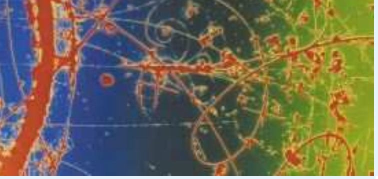
Two baryons on a lattice

Phase shifts and beyond

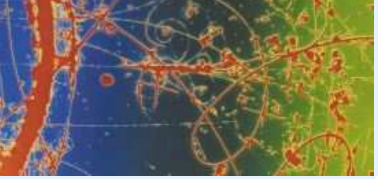
Will Detmold

University of Washington



Outline

- Nuclear physics from lattice QCD
- Electroweak properties of two nucleon systems
  [WD & Martin Savage, hep-lat/0403007]
  - Two particle energy levels in background fields
  - Radiative capture $n\,p \longrightarrow d\,\gamma$
  - Deuteron breakup: $\overline{\nu}_e\,d \longrightarrow n\,n\,e^+$
  - Deuteron magnetic moment
- Other recent highlights
  - Twisted boundary conditions/Aharonov-Bohm effect
    [Bedaque '04]
  - Hyperon nucleon scattering
    [Beane, Bedaque, Parreño & Savage '03]



Nuclear physics from lattice QCD

NPLQCD
- Nuclear physics
- Two nucleon sector

Electroweak NN properties

Scattering in background fields

Aharonov-Bohm effect

ΛN scattering

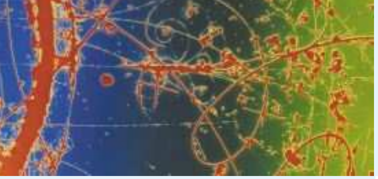
Summary

========================

- Traditional nuclear physics: based on meson exchange models, generic functional forms etc
  - Not really clear what questions to ask of lattice QCD
  - NN potential not an observable/measurable quantity
- Effective field theory (modern) approach to nuclear physics *[Weinberg 90,91]* is more systematic
  - Clear what we want to know from the lattice: fix counterterms via matching. E.g.

$$\mu_d = f(m_\pi, \kappa_0, \gamma, L_2)$$

  - Nuclear physics from first principles
- Particularly import in context of unphysical masses and partially-quenched simulations
- Once counterterms are fixed, the physical limit can be taken



Two nucleon sector

- Two nucleon scattering states: quantum numbers of angular momentum (J) and isospin (I)
- NN scattering S-matrix described in terms of phase shifts and defined by scattering amplitude (p: COM momentum)

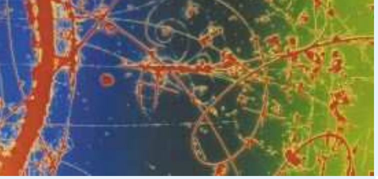
$$S \equiv \exp[2i\delta(p)] \equiv 1 + \frac{i\,p\,M}{2\pi}\mathcal{A}$$

$$\mathcal{A} = \frac{4\pi}{M}\frac{1}{p\cot\delta(p) - i\,p}$$

Two nucleon sector

- Two nucleon scattering states: quantum numbers of angular momentum (J) and isospin (I)
- NN scattering S-matrix described in terms of phase shifts and defined by scattering amplitude (p: COM momentum)
- Effective range expansion (ERE): scattering length(a), effective range (r), shape parameters (q_i)

$$p \ \cot\delta(p) = -\frac{1}{a} + \frac{r}{2}p^2 + \sum_i q_i p^{2i} + \ldots$$

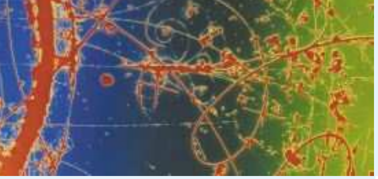


Two nucleon sector

- Two nucleon scattering states: quantum numbers of angular momentum (J) and isospin (I)
- NN scattering S-matrix described in terms of phase shifts and defined by scattering amplitude (p: COM momentum)
- Effective range expansion (ERE): scattering length(a), effective range (r), shape parameters (q_i)

$$p\ \cot\delta(p) = -\frac{1}{a} + \frac{r}{2}p^2 + \sum_i q_i p^{2i} + \ldots$$

- ${}^1S_0$: $J = 0$, $I = 1$
  - $a_1 = -23.714$ fm, $r_1 = 2.73$ fm
- ${}^3S_1$: $J = 1$, $I = 0$
  - mixes with ${}^3D_1$ channel
  - $a_3 = 5.425$ fm, $r_3 = 1.75$ fm
  - deuteron bound state: $B = 2.2$ MeV, $\gamma = 45$ MeV



Neutrino masses and mixing

- SNO and SuperK have given conclusive evidence for non-electron neutrino components in ^{8}B solar neutrino flux

  $\Longrightarrow$ **NEUTRINO OSCILLATIONS!!!**

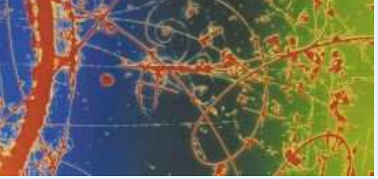
- Results based on three reactions ($x = e, \mu, \tau$):

$$\begin{array}{lll} \sigma_{CC}: & \nu_e\, d \longrightarrow p\, p\, e^- & Charged\ current \\ \sigma_{NC}: & \nu_x\, d \longrightarrow p\, n\, \nu_x & Neutral\ current \\ \sigma_{ES}: & \nu_x\, e^- \longrightarrow \nu_x\, e^- & Elastic\ scattering \end{array}$$

- Event rates $\Longrightarrow$ neutrino fluxes (f_{ν_i}):

$$R_{CC/NC/ES} \sim \int d\,E\, \sigma_{CC/NC/ES}\, f_{\nu_{e/x/x}}$$

- Cross sections are a major source of theoretical uncertainty in determination of fluxes



Nuclear cross-sections

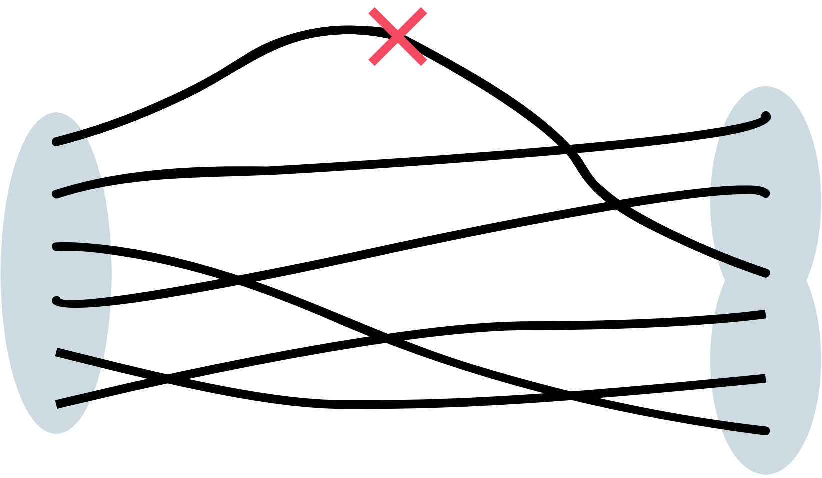
- Elastic scattering cross-section is well known, but the CC and NC cross-sections have large uncertainties
- Difficulties are because of the poorly constrained two-body interaction with the external current
  - Meson exchange currents (MEC's) in potential models
  - Effective Field Theory: multi-nucleon – current operators
- Most relevant operator is zero-derivative, isovector axial two-body current $\sim L_{1,A}$
- In terms of this coupling (at $E_\nu = 10$ MeV):

$$\sigma_{CC} = 4.07 + 0.12\ L_{1,A} \qquad \sigma_{NC} = 1.76 + 0.056\ L_{1,A}$$

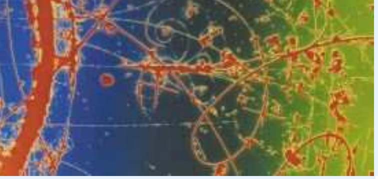
- $L_{1,A}$ is only poorly known: $L_{1,A} = 4 \pm 6$ fm^3 from experiment *[Chen, Heeger & Robertson, '03]*
- A lattice determination of it would be very useful

$\sigma_{CC/NC}$ from the lattice

- How does one determine these hadronic cross-sections (tow-body counterterms) from the lattice?
- Measure $\langle d(\mathbf{p}=0)|J^{\mu}_{5,a}|n(\mathbf{p}=0)p(\mathbf{p}=0)\rangle$ at unphysical kinematics

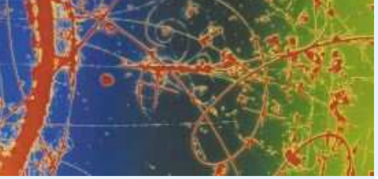


- Would allow determination of the coefficients ($L_{1,A}$ etc) in the low energy EFT to give an ab initio calculation from QCD
- Extremely difficult: one or two orders of magnitude harder than two-particle energies



Two-particle energies in a background field

- A simpler way . . .
- By measuring the proton mass on a lattice with a background magnetic field, one can extract the magnetic moment *[Bernard et al.; Martinelli et al.]* and even polarisability *[F. Lee et al.]*
- Using same philosophy: by calculating two-particle energies in background magnetic field one can probe eg: deuteron magnetic moment
- Background (Z_0) axial field (a flux of neutrinos across the lattice): extract axial, isovector two-nucleon coupling $L_{1,A}$, weak moment of the deuteron, . . .
- Need additional ensembles of gauge configurations
- Computational advantage for isosinglet: disconnected contributions automatically included



Pionless effective field theory

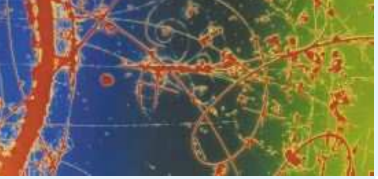
- ERE describes NN data but not eg: $d\,\gamma \longrightarrow n\,p$
  Improvements:
  - Traditional potential models: meson-exchange currents (MECs)
  - EFT provides a model independent approach (but requires parameters to be determined)

Pionless effective field theory

- For $|\mathbf{p}| \ll m_\pi/2$, only effective degrees of freedom are nucleons and external electroweak currents: $\not\pi$EFT

Pionless effective field theory

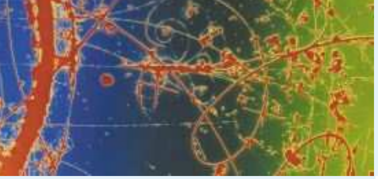
- For $|\mathbf{p}| \ll m_\pi/2$, only effective degrees of freedom are nucleons and external electroweak currents: $\not\pi$EFT
- Lagrangian involves all possible interactions with correct symmetries (M_N phased away) *[Chen, Savage, Rupak]*:



Pionless effective field theory

- For $|\mathbf{p}| \ll m_\pi/2$, only effective degrees of freedom are nucleons and external electroweak currents: $\not\pi$EFT
- Lagrangian involves all possible interactions with correct symmetries (M_N phased away) *[Chen, Savage, Rupak]*:

$$\mathcal{L}_1 = N^\dagger \left[iD_0 + \frac{|\mathbf{D}|^2}{2M} \right] N$$



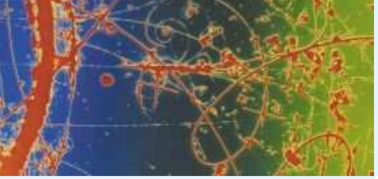
Pionless effective field theory

- For $|\mathbf{p}| \ll m_\pi/2$, only effective degrees of freedom are nucleons and external electroweak currents: $\not{\pi}$EFT
- Lagrangian involves all possible interactions with correct symmetries (M_N phased away) *[Chen, Savage, Rupak]*:

$$\mathcal{L}_1 = N^\dagger \left[iD_0 + \frac{|\mathbf{D}|^2}{2M} \right] N$$

$$\mathcal{L}_2^{^3S_1} = C_0 (N^T P_i N)^\dagger (N^T P_i N) + C_2 (N^T P_i N)^\dagger (N^T P_i \mathbf{D}^2 N) + \ldots$$

$$\mathcal{L}_2^{^1S_0} = \hat{C}_0 (N^T \bar{P}^a N)^\dagger (N^T \bar{P}^a N) + \hat{C}_2 (N^T \bar{P}^a N)^\dagger (N^T \bar{P}^a \mathbf{D}^2 N) + \ldots$$



