*"Matching Light Quarks to Hadrons"*

Benasque Center for Science, 2nd August 2004

# Weak Matrix Elements ($B_K$, $Q_8$, $\varepsilon'/\varepsilon$, etc.)

## -- which of these quantities will ever be calculated reliably?


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Based on recent work by ***Samuel Friot***, ***David Greynat*** and ***Oscar Catà,*** and on published work and many discussions with ***S. Peris, M. Knecht, M. Perrottet, T. Hambye, M. Golterman, A. Nyffeler*** and ***B. Phily***

$$\int d^4x\ e^{iq.x} \langle 0|T\left\{\bar{u}_L\gamma^\mu d_L(x)\ \bar{d}_R\gamma^\nu u_R(0)\right\}|0\rangle = \frac{1}{2i}\left(q^\mu q^\nu - q^2 g^{\mu\nu}\right)\underline{\Pi_{LR}(Q^2)}$$

# $\Pi_{LR}(Q^2)$ in Large Nc QCD

$$\Pi_{LR}(Q^2) = \sum_V \frac{f_V^2 M_V^2}{Q^2 + M_V^2} - \sum_A \frac{f_A^2 M_A^2}{Q^2 + M_A^2} - \frac{F_0^2}{Q^2}$$

No $1/Q^2$ term in OPE → $\sum_V f_V^2 M_V^2 - \sum_A f_A^2 M_A^2 - F_0^2 = 0$

*1st Weinberg Sum Rule*

No $1/Q^4$ term in OPE → $\sum_V f_V^2 M_V^4 - \sum_A f_A^2 M_A^4 = 0$

*2nd Weinberg Sum Rule*

In Large-$N_c$ QCD, for a finite number of narrow states, one can also write other sum rules which relate **Hadronic Masses** and **Couplings** to Local Order Parameters of the OPE and to Non-Local Order Parameters of the Chiral Lagrangian:

Matching $1/Q^6$ term to OPE → $\sum_V f_V^2 M_V^6 - \sum_A f_A^2 M_A^6 = \left[-4\pi\alpha_s + \mathcal{O}(\alpha_s^2)\right] \langle\bar{\psi}\psi\rangle^2$

Matching to $O(p^4)$ in ChPT → $\sum_V f_V^2 - \sum_A f_A^2 = -4L_{10}$

# The Effective Chiral Lagrangian

$$\mathcal{L}_{\rm eff} = \underbrace{\frac{1}{4}F_0^2 \ {\rm tr}\left(D_\mu U D^\mu U^\dagger\right)}_{\pi\pi\to\pi\pi\,,\quad K\to\pi e\nu} + \underbrace{L_{10}\ {\rm tr}\left(U^\dagger F_{R\mu\nu} U F_L^{\mu\nu}\right)}_{\pi\to e\nu\gamma} + \cdots$$

$$+e^2\ C\ \underbrace{{\rm tr}\left(Q_R U Q_L U^\dagger\right)}_{-\frac{2}{F_0^2}(\pi^+\pi^- + K^+K^-)} \quad \underbrace{-\frac{G_F}{\sqrt{2}} V_{ud}V_{us}^*\ g_8 F_0^4 \left(D_\mu U^\dagger D^\mu U\right)_{23}}_{K\to\pi\pi\,,\quad K\to\pi\pi\pi} \quad + \quad \cdots$$

Two Crucial Observations

- The low-energy constants of the *Strong Lagrangian*, like $F_0^2$ and $L_{10}$, are the coefficients of the *Taylor Expansion* of QCD Green's Functions.

- The low-energy constants of the *ElectroWeak Lagrangian*, like C and $g_8$, are *Integrals* of QCD Green's Functions.

# Large-$N_C$ QCD

**Green's Functions** $W(z)$ are **Meromorphic Functions** of $z = \frac{Q^2}{M_V^2}$:

$$W(z) = A_N \prod_{i=1}^{P} \frac{1}{(z+\rho_i)} \prod_{j=1}^{N} (z+\sigma_j)$$

If the OPE in QCD gives $W(z) \underset{z\to\infty}{\sim} z^{-\mathcal{P}_{OPE}}$ then, matching, requires:

