

The status of the scalar channel in chiral dynamics

Sébastien Descotes-Genon

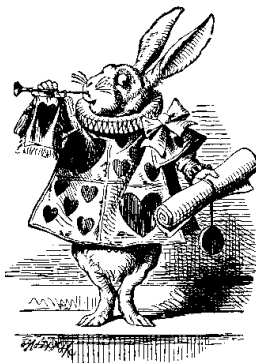
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or

A chiralist's adventures in Scalarland



Contents

The scalar exception

Scalars and the QCD vacuum

The spectrum of the Dirac operator

Consequences for three-flavour χ PT

Conclusions

Scalars : simple, elusive ...

Conceptually, scalars are very simple objects

- ▶ Simplest under space-time transformations
- ▶ Quantum numbers of the vacuum

But the light scalars do not seem to behave very nicely

- ▶ No scalar source provided by nature (\neq vector, axial)
- ▶ Mixture of broad and narrow resonances, $\sigma(\sim 600)$, $f_0(980) \dots$
- ▶ Hard to “interpret” : $q\bar{q}$, $q\bar{q}q\bar{q}$, $K\bar{K}$ molecule, glueballs. ...

...and hard to put together again

Scalar “multiplet” completely baffling

$I = 0$: $\sigma(\sim 600)$, $f_0(980)$, $f_0(1370)$, $f_0(1500) \dots$

$I = 1$: $a_0(980)$, $a_0(1450) \dots$ $I = 1/2$: $\kappa(\sim 800)$, $K_0^*(1430) \dots$

Mass splittings ? Gell-Mann-Okubo formula ? Ideal mixing ?

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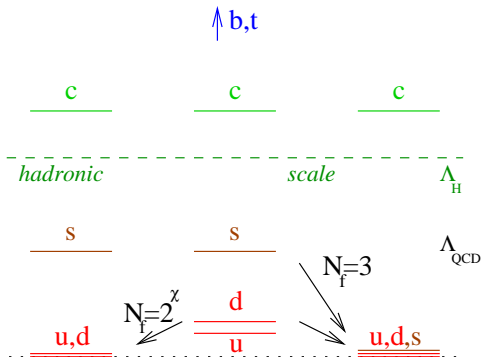
*Humpty Dumpty sat on a wall
Humpty Dumpty had a great fall
All the king's horses and all the king's men
Couldn't put Humpty together again*

Scalar channel has the quantum numbers of the vacuum



Nonperturbative properties of QCD vacuum
related to all this scalar nonsense ?

Three chiral limits of interest



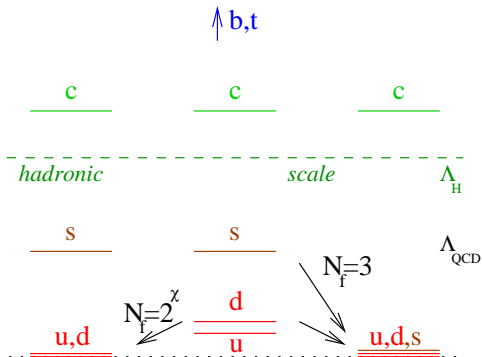
$$m_u, m_d \rightarrow 0$$

$$N_f = 3 : m_s \rightarrow 0$$

$$N_f = 2^\chi : m_s \text{ physical}$$

$$N_f = 2^{\text{lat}} : \text{no dynamical } s$$

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Two versions
of χ PT

$$N_f = 2^\chi : \pi \text{ only d.o.f} \quad (\text{few param. \& processes})$$

$$N_f = 3 : \pi, K, \eta \text{ d.o.f} \quad (\text{more param. \& processes})$$

From 2 to 3 massless flavours

$$\Sigma(2; m_s) = \lim_{m_u, m_d \rightarrow 0} -\langle 0 | \bar{u} u | 0 \rangle \quad \left\{ \begin{array}{ll} \Sigma(3) & = \Sigma(2; 0) \\ \Sigma(2^\chi) & = \Sigma(2; m_s^{\text{phys}}) \\ \Sigma(2^{\text{lat}}) & = \Sigma(2; \infty) \end{array} \right.$$

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$$\Sigma(2^x) = \Sigma(3) + m_s^{\text{phys}} \lim_{m_u, m_d \rightarrow 0} i \int d^4 x \langle 0 | \bar{u} u(x) \bar{s} s(0) | 0 \rangle + O(m_s^2)$$

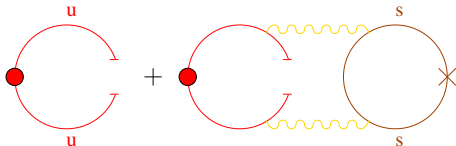
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$\Sigma(2^x)$ contains