Pionless effective field theory

- For $|\mathbf{p}| \ll m_\pi/2$, only effective degrees of freedom are nucleons and external electroweak currents: $\not\pi$EFT
- Lagrangian involves all possible interactions with correct symmetries (M_N phased away) *[Chen, Savage, Rupak]*:

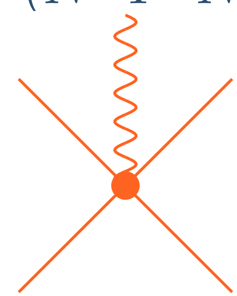
$$\mathcal{L}_1 = N^\dagger \left[iD_0 + \frac{|\mathbf{D}|^2}{2M} \right] N$$

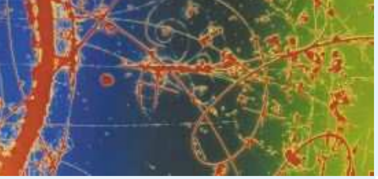
$$\mathcal{L}_2^{^3S_1} = C_0 (N^T P_i N)^\dagger (N^T P_i N) + C_2 (N^T P_i N)^\dagger (N^T P_i \mathbf{D}^2 N) + \ldots$$

$$\mathcal{L}_2^{^1S_0} = \hat{C}_0 (N^T \bar{P}^a N)^\dagger (N^T \bar{P}^a N) + \hat{C}_2 (N^T \bar{P}^a N)^\dagger (N^T \bar{P}^a \mathbf{D}^2 N) + \ldots$$

$$\mathcal{L}_{EM} = \frac{e}{2M} N^\dagger (\kappa_0 + \kappa_1 \tau_3) \sigma_i B_i N + e L_1 (N^T P_i N)^\dagger (N^T \bar{P}^3 N) \mathbf{B}_i + e L_2 \epsilon_{ijk} (N^T P_i N)^\dagger (N^T P_j N) \mathbf{B}_k + \ldots$$

$$\kappa_0 = \frac{1}{2}(\kappa_p + \kappa_n) \qquad \kappa_1 = \frac{1}{2}(\kappa_p - \kappa_n)$$





Pionless effective field theory

NPLQCD

Electroweak NN properties
- Neutrino masses and mixing
- Nuclear cross-sections
- Lattice cross-sections
- Two particle energies
- $\pi\!\!\!/$EFT
- EW properties in $\pi\!\!\!/$EFT

Scattering in background fields

Aharonov-Bohm effect

ΛN scattering

Summary

========================

- For $|\mathbf{p}| \ll m_\pi/2$, only effective degrees of freedom are nucleons and external electroweak currents: $\pi\!\!\!/$EFT
- Lagrangian involves all possible interactions with correct symmetries (M_N phased away) *[Chen, Savage, Rupak]*:

$$\mathcal{L}_1 = N^\dagger \left[iD_0 + \frac{|\mathbf{D}|^2}{2M} \right] N$$

$$\mathcal{L}_2^{^3S_1} = C_0 (N^T P_i N)^\dagger (N^T P_i N) + C_2 (N^T P_i N)^\dagger (N^T P_i \mathbf{D}^2 N) + \ldots$$

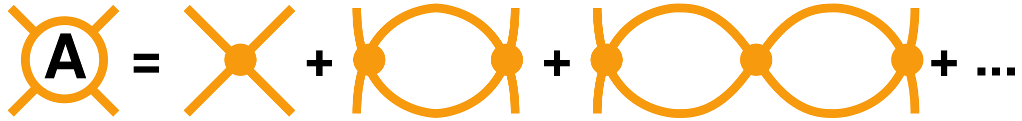
$$\mathcal{L}_2^{^1S_0} = \hat{C}_0 (N^T \bar{P}^a N)^\dagger (N^T \bar{P}^a N) + \hat{C}_2 (N^T \bar{P}^a N)^\dagger (N^T \bar{P}^a \mathbf{D}^2 N) + \ldots$$

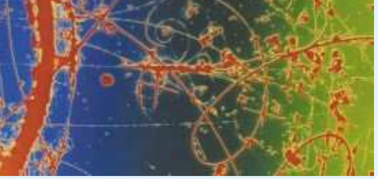
$$\mathcal{L}_{EM} = \frac{e}{2M} N^\dagger (\kappa_0 + \kappa_1 \tau_3) \sigma_i B_i N + e L_1 (N^T P_i N)^\dagger (N^T \bar{P}^3 N) \mathbf{B}_i + e L_2 \epsilon_{ijk} (N^T P_i N)^\dagger (N^T P_j N) \mathbf{B}_k + \ldots$$

$$\mathcal{L}_{EW} = -gW \frac{g_A}{2} N^\dagger \sigma^z \tau^3 N - \frac{gW\ L_{1,A}}{2M} (N^T P_3 N)^\dagger (N^T \bar{P}^3 N) + \ldots$$

Pionless effective field theory

- Large scattering lengths lead to interesting power counting: small expansion parameter is $Q \sim p/m_\pi,\ \gamma/m_\pi,\ a_1^{-1}/m_\pi$
- Reproduces ERE and describes low energy EW processes
- Mixing with higher partial waves is suppressed by m_π^{-J}
- Exact scattering amplitude:





Pionless effective field theory

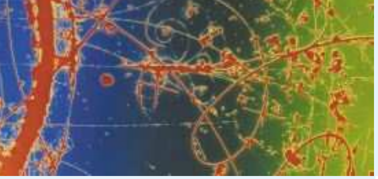
- Large scattering lengths lead to interesting power counting: small expansion parameter is $Q \sim p/m_\pi,\ \gamma/m_\pi,\ a_1^{-1}/m_\pi$
- Reproduces ERE and describes low energy EW processes
- Mixing with higher partial waves is suppressed by m_π^{-J}
- Exact scattering amplitude:

$$\mathcal{A} = \left[\sum_n C_{2n} p^{2n}\right] + \left[\sum_n C_{2n} p^{2n}\right] I_0 \left[\sum_n C_{2n} p^{2n}\right] + \ldots$$

$$= \frac{\sum_n C_{2n} p^{2n}}{1 - I_0(p) \sum_n C_{2n} p^{2n}}$$

- Loop integral *[Kaplan, Savage & Wise]*

$$I_0^{PDS} = \left(\frac{\mu}{2}\right)^{4-D} \int \frac{d^{D-1}\mathbf{k}}{E - |\mathbf{k}|^2/M + i\epsilon} = -\frac{M}{4\pi}(\mu + ip)$$



Electroweak properties in $\not\pi$EFT

- All low energy EW properties of two nucleon states are described
- Eg: deuteron magnetic moment

$$\mu_d = \frac{2}{1-\gamma r_3}\left(\gamma L_2 + \kappa_0\right)$$

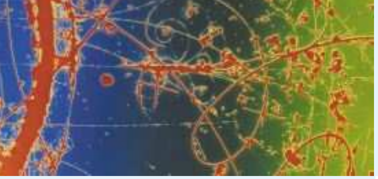
- Eg: radiative capture $n\,p \to d\gamma$ near threshold ($^1S_0$ to $^3S_1$)

$$\sigma(np \to d\gamma) = \frac{4\pi\alpha\left(\gamma_0^2+|\mathbf{p}|^2\right)^3}{M^4\gamma_0^3|\mathbf{p}|}\left[\,|\tilde{X}_{M1}|^2 + |\tilde{X}_{E1}|^2 \; + \; \ldots\right]$$

$$\begin{aligned}\tilde{X}_{M1} \quad &= \quad \frac{1}{\sqrt{1-\gamma_0 r_3}}\,\frac{1}{-\frac{1}{a_1}+\frac{1}{2}r_1|\mathbf{p}|^2 - i|\mathbf{p}|} \\ &\quad \times\left[\frac{\kappa_1\gamma_0^2}{\gamma_0^2+|\mathbf{p}|^2}\left(\gamma_0 - \frac{1}{a_1}+\frac{1}{2}r_1|\mathbf{p}|^2\right) + L_1\frac{\gamma_0^2}{2}\;\right]\end{aligned}$$

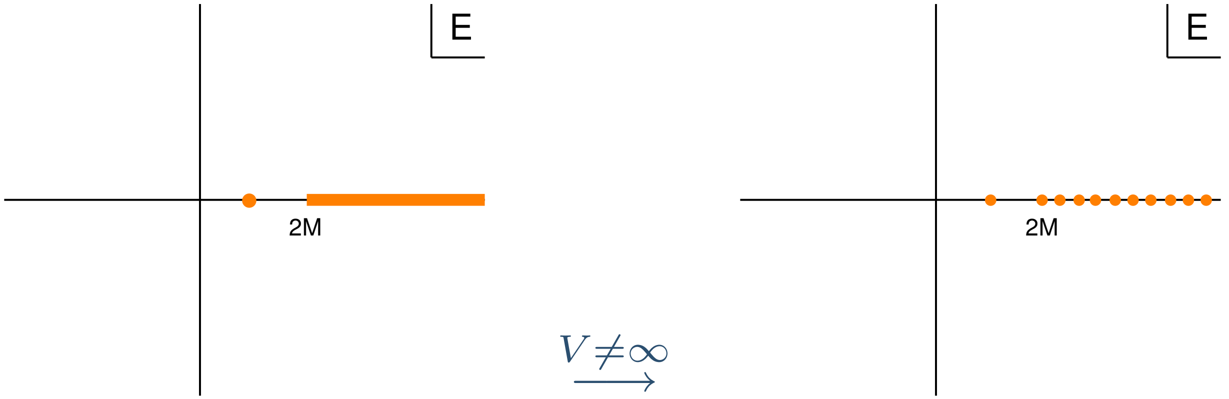
Scattering in Euclidean space

- Maiani-Testa theorem: cannot obtain S-matrix elements away from kinematic thresholds from infinite volume Euclidean space calculations



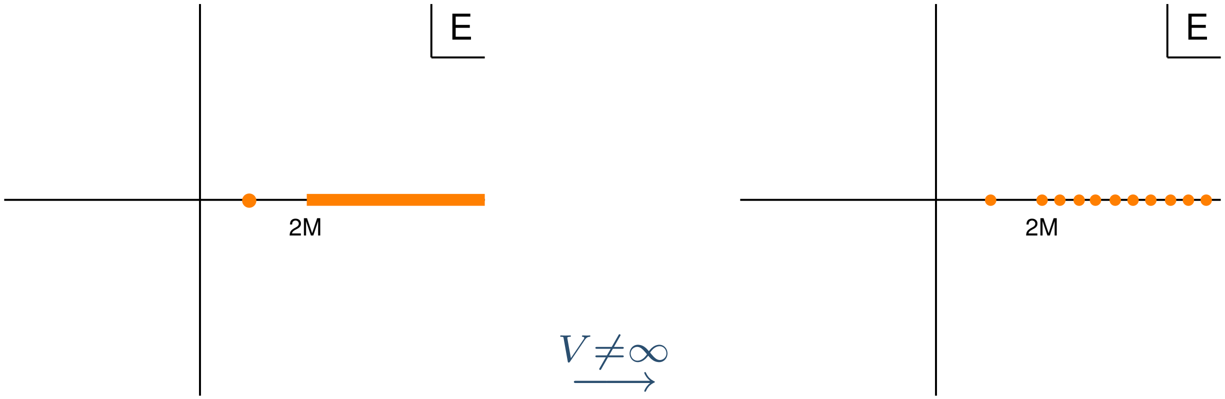
Scattering in Euclidean space

- Maiani-Testa theorem: cannot obtain S-matrix elements away from kinematic thresholds from infinite volume Euclidean space calculations
- Lüscher *['86]*: elastic scattering amplitude uniquely related to the energy shifts of two particle states at finite volume

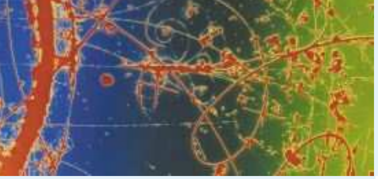


Scattering in Euclidean space

- Maiani-Testa theorem: cannot obtain S-matrix elements away from kinematic thresholds from infinite volume Euclidean space calculations
- Lüscher *['86]*: elastic scattering amplitude uniquely related to the energy shifts of two particle states at finite volume

