Proton Decay Matrix Elements from Lattice QCD

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for CP-PACS/JLQCD Collabs.

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0. CP-PACS/JLQCD Collabs.

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projects

- 2+1 flavor full QCD (SW + RG) $\quad m_s = 80.2(2.6)^{+8.6}_{-0.5}$ MeV
  ● [illegible]

  $$\left.\begin{array}{ll} a \approx 0.12\ \text{fm} & 16^3 \times 32 \\ a \approx 0.1\ \text{fm} & 20^3 \times 40 \\ \hline a \approx 0.07\ \text{fm} & 28^3 \times 56 \end{array}\right\} \quad m_\pi/m_\rho \approx 0.6 - 0.8$$

- heavy quark physics
  charm physics using relativistic heavy quark action

- proton decay matrix elements

- π-π scattering
  phase shift in $I = 2$ channel

- NEDM
  some preliminary results, not yet completed

- running coupling, nonperturbative Z
  steady progress

I. Introduction

exciting prediction of (SUSY-)GUTs

Experiment

efforts to push lower limit on partial life time of nucleon

Shiozawa, ICHEP2000

- $p \to \pi^0 e^+$ $\quad \tau/B(p \to \pi^0 e^+) > 4.4 \times 10^{33}$ yrs
- $p \to K^+ \bar{\nu}$ $\quad \tau/B(p \to K^+ \bar{\nu}) > 1.9 \times 10^{33}$ yrs

improvement by Super-Kamiokande

$\to$ strong constraints on (SUSY-)GUTs

Theory

one of the main sources of uncertainties

$$\langle PS | \mathcal{O}^{\not B} | N \rangle$$

predictions with various QCD models scatter over wide range

$$\mathrm{max}/\mathrm{min} \sim 10 \quad , \quad \Gamma \propto |\langle PS | \mathcal{O}^{\not B} | N \rangle|^2$$

$\to$ important to determine $\langle PS | \mathcal{O}^{\not B} | N \rangle$ from first principles

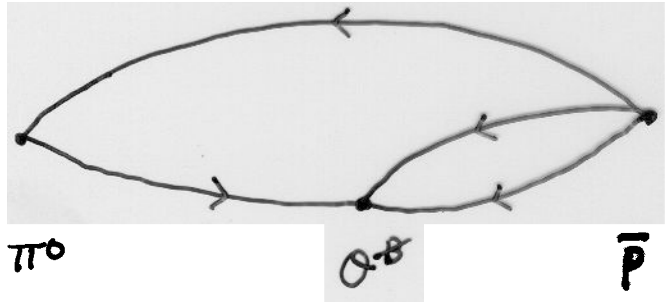
using lattice QCD

Our purpose

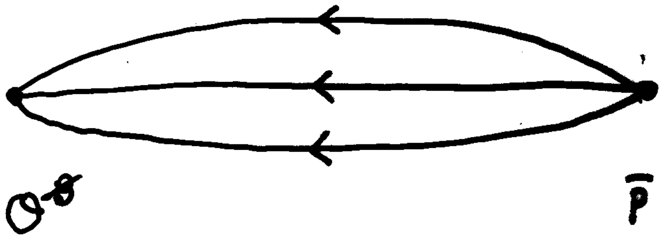
- comparison of direct and indirect methods

  ex. $p \to \pi^0 e^+$ decay mode

$$\langle \pi^0 | \mathcal{O}^B | p \rangle = \langle 0 | \mathcal{O}^B | p \rangle \frac{1}{\sqrt{2} f} (1 + F + D) + O(p_\pi^2)$$



$\to \langle \pi^0 | \mathcal{O}^B | p \rangle$



$\to \langle 0 | \mathcal{O}^B | p \rangle$

- GUT-model-independent calculation of $\langle PS | \mathcal{O}^B | N \rangle$

$p \to \pi^0 e^+$	minimal SU(5) GUT
$p \to K^+ \bar{\nu}$	minimal SUSY SU(5) GUT
⋮	⋮

$\to$ enumerate independent matrix elements

for $(p, n) \to (\pi, K, \eta) + (\bar{\nu}, e^+, \mu^+)$

II. GUT-Model-Independent Analysis on $\langle PS|\mathcal{O}^{B}|N\rangle$

independent matrix elements for nucleon decays

$(p, n) \rightarrow (\pi, K, \eta) + (\bar{\nu}, e^{+}, \mu^{+})$

III. Indirect Method

chiral lagrangian + $\langle 0|\mathcal{O}^{B}|N\rangle$ with lattice QCD

IV. Direct Method

$\langle PS|\mathcal{O}^{B}|N\rangle$ from 3-point function

V. Continuum limit of $\langle 0|\mathcal{O}^{B}|N\rangle$ in quenched QCD

VI. Summary

II. GUT-Model-Independent Analysis on $\langle PS | \mathcal{O}^{B} | N \rangle$

Complete set of dim.=6 operators

low energy effective Hamiltonian is described by

$$\underbrace{SU(3)}_{\text{strong}} \times \underbrace{SU(2) \times U(1)}_{\text{electroweak}} \text{ gauge symmetry}$$

dim.=6 is lowest

$$\varepsilon_{ijk} Q^{i} Q^{j} Q^{k} L$$

Weinberg
Wilczek-Zee
Abbott-Wise

$$\mathcal{O}^{(1)}_{abcd} = (\bar{D}^{c}_{iaR} U_{jbR})(\bar{Q}^{c}_{k\alpha cL} L_{\beta dL})\, \varepsilon_{ijk} \varepsilon_{\alpha\beta}$$

$$\mathcal{O}^{(2)}_{abcd} = (\bar{Q}^{c}_{i\alpha aL} Q_{j\beta bL})(\bar{U}^{c}_{kcR} L_{dR})\, \varepsilon_{ijk} \varepsilon_{\alpha\beta}$$

$$\mathcal{O}^{(3)}_{abcd} = (\bar{Q}^{c}_{i\alpha aL} Q_{j\beta bL})(\bar{Q}^{c}_{k\gamma cL} L_{\delta dL})\, \varepsilon_{ijk} \varepsilon_{\alpha\delta} \varepsilon_{\beta\gamma}$$