$$\boxed{N - P = -\mathcal{P}_{OPE}}$$

Here

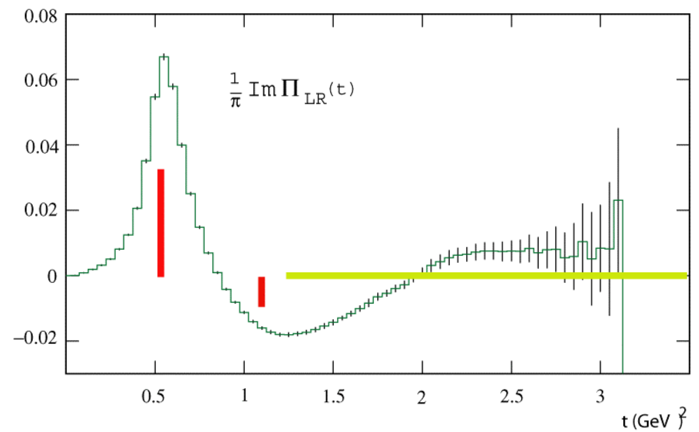
$$W(z) = -\frac{Q^2}{M_V^2}\Pi_{LR}(Q^2) \underset{z\to\infty}{=} -\frac{1}{2}\frac{\langle \mathcal{O}_6 \rangle}{M_V^6 z^2} - \frac{1}{2}\frac{\langle \mathcal{O}_8 \rangle}{M_V^8 z^4} + \cdots$$

Therefore

$$N - P = -2$$

*Minimal Hadronic Approximation* ⟷ $N = 0$

# The Real World versus Large-$N_c$ (MHA)

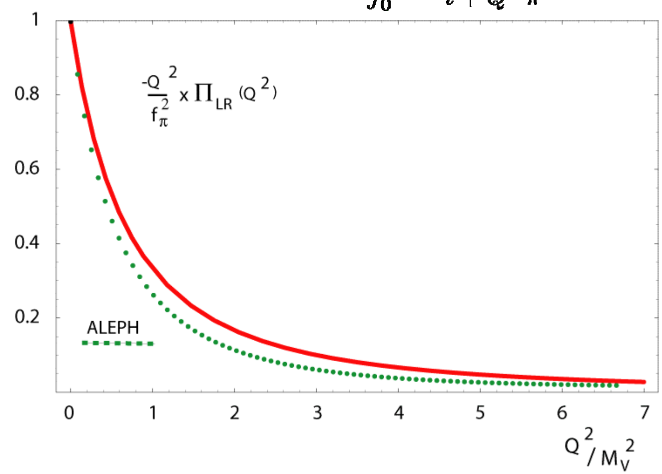



**M**iminimal **H**adronic **A**pproximation

$$\frac{-Q^2}{F_0^2}\Pi_{LR}(Q^2) = \frac{\rho_A}{(z+1)(z+\rho_A)}$$

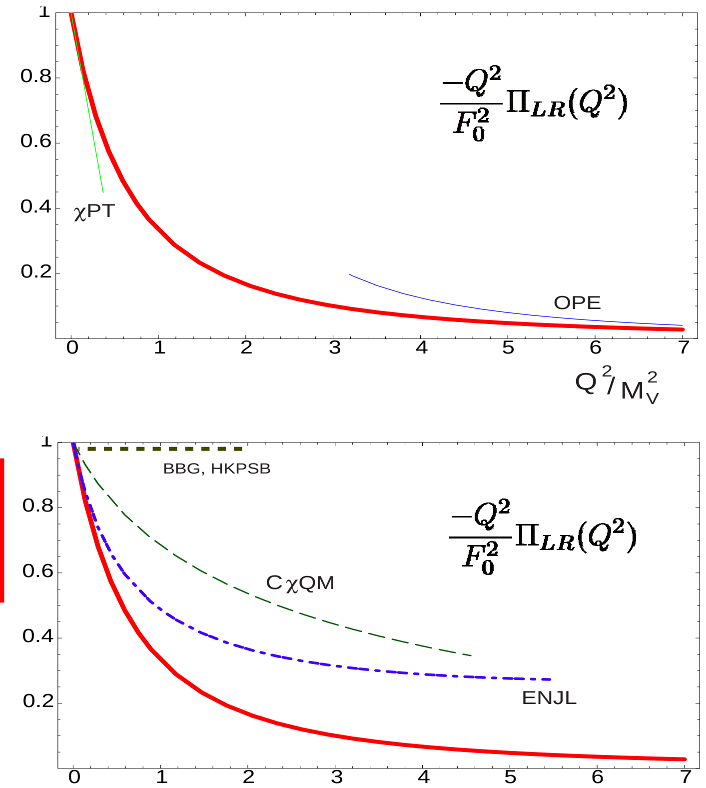
$$z = \frac{Q^2}{M_\rho^2} \quad \text{and} \quad \rho_A = \frac{M_A^2}{M_\rho^2}$$

$$\Pi_{LR}(Q^2) = \int_0^\infty dt \frac{1}{t+Q^2}\frac{1}{\pi}\mathrm{Im}\Pi_{LR}(t)$$



