# MESON CHPT AT TWO LOOPS

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- general
- 2 flavour
- 3 flavour
- 3 valence + 3 sea flavour: partially quenched
  ⇒ first results
- 3 flavours:
  - general fit + some remarks
  - scalar form factors, $\pi\pi$, $\pi K$
  - $F_\pi^V$, $K_{\ell 3}$

# GENERAL

## Structure of the lagrangian

| | 2 flavour | | 3 flavour | | 3+3 PQ | |
|---|---|---|---|---|---|---|
| $p^2$ | $F, B$ | 2 | $F_0, B_0$ | 2 | $F_0, B_0$ | 2 |
| $p^4$ | $l_i + h_i$ | 7+3 | $L_i + H_i$ | 10+2 | $\hat{L}_i + \hat{H}_i$ | 11+2 |
| $p^6$ | $c_i$ | 53+4 | $C_i$ | 90+4 | $K_i$ | 112+3 |

$p^4$: Gasser, Leutwyler 84, 85

$p^6$: JB, Colangelo, Ecker

All infinities known

Note: • replica method $\Rightarrow$ $N_F$ flavour can be used to obtain PQ quantities

• 3 flavour is a special case of 3+3 PQ
the $L_i, C_i$ can be had from $\hat{L}_i, K_i$
using Cayley-Hamilton relations

# Two-flavour

MOST CALCULATIONS DONE

$\gamma\gamma \to \pi^0\pi^0$ Bellucci, Gasser, Sainio

$\gamma\gamma \to \pi^+\pi^-$ Bürgi

$F_\pi$ Bürgi; JB, Colangelo, Ecker, Gasser, Sainio

$m_\pi^2$ Bürgi; "

" " scattering "

$F_\pi^V$ $F_\pi^S$ JB, Colangelo, Talavera

" $\ell\nu\gamma$ JB, Talavera

In general: reasonable convergence

$c_i$ not important for many threshold quantities

→ combined with dispersive [illegible], Roy equations

etc... for very precise predictions

# Three flavour

VV two point functions : Kambor, Golowich
Kambor, Dürr
Amorós, JB, Talavera (ABT)

AA two point functions, masses, decay constants : $\pi$, $\eta$, K
Kambor, Golowich ; ABT

SS two point function : Moussallam

$K_{\ell 4}$ ABT

masses & decay constants $m_u \neq m_d$ : ABT

Vector form factors : Post, Schilcher ; JB, Talavera

$K_{\ell 3}$ : " ; JB, Talavera

Scalar form factors : JB, Dhonte

$\pi\pi$ : JB, Dhonte, Talavera

$\pi K$ : "

$K, \pi \to \ell \nu \gamma$ : Geng, Ho, Wu

- 3 different schemes in use
- most not JB et al. : take an old set of $L_i$ + give some plots

# Partially Quenched

$m^2_{\pi^+}$

- valence equal mass
- 3 sea equal mass

JB, Danielsson, Lähde
hep-lat/0406017

$F_{\pi^+}$

- valence equal mass
- 3 sea equal mass

JB, Lähde
to be published

**Planned:**

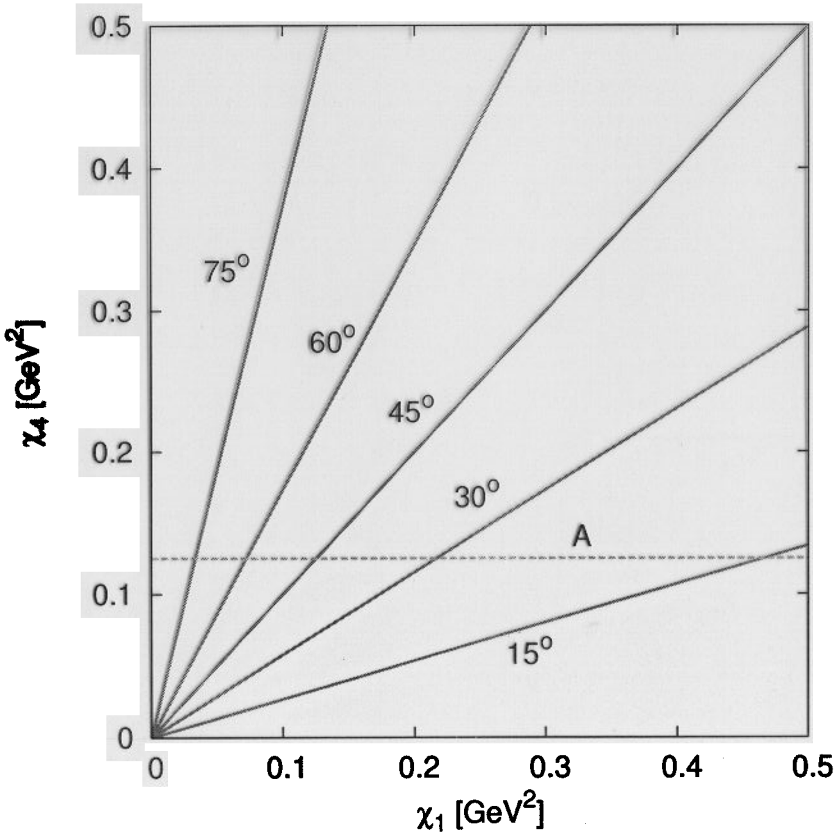
- different valence masses
- 2 equal + 1 sea masses

both for masses and decay constants

Actual calculations:

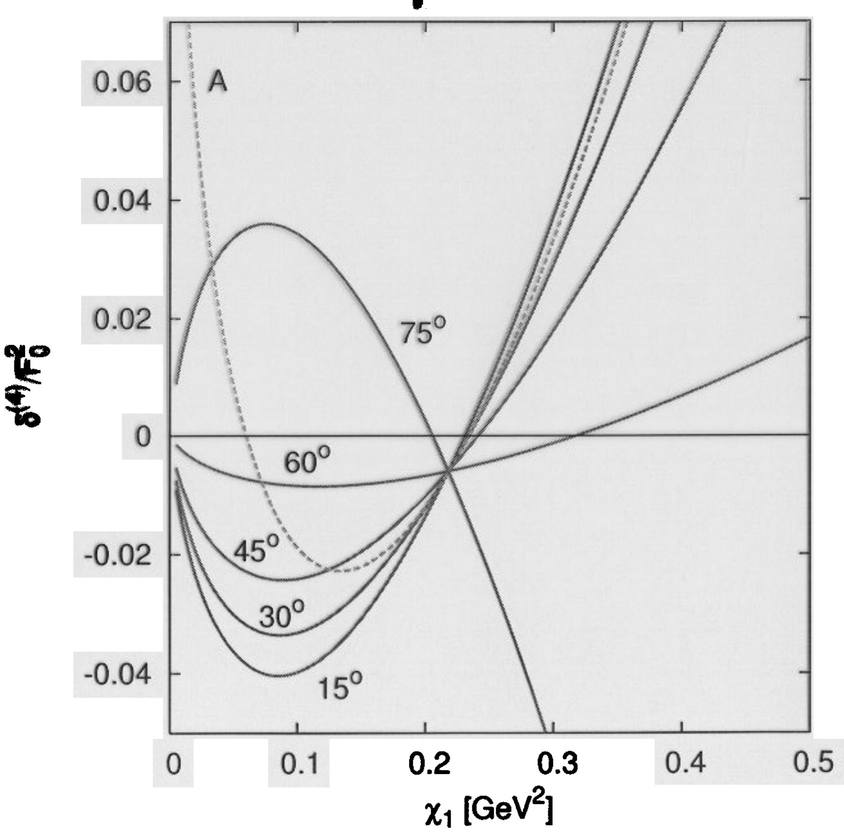
- heavy use of FORM
- use PQ without the super $\Phi_0$ within the supersymmetric formalism
- How important is $N_{SEA} = 2$ ?