$$\mathcal{O}^{(4)}_{abcd} = (\bar{D}^{c}_{iaR} U_{jbR})(\bar{U}^{c}_{kcR} L_{dR})\, \varepsilon_{ijk}$$

$$\bar{\psi}^{c} = \psi^{T} C, \quad P_{R,L} = \frac{1 \pm \gamma_5}{2}$$

i, j, k : $SU(3)$

$\alpha, \beta, \gamma, \delta$: $SU(2)$

a, b, c, d : generation

cf. Fierz transf. for vector and tensor structures

Relevant operators for non-strange final states

$$O_d^{(1)} = (\bar{d}^c_{iR} u_{jR})(\bar{u}^c_{kL} e_{dL} - \bar{d}^c_{kL} \nu_{dL}) \varepsilon_{ijk}$$

$$O_d^{(2)} = (\bar{d}^c_{iL} u_{jL})(\bar{u}^c_{kR} e_{dR}) \varepsilon_{ijk}$$

$$O_d^{(3)} = (\bar{d}^c_{iL} u_{jL})(\bar{u}^c_{kL} e_{dL} - \bar{d}^c_{kL} \nu_{dL}) \varepsilon_{ijk}$$

$$O_d^{(4)} = (\bar{d}^c_{iR} u_{jR})(\bar{u}^c_{kR} e_{dR}) \varepsilon_{ijk}$$

Relevant operators for strange final states

$$\tilde{O}_d^{(1)} = (\bar{s}^c_{iR} u_{jR})(\bar{u}^c_{kL} e_{dL} - \bar{d}^c_{kL} \nu_{dL}) \varepsilon_{ijk}$$

$$\tilde{O}_d^{(2)} = (\bar{s}^c_{iL} u_{jL})(\bar{u}^c_{kR} e_{dR}) \varepsilon_{ijk}$$

$$\tilde{O}_d^{(3)} = (\bar{s}^c_{iL} u_{jL})(\bar{u}^c_{kL} e_{dL} - \bar{d}^c_{kL} \nu_{dL}) \varepsilon_{ijk}$$

$$\tilde{O}_d^{(4)} = (\bar{s}^c_{iR} u_{jR})(\bar{u}^c_{kR} e_{dR}) \varepsilon_{ijk}$$

$$\tilde{O}_d^{(5)} = (\bar{d}^c_{iR} u_{jR})(\bar{s}^c_{kL} \nu_{dL}) \varepsilon_{ijk}$$

$$\tilde{O}_d^{(6)} = (\bar{d}^c_{iL} u_{jL})(\bar{s}^c_{kL} \nu_{dL}) \varepsilon_{ijk}$$

where d denotes generation

$$e_1 = e, \quad \nu_1 = \nu_e$$

$$e_2 = \mu, \quad \nu_2 = \nu_\mu$$

Independent matrix elements

$$N \to PS + \bar{l}$$
$$(p, n) \quad (\pi, K, \eta) \quad (\bar{\nu}, e^+, \mu^+)$$

assumption of isospin symmetry ($m_u = m_d$)

$$\langle \pi^0 | \varepsilon_{ijk}(u^i C P_{R,L} d^j) P_L u^k | p \rangle$$
$$\langle \pi^+ | \varepsilon_{ijk}(u^i C P_{R,L} d^j) P_L d^k | p \rangle$$
$$\langle K^0 | \varepsilon_{ijk}(u^i C P_{R,L} s^j) P_L u^k | p \rangle$$
$$\langle K^+ | \varepsilon_{ijk}(u^i C P_{R,L} s^j) P_L d^k | p \rangle$$
$$\langle K^+ | \varepsilon_{ijk}(u^i C P_{R,L} d^j) P_L s^k | p \rangle$$
$$\langle K^0 | \varepsilon_{ijk}(u^i C P_{R,L} s^j) P_L d^k | n \rangle$$
$$\langle \eta | \varepsilon_{ijk}(u^i C P_{R,L} d^j) P_L u^k | p \rangle$$

$|p\rangle \to -|n\rangle, \; |n\rangle \to -|p\rangle$

$\langle \pi^+ | \to \langle \pi^- |, \; \langle \pi^0 | \to -\langle \pi^0 |$

$\langle K^+ | \to \langle K^0 |, \; \langle K^0 | \to \langle K^+ |$

$\langle \eta | \to \langle \eta |$

where

$$\left.\begin{aligned} \langle PS | \mathcal{O}_{LR} | N \rangle &= \langle PS | \mathcal{O}_{RL} | N \rangle \\ \langle PS | \mathcal{O}_{RR} | N \rangle &= \langle PS | \mathcal{O}_{LL} | N \rangle \end{aligned}\right\} \text{parity conservation}$$

All we have to calculate in lattice QCD are these 14 matrix elements

indirect method — chiral lagrangian + $\langle 0 | \mathcal{O}^{\not{B}} | N \rangle$ with lattice

direct method — $\langle PS | \mathcal{O}^{\not{B}} | N \rangle$ with lattice

III. Indirect Method

III-1. Tree-level results of chiral lagrangian

Claudson-Wise-Hall

fields

$$\Sigma = \exp\left(\frac{2i\phi}{f}\right) \quad \text{with} \quad \phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$$

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda^0 \end{pmatrix}$$

transformations

$$\Sigma \to L\Sigma R^\dagger \qquad L \in SU(3)_L,\ R \in SU(3)_R$$

$$B \to UBU^\dagger \qquad U \in SU(3)_V$$

Action

lowest order of $SU(3)_L \times SU(3)_R$ invariant chiral lagrangian

$$\mathcal{L}_0 = \frac{f^2}{8} \mathrm{Tr}(\partial_\mu \Sigma)(\partial_\mu \Sigma^\dagger) + \mathrm{Tr}\,\bar{B}(\gamma_\mu \partial_\mu + M_B)B$$

$$+ \frac{1}{2}\mathrm{Tr}\,\bar{B}\gamma_\mu[\xi \partial_\mu \xi^\dagger + \xi^\dagger \partial_\mu \xi]B + \frac{1}{2}\mathrm{Tr}\,\bar{B}\gamma_\mu B[(\partial_\mu \xi)\xi^\dagger + (\partial_\mu \xi^\dagger)\xi]$$

$$- \frac{1}{2}(D-F)\,\mathrm{Tr}\,\bar{B}\gamma_\mu\gamma_5 B[(\partial_\mu \xi)\xi^\dagger - (\partial_\mu \xi^\dagger)\xi]$$

$$+ \frac{1}{2}(D+F)\,\mathrm{Tr}\,\bar{B}\gamma_\mu\gamma_5[\xi \partial_\mu \xi^\dagger - \xi^\dagger \partial_\mu \xi]B$$

