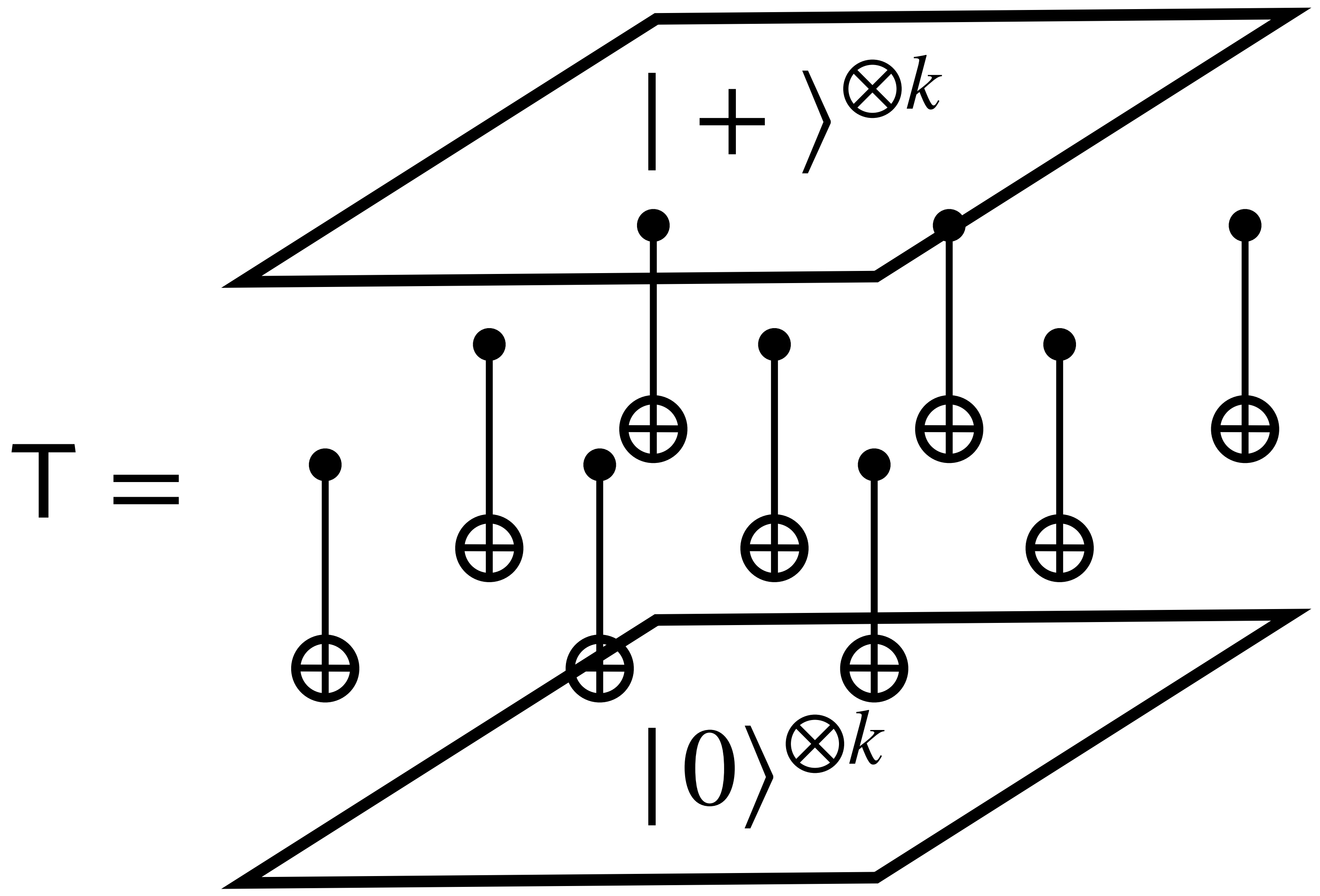

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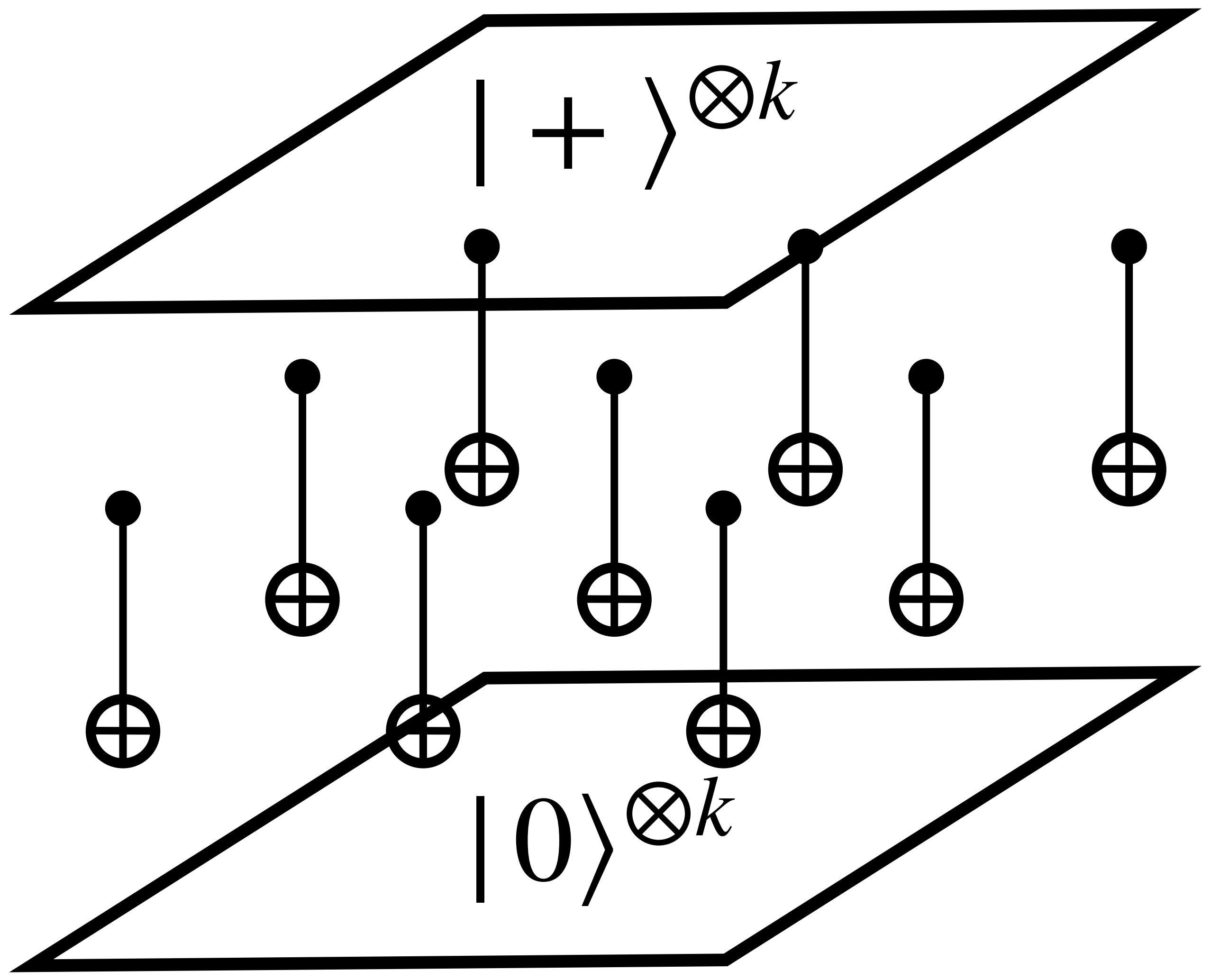
T = 

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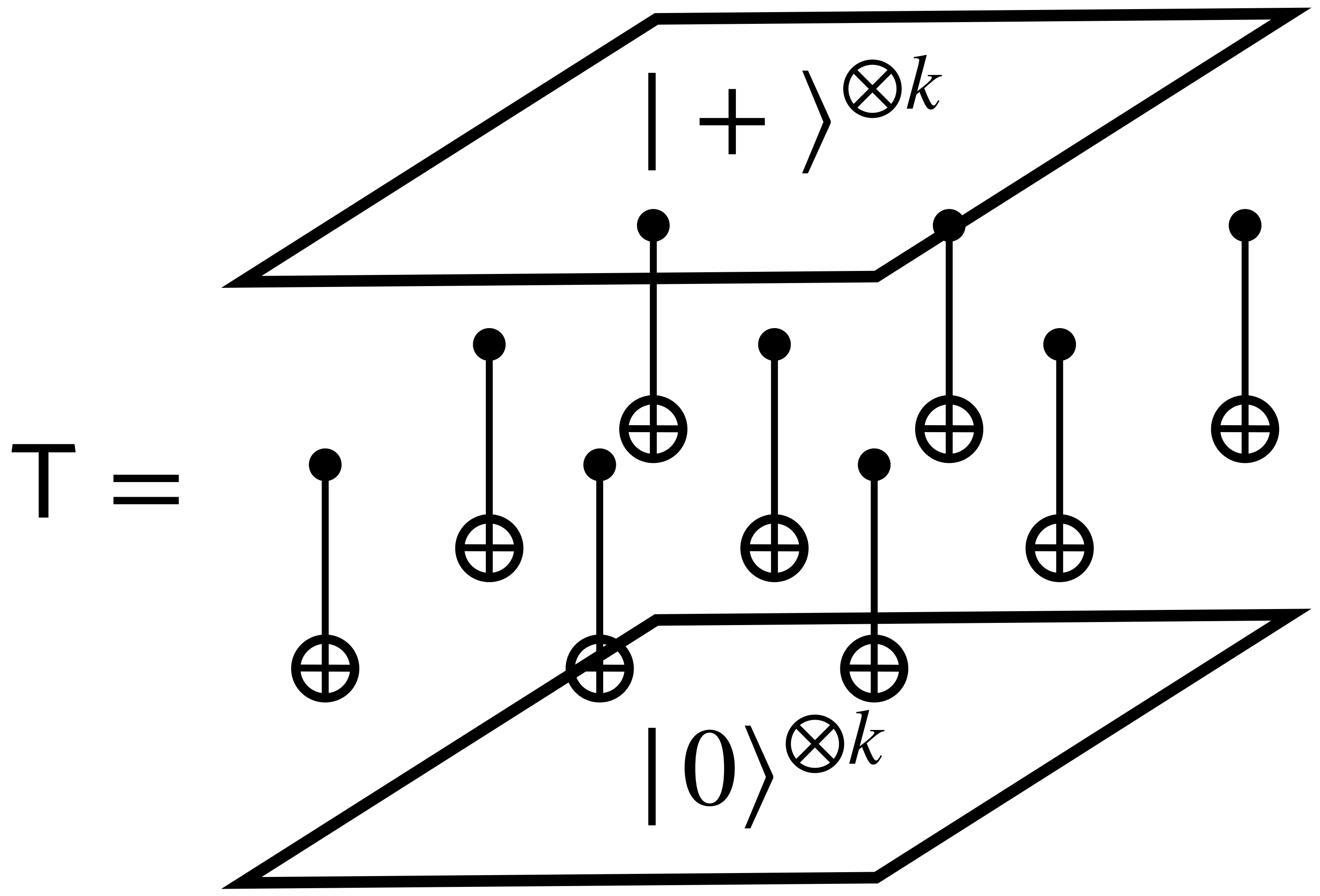