sea



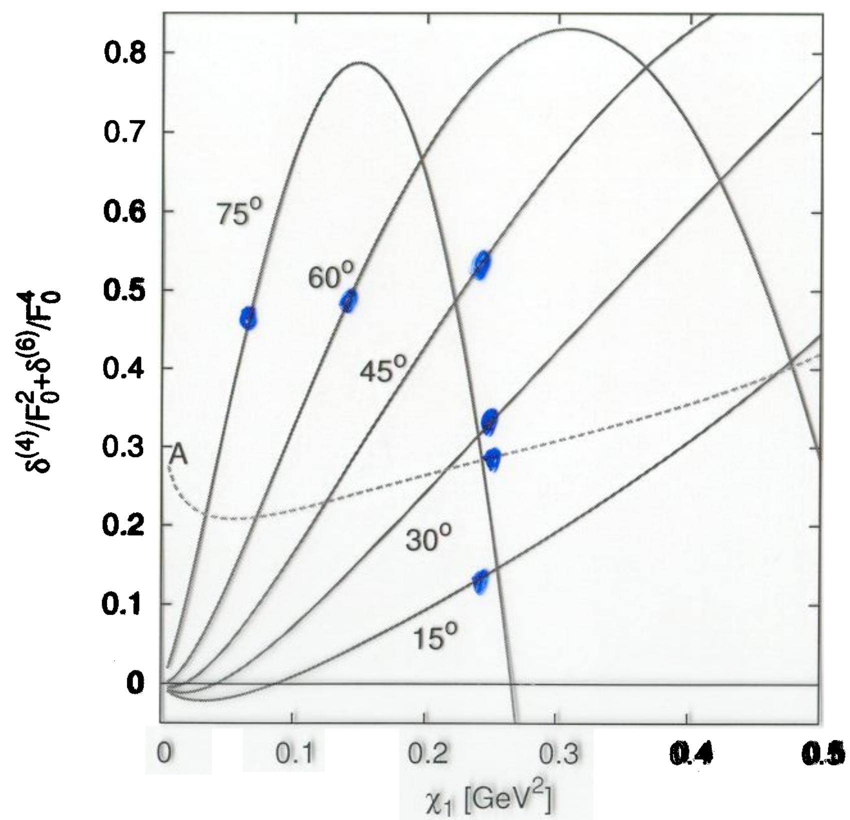

valence

p⁴

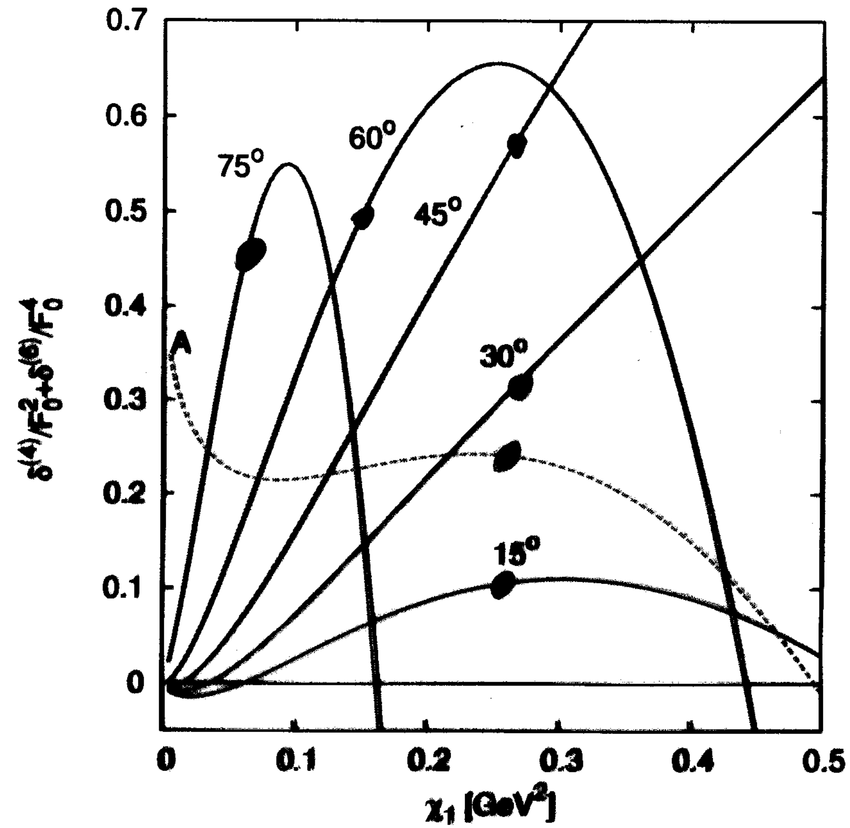



$0.25 = (500 \text{ MeV})^2$

$p^4 + p^6 \quad L_i = K_i = 0$



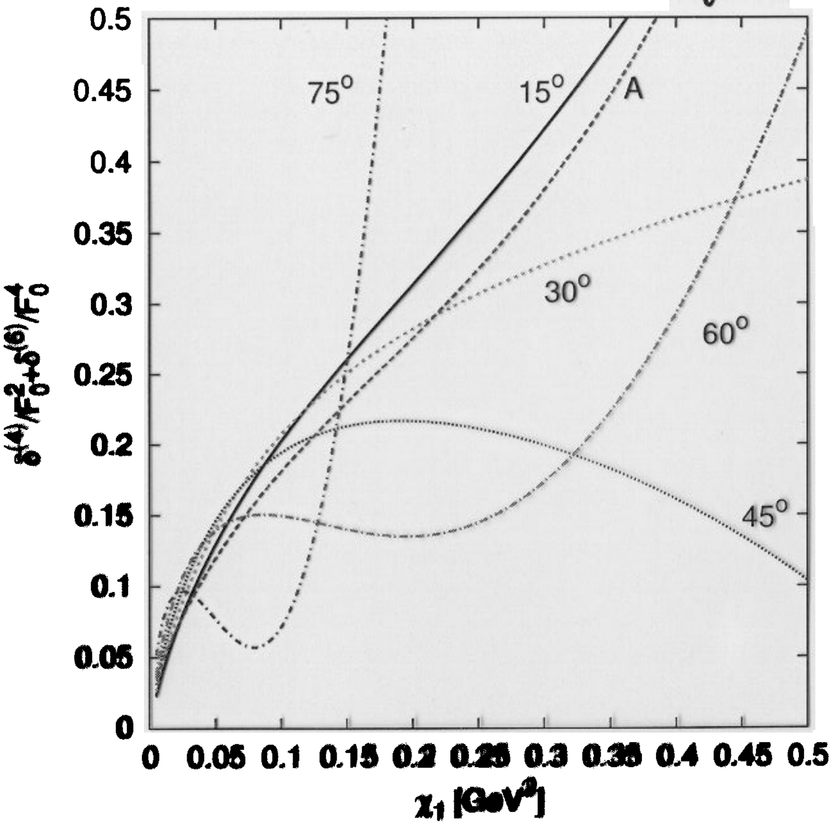

$p^4 + p^6 \quad K_i = 0$



$p^4 + p^6$

$L_i \neq 0$

$K_i = 0$




preliminary

# General strategy + some comments

- Find enough inputs: experiment etc
- $C_i^r$ : • kinematical dependence

  ⇒ fits well with single resonance saturation

  • quark mass dependence

  ⇒ if vector dominated: naive VMD seems to be OK (see $K_{\ell 3}$ discussion)

  ⇒ if scalar dominated

  if $\bar{u}d$ : $a_0$ ~ 980 MeV
  $\bar{u}s$ : $K$ ~ 1400 MeV

  ⇒ estimates of $C_i^r$ unrealistically large

  ⇒ at present set to zero.