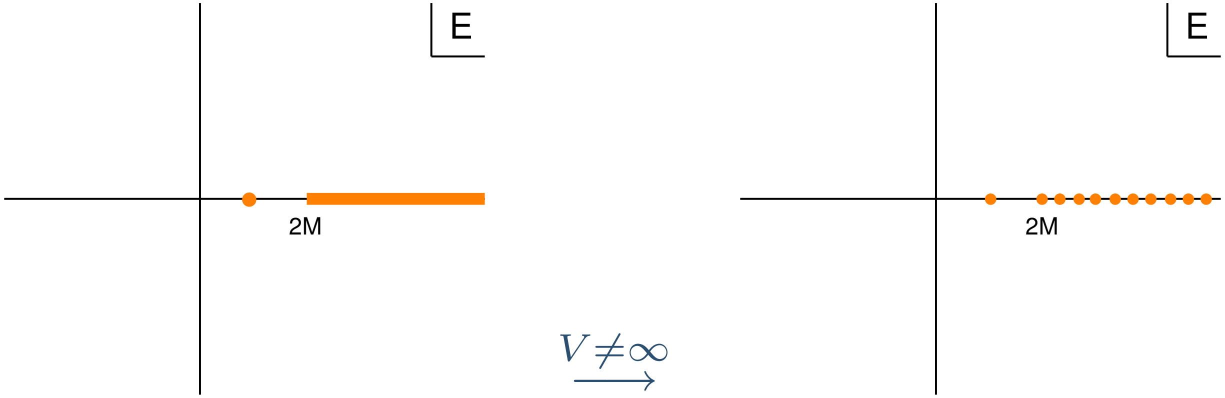


- Two particle energies are given by poles in scattering amplitude: solve for eigenvalues $\mathcal{A}^{-1} = 0$



Scattering in Euclidean space

- Maiani-Testa theorem: cannot obtain S-matrix elements away from kinematic thresholds from infinite volume Euclidean space calculations
- Lüscher *['86]*: elastic scattering amplitude uniquely related to the energy shifts of two particle states at finite volume



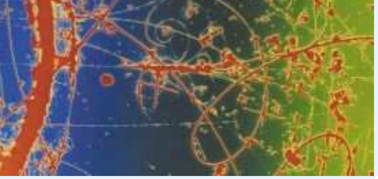
- Two particle energies are given by poles in scattering amplitude: solve for eigenvalues $\mathcal{A}^{-1} = 0$

$$0 = \mathcal{A}^{-1}(L) = p \cot \delta(p) - \frac{M\mu}{4\pi} - I_0^{PDS}(L)$$

Scattering on a finite volume

- On a finite volume $L \otimes L \otimes L$: $\int d^3k \longrightarrow \frac{1}{L^3} \sum_{\mathbf{k}}$

$$I_0^{PDS}(L) = \frac{1}{L^3} \sum_{\mathbf{k}}^{PDS} \frac{1}{E - |\mathbf{k}|^2/M}$$



Scattering on a finite volume

- On a finite volume $L \otimes L \otimes L$: $\int d^3k \longrightarrow \frac{1}{L^3}\sum_{\mathbf{k}}$

$$I_0^{PDS}(L) \quad = \quad \frac{1}{L^3}\sum_{\mathbf{k}}^{\Lambda}\frac{1}{E-|\mathbf{k}|^2/M} - \int^{\Lambda}\frac{d^3k}{|\mathbf{k}|^2/M} + \int_{PDS}\frac{d^3k}{|\mathbf{k}|^2/M}$$

Scattering on a finite volume

- On a finite volume $L \otimes L \otimes L$: $\int d^3k \longrightarrow \frac{1}{L^3} \sum_{\mathbf{k}}$

$$I_0^{PDS}(L) = \frac{1}{L^3} \sum_{\mathbf{k}}^{\Lambda} \frac{1}{E - |\mathbf{k}|^2/M} - \int^{\Lambda} \frac{d^3k}{|\mathbf{k}|^2/M} + \int_{PDS} \frac{d^3k}{|\mathbf{k}|^2/M}$$

$$= \frac{M}{4\pi} \left[-\frac{1}{\pi L} \sum_{\mathbf{n}}^{\Lambda_n} \frac{1}{|\mathbf{n}|^2 - \tilde{p}^2} + 4\frac{\Lambda_n}{L} - \mu \right]$$

- $\mathbf{n} \left(= \frac{L}{2\pi}\mathbf{k}\right) \in \mathbb{Z}_3$, $\tilde{p}^2 = \frac{L^2}{4\pi} E\ M$
- Defines dimensionally regulated (PDS) sum
- Equivalent to Lüscher's analytically continuation of generalized ζ-function

Scattering on a finite volume

- On a finite volume $L \otimes L \otimes L$: $\int d^3k \longrightarrow \frac{1}{L^3} \sum_{\mathbf{k}}$

$$I_0^{PDS}(L) = \frac{1}{L^3} \sum_{\mathbf{k}}^{\Lambda} \frac{1}{E - |\mathbf{k}|^2/M} - \int^{\Lambda} \frac{d^3k}{|\mathbf{k}|^2/M} + \int_{PDS} \frac{d^3k}{|\mathbf{k}|^2/M}$$

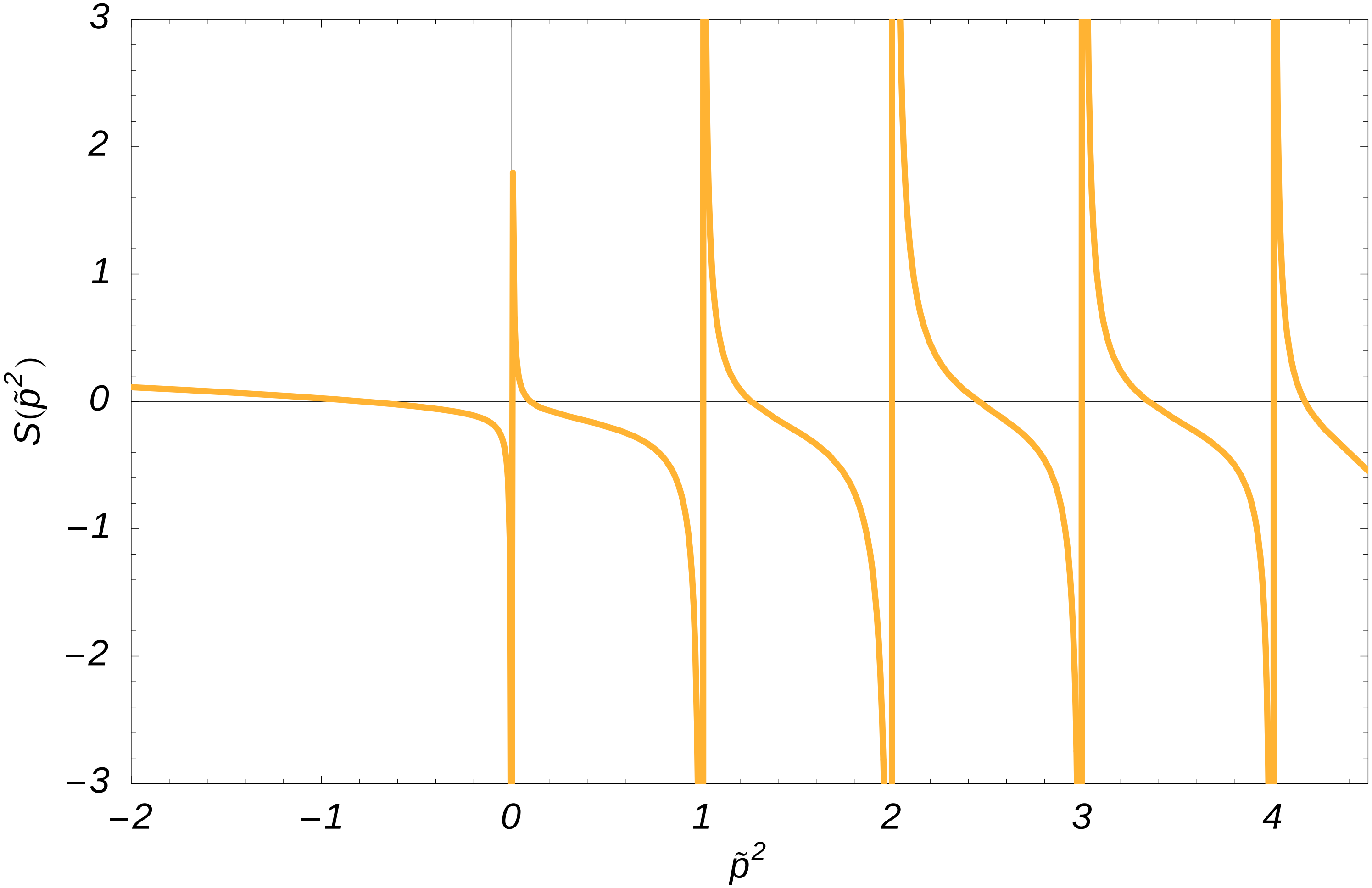
$$= \frac{M}{4\pi} \left[-\frac{1}{\pi L} \sum_{\mathbf{n}}^{\Lambda_n} \frac{1}{|\mathbf{n}|^2 - \tilde{p}^2} + 4\frac{\Lambda_n}{L} - \mu \right]$$

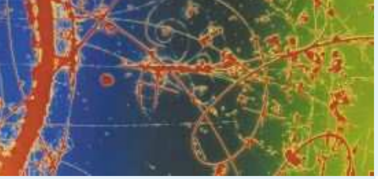
- $\mathbf{n} \left(= \frac{L}{2\pi}\mathbf{k}\right) \in \mathbb{Z}_3$, $\tilde{p}^2 = \frac{L^2}{4\pi} E\ M$
- Defines dimensionally regulated (PDS) sum
- Equivalent to Lüscher's analytically continuation of generalized ζ-function
- Numerically determine energy eigenstates:

$$\boxed{p \cot\delta(p) - \frac{1}{\pi L} S(\tilde{p}^2) = 0}$$

$$\boxed{S(\tilde{p}^2) = \sum_{\mathbf{n}}^{\Lambda_n} \frac{1}{|\mathbf{n}|^2 - \tilde{p}^2} - 4\pi\Lambda_n}$$

Scattering on a finite volume





Scattering on a finite volume

- Asymptotic expansions *[Lüscher]*: $L \to \infty$

$$E_0 = \frac{4\pi a}{ML^3}\left[1 - c_1\left(\frac{a}{L}\right) + c_2\left(\frac{a}{L}\right)^2 + \ldots\right]$$

$$E_1 = \frac{4\pi^2}{ML^2} - \frac{12\tan\delta_0}{ML^2}\left[1 + c_1'\tan\delta_0 + c_2'\tan\delta_0 + \ldots\right]$$

$$E_{-1}^{(^3S_1)} = -\frac{\gamma_0^2}{M}\left[1 + \frac{12}{\gamma_0 L(1-\gamma_0 r_3)}e^{-\gamma_0 L} + \ldots\right]$$

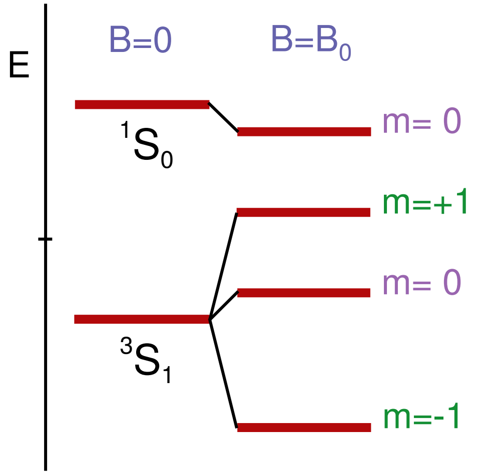
- Small L expansion also exists: $L/a \ll 1$ *[Beane et al.]*
- Minimum L set by range of interaction: $L > r \sim m_\pi^{-1}$
- Two-particle energy levels have been studied on the lattice:
  - Scalar theory *[Göckeler et al. '94]*
  - NN, KN, πN quenched, large masses *[Aoki et al., '95]*
  - $\pi\pi$ (I=2) unquenched *[Aoki et al., '03]*
- So far (hadronic results) not realistic