- ▶ A “genuine” condensate $\Sigma(3)$
- ▶ An “induced” condensate $m_s \times$ (scalar N_c -suppressed)
[effect from sea $s\bar{s}$ -pairs]



$$\Sigma(2^x)$$

In the isospin limit $m = m_u = m_d$

$$SU(2^x) : F_\pi^2 M_\pi^2 = 2m\Sigma(2^x) + O(m^2)$$

$$SU(3) : F_\pi^2 M_\pi^2 = 2m\Sigma(3) + O(m_s^2, mm_s, m^2)$$

$$\Sigma(2^\chi)$$

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$\pi\pi$ scattering data from K_{e4}^+ decay
+ analyticity, unitarity, crossing

E865 Brookhaven (2001)
B.Ananthanarayan et al.

$$\frac{2m\Sigma(2^\chi)}{F_\pi^2 M_\pi^2} = 0.81 \pm 0.08$$

SDG et al.

... even larger G.Colangelo et al.

$\implies N_f = 2^\chi$ χ PT framework experimentally confirmed

$$\Sigma(2^\chi) - \Sigma(3)$$

$$\Sigma(2^\chi) - \Sigma(3) \propto \Pi(0) \quad \Pi(k^2) = \lim_{m_u, d \rightarrow 0} \int d^4x \, e^{-ik \cdot x} \langle \bar{u}u(x) \, \bar{s}s(0) \rangle_c$$

$\Pi(0)$ from superconvergent sum rule

B. Moussallam

$$\text{Im } \Pi(s) \propto \sum_{\sigma} \sqrt{\frac{s - M_{\sigma}^2}{s}} \theta(s - M_{\sigma}^2) \langle 0 | \bar{u}u | \sigma \rangle \langle \sigma | \bar{s}s | 0 \rangle \quad \sigma = (\pi\pi, K\bar{K}, \dots)_{I=0}^{\ell=0}$$

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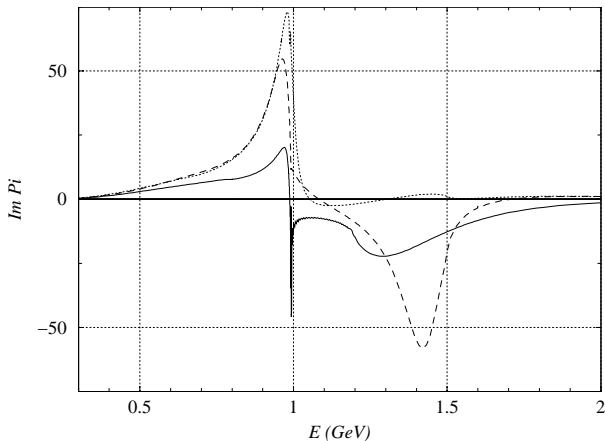
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- ▶ Requires π and K scalar form factors...
- ▶ ...in some cases, out of the physical region

\implies Models for the interactions in $\pi\pi$ and $K\bar{K}$ channels



Various **models** for
the **scalar** spectral
function $\text{Im } \Pi$
suggest

- ▶ $f_0(980)$ strongly coupled to nonstrange and strange scalar densities
- ▶ $\langle (\bar{u}u)(\bar{s}s) \rangle$ large and positive
- ▶ $\Sigma(3)$ could be as small as $\Sigma(2^\chi)/2$

B. Moussallam, SDG

$\Sigma(3) :$

$$\underbrace{\langle \bar{u}u \rangle|_{m_u=m_d=0, m_s \text{ phys}}}_{\text{sizeable } \Sigma(2^\chi)} = \underbrace{\langle \bar{u}u \rangle|_{m_u=m_d=m_s=0}}_{\Sigma(3)} + m_s \langle (\bar{u}u)(\bar{s}s) \rangle + \dots$$

$\Sigma(3)$: a fat cat,

$$\underbrace{\langle \bar{u}u \rangle|_{m_u=m_d=0, m_s \text{ phys}}}_{\text{sizeable } \Sigma(2^\chi)} = \underbrace{\langle \bar{u}u \rangle|_{m_u=m_d=m_s=0}}_{\Sigma(3)} + m_s \langle (\bar{u}u)(\bar{s}s) \rangle + \dots$$



$\Sigma(3) \simeq \Sigma(2^\chi)$
 $\langle (\bar{u}u)(\bar{s}s) \rangle$ small
 Large- N_c picture

$\Sigma(3)$: a fat cat, or just a grin ?

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$\Sigma(3) < \Sigma(2^\chi)$
 $\langle (\bar{u}u)(\bar{s}s) \rangle$ large
 Big Zweig-rule violation

Well, I understand large- N_c pretty well. . .