- In the $p^6$ parts: lowest order or physical masses make some difference: $\frac{m_K^4}{F_\pi^4}$ $\frac{m_{K0}^4}{F_0^4}$ $\frac{m_{K0}^4}{F_\pi^4}$

  introduce an uncertainty

- kinematics can require physical masses to get threshold singularities in the right place + obey things like $s_1 + s_2 + s_3 = 4 m_\pi^2$

## Strategy

### Inputs:

$K_{\ell 4}$ (E865) : $F(0)$
$G(0)$
$\lambda$

$m^2_{\pi^0}$
$m^2_{K^0}$
$m^2_{\eta}$
$m_{K^+}$ em with Dashen violation (also without)

$F_{\pi^+}$

$F_{K^+}/F_{\pi^+}$

$m_s/\hat{m}$ : mainly used 24, 26 gives changes within the error

$L_4, L_6$

$C_i^r$ as estimated above

| | fit 10 | same $p^4$ | fit B |
|---|---|---|---|
| $10^3 L_1$ | $0.43 \pm 0.12$ | 0.38 | |
| $10^3 L_2$ | $0.43 \pm 0.12$ | 1.59 | |
| $10^3 L_3$ | $-2.53 \pm 0.37$ | $-2.91$ | |
| $10^3 L_4$ | $\equiv 0$ | $\equiv 0$ | $\equiv 0.5$ |
| $10^3 L_5$ | $0.94 \pm 0.11$ | 1.46 | |
| $10^3 L_6$ | $\equiv 0$ | $\equiv 0$ | $\equiv 0.1$ |
| $10^3 L_7$ | $-0.31 \pm 0.14$ | $-0.49$ | |
| $10^3 L_8$ | $0.60 \pm 0.18$ | 1.00 | |
| $2 B_0 \hat{m} / m^2_{\pi^0}$ | 0.736 | 0.986 | 1.129 |
| $m_u / m_d$ | $0.45 \pm 0.05$ | 0.52 | |
| $F_0$ | 87.7 | 81.1 | 70.4 |
| $\frac{m_\pi^{2(4)}}{m_\pi^2}$, $\frac{m_\pi^{2(6)}}{m_\pi^2}$ | 0.006, 0.258 | | $-0.138$, 0.009 |
| $\frac{m_K^{2(4)}}{m_K^2}$, $\frac{m_K^{2(6)}}{m_K^2}$ | 0.007, 0.306 | | $-0.149$, 0.094 |
| $\frac{m_\eta^{2(4)}}{m_\eta^2}$, $\frac{m_\eta^{2(6)}}{m_\eta^2}$ | $-0.052$, 0.318 | | $-0.197$, 0.073 |
| $\frac{F_K}{F_\pi}$ $p^4$, $p^6$ | 0.169, 0.051 | 0.22, $\equiv 0$ | 0.153, 0.067 |

Note: fit B chosen to have $F_S(0)$ reasonable

# Remarks:

- errors are very correlated
- $\mu = 0.77$ GeV
  - if RS at $\mu = 0.55$ or $1$ GeV results change within error.
  - varying RS with factor of 2 stay within error
- $K_{\ell 4}$: Rosselet versus E865 is outside error

plots in remainder (mainly)

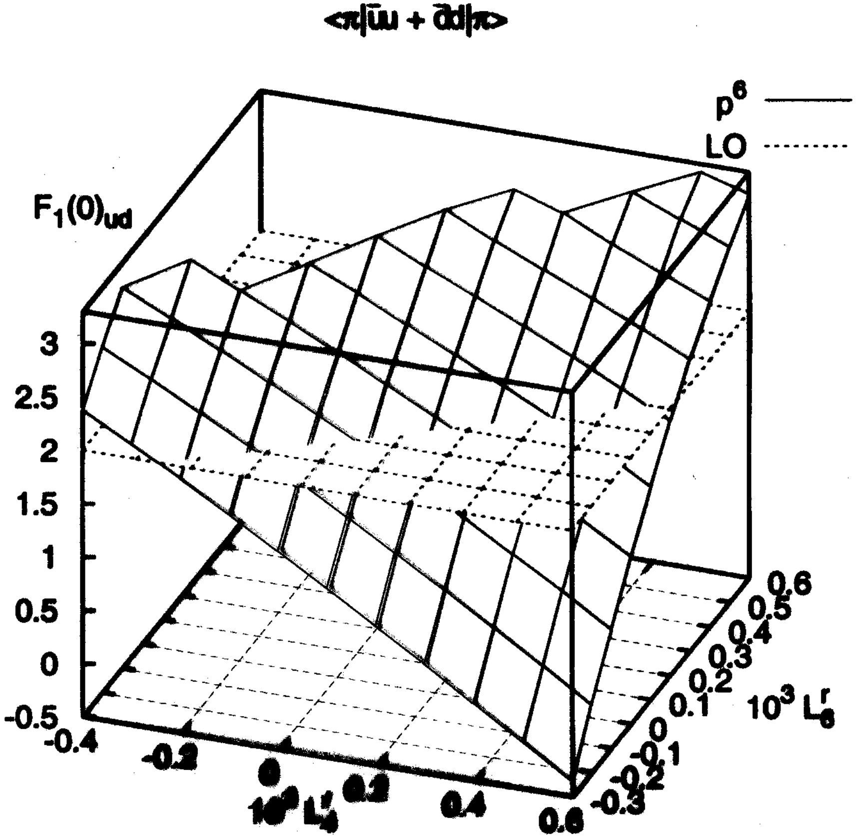
vary $L_4, L_6 \Rightarrow$ refit other $L_i$ : { $\chi^2$ OK except top left

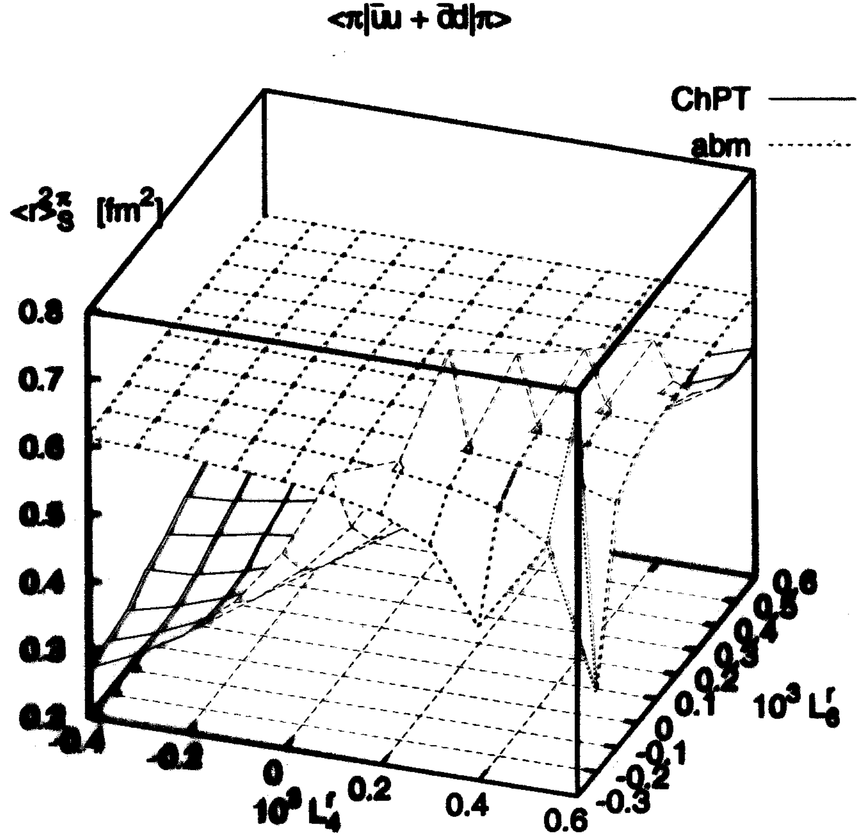
$\Rightarrow$ plot other quantities

From dispersion theory:

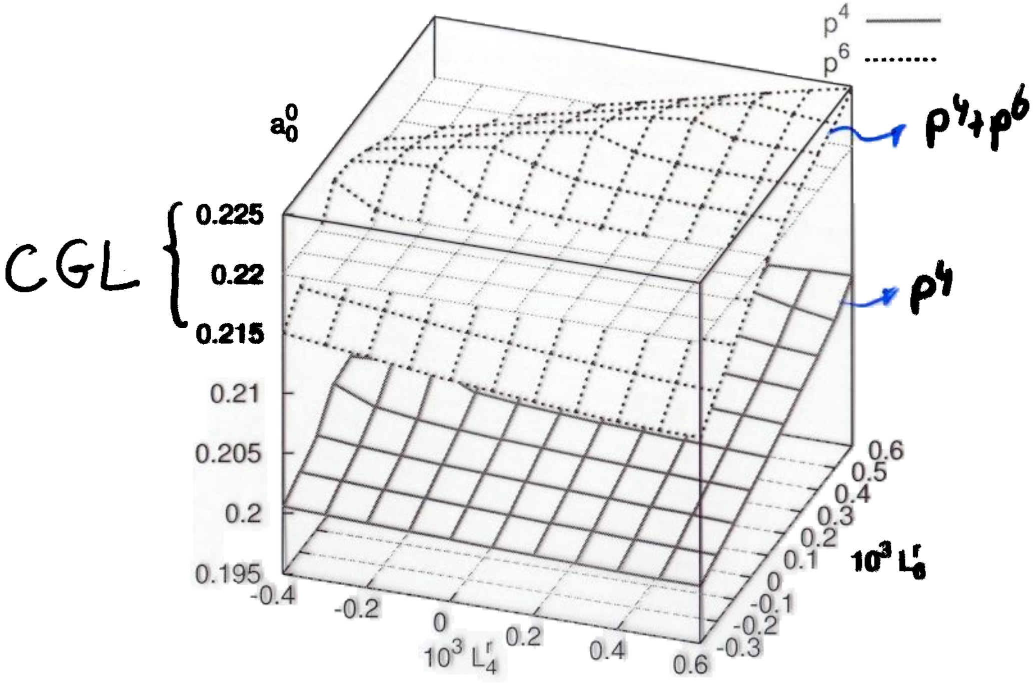
From $\langle\pi|J|\pi\rangle(0)$, $\langle K|J|K\rangle(0)$ and the phases, we know the $t$ dependence

$J = \bar{u}u + \bar{d}d$

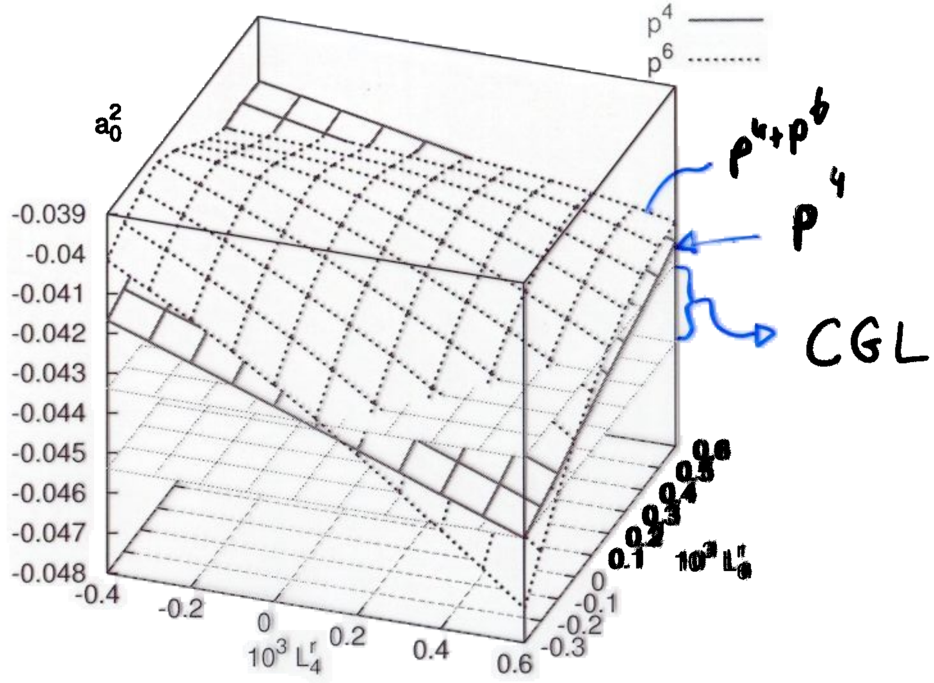


ππ



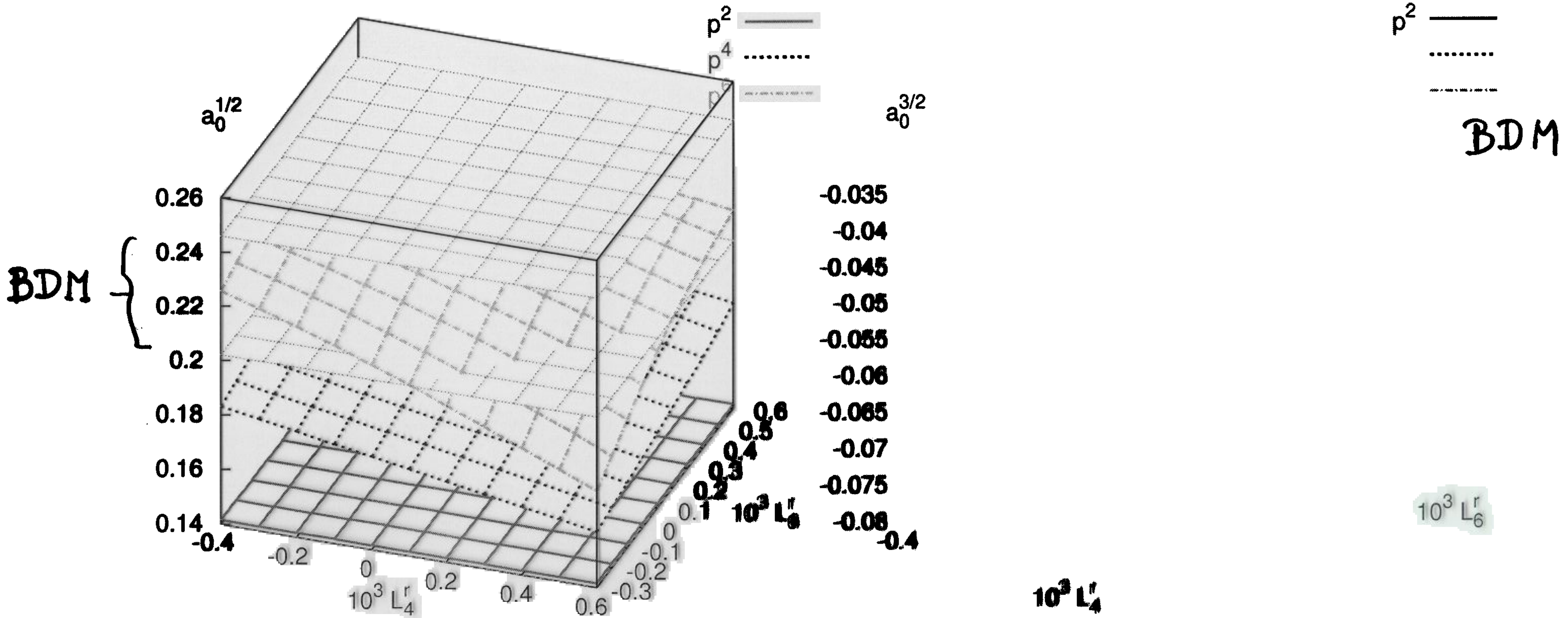
p4
p6
a00
p4+p6
p4
CGL
0.225
0.22
0.215
0.21
0.205
0.2
0.195
-0.4
-0.2
0
0.2
0.4
0.6
103 L4r
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
103 L6r