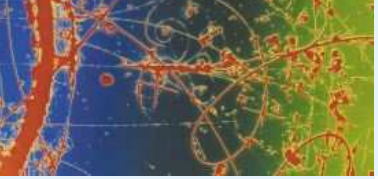
Background fields in QCD

- Magnetic field shifts single particle mass eigenstates

$$M_{p\uparrow}(B_0) = M_0 + \mu_p B_0 + 4\pi \frac{\beta_p}{2} B_0^2 + \dots$$

- Also split two particle mass eigenstates





Background fields in QCD

- Magnetic field shifts single particle mass eigenstates

$$M_{p\uparrow}(B_0) = M_0 + \mu_p B_0 + 4\pi \frac{\beta_p}{2} B_0^2 + \ldots$$

- Also split two particle mass eigenstates
- Magnetic fields on the lattice: $U_\mu(x) \longrightarrow U_\mu^{\rm ext}(x) U_\mu(x)$

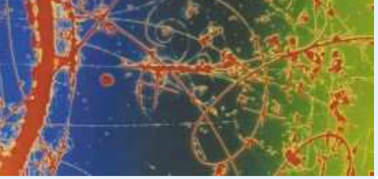
$$U_{0,3}^{ext}(x) = 1 \qquad U_1^{ext}(x) = e^{+i\beta x_2} \qquad U_2^{ext}(x) = e^{-i\beta x_1}$$

  - generates $\mathbf{B} = B_0 \hat{\mathbf{z}}$, $eB_0 a^2 \sim \beta$
  - Homogeneity requires $eB_0 L_\perp^2 / 2\pi \in \mathbb{Z}$
- Weak background fields can be implemented: couple differently to left- and right- chirality quarks
  - eg: DW fermions with background field varying in 5th direction
  - Similar shifts in two particle states

Asymmetric boxes

- Landau levels: in magnetic field, single particle states are not plane waves in transverse directions
- Two particle states also have analogous effects
- Modifications small if

$$|eB_0| \ll \frac{8\sqrt{3}\pi}{L_\perp^2}$$

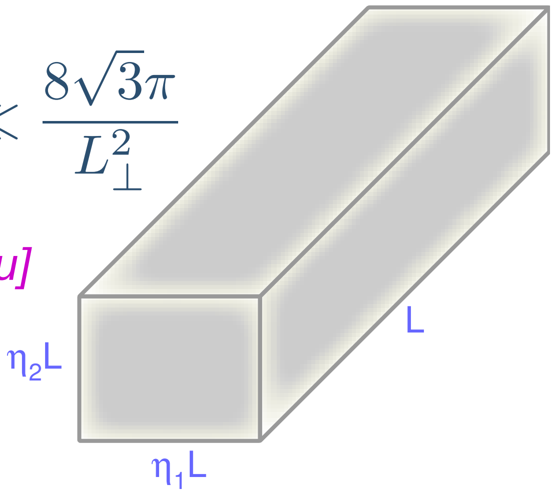


Asymmetric boxes

- Landau levels: in magnetic field, single particle states are not plane waves in transverse directions
- Two particle states also have analogous effects
- Modifications small if

$$|eB_0| \ll \frac{8\sqrt{3}\pi}{L_\perp^2}$$

- Use an asymmetric box *[Li & Liu]*
- Redo Lüscher analysis



- Small transverse directions (~ 4 fm) move transverse levels up in spectrum
- Low lying excitations are all longitudinal

Deuteron magnetic moment

- Eigenvalue equation in presence of background magnetic field

$$p\cot\delta_3 - \frac{1}{\pi L} S(\tilde{p}^2 \pm \tilde{u}_0^2; \eta_1, \eta_2) \mp \frac{eB_0}{2}(L_2 - r_3\kappa_0) = 0$$

where $\tilde{u}_0^2 = \frac{L^2}{4\pi^2} eB_0\kappa_0$ and $\mp$ correspond to ${}^3S_1$ $(m = \pm 1)$ states

- Asymptotic behaviour of first continuum [$\mathbf{p} = (0,0,0)$]:

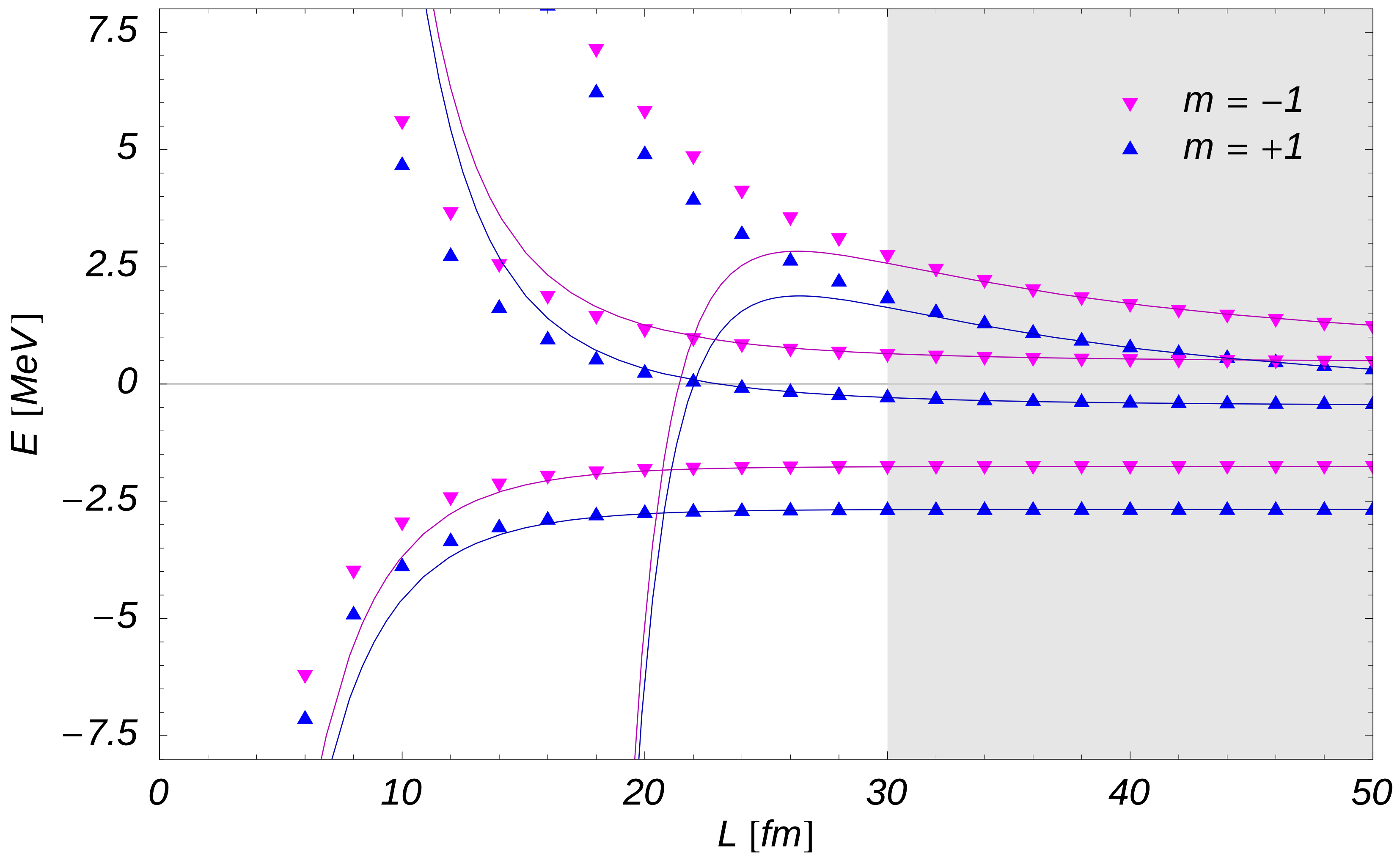
$$E_0^{(m=\pm1)} = \mp\frac{eB_0}{M}\kappa_0 + \frac{4\pi A_3}{ML^3}\left[1 - c_1\frac{A_3}{L} + c_2\left(\frac{A_3}{L}\right)^2 + \ldots\right]$$

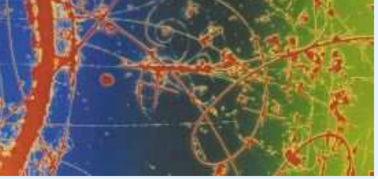
- Effective scattering length:

$$\frac{1}{A_3} = \frac{1}{a_3} \pm \frac{eB_0}{2} L_2$$

Deuteron magnetic moment

$\eta_1 = \eta_2 = 1$, $|eB_0| = 1000$ MeV^2





L_1 counterterm: $n\,p \longrightarrow d\,\gamma$

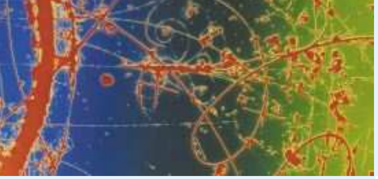
- L_1 induces mixing between ${}^1S_0$ and ${}^3S_1$ states
- Need to diagonalize coupled channel system
- Eigenvalue equation:

$$\left[p\cot\delta_1 - \frac{S_1 + S_2}{2\pi L}\right]\left[p\cot\delta_3 - \frac{S_1 + S_2}{2\pi L}\right] = \left[\frac{eB_0 L_1}{2} + \frac{S_1 - S_2}{2\pi L}\right]^2$$

$$S_1 = S(\eta_1, \eta_2; \tilde{p}^2 + \tilde{u}_1^2) \quad , \quad S_2 = S(\eta_1, \eta_2; \tilde{p}^2 - \tilde{u}_1^2)$$

$$\tilde{u}_1^2 = \frac{L^2}{4\pi^2}\, eB_0\, \kappa_1$$

- How could we determine L_1?
- Bound state:
  - Fine tuning between kinetic and potential terms
  - Very sensitive to short distance physics (L_1)



L_1 counterterm: $n\,p \longrightarrow d\,\gamma$

NPLQCD

Electroweak NN properties

Scattering in background fields
- Scattering in Euclidean space
- Scattering on a finite volume
- Asymptotic Forms
- Background fields
- Asymmetric boxes
- Deuteron magnetic moment
- $n\ p \longrightarrow d\ \gamma$
- $\overline{\nu}_e\ d \longrightarrow n\ n\ e^+$
- Mass dependence

