

Numerical relativity and waveform modeling for XG detectors

Nils Deppe, July 22, 2026
New Frontiers in Strong Gravity



Cornell University

Afram, Blackman, Bonilla, Boyle, Carpenter, Ceja, Corso, Duez, Field, Foucart, Giesler, Habib, Hebert, Jones, Kidder, Kim, Kumar, Lara, Legred, Lovelace, Macedo, Melchor, Mendes, Mitman, Morales, Most, Moxon, Nelli, O'Shea, Pannone, Pfeiffer, Ramirez-Aguilar, Ravishankar, Sanchez, Scheel, Teukolsky, Throwe, Varma, Vu, Wittek, Yoo +++

github.com/sxs-collaboration/spectre



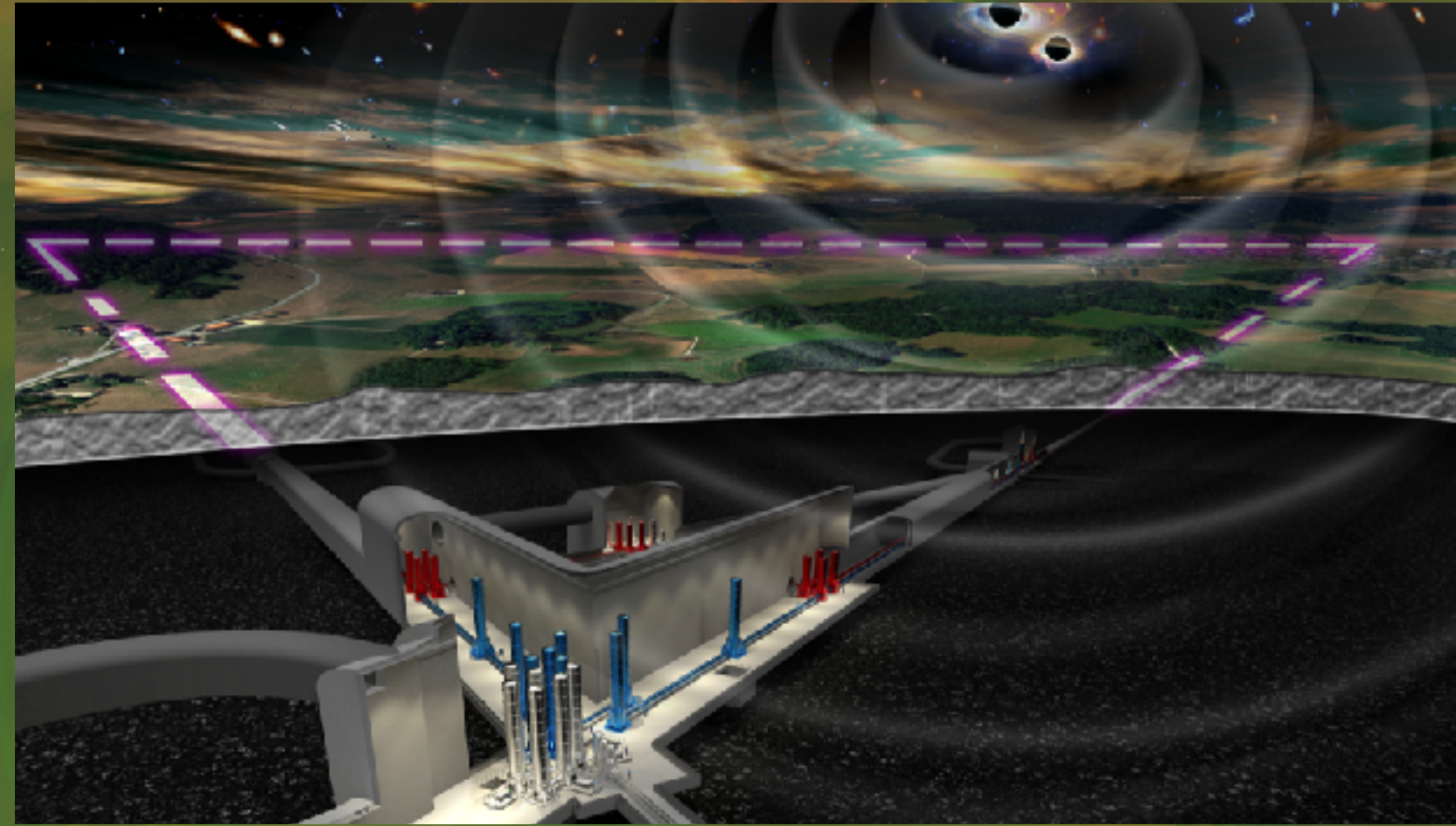


XG GW Observatories



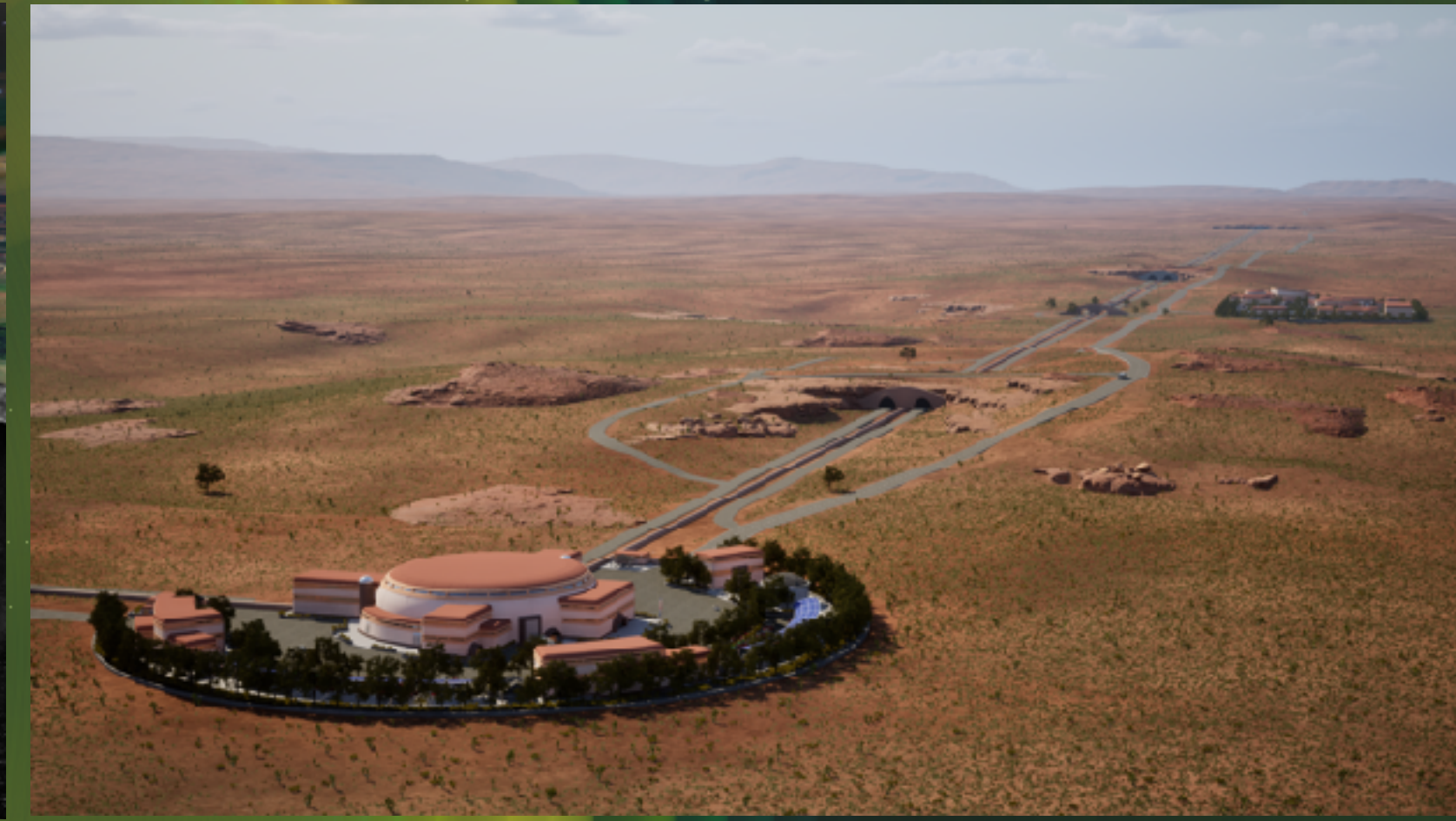
LISA

- 2.5 million km triangular interferometer in space



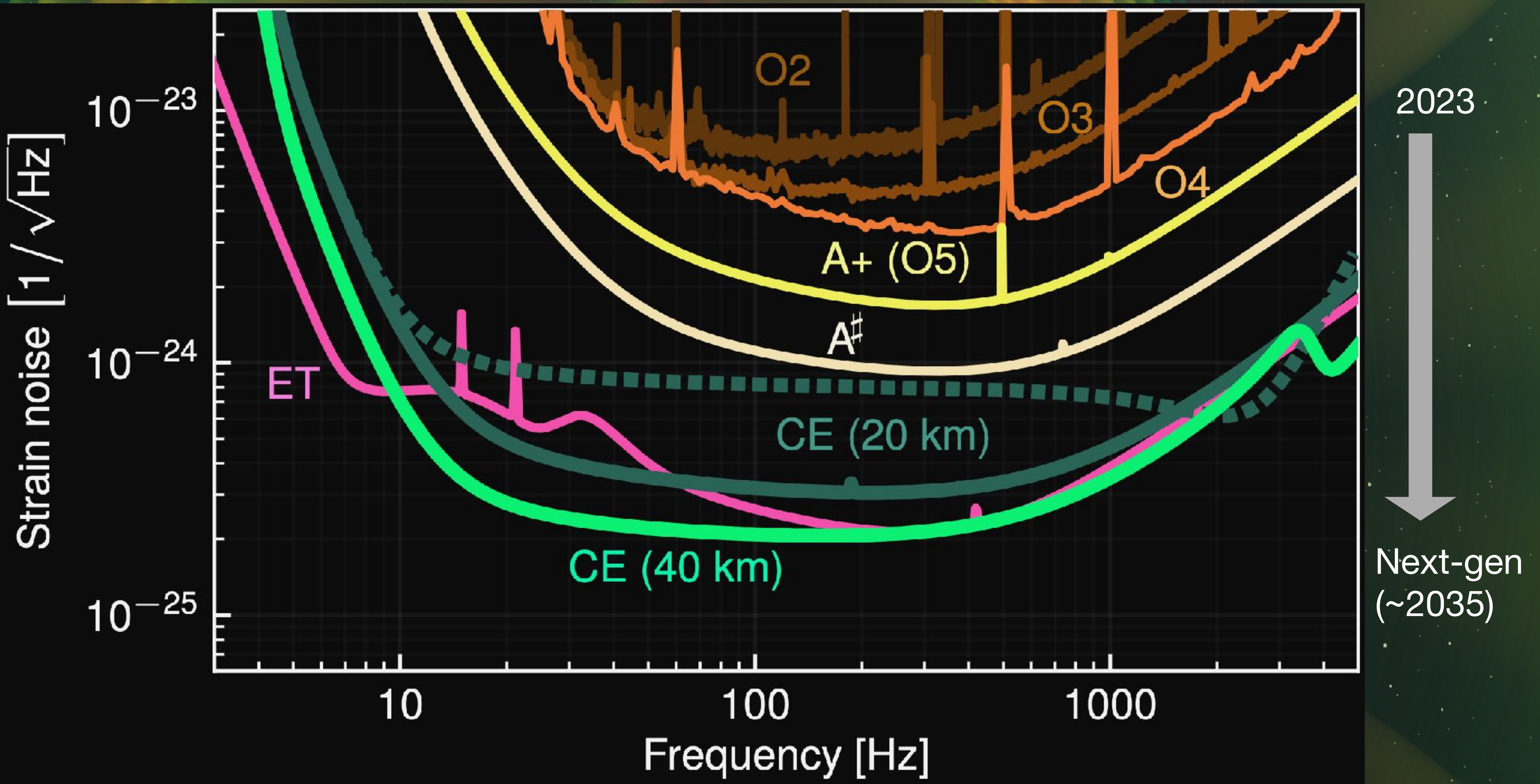
Einstein Telescope (ET)

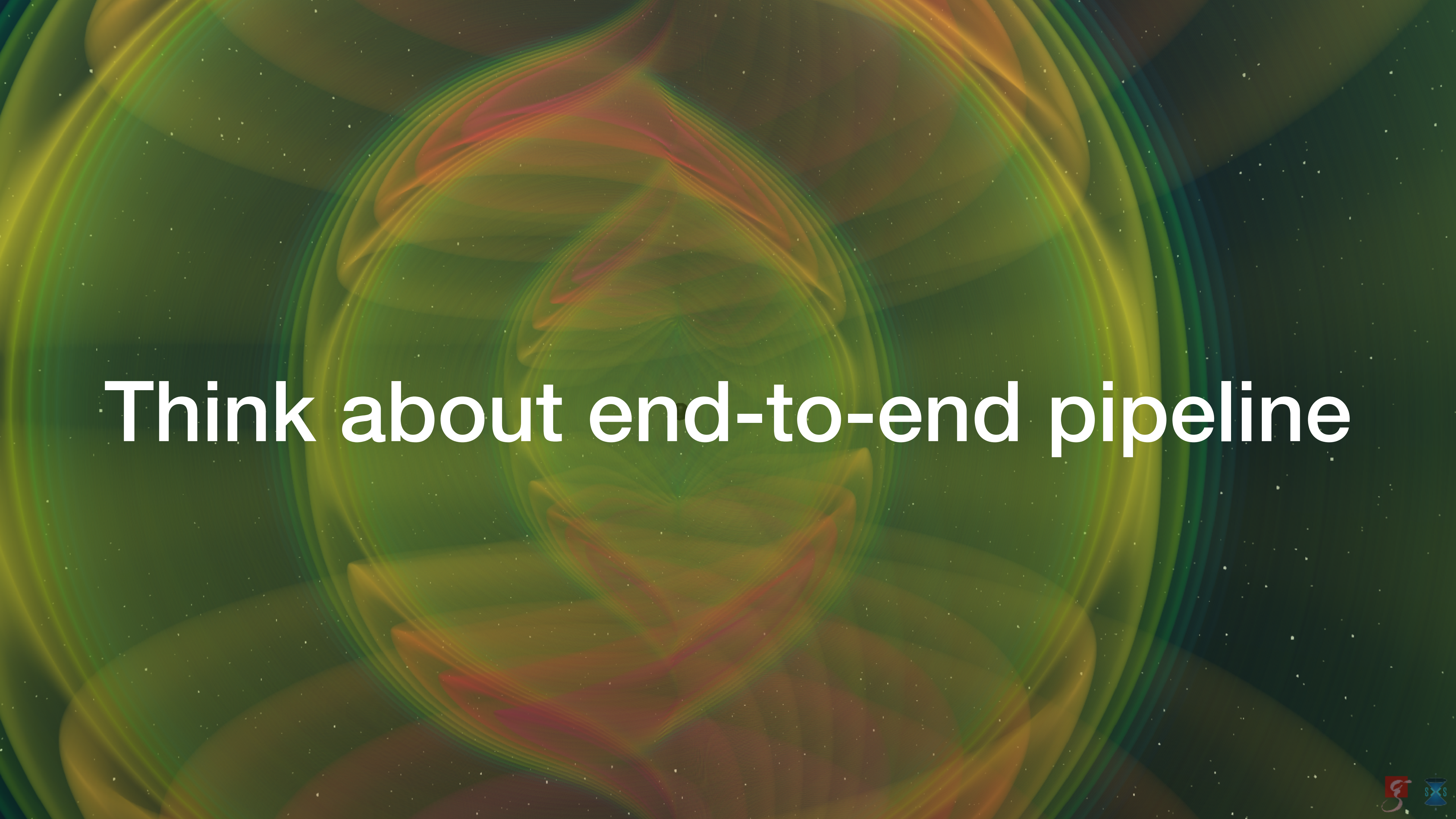
- 10 km underground triangle



Cosmic Explorer (CE)

- 20 km and 40 km L-shaped surface observatories
- scaled up A+ technology & enhancements

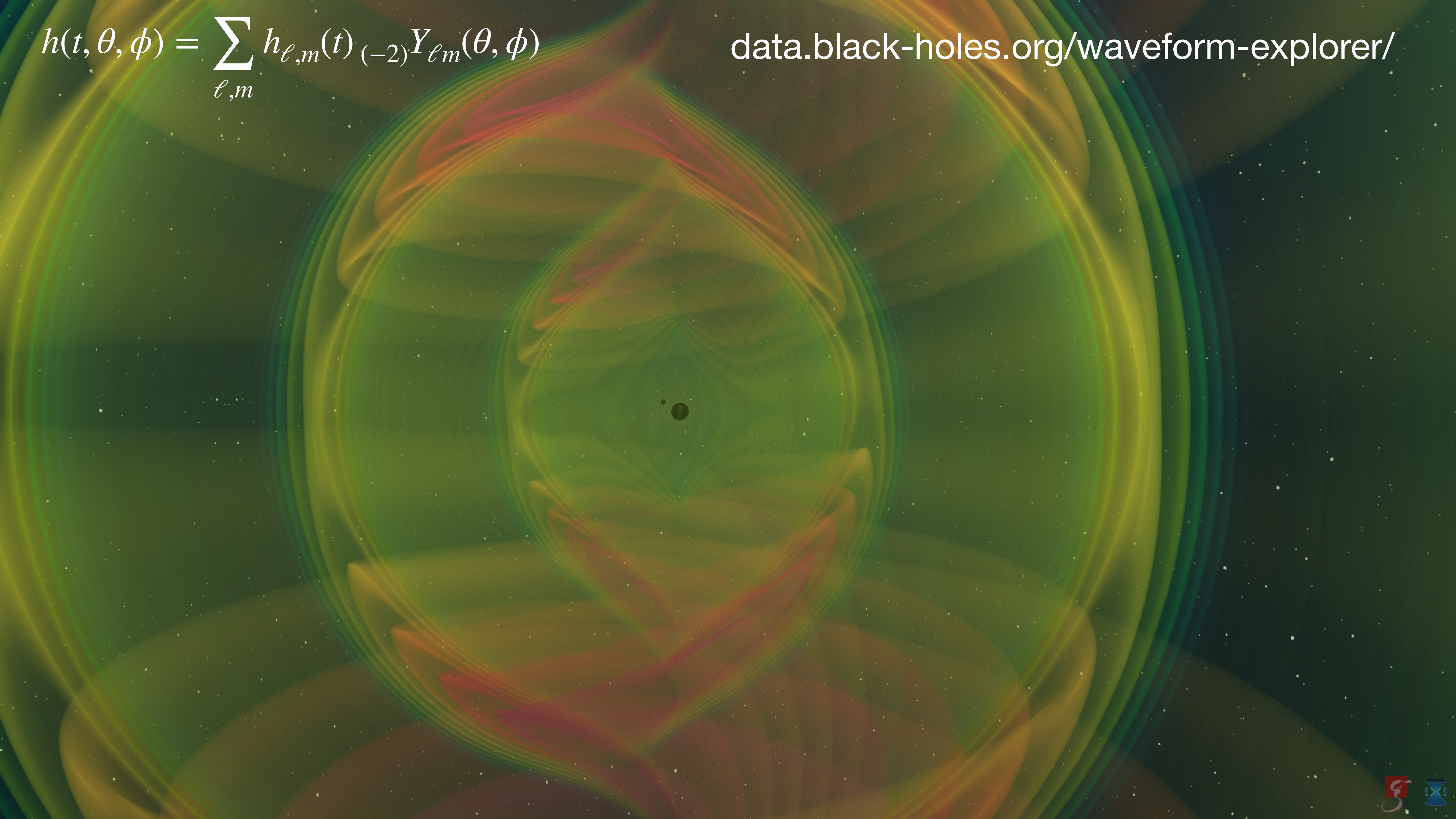




Think about end-to-end pipeline

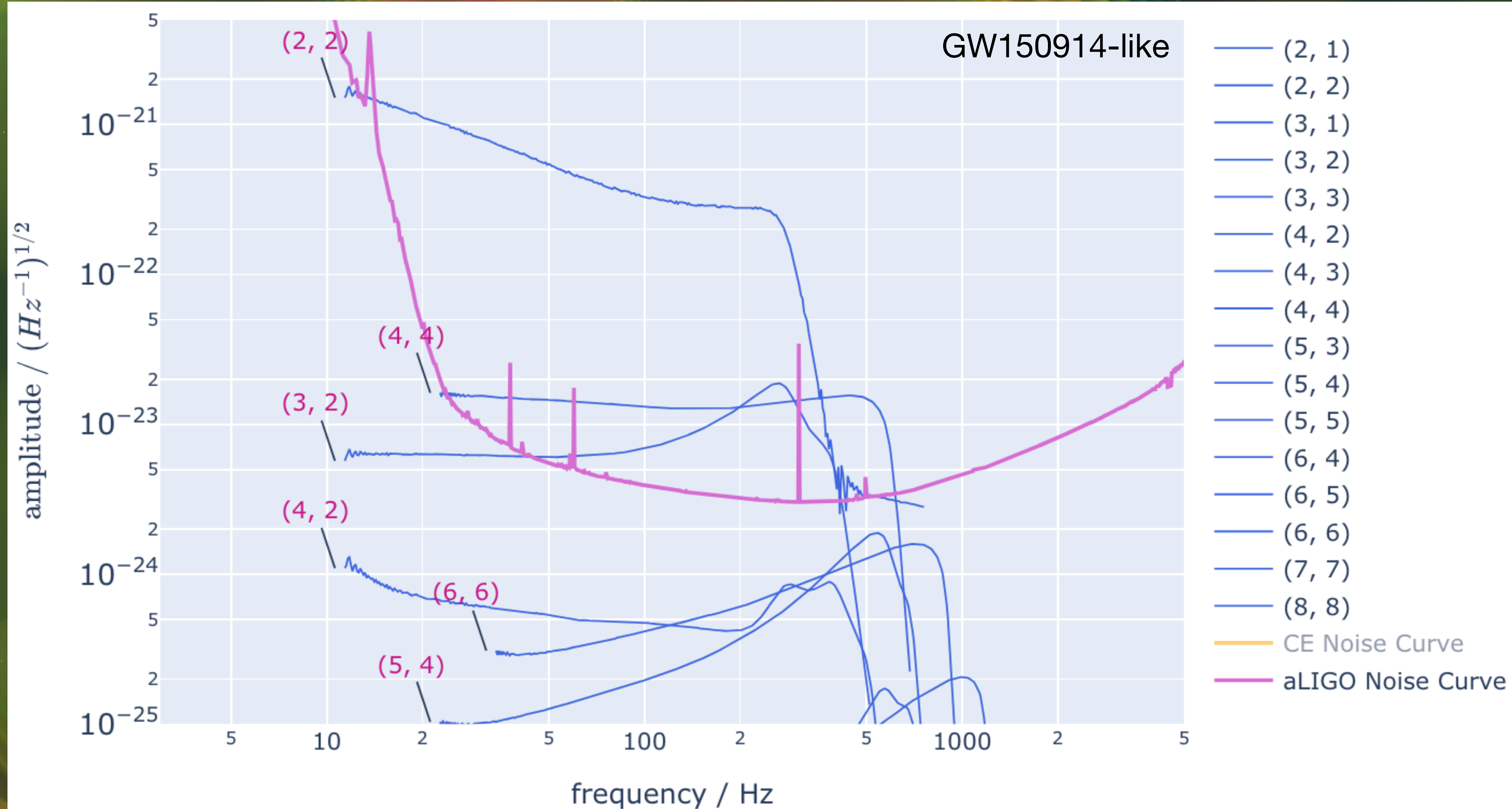
$$h(t, \theta, \phi) = \sum_{\ell, m} h_{\ell, m}(t) {}_{(-2)}Y_{\ell m}(\theta, \phi)$$