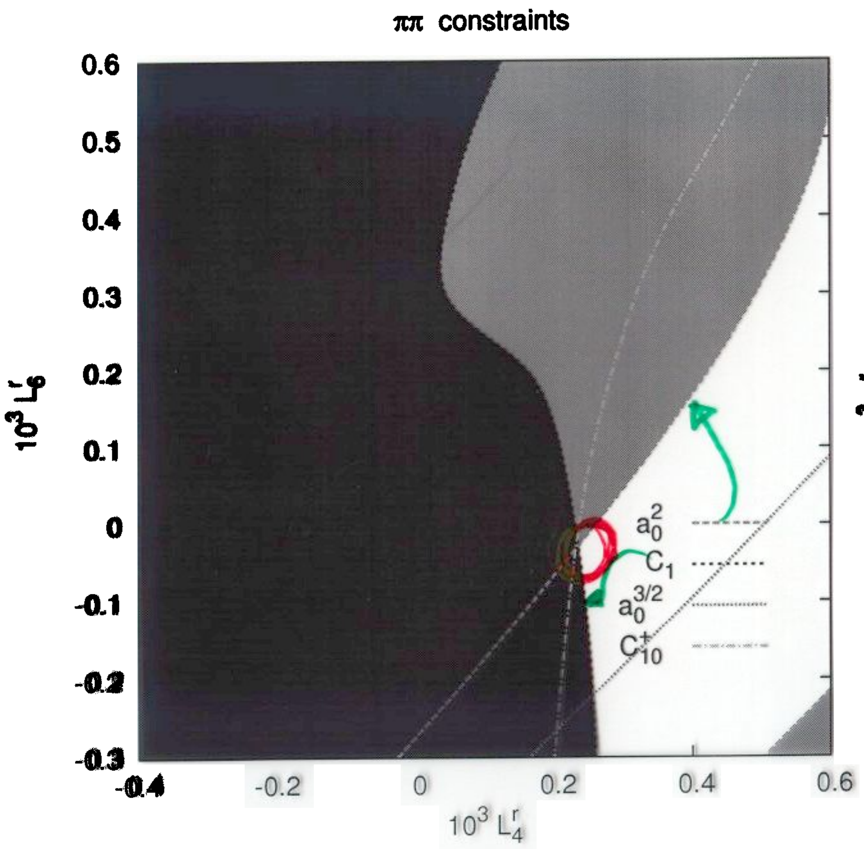
p4
p6
a02
p4+p6
p4
CGL
-0.039
-0.04
-0.041
-0.042
-0.043
-0.044
-0.045
-0.046
-0.047
-0.048
-0.4
-0.2
0
0.2
0.4
0.6
103 L4r
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
103 L6r