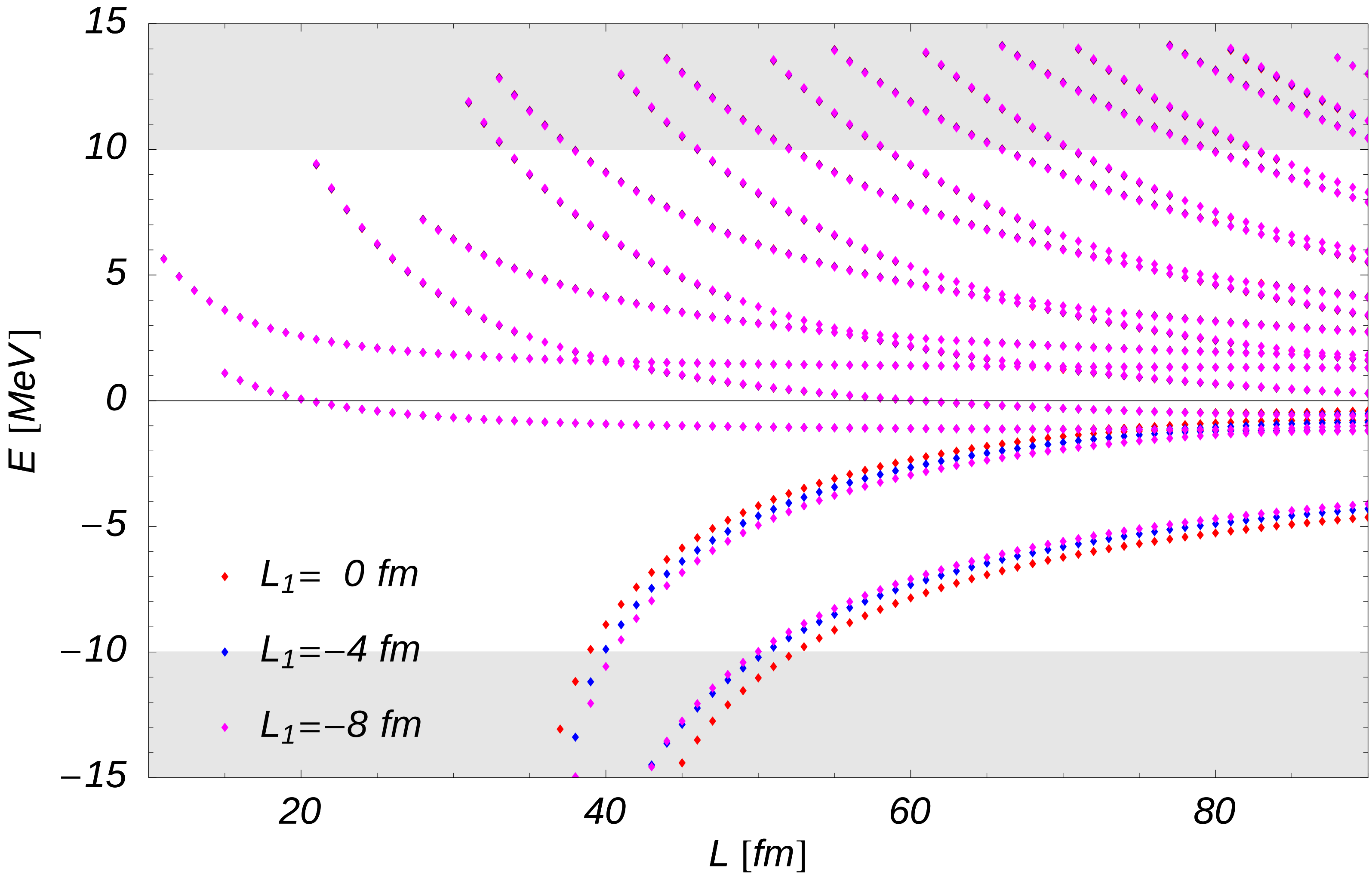
Aharonov-Bohm effect

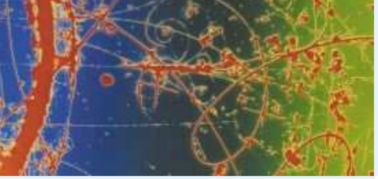
ΛN scattering

Summary

========================

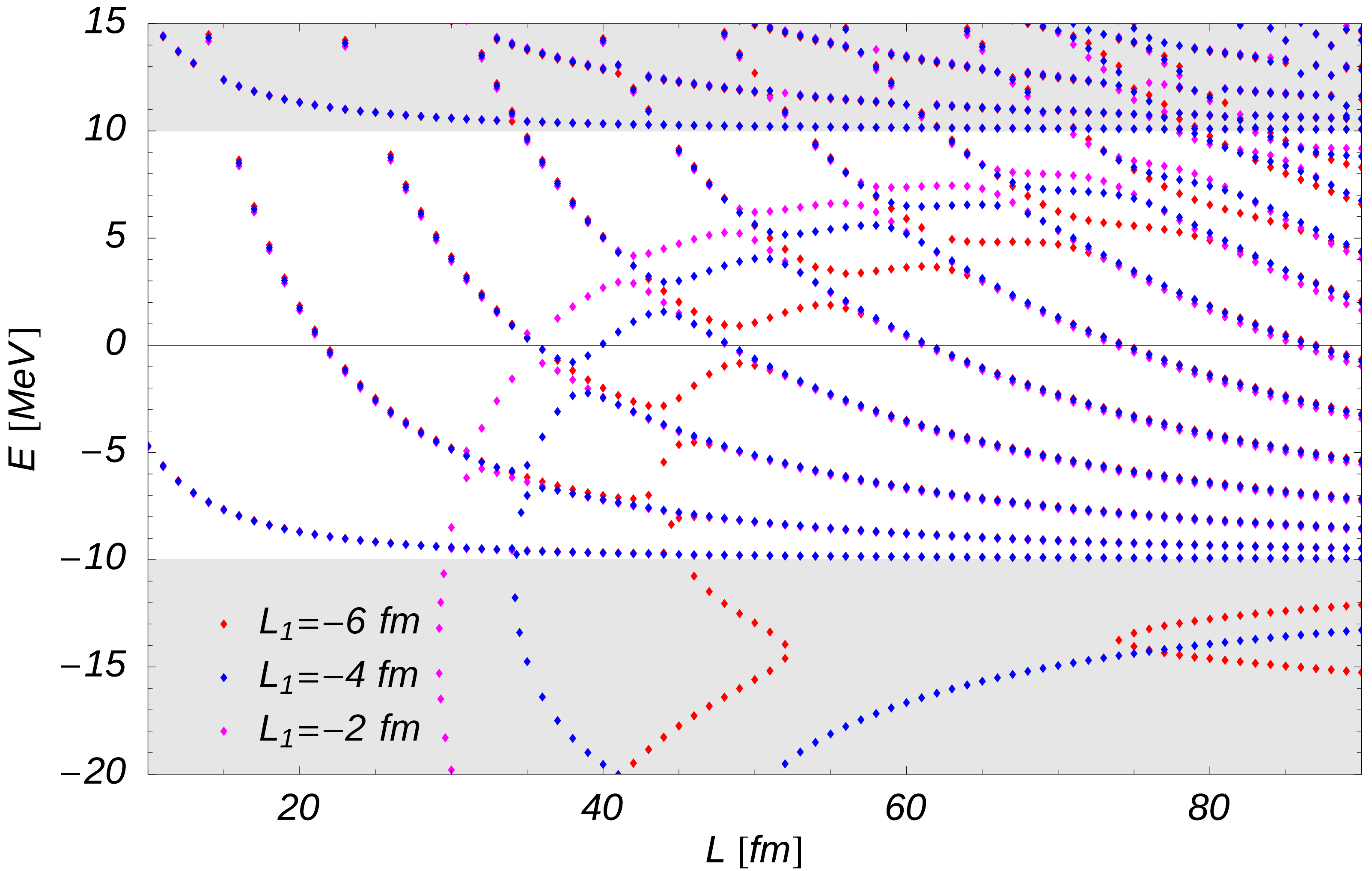
$\eta_1 = \eta_2 = 0.1$, $|eB_0| = 500$ MeV^2





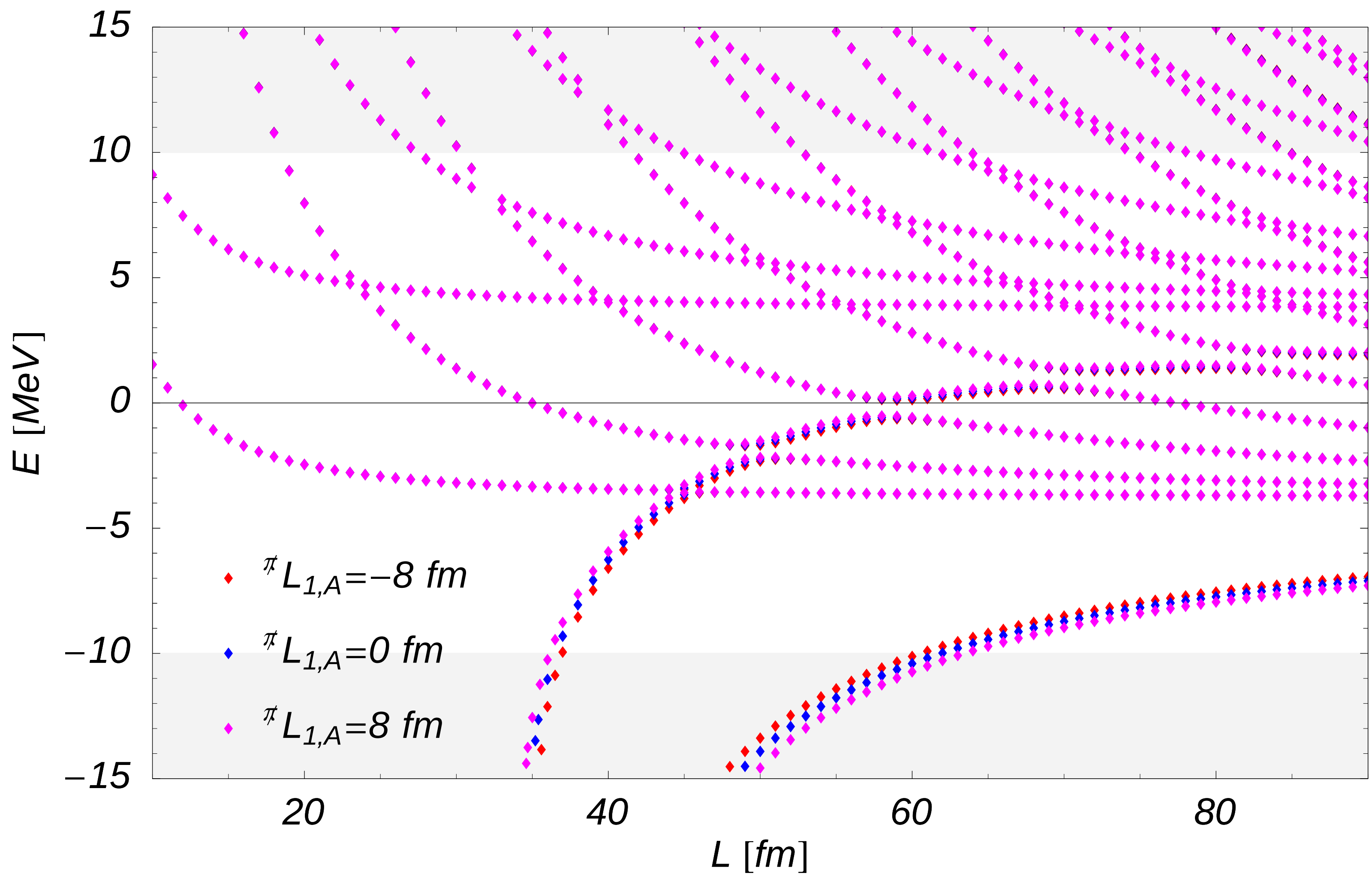
L_1 counterterm: $n\,p \longrightarrow d\,\gamma$

$\eta_1 = \eta_2 = 0.1,\ |eB_0| = 4000$ MeV^2



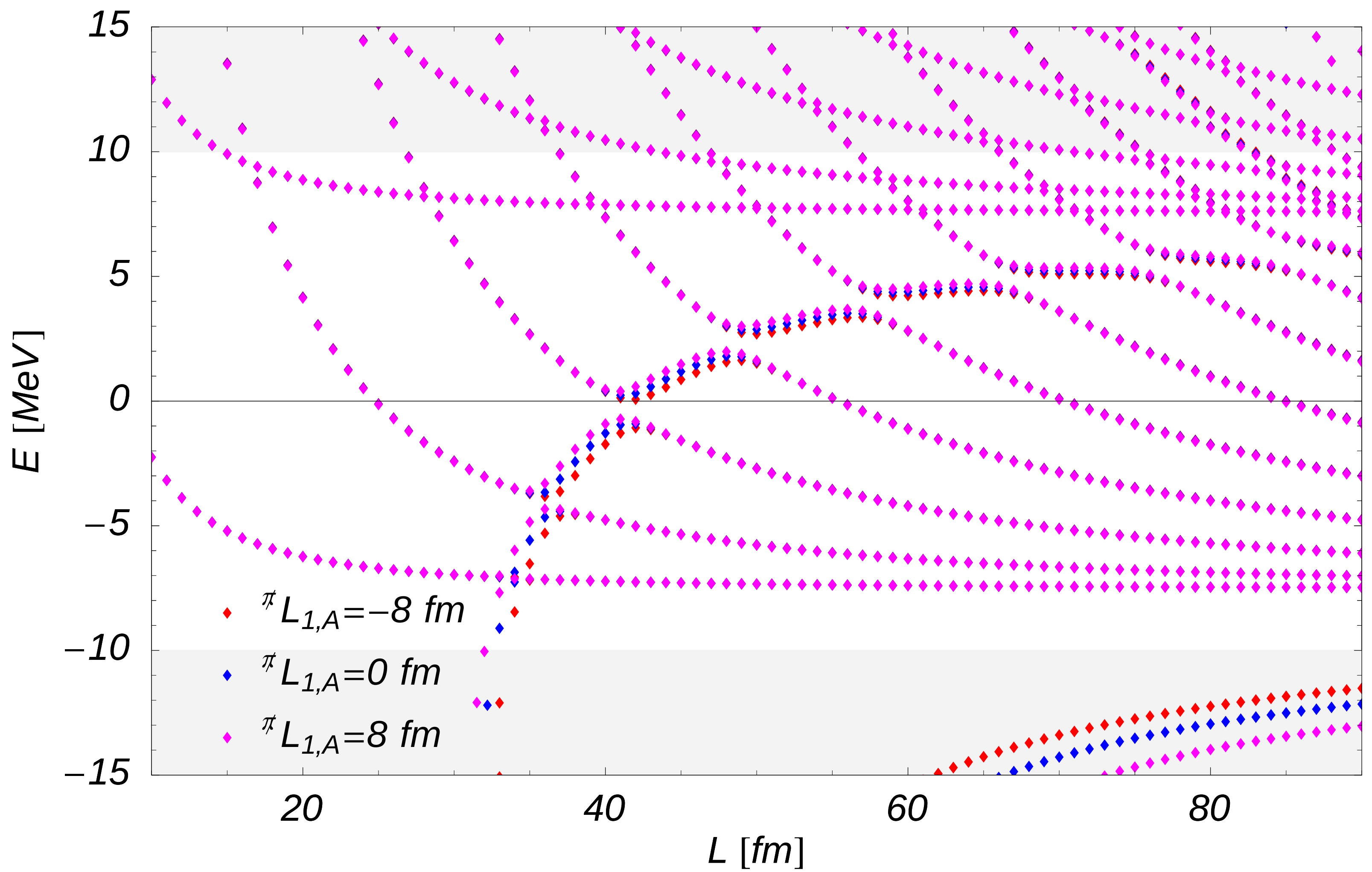
Deuteron breakup: $\overline{\nu}_e\, d \longrightarrow n\, n\, e^+$