$\xi^2 = \Sigma$, $\xi \to L\xi U^\dagger = U\xi R^\dagger$ under $SU(3)_L \times SU(3)_R$

Symmetry-breaking term due to quark mass

$$\mathcal{L}_1 = -v^3\,\mathrm{Tr}[\Sigma^\dagger M_q + M_q \Sigma]$$

$$- a_1 \mathrm{Tr}\,\bar{B}(\xi^\dagger M_q \xi^\dagger + \xi M_q \xi)B - a_2 \mathrm{Tr}\,\bar{B}B(\xi^\dagger M_q \xi^\dagger + \xi M_q \xi)$$

$$- b_1 \mathrm{Tr}\,\bar{B}\gamma_5(\xi^\dagger M_q \xi^\dagger - \xi M_q \xi)B - b_2 \mathrm{Tr}\,\bar{B}\gamma_5 B(\xi^\dagger M_q \xi^\dagger - \xi M_q \xi)$$

Semileptonic baryon decays $\longrightarrow F=0.47,\ D=0.80$

$$v = \frac{f\, m_\pi^2}{4(m_u + m_d)}$$

octet baryon mass splitting $\longrightarrow a_1 \approx -0.$[illegible], $a_2 \approx 0.$[illegible]

b_1, b_2 are not well determined

Construction of operators

under $SU(3)_L \times SU(3)_R$

$$\left.\begin{array}{l} (3,\bar{3}) : \mathcal{O}_d^{(1)},\ \tilde{\mathcal{O}}_d^{(1)},\ \tilde{\mathcal{O}}_d^{(5)} \\ (\bar{3},3) : \mathcal{O}_d^{(2)},\ \tilde{\mathcal{O}}_d^{(2)} \end{array}\right\} \text{parameter } \alpha$$

$$\left.\begin{array}{l} (8,1) : \mathcal{O}_d^{(3)},\ \tilde{\mathcal{O}}_d^{(3)},\ \tilde{\mathcal{O}}_d^{(6)} \\ (1,8) : \mathcal{O}_d^{(4)},\ \tilde{\mathcal{O}}_d^{(4)} \end{array}\right\} \text{parameter } \beta$$

$\xi B \xi \in (3,\bar{3})$, $\xi^\dagger B \xi^\dagger \in (\bar{3},3)$, $\xi B \xi^\dagger \in (8,1)$, $\xi^\dagger B \xi \in (1,8)$

ex.

$$\mathcal{O}_d^{(1)} = \alpha\left[\bar{e}^c_{dL}\,\mathrm{Tr}(F\xi B_L \xi) - \bar{\nu}^c_{dL}\,\mathrm{Tr}(F'\xi B_L \xi)\right]$$

$$\mathcal{O}_d^{(2)} = \alpha\,\bar{e}^c_{dR}\,\mathrm{Tr}(F\xi^\dagger B_R \xi^\dagger)$$

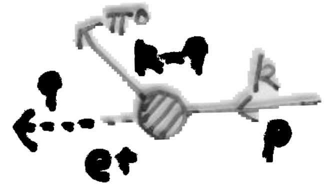
$$\mathcal{O}_d^{(3)} = \beta\left[\bar{e}^c_{dL}\,\mathrm{Tr}(F\xi B_L \xi^\dagger) - \bar{\nu}^c_{dL}\,\mathrm{Tr}(F'\xi B_L \xi^\dagger)\right]$$

$$\mathcal{O}_d^{(4)} = \beta\,\bar{e}^c_{dR}\,\mathrm{Tr}(F\xi^\dagger B_R \xi)$$

$$F = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad F' = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

Results for matrix elements

ex. $p \to \pi^0 e^+$, $\mathcal{O}_d^{(1)} = \alpha \bar{e}^c_{dL} \mathrm{Tr}(F \xi B_L \xi)$



$\langle \pi^0 | \alpha \mathrm{Tr}(F \xi B_L \xi) | p \rangle$

$$= \alpha P_L u_p \left[\frac{1}{\sqrt{2} f} - \frac{D+F}{\sqrt{2} f} \frac{-q^2 + m_N^2}{-q^2 - m_N^2} - \frac{4 b_1}{\sqrt{2} f} \frac{m_u m_N}{-q^2 - m_N^2} \right]$$

$$- \alpha P_L i \slashed{q} u_p \left[\frac{D+F}{\sqrt{2} f} \frac{2 m_N}{-q^2 - m_N^2} + \frac{4 b_1}{\sqrt{2} f} \frac{m_u}{-q^2 - m_N^2} \right]$$

on-shell condition for out-going leptons

$$-q^2 = m_e^2, \quad i \slashed{q} v_e = m_e v_e$$

$\bar{e}^c \langle \pi^0 | \alpha \mathrm{Tr}(F \xi B_L \xi) | p \rangle$

$$= \alpha v_e C P_L u_p \left[\frac{1}{\sqrt{2} f} - \frac{D+F}{\sqrt{2} f} \frac{m_e^2 + m_N^2}{m_e^2 - m_N^2} - \frac{4 b_1}{\sqrt{2} f} \text{[redacted]} \right]$$

$$+ \alpha v_e C P_R u_p \left[\frac{1}{\sqrt{2} f} \frac{2 m_N m_e}{m_e^2 - m_N^2} + \frac{4 b_1}{\sqrt{2} f} \frac{m_u m_e}{m_e^2 - m_N^2} \right]$$

$m_u, m_e \ll m_N$, assume $b_1, b_2 \sim O(1)$

$$\bar{e}^c \langle \pi^0 | \alpha \mathrm{Tr}(F \xi B_L \xi) | p \rangle \simeq \alpha v_e C P_L u_p \frac{1}{\sqrt{2} f} (1 + D + F)$$

$\to$ unknown parameters α, β?