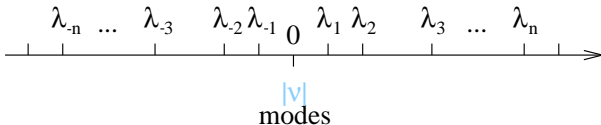


But how could the other alternative occur ?

The Dirac operator

In Euclidean QCD on a torus L^4 , the Dirac operator can be diagonalised

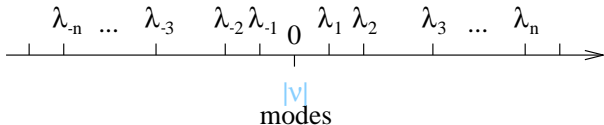
$$H[G] \equiv \not{D} = \gamma_\mu (\partial_\mu + i G_\mu) \quad H\phi_n = \lambda_n[G]\phi_n$$



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A correlation function expressed as a statistical average over G

$$\langle\langle \Gamma \rangle\rangle \propto \int dG \, e^{-S_{YM}[G]} \prod_j \Delta(m_j|G) \, \Gamma$$

with the fermionic determinant

$$\Delta(m_j|G) \propto m^{|\nu[G]|} \prod_{n>0} (m_j^2 + \lambda_n^2)$$

Order parameters and IR Dirac spectrum

Scalar density $\bar{q}q$	Eigenvalue density $\rho(\lambda) = \sum_n \delta(\lambda - \lambda_n[G])$
$\Sigma(N_f)$	Average e.v. density around 0
$\langle(\bar{u}u)(\bar{s}s)\rangle$	Fluctuation of e.v. density around 0

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For order parameters dominated by lowest Dirac e.v. like $\Sigma(N_f)$

- ▶ Dependence on m_s through fermionic determinant $\Delta(m_s|G)$
- ▶ IR end of fermionic determinant increasing function of m_s

$$\Delta_{IR}(m_s|G) \propto m_s^{|\nu[G]|} \prod_{n>0}^K (m_s^2 + \lambda_n^2)$$

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$$\Sigma(2,0) < \Sigma(2, m_s)$$

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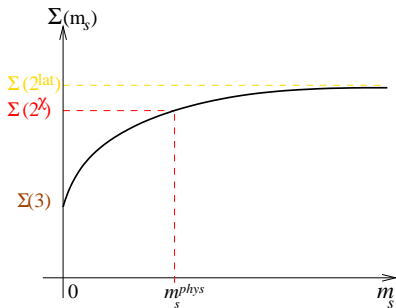
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$$\Sigma(3) < \Sigma(2^x)$$

Decrease of Σ due to sea $s\bar{s}$ -pairs
(similar effect for $F^2 = \lim_{N_f} F_\pi^2$)

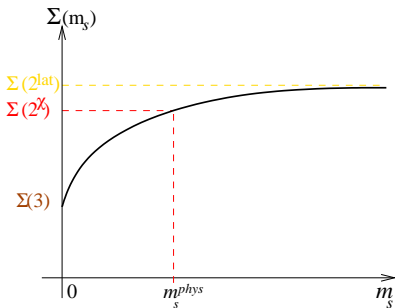
Vacuum fluctuations of $s\bar{s}$ pairs



$$\Sigma(2^x) \sim \underbrace{\Sigma(3)}_{\text{Mean}} + \underbrace{\langle \bar{u}u \bar{s}s \rangle}_{\text{Fluctuations}}$$

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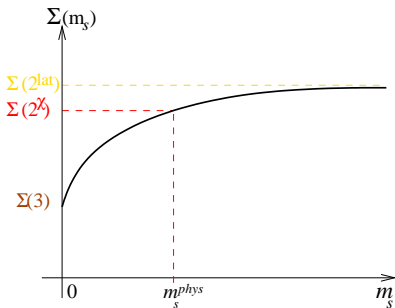
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Mean-field

- ▶ Large average, small fluct.
- ▶ Zweig rule not violated in 0^+
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Near a critical point

- ▶ Small average, large fluct.
- ▶ Large violation of Zweig rule
- ▶ Strange sea quarks important

$$\Sigma(3) < \Sigma(2^x)$$

$\Sigma(3)$ might be smaller than usually assumed. . .



So what ?