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Very mild requirement on the code (transversal CNOT) to generate magic states in situ.
In principle, any CSS QLDPC code could be turned into a magic state factory.

Beyond magic state preparation: control-T gadget

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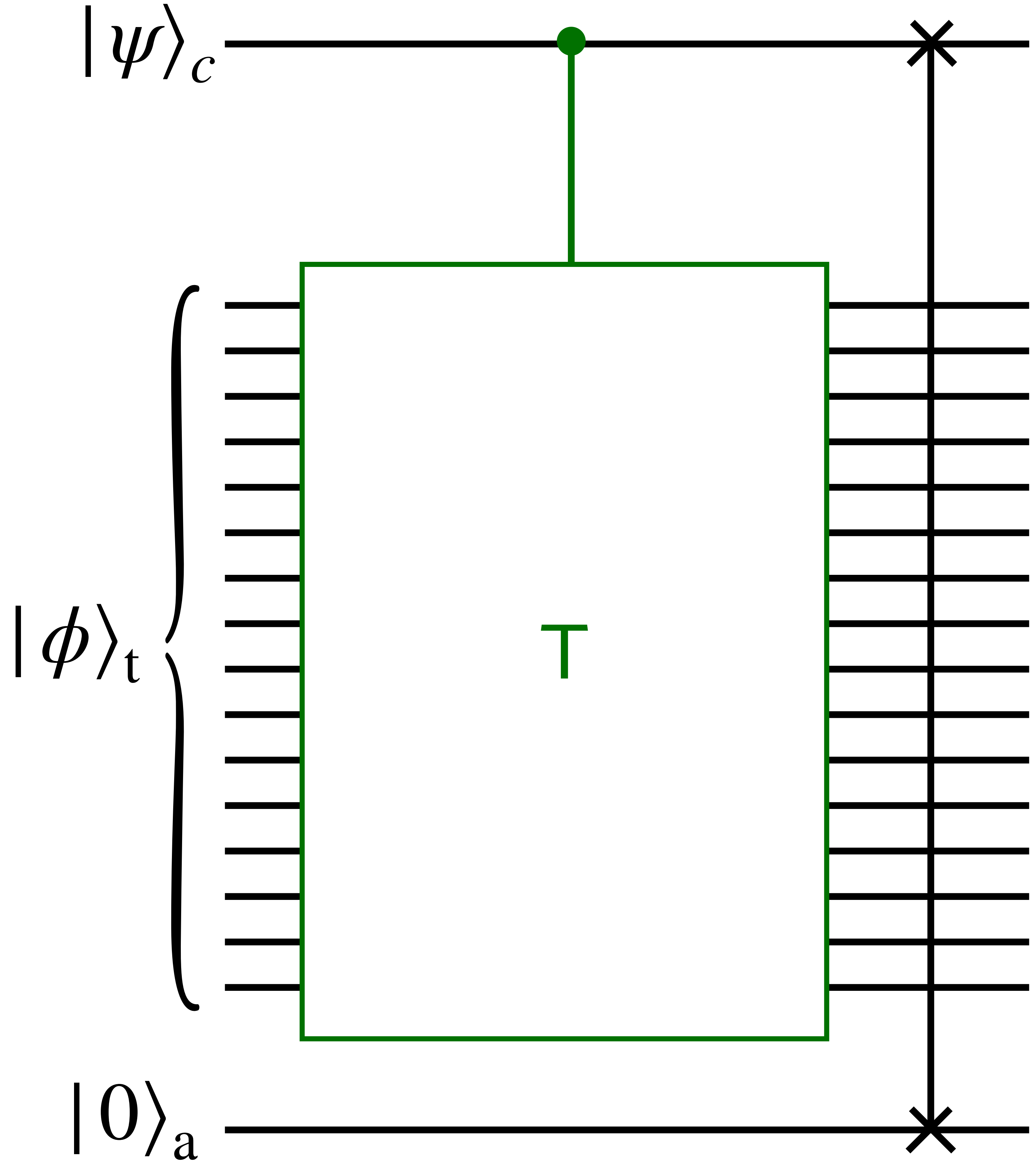

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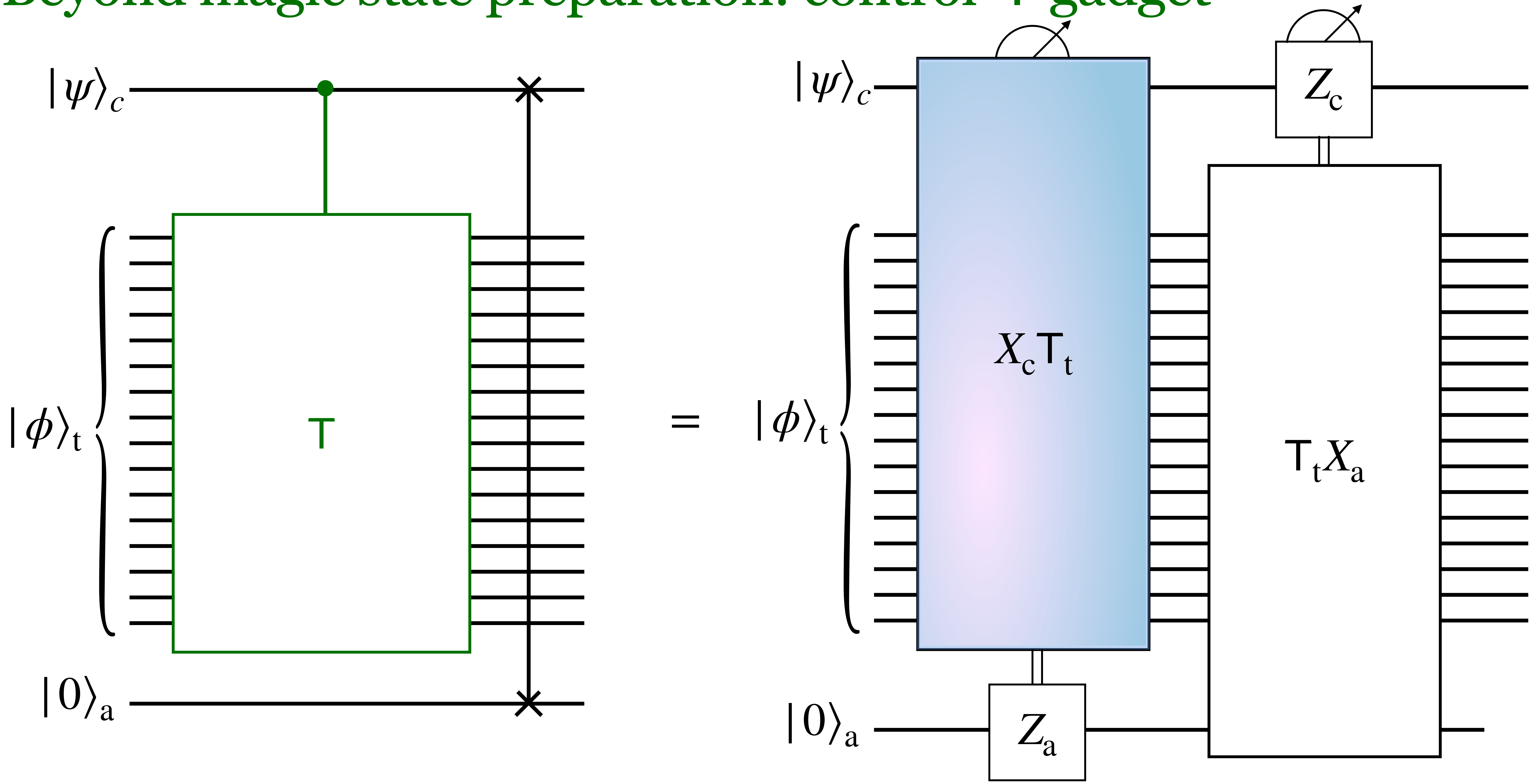