Definitions of α, β

$$
\mathcal{O}_d^{(1)} = \begin{cases} (\bar{d}^c_{iR} u_{jR})(\bar{u}^c_{kL} e_{dL} - \bar{d}^c_{kL} \nu_{dL})\, \varepsilon_{ijk} & \text{in quark fields} \\ \alpha \left[\bar{e}^c_{dL} \mathrm{Tr}(F \xi B_L \xi) - \bar{\nu}^c_{dL} \mathrm{Tr}(F' \xi B_L \xi) \right] & \text{in chiral lagrangian} \end{cases}
$$

$$
\langle 0 | \mathcal{O}_d^{(1)} | P \rangle \to \langle 0 | \varepsilon_{ijk} (u^{iT} C P_R d^j) P_L u^k | P^{(s)} \rangle = \alpha P_L u^{(s)}
$$

$$
\langle 0 | \mathcal{O}_d^{(3)} | P \rangle \to \langle 0 | \varepsilon_{ijk} (u^{iT} C P_L d^j) P_L u^k | P^{(s)} \rangle = \beta P_L u^{(s)}
$$

We can determine α, β with lattice QCD

NLO effects in indirect method

$$
\left(\frac{\vec{p}_\pi}{\Lambda_\chi} \right)^2 \simeq \left(\frac{m_N/2}{4\pi f} \right)^2 \sim 0.25
$$

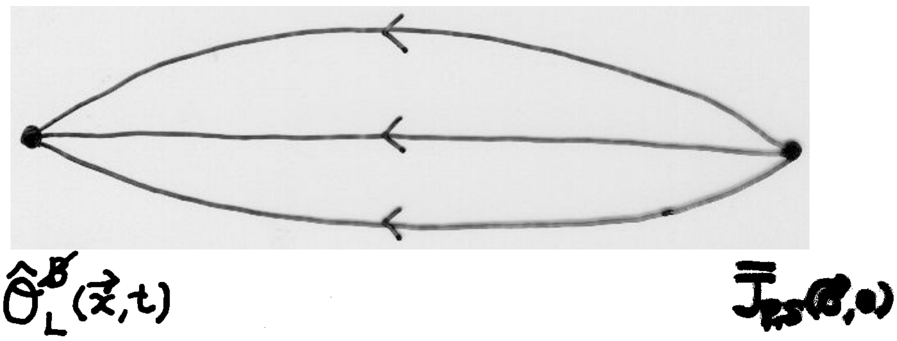
Simulation parameters

Wilson quark action + plaquette gauge action, quench

$\beta = 6.0$, $L_x \times L_y \times L_z \times L_t = 28 \times 28 \times 48 \times 80$, 100 config.

$K = 0.15620,\ 0.15568,\ 0.15516,\ 0.15464\ (\approx \frac{3}{8} m_s - \frac{9}{8} m_s)$

$a^{-1} = 2.30(4)$ GeV from $m_\rho = 770$ MeV



Results for α, β

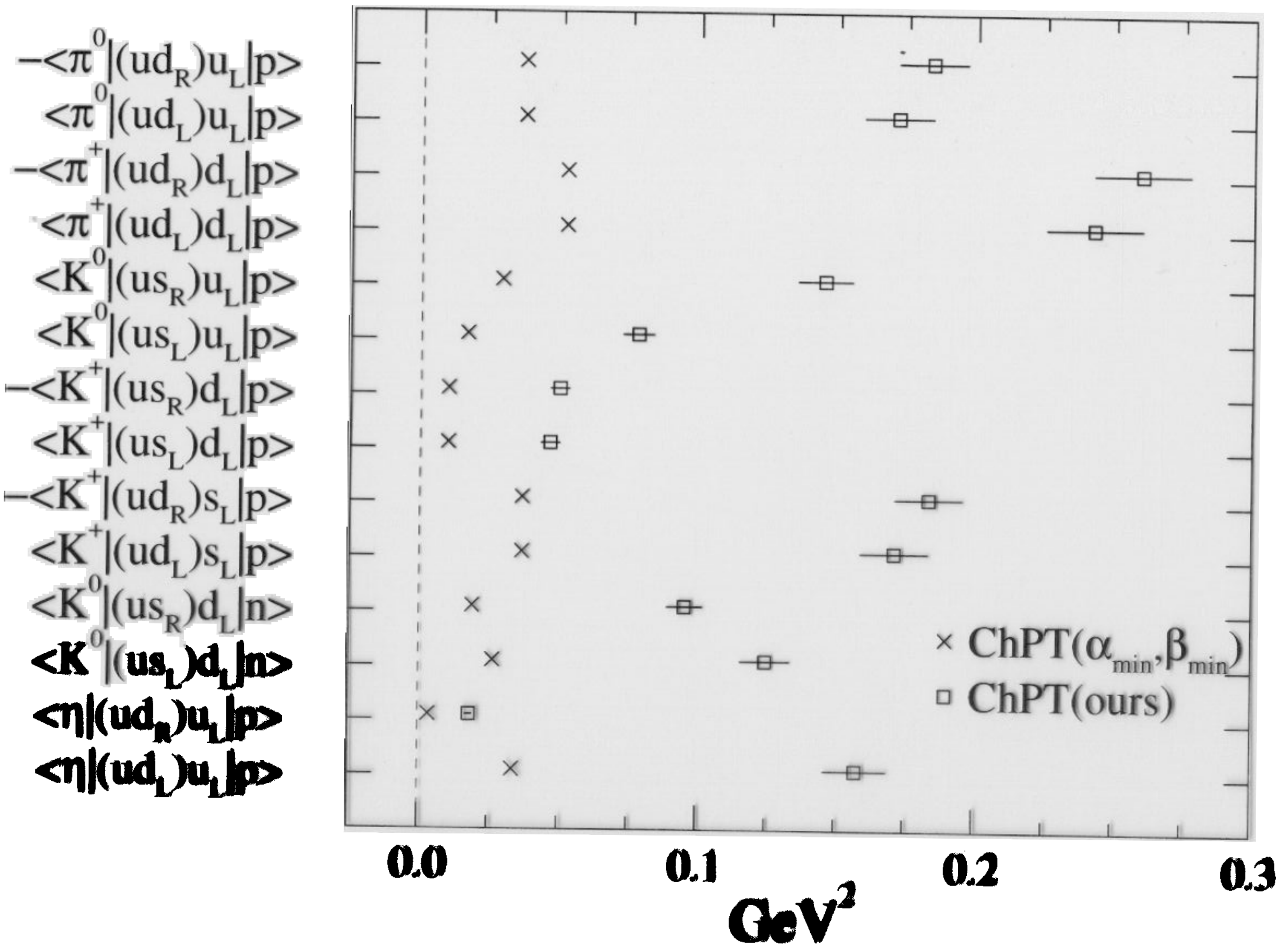
$\alpha(NDR, \frac{1}{a}) = -0.015(1)$ GeV^3

