Comparing magic state methods (moderated by Katie)

Topics of conversation:

- What are interesting magic states beyond the generic ones, i.e. T and CCZ? Maybe on more than O(1) logical qubits
- What are the different ways we can get magic into a computer
- What metric should we use to evaluate schemes, e.g. used to be gamma/beta parameters etc
- Architecture-dependent evaluation metrics? Rejection-rate? Space-time footprint?
- How practical is it to have transversal non-Clifford high-rate codes
- what methods are practical in the finite size regime
- Post-selection versus unitary methods
- How useful are different methods algorithmically?
- How to categorise different methods
- Post-selection vs measurement-based protocols?
- How to estimate the performance of these methods?
- How do we efficiently simulate magic state protocols?
- Analytical methods for arguing about fault-tolerance
- Synthillation practical for distance 2 but not so beyond
- Trilinear pairings beyond the cup-product
- Good factory code candidates?

- Should we have a protocol "zoo"? Potential database exists, will be publicly released "soon" from Austin Fowler's TQEC working group

Discussion

Methods for generating/consuming magic

- Non-FT magic state injection
    - e.g. catalytic embedding

- Non-FT small angle dependent (regime-dependent as scalability issue)

- Magic state distillation (T, CCZ, mixed - e.g. shared-ctr-SWAP, diagonal Clifford hierarchy)
    - Cultivation, flag-based circuits (checking the eigenvalue of a Clifford operator)
    - Clifford surgery
    - Finite-depth unitaries
    - Code switching (dimension jumping, teleportation gadgets)

Metrics

- Finite size regime (10^3 physical qubits max)
- Expected quantum physical space, and time, or space-time volume

per magic state (not state) given target logical fidelity (expected meaning includes post-selection cost)
    - What's a reasonable time overhead/throughput (Gidney: 200k qubits, 1 per logical cycle)
        - How many magic states do we want per logical cycle? Related to the active volume picture, architecture dependent?
- Decoding difficulty

- What target LERs do we care about?
    - Mega-QUOP regime 10^-6 - Achievable with cultivation
    - Some cryptographic applications 10^-9 Distillation from here?
    - chemistry simulations 10^(-12) - 10^(-15)
    - 10^(-18) / 10^(-20) and beyond useful?
    - What applications for what LERs?
- Logical space-time volume better suited for distillation schemes, versus physical for cultivation schemes.
    - Or is it? Big topic of discussion/point of confusion in the room!
    - NB: Given a target factory code for distillation, we can transfer between physical and logical.
- Target factory code
    - Assumed physical gates: arbitrary single-qubit, 2-qubit gates, CCZ
    - Assumed noise model
        - Physical 2-qubit gates noisier than single-qubit
        - Single and two-qubit gates, CCZ with noise rate p
        - K-qubit gate
        - Injecting k-qubit magic states should be more expensive than injecting single-qubit magic states
- Code switching between two CSS codes is easier

Open questions

- Clifford equivalence of hypergraph (CCZ) states
    - Refs: magic tricycles, cup product
    - Similar to choosing a good logical basis
    - Having a theory for what can and cannot be converted
- MSD beyond diagonal Clifford hierarchy
- Generalised synthillation
- Codes with unitary, addressable non-Clifford gates in the finite-sized regime
- "Hierarchical MSD? Hybrid MSD? Cultivated MSD?":
    - Cultivate a little to 10^-6, then do high-rate distillation at d = 2, 3
    - Or Clifford surgery on a small code then MSD
    - Or small CCZ codes with transversal CCZ
    - Multiple of these small gadgets feeding into qLDPC codes