## The Large–$N_c$ Minimal Hadronic Approximation

### versus other *Analytic Approaches*




**Large-$N_c$ Framework**

**Bardeen et al '87**

**Resonance Dominance**

**Ecker et al '89**

BBG: *Bardeen, Buras, Gérard 87* HKPSB: *Hambye, Köhler, Paschos, Soldan, Bardeen '98*

CχQM: *Bertolini, Eeg, Fabbrichesi '00*

ENJL: *Bijnens, Prades '95, '00*

## A Simple Example of "short-distance" to "long-distance" Matching

In the Standard Model:

$$(m_{\pi^+}^2)_{\mathrm{ew}} = \frac{\alpha}{\pi}\frac{3}{4F_0^2}\int_0^\infty dQ^2\left(1-\frac{Q^2}{Q^2+M_Z^2}\right)\left(-Q^2\Pi_{LR}(Q^2)\right)$$

In the MHA to Large-$N_c$ QCD:

$$-Q^2\Pi_{LR}(Q^2) = F_0^2\frac{M_V^2 M_A^2}{(Q^2+M_V^2)(Q^2+M_A^2)}$$

# Integrating out the Heavy $Z$–Field

*Knecht–Peris–de Rafael '98*

$$\mathcal{L} \Rightarrow \mathcal{H}_{\rm eff} = -\frac{e^2}{M_Z^2}\underbrace{(\bar{q}_L\gamma_\mu Q_L q_L)(\bar{q}_R\gamma^\mu Q_R q_R)}_{Q_{LR}(M_Z^2)} + \cdots$$

Evolution from $M_Z^2$ to $\mu^2$, *(pQCD evolution)*

$$Q_{LR}(M_Z^2) \to \underbrace{Q_{LR}(\mu^2)}\; \underbrace{-\frac{3}{2}\frac{\alpha_s(\mu^2)}{\pi}\log\frac{M_Z^2}{\mu^2}\underbrace{\sum_{q,q'} e_q e_{q'}(\bar{q}'_L q_R)(\bar{q}'_R q_L)}_{D_{RL}(\mu^2)}}$$

Bosonization to $\mathcal{O}(N_c)$ and to $\mathcal{O}(p^0)$

$$\left[\int_0^\infty dQ^2\left(-\frac{Q^2}{M_Z^2}\right)\left(-Q^2\Pi_{LR}(Q^2)\right)\right]_{\overline{\rm MS}}$$

$$-\frac{3}{2}\log\frac{M_Z^2}{\mu^2}\;\underbrace{\frac{\alpha_s(\mu^2)}{4\pi}\langle\bar\psi\psi\rangle^2}_{\frac{1}{16\pi^2}\left(\sum f_A^2M_A^6-\sum f_V^2M_V^6\right)}\;{\rm tr}(UQ_LU^\dagger Q_R)$$

Mesonic Evolution of $\underbrace{Q_{LR}(\mu^2)}$ to $\mathcal{O}(N_c)$ and to $\mathcal{O}(p^0)$

$$\frac{3}{32\pi^2}\left(\sum f_A^2M_A^6\log\frac{M_A^2}{\mu^2}-\sum f_V^2M_V^6\log\frac{M_V^2}{\mu^2}\right){\rm tr}(UQ_LU^\dagger Q_R)$$

- *Short–Distance* scale $\mu^2$ and *Long–Distance* scale $\mu^2$ Cancel
- Restriction to the *MHA approximation* $\Rightarrow$ previous result

# Methodology

1. Identify the underlying QCD Green's Functions.

2. Work out the leading OPE behavior of the relevant Green's Functions and their leading ChPT behavior.

3. Introduce the *Minimal Hadronic Approximation (MHA)* to Large-Nc QCD for the relevant Green's Functions ( $N - P = -\mathcal{P}_{OPE}$ )

4. The *MHA* is, "in principle", improvable: more ChPT constraints and more OPE constraints.

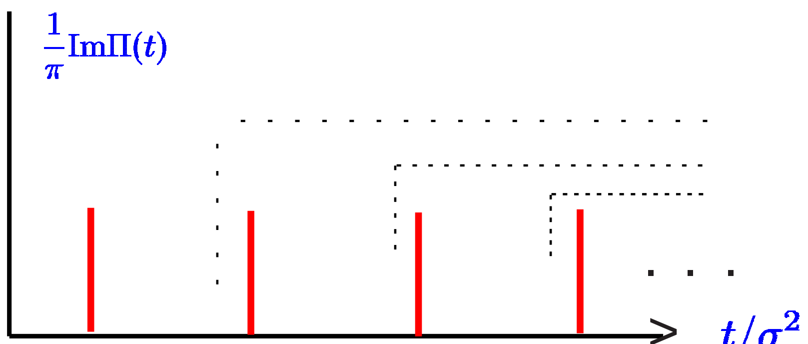
# Large-$N_c$ Toy Model *(Shifman, Golterman et al '01)*