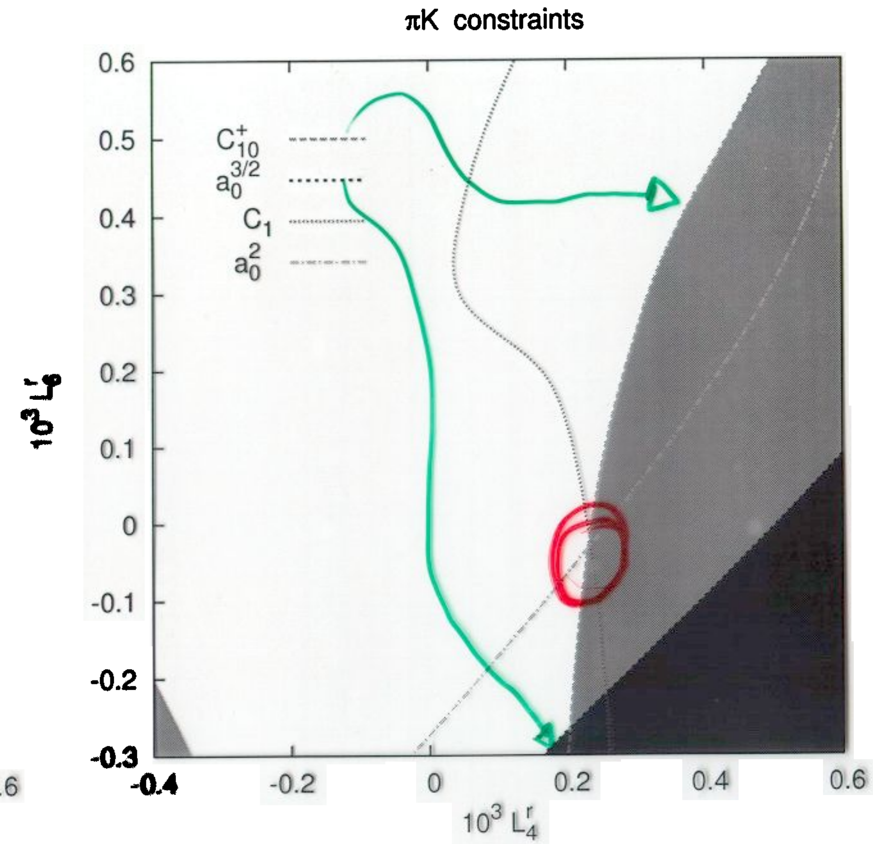

$p^2 = 0.16$

πK



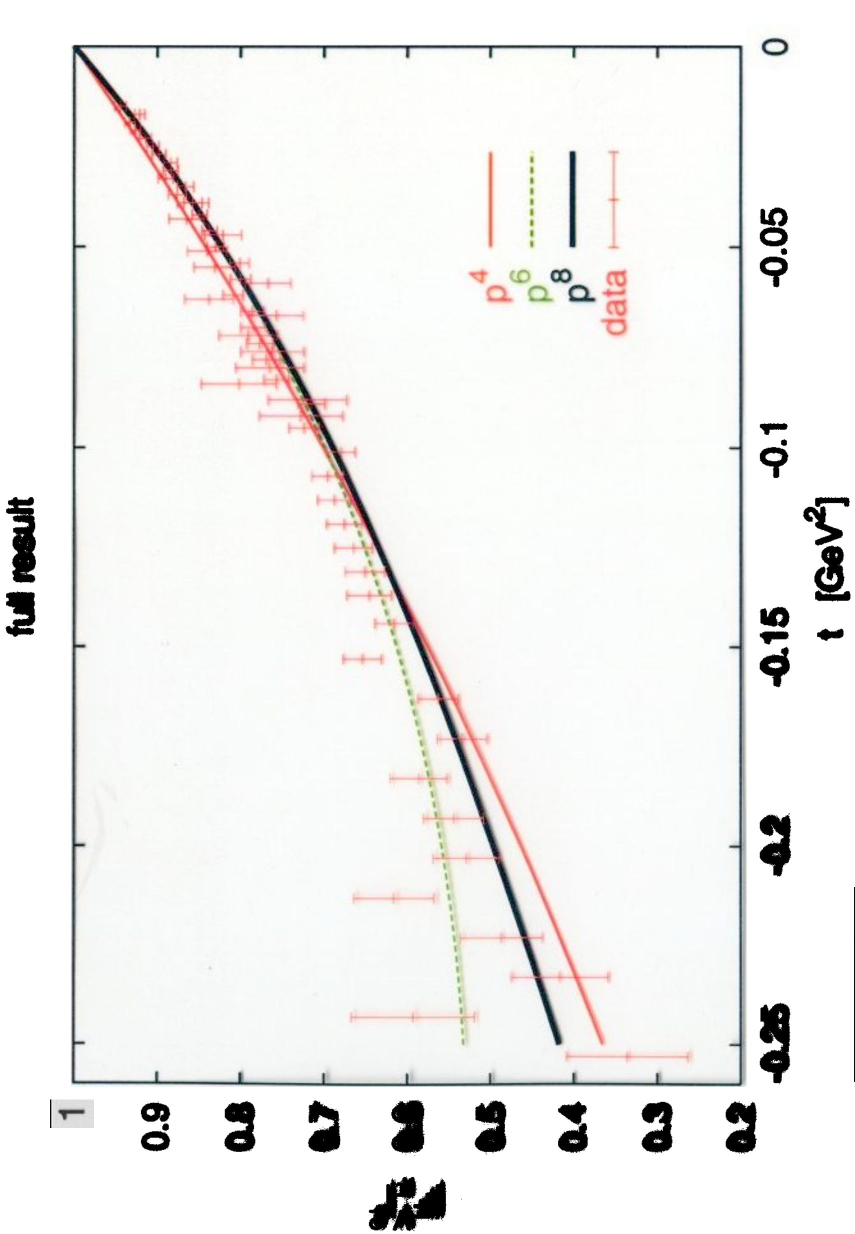
BDM
$a_0^{1/2}$
0.26
0.24
0.22
0.2
0.18
0.16
0.14
-0.4
-0.2
0
0.2
0.4
0.6
$10^3 L_4^r$
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
$10^3 L_6^r$
$p^2$
$p^4$
$a_0^{3/2}$
-0.035
-0.04
-0.045
-0.05
-0.055
-0.06
-0.065
-0.07
-0.075
-0.08
-0.4
$10^3 L_4^r$
$p^2$
BDM
$10^3 L_6^r$



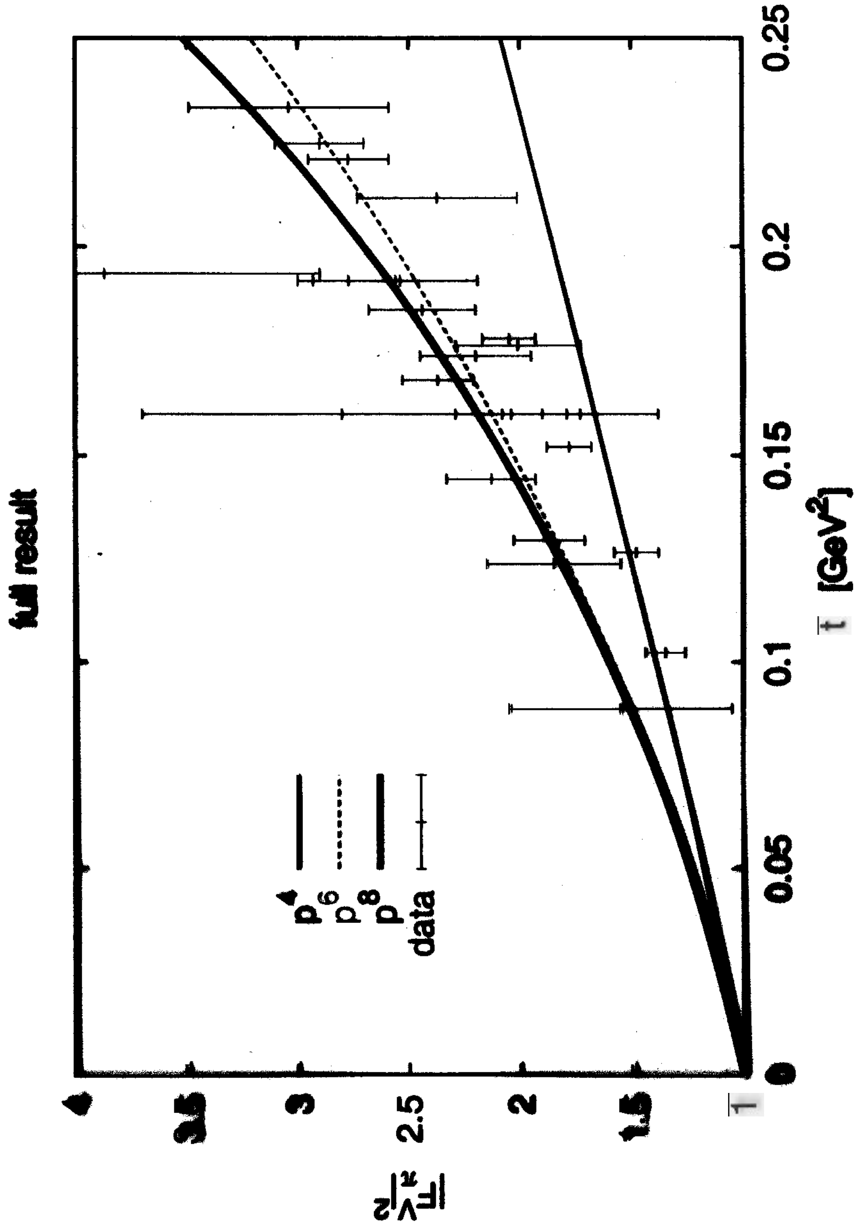




Preferred region

# Pion Electromagnetic Form Factor




Bubble chamber $\pi e \to \pi e$




$e^+e^- \to \pi^+\pi^-$ and $\tau^\pm \to \pi^\pm\pi^0\nu$

# $K_{\ell 3}$ Definitions and $V_{us}$

Scalar formfactor: $$f_0(t) = f_+(t) + \frac{t}{m_K^2 - m_\pi^2} f_-(t)$$

Usual parametrization: $$f_{+,0}(t) = f_+(0)\left(1 + \lambda_{+,0}\frac{t}{m_\pi^2}\right)$$

$|V_{us}|$:

- Know theoretically $f_+(0) = 1 + \cdots$
  - Short distance correction to $G_F$ from $G_\mu$ Marciano-Sirlin
  - Ademollo-Gatto-Behrends-Sirlin theorem: $(m_s - \hat{m})^2$
  - Isospin Breaking Leuwyler-Roos $\frac{f_+^{K^+\pi^0}(0)}{f_+^{K^0\pi^-}(0)} = 1.022$ In Progress
- Know experimentally $f_+(0)$
  - Radiative Corrections: use generalized formfactors Cirigliano et al., hep-ph/0110153
  - Parametrize form-factor: is linear enough for $f_+(t)$ ?