data.black-holes.org/waveform-explorer/



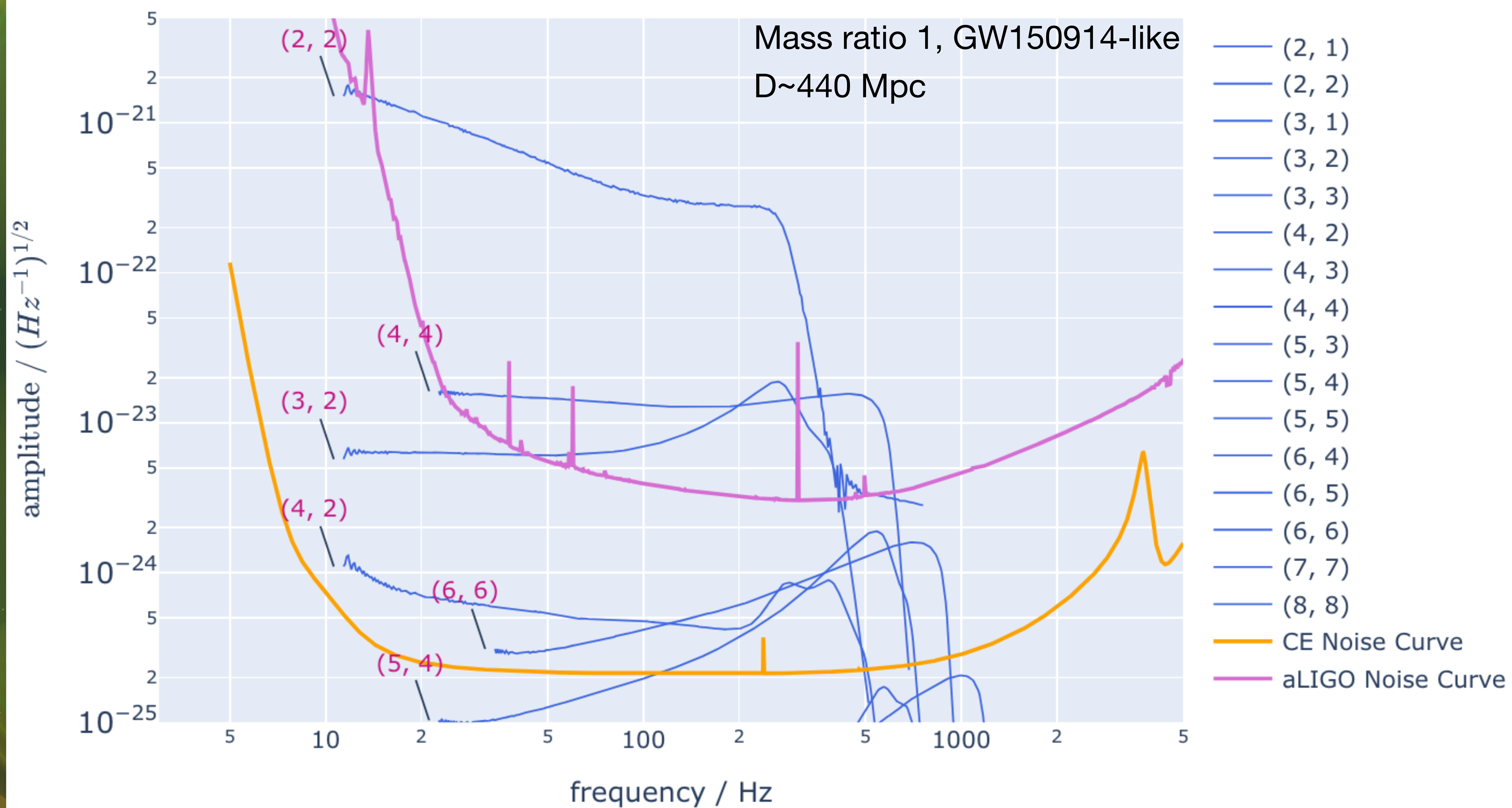
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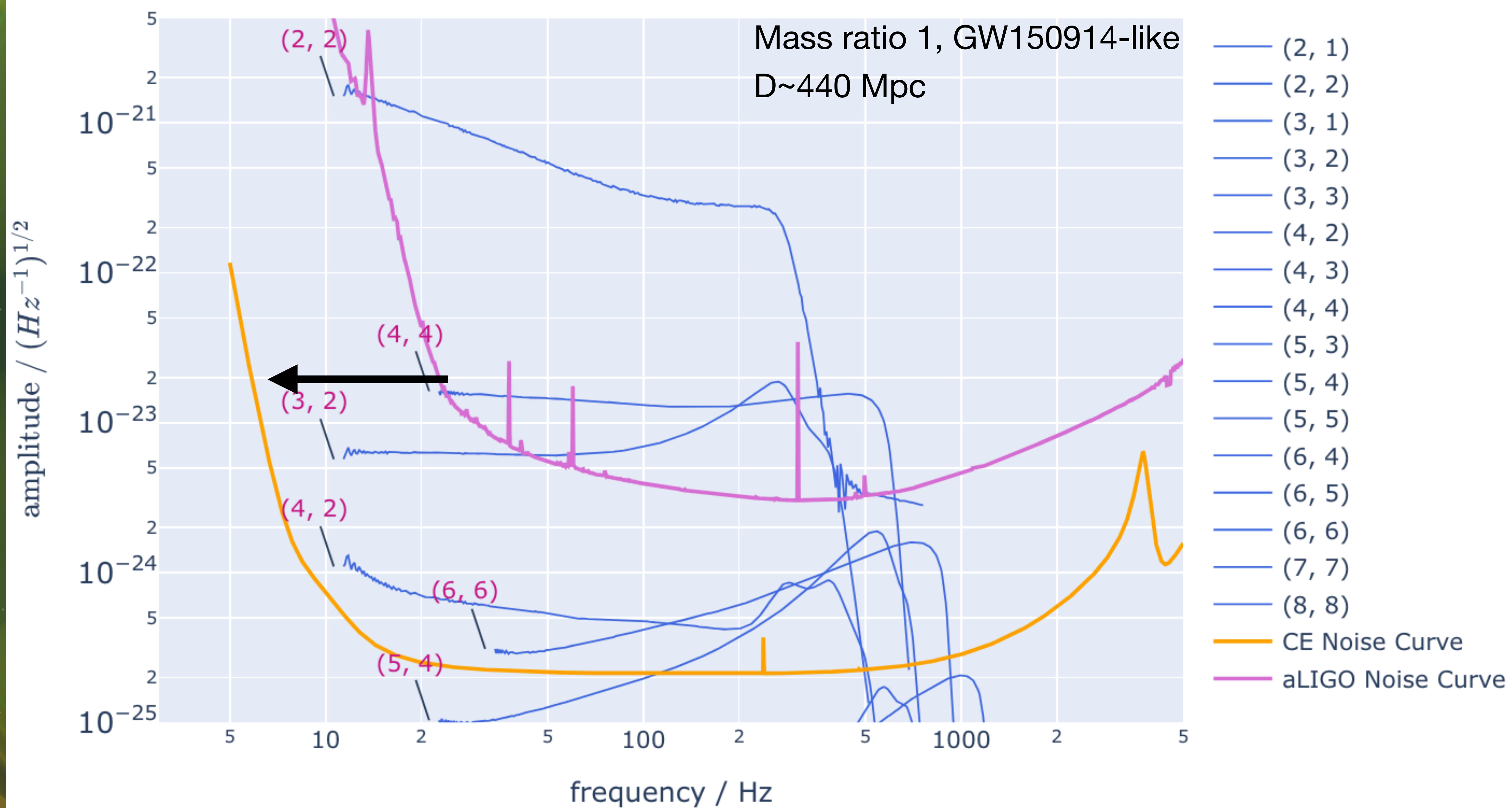
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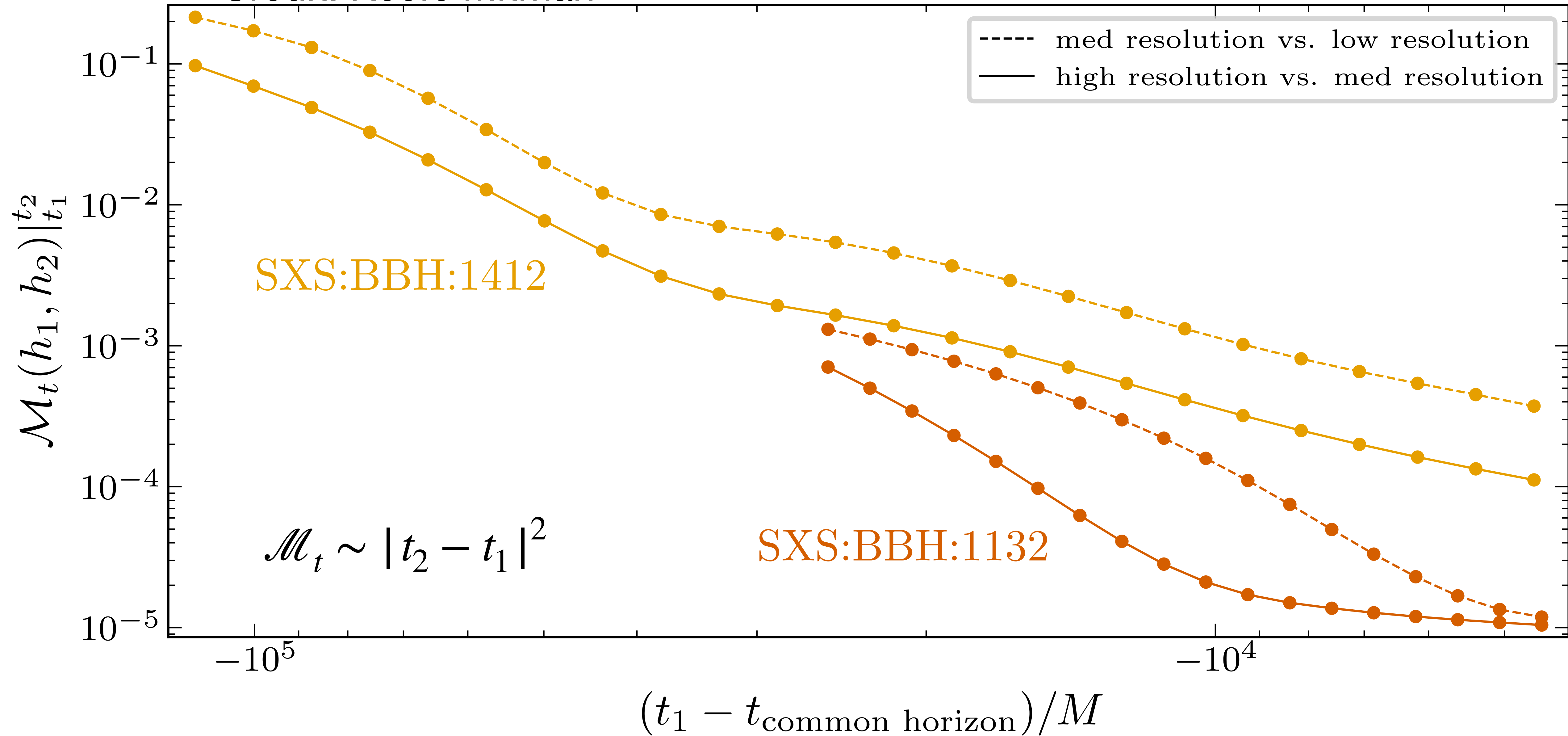


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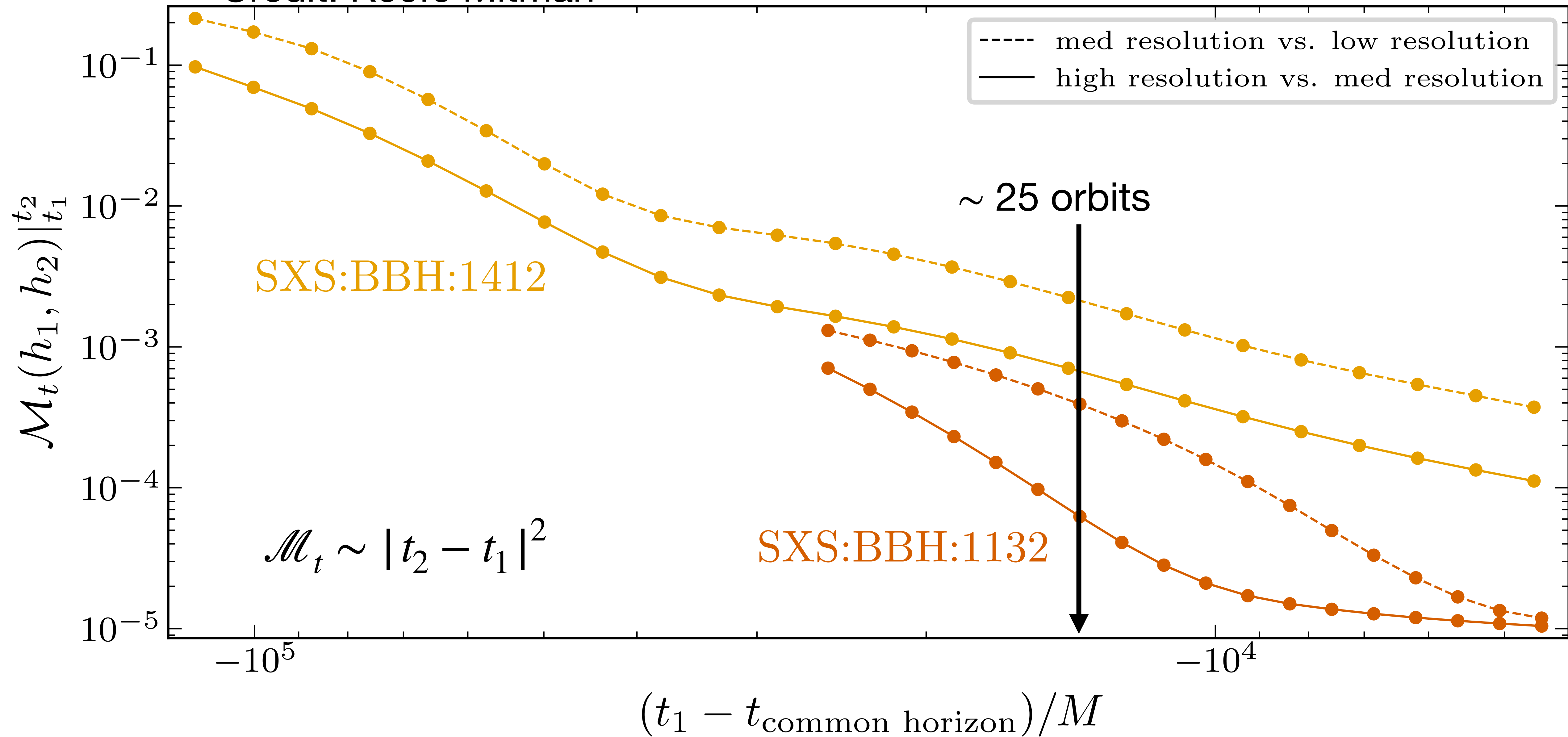
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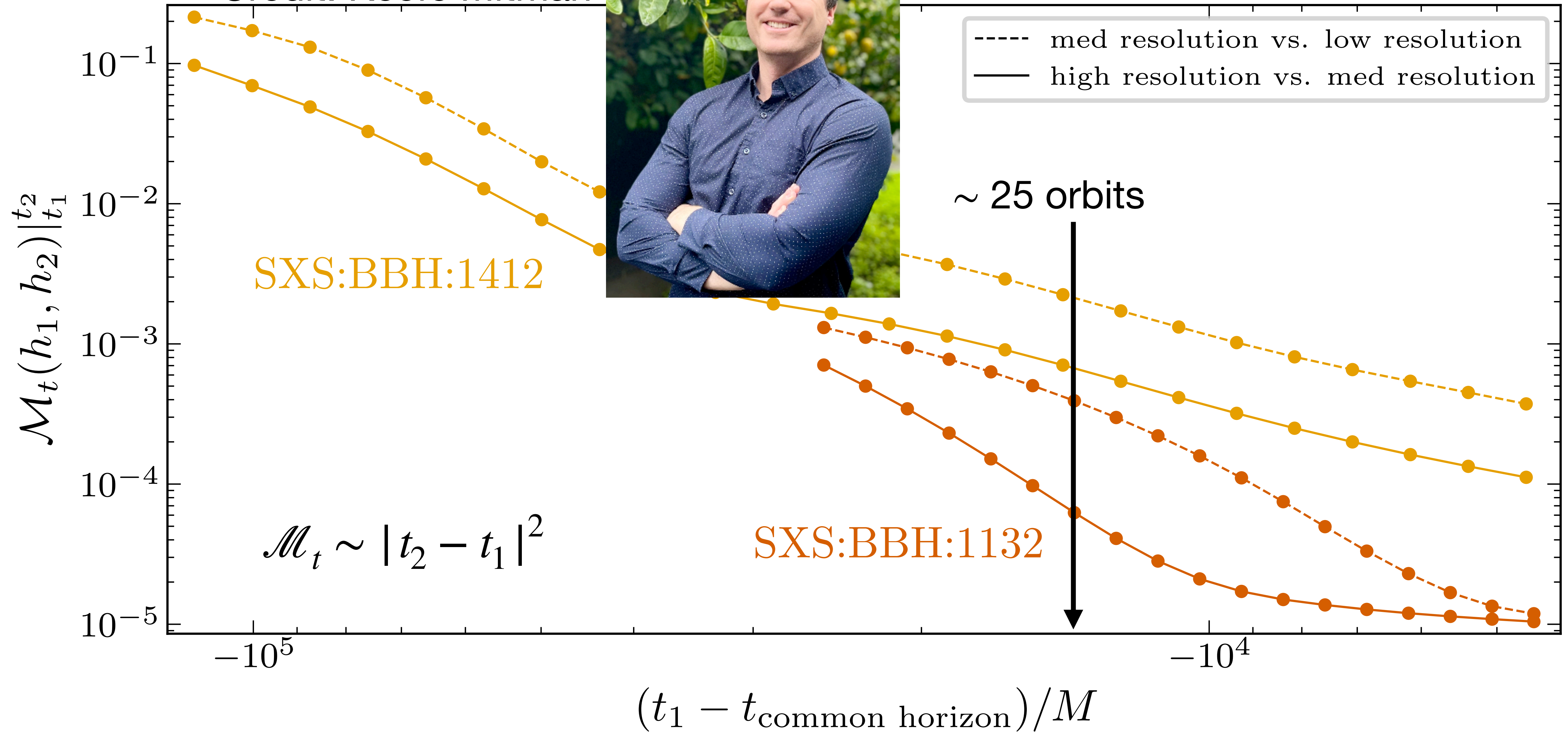
Credit: Keefe Mitman

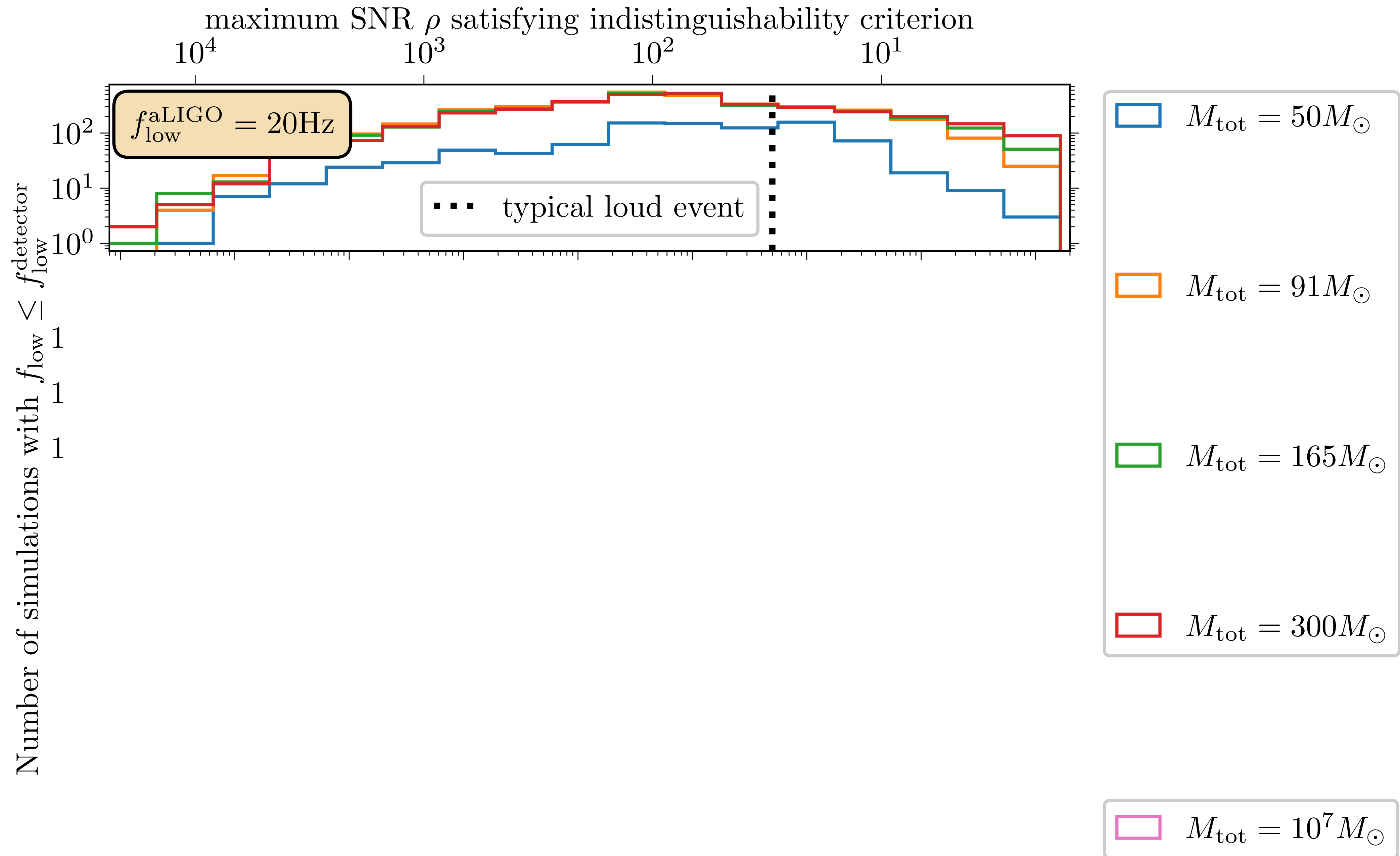


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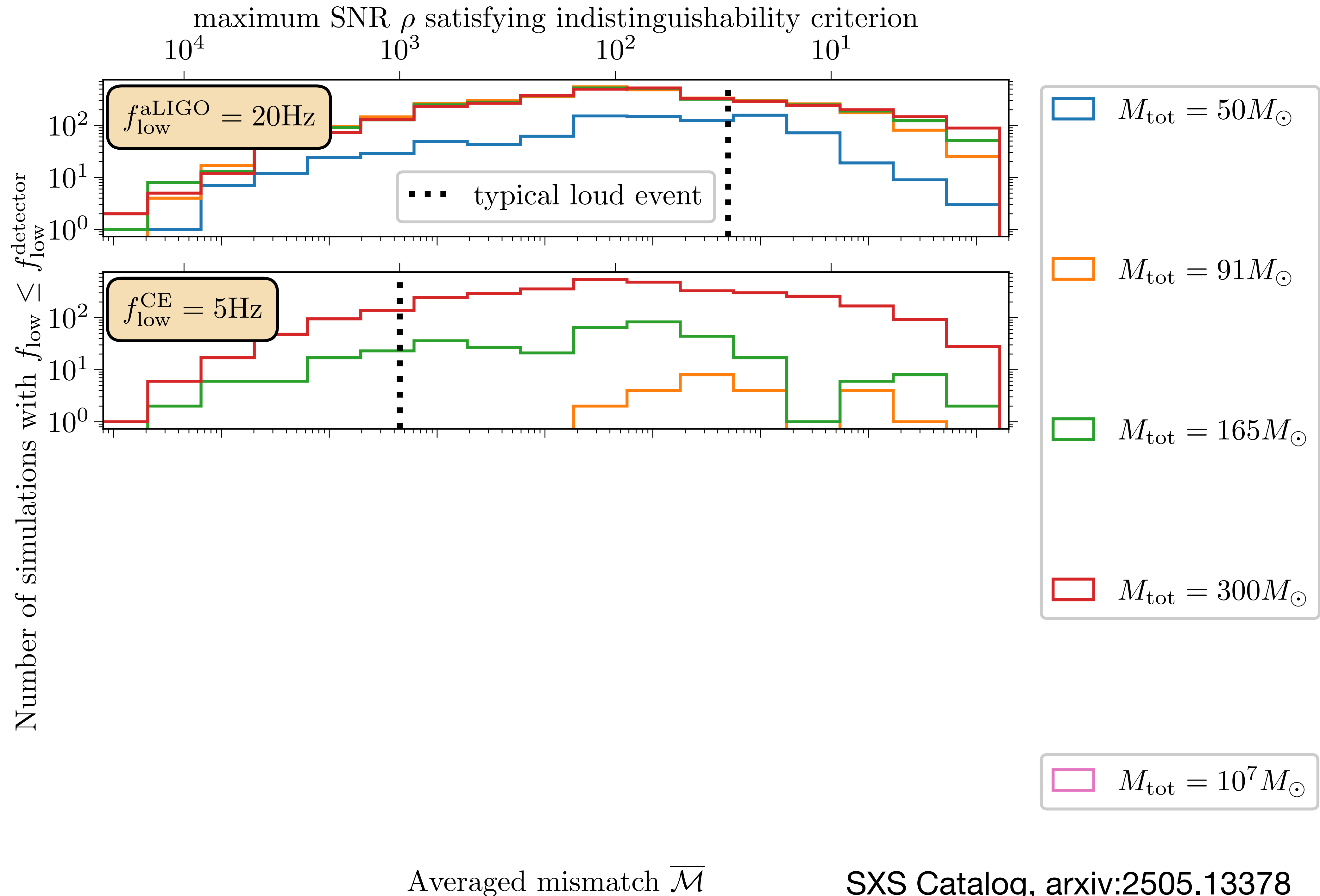


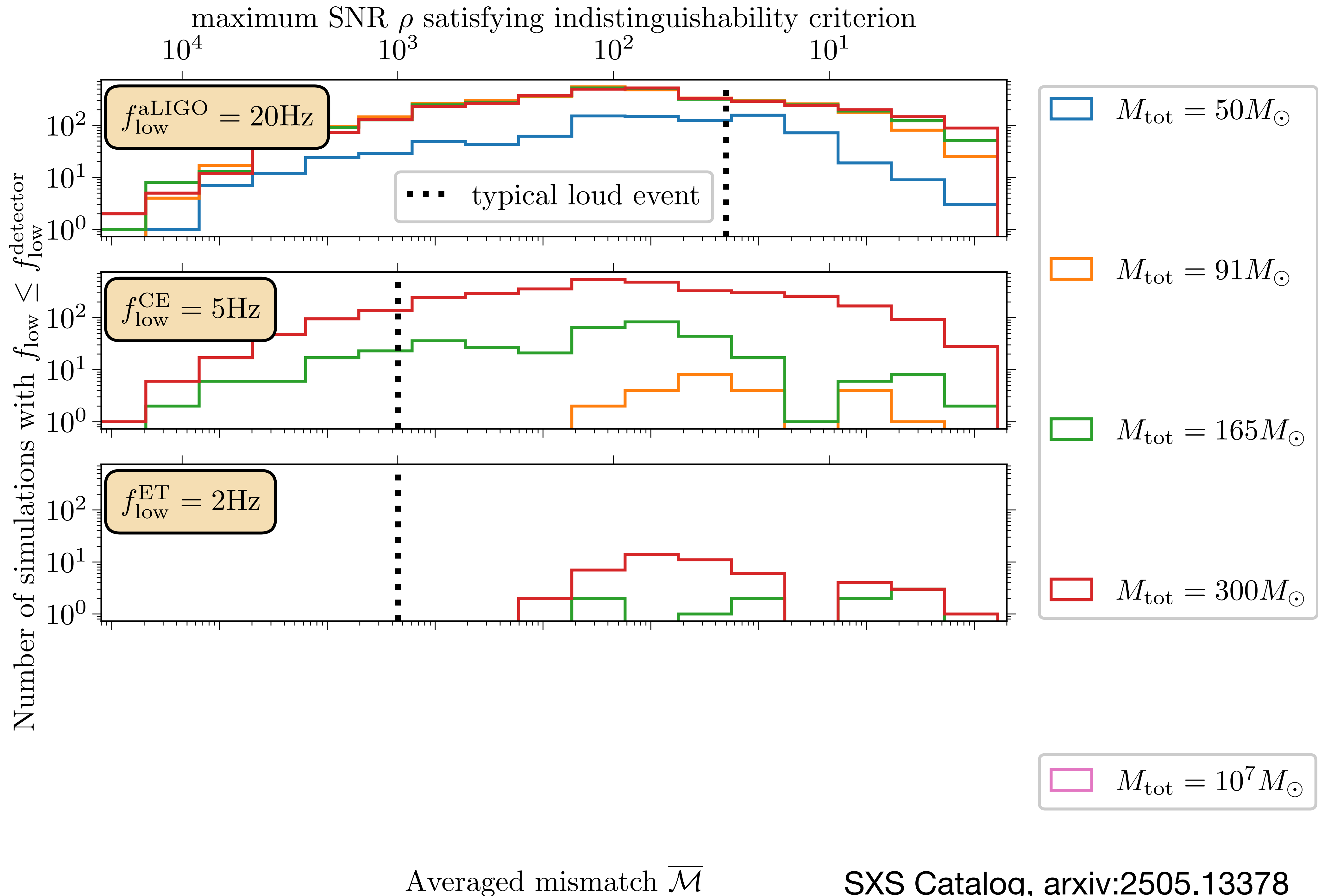


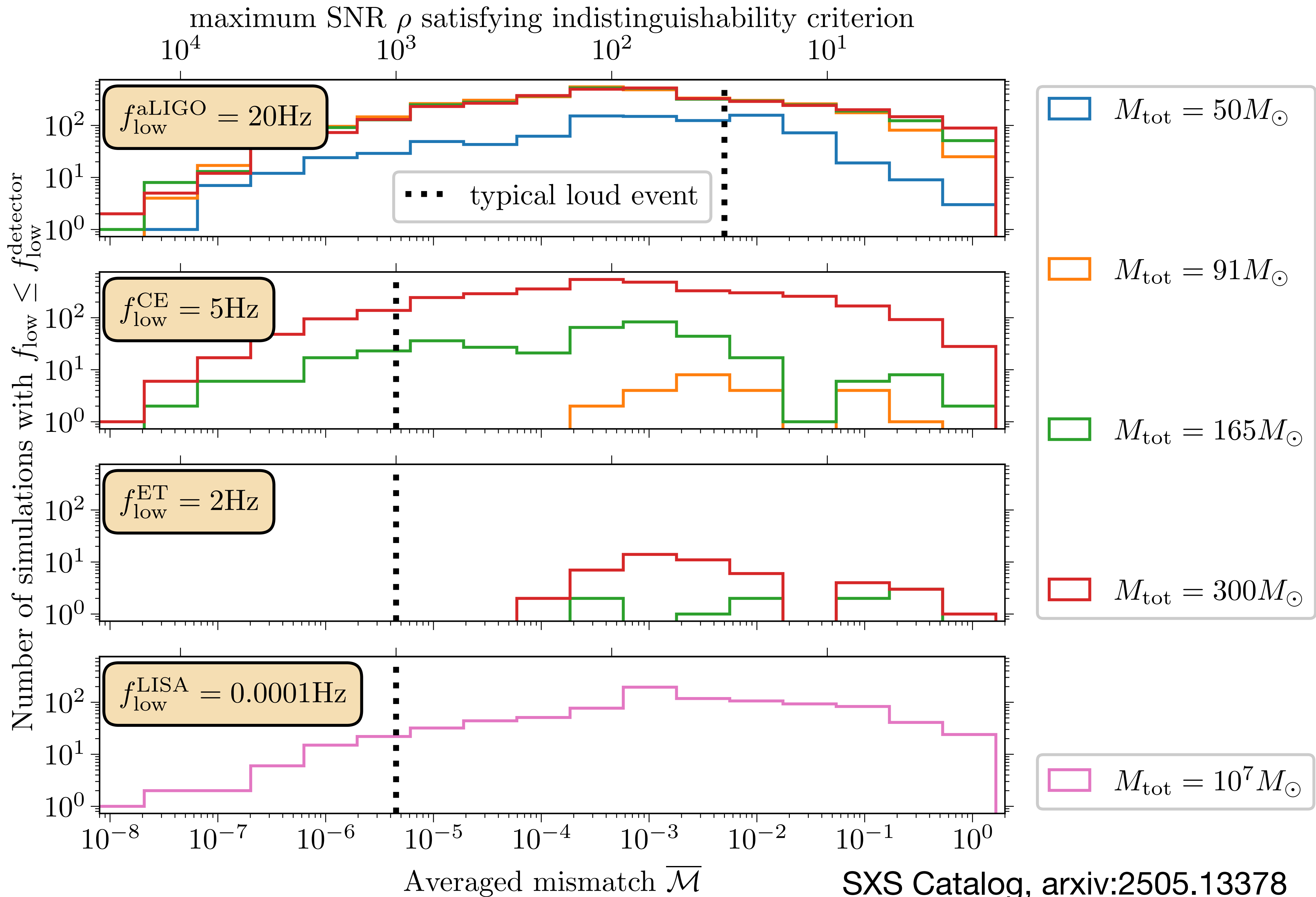
Averaged mismatch $\overline{\mathcal{M}}$

SXS Catalog, arxiv:2505.13378









Recap

- Need longer waveforms
- NR: $\mathcal{M}_t \sim |t_2 - t_1|^2$
- Early orbits longer

Algorithms

Algorithms

- It's all that matters.