$\beta(NDR, \frac{1}{a}) = 0.014(1)$ GeV^3

five times larger than $\underbrace{|\alpha| = |\beta| = 0.003 \text{ GeV}^3}$

Smallest among various QCD models

MIT bag, Donoghue-Golowich ('82)

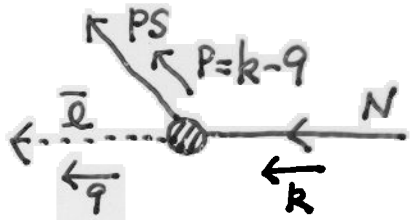


$|\alpha_{min}| = |\beta_{min}| = 0.003\ GeV^3$

IV. Direct Method

defects of indirect method can be overcome

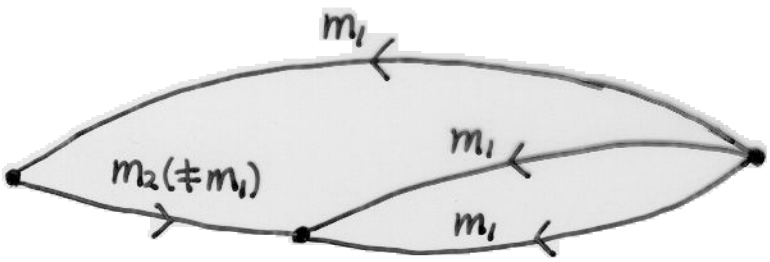
Form factors



$$\langle PS(\vec{p})|\mathcal{O}_L^{\not B}|N^{(S)}(\vec{k})\rangle = P_L W_0(q^2)u^{(S)} - P_L W_q(q^2)\, i\not{q}\, u^{(S)}$$

W_q term is negligible after the multiplication of $\bar{\nu}_L$ ([illegible])

$W_0(q^2)$ and $W_q(q^2)$ should be disentangled in lattice calculation



$J_{PS}(\vec{x}',t')$ $\qquad$ $\hat{\mathcal{O}}_L^{\not B}(\vec{x},t)$ $\qquad$ $\bar{J}_N(\vec{0},0)$