PDG2002: $|V_{ud}| = 0.9734 \pm 0.0008$ $\qquad$ $|V_{us}| = 0.2196 \pm 0.0026$

$$|V_{ud}|^2 + |V_{us}|^2 = (0.9475 \pm 0.0016) + (0.0482 \pm 0.0011) = 0.9957 \pm 0.0019$$

($V_{ud}$ from neutron decay only $\Rightarrow$ a bit worse)

$$f_+(t) = 1 + f_+^{(4)}(t) + f_+^{(6)}(t)$$

$$f_+^{(4)}(t) = \frac{t}{2F_\pi^2} L_9^r + \text{loops}$$

$$f_+^{(6)}(t) = -\frac{8}{F_\pi^4}(C_{12}^r + C_{34}^r)\left(m_K^2 - m_\pi^2\right)^2 + \frac{t}{F_\pi^4} R_{+1}^{K\pi} + \frac{t^2}{F_\pi^4}(-4C_{88}^r + 4C_{90}^r) + \text{loops}(L_i^r)$$

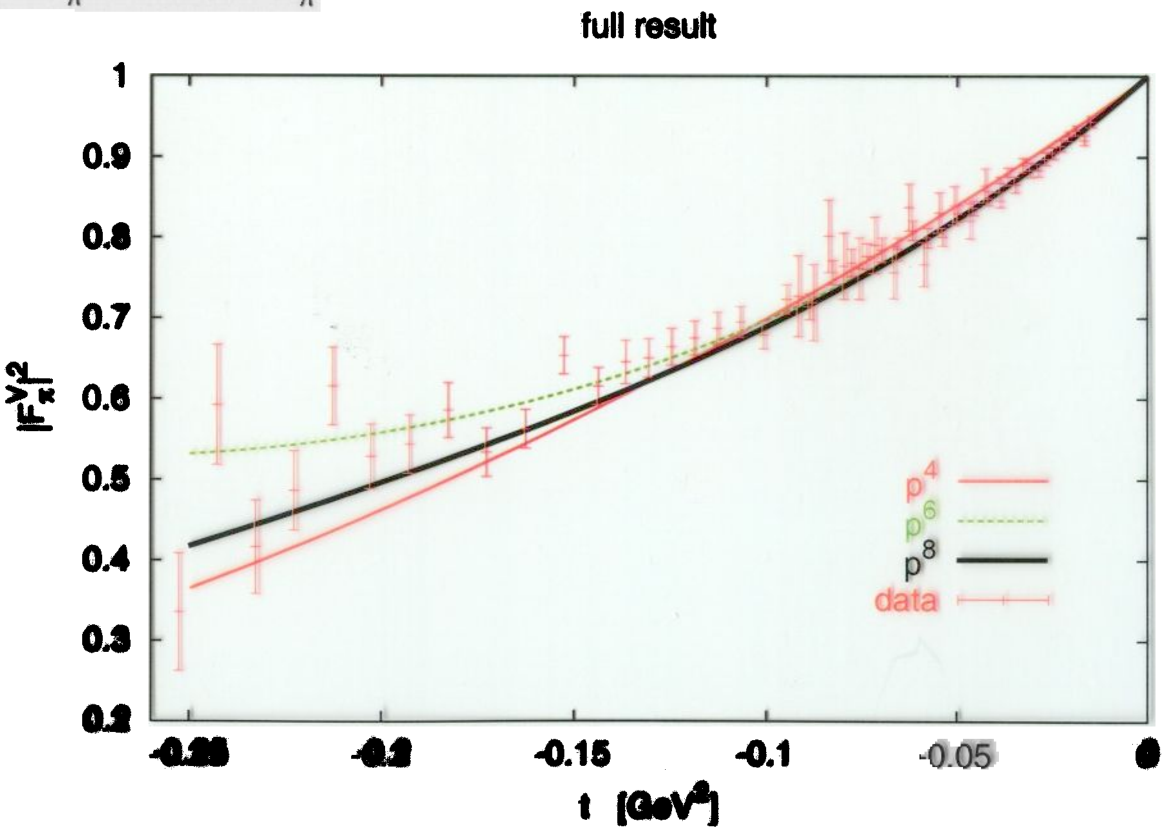
Pion electromagnetic Form factor:

JB, Talavera

$$L_9^r = 0.00593 \pm 0.00043$$

$$-4C_{88}^r + 4C_{90}^r = 0.00022 \pm 0.00002$$

VMD: $R_{+1}^{K\pi} \approx -4\ 10^{-5}$ GeV$^2$



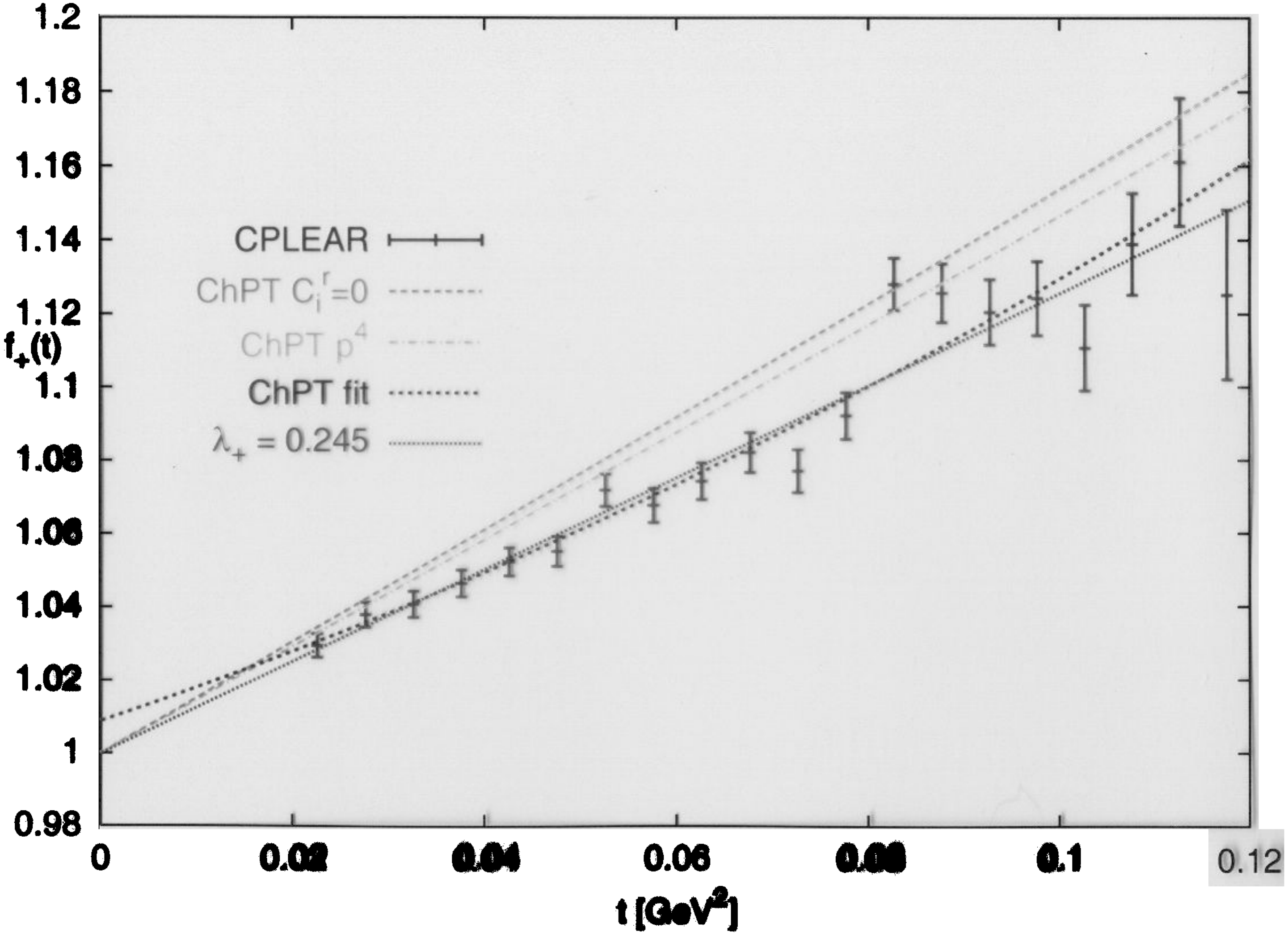
# ChPT fit to $f_+(t)$

$\Rightarrow R^{K\pi}_{+1} =$
$-(4.7 \pm 0.5)\ 10^{-5}\ \text{GeV}^2$

$\left(c_+ = 3.2\ \text{GeV}^{-4}\right)$

$\Rightarrow a_+ = 1.009 \pm 0.004$

$\Rightarrow \lambda_+ = 0.0170 \pm 0.0015$



# ChPT fit to $f_+(t)$

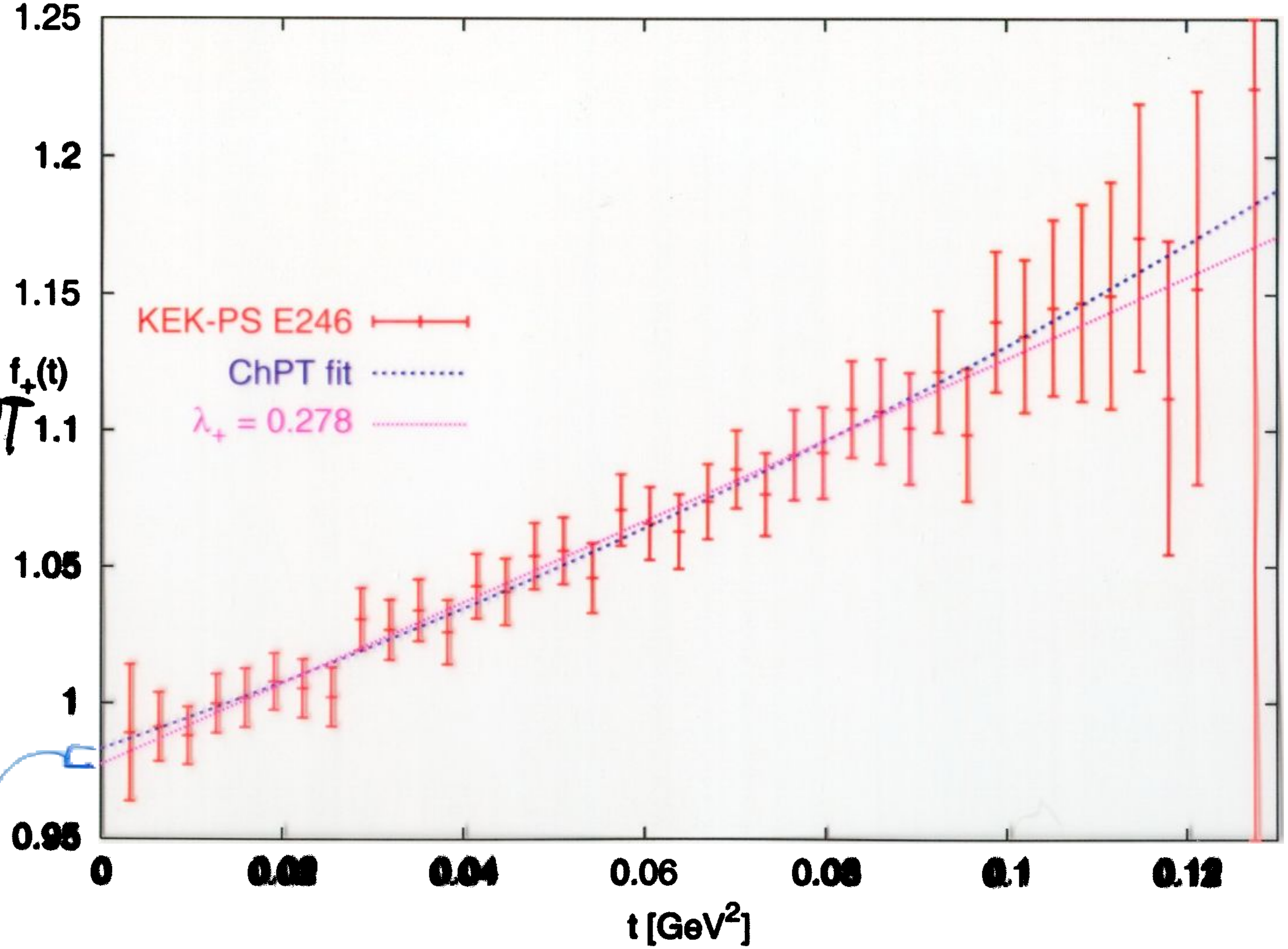
$\Rightarrow R^{K\pi}_{+1} =$
$-2.5\ 10^{-5}\ \text{GeV}^2$

$(c_+ = 3.2\ \text{GeV}^{-4})$ CURVATURE
↑ FIXED BY CHPT

$\Rightarrow a_+ = 1.006$

$\Rightarrow \lambda_+ = 0.0214 \pm 0.0018$

0.6 %



Main Result:

$$f_0(t) = 1 - \frac{8}{F_\pi^4}(C^r_{12} + C^r_{34})\left(m_K^2 - m_\pi^2\right)^2 + 8\frac{t}{F_\pi^4}(2C^r_{12} \quad C^r_{34})\left(m_K^2 + m_\pi^2\right) + \frac{t}{m_K^2 - m_\pi^2}(F_K/F_\pi - 1) - \frac{8}{F_\pi^4}t^2 C^r_{12} + \overline{\Delta}(t) + \Delta(0)$$

$\overline{\Delta}(t)$ and $\Delta(0)$ contain NO $C_i^r$ and only depend on the $L_i^r$ at order $p^6$

$\Longrightarrow$

All needed parameters can be determined experimentally

$$\Delta(0) = -0.0080 \pm 0.0057[\text{loops}] \pm 0.0028[L_i^r]\,.$$

$= -0.0224\ (p^4) + 0.0113\ (p^6 \text{ pure loop}) + 0.0033\ (p^6, L_i^r)$

# Conclusions

- 2 flavour ChPT at 2 loops (almost) finished subject
- 3 flavour ChPT at 2 loops
  - many calculations done
  - things seem to work but convergence is fairly slow (very bad for some $\pi K$ quantities)
  - "kinematical" and "vector" $C_i^r$ seem to be OK
  - $L_4, L_6$ nonzero but reasonable for $1/N_c$
  - $\eta \to 3\pi$, $K_{\ell 3}$ isobreaking: bits and pieces done
- PQChPT at 2 loops subject just beginning
- $K_{\ell 3}$ an example of results even with all the $C_i^r$