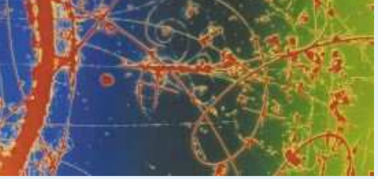
$\eta_1 = \eta_2 = 0.1,\ gW = 3$ MeV



Deuteron breakup: $\overline{\nu}_e \, d \longrightarrow n \, n \, e^+$

$\eta_1 = \eta_2 = 0.1$, $gW = 6$ MeV



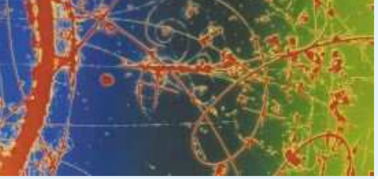


m_q dependence of NN scattering

- Above discussion all at $m_q \simeq m_q^{\rm phys}$
- Explicitly makes use of a near-threshold state (large scattering lengths)
- For $m_q = m_q^{\rm lattice} \gg m_q^{\rm phys}$, scattering lengths will relax to natural size *[Beane & Savage; Epelbaum, Meißner & Glöckle]*:

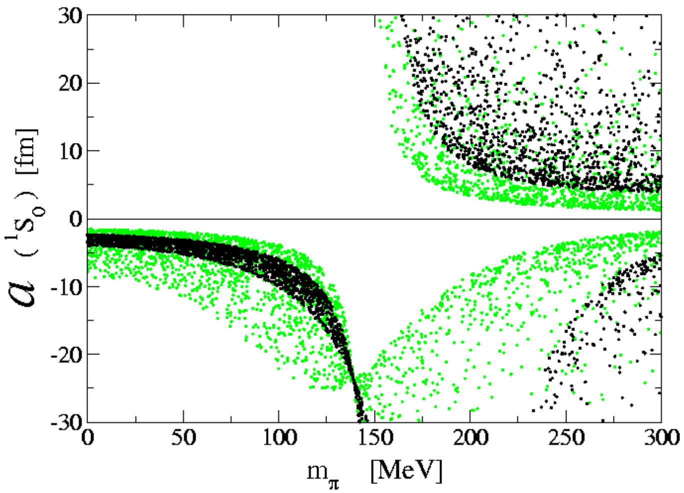
$$a_{^1S_0}^{-1} = \gamma + \frac{g_A^2 M}{8\pi f_\pi^2}\left[m_\pi^2 \log\frac{\mu}{m_\pi} + (\gamma - m_\pi)^2 - (\gamma-\mu)^2\right] - \frac{M m_\pi^2}{4\pi}(\gamma-\mu)^2 D_2(\mu) \qquad [\gamma = \mu + 4\pi/MC_0(\mu)]$$

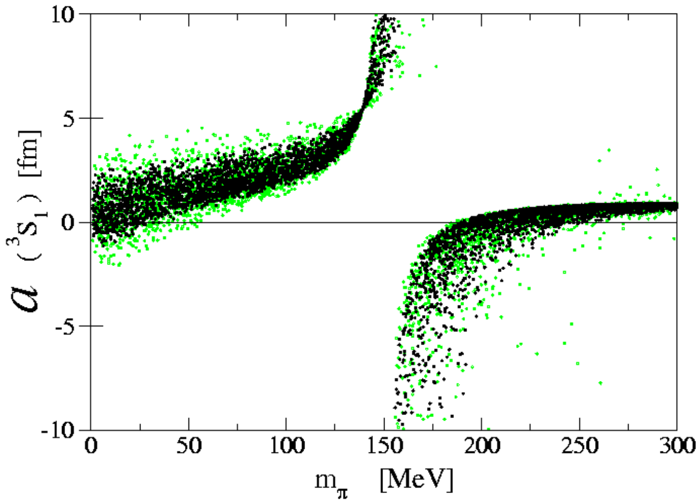
(NLO analysis)

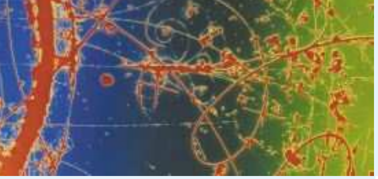


m_q dependence of NN scattering

- Above discussion all at $m_q \simeq m_q^{\rm phys}$
- Explicitly makes use of a near-threshold state (large scattering lengths)
- For $m_q = m_q^{\rm lattice} \gg m_q^{\rm phys}$, scattering lengths will relax to natural size *[Beane & Savage; Epelbaum, Meißner & Glöckle]*:

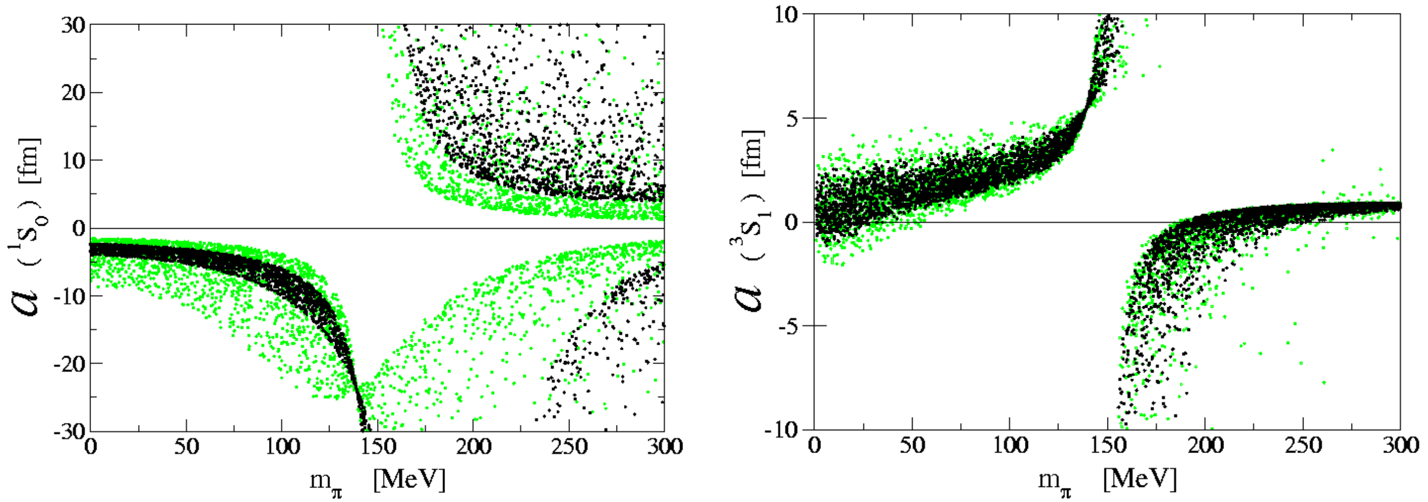






m_q dependence of NN scattering

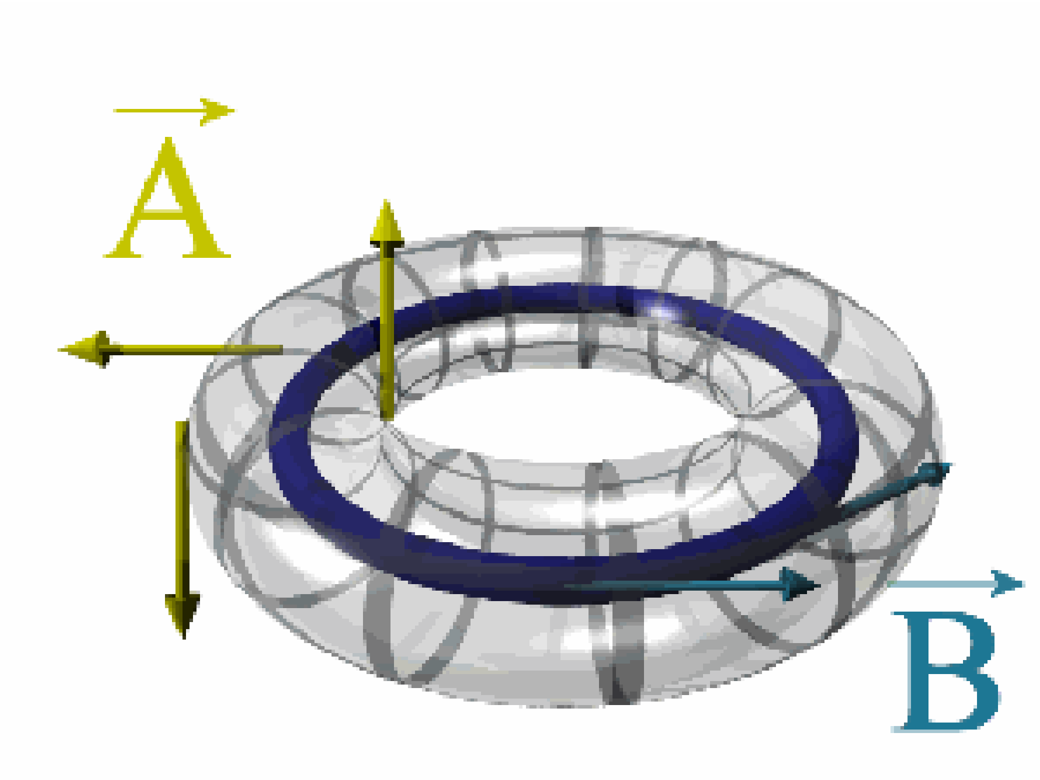
- Above discussion all at $m_q \simeq m_q^{\rm phys}$
- Explicitly makes use of a near-threshold state (large scattering lengths)
- For $m_q = m_q^{\rm lattice} \gg m_q^{\rm phys}$, scattering lengths will relax to natural size *[Beane & Savage; Epelbaum, Meißner & Glöckle]*:

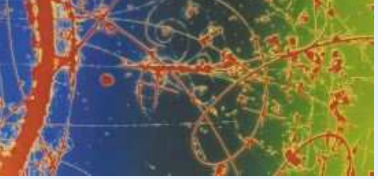


- No longer see extreme sensitivity to short-distance physics

Aharonov-Bohm effect *[Bedaque]*

- Background field: $A_\mu = (0, 0, 0, \frac{\phi}{3L})$
- Modifies two-particle energy levels (Aharonov-Bohm effect) even though $\mathbf{E} = \mathbf{B} = 0$





Aharonov-Bohm effect *[Bedaque]*

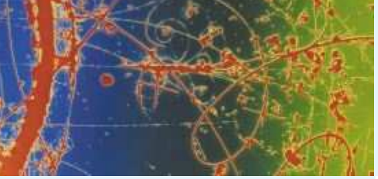
- Background field: $A_\mu = (0, 0, 0, \frac{\phi}{3L})$
- Modifies two-particle energy levels (Aharonov-Bohm effect) even though $\mathbf{E} = \mathbf{B} = 0$
- Equivalent to twisted boundary conditions (c.f. *[Petronzio et al.]*)

$$q(x + \hat{\mathbf{z}}L) = \exp(i\phi/3)q(x)$$

- Phase shifts can be probed for continuous (NN COM) momenta

$$\mathbf{q} = \frac{2\pi}{L}\left(n_1, n_2, n_3 + \frac{\phi}{2\pi}\right)$$

- Allows ground state energy level to be tuned to be within range of $\not\pi$EFT for smaller lattices
- Perhaps tune $m_q = m_q^{\text{lattice}}$ systems to large (infinite) scattering lengths too (Feshbach resonances) $\Rightarrow$ EW properties



ΛN scattering

- Important in hypernuclear structure and decays: ${}^{5}_{\Lambda}$He, ${}^{7}_{\Lambda}$Li, ${}^{12}_{\Lambda}$C *[CERN,BNL,KEK,DAΦNE]*
- Very poorly known *[CERN '60s]*

$$0 > a^{\Lambda}_{{}^1S_0} > -15\,\mathrm{fm} \qquad 0 < r^{\Lambda}_{{}^1S_0} < 15\,\mathrm{fm}$$

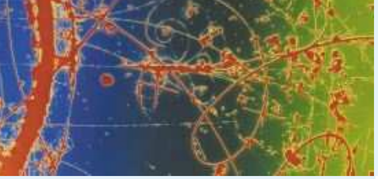
$$-0.6 > a^{\Lambda}_{{}^3S_1} > -3.2\,\mathrm{fm} \qquad 2.5 < r^{\Lambda}_{{}^3S_1} < 15\,\mathrm{fm}$$

but no reason to expect scattering lengths and effective ranges to be unnaturally large

- EFT developed *[Beane, Bedaque, Parreño & Savage]*. E.g.