Algorithms

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- Hardware = linear

Algorithms

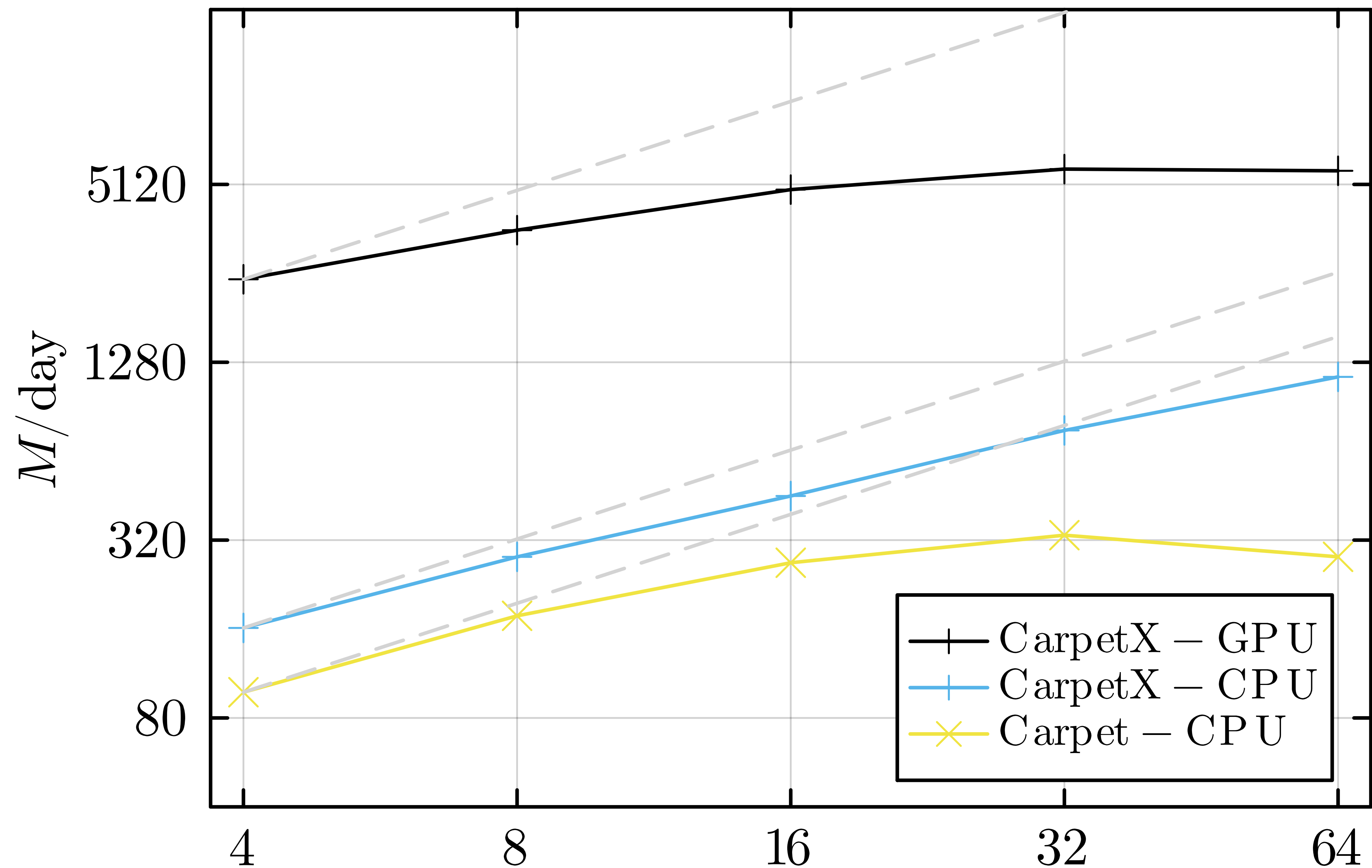
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Algorithms

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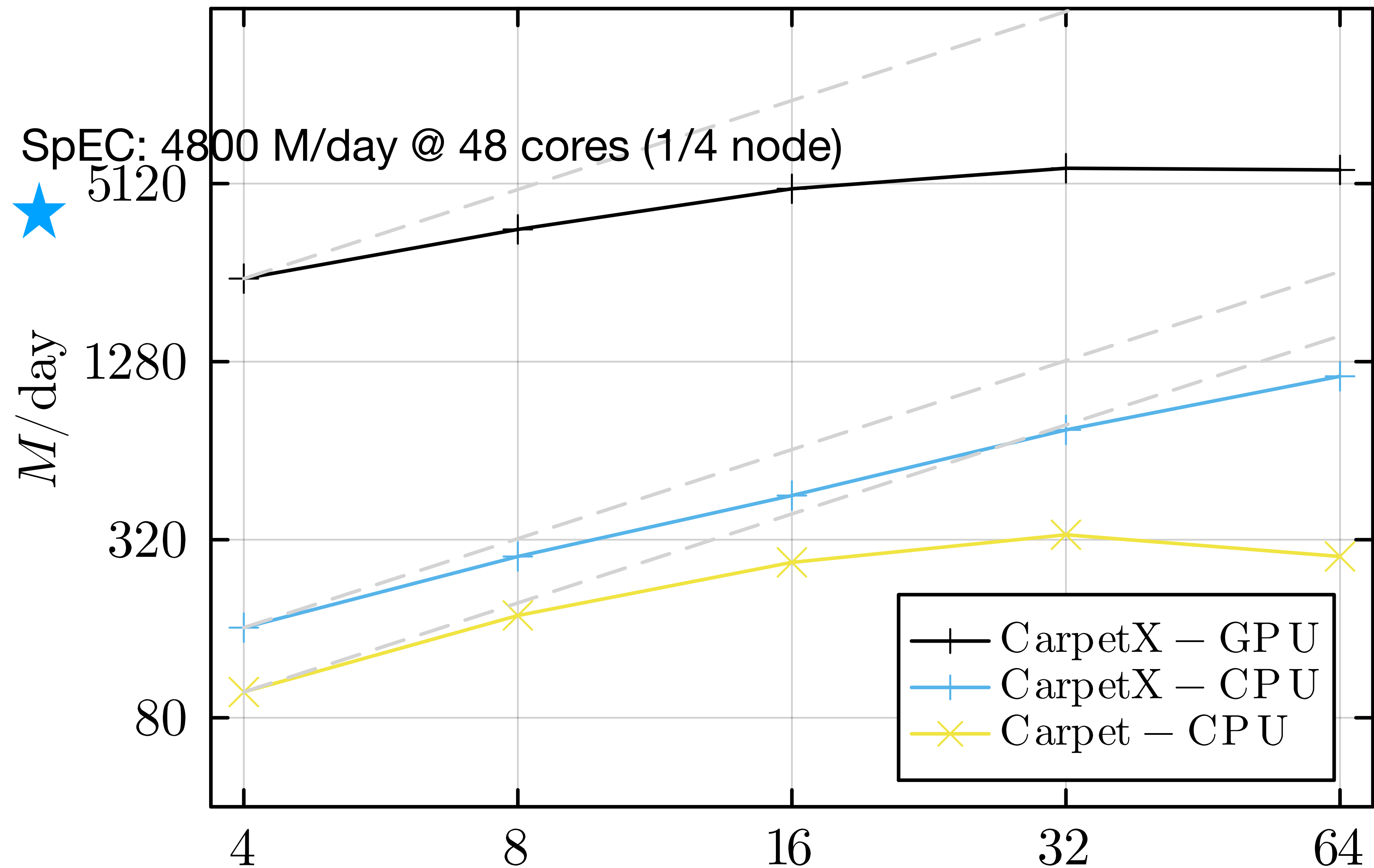
- It's all that matters.
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- Algorithms = exponential
- Algorithms are difficult
- Fast=more science



CPU: Frontera (7 yo), 56/node
GPU: 1H200/node

Number of Nodes

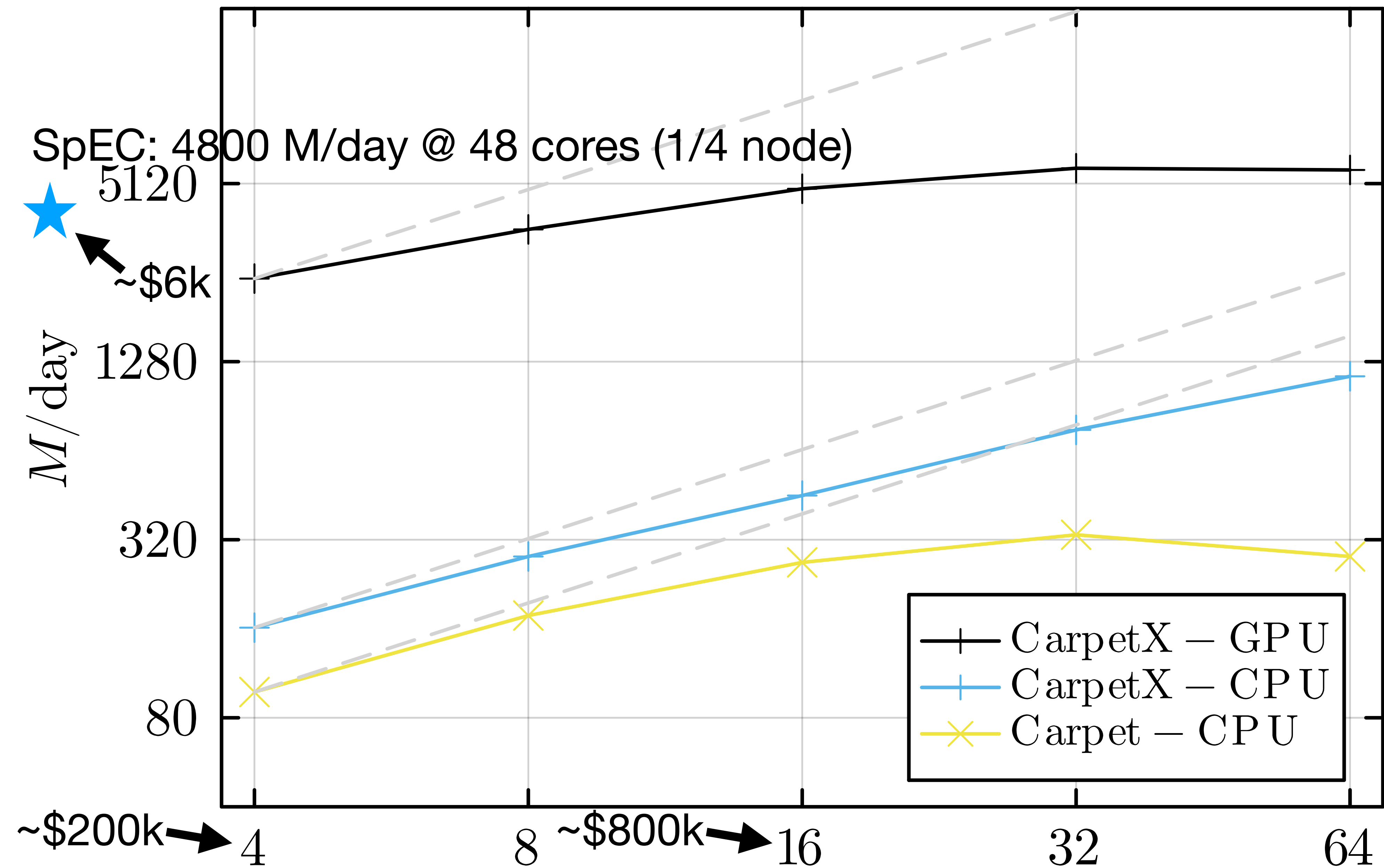
From arXiv 2503.09629, Ji+



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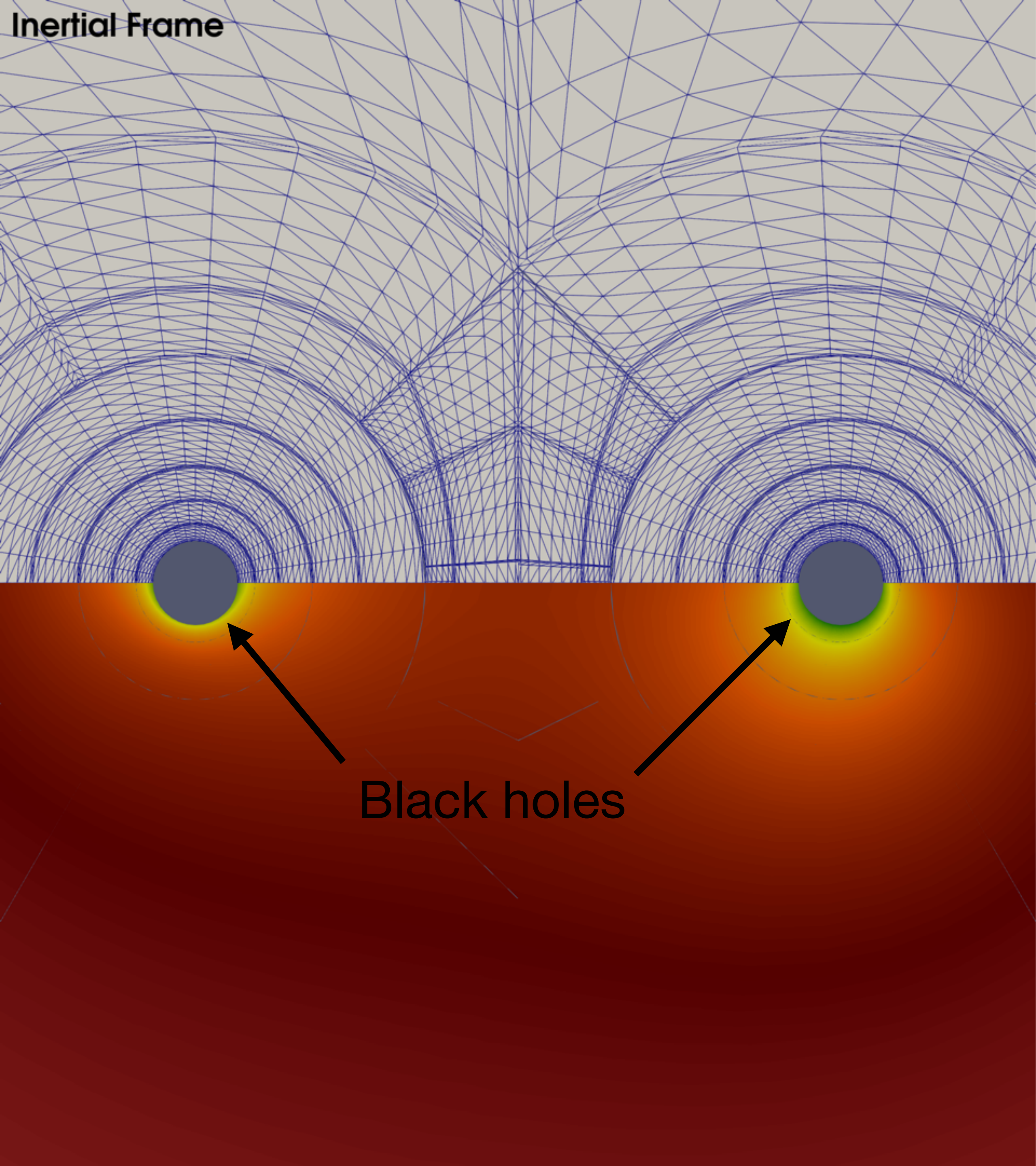
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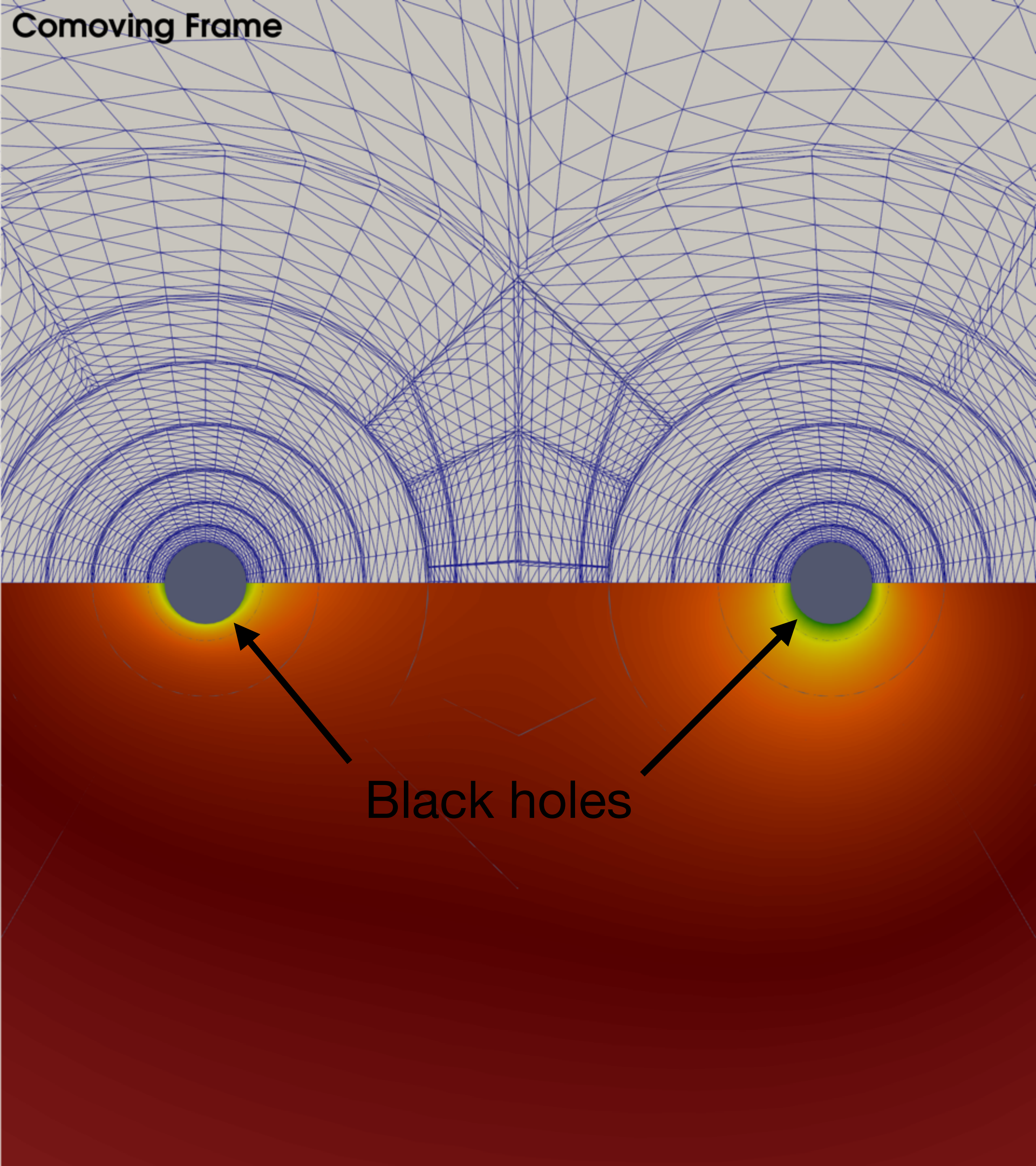
Algorithms

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- Hardware = linear **AT BEST**
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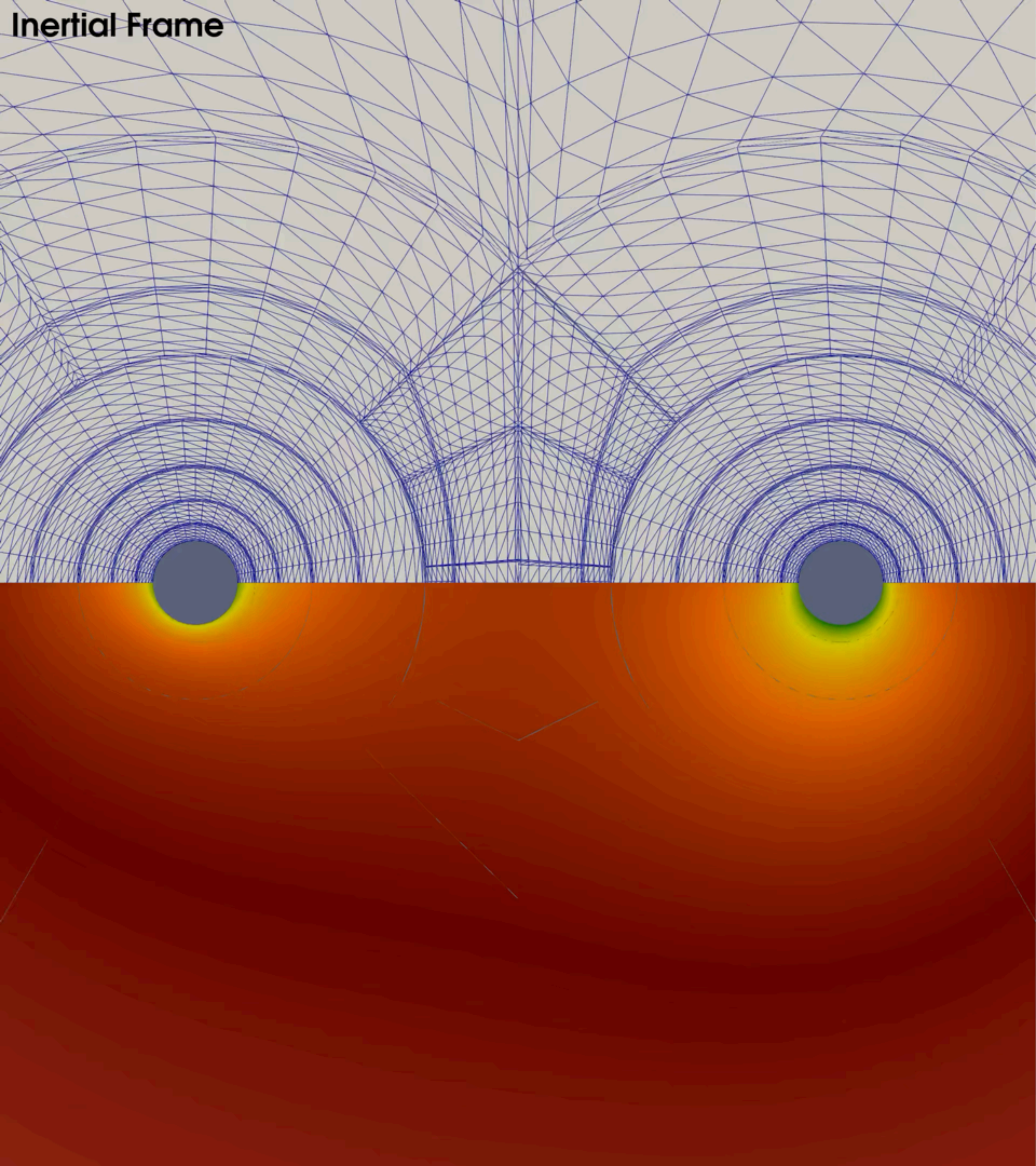
Inertial Frame