Spectral Function:

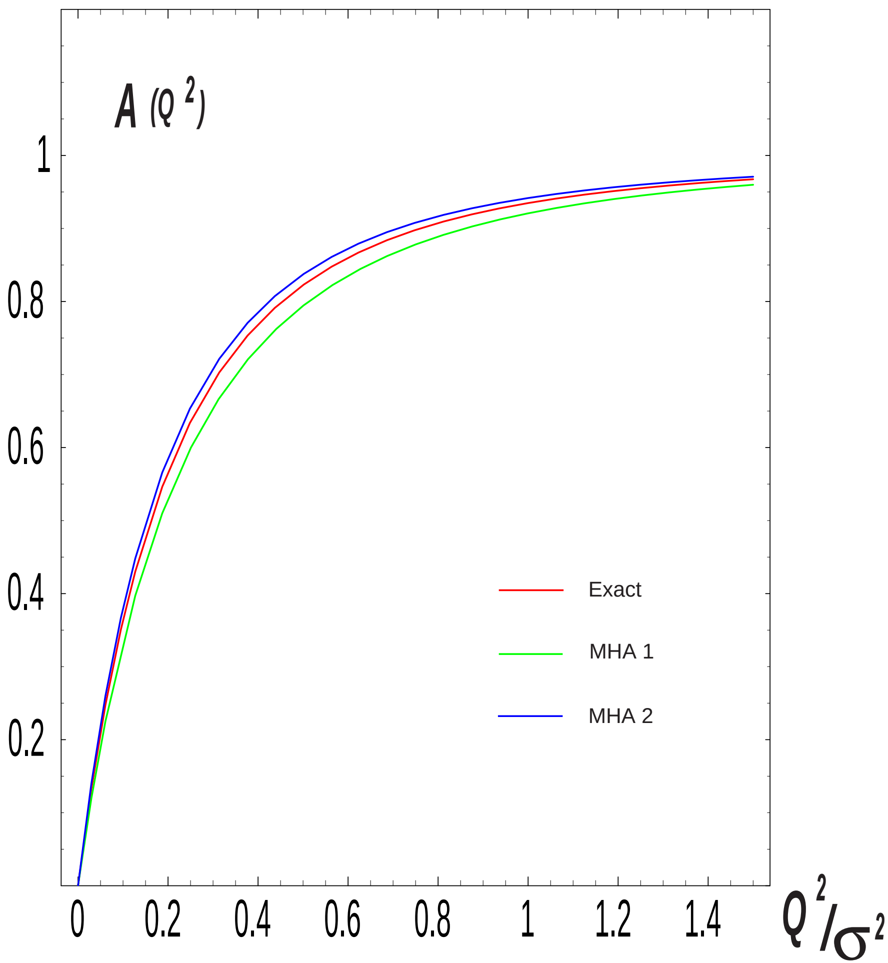
$$\frac{1}{\pi}\mathrm{Im}\Pi(t) = A\sigma^2 \sum_{n=0}^{\infty} \delta(t - M_0^2 - n\sigma^2)$$

Adler Function:

$$\mathcal{A}(Q^2) = \int_0^\infty dt \frac{Q^2}{(t+Q^2)^2} \frac{1}{\pi}\mathrm{Im}\Pi(t) = A\frac{Q^2}{\sigma^2}\zeta\left(2, \frac{Q^2+M_0^2}{\sigma^2}\right)$$



# Toy Model's Adler Function



# The $B_K$–Factor of $K^0 - \bar{K}^0$ Mixing

$$\mathcal{L} \Rightarrow \cdots C_{\Delta S=2}(\mu) \underbrace{(\bar{s}_L \gamma^\mu d_L)(\bar{s}_L \gamma_\mu d_L)(x)}_{Q_{\Delta S=2}(x)}$$

- Definition: $< \bar{K}^0 | Q_{\Delta S=2}(0) | K^0 > \equiv \frac{4}{3} f_K^2 M_K^2 \underline{B_K}(\mu)$

- To lowest order in the chiral expansion, $\mathcal{O}(p^2)$:

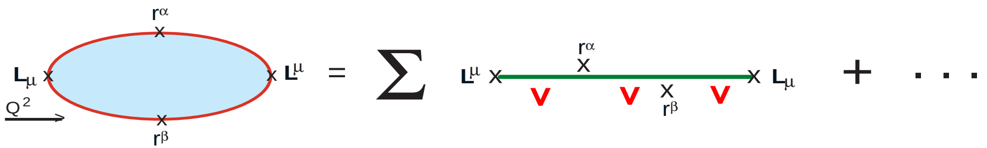
$$Q_{\Delta S=2}(x) \Rightarrow -\frac{F_0^4}{4} \underline{g_{\Delta S=2}}(\mu) \underbrace{\mathrm{tr}\left[\lambda_{32}(D^\mu U^\dagger) U \lambda_{32} (D_\mu U^\dagger) U\right]}_{\frac{2}{F_0^2}\partial^\mu K^0 \partial_\mu K^0 + \cdots - \underbrace{\mathrm{tr}\left[\lambda_{32} r^\mu \lambda_{32} r_\mu\right]} + \cdots}$$

- Definition of the invariant $\hat{B}_K$–factor, *in the chiral limit*

$$\hat{B}_K = \frac{3}{4} C_{\Delta S=2}(\mu) \times \underline{g_{\Delta S=2}}(\mu)$$

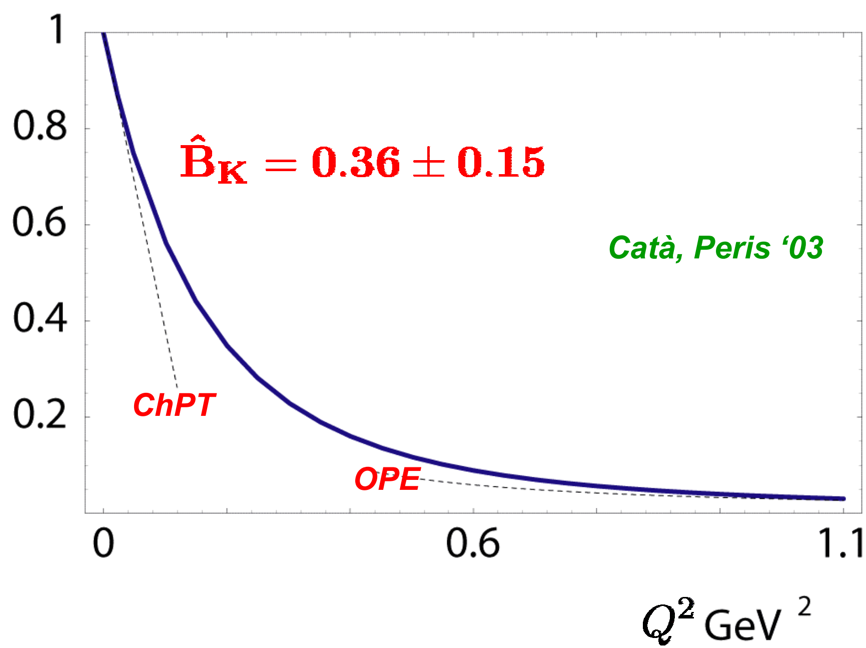
- Underlying QCD Green's Function

*Peris, de Rafael '00; Catà, Peris '03*




$$\underline{g_{\Delta S=2}}(\mu) = 1 - \frac{1}{32\pi^2 F_0^2} \left[ \frac{(4\pi\mu^2)^{\epsilon/2}}{\Gamma(2-\epsilon/2)} \int_0^\infty \frac{dQ^2}{\left(Q^2\right)^{\epsilon/2}} W_{LRLR}(Q^2) \right]_{\overline{\mathrm{MS}}}$$

$Q^2 W_{LRLR}(Q^2)$ normalized at $Q^2 = 0$




★ Reproduces, within errors, $\Gamma[K^+ \to \pi^+ \pi^0]_{exp.}$ *(Donoghue, Golowich, Holstein '82)*

★ Correlated to $\Delta I = 1/2$ Enhancement *(Pich, de Rafael '96)*

# Matrix Elements of ElectroWeak Penguin-like Operators

$$Q_7 = 6(\bar{s}_L\gamma^\mu d_L)\sum_{q=u,d,s} e_q(\bar{q}_R\gamma_\mu q_R) \quad \text{and} \quad Q_8 = -12\sum_{q=u,d,s} e_q(\bar{s}_L q_R)(\bar{q}_R d_L)$$

## *Relation to* $\Pi^{\mu\nu}_{LR}(q)$ *for* $Q_7$

$$\langle(\pi\pi)_{I=2}|Q_7|K^0\rangle = -\frac{4}{F_0^3}\langle O_1\rangle \qquad \textit{with} \qquad \langle O_1\rangle \equiv \langle 0|(\bar{s}_L\gamma^\mu d_L)(\bar{d}_R\gamma_\mu s_R)|0\rangle$$

$$\langle O_1\rangle = \frac{1}{6}\left(-3ig_{\mu\nu}\int\frac{d^4q}{(2\pi)^4}\Pi^{\mu\nu}_{LR}(q)\right)_{\overline{\rm MS}}$$