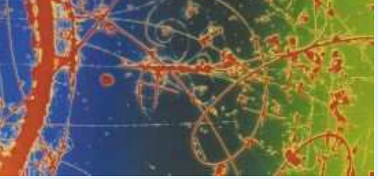
$$a^{\Lambda}_{{}^1S_0} = -\frac{\mu_{\Lambda N}}{2\pi}\left[{}_{\Lambda\Lambda}C_0^{{}^1S_0} + \ldots - \frac{3g^2_{\Sigma\Lambda}g_A^2\mu_{\Lambda N}}{4\pi f^4} f(m_\pi, \Delta_{\Lambda\Sigma})\right]$$

- Lattice calculation of ΛN energy levels (even at current masses) would improve this dramatically
- Partial-quenching might be an obstacle



Summary

- Can use lattice QCD to calculate electroweak nuclear properties of two baryon states:
  - Measure two-nucleon energy levels in background field
  - Match $\not{\pi}$EFT to energy levels to fix two-body counterterms
  - Calculate observables in $\not{\pi}$EFT
- Will allow extraction of NC and CC cross-sections for SNO
- Just a first step, much to be done:
  - Analyze higher orders (include Landau levels)
  - Extend to larger energies (include pions)
  - Polarisibilities
  - Three nucleon system
  - . . .
- Twisted boundary conditions, ΛN scattering



$\not{\pi}$EFT details

Projectors

$$P_3^j = \frac{1}{\sqrt{8}}\,\tau_2 \otimes \sigma_2 \sigma^j \qquad P_1^a = \frac{1}{\sqrt{8}}\,\tau_2 \tau^a \otimes \sigma_2$$

Dibaryon formalism

$$y_{1,3}^2 = \frac{8\pi}{M^2 r_{1,3}} \qquad \Delta_{1,3} = \frac{2}{M r_{1,3}}\left(\frac{1}{a_{1,3}} - \mu\right)$$

Binding momentum and energy of the deuteron

$$\gamma_0 - \frac{1}{a_3} - \frac{1}{2} r_3 \gamma_0^2 = 0$$

$$B = -\gamma_0^2 / M$$

Background Electroweak Fields

- Magnetic field shifts single particle mass eigenstates

$$M_{p\uparrow}(B_0) = M_0 + \mu_p B_0 + 4\pi \frac{\beta_p}{2} B_0^2 + \ldots$$

Background Electroweak Fields

- Magnetic field shifts single particle mass eigenstates

$$M_{p\uparrow}(B_0) = M_0 + \mu_p B_0 + 4\pi \frac{\beta_p}{2} B_0^2 + \ldots$$

- Landau levels: single particle states are not plane waves in transverse directions

Background Electroweak Fields

- Magnetic field shifts single particle mass eigenstates

$$M_{p\uparrow}(B_0) = M_0 + \mu_p B_0 + 4\pi \frac{\beta_p}{2} B_0^2 + \ldots$$

- Landau levels: single particle states are not plane waves in transverse directions
- Infinite volume particle moves in SHO potential:

$$\hat{H} = \frac{|\hat{\mathbf{p}}|^2}{2M} + \frac{1}{2} M\omega^2(\hat{x}^2 + \hat{y}^2) + \frac{eB_0}{2M}\,\hat{l}_z \qquad \omega = |\frac{eB_0}{2M}|$$

- Tower of eigenstates with energy shifts $\sim B_0$:

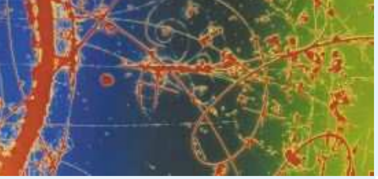
$$E^{(n)}_{p\uparrow}(B_0) = M + \frac{|eB_0|}{M}\left(n + \frac{1}{2}\right) + \frac{p_z^2}{2M} + \mu_p B_0 + 4\pi \frac{\beta_p}{2} B_0^2$$

Background Electroweak Fields

- Magnetic field shifts single particle mass eigenstates

$$M_{p\uparrow}(B_0) = M_0 + \mu_p B_0 + 4\pi \frac{\beta_p}{2} B_0^2 + \ldots$$

- Landau levels: single particle states are not plane waves in transverse directions

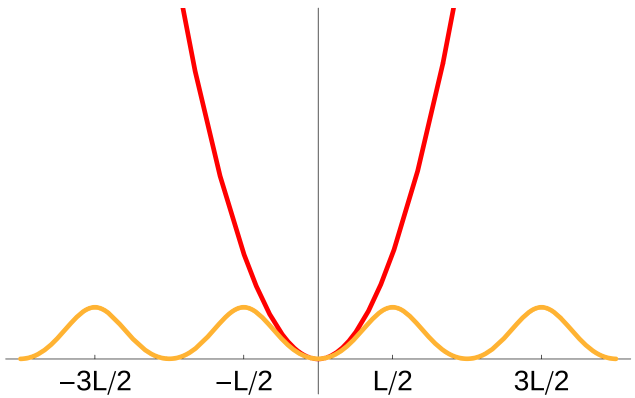


Background Electroweak Fields

- Magnetic field shifts single particle mass eigenstates

$$M_{p\uparrow}(B_0) = M_0 + \mu_p B_0 + 4\pi \frac{\beta_p}{2} B_0^2 + \ldots$$

- Landau levels: single particle states are not plane waves in transverse directions
- At finite volume potential is bounded and perturbative: shift is $\sim B_0^2$



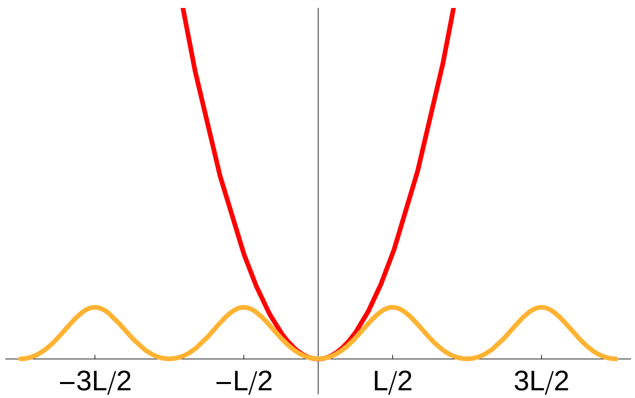
Background Electroweak Fields

NPLQCD
Electroweak NN properties
Scattering in background fields
Aharonov-Bohm effect
ΛN scattering
Summary
========================
- $\not\pi$EFT details
- Background EW fields
- Background EW fields
- BF in Lattice QCD
- Asymmetric boxes
- Asymmetric scattering
- Integral representation
- Asymmetric asymptotics
- Magic boxes

- Magnetic field shifts single particle mass eigenstates

$$M_{p\uparrow}(B_0) = M_0 + \mu_p B_0 + 4\pi \frac{\beta_p}{2} B_0^2 + \ldots$$

- Landau levels: single particle states are not plane waves in transverse directions
- At finite volume potential is bounded and perturbative: shift is $\sim B_0^2$

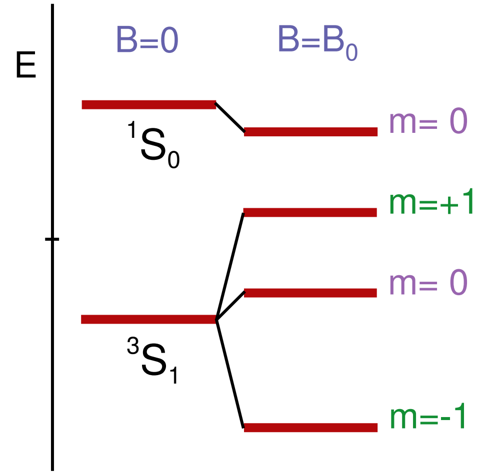


- Need ΔE_{LL} to be small compared to spacing of energy levels

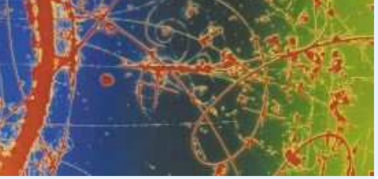
$$|eB_0| \ll \frac{8\sqrt{3}\pi}{L_\perp^2}$$

Background Electroweak Fields

- Magnetic fields: split two particle mass eigenstates



- Two-particle states also have Landau like effects
- Deuteron deformed
- Weak background field gives similar shifts
- No weak Landau levels



Background Fields in Lattice QCD

- Magnetic fields

$$U_\mu(x) \longrightarrow U_\mu^{\rm ext}(x) U_\mu(x)$$

$$U_{0,3}^{ext}(x) = 1 \qquad U_1^{ext}(x) = e^{+i\beta x_2} \qquad U_2^{ext}(x) = e^{-i\beta x_1}$$

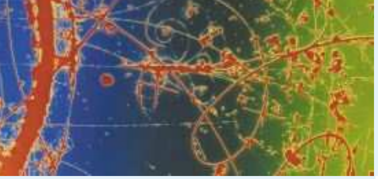
- generates $\mathbf{B} = B_0 \mathbf{e}_z$, $eB_0 a^2 \sim \beta$
- Homogeneity requires $eB_0 A_{xy}/2\pi \in \mathbb{Z}$
- Have been used to compute:
  - Magnetic moments
  - Electric and magnetic polarisabilities
  - Neutron EDM (unsuccessfully)
- Weak background fields can be implemented: couple differently to left- and right- chirality quarks
  - eg: DW fermions with background field varying in 5th direction
  - used in first calculation of g_A

Asymmetric boxes

- Magnetic field introduce a problem: Landau levels

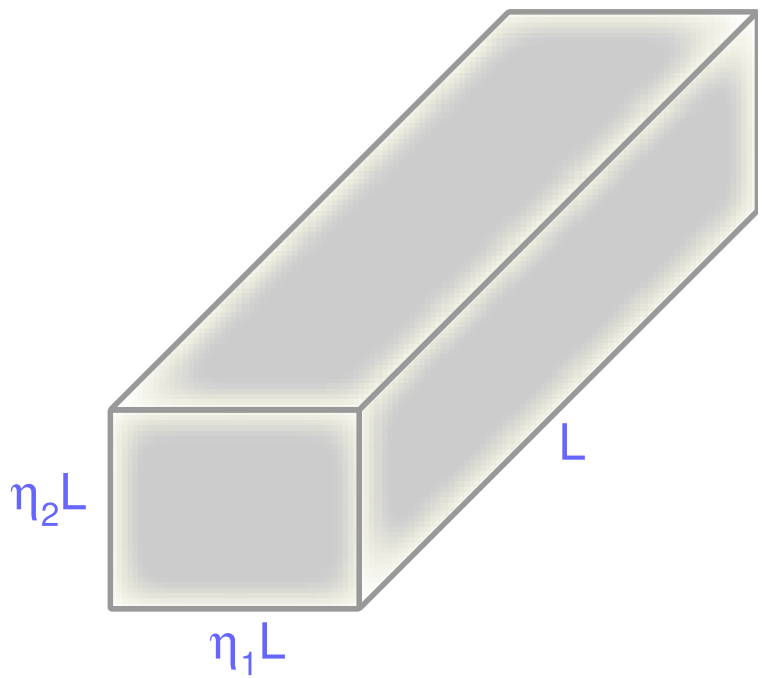
Asymmetric boxes

- Magnetic field introduce a problem: Landau levels
- These effects can be controlled by . . .