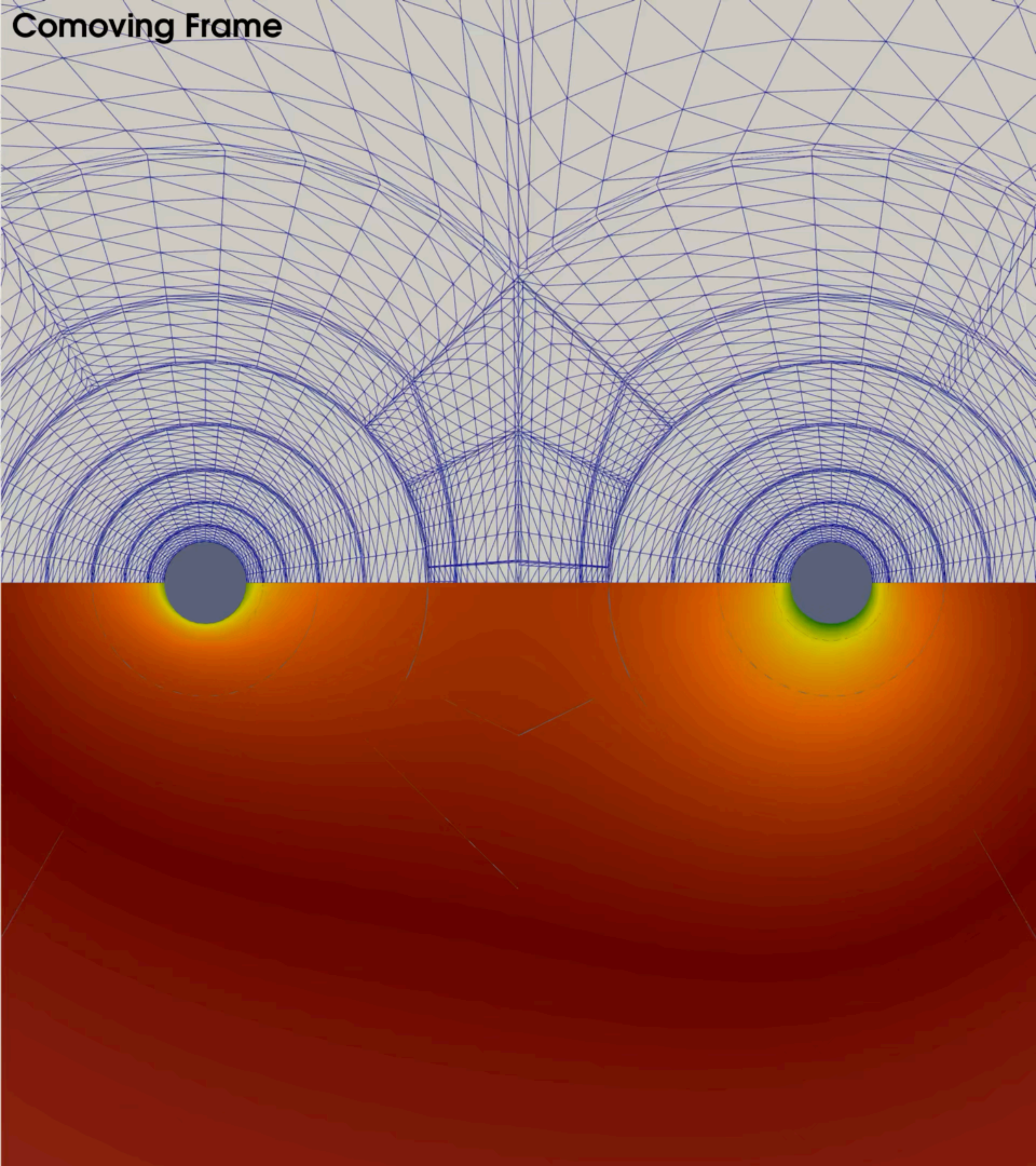
Comoving Frame



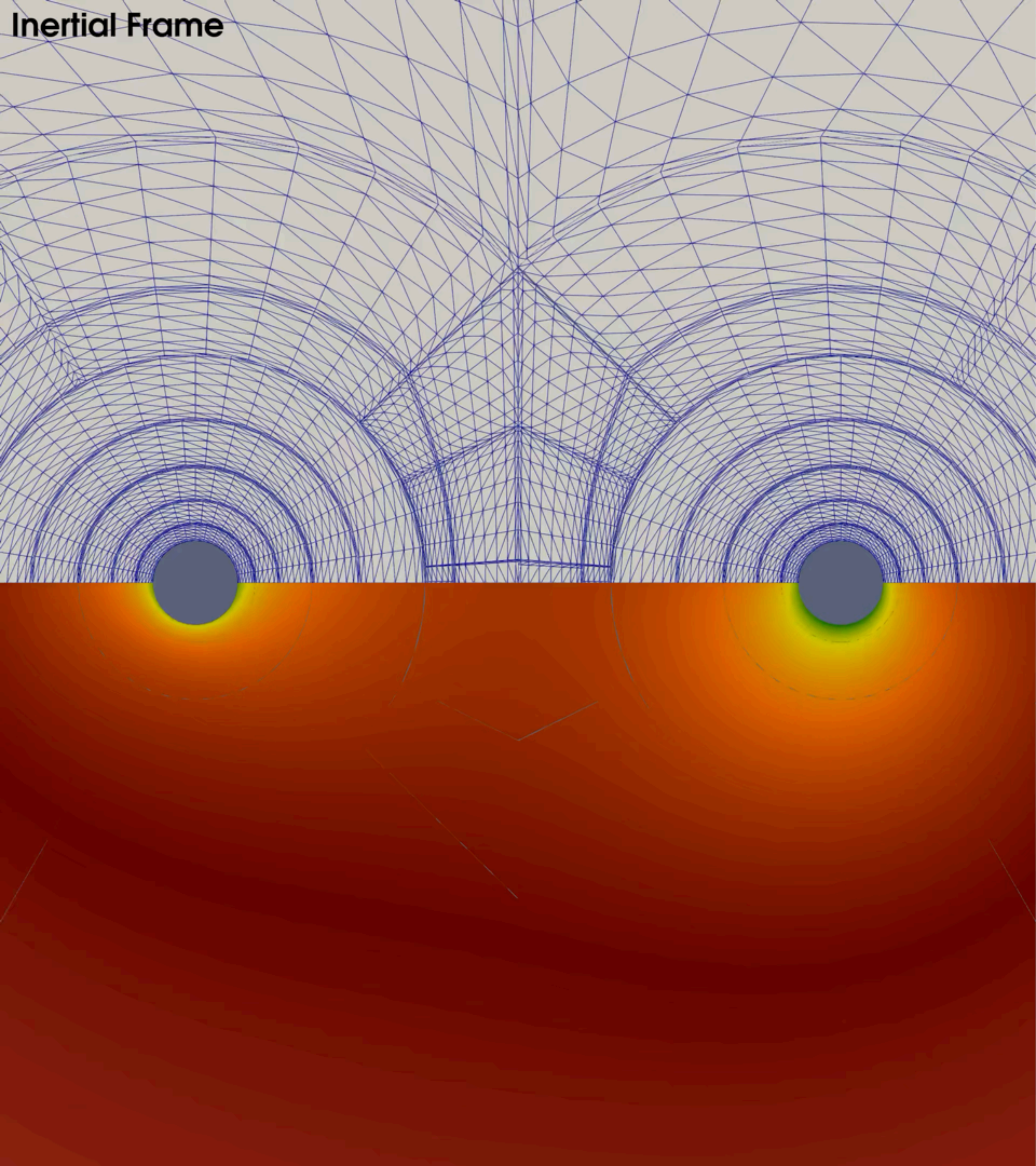
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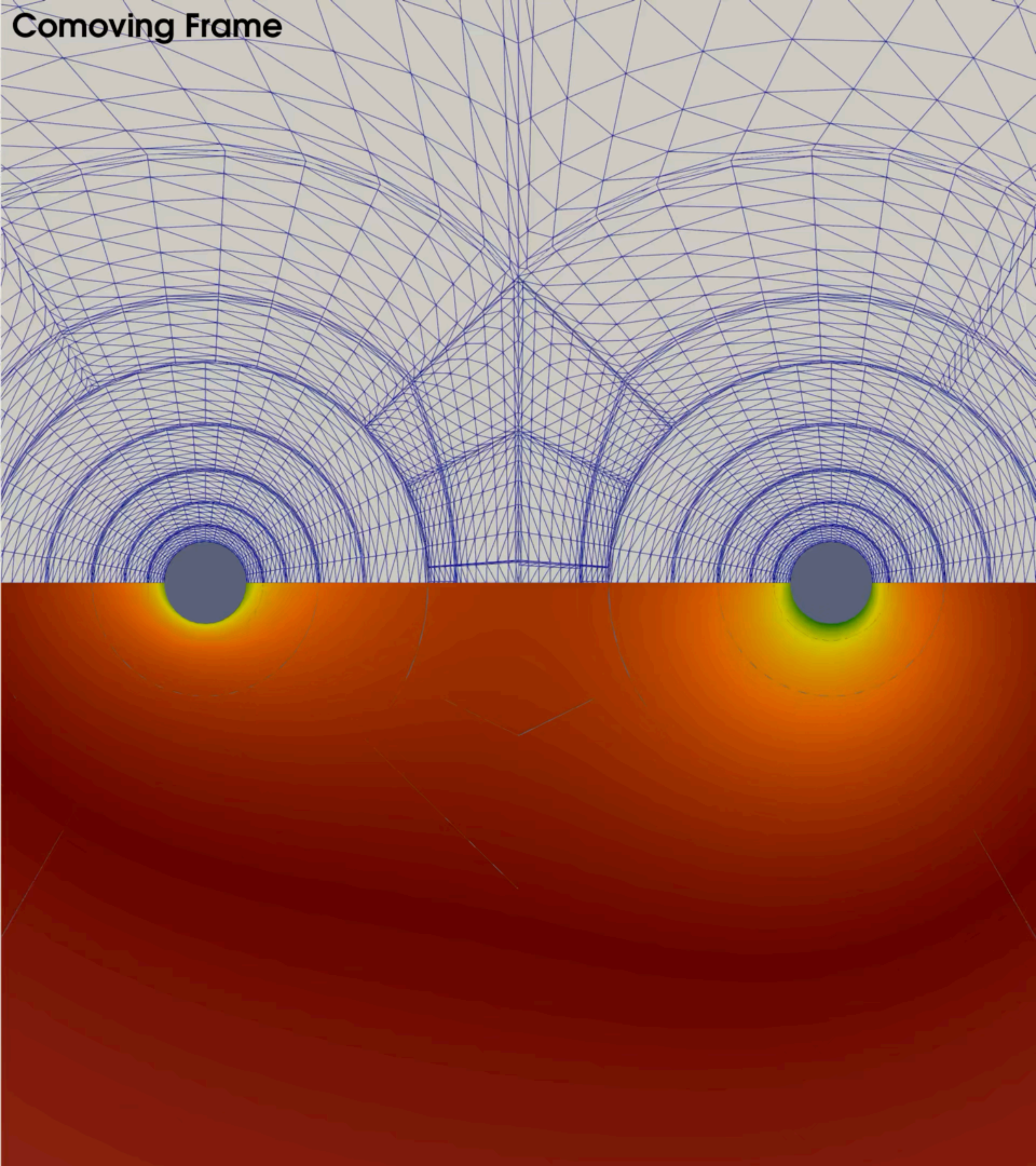
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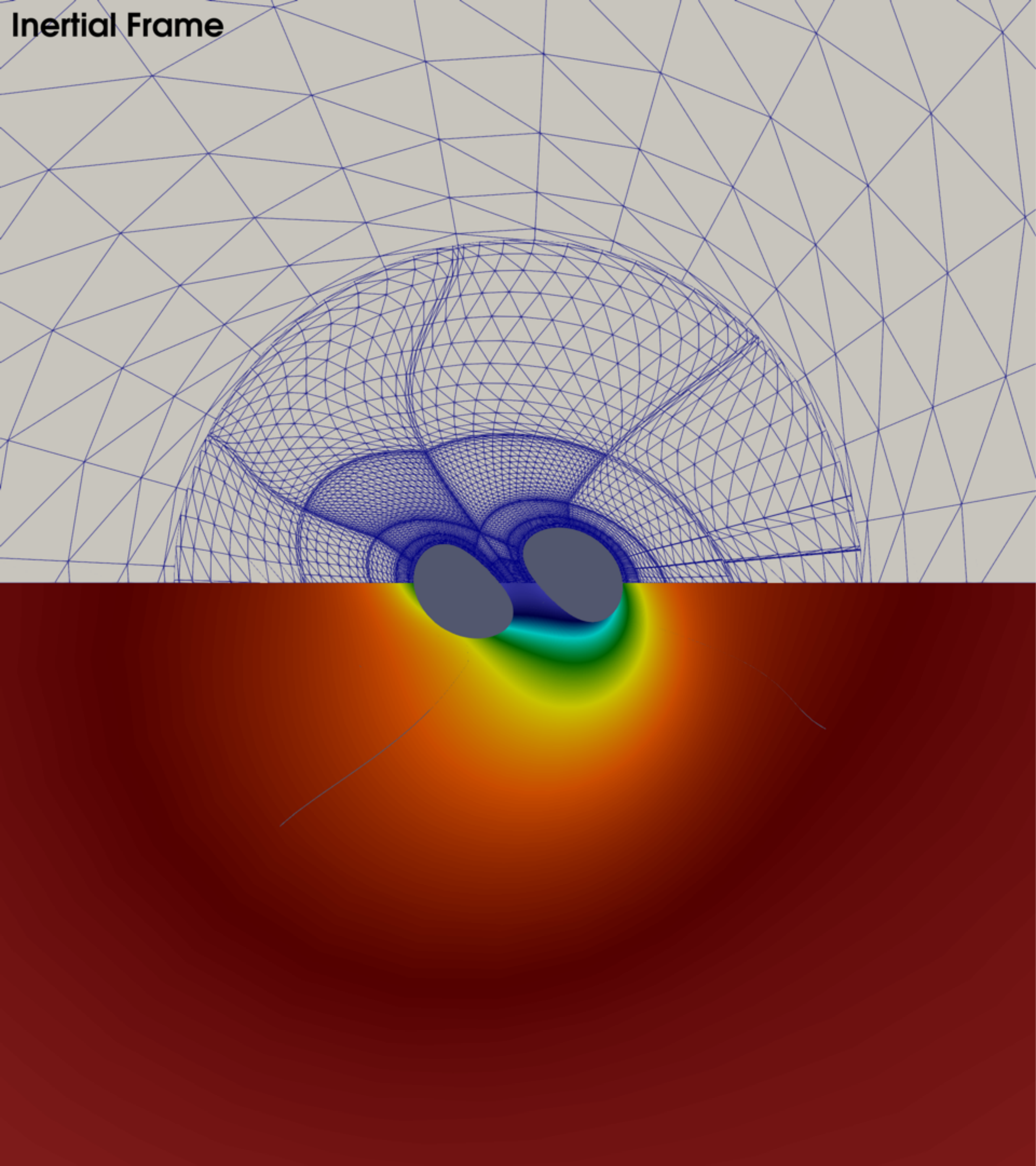
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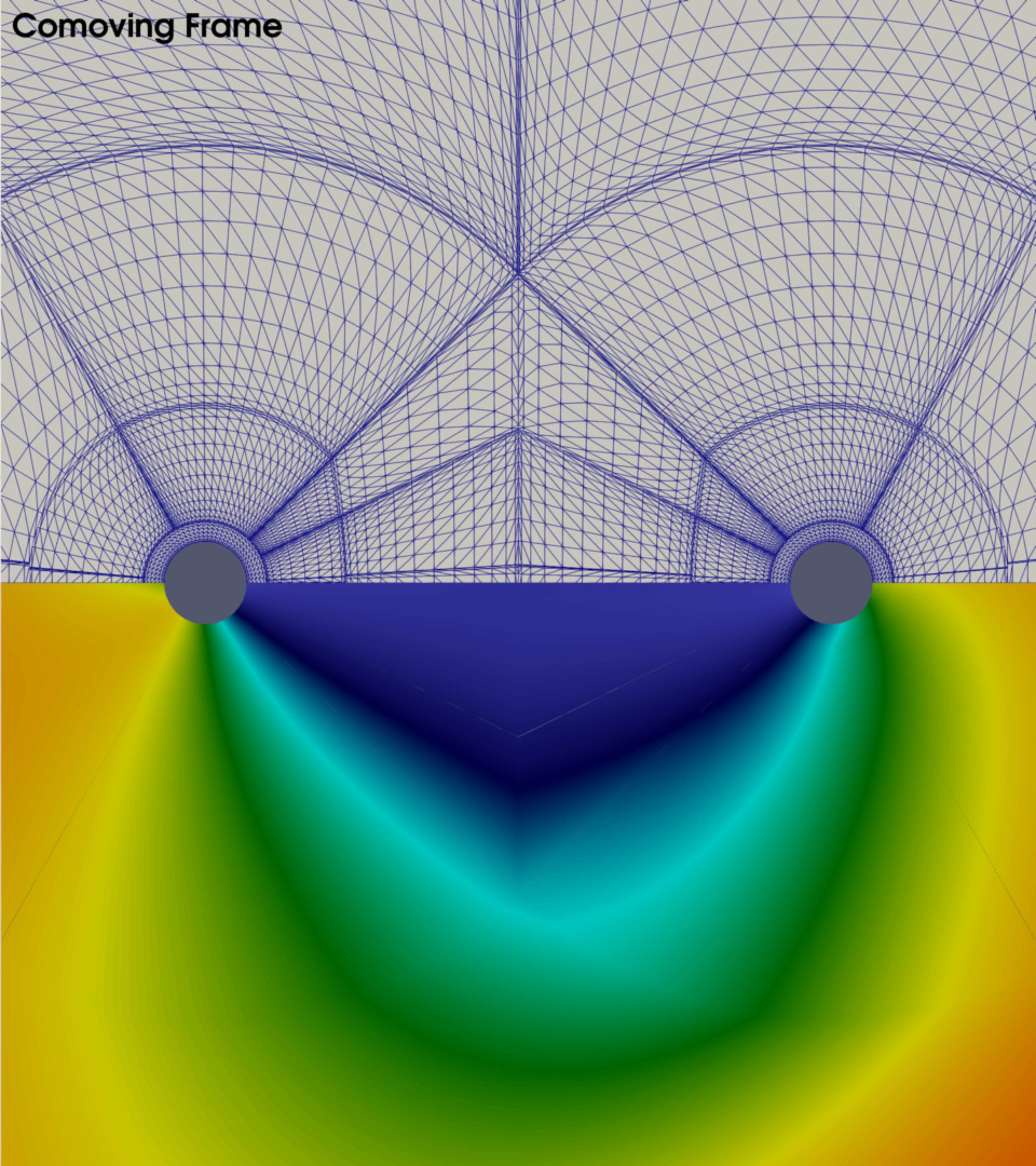
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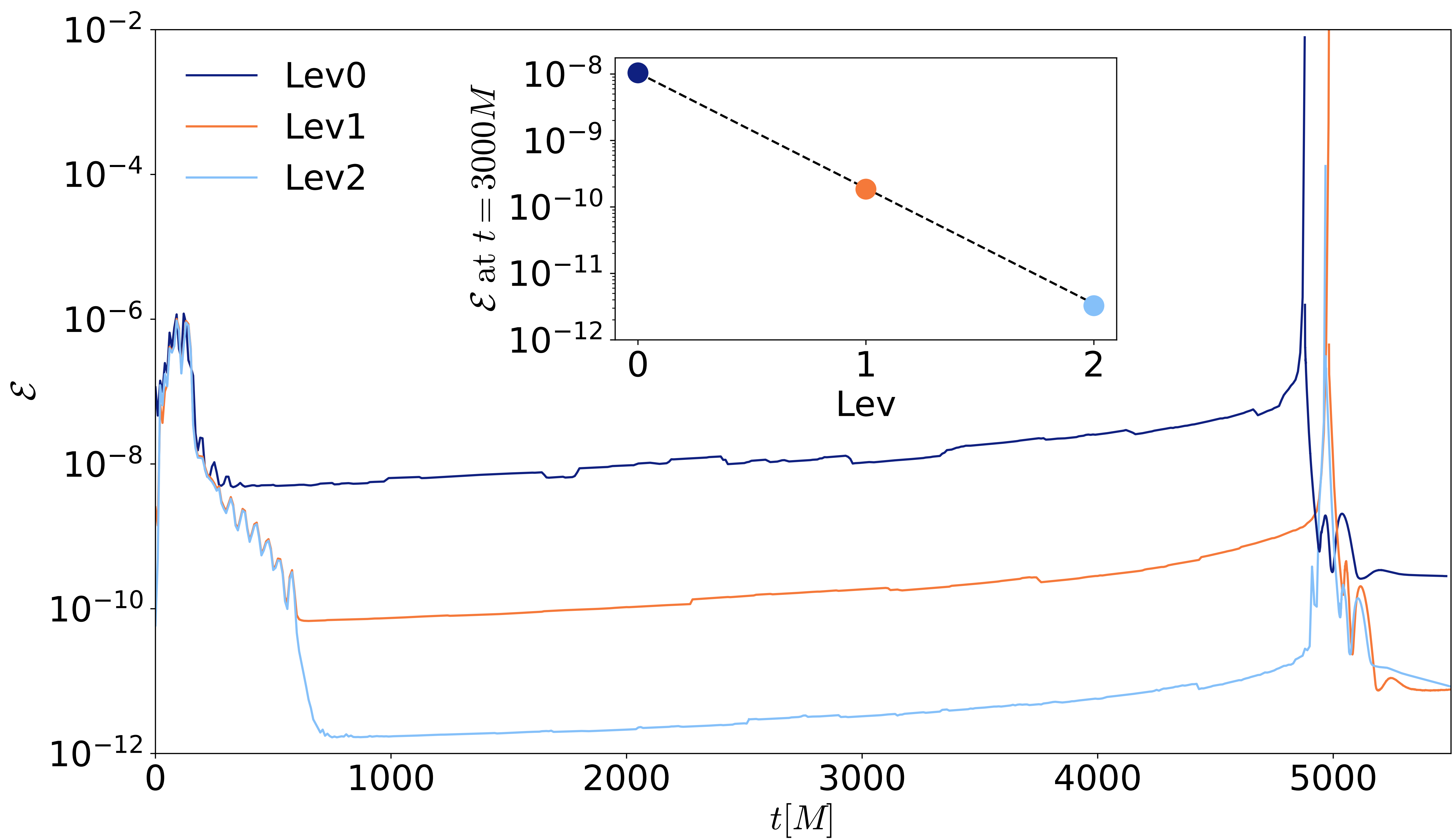


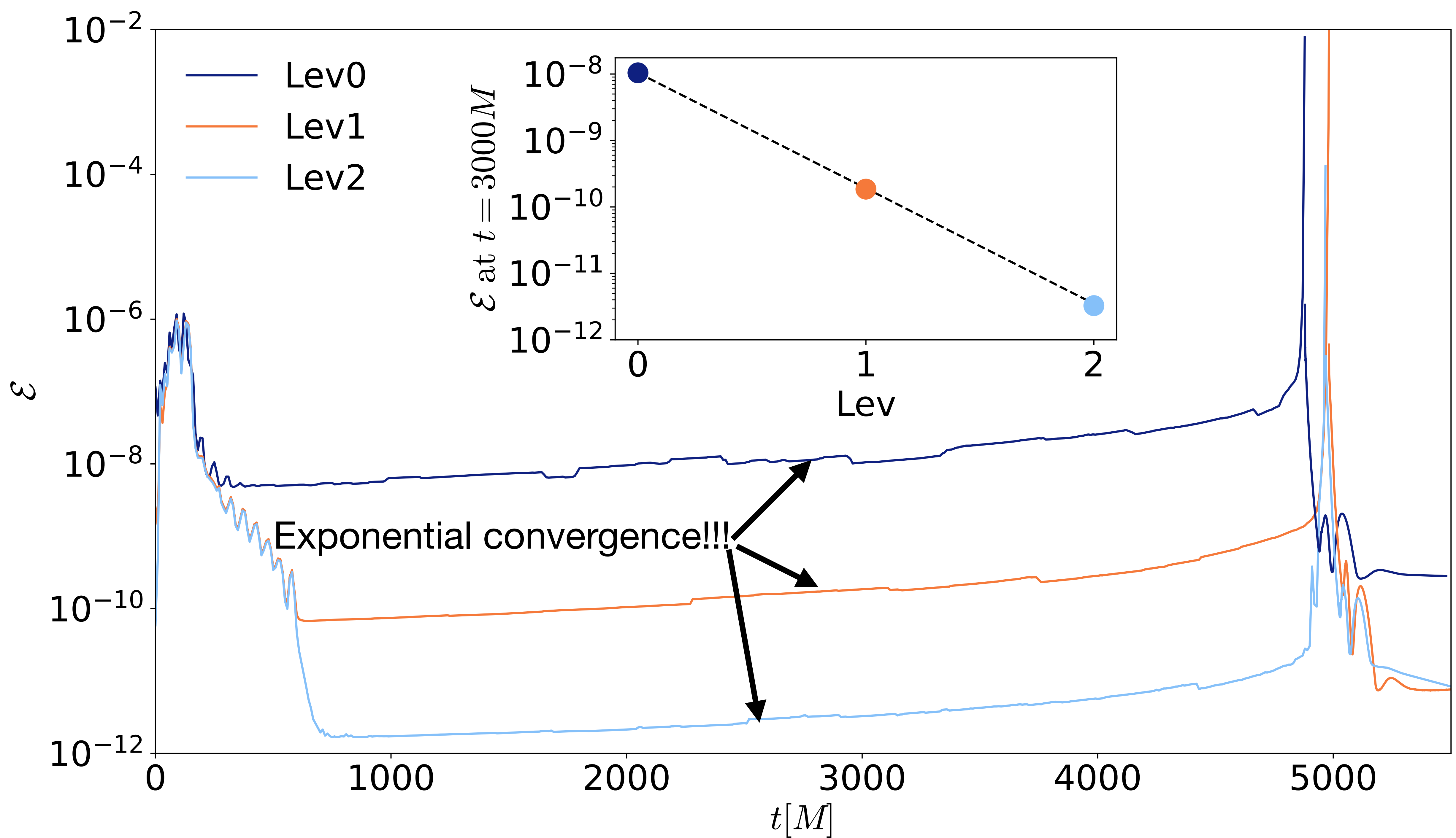
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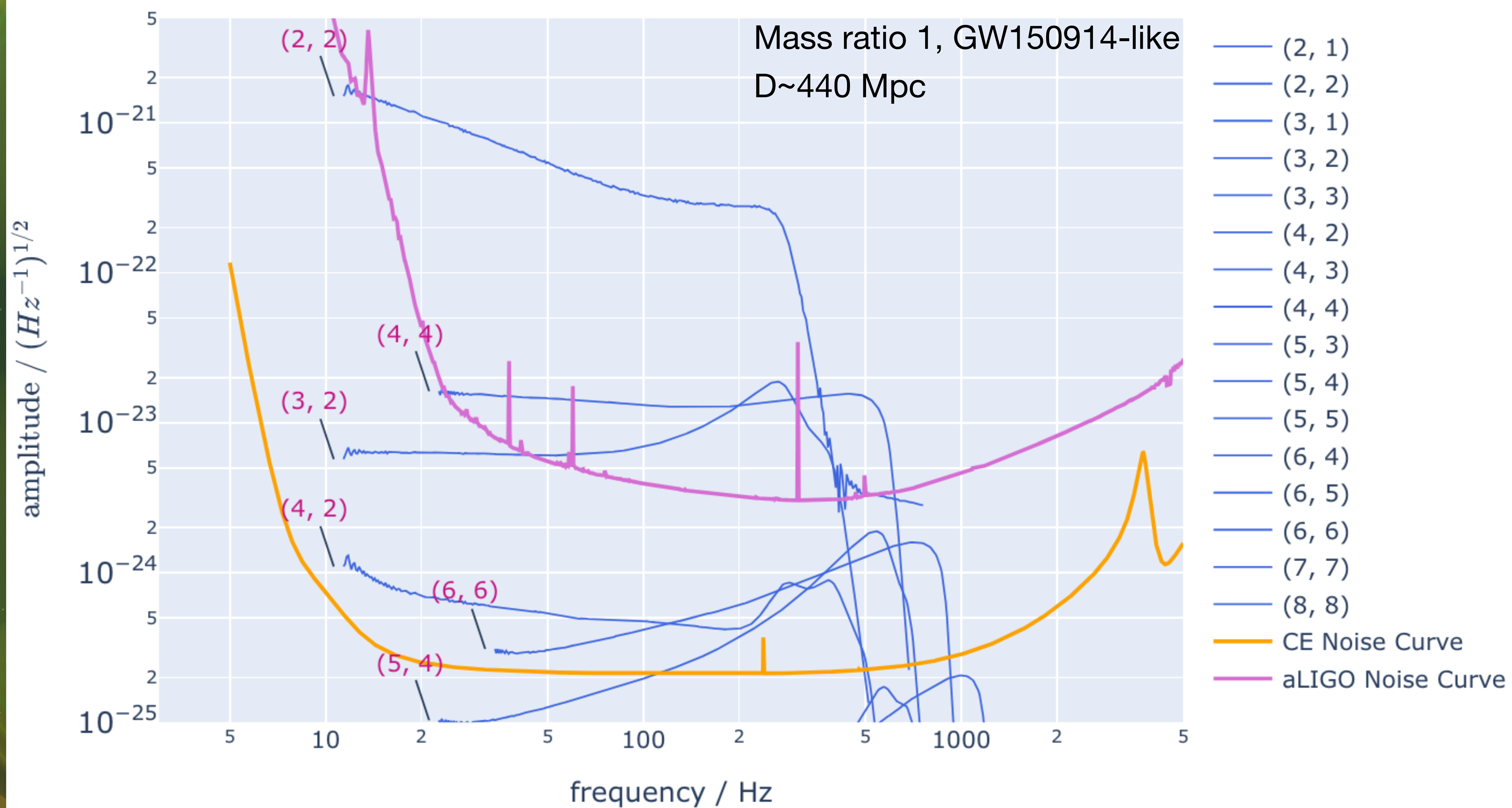