## *Relation for* $Q_8$

$$\lim_{Q^2\to\infty}\left(-Q^2\Pi_{LR}(Q^2)\right)\times Q^4 = 4\pi^2\frac{\alpha_s}{\pi}\left(4\langle O_2\rangle + \frac{2}{N_c}\langle O_1\rangle\right) + \mathcal{O}\left(\frac{\alpha_s}{\pi}\right)^2$$

$$\langle(\pi\pi)_{I=2}|Q_8|K^0\rangle = \frac{8}{F_0^3}\langle O_2\rangle$$

*Donoghue, Golowich '00; Knecht, Peris, de Rafael '98, '01; ...*

$$\langle O_2(\mu)\rangle \simeq \frac{1}{16\pi\alpha_s(\mu)}\left(\sum_A f_A^2 M_A^6 - \sum_V f_V^2 M_V^6\right)\times\left[1-\left(\begin{array}{c}25/8\ (\text{NDR})\\ 21/8\ (\text{HV})\end{array}\right)\frac{\alpha_s}{\pi}\right]$$

## Comparison with other Phenomenological Determinations

$M_7$ and $M_8$ at $\mu = 2$ GeV

| | $\langle O_6 \rangle$ $\times 10^3 \mathrm{GeV}^6$ | $\langle O_8 \rangle$ $\times 10^3 \mathrm{GeV}^8$ | $M_7$ $(\mathrm{GeV}^3)$ | | $M_8$ $(\mathrm{GeV}^3)$ | |
|---|---|---|---|---|---|---|
| | | | NDR | HV | NDR | HV |
| MHA+V' **new!** **(preliminary)** | **- 7.9 ±0.5 ±1.6** | **+11.7 ±1.1 ±2.5** | **0.12 ±0.00 ±0.01** | **0.59 ±0.02 ±0.06** | **2.00 ±0.09 ±0.20** | **2.15 ±0.10 ±0.22** |
| MHA | - 9.5 ± 3 | +16.2 ± 5 | 0.11 ± 0.03 | 0.67 ± 0.20 | 2.3 ± 0.7 | 2.5 ± 0.8 |
| Aleph | - 7.7 ± 0.8 | +11 ± 1 | | | | |
| Opal | - 6.0 ± 0.6 | +7.5 ± 1.3 | | | | |
| Davier *et al.* | - 6.4 ± 1.6 | +8.7 ± 2.4 | | | | |
| Ioffe *et al.* | - 6.8 ± 2.1 | +7 ± 4 | | | | |
| Zyablyuk | - 7.1 ± 1.5 | + 7.8 ± 3.0 | | | | |
| Bijnens *et al.* | - 3.2 ± 2.0 | **-12.4 ±9.0** | 0.24 ± 0.03 | 0.37 ± 0.08 | 1.2 ± 0.8 | 1.3 ± 0.8 |
| Cirigliano *et al.* | - 4.45 ± 0.7 | **- 6.2 ± 3.2** | 0.22 ± 0.05 | | 1.5 ± 0.3 | |
| Rojo-Latorre | - 4 ± 2 | **- 12 $^{+7}_{-11}$** | | | | |
| Narison | | | 0.21 ± 0.05 | | 1.4 ± 0.3 | |

# Comparison with Lattice Results

$M_7$ and $M_8$ at $\mu = 2$ GeV

| | $M_7$ (GeV$^3$) | | $M_8$ (GeV$^3$) | |
|---|---|---|---|---|
| | NDR | HV | NDR | HV |
| MHA+V' **(preliminary!)** (Friot-Greynat-de Rafael) | **0.12 ± 0.00 ±0.01** | **0.59 ±0.02 ±0.06** | **2.00 ±0.09 ±0.20** | **2.15 ±0.10 ±0.22** |
| MHA | 0.11 ± 0.03 | 0.67 ± 0.20 | 2.3 ± 0.7 | 2.5 ± 0.8 |
| Bhattacharya *et al.* | 0.32 ± 0.06 | | 1.2 ± 0.2 | |
| Donini *et al.* | 0.11 ± 0.04 | 0.18 ± 0.06 | 0.51 ± 0.1 | 0.62 ± 0.12 |
| RBC - *Collaboration* | 0.27 ± 0.03 | | 1.1 ± 0.2 | |
| CP-PACS -*Collaboration* | 0.24 ± 0.03 | | 1.0 ± 0.2 | |
| SPQCDR | 0.24 ± 0.02 | | 1.05 ±0.10 | |

