Comparison between overlap and twisted mass fermions toward the chiral limit

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Motivations

- Problems of the Wilson formulation:
  - Explicit breaking of chiral symmetry $\Rightarrow$ operator mixing
  - Unphysical small eigenvalues $\Rightarrow$ exceptional configurations
  - Large discretization errors $\Rightarrow$ Symanzik improvement needed
- Overcome these problems toward unquenched simulations:
  - overlap fermions: exact chiral symmetry at finite $a \Rightarrow$ approach to the chiral limit + better ultraviolet properties of composite operators + simple $\mathcal{O}(a)$ improvement
  - twisted mass fermions: protection from unphysical small eigenvalues + possible siplifications in operator mixing + automatic $\mathcal{O}(a)$ improvement for $\omega = \frac{\pi}{2}$ [Frezzotti & Rossi, '03] + computationally cheap

Overlap fermions

$$S_F^{\text{ov}} = a^4 \sum_x \bar\psi^\alpha(x) \left[D + M_{\alpha\beta} \left(1 - \frac{\bar a D}{2} \right) \right] \psi^\beta(x), \qquad \bar a = a/(1+s),$$

$$M = \text{diag}(m_\alpha, m_\beta, \ldots), \quad D = \frac{1}{\bar a}[1 + \gamma_5 \text{sign}(Q)], \quad Q = \gamma_5 (a D_{\text{w}} - 1 - s).$$

- S_F^{ov} invariant under $\delta\psi = \omega_a T^a \gamma_5 (1 - \bar a D)\psi$, $\delta\bar\psi = \bar\psi\gamma_5 T^a \omega_a$ and $\mathcal{O}(a)$ improved.
- $\mathcal{O}(a)$ improved bilinears (inside correlators at non-zero distance)

$$O_\Gamma^a = \bar\psi^\alpha T^a_{\alpha\beta} \Gamma \left(1 - \frac{\bar a D}{2} \right) \psi^\beta = \frac{1}{1 - \frac{\bar a m_\beta}{2}} \left(\bar\psi^\alpha T^a_{\alpha\beta} \Gamma \psi^\beta \right)$$

- PCAC relation:

$$\langle Z_A \partial^*_\mu A^a_\mu \, O \rangle = \langle \bar\psi \gamma_5 \left(1 - \frac{\bar a D}{2} \right) \{ T^a, M \} \psi \, O \rangle$$

(Two flavour) twisted mass QCD (1)

- "physical" basis

$$S_F^{\text{ph}} = a^4 \sum_x \bar{\psi}(x) \left[K + (W + M_{\text{cr}}) e^{-i\omega\gamma_5\tau_3} + m_q \right] \psi(x)$$

- A change of variables in the functional integral

$$\psi = e^{i\omega\gamma_5 \frac{\tau_3}{2}} \chi, \qquad \bar{\psi} = \bar{\chi} e^{i\omega\gamma_5 \frac{\tau_3}{2}}$$

leads to the "twisted" basis [Frezzotti, Grassi, Sint, Weisz, '00] (implemented in the simulations)

$$S_F^{\text{tm}} = a^4 \sum_x \bar{\chi}(x) \left[K + (W + M_{\text{cr}}) + m_q' + i\mu_q\gamma_5\tau_3 \right] \chi(x)$$

with $m_q' = m_q \cos\omega$ and $\mu_q = \sin\omega$.

- At $\omega = \pi/2$ ($m_q' = 0$), hadronic masses and matrix elements are automatically $\mathcal{O}(a)$ improved, without need of Symanzik improvement [Frezzotti & Rossi, '03].

(Two flavour) twisted mass QCD (2)

- The massless “twisted” Wilson operator (in the “physical” basis) is now antihermitean $\Rightarrow$ mass protects from zero modes.

- Bilinears in the two basis are realted by

$$A_\mu^{\text{phys},a} = \begin{cases} \cos(\omega) A_\mu^a + \epsilon^{3ab} \sin(\omega) V_\mu^b & a = 1,2 \\ A_\mu^3 & a = 3 \end{cases}$$

$$P_a^{\text{phys}} = \begin{cases} P_a & a = 1,2 \\ \cos(\omega) P_3 + \sin(\omega) \frac{i}{2} \bar{\chi}\chi & a = 3 \end{cases}$$

- PCAC relation in the twisted basis at $\omega = \pi/2$ is

$$\epsilon^{3ab} \langle Z_V \partial_\mu^* V_\mu^b \, O \rangle = 2\mu_q \langle P^a \, O \rangle \qquad a = 1,2$$

(Two flavour) twisted mass QCD (3)

- Chiral symmetry still broken by the Wilson term $\Rightarrow$ chiral limit to be taken after continuum limit.
- at fixed lattice spacing

$$a\Lambda^5 \ll m_q \Lambda^3 \qquad \Rightarrow \qquad (a\Lambda)^2 \ll m_q a$$

or (if correlators are $\mathcal{O}(a)$ improved)

$$(a\Lambda)^3 \ll m_q a$$

- our lowest two masses do not satisfy the first (stronger) condition while satisfy the weaker form of it.

Numerical details

$\beta = 5.85 \Rightarrow a^{-1} = 1.6$ GeV ($a = 0.123$ fm) from $r_0 = 0.5$ fm.

- Overlap:
  - 140 conf. on $12^3 \times 24$ ($L \sim 1.5$ fm)
  - $m_q a = (0.01, 0.02, 0.04, 0.06, 0.08, 0.10)$
  - $M_\pi L = (1.7,\ 2.4,\ 3.4,\ 4.2, ...)$ and $M_\pi \in [230, 730]$ MeV

- Twisted mass at $\omega = \pi/2$ ($m_q' = 0$):
  - $\kappa_{cr} = 0.16166(2)$ determined from Wilson fermions
  - 60 conf. on $12^3 \times 24$ ($L_1 \sim 1.5$ fm)
  - 140 conf. on $14^3 \times 32$ ($L_2 \sim 1.75$ fm)
  - 175 conf. on $16^3 \times 32$ ($L_3 \sim 2.0$ fm)
  - $\mu_q a = 0.01, 0.02, 0.04, 0.06, 0.08, 0.10$
  - $M_\pi L_3 = (3.6,\ 5.0, ...)$ and $M_\pi \in [360, 1180]$ MeV

Meson masses

- Extraction of M_{mes} (with and without sink smearing) and of $Z_{O^a_\Gamma} = |\langle 0|O^a_\Gamma|\text{mes}\rangle|^2$ by fitting in $[t_{\text{min}}, \frac{T}{2}]$ (with $1 \ll t_{\text{min}} \ll \frac{T}{2}$)

$$\sum_{\mathbf{x}} \langle O^a_\Gamma(x) O^a_\Gamma(0)\rangle \rightarrow \frac{Z_{O^a_\Gamma}}{M_{\text{mes}}} e^{-M_{\text{mes}}\frac{T}{2}} \cosh\left[M_{\text{mes}}\left(x_0 - \frac{T}{2}\right)\right]$$

  t_{min} is chosen by looking at the effective mass, at the dependence of the fit from t_{min} and by comparing with a two-state fit.

- For the pseudoscalar (degenerate and non-degenerate) we use

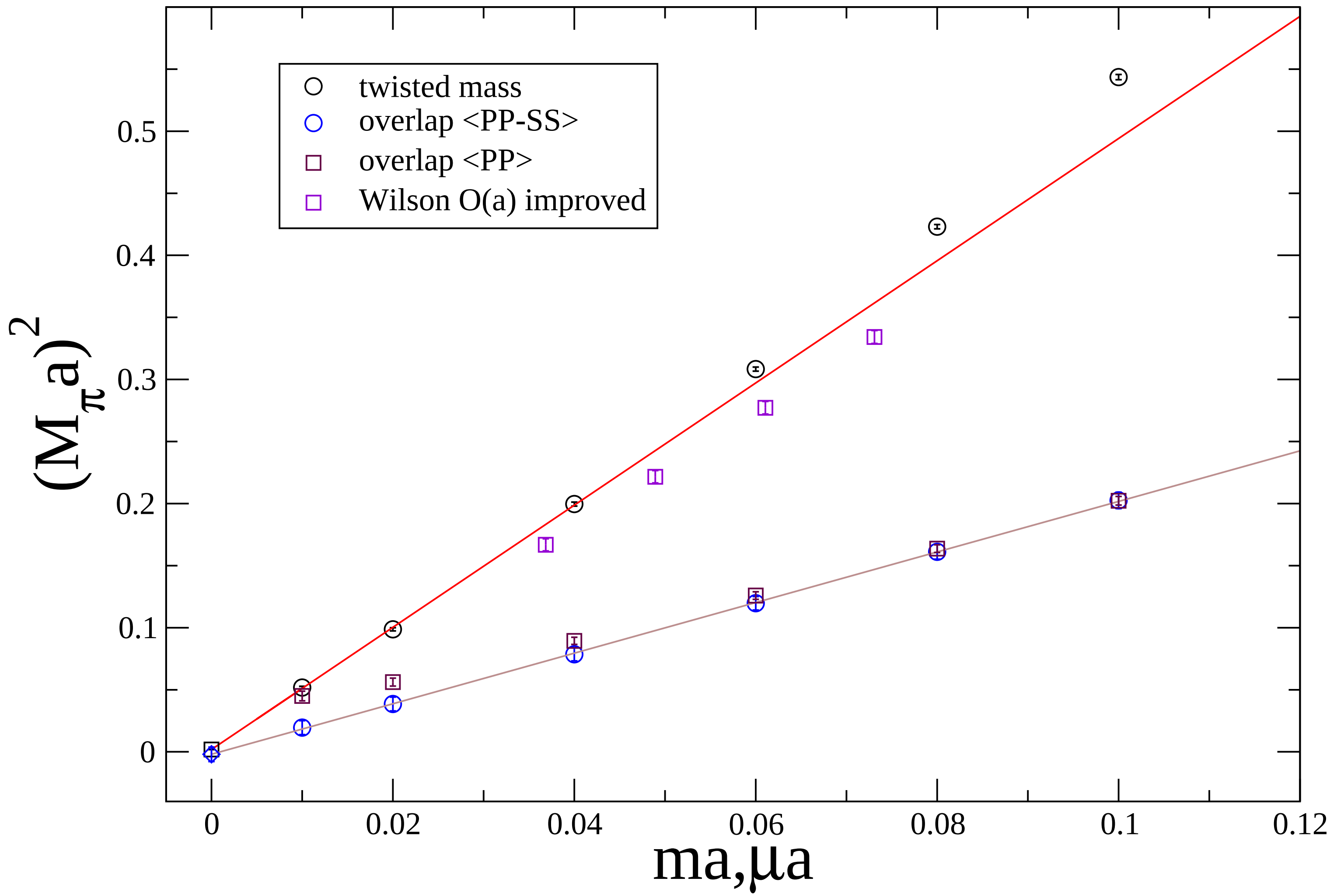
$$\sum_{\mathbf{x}} \langle P^a(x) P^a(0)\rangle \quad \text{and} \quad \sum_{\mathbf{x}} \langle P^a(x) P^a(0) - S^a(x) S^a(0)\rangle \quad a = 1, 2$$