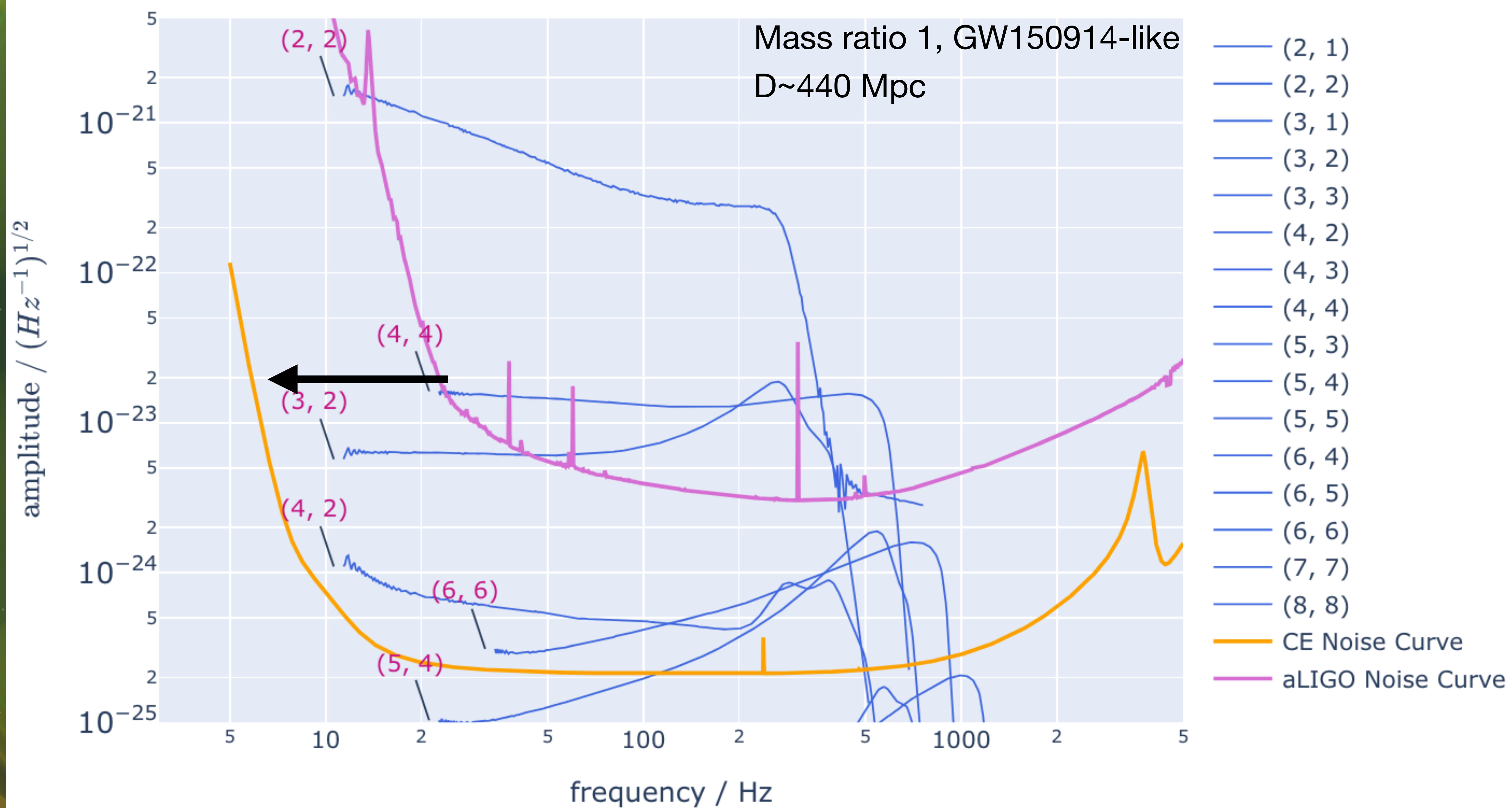
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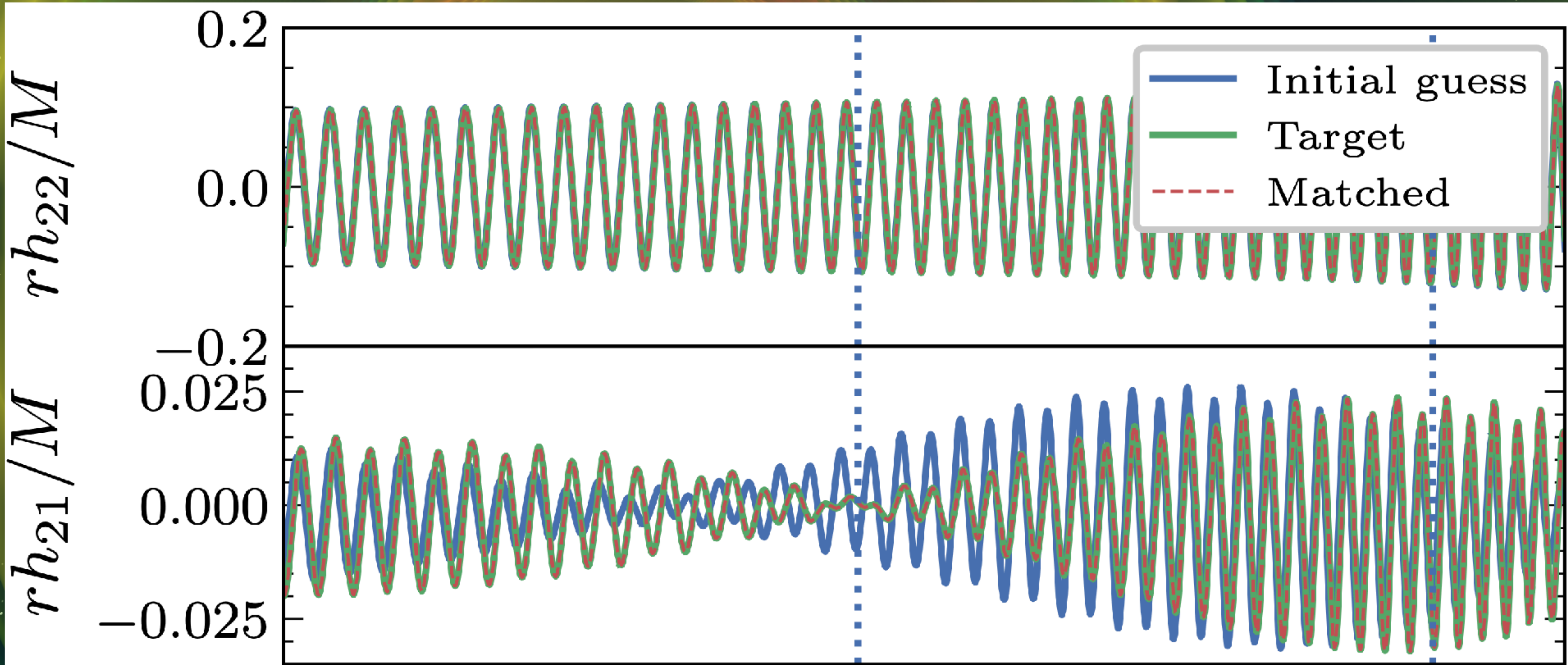


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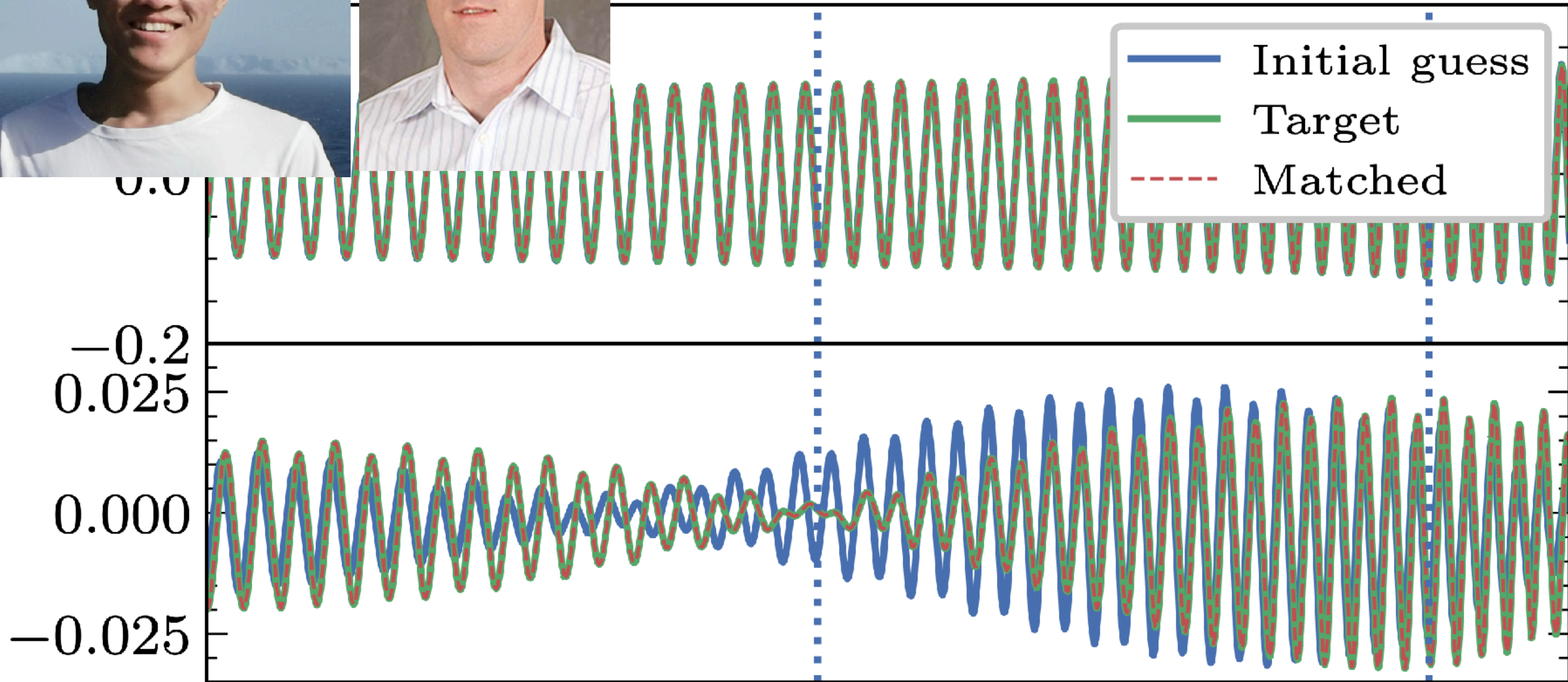
PN+NR





PN+NR

rh_{22}/M
 rh_{21}/M

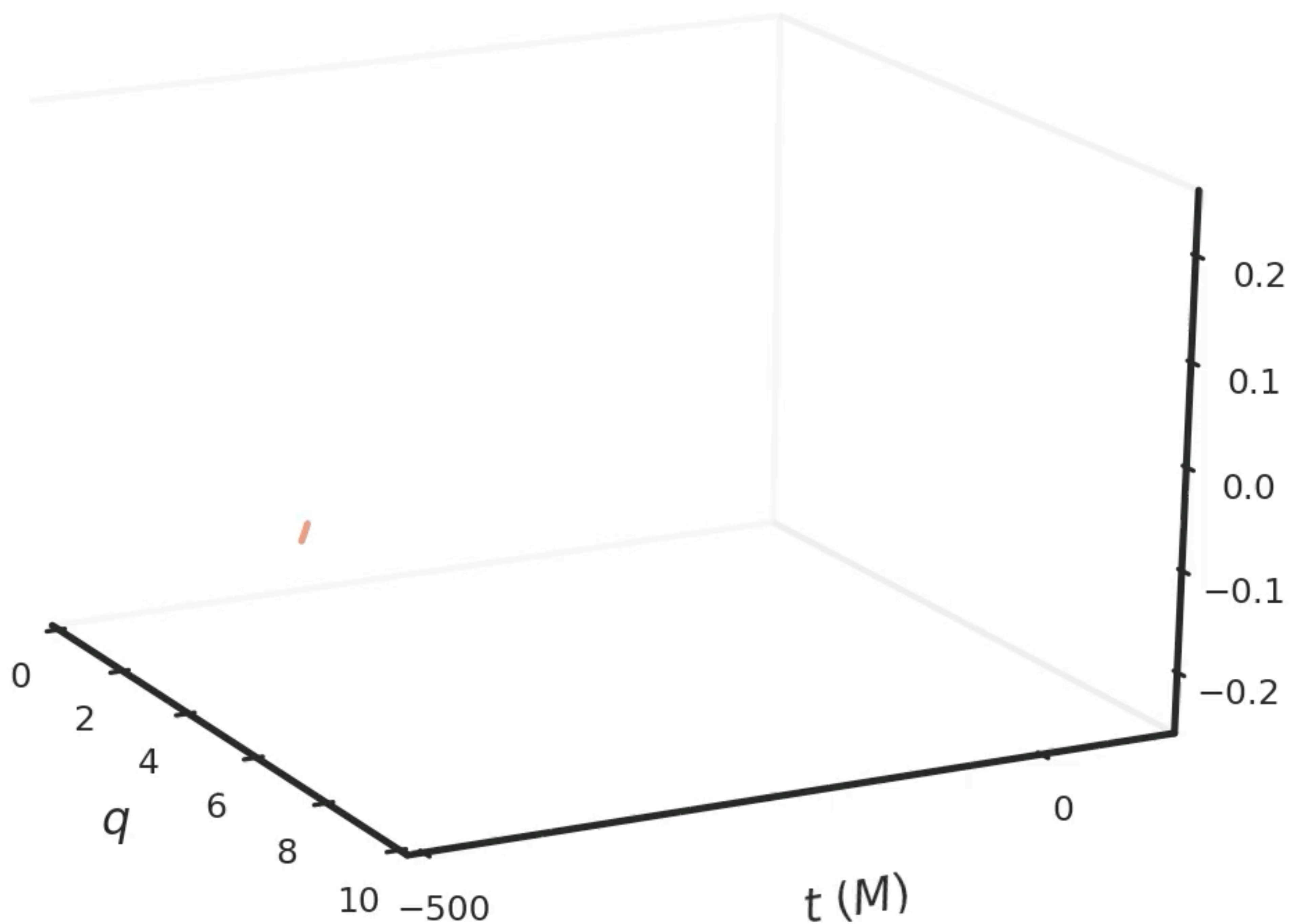


PN+NR Algorithms

- github.com/moble/PostNewtonian.jl
- github.com/moble/Scri.jl
- $\sim >20x$ speedup, less RAM
- Differentiable
- PINNs for PN: 2511.10522

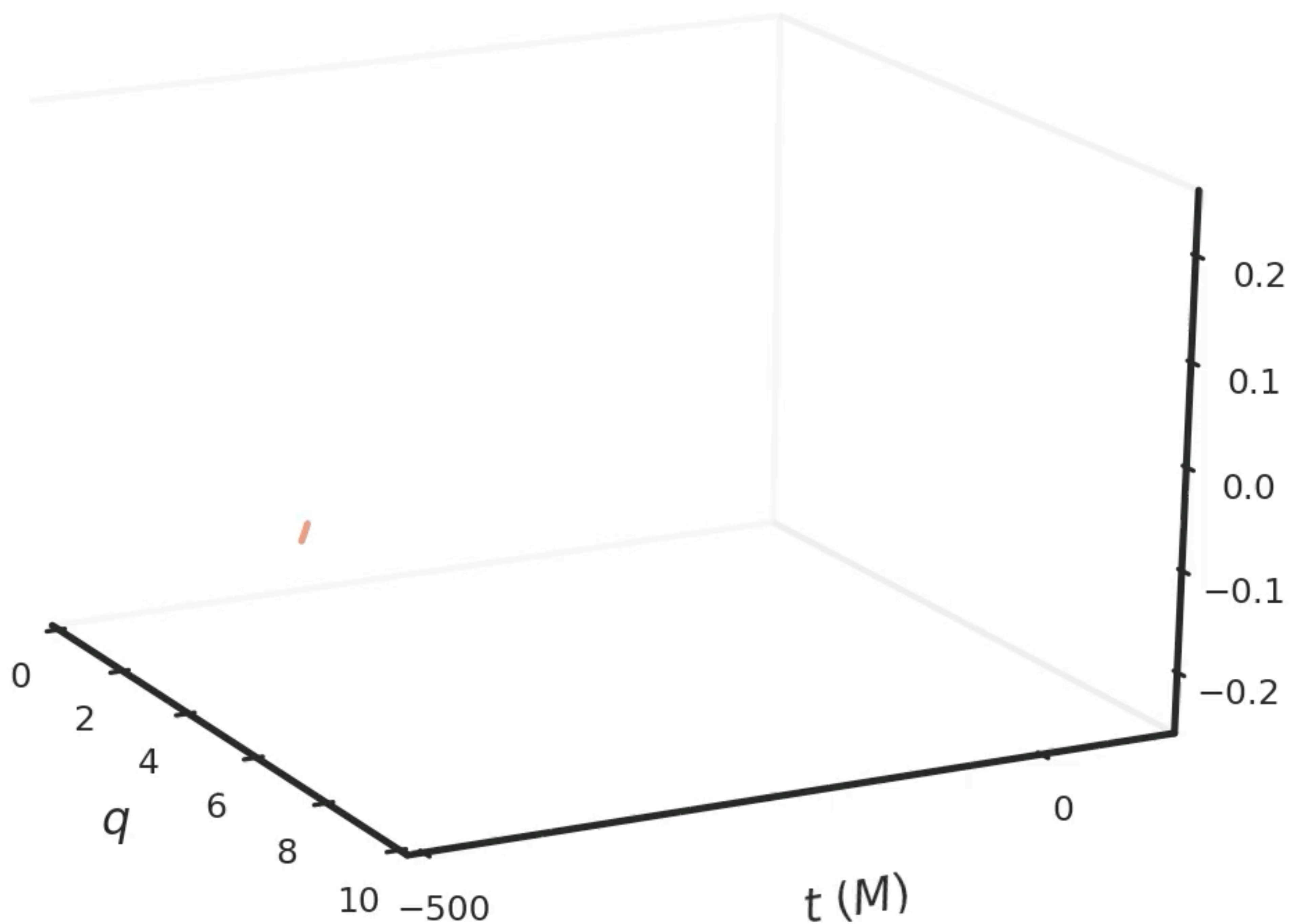


(Numerical relativity) surrogates



Credit: Vijay Varma, <https://vijayvarma392.github.io/SurrogateMovie/>





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Numerical relativity surrogates

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- NR slow...