# $\Pi_{LR}$ in the case of a $\pi - V - A - V'$ Spectrum

$$-\frac{Q^2}{M_V^2}\Pi_{LR}(Q^2) \equiv W[z] = A_1 \frac{z+\sigma}{(z+1)(z+\rho_A)(z+\rho_{V'})},$$

$$A_1 \frac{\sigma}{\rho_A \rho_{V'}} = \frac{F_0^2}{M_V^2} \equiv \rho_F \qquad \text{and} \qquad \langle \mathcal{O}_6 \rangle = -M_V^6 \frac{2}{\sigma} \rho_F \rho_A \rho_{V'}\,.$$

Linear Constraint $\qquad \sigma - (1+\rho_A+\rho_{V'}) = \frac{1}{M_V^2}\frac{\langle \mathcal{O}_8 \rangle}{\langle \mathcal{O}_6 \rangle}$

Slope Constraint $\qquad 4L_{10} = \rho_F \left[\frac{1}{\sigma} - \left(1 + \frac{1}{\rho_A} + \frac{1}{\rho_{V'}}\right)\right]$

**Spectral Functions**

$$\frac{1}{\pi}\mathrm{Im}\Pi_A(t) = F_0^2\delta(t) + F_0^2\frac{\rho_{V'}}{\sigma}\frac{\sigma-\rho_A}{(\rho_A-1)(\rho_{V'}-\rho_A)}\delta(t-M_A^2) + \frac{N_c}{16\pi^3}\frac{2}{3}\theta(t-s_0)(1+\cdots)$$

$$\frac{1}{\pi}\mathrm{Im}\Pi_V(t) = F_0^2\frac{\rho_A}{\sigma}\left\{\frac{\rho_{V'}(\sigma-1)}{(\rho_A-1)(\rho_{V'}-1)}\delta(t-M_V^2) + \frac{\sigma-\rho_{V'}}{(\rho_{V'}-1)(\rho_{V'}-\rho_A)}\delta(t-M_{V'}^2)\right\} + \frac{N_c}{16\pi^3}\frac{2}{3}\theta(t-s_0)(1+\cdots)$$

# Input Quantities

$$\frac{m_{\pi^+} + m_{\pi^0}}{M_\rho^2}\delta m_\pi \qquad \frac{1}{M_\rho}\Gamma_{\rho\to e^+e^-} \qquad \frac{1}{M_\rho}\Gamma_{A\to\pi\gamma} \qquad \frac{F_0^2}{M_\rho^3}\Gamma_{\rho\to\pi\pi}$$

$$L_{10} = (-5.13 \pm 0.19) \times 10^{-3} \qquad L_9 = (6.9 \pm 0.7) \times 10^{-3}$$

$$\rho_F = \frac{F_0^2}{M_\rho^2} = (12.4 \pm 0.5) \times 10^{-3} \qquad \rho_A = 1.47 \pm 0.02$$

$$\rho_{V'} = 2.63 \pm 0.01 \qquad \sigma = 2.64 \pm 0.01$$

$$\chi^2_{\mathrm{min}}/d.o.f. = 0.60/1$$

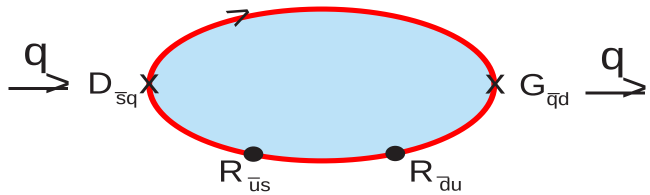
(Statistical errors only!)

$M_V = 776$ MeV → $F_0 \simeq 86$ MeV, $M_A \simeq 939$ MeV, $M_{V'} \simeq 1258$ MeV, $s_0 \simeq 1.3$ GeV

# Bosonization of QCD–Penguins

**to $\mathcal{O}(N_c^2)$ and $\mathcal{O}(N_c^2 \frac{n_f}{N_c})$**

*Hambye, Peris, de Rafael '03*




$$g_{\underline{8}}|_{Q_6,Q_4} = C_6(\mu)\left\{ -16 L_5 \frac{\langle\bar{\psi}\psi\rangle^2}{F_0^6} + \right.$$

$$\frac{8n_f}{16\pi^2 F_0^4}\left[\frac{(4\pi\mu^2)^{\epsilon/2}}{\Gamma(2-\epsilon/2)}\underbrace{\int_0^\infty dQ^2 (Q^2)^{1-\epsilon/2}\mathcal{W}_{DG}(Q^2)}_{\sim B^2F_0^2}\right]_{\overline{\text{MS}}}$$

$$\left. +C_4(\mu)\left\{1 - \frac{4n_f}{16\pi^2 F_0^4}\left[\frac{(4\pi\mu^2)^{\epsilon/2}}{\Gamma(2-\epsilon/2)}\underbrace{\int_0^\infty dQ^2 (Q^2)^{1-\epsilon/2}\mathcal{W}_{LL}(Q^2)}_{\sim F_0^2M_R^2}\right]_{\overline{\text{MS}}}\right\}\right.$$

- Notice that $\qquad \frac{\text{unfactorized}}{\text{factorized}} \sim \frac{M_R^2}{16\pi^2 F_0^2} \qquad$ !!