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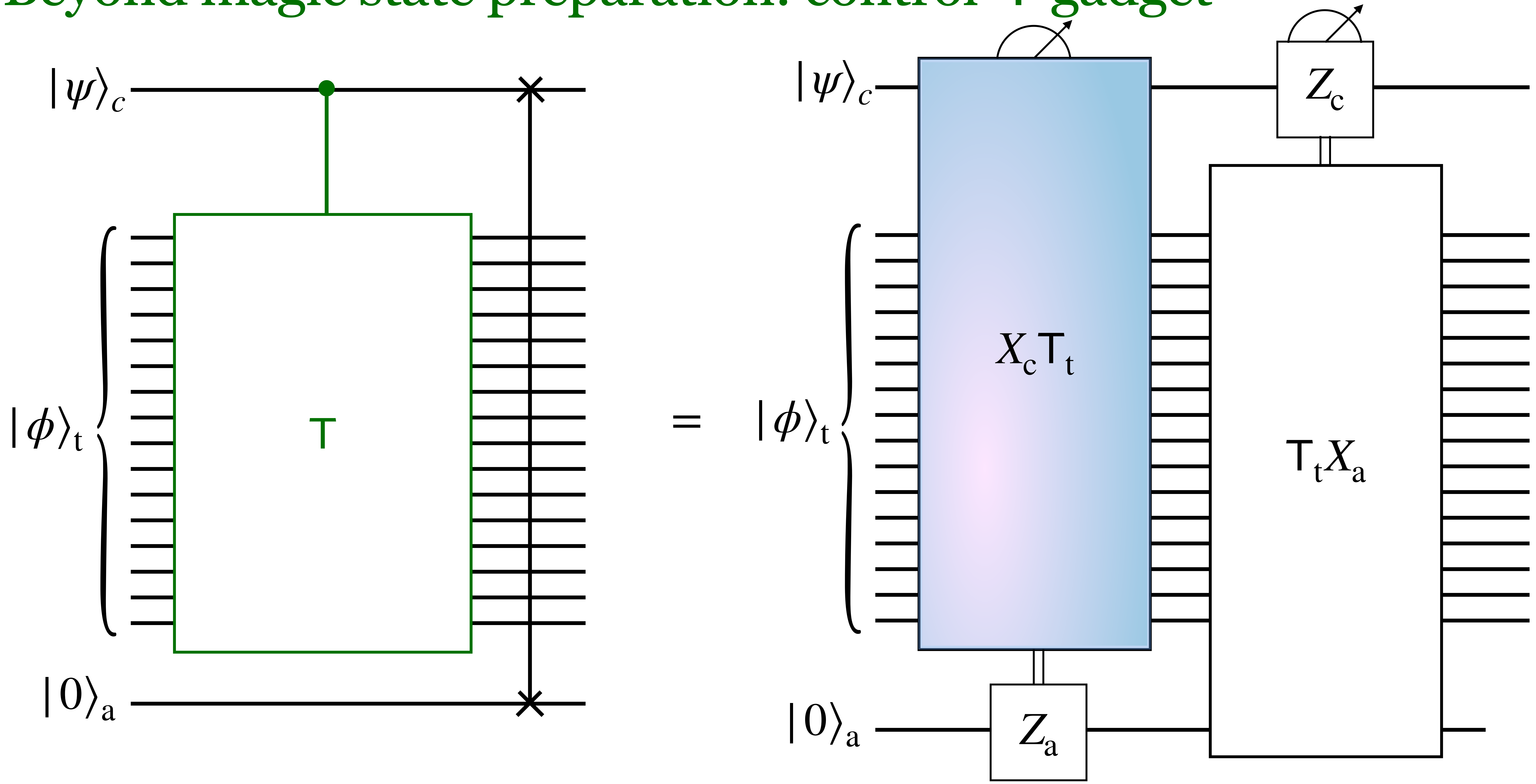

One Clifford measurement + feedforward Paulis and Cliffords *deterministically* performs the control-T

Data lookup in superposition

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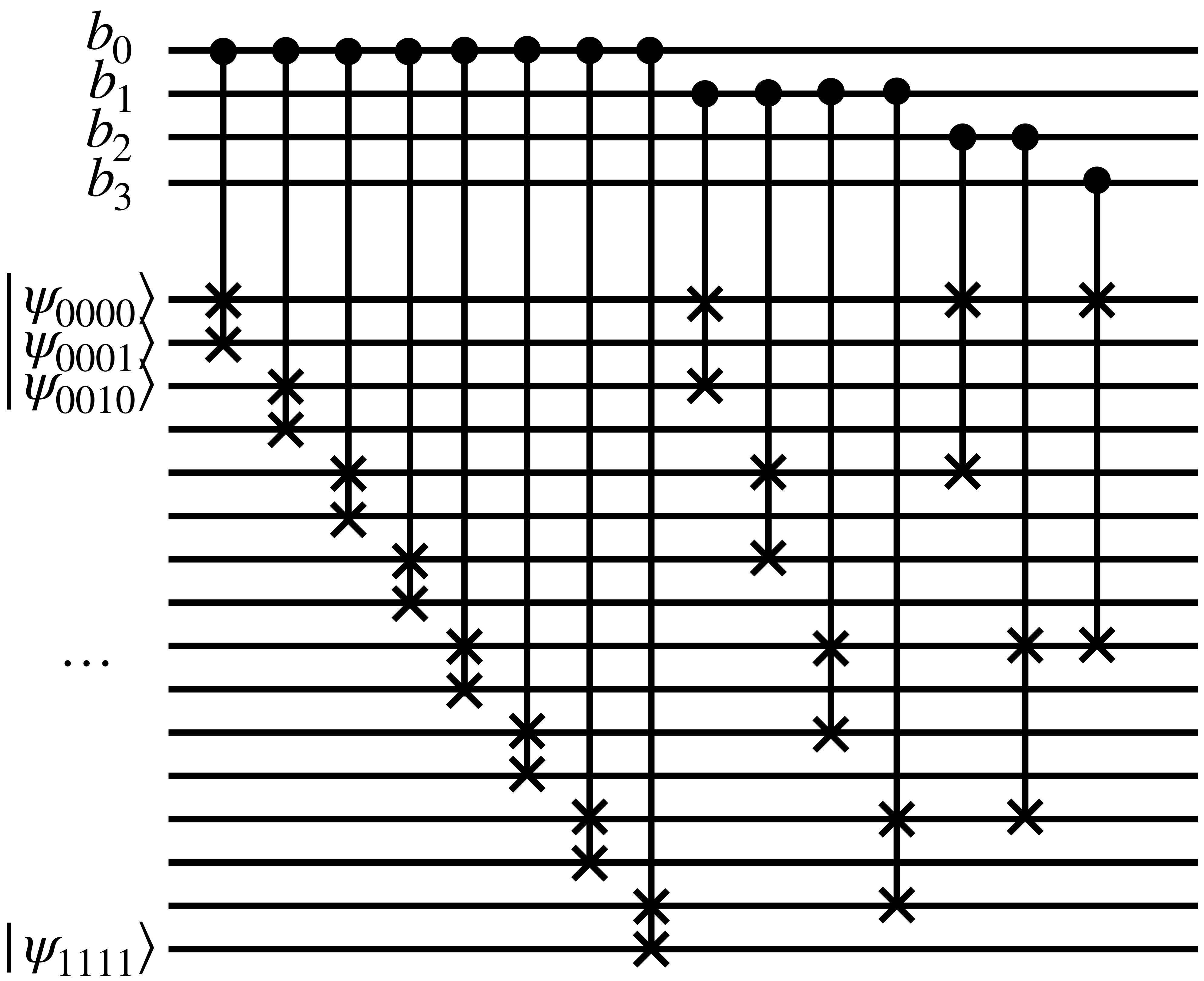