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- PN inaccurate

Numerical relativity surrogates

- NR slow...
- PN inaccurate
- Goal: go fast & accurate

Surrogates

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- “Boring” data

Surrogates

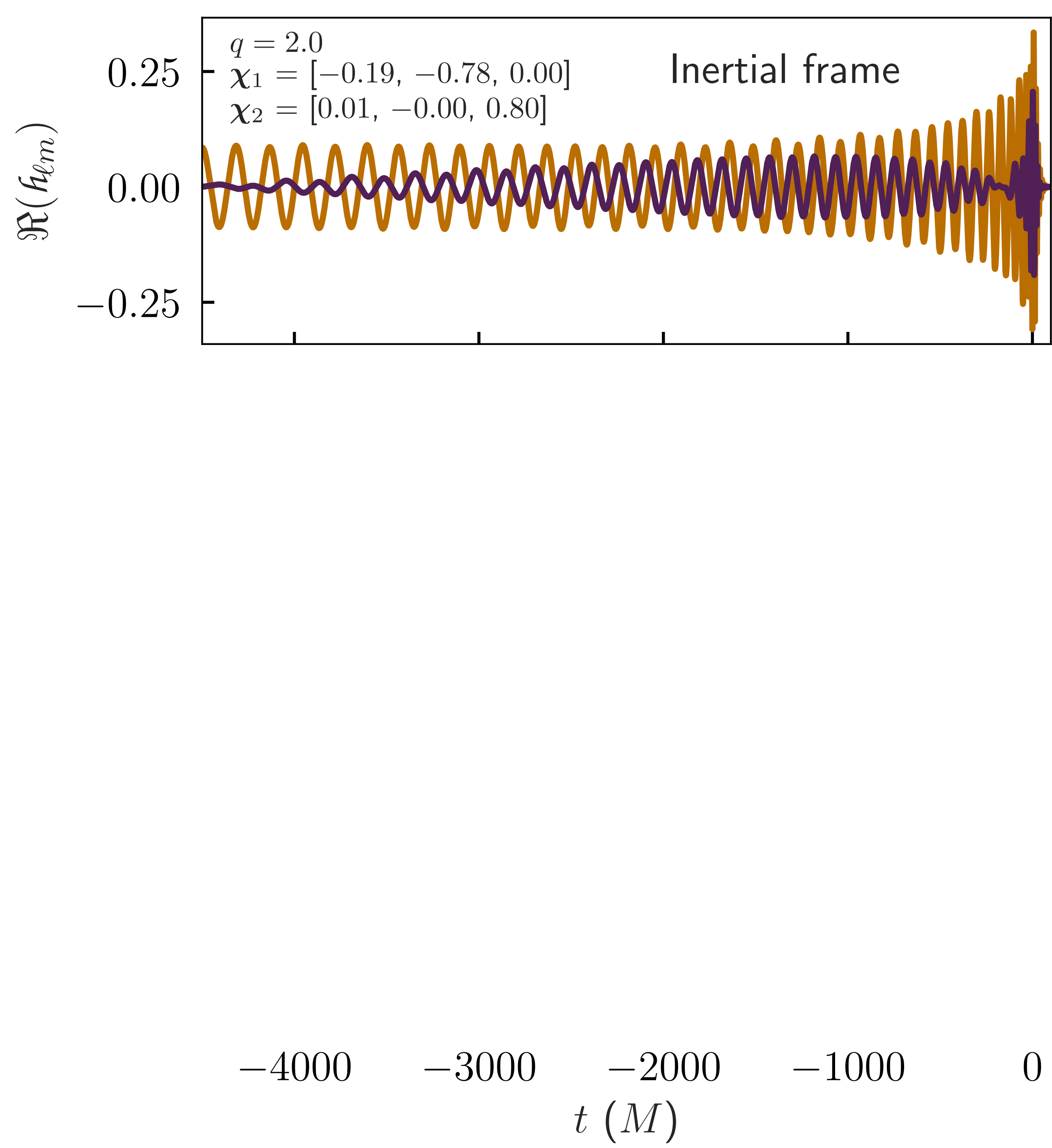
- “Boring” data
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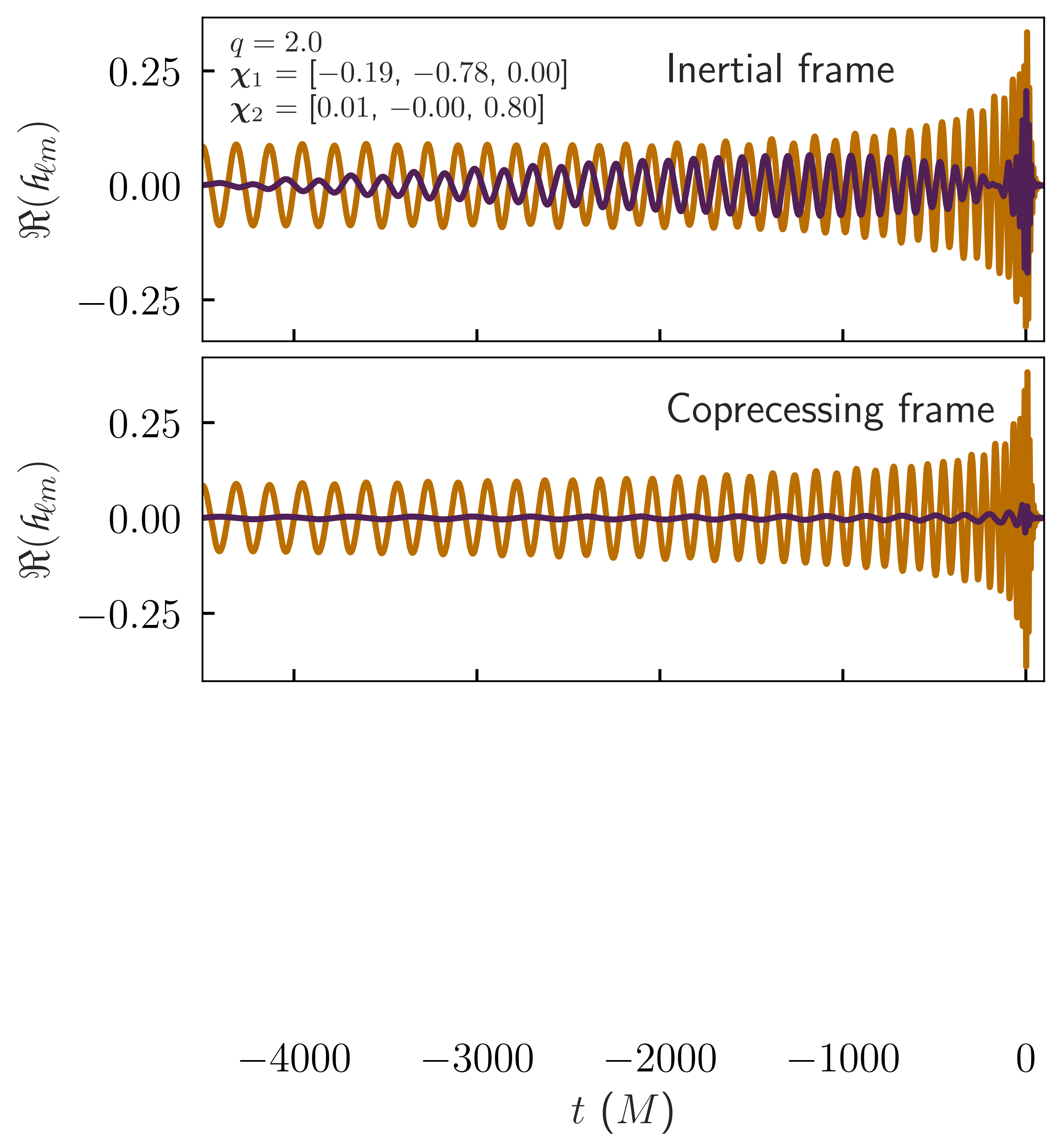
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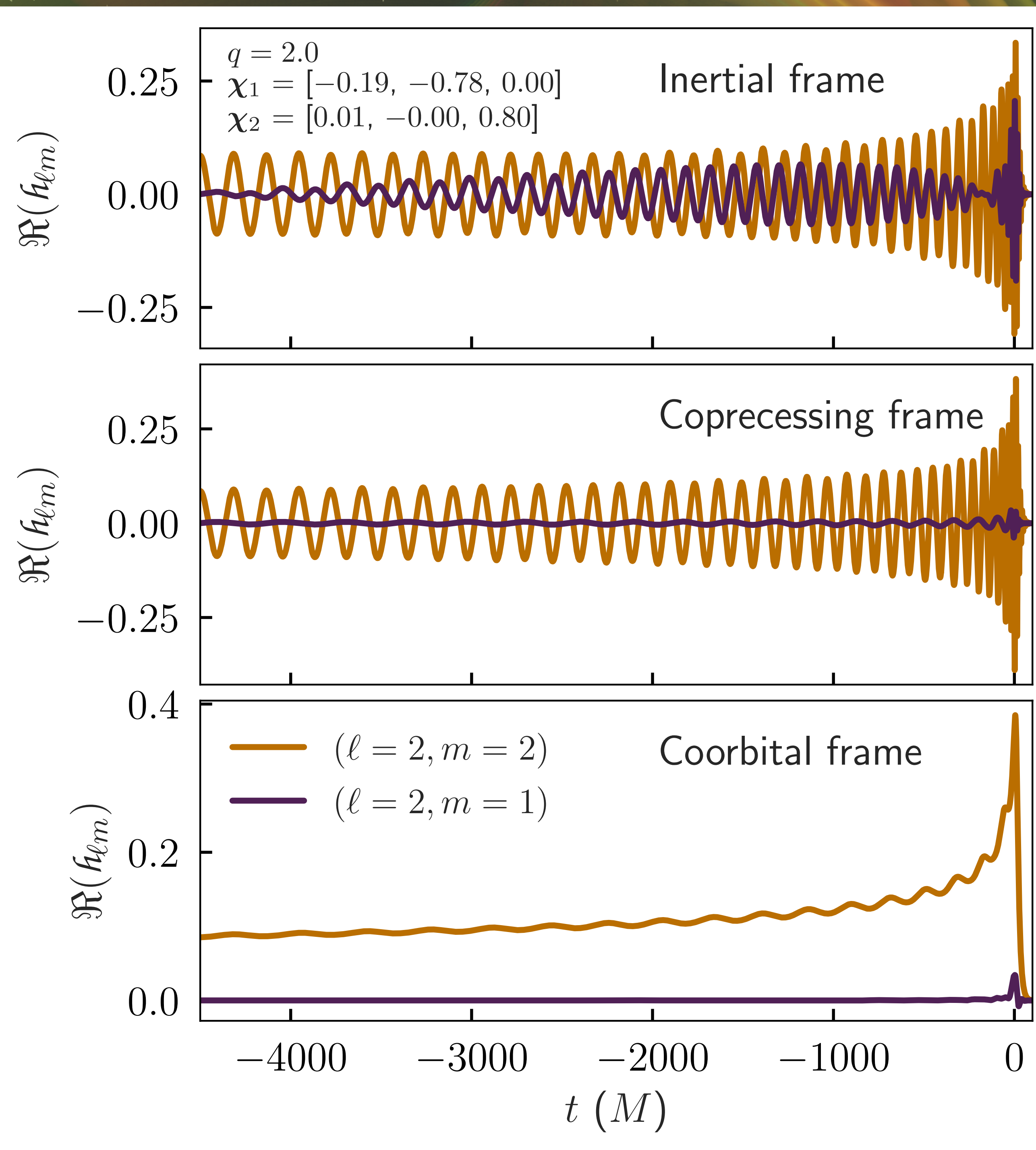
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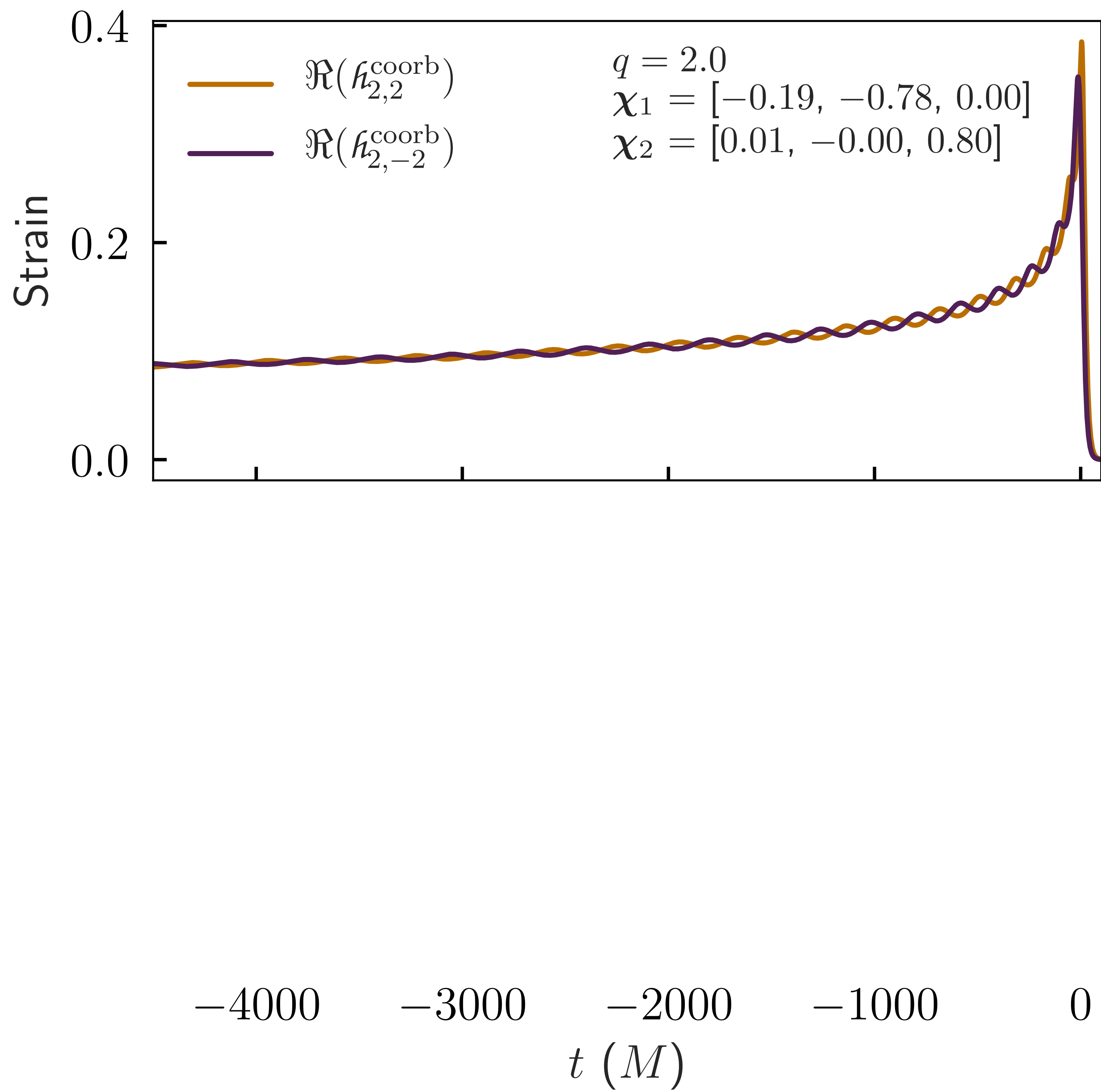
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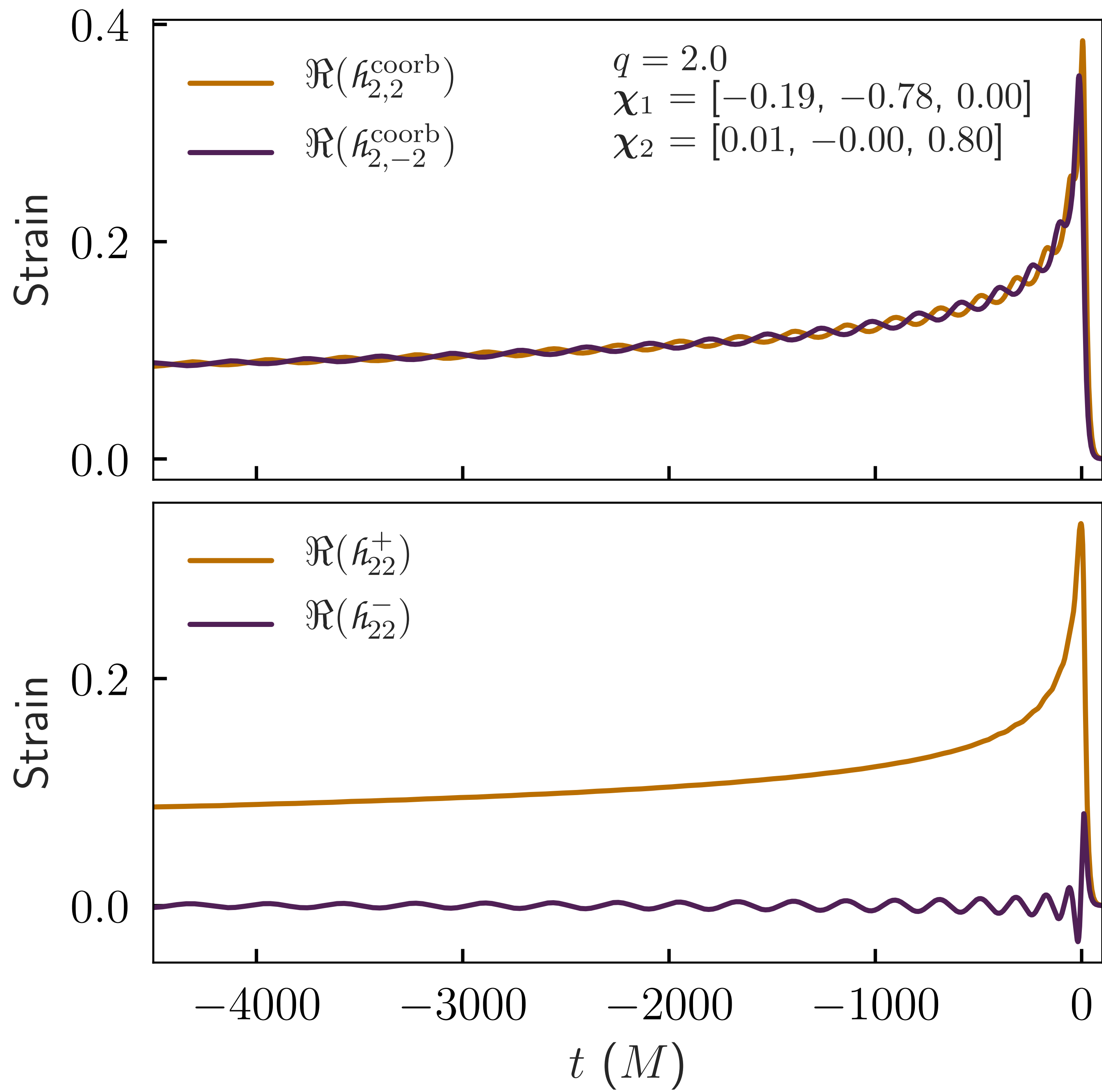
- “Boring” data
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- Waveform: Inertial NR $\rightarrow$ Coprecessing $\rightarrow$ Coorbital
- Dynamics: Spins + frames











Orbital dynamics:

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$$\frac{d\varphi}{dt} = \omega(\Lambda(t)),$$

$$\frac{d\vec{\chi}_{\text{copr}}^{1,2}}{dt} \sim \dot{\vec{\chi}}^{1,2}(\Lambda(t)),$$

- $$\frac{dq}{dt} \sim \frac{1}{2} q \bar{\Omega}^{\text{cp}}(\Lambda(t)).$$

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- Universal DEs (arxiv:2001.04385)

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- Neural DEs (res net: 1512.03385, neural DE: 1806.07366)

How fast can you go?

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- Waveform eval: 0.05ms (fixed)

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- Waveform rotation: 0.05ms (skip??)
- Waveform interpolation: 0.16ms (skip!??!)
- Dynamics: 0.37ms (~10x reduction)
- Potential frequency-domain: ~0.2ms

Bonus: forward sensitivity analysis

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Yet Another Algorithms Example (YAAE?)

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- NR catalog: ~10TB

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- “Uncompressed”: ~50TB

Binary neutron stars?

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- Length?

Binary neutron stars?

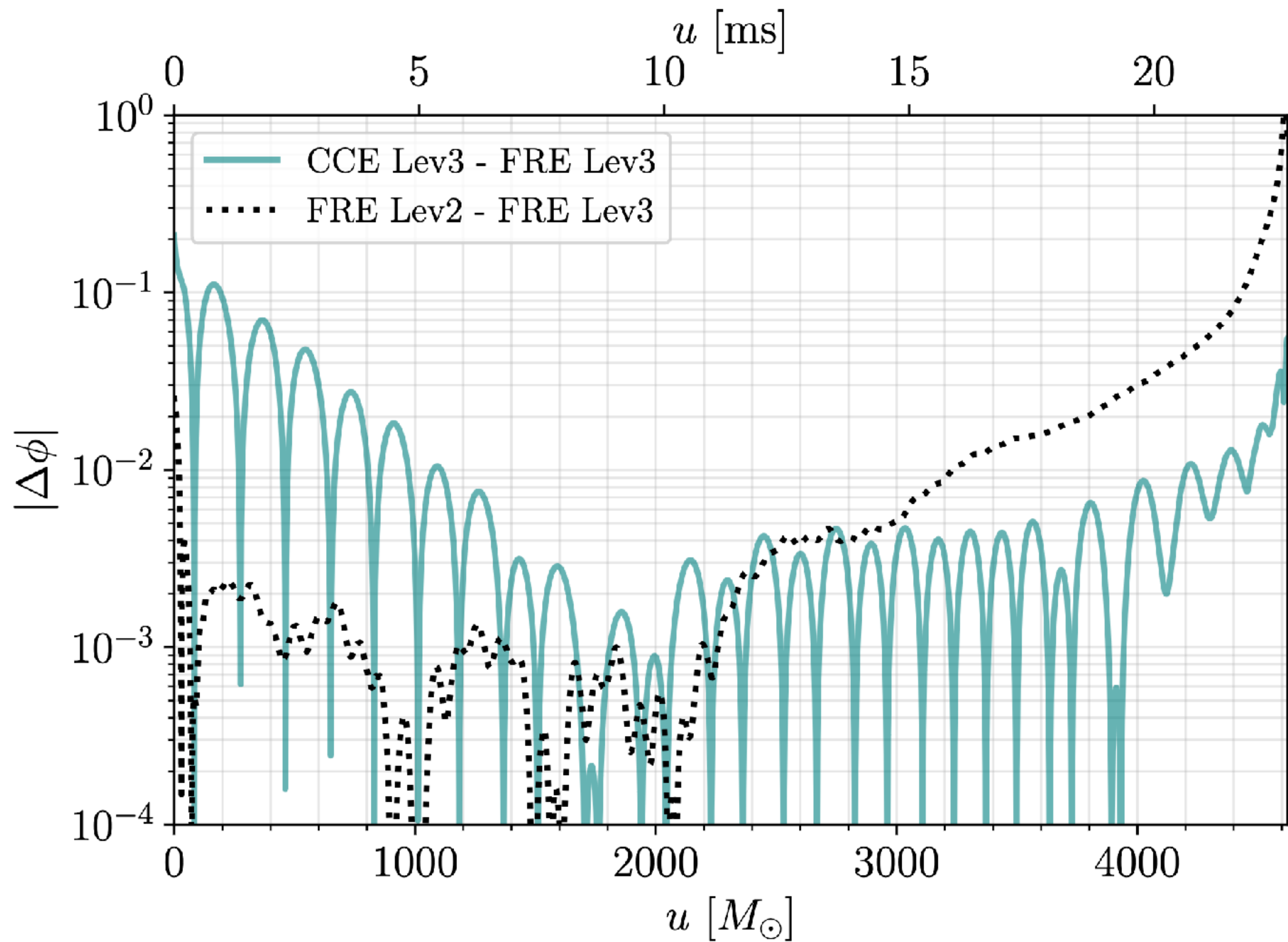
- Length?
- Mismatch?

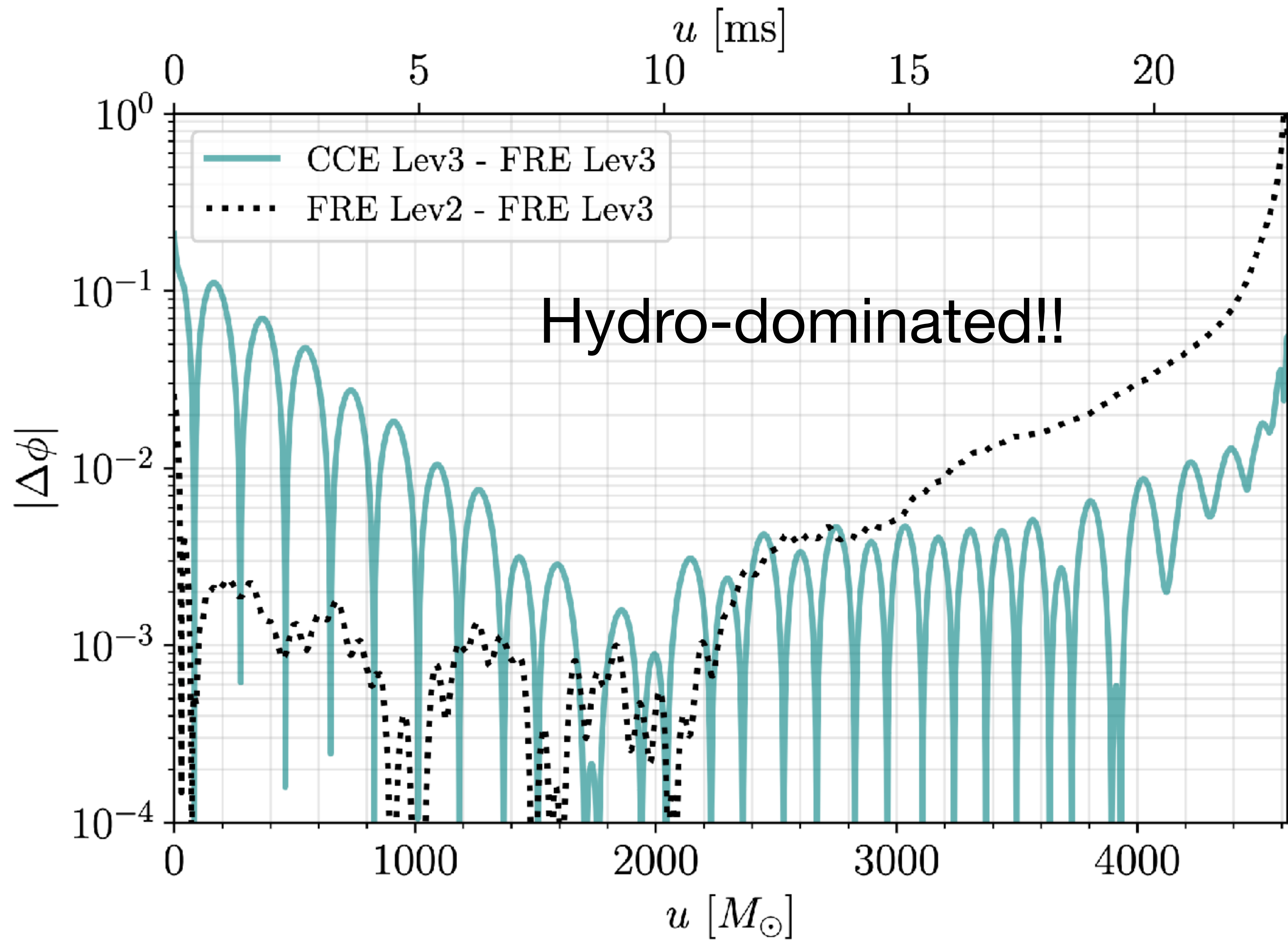
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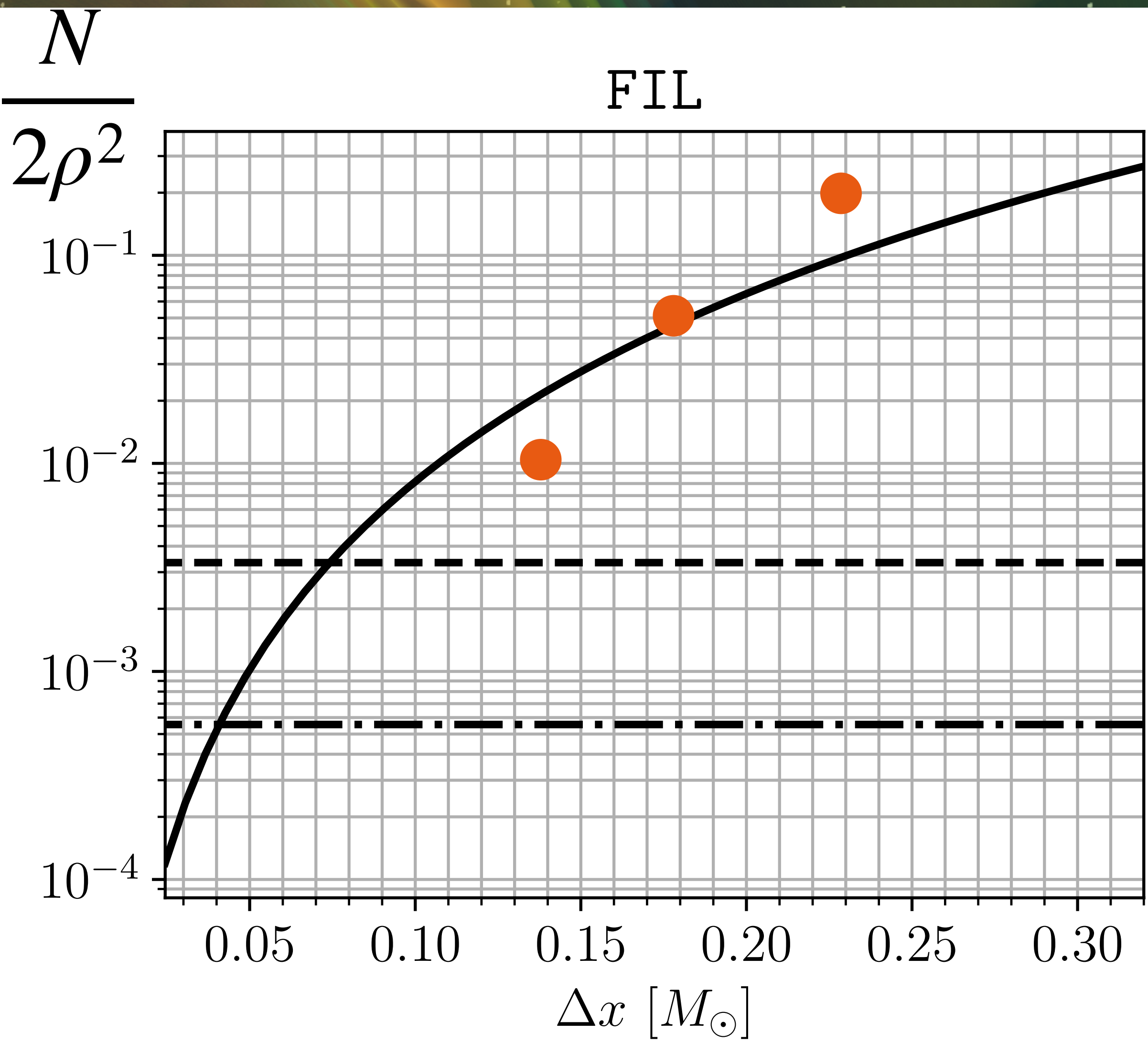
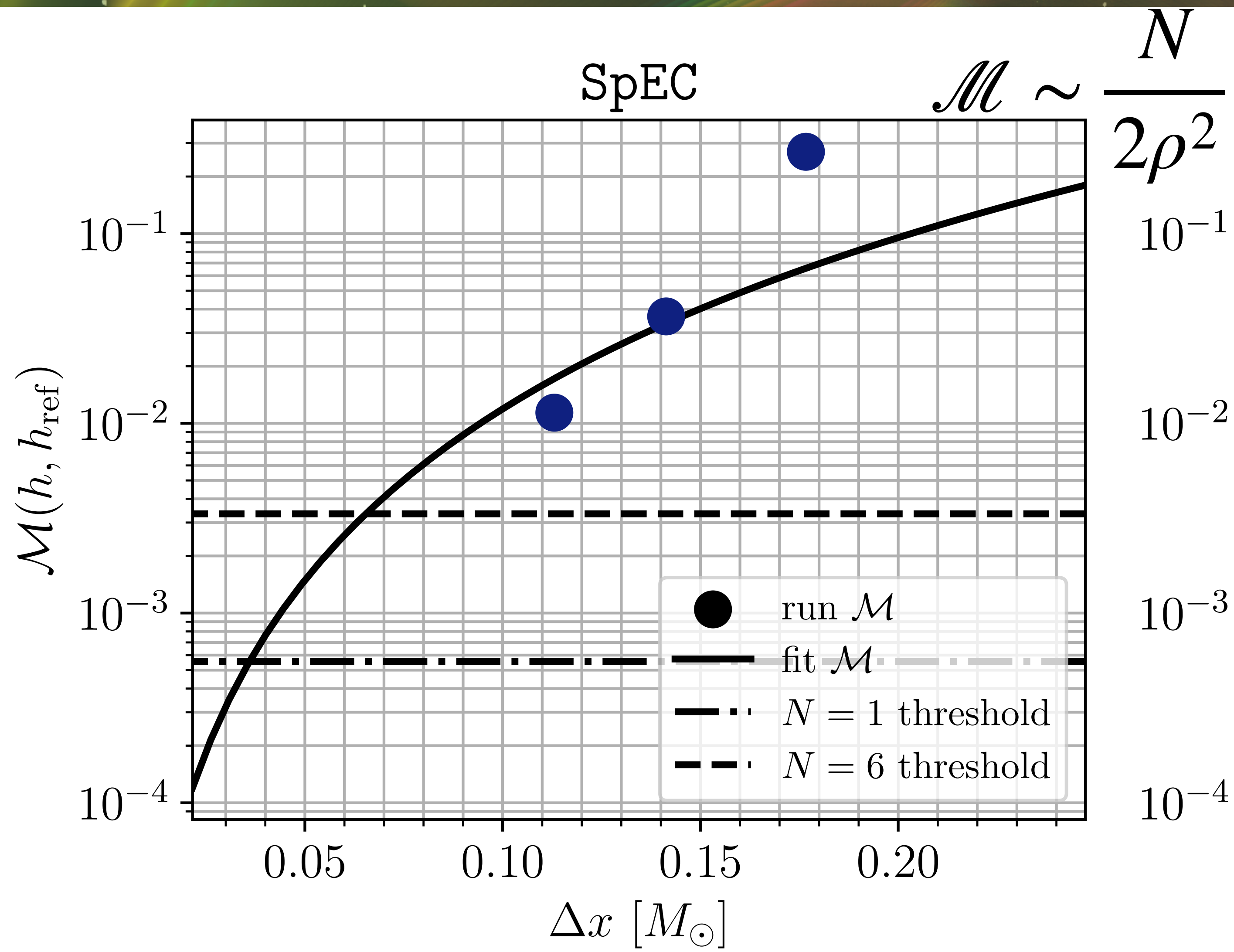
- Length?
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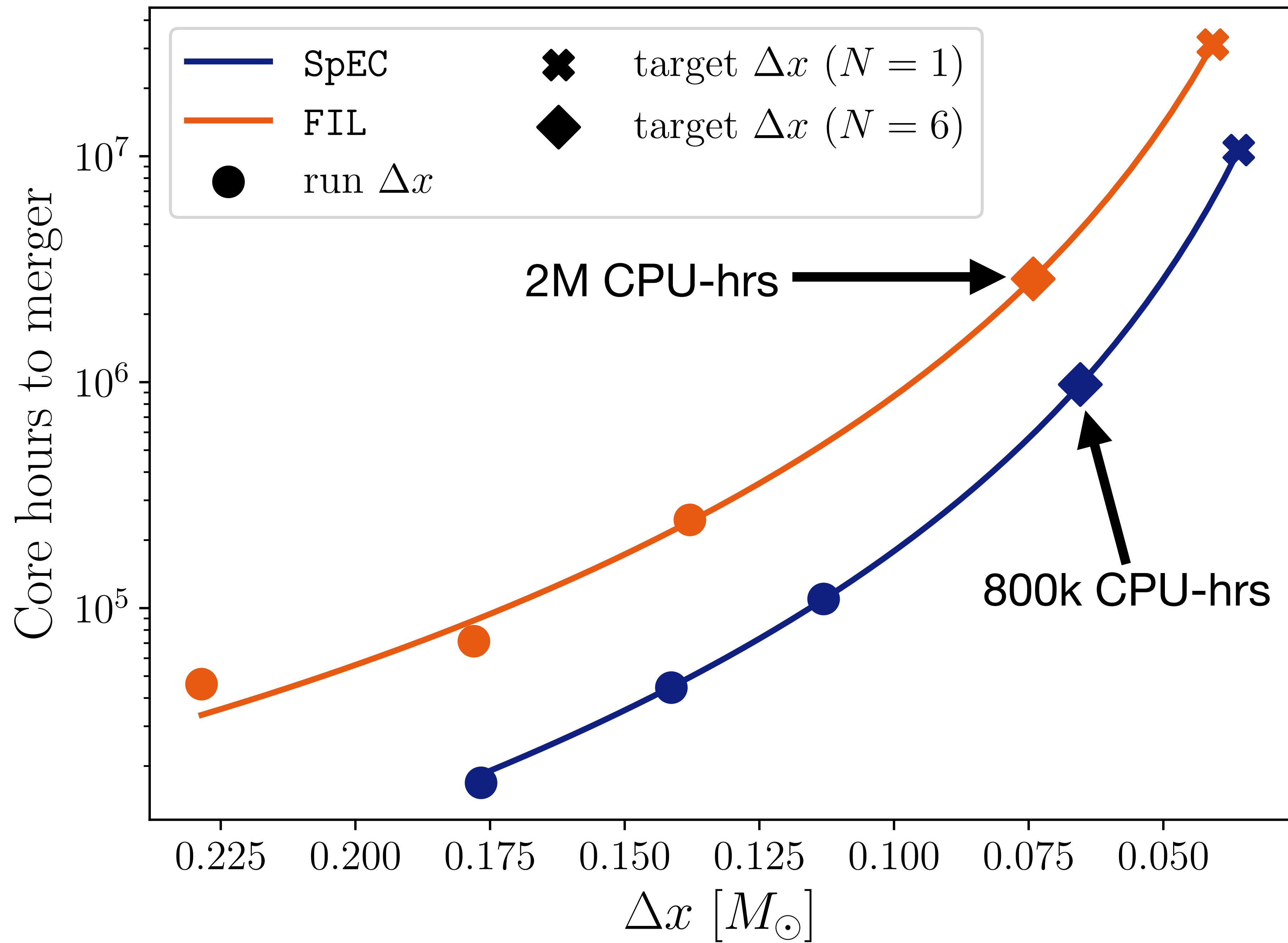
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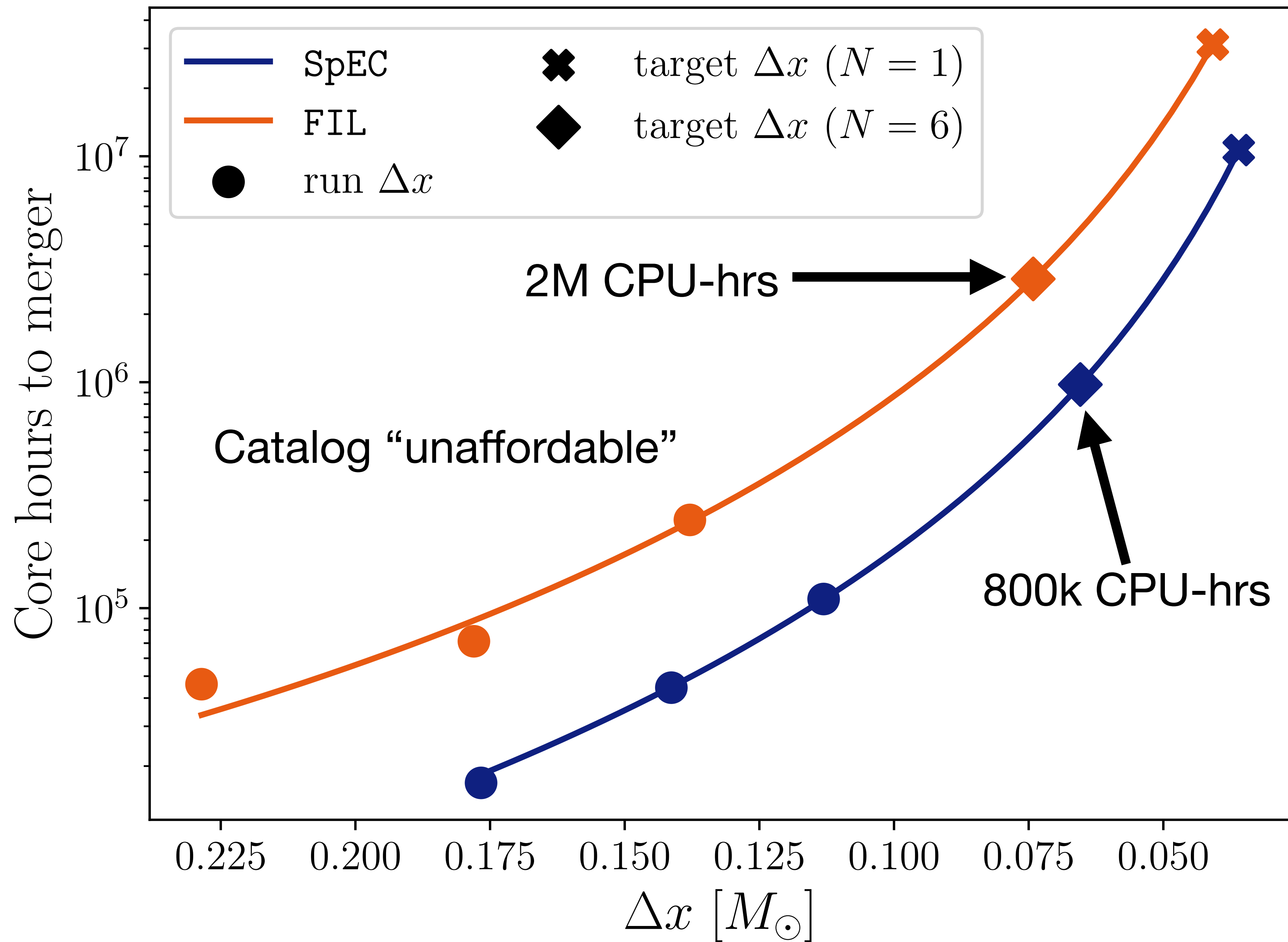
- Length?
- Mismatch?
- IM?
- Microphysics?