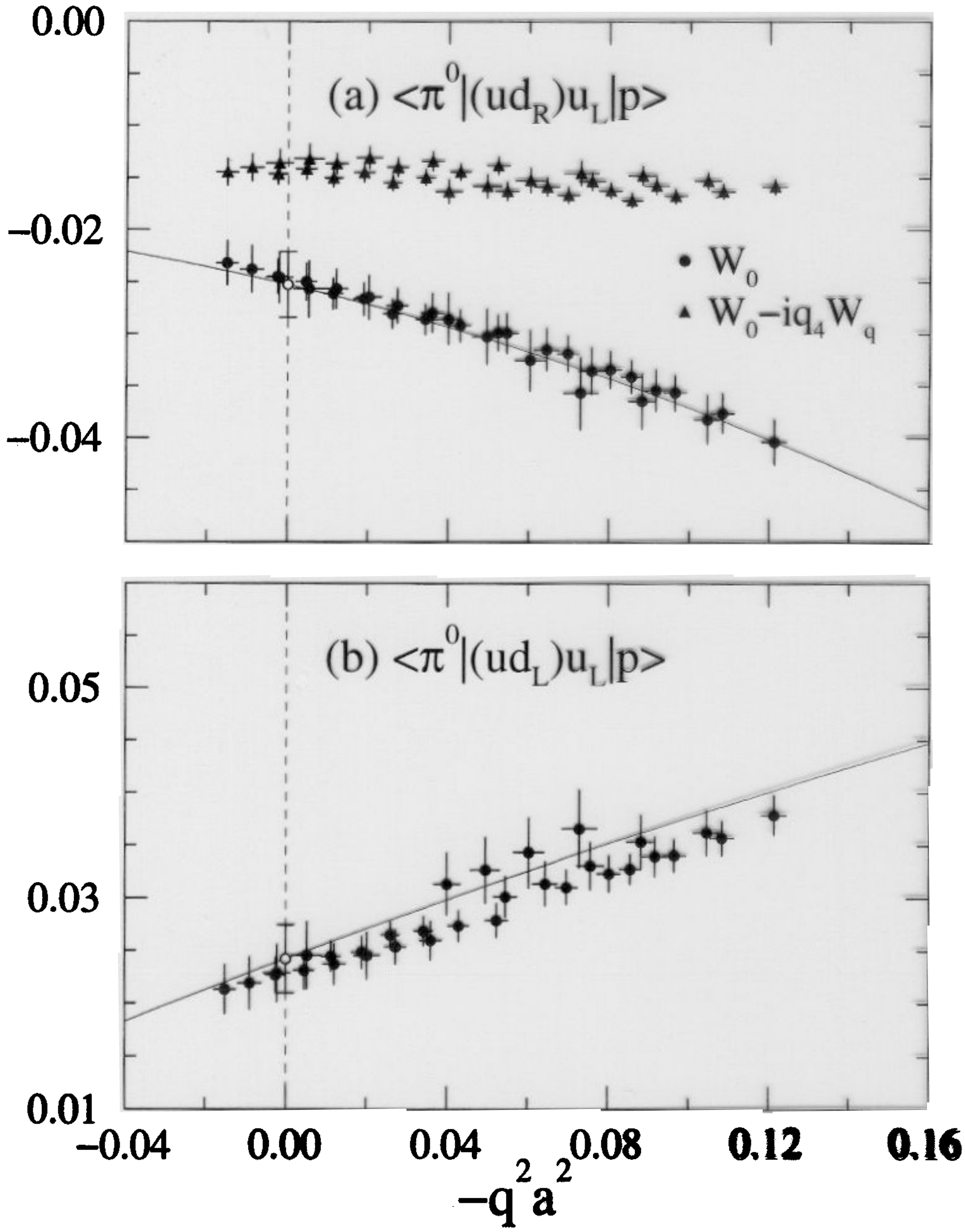
Important technical points

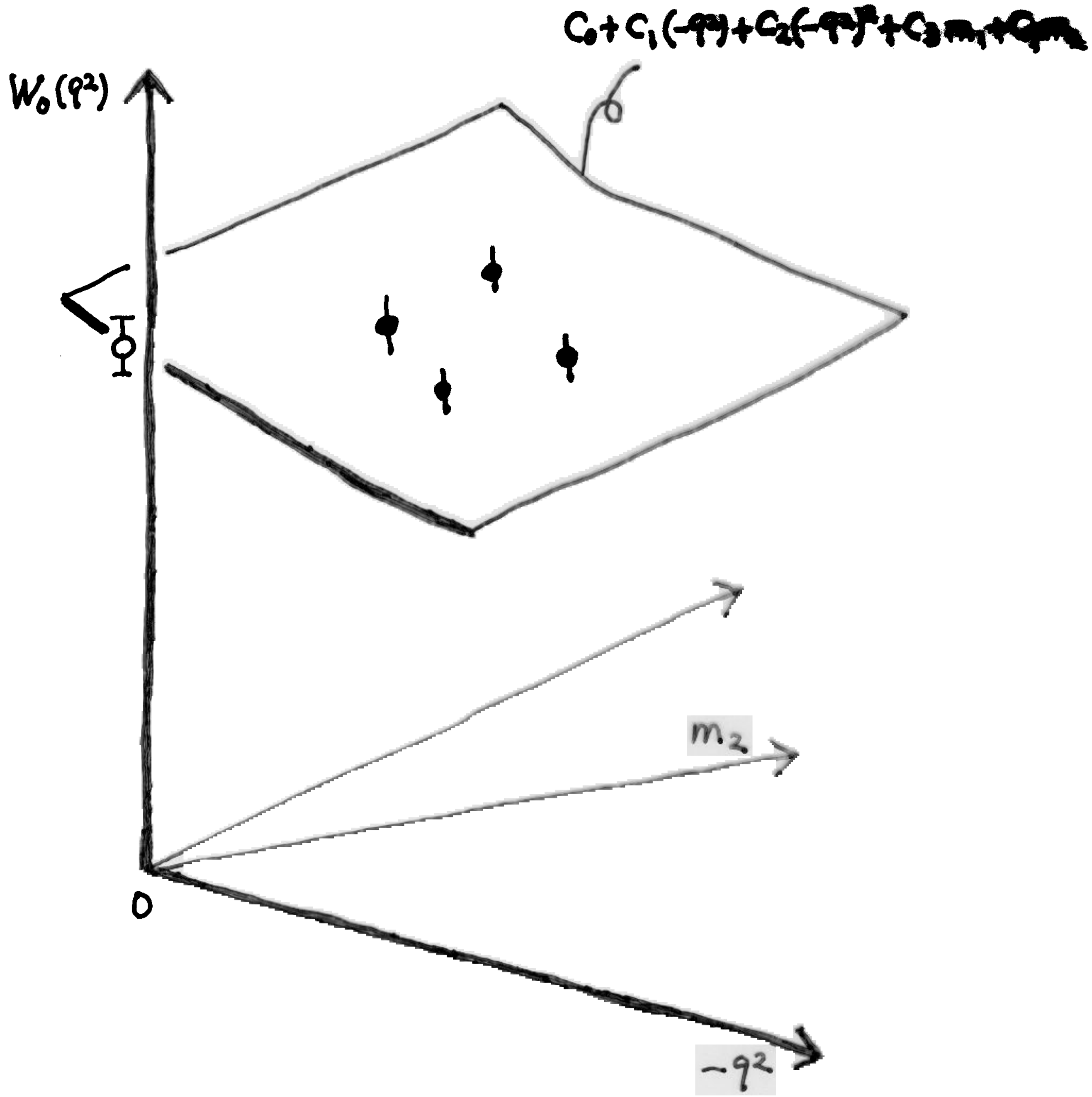
- $m_s \neq m_{ud}$ by $m_2 \neq m_1$
- $28^2 \times 48 \times 80$ allows $|\vec{p}|a = \frac{\pi}{14}, \frac{\pi}{24}$
  - investigate the q^2 dependence
  - interpolate the results to $q^2=0$ point