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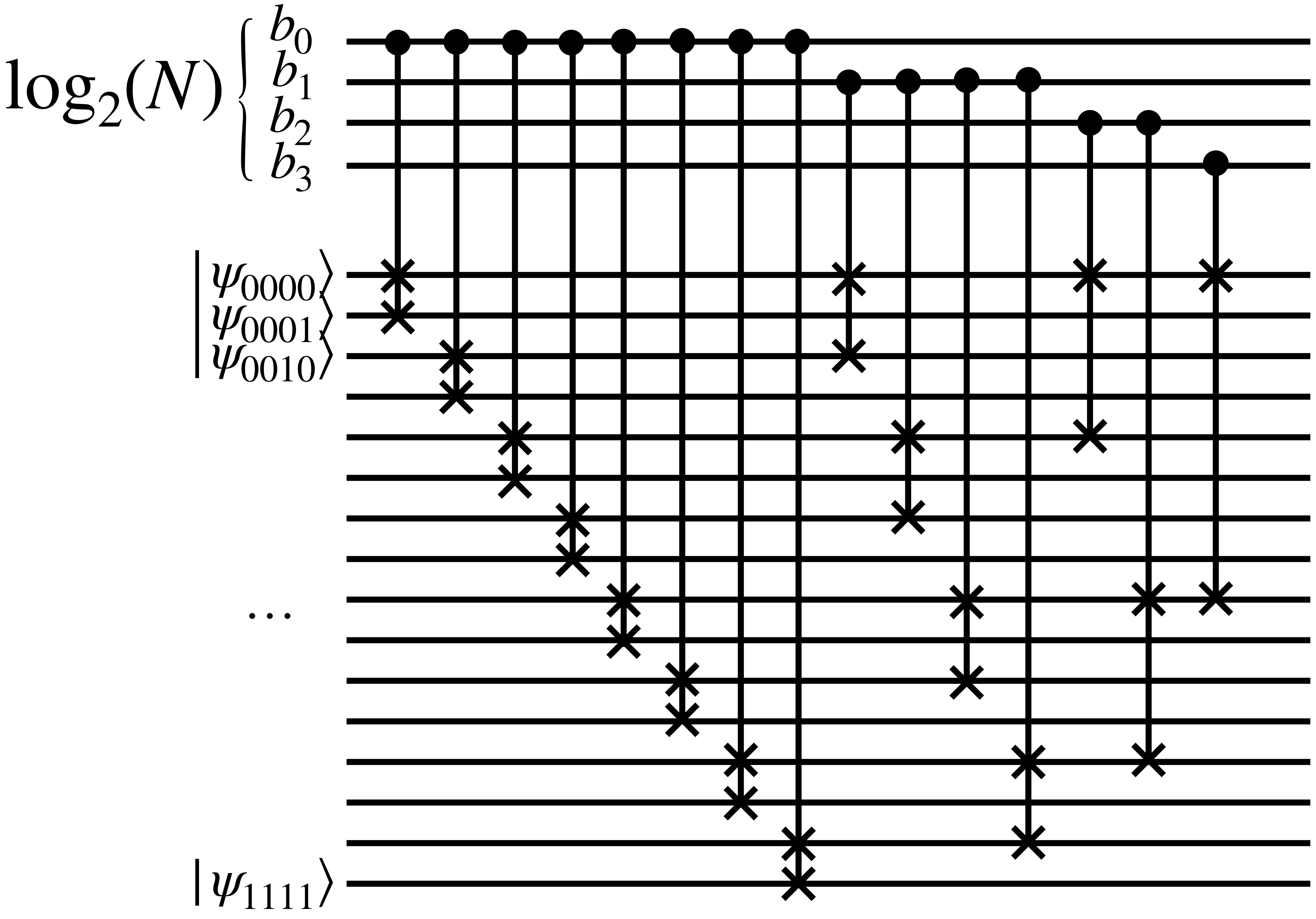

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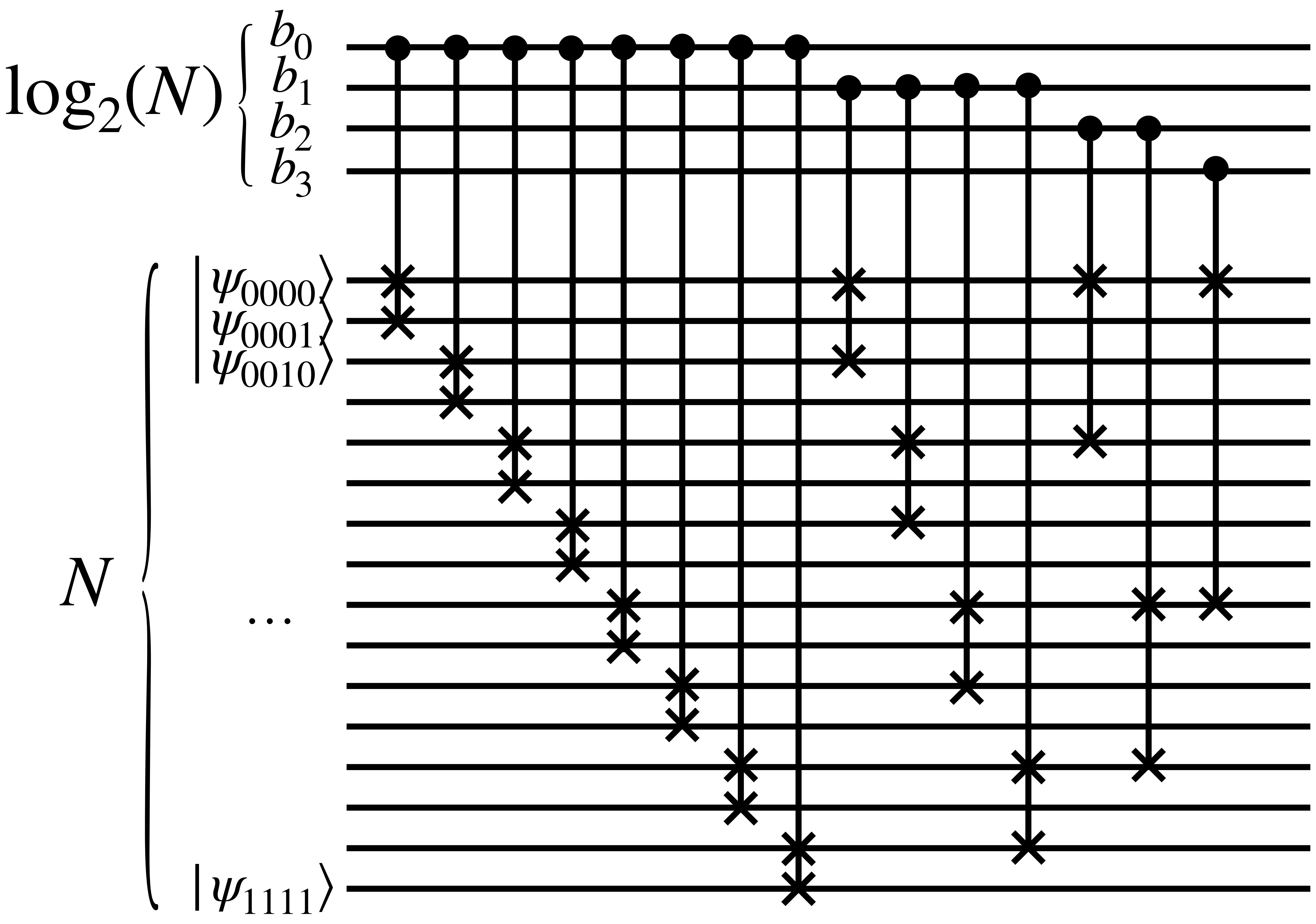

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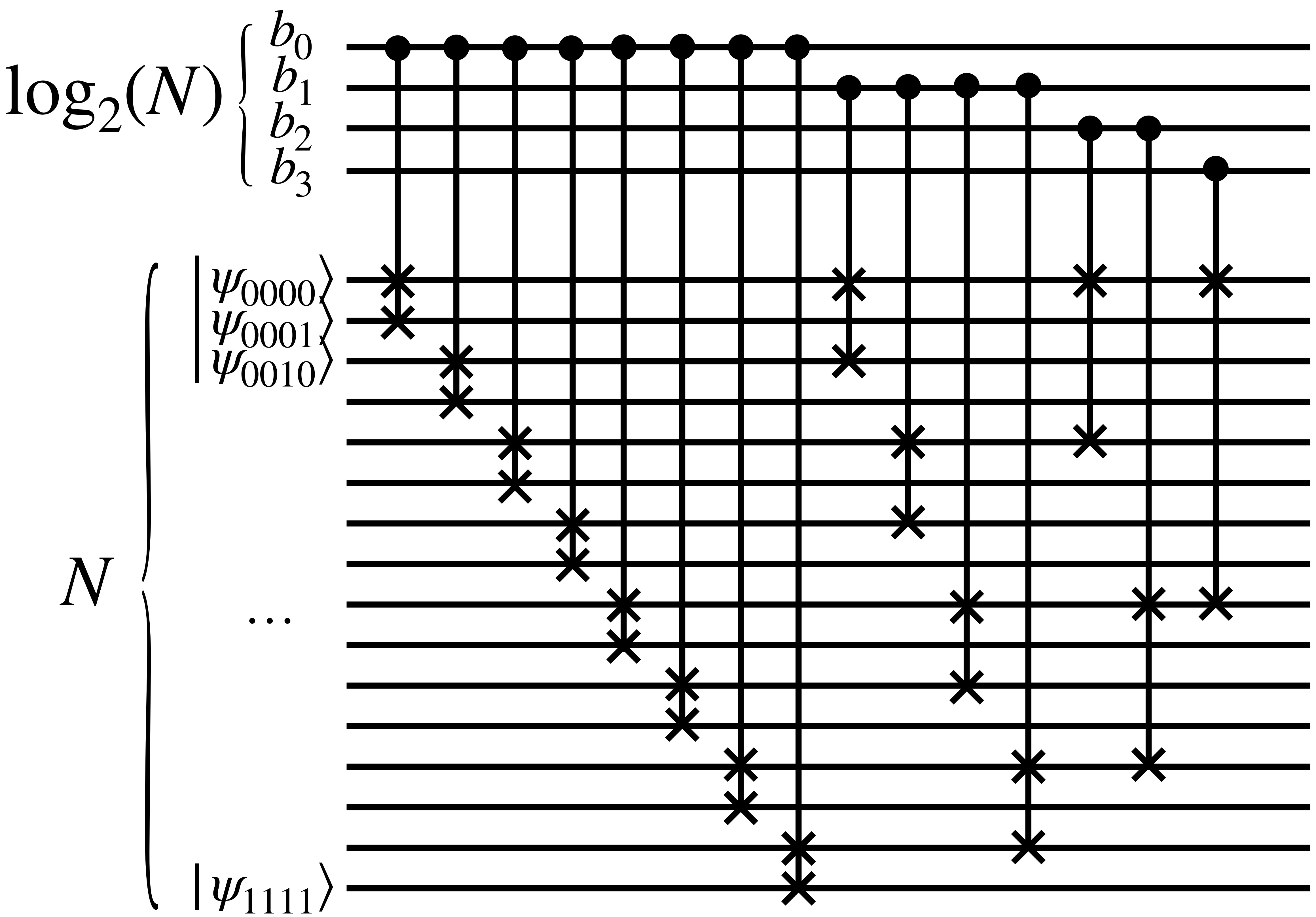