Convergence and instabilities

$$F_\pi^2 M_\pi^2 = 2m\Sigma(3) + 64m(m_s + 2m)B_0^2 \Delta L_6 + 64m^2 B_0^2 \Delta L_8 + O(m_q^2)$$

- ▶ $B_0 = -\lim_{m_u, m_d, m_s \rightarrow 0} \langle \bar{u}u \rangle / F_\pi^2$
- ▶ $\Delta L_8 = L_8(M_\rho) + 0.20 \cdot 10^{-3} = O(p^4) \text{ LEC} + \chi \log$
- ▶ $\Delta L_6 = L_6(M_\rho) + 0.26 \cdot 10^{-3} = O(p^4) \text{ LEC} + \chi \log$

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L_6 is the awesome guy here

- ▶ Enhanced by m_s , related to $\langle (\bar{u}u)(\bar{s}s) \rangle \dots$
- ▶ ...and “guestimated” **assuming** Zweig rule in scalar sector

Numerical competition between $O(p^2)$ and $O(p^4)$?

Source of instability in chiral series ?

A usual (and dangerous) treatment of $N_f = 3$ series

$$X(3) \equiv \frac{2m\Sigma(3)}{F_\pi^2 M_\pi^2} = \frac{2m\Sigma(3)}{2m\Sigma(3) + \Delta L_{6,8} + O(p^6)}$$

$$\rightarrow 1 - 16 \frac{2M_K^2 + M_\pi^2}{F_\pi^2} \Delta L_6 - \frac{M_\pi^2}{F_\pi^2} \Delta L_8 + \dots$$

using $1/(1+x) = 1 - x + \dots$ and reexpressing mB_0 and $m_s B_0$

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For $r = 25$, $L_6(M_\rho) = 0.4 \cdot 10^{-3}$, $L_8(M_\rho) = 0.9 \cdot 10^{-3}$

$$X(3) = 1 - 0.62 + 0.04 + O(p^6)$$

- ▶ No fluctuations : $\Delta L_6 = L_6(M_\rho) + 0.23 \cdot 10^{-3}$ **very** close to 0
- ▶ If (m_s -enhanced) fluctuations, $N_f = 3$ series require **extra care**

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There **may** be large fluctuations of $s\bar{s}$ pairs with a dramatic impact on the convergence of $SU(3)$ chiral expansion

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H.Bijnens et al. @ $O(p^6)$

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- ▶ Determine $\Sigma(3)$ and fluctuations from experiment

SDG, L.Girlanda, N.Fuchs, J.Stern

- ▶ Assume overall convergence for a subset of observables
- ▶ Leave open a numerical competition between LO and NLO
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$\pi\pi$ scattering $\implies 2m_s/(m_u + m_d) \geq 14$ @ 95 % CL

Lattice connections

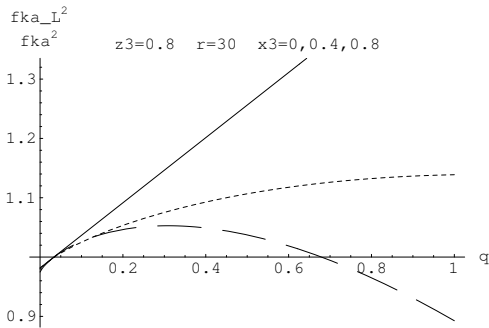
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- ▶ Trading chiral couplings for observables delicate $2mB_0 \rightarrow M_\pi^2$
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(2+1) simulation
 $(m_{\text{lat}}, m_{\text{lat}}, m_s)$

$$\frac{F_K^2|_{\text{lat}}}{F_K^2} = F\left(q \equiv \frac{m_{\text{lat}}}{m_s}\right)$$

for some choice
 of $F(3)$ and m_s/m

Typical sea-quark/unquenched effect

Be careful when you use $N_f = 3$ χ PT for chiral extrapolations. . .

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It might prove a tool more difficult to use than expected !

Conclusions

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 \leftrightarrow Zweig rule violation in scalar sector

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- ▶ N_f -dependence of order parameters \leftrightarrow Role of sea $s\bar{s}$ -pairs
 \leftrightarrow Zweig rule violation in scalar sector
- ▶ In terms of the spectrum of the Dirac operator $\Sigma(2^\chi) \geq \Sigma(3)$
$$\left. \begin{array}{l} \Sigma(3) \text{ average density} \\ \langle (\bar{u}u)(\bar{s}s) \rangle \text{ fluctuations} \end{array} \right\} \text{ of eigenvalues around 0}$$
$$\implies \text{Possibility of small } \Sigma(3) \text{ and large fluctuations}$$

Conclusions (2)

- ▶ In $N_f = 3$ $O(p^4)$ χ PT, fluctuations mirrored by L_4 and L_6
 L_6 (L_4) related to N_f -dependence of Σ (F^2)

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- ▶ Due to m_s -enhancement, $L_{4,6}(M_\rho)$ a few 10^{-3} larger than

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We must assess experimentally
how large the vacuum fluctuations of $s\bar{s}$ -pairs are !

The scalar channel is definitely very talkative...



but its shouting is sometimes hard to understand !