- For the vector meson we use

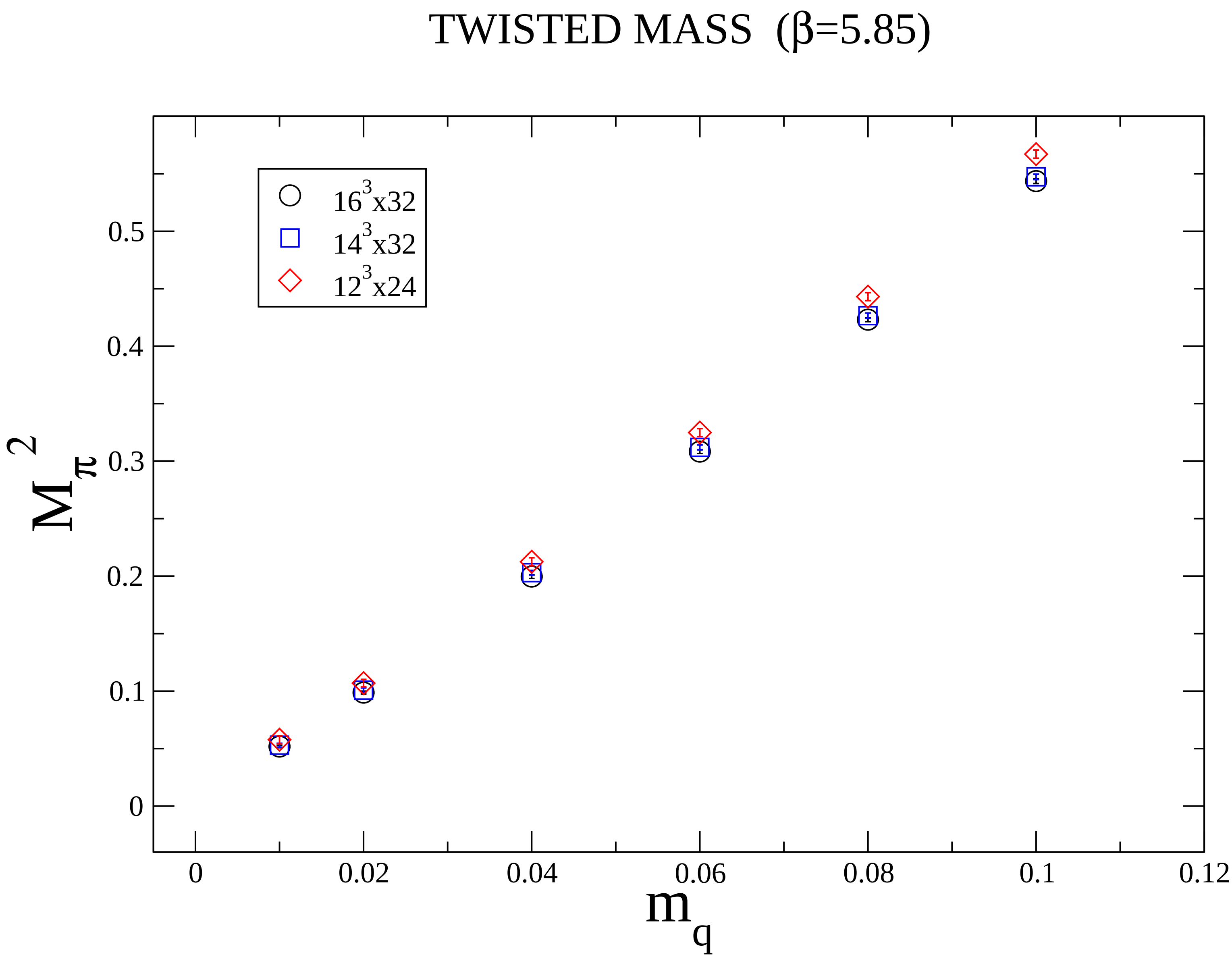
$$\sum_{\mathbf{x}} \langle A^a_k(x) A^a_k(0)\rangle \qquad a = 1, 2 \quad \text{for tm (twisted basis)}$$

$$\sum_{\mathbf{x}} \langle V^a_k(x) V^a_k(0)\rangle \qquad a = 1, 2, 3, \ldots \quad \text{for overlap}$$