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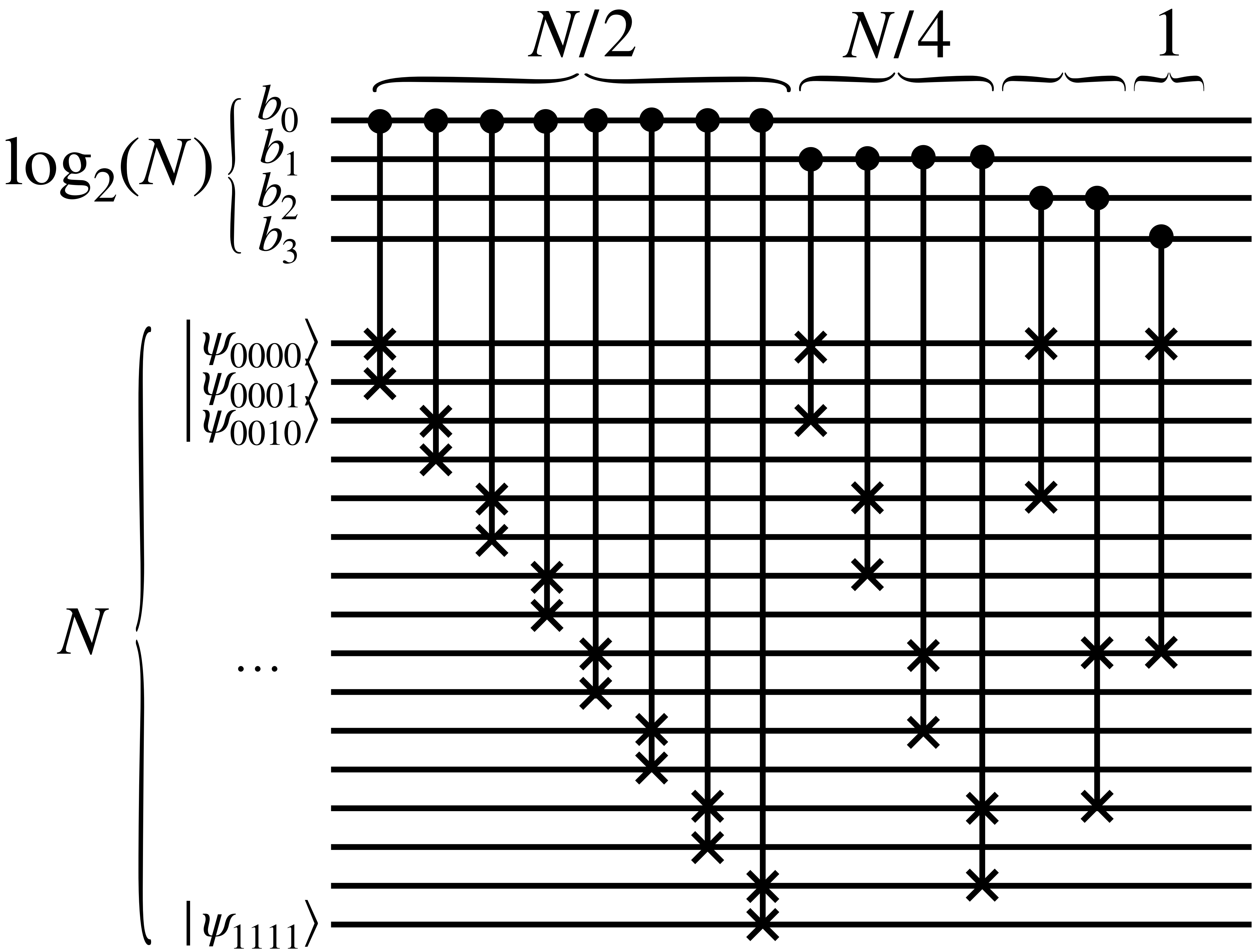

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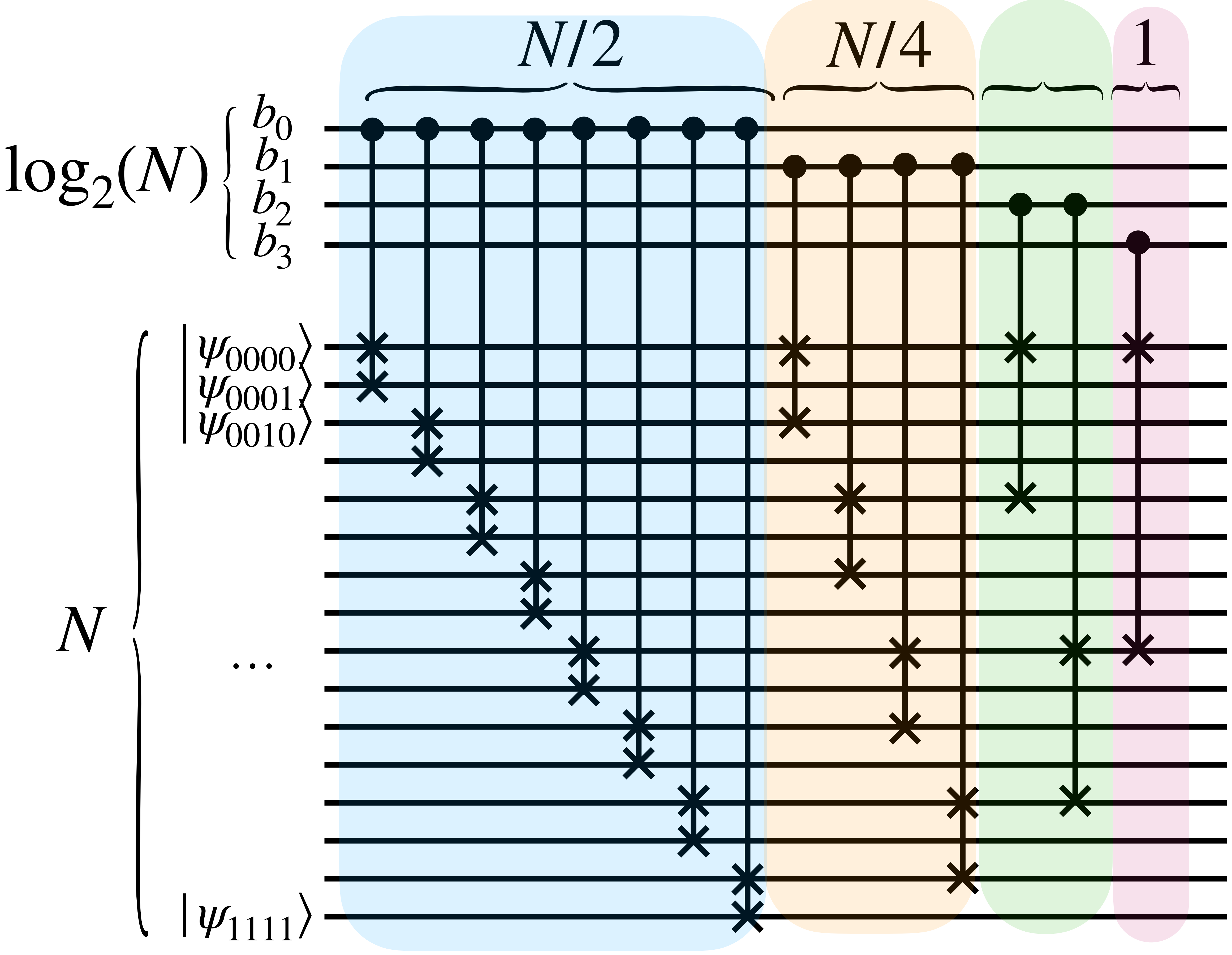

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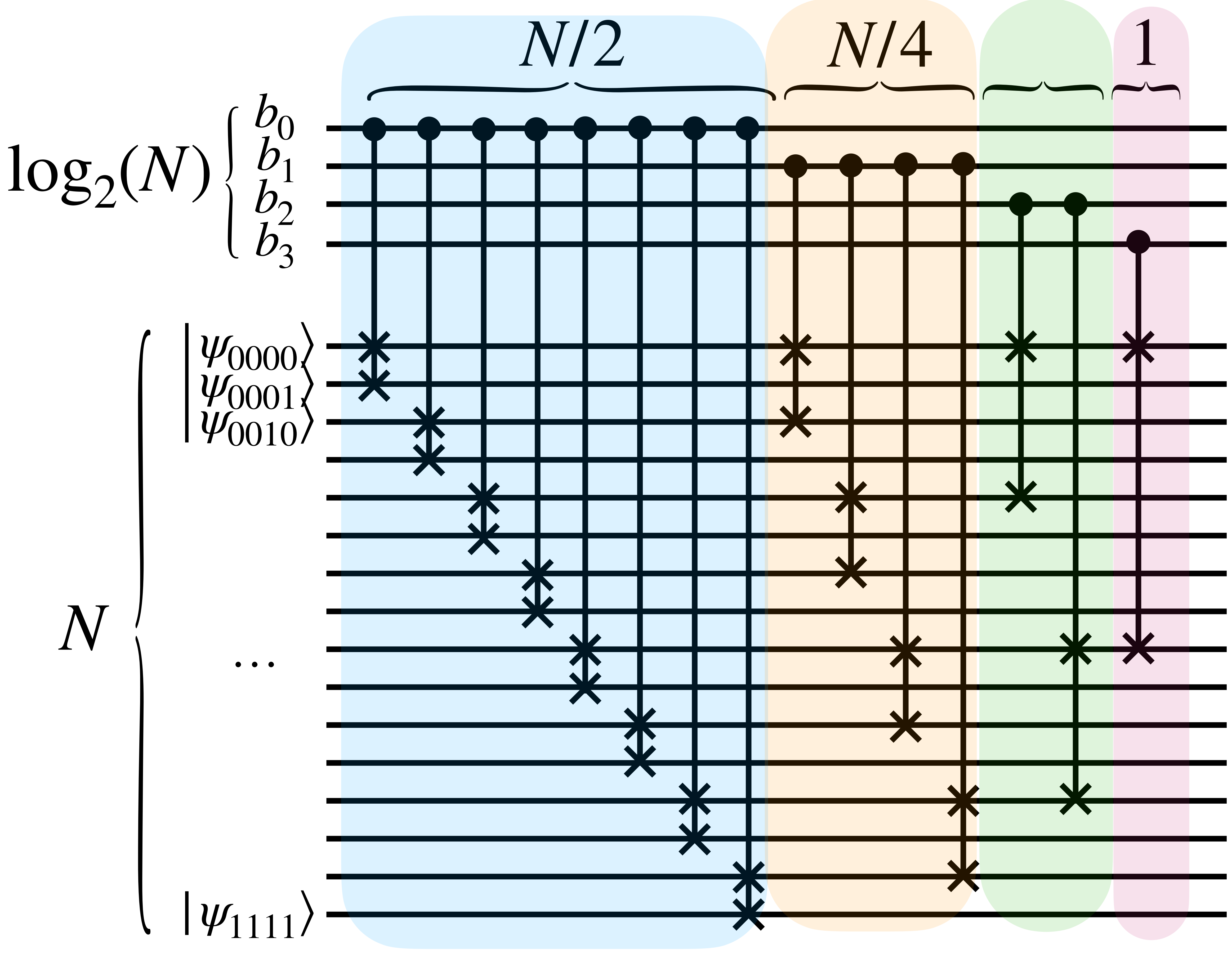