Asymmetric boxes

- Magnetic field introduce a problem: Landau levels
- These effects can be controlled by . . .



- Small transverse directions (~ 4 fm) move Landau levels up in spectrum
- Low lying excitations are all longitudinal

Scattering in asymmetric boxes

- Momentum mode sum:

$$S(\tilde{p}^2) \longrightarrow S(\tilde{p}^2; \eta_1, \eta_2) = \frac{1}{\eta_1 \eta_2} \sum_{\tilde{\mathbf{n}}}^{\Lambda_n} \frac{1}{|\tilde{\mathbf{n}}|^2 - \tilde{p}^2} - 4\pi \Lambda_n$$

where

$$\tilde{\mathbf{n}} = (\frac{1}{\eta_1} n_1, \frac{1}{\eta_2} n_2, n_3)$$

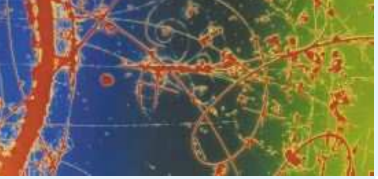
- Energy levels now given by eigenvalues of

$$p \ \cot \delta(p) - \frac{1}{\pi L} S(\tilde{p}^2; \eta_1, \eta_2) = 0$$

Integral representation of ζ functions

- Momentum mode sums are slowly convergent (or divergent)

$$S(\tilde{p}^2; \eta_1, \eta_2; s) = \sum_{|\tilde{\mathbf{n}}|^2 \neq \tilde{p}^2} \frac{1}{(|\tilde{\mathbf{n}}|^2 - \tilde{p}^2)^s}$$



Integral representation of ζ functions

- Momentum mode sums are slowly convergent (or divergent)

$$S(\tilde{p}^2;\eta_1,\eta_2;s) = \sum_{|\tilde{\mathbf{n}}|^2\neq\tilde{p}^2} \frac{1}{(|\tilde{\mathbf{n}}|^2-\tilde{p}^2)^s}$$

- Using Poisson summation formula $\Longrightarrow$

$$S(\tilde{p}^2;\eta_1,\eta_2;s) = \sum_{|\tilde{\mathbf{n}}|^2\neq\tilde{p}^2} \frac{E_{1-s}\left(|\tilde{\mathbf{n}}|^2-|\mathbf{m}|^2\right)}{\Gamma(s)} - \frac{n_{\rm deg}(\eta_1,\eta_2)}{s\,\Gamma(s)}$$

$$+\frac{\eta_1\eta_2\pi^{\frac{3}{2}}}{\Gamma(s)}\left\{\int_0^1 dt\, t^{s-\frac{5}{2}} e^{t\tilde{p}^2} \sum_{|\mathbf{n}|^2\neq 0} e^{-\frac{\pi^2|\overline{\mathbf{n}}|^2}{t}} + \frac{{}_1F_1(s-\frac{3}{2},s-\frac{1}{2},\tilde{p}^2)}{s-3/2}\right\}$$

where $n_{\rm deg}(\eta_1,\eta_2)=\sum_{|\tilde{\mathbf{n}}|^2=\tilde{p}^2}$ and $|\overline{\mathbf{n}}|^2=\eta_1^2 n_1^2+\eta_2^2 n_2^2+n_3^2$

- Remaining sums converge exponentially

Asymmetric asymptotics

- Asymptotic expansions are simple generalisations
- Bound states:

$$E_{-1}^{(^3S_1)} = \frac{\gamma_0^2}{M}\left[1 + \frac{4}{\gamma_0 L(1-\gamma_0 r_3)}\left(e^{-\gamma_0 L} + \frac{e^{-\gamma_0\eta_1 L}}{\eta_1} + \frac{e^{-\gamma_0\eta_2 L}}{\eta_2}\right)\right]$$

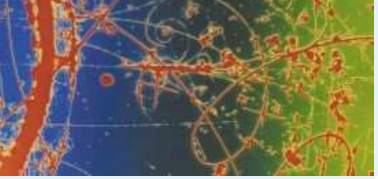
- Lowest continuum [$\mathbf{p} = (0,0,0)$]

$$E_0 = \frac{4\pi a}{\eta_1\eta_2 M L^3}\left[1 - c_1(\eta_1,\eta_2)\left(\frac{a}{L}\right) + c_2(\eta_1,\eta_2)\left(\frac{a}{L}\right)^2 + \ldots\right]$$

- Second continuum [$\mathbf{p} = (0,0,\pm\frac{2\pi}{L})$ for $\eta_{1,2} \leq 1$]

$$E_1 = \frac{4\pi^2}{ML^2} - \frac{4d\tan\hat{\delta}}{\eta_1\eta_2 M L^2}\left[1 + c_1'(\eta_1,\eta_2)\tan\hat{\delta} + c_2'(\eta_1,\eta_2)\tan^2\hat{\delta} + ..\right]$$

[$\hat{\delta} = \delta(|\mathbf{p}|^2 = 4\pi^2/L^2)$, and d is a degeneracy factor]



Asymmetric asymptotics

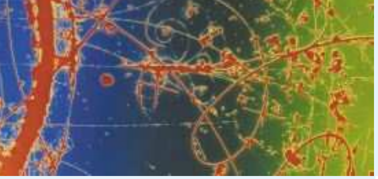
- Lüscher (Zucker) coefficients:

$$c_1(\eta_1,\eta_2) = \frac{1}{\pi}\left(\frac{1}{\eta_1\eta_2}\sum_{\tilde{\mathbf{n}}\neq 0}^{\Lambda_n}\frac{1}{|\tilde{\mathbf{n}}|^2} - 4\pi\Lambda_n\right)$$

$$c_2(\eta_1,\eta_2) = c_1^2(\eta_1,\eta_2) - \frac{1}{\pi^2\eta_1^2\eta_2^2}\sum_{\tilde{\mathbf{n}}\neq 0}^{\Lambda_n}\frac{1}{|\tilde{\mathbf{n}}|^4}$$

$$c_1'(\eta_1,\eta_2) = \frac{1}{2\pi^2}\left(\frac{1}{\eta_1\eta_2}\sum_{\tilde{\mathbf{n}}\neq 1}^{\Lambda_n}\frac{1}{|\tilde{\mathbf{n}}|^2 - 1} - 4\pi\Lambda_n\right)$$

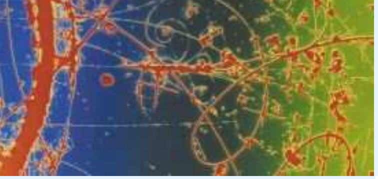
$$c_2'(\eta_1,\eta_2) = (c_1'(\eta_1,\eta_2))^2 - \frac{1}{2\pi^4\eta_1^2\eta_2^2}\sum_{\tilde{\mathbf{n}}\neq 1}^{\Lambda_n}\frac{1}{(|\tilde{\mathbf{n}}|^2 - 1)^2}$$



Asymmetric asymptotics

- Lüscher (Zucker) coefficients:

η_1, η_2	$c_1(\eta_1, \eta_2)$	$c_2(\eta_1, \eta_2)$	$c_1'(\eta_1, \eta_2)$	$c_2'(\eta_1, \eta_2)$	d
0.1 , 0.1	65.7171	2111.12	-3.60224	-52.944	1
0.2 , 0.2	6.67861	-99.6627	-2.3242	0.950737	1
0.5 , 0.5	-3.61168	6.65945	0.73400	0.24642	1
1 , 1	-2.837297	6.375183	-0.061367	-0.354156	3
2 , 2	-1.466654	1.278623			2
2 , 1	-1.805872	1.664979			1
1.5 , 1.25	-2.200918	3.647224			1



Magic boxes

NPLQCD

Electroweak NN properties

Scattering in background fields

Aharonov-Bohm effect

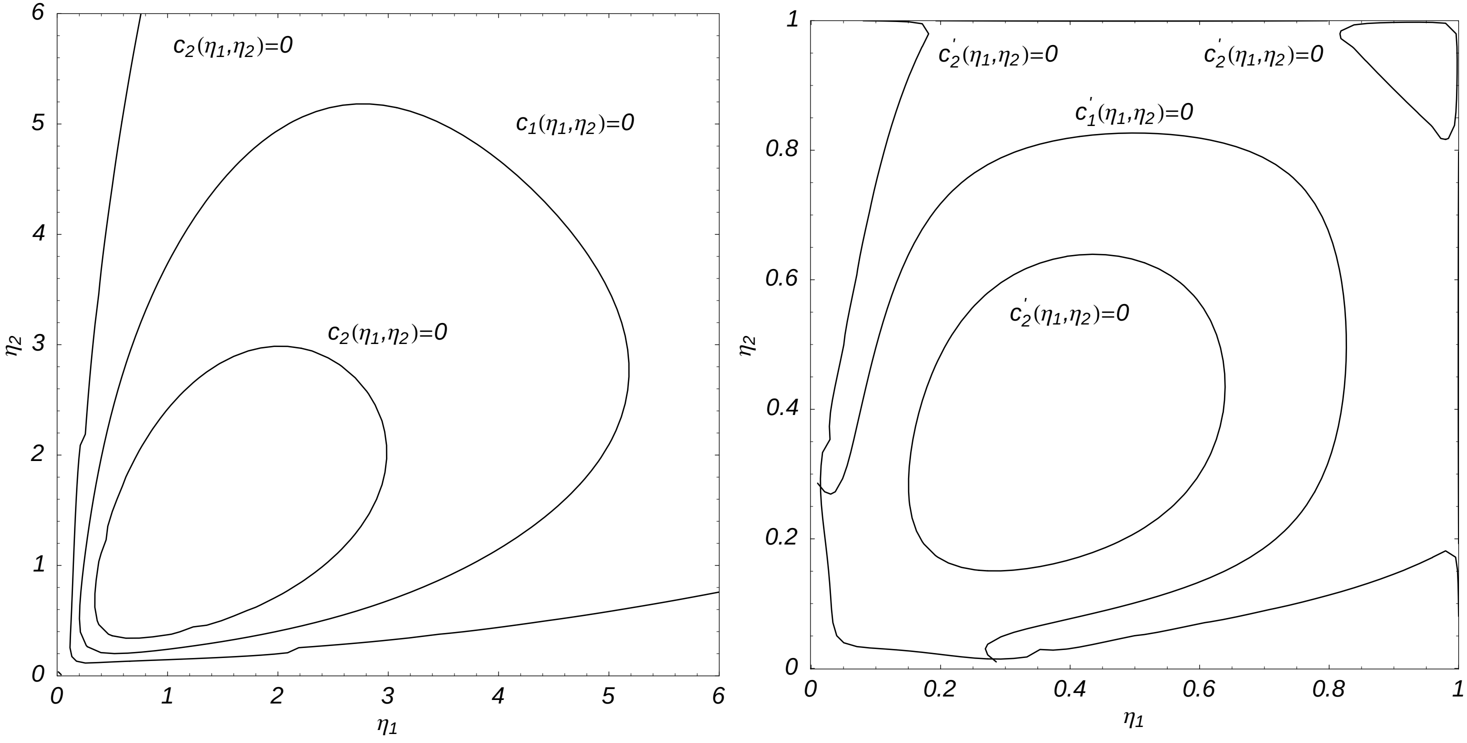
ΛN scattering

Summary

========================

- $\not\pi$EFT details
- Background EW fields
- Background EW fields
- BF in Lattice QCD
- Asymmetric boxes
- Asymmetric scattering
- Integral representation
- Asymmetric asymptotics
- Magic boxes

- Can find magic boxes such that subleading finite volume effects are supressed: $c_i^{(\prime)}(\eta_1\eta_2) = 0$



$\Lambda_Q \Lambda_Q$ potential

- For infinitely heavy Qqq baryons, a potential is well-defined
- LHP collaboration will hopefully measure it ($Q\bar{q}$–$Q\bar{q}$ already studied *[Richards et al.; Markum et al.]*)
- NLO pionful EFT analysis *[Arndt, Beane & Savage '03]*