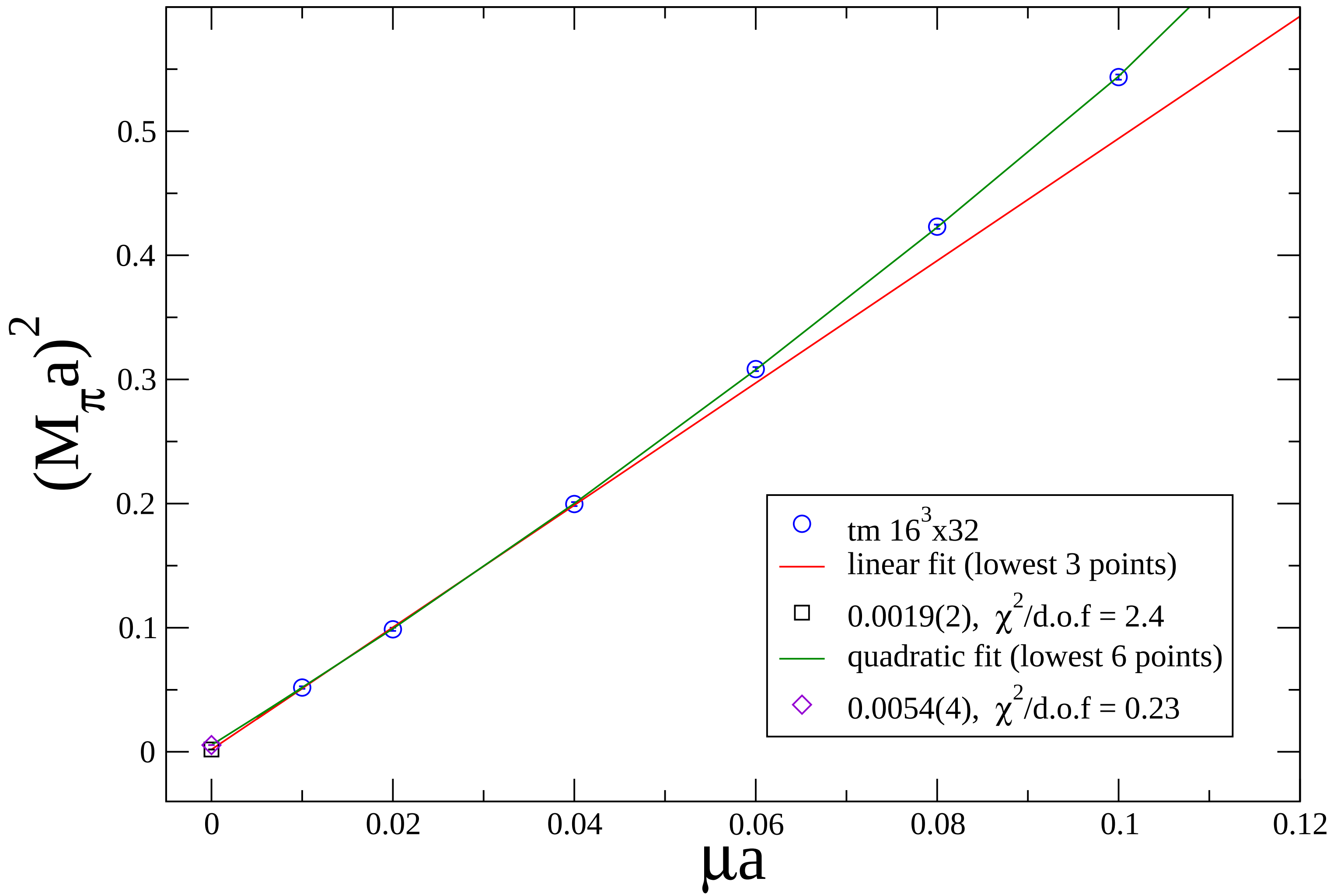
Pseudoscalar masses



Pseudoscalar masses



Pseudoscalar masses



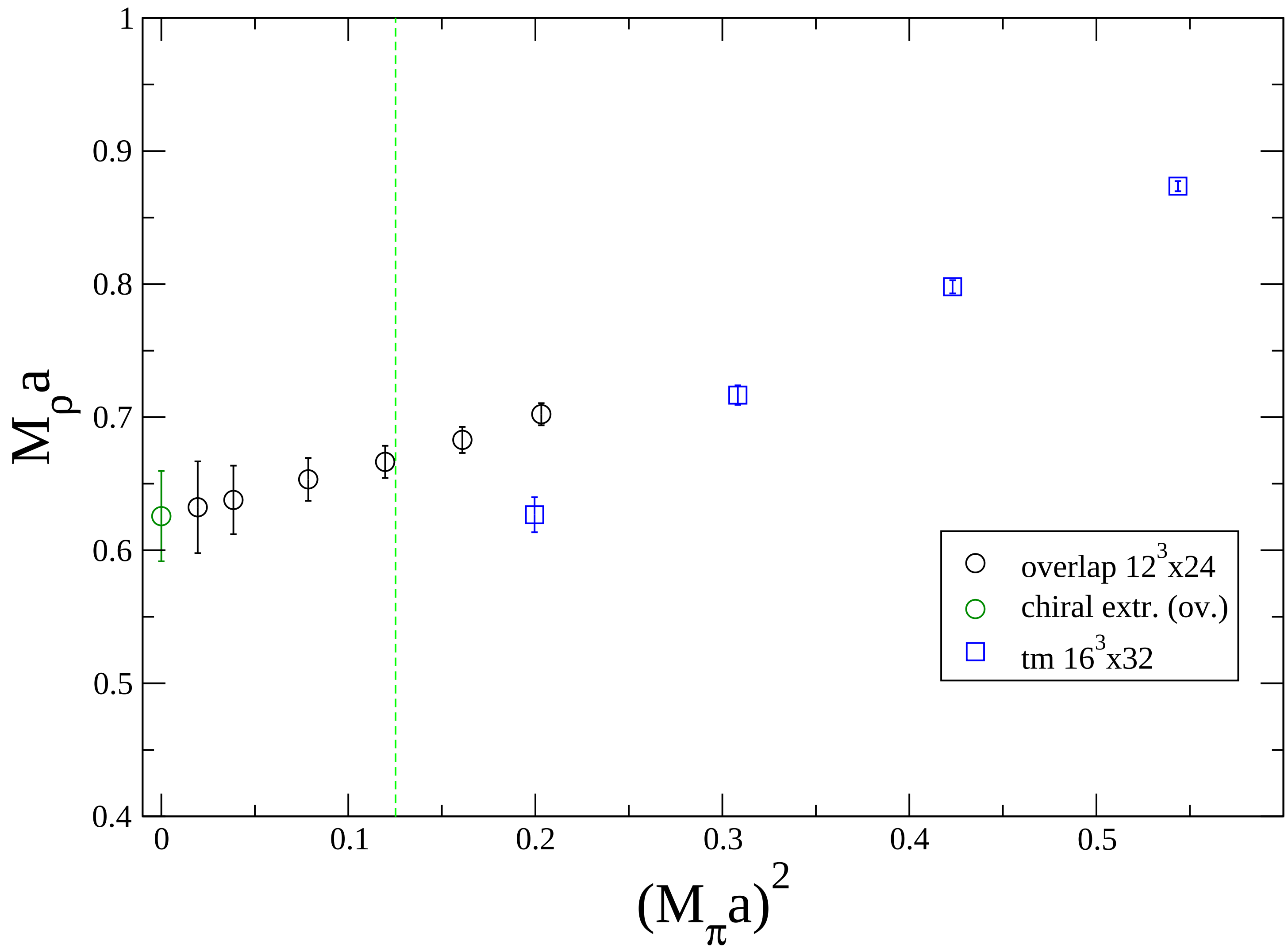
Pseudoscalar masses

We use the scheme defined in [Hernandez,Jansen,Lellouch,Wittig, '01] to compute Z_m^{RGI} by matching our data with the continuum limit of $\mathcal{O}(a)$ NP-improved Wilson fermions [see A. Shindler's talk]. We get

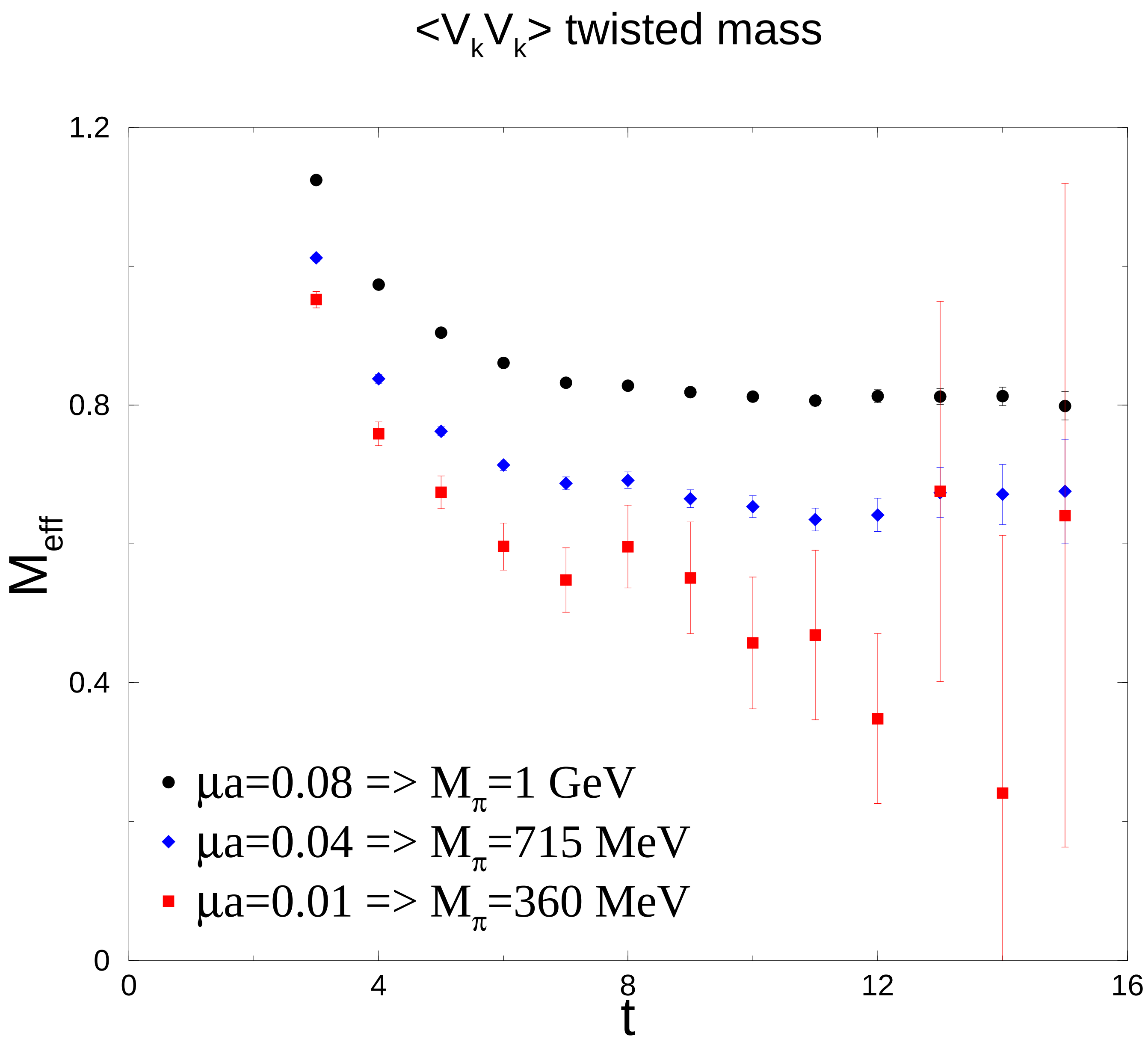
$$Z_m^{\text{RGI,tm}} = 2.52(5) \qquad Z_m^{\text{RGI,ov}} = 0.96(7)$$

which is reflected in the slope of the previous curves.