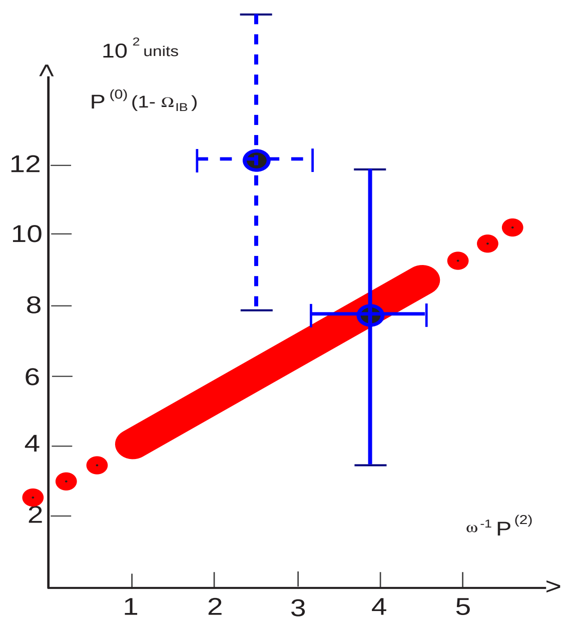
# Implications for $\epsilon'/\epsilon$

$$\mathrm{Re}(\epsilon'/\epsilon)_{\mathrm{exp.}} = (1.66 \pm 0.16) \times 10^{-3}$$

World average from NA31, NA48, KTeV

$$\frac{\varepsilon'}{\varepsilon} = \underbrace{\mathrm{Im}\left(V_{ts}^* V_{td}\right) \frac{G_{\mathrm{F}}\omega}{2|\varepsilon|\left|\mathrm{Re}A_0\right|}}_{0.055\pm0.008} \left[ P^{(0)} \underbrace{(1 - \Omega_{\mathrm{IB}})}_{0.94\pm0.08} - \underbrace{\frac{1}{\omega}}_{22.2} P^{(2)} \right]$$

$$P^{(I)} = \sum_i y_i(\mu) \langle (\pi\pi)_I | Q_i(\mu) | K^0 \rangle\,, \quad \text{for} \quad I = 0, 2$$



# Conclusions

★ **$B_K$ in the Chiral Limit:**
**Good agreement among Analytic Approaches**
**Good** prospects for an $O(p^4)$ Calculation

★ **Physics related to the LR-Correlator ($Q_7$ and $Q_8$):**
**Problems** when comparing MHA with ***Certain*** Phenomenological Determinations and with ***Most*** of the Lattice Results
**Very Good Stability** of the MHA results when a **V'** is added **!**

★ **The Large-Nc approach has shown that:**
**Unfactorized** contributions (four-quark operators) are **Large**
Slow progress towards "convincing" $\Delta I = 1/2$ **and** $\epsilon'/\epsilon$

★ **Good** prospects for **Rare Kaon Decay Physics** within the **Large-Nc Framework**

# The Onset of the pQCD Continuum

The Vector Spectral Function:

$$\frac{1}{\pi}\mathrm{Im}\Pi_V(t) = F_0^2\frac{\rho_A}{\sigma}\left\{\frac{\rho_{V'}(\sigma-1)}{(\rho_A-1)(\rho_{V'}-1)}\delta(t-M_V^2)+\frac{\sigma-\rho_{V'}}{(\rho_{V'}-1)(\rho_{V'}-\rho_A)}\delta(t-M_{V'}^2)\right\} \;+\frac{1}{\pi}\mathrm{Im}\Pi(t)_{\mathrm{pQCD}}\,\theta(t-s_0)$$

**Duality constraint** from the requirement that the **Adler function**:

$$\mathcal{A}(Q^2) = \int_0^\infty dQ^2\frac{Q^2}{(Q^2+t)^2}\frac{1}{\pi}\mathrm{Im}\Pi_V(t)$$

has no $1/Q^2$ term in the **OPE** (*in the chiral limit*)

$$\rho_F\left(1+\frac{\rho_{V'}}{\sigma}\frac{\sigma-\rho_A}{(\rho_A-1)(\rho_{V'}-\rho_A)}\right) = \frac{N_c}{16\pi^2}\frac{2}{3}\frac{s_0}{M_V^2}\left(1+\frac{3}{8}\frac{N_c\alpha_s(s_0)}{\pi}+\cdots\right)$$