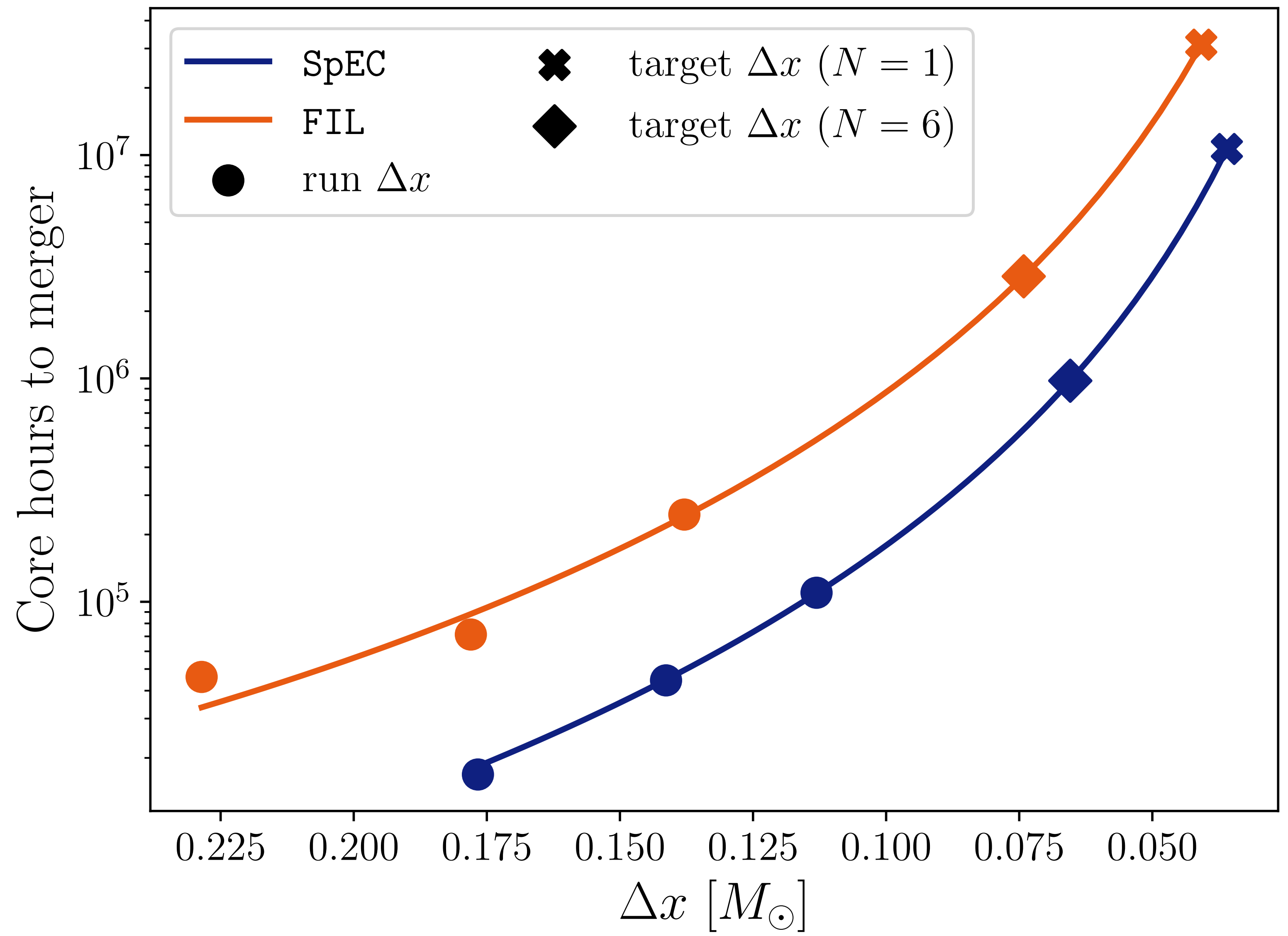




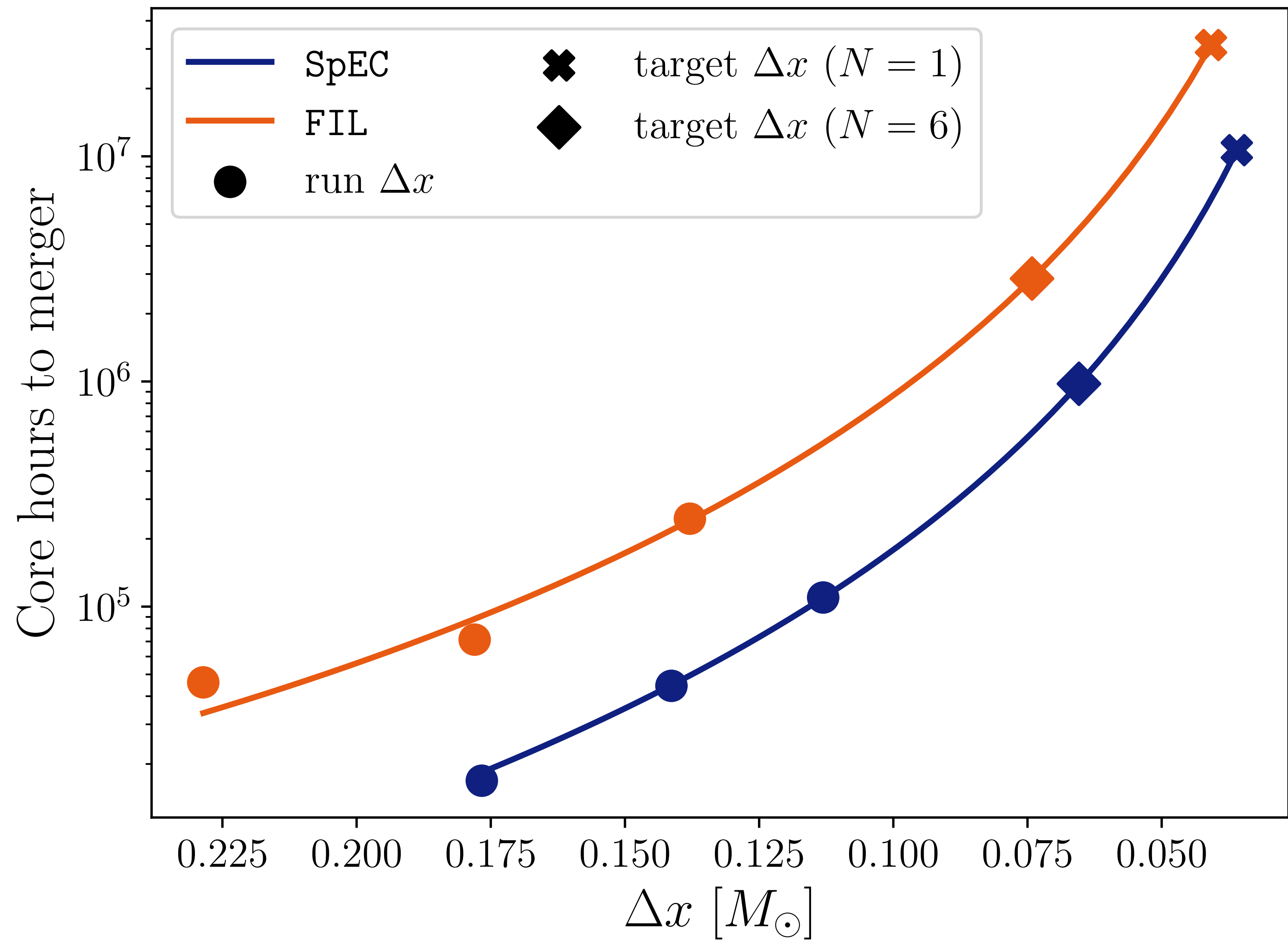




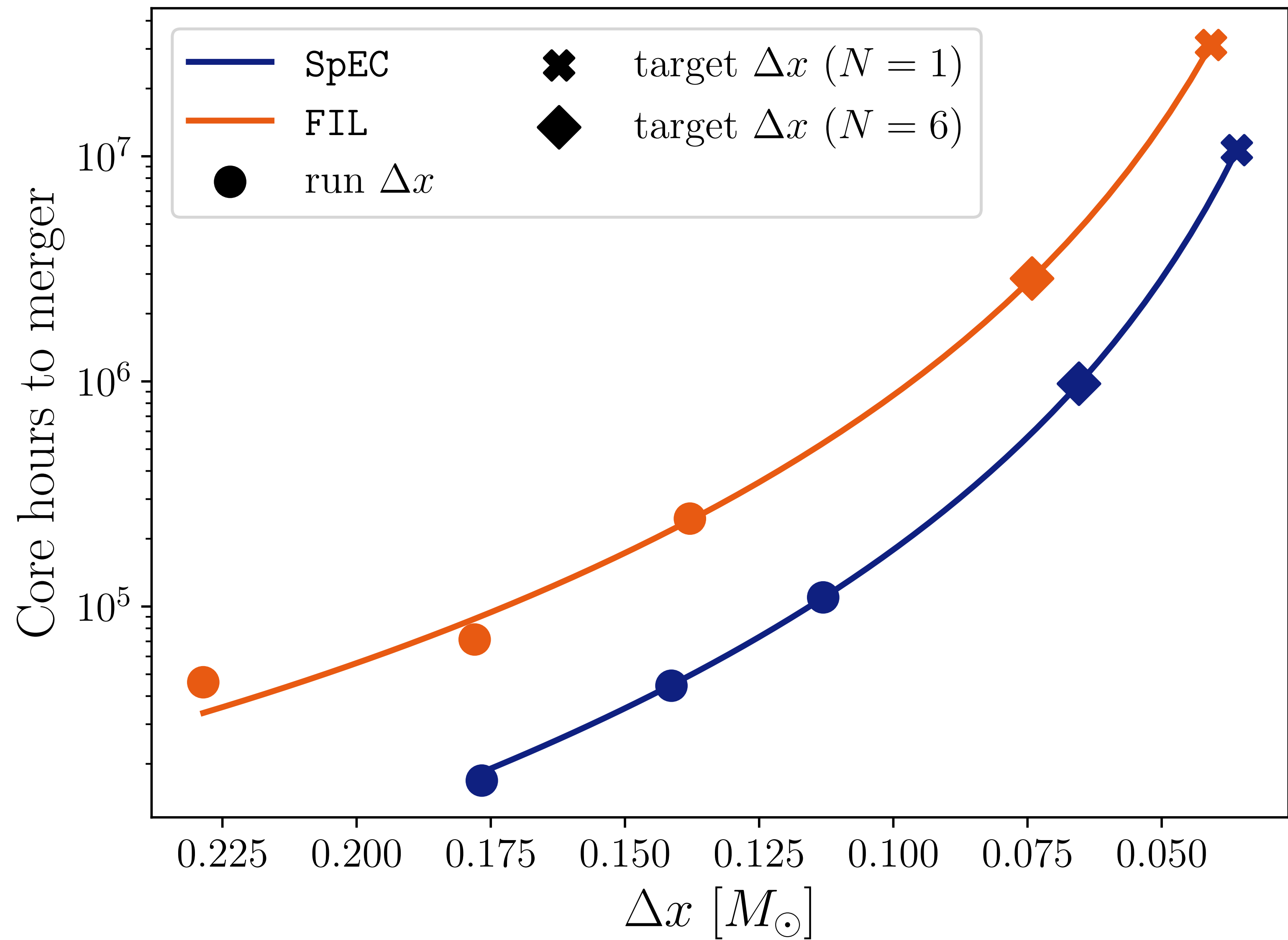




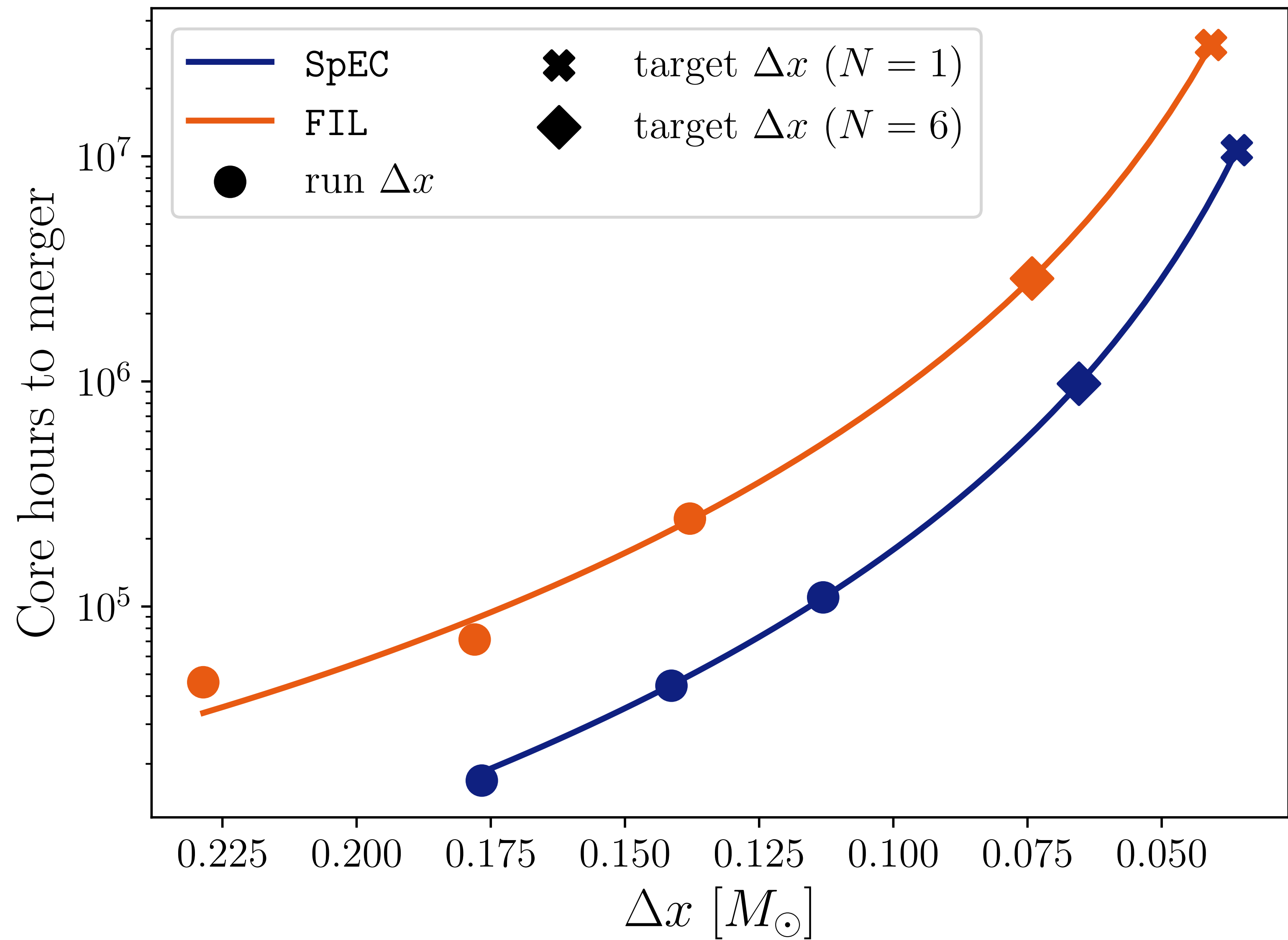
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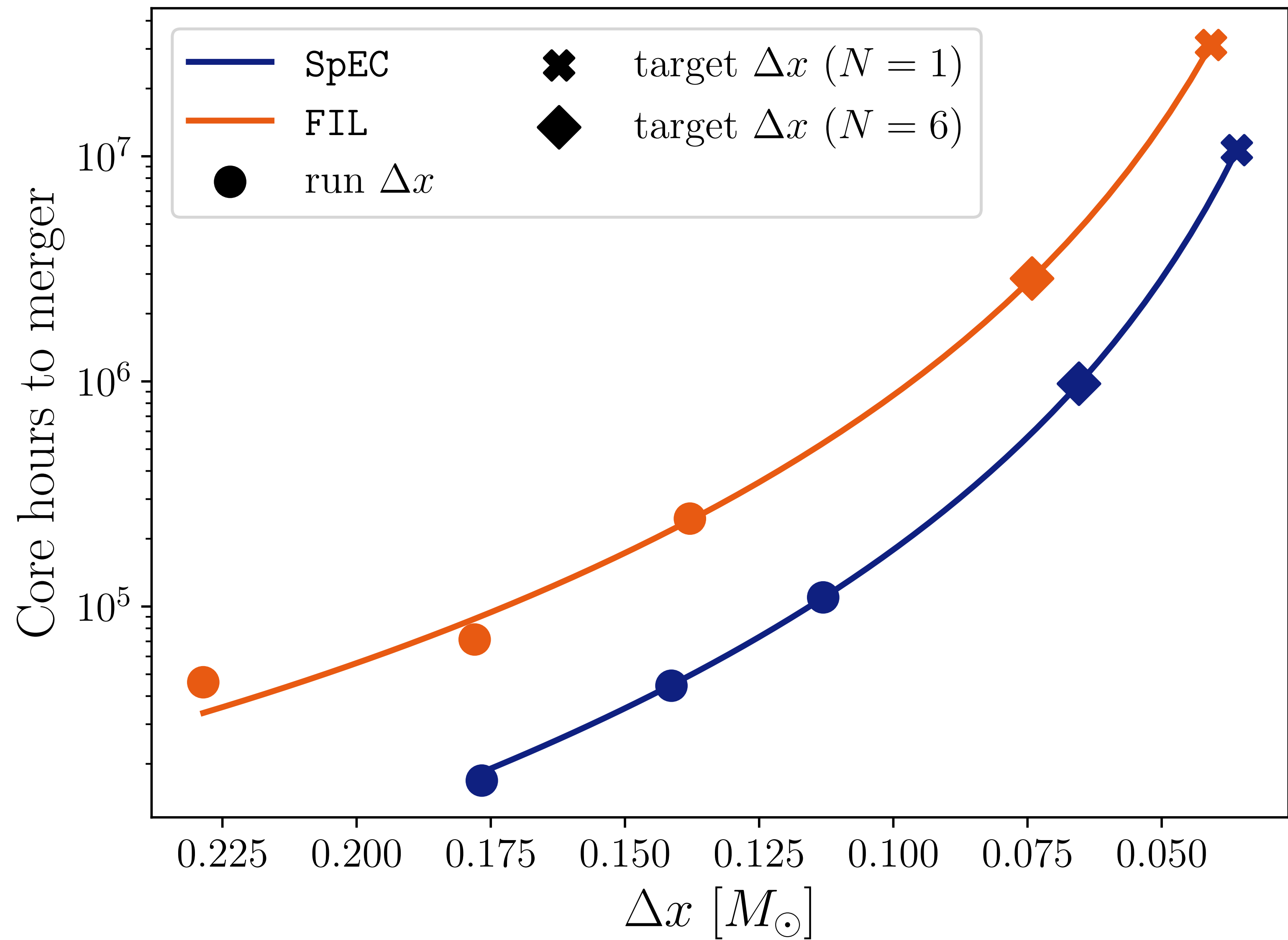
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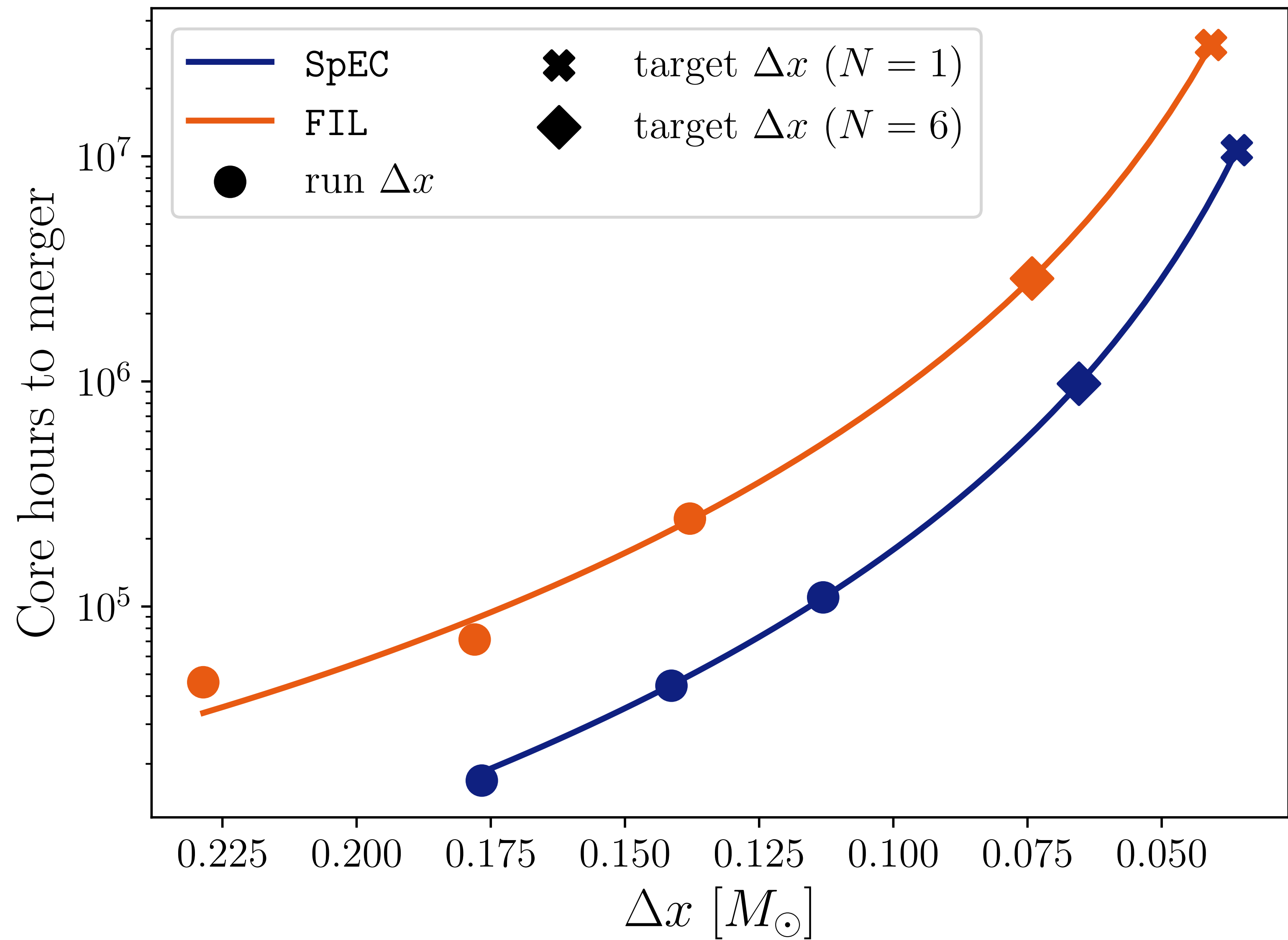
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- Mismatch: $\sim 10^{-3}$
- PN hybrid?
- GW post-merger?
- Inspiral microphysics?