Vector meson masses



Vector meson masses



Pion decay constant

- Using the PCAC relation one can compute f_π without need of renormalization constants:

$$f_\pi^{\text{ov}} = \frac{2m_q}{M_\pi^2}|\langle 0|P^a|\pi\rangle| \qquad f_\pi^{\text{tm}} = \frac{2\mu_q}{M_\pi^2}|\langle 0|P^a|\pi\rangle| \qquad a = 1,2$$

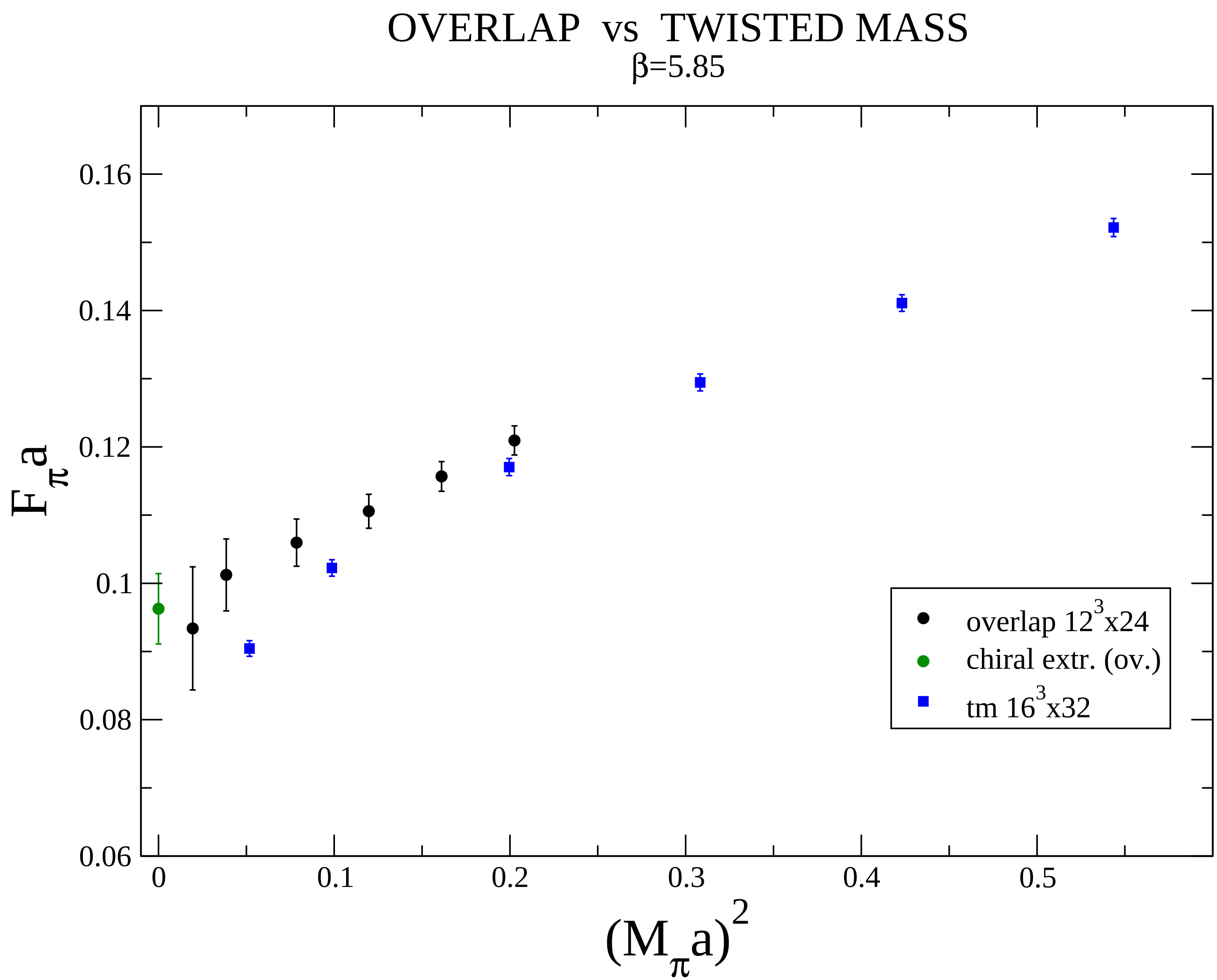
- The direct definition requires the determination of Z_A^{ov} and of $Z_V^{\text{tm}} = Z_V^{\text{Wi}}$ (at $\omega = \frac{\pi}{2}$) :

$$f_\pi^{\text{ov}} = \frac{Z_A^{\text{ov}}|\langle 0|A_0^a|\pi\rangle|}{M_\pi} \qquad f_\pi^{\text{tm}} = \frac{Z_V^{\text{Wi}}|\langle 0|V_0^a|\pi\rangle|}{M_\pi} \qquad a = 1,2$$

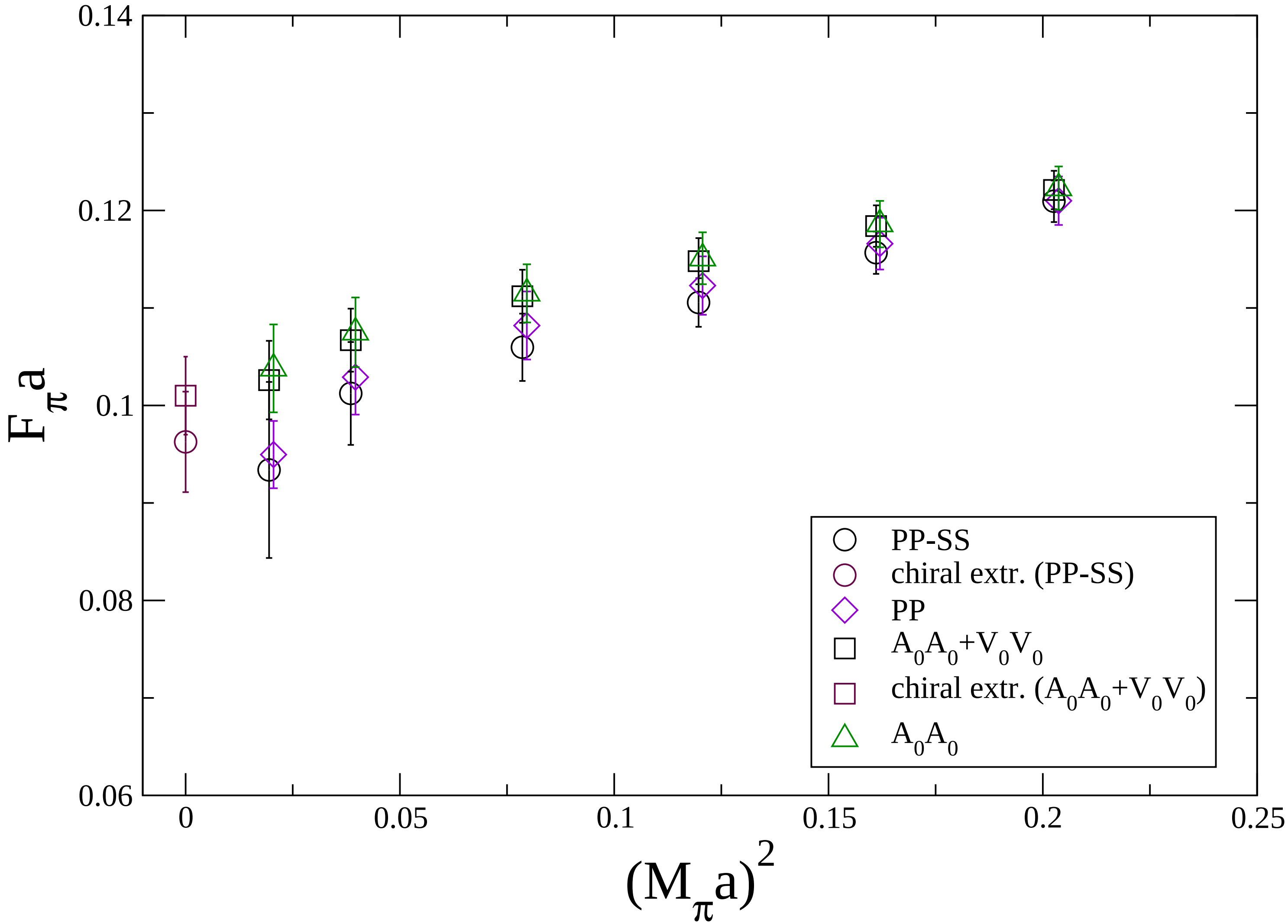
- At one loop in quenched chiral perturbation theory (qChPT), f_π has neither chiral logarithms nor FV effects:

$$f_\pi = f\left(1 + \frac{\alpha_5}{(4\pi f)^2}M_\pi^2\right)$$

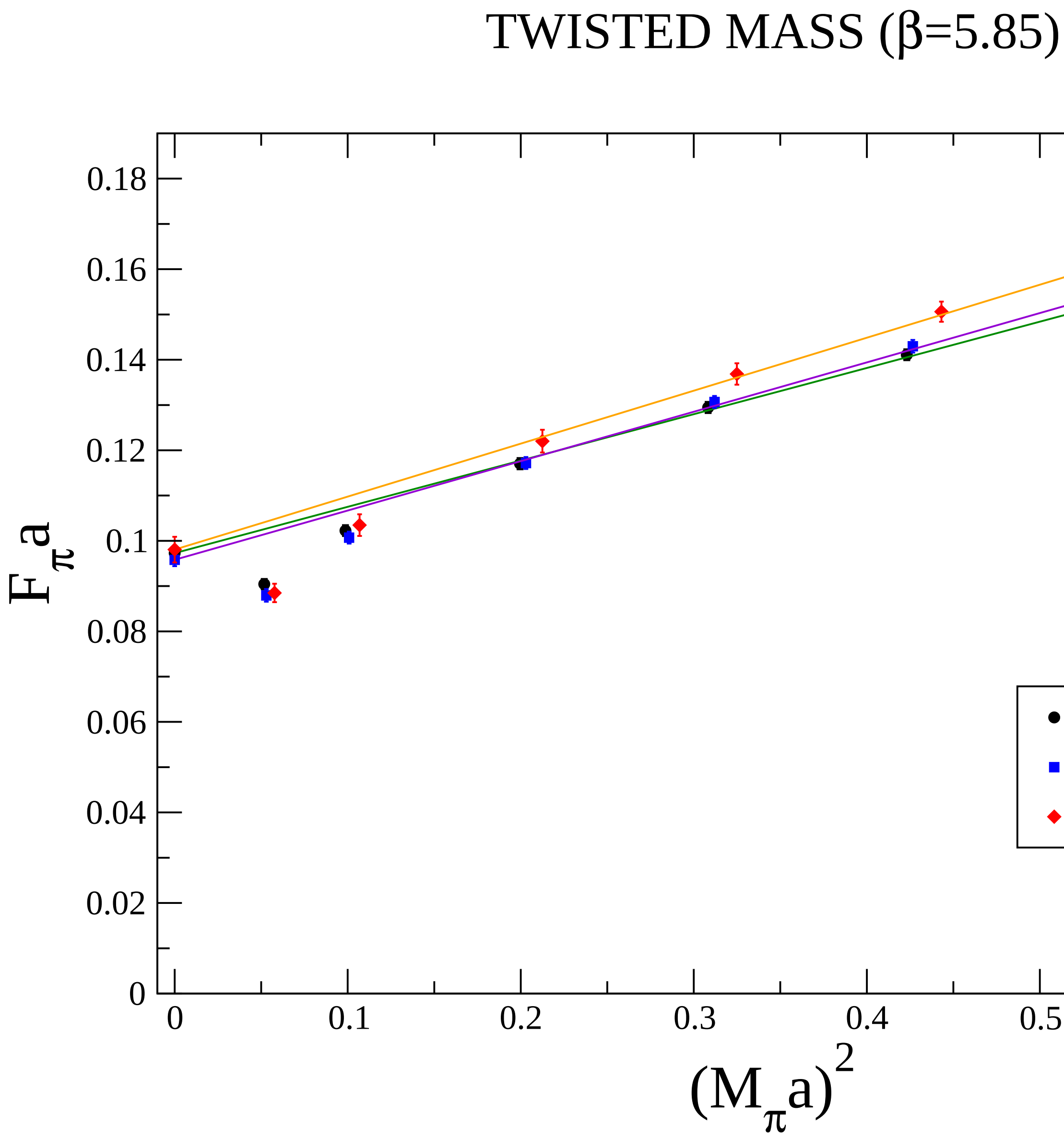
Pion decay constant



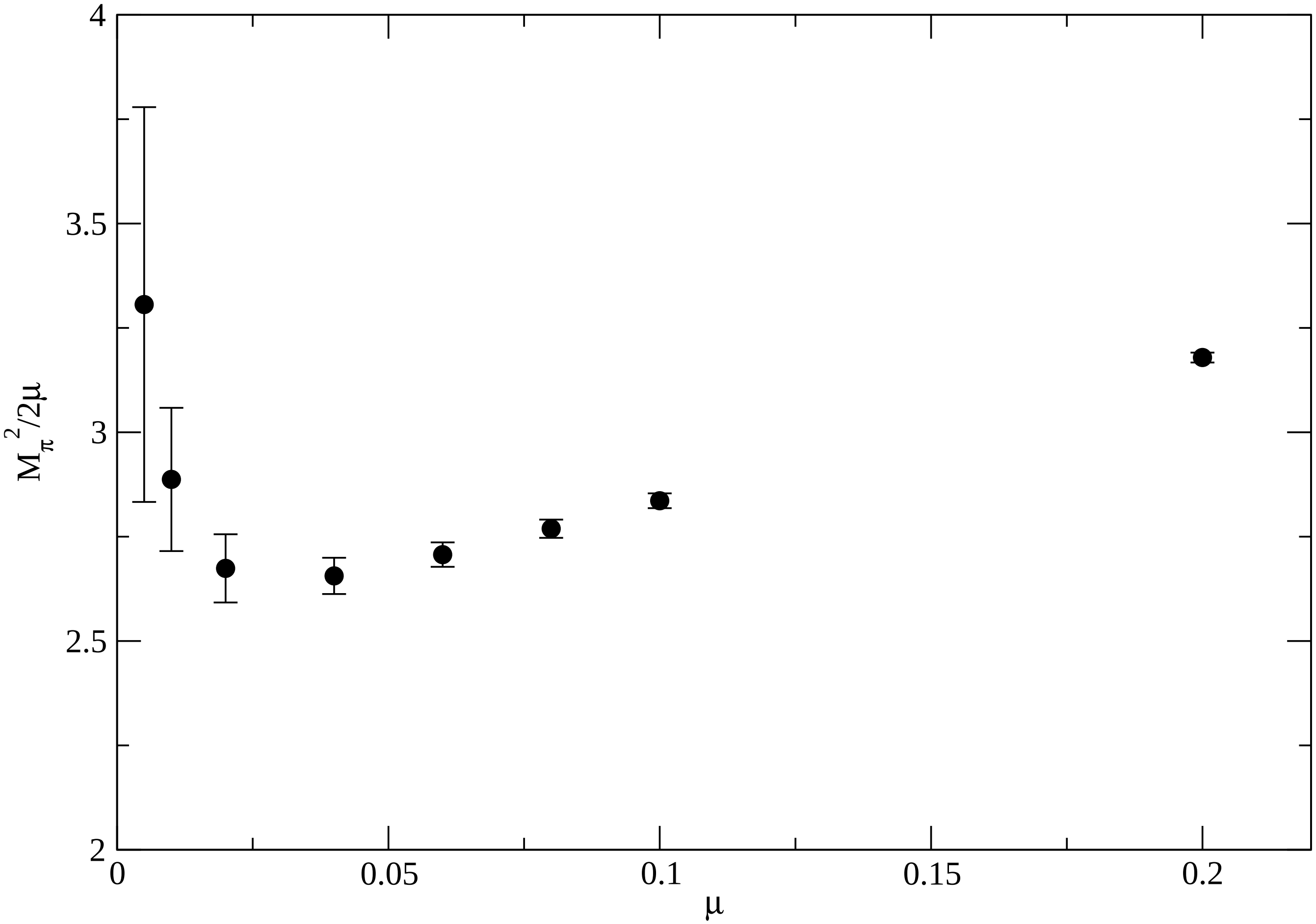
Pion decay constant



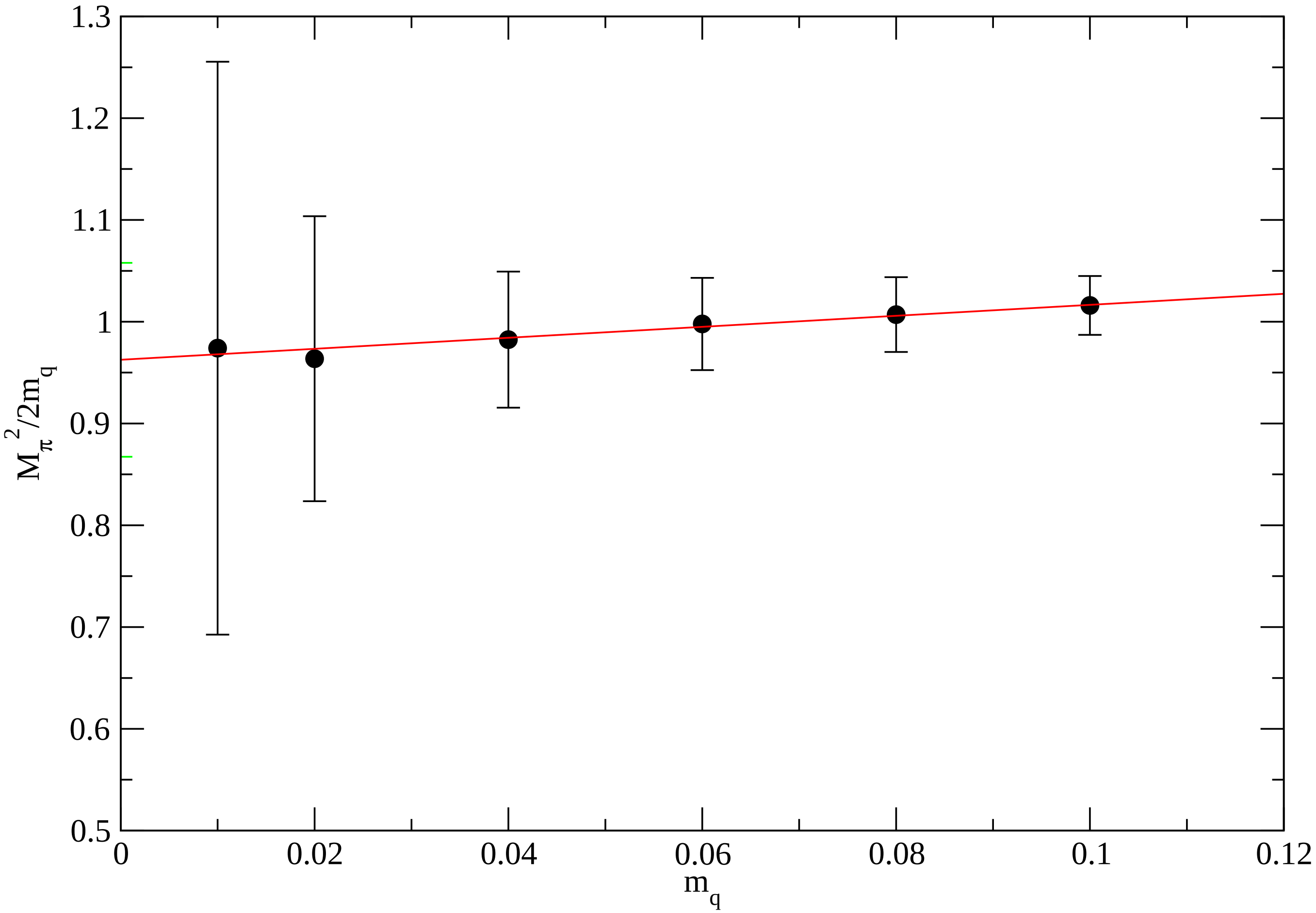
Pion decay constant



Pseudoscalar masses



Pseudoscalar masses



Kaon decay constant

- The behaviour of f_π^{ov} is perfectly linear, as predicted by qChPT. We now extract f_K again by using the PCAC:

$$f_K^{\text{ov}} = \frac{m_1 + m_2}{M_K^2} |\langle 0|P^a|K\rangle|$$

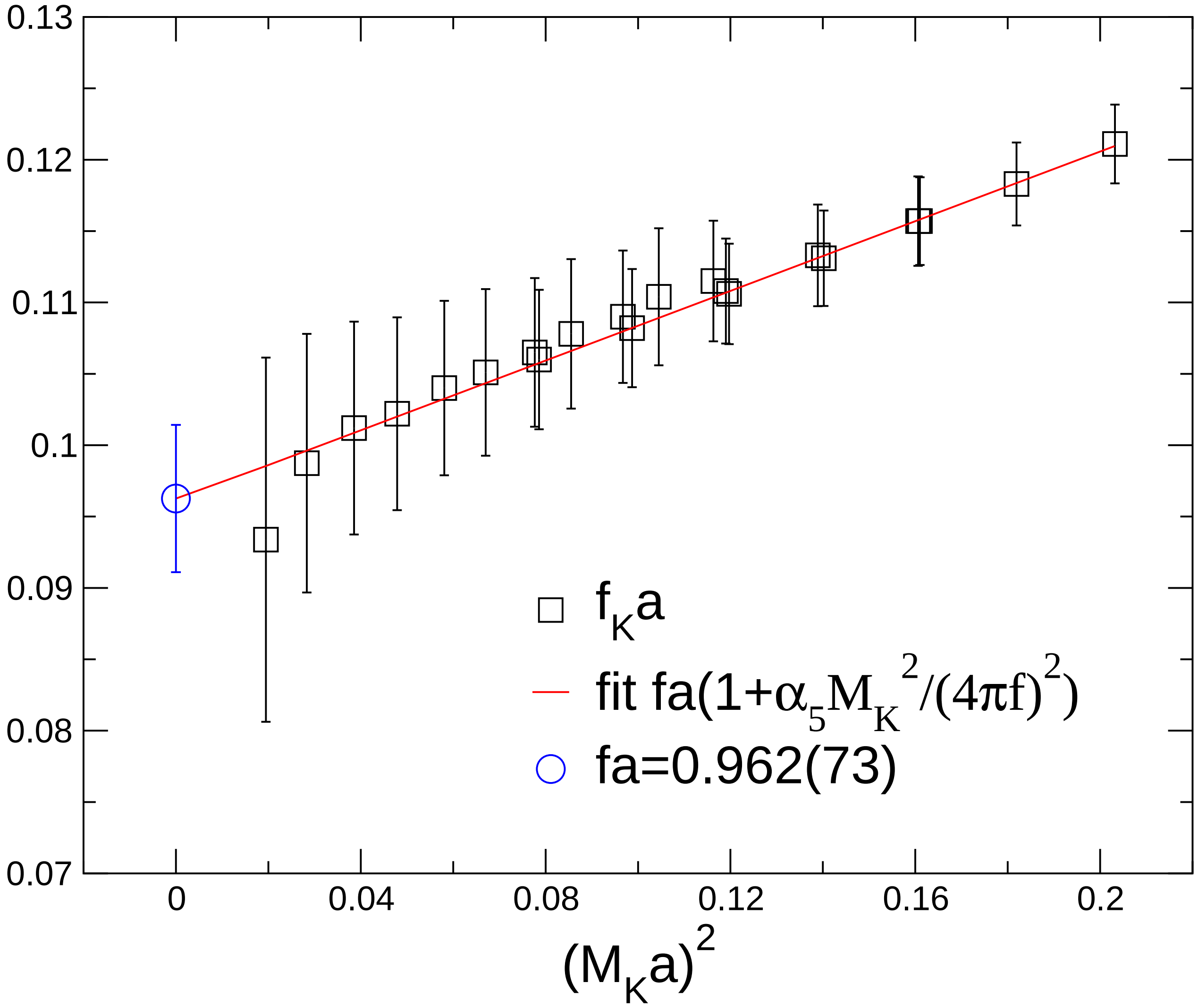
- At one loop in qChPT, f_K has the form

$$f_K = f\left(1 + \frac{\alpha_5}{(4\pi f)^2} M_K^2 + \text{FV}(M_\pi^2, M_K^2) + \log(M_\pi^2, M_K^2)\right)$$

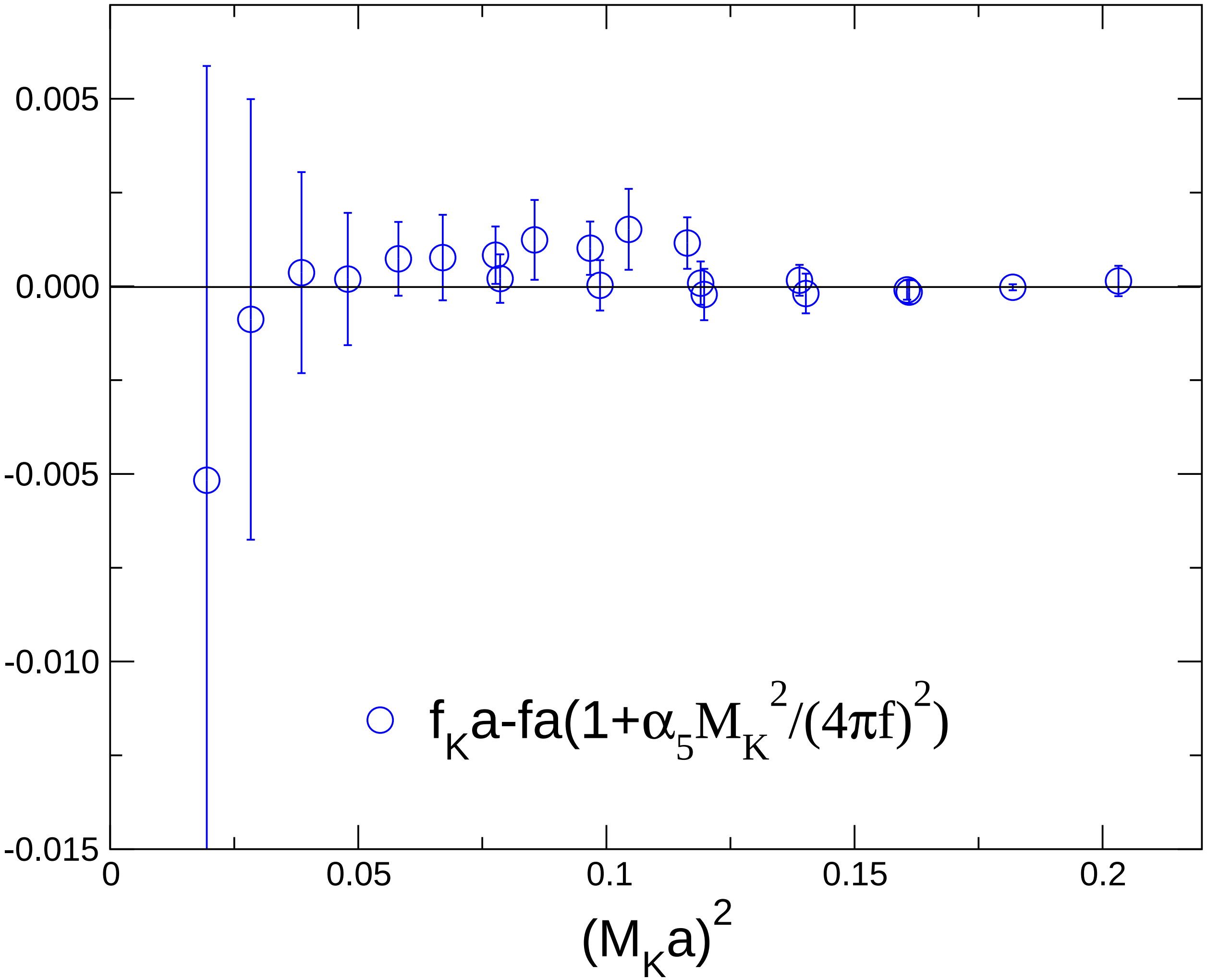
  where α_5 is the same of f_π and "FV" and "log" are linear in the quenched parameters δ and α.

- Try to fit these three parameters (or determining α_5 from f_π and the remaining two from f_K). Within the errors we see only a good linear behaviour with M_K^2, with α_5 in perfect agreement with the determination from f_π. We get $f_\pi = 155(11)$ MeV, $f_K = 173(8)$ MeV, $f_K/f_\pi = 1.11(3)$.