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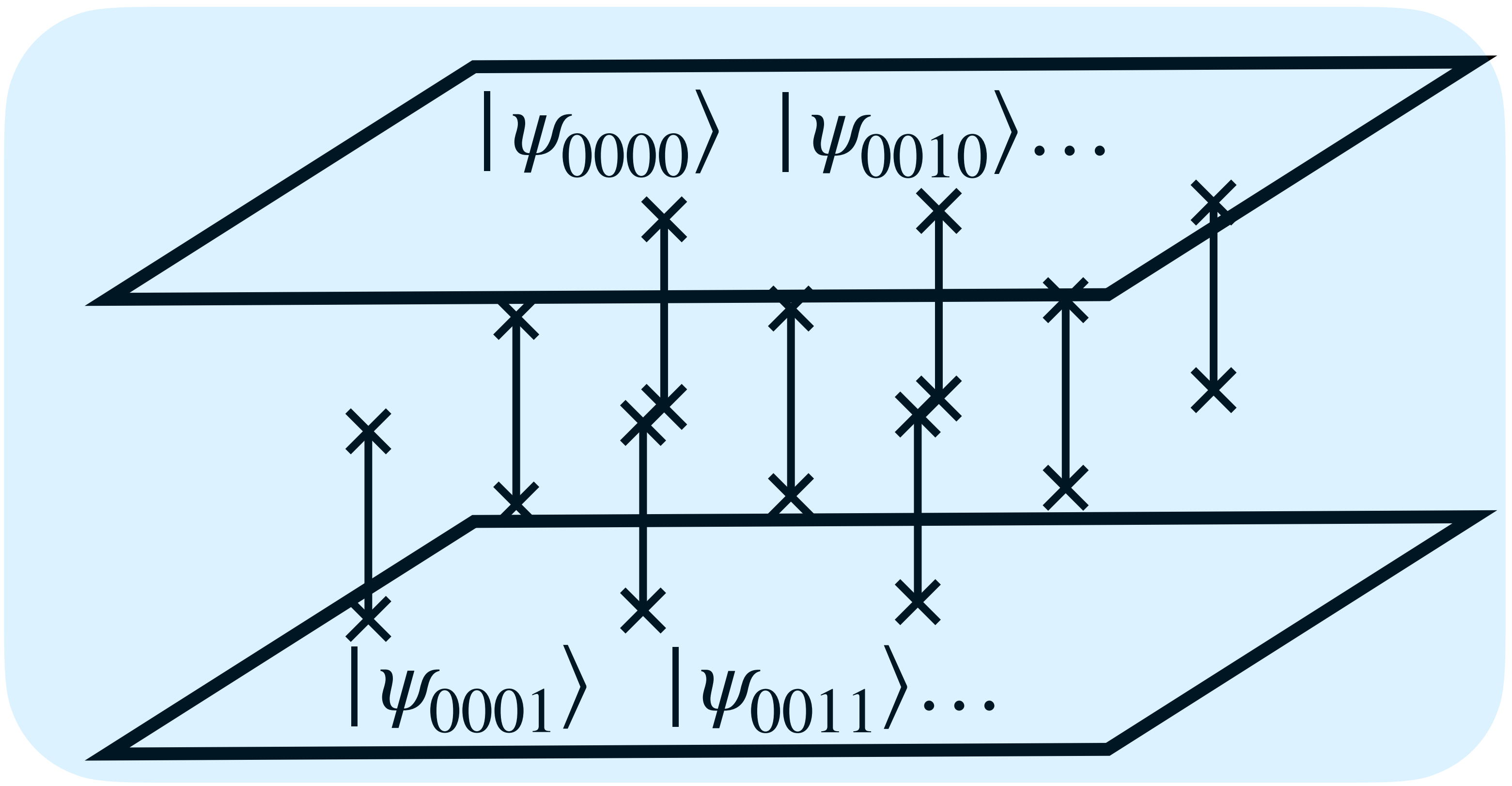

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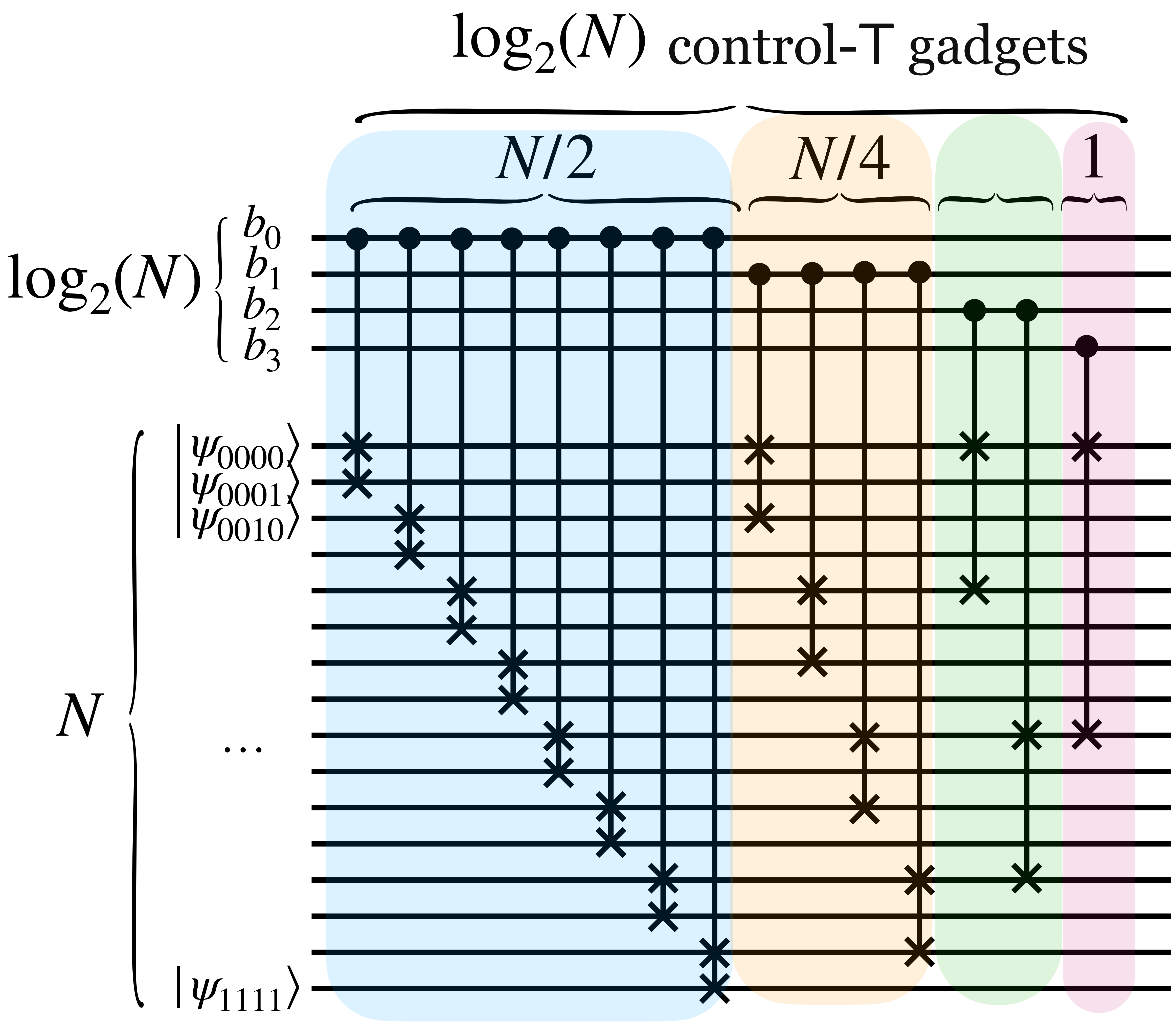