Thank you!



Afram, Blackman, Bonilla, Boyle, Carpenter, Ceja, Corso, Duez, Field, Foucart, Giesler, Habib, Hebert, Jones, Kidder, Kim, Kumar, Lara, Legred, Lovelace, Macedo, Melchor, Mendes, Mitman, Morales, Most, Moxon, Nelli, O'Shea, Pannone, Pfeiffer, Ramirez-Aguilar, Ravishankar, Sanchez, Scheel, Teukolsky, Throwe, Varma, Vu, Wittek, Yoo +++

$$L = \lambda_w L_w + \lambda_v L_v + \lambda_{v \rightarrow 0} L_{v \rightarrow 0} + \lambda_{q \rightarrow 1} L_{q \rightarrow 1} + \lambda_{L^2} L_{L^2}$$

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- L_{L^2} weight regularization

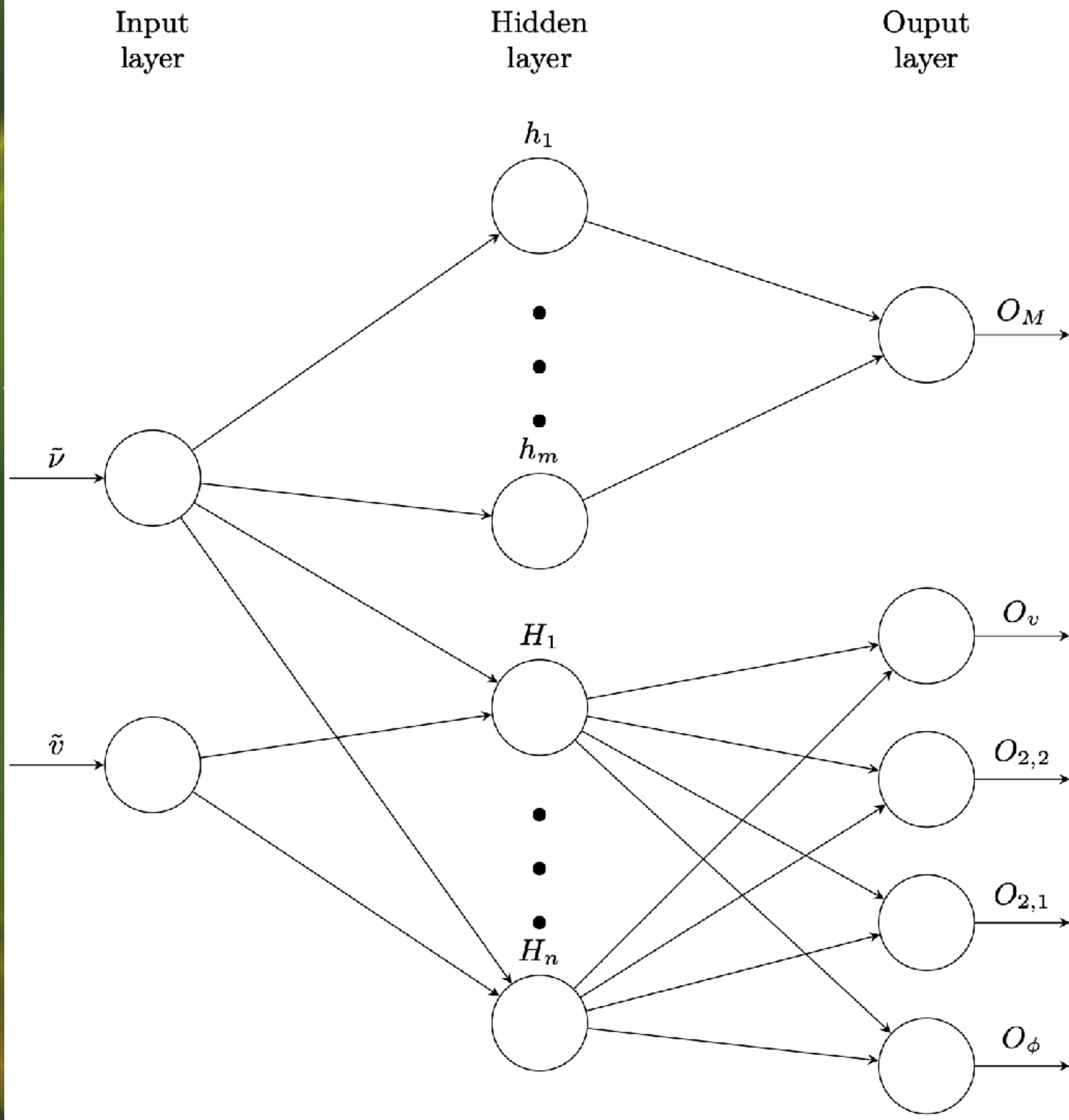
PINNs

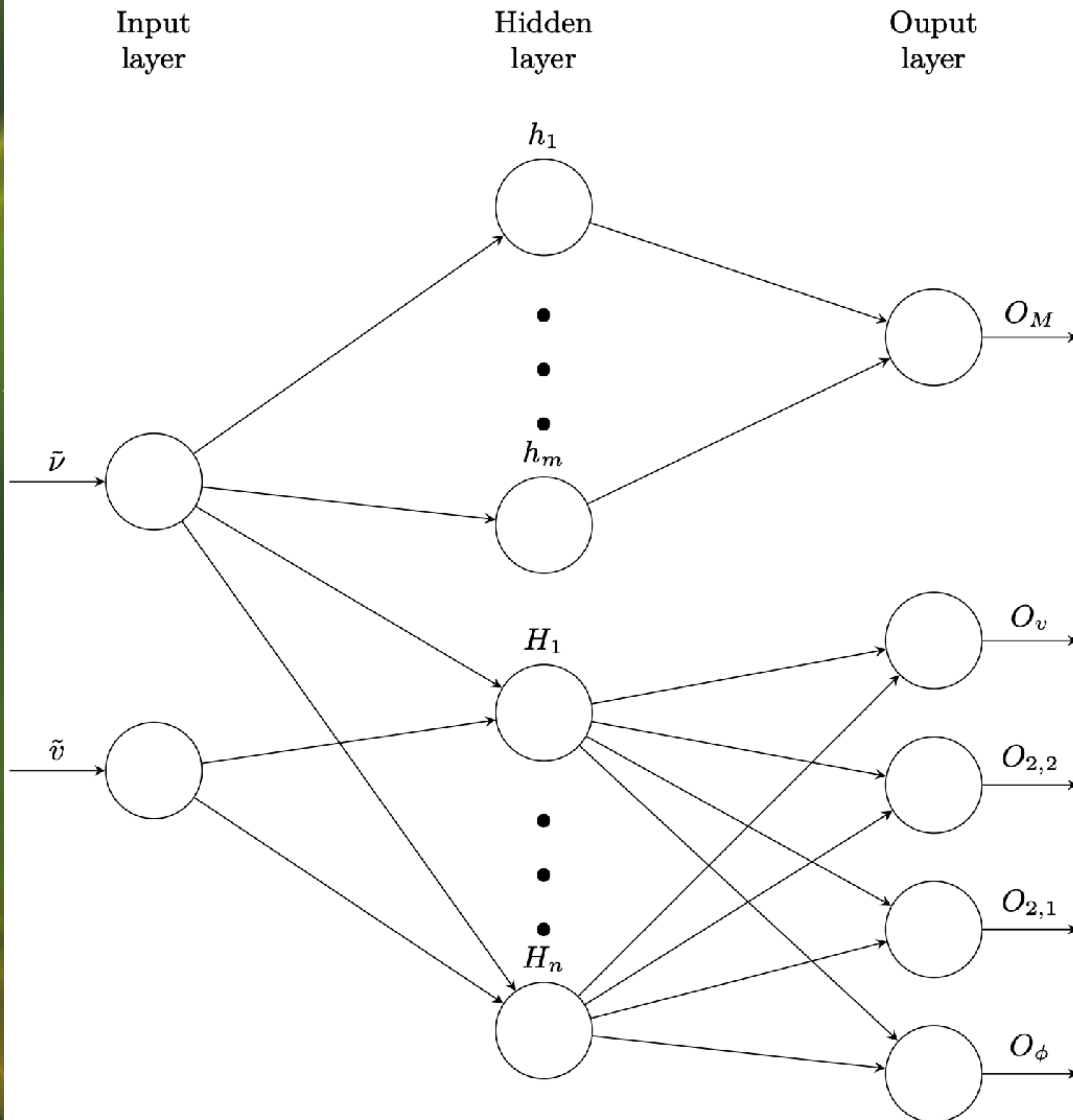
$$\frac{dv}{dt} = - \frac{\mathcal{F}_{\text{GW}}}{\frac{d\mathcal{E}_{\text{binding}}}{dv}} + \text{NN}_1$$

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$$h'_{\ell,m} = h_{\ell,m} \left(1 + \text{NN}_2 \right) e^{-im\text{NN}_3}$$



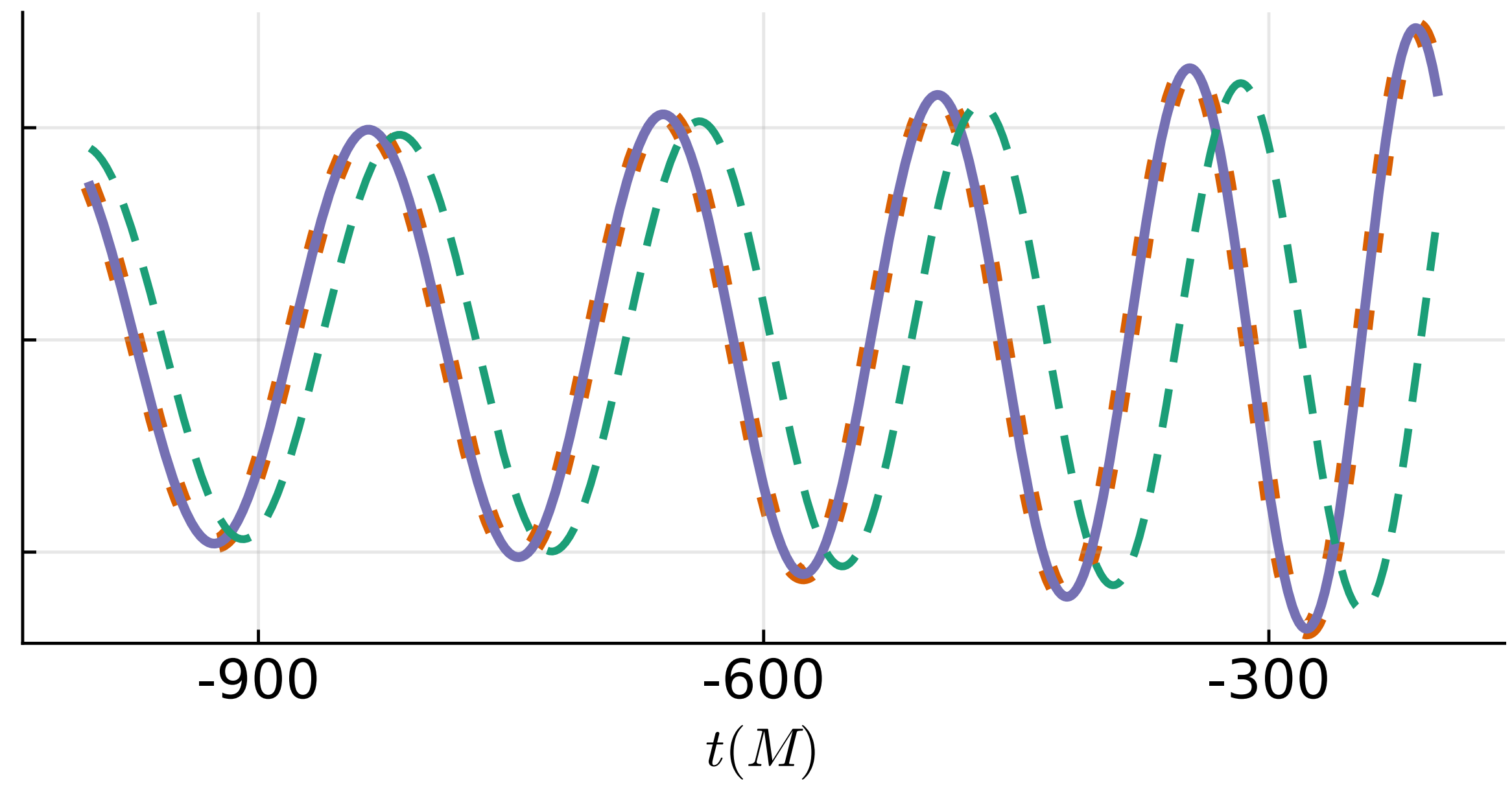
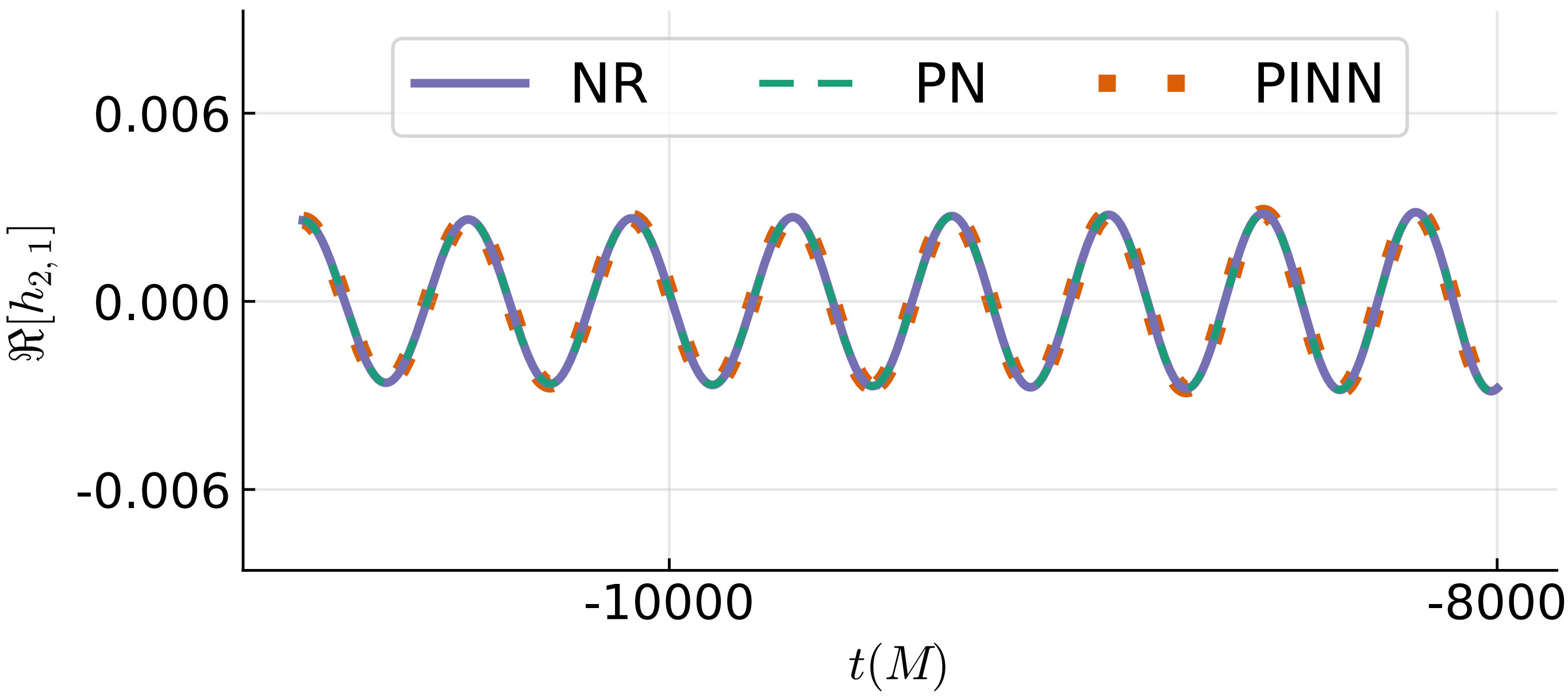
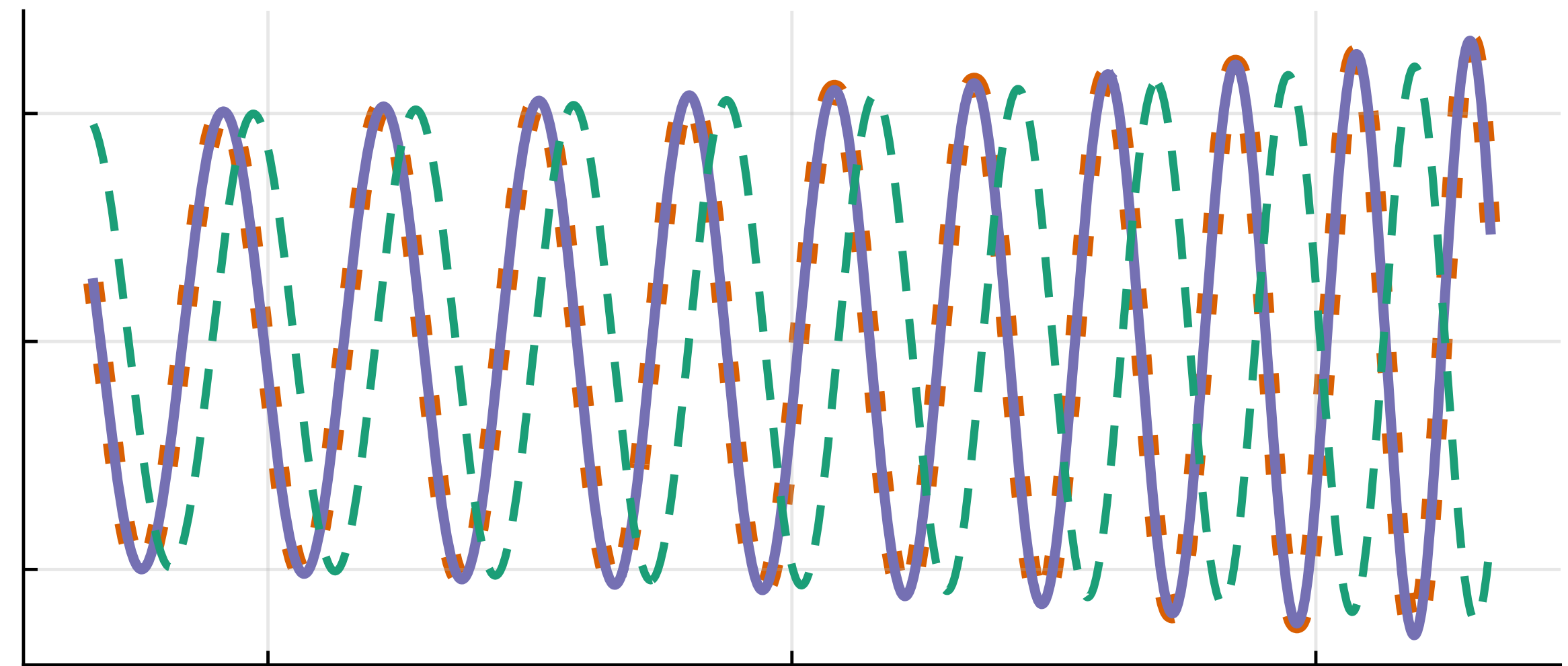
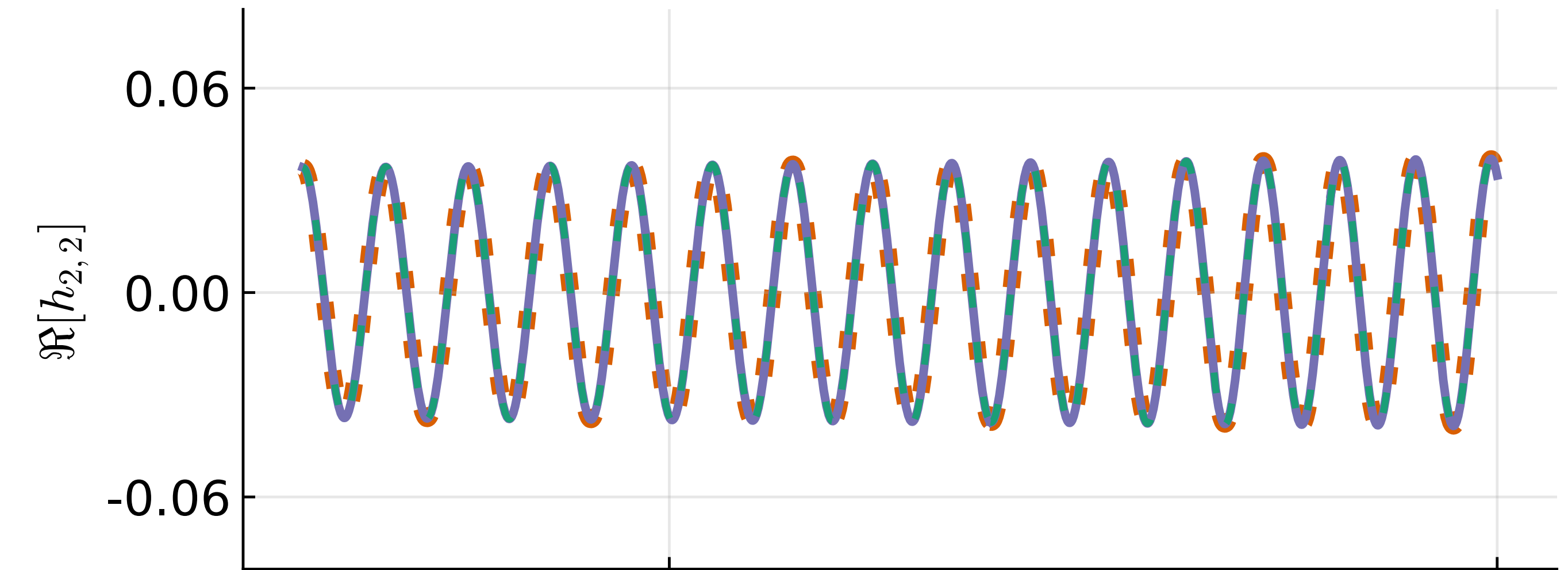


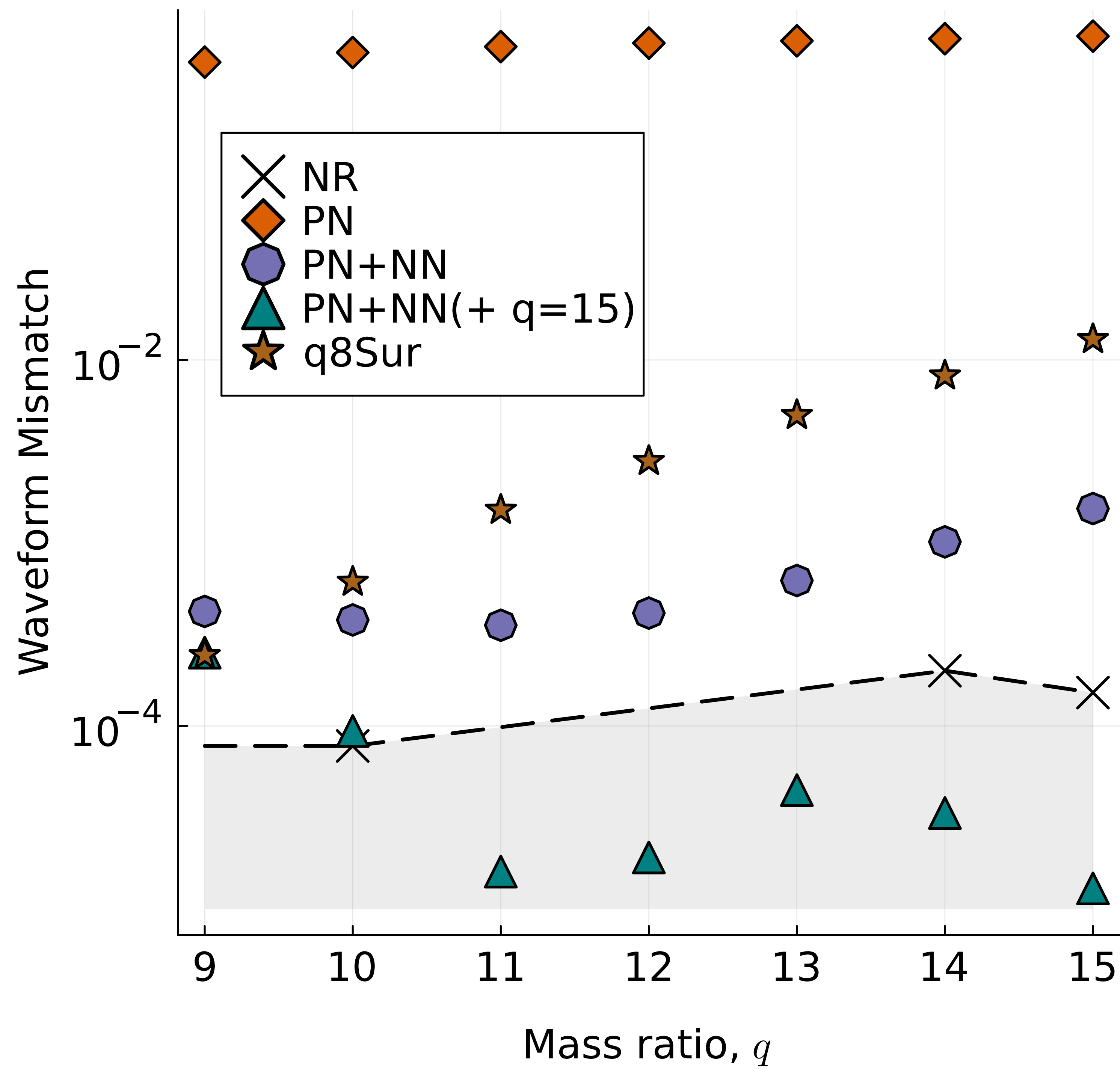
Tiny network!

101 parameters!

120 CPU hours

8 waveforms





Future of PINNs

- z-spins
- Precession
- Eccentricity
- Merger/ringdown
- “Extreme” q