Kaon decay constant



Kaon decay constant



PCAC

- We extract the PCAC mass ρ from the ratios

$$\rho^{\text{ov}} = \frac{\sum_{\mathbf{x}} \langle \partial_0 A_0^a(x)\ P^a(0)\rangle}{\sum_{\mathbf{x}} \langle P^a(x) P^a(0)\rangle} \qquad \rho^{\text{tm}} = \frac{\epsilon^{3ab} \sum_{\mathbf{x}} \langle \partial_0 V_0^b(x)\ P^a(0)\rangle}{\sum_{\mathbf{x}} \langle P^a(x) P^a(0)\rangle} \quad a = 1, 2$$

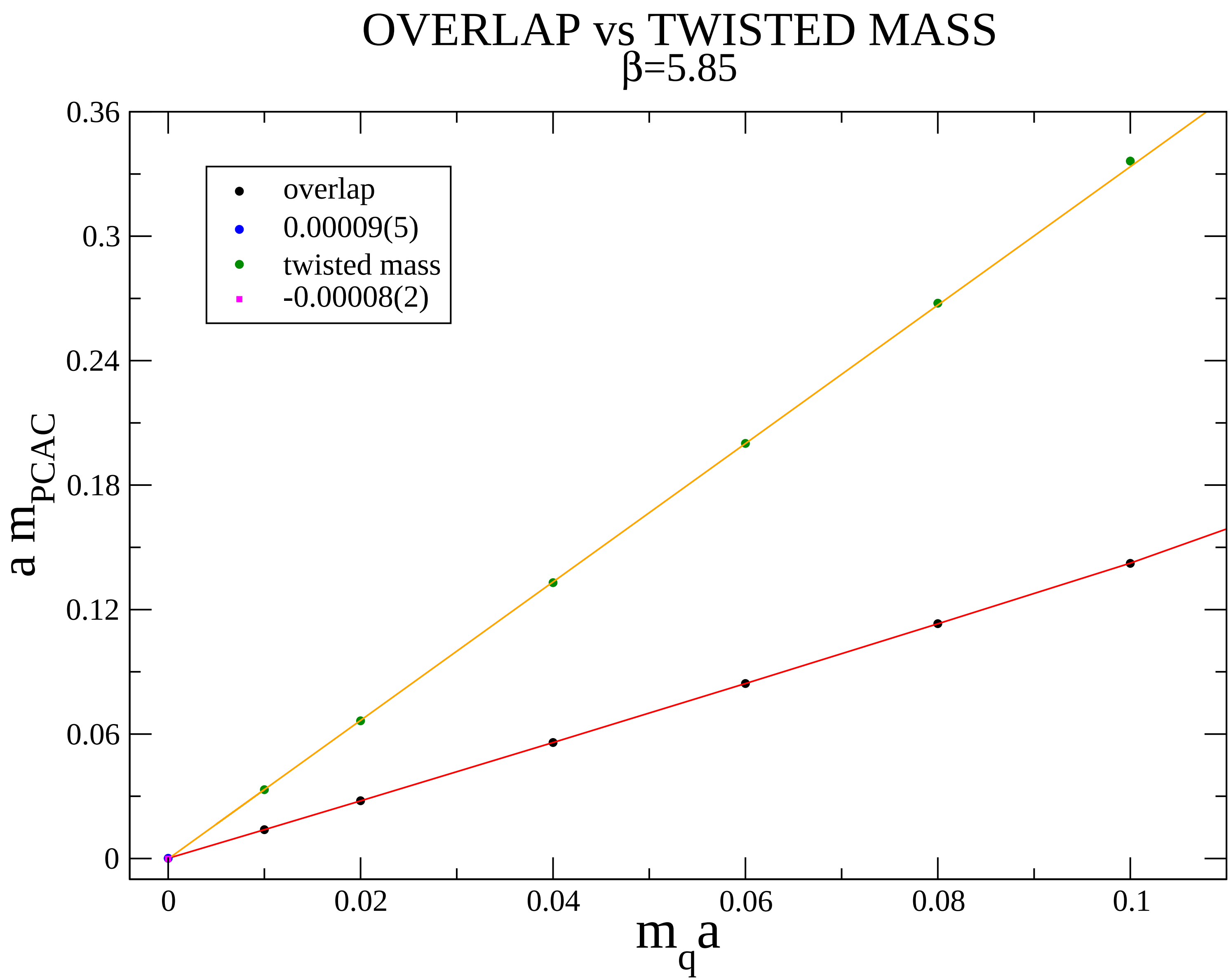
- We extract Z_A^{ov} and Z_V^{tm} from

$$Z_A^{\text{ov}} = \frac{2m_q}{\rho^{\text{ov}}} \qquad Z_V^{\text{tm}} = \frac{2\mu_q}{\rho^{\text{tm}}}$$

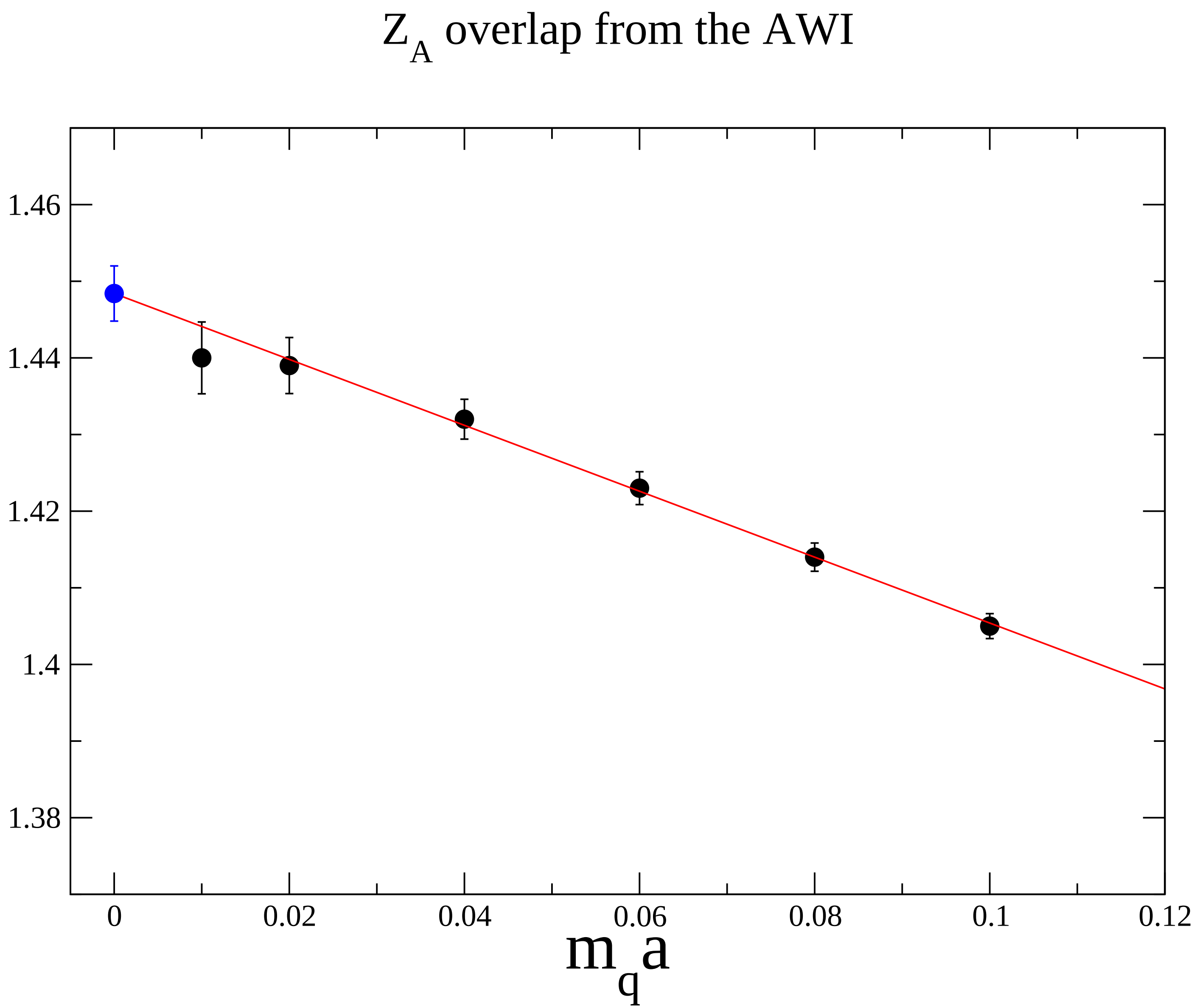
  at the various quark masses and then we perform the chiral extr.

- remark: in the twisted mass case, only the imaginary part of the correlator $\langle \bar{d}\gamma_0 u \bar{u}\gamma_5 d - \bar{u}\gamma_0 d \bar{d}\gamma_5 u\rangle$ contributes.