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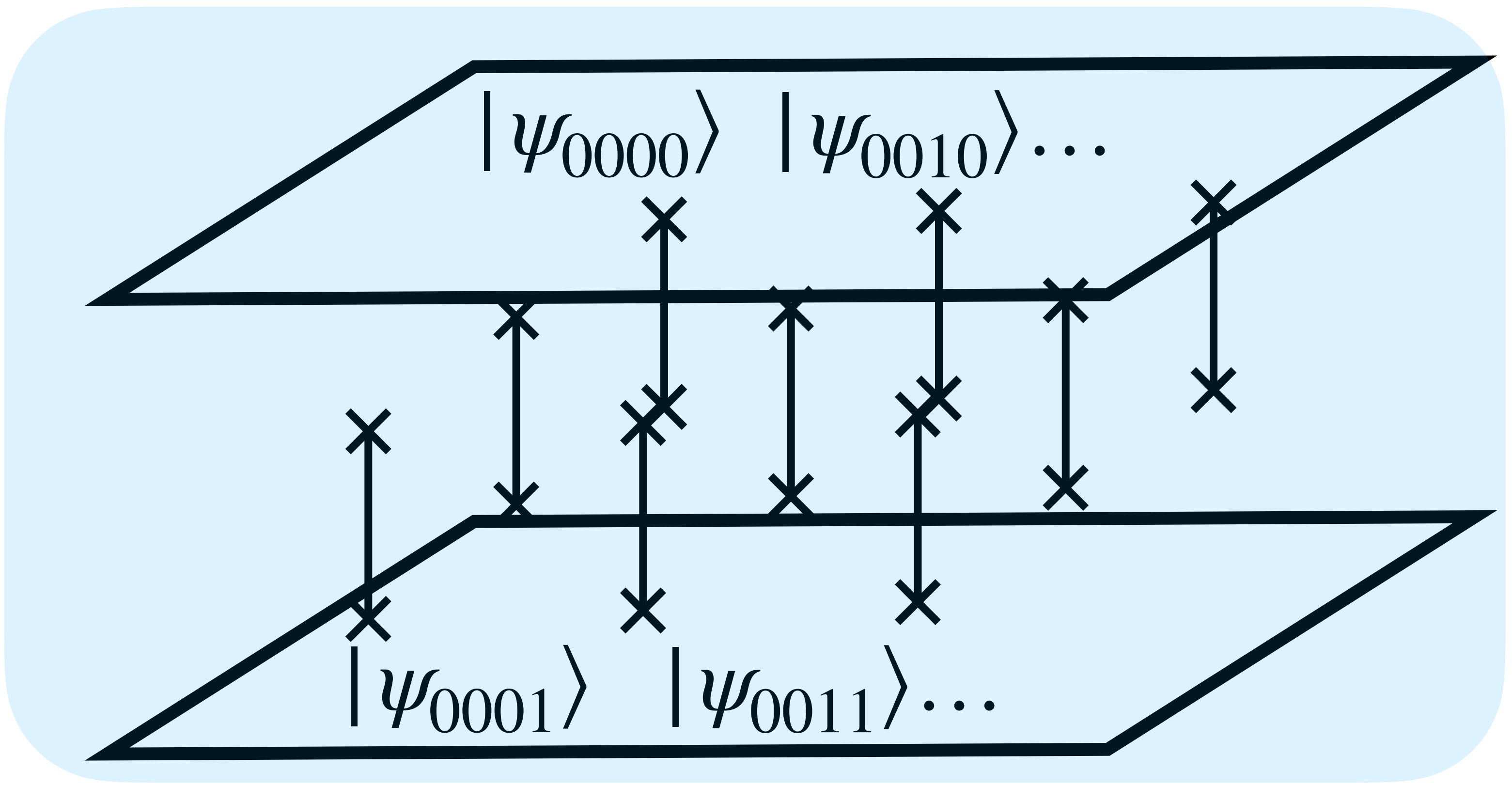

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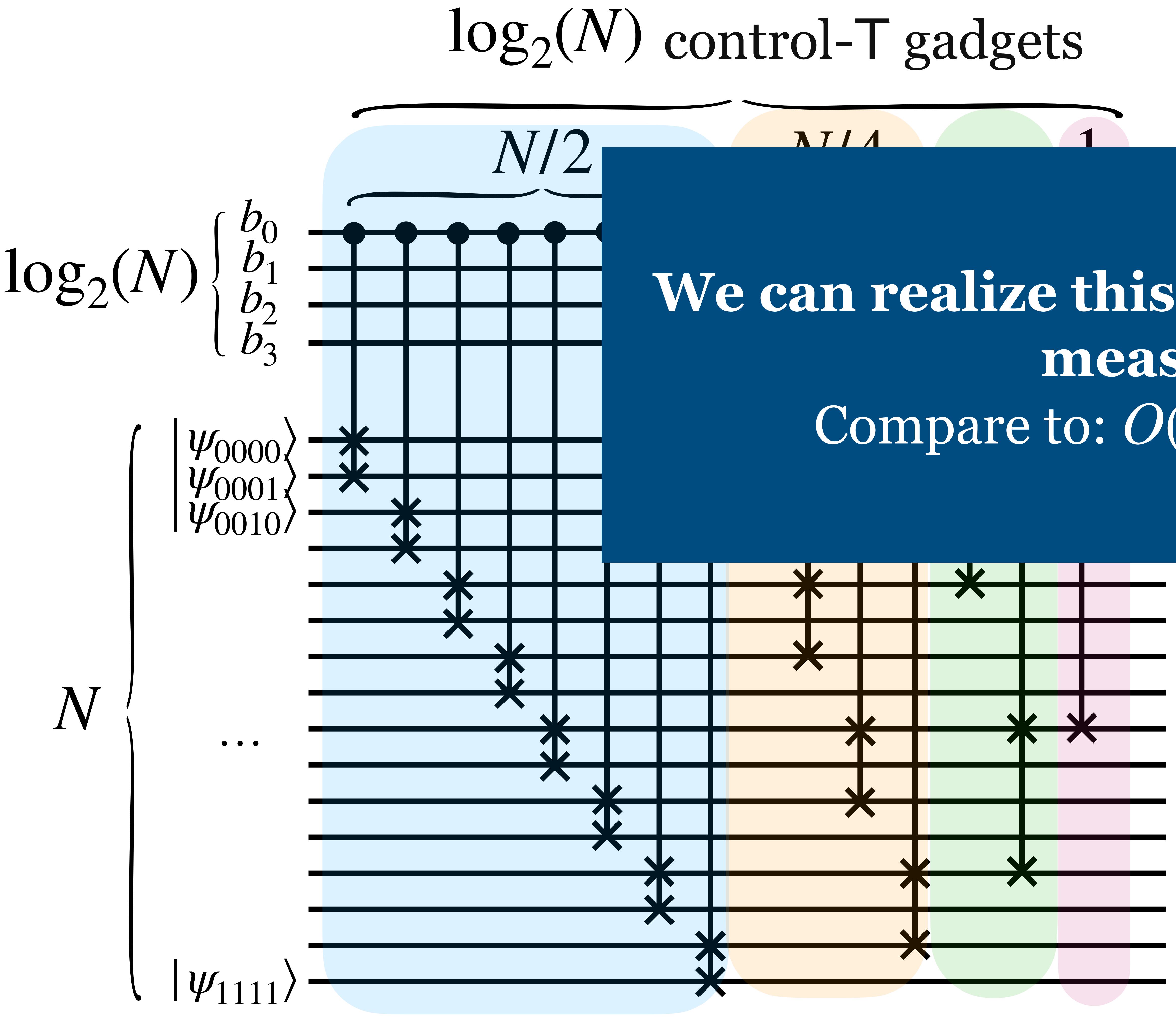

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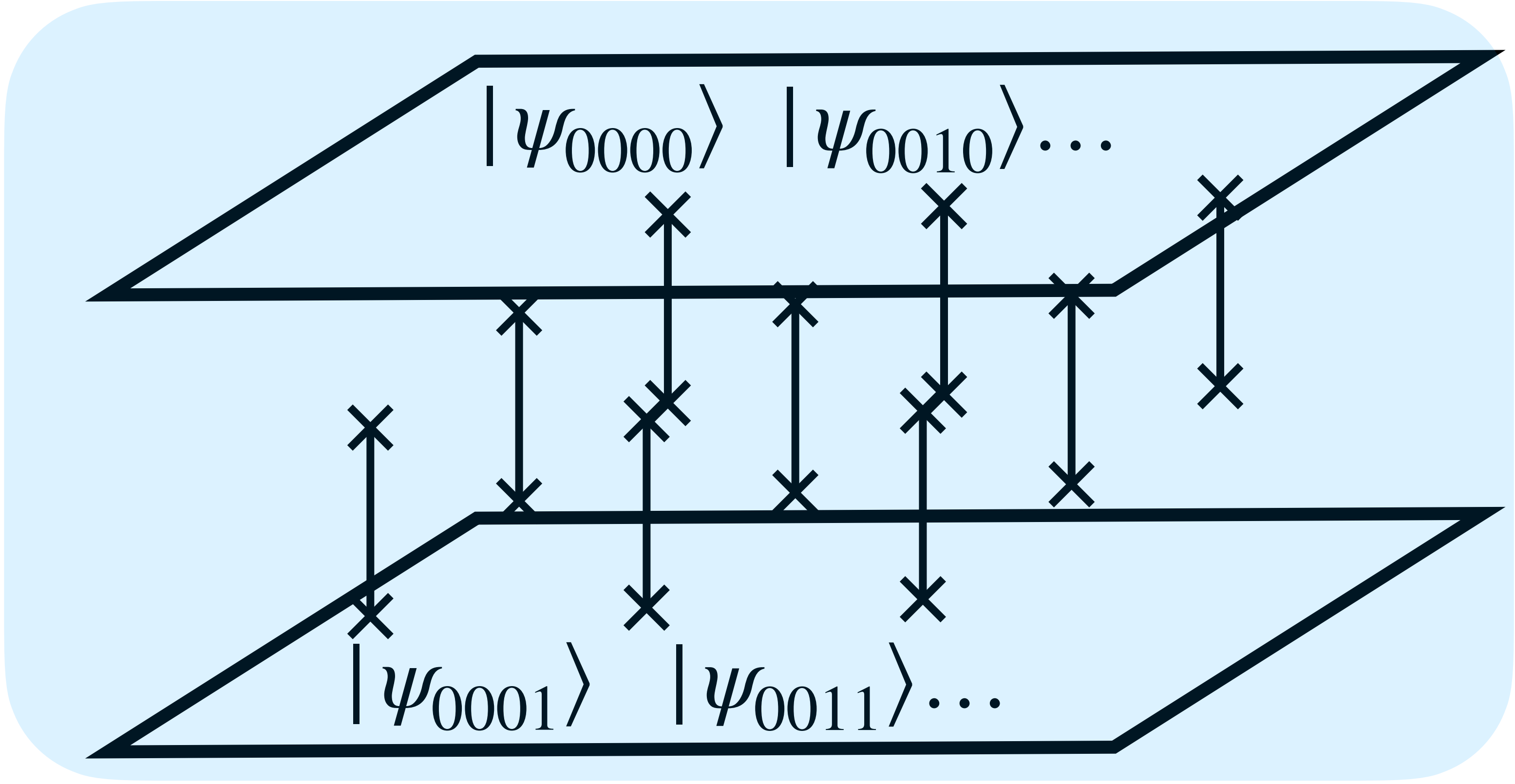