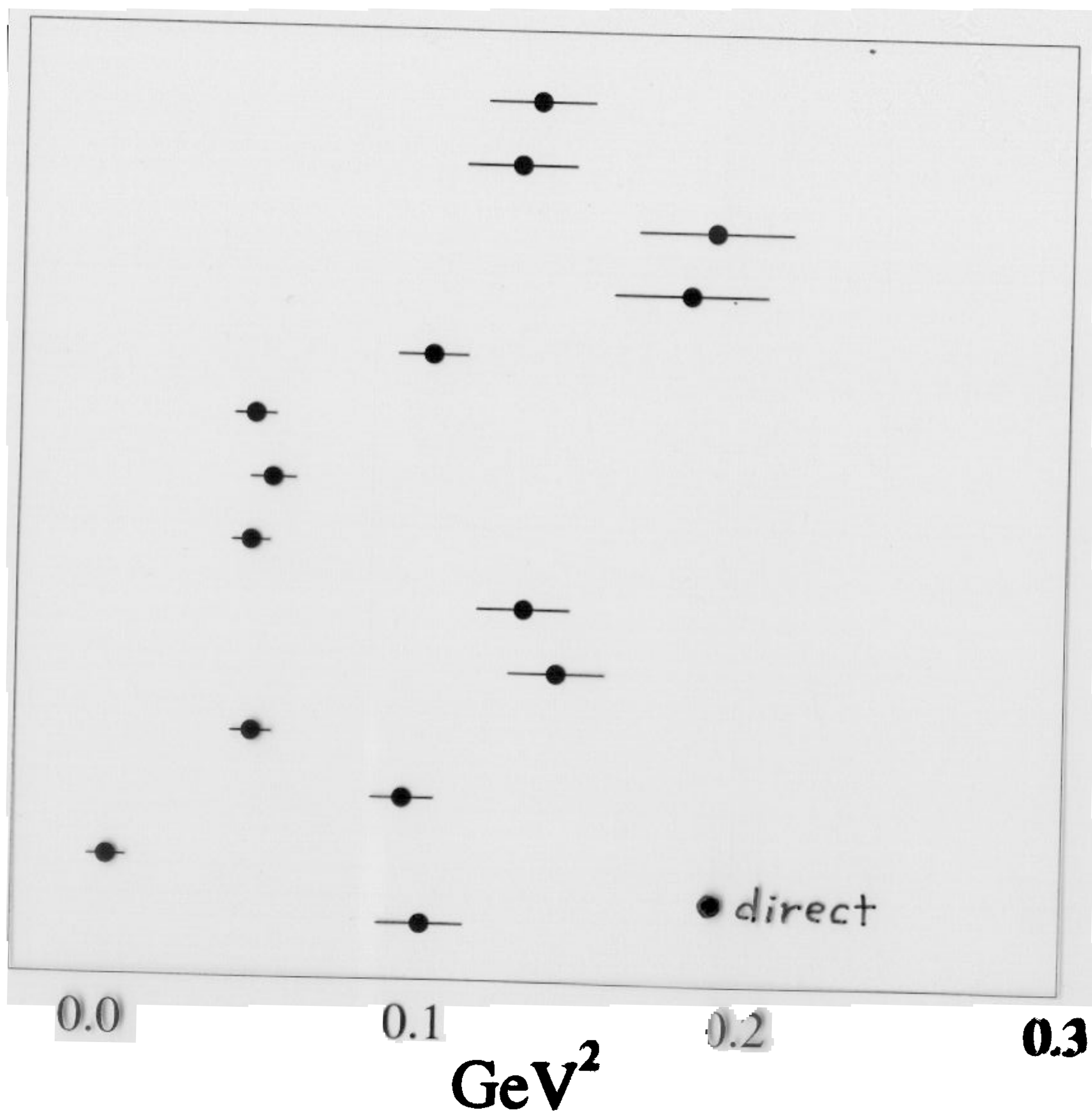
fitting function inspired by chiral lagrangian

$$C_0 + C_1(-q^2) + C_2(-q^2)^2 + C_3 m_1 + C_4 m_2$$

relevant form factor $W_0(q^2)$




$C_0+C_1(-q^2)+C_2(-q^2)^2+C_3m_1+C_4m_2$
$W_0(q^2)$
m_2
0
$-q^2$



direct
0.0
0.1
0.2
0.3
GeV^2

Results for direct method

- roughly comparable with the results for indirect method
- 3–5 times larger than the tree-level predictions of Chiral lagrangian with $|\alpha|=|\beta|=0.003\ \mathrm{GeV}^3$

$\rightarrow$ enhancement of the partial decay width by a factor of ~ 10

$$\Gamma \propto |\langle PS|\mathcal{O}^{\not B}|N\rangle|^2$$

more stringent constraints on GUT models

V. Continuum limit of $\langle 0|\mathcal{O}^{B}|N\rangle$ in quenched QCD

Systematic errors

- scaling violation
  could be large for the Wilson quark action
- quenched approximation
- NLO of ChPT for α, β parameters

$\rightarrow$ remove scaling violation effects for α, β params

Simulation params.

β	$L^3 \times T$	a[fm]	#conf
5.9	$32^3 \times 56$	0.1020(8)	300/800
6.1	$40^3 \times 70$	0.0777(7)	200/600
6.25	$48^3 \times 84$	0.0642(7)	140/420

$L \approx 3\,\mathrm{fm}$ $\quad m_\pi/m_\rho \approx 0.5 - 0.75$ with 4 quark masses

same parameters as the quenched light hadron spectrum
calculated by CP-PACS Collab
PRD67(2003)034503

except that we drop the largest β and the lightest
quark mass at each β

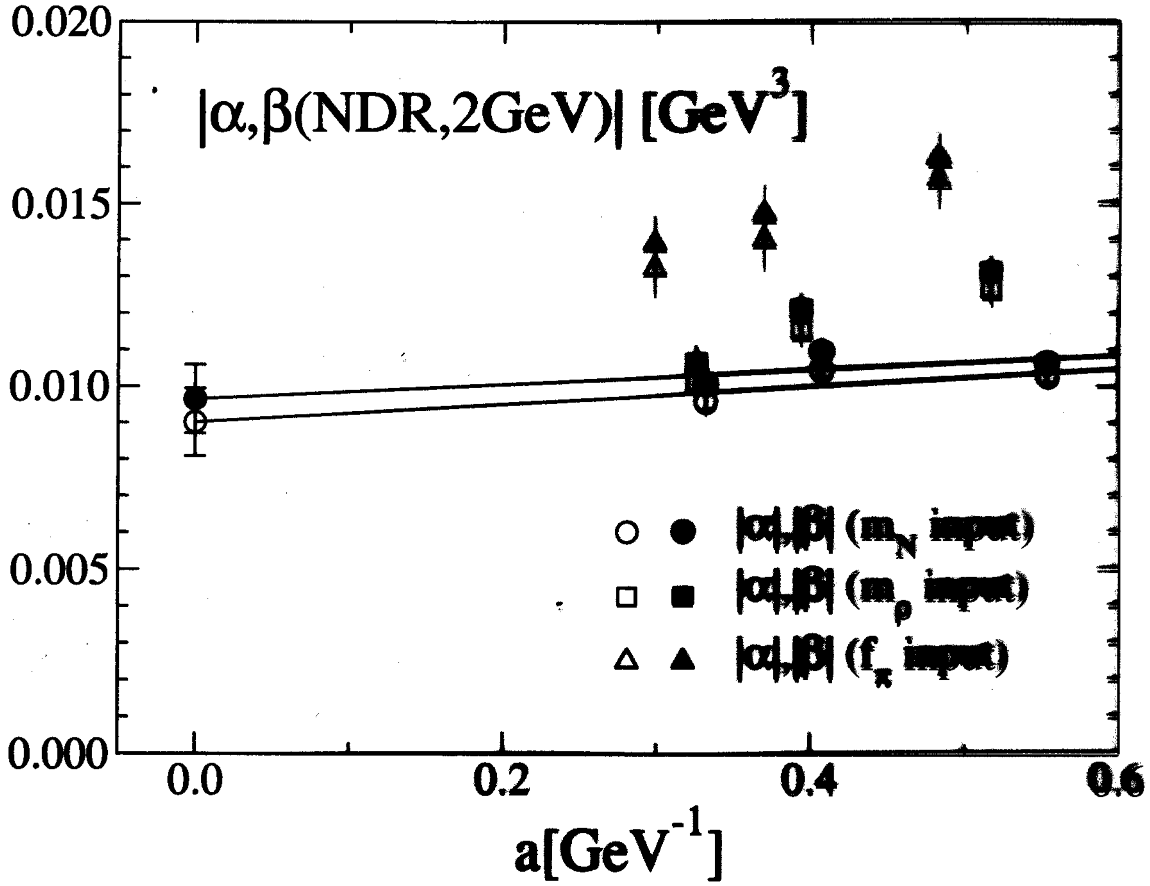
Systematic error coming from scale setting is large