PCAC

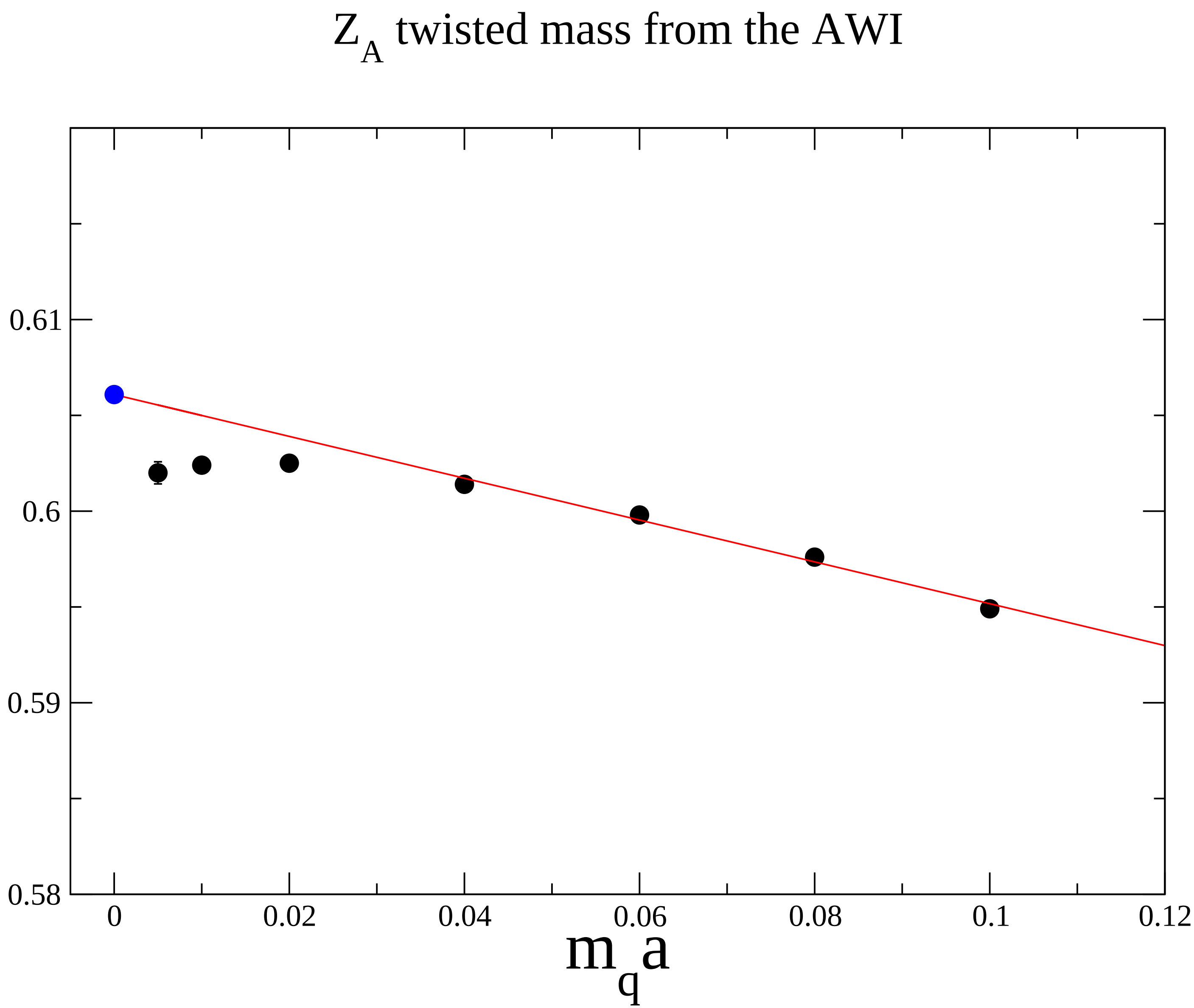

OVERLAP vs TWISTED MASS
β=5.85
overlap
0.00009(5)
twisted mass
-0.00008(2)
a m_PCAC
m_q a
0.36
0.3
0.24
0.18
0.12
0.06
0
0
0.02
0.04
0.06
0.08
0.1

PCAC



Z_A overlap from the AWI
1.46
1.44
1.42
1.4
1.38
0
0.02
0.04
0.06
0.08
0.1
0.12
m_q a

PCAC



Z_A twisted mass from the AWI
0.61
0.6
0.59
0.58
0
0.02
0.04
0.06
0.08
0.1
0.12
$m_q a$

Baryons

- Interpolating operators for the octet and the decuplet:

$$
\begin{aligned}
B^{\text{oct}}_{\alpha} &= \epsilon^{ABC}\left[((d^A)^T C\gamma_5 u^B)u^C_\alpha - ((u^A)^T C\gamma_5 d^B)u^C_\alpha\right] \\
B^{\text{dec}}_{k,\alpha} &= \epsilon^{ABC}((u^A)^T C\gamma_k u^B)u^C_\alpha \quad k=1,2,3
\end{aligned}
$$

  $k=1,2,3$ are equivalent. We choose 1 for definitness.

- For correlators at zero momentum, in the overlap case

$$
\sum_{\mathbf{x}} \langle \bar{B}^{\text{oct,dec}}_{\alpha}(x) B^{\text{oct,dec}}_{\beta}(0)\rangle \propto (1+\gamma_0)_{\alpha\beta} e^{-Mx_0} \qquad 1 \ll x_0 \leq \frac{T}{2}
$$

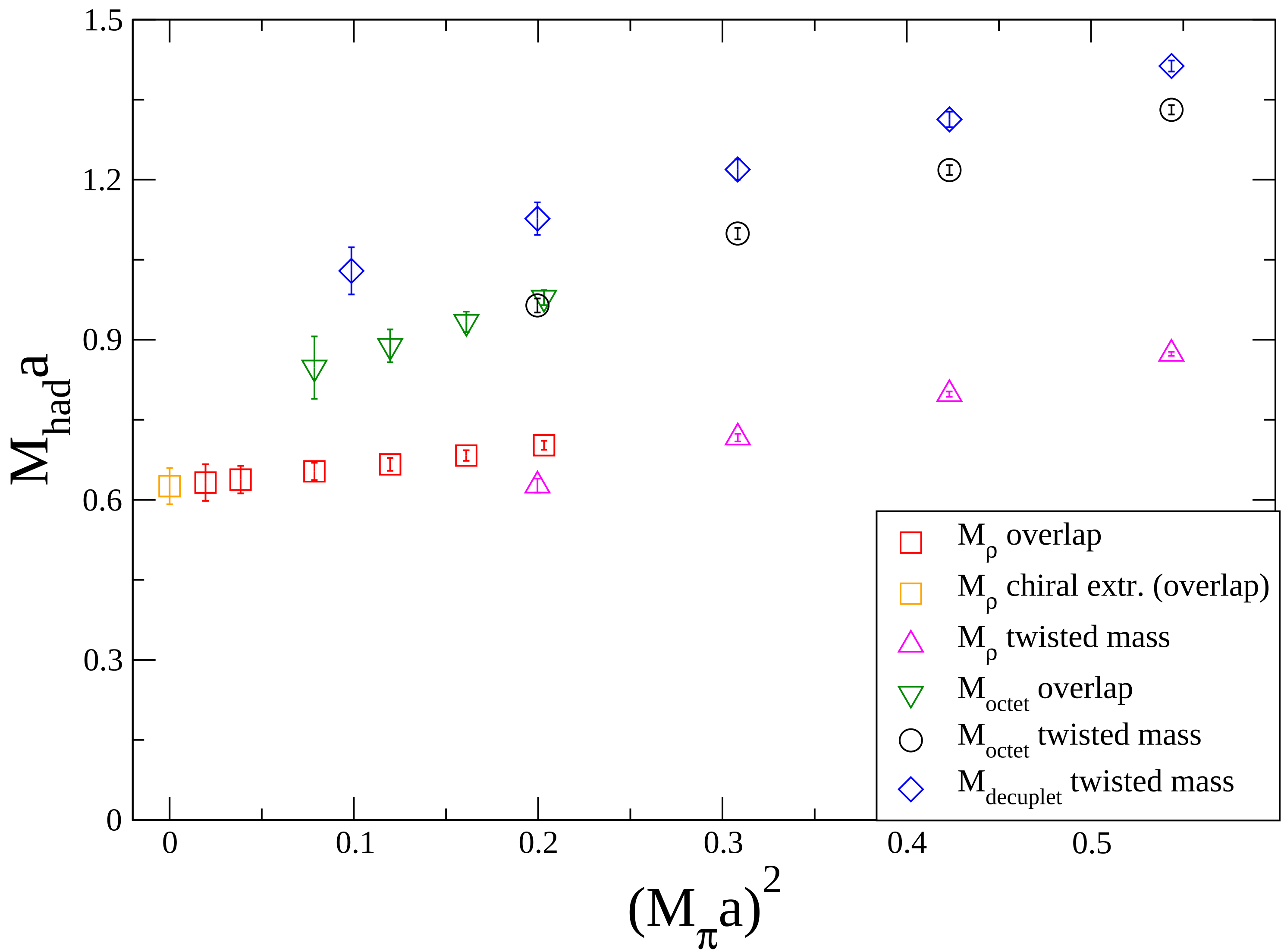
- In the twisted mass case it's easy to show that

$$
\langle \bar{B}^{\text{oct,dec}}_{\alpha}(x) B^{\text{oct,dec}}_{\beta}(0)\rangle^{\text{phys}} = \frac{1}{2}(1+i\gamma_5)_{\alpha\gamma}\langle \bar{B}^{\text{oct,dec}}_{\gamma}(x) B^{\text{oct,dec}}_{\delta}(0)\rangle^{\text{tm}}(1+i\gamma_5)_{\delta\beta}
$$

Baryons

- remark: two propagators are needed for the octet: one corresponding to a twisted term $+i\mu_q\gamma_5$ (let's call it the u propagator) and the other to a twisted term $-i\mu_q\gamma_5$ (the "d propagator").

Baryons



1.5
1.2
0.9
0.6
0.3
0
0
0.1
0.2
0.3
0.4
0.5
$M_{had}a$
$(M_\pi a)^2$
M_ρ overlap
M_ρ chiral extr. (overlap)
M_ρ twisted mass
M_{octet} overlap
M_{octet} twisted mass
$M_{decuplet}$ twisted mass

Conclusions and outlook

- Reached pion masses down to 230 MeV with overlap and 360 MeV (and even lower) with twisted mass.
- Preliminary results for twisted mass at quark masses such that $(a\Lambda)^2 \ll m_q a$ looks promising.
- Preliminary results for twisted mass at lower quark masses deserve further investigation (Aoki phase?).
- Of particular relevance are the (presently on-going) scaling tests (on the range $\beta \in [5.7, 6.4]$) which should clarify the scaling region (as a function of the smallest quark masses) from which critically depends the estrapolation to the continuum limit.
- At this value of beta, overlap simulation are $\mathcal{O}(20) - \mathcal{O}(50)$ more expensive than twisted mass simulation but of course they allow studies in the chiral region even at finite lattice spacing. Room for algorithmic improvement is left in both cases