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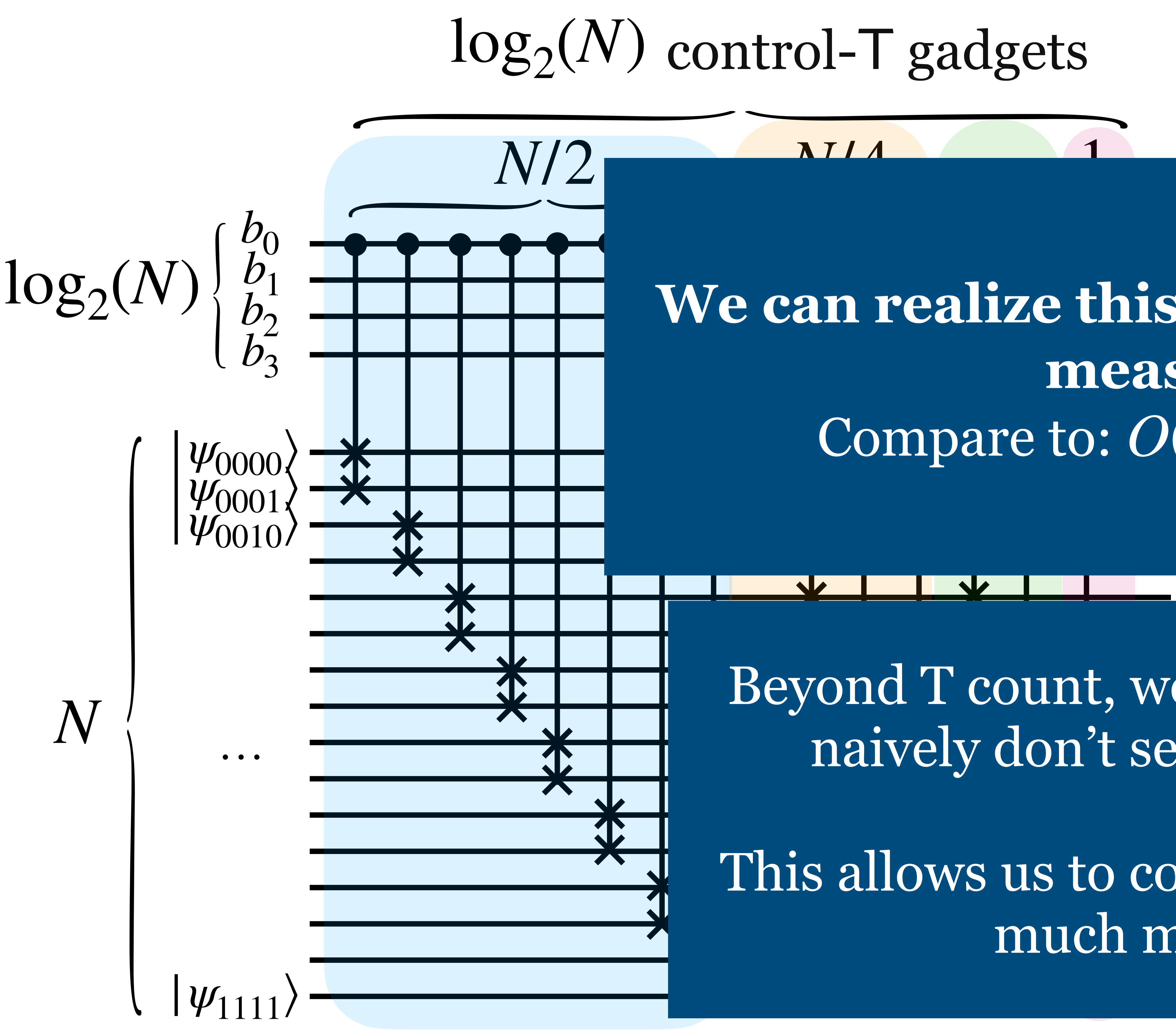

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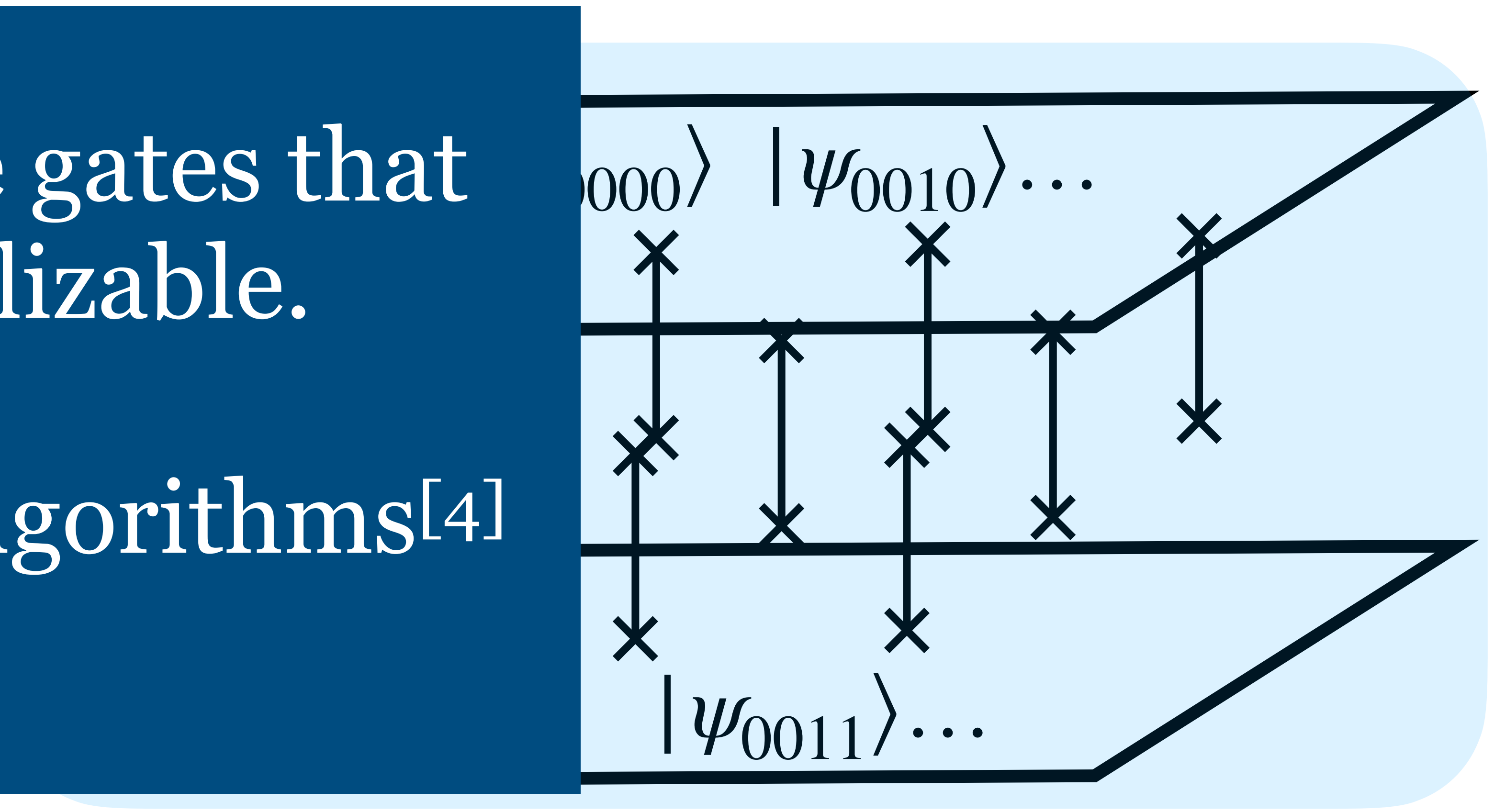
Beyond T count, we can parallelize gates that naively don't seem very parallelizable.

This allows us to compile certain algorithms[4] much more efficiently.



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