m_N, m_ρ, f_π

$$|\alpha(\mathrm{NDR}, 2\mathrm{GeV})| = 0.0090(09)\binom{+5}{-19}\ \mathrm{GeV}^3$$

$$|\beta(\mathrm{NDR}, 2\mathrm{GeV})| = 0.0096(09)\binom{+6}{-20}\ \mathrm{GeV}^3$$

$\longrightarrow$ still larger than $|\alpha| = |\beta| = 0.003\ \mathrm{GeV}^3$



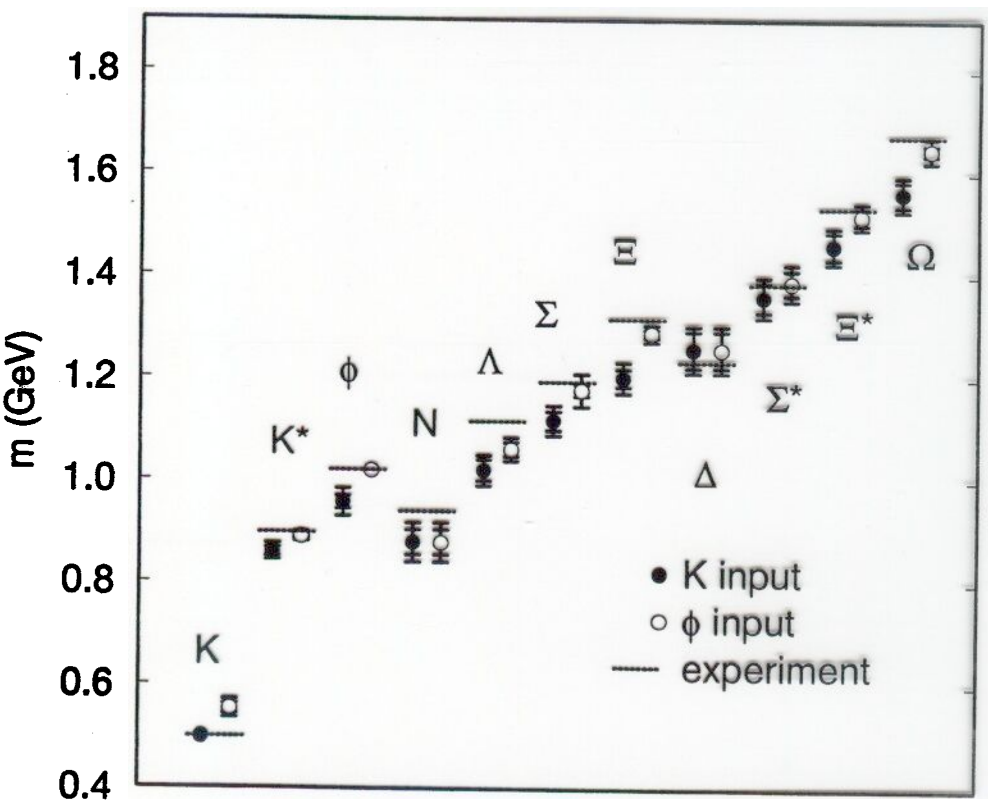
Once we set the lattice spacing by m_ρ

-7% deviation for m_N

$+2\%$ for f_π

$\rightarrow$ quenching effects

CP-PACS, PRD67(2003)034503



V. Summary

- GUT-model-independent calculation of $\langle PS|O^{B}|N\rangle$

$$(P,n) \to (\pi, K, \eta) + (\bar{\nu}, e^+, \mu^+)$$

- $\langle PS|O^{B}|N\rangle_{direct}$ are roughly comparable with $\langle PS|O^{B}|N\rangle_{indirect}$ using α, β determined on the same lattice

- $\langle PS|O^{B}|N\rangle_{direct}$ are 3–5 times larger than $\langle PS|O^{B}|N\rangle_{indirect}$ using $|\alpha| = |\beta| = 0.003\ GeV^3$
  $\to$ stronger constraints on GUT models

- Continuum limit in quenched QCD

$$|\alpha(NDR, 2GeV)| = 0.0090(09)\binom{+5}{-19}\ GeV^3$$

$$|\beta(NDR, 2GeV)| = 0.0096(09)\binom{+6}{-20}\ GeV^3$$

  $\to$ Scaling study for $\langle PS|O^{B}|N\rangle_{direct}$ is required

<u>next step</u>

- remove quenching effects
  now possible with 2- and 3-flavor QCD

- one-loop calculation of chiral lagrangian