Some optimal shape problems related to river fishways ⋆

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Abstract

Fishways are hydraulic structures that enable fish to overcome obstructions to their spawning and other migrations in rivers. In this paper we first introduce a mathematical formulation of the optimal design problem for a vertical slot fishway, where the state system is given by the shallow water equations determining the height of water and its velocity, the design variables are the geometry of the slots, and the objective function is related to the existence of rest areas for fish and a water velocity suitable for fish leaping and swimming capabilities. We also obtain an expression for the gradient of the objective function *via* the adjoint system. From the numerical point of view, we present a characteristic-Galerkin method for solving the shallow water equations, and an optimization algorithm for the computation of the optimal design variables. Finally, we give numerical results obtained for a standard ten pools channel.

Key words: Optimal shape design, Fishway, River, Shallow water

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1 Introduction

The need to preserve and enhance natural stocks of diadromous and resident fish: salmon (*salmo salar*), trout (*salmo trutta*), eel (*anguilla anguilla*), sturgeon (*acipenser sturio*), lamprey (*lampetra fluviatilis*, *petromyzon marinus*), barbel (*barbus bocagei*), etc.) have been recognized for, at least, the past century. Diadromous fish are fish that migrate between freshwater and saltwater. Their migration patterns differ for each species: some diadromous fish migrate great distances, while others migrate much shorter ones. In both cases, fish undergo physiological changes that allow them to survive as they migrate. There are three types of diadromous fish, depending on their specific migration patterns: anadromous, catadromous and amphidromous, which spend most of their adult lives, respectively, in saltwater, freshwater or both of them.

Fishways are hydraulic structures that enable fish to overcome obstructions (for instance, dams or falls) to their spawning and other river migrations, and are built whenever they are required, based on ecological, economical, or legal considerations. Fishways are generally divided into three groups: pool and weir type [4], Denil type [5], and vertical slot type [11]. Pool and weir fishways were the earliest type constructed and are still built with the addition of orifices in their walls. A pool and weir fishway consists of a number of pools formed by a series of weirs. The fish passes over a weir by swimming at burst speed (or in some cases - salmon, trout, etc. - by jumping over it). The fish then rests in the pool, then passes over the next weir, and so on, till it completes the ascent. The success of this type of fishway depends on the maintenance of water levels, which can be facilitated by the provision of a set of orifices in the weir walls close to the floor. The Denil fishway is essentially a straight rectangular flume provided with closely spaced baffles or vanes on the bottom and sides. (The first of the classical works of G. Denil on the scientific design of fish-passes was already published in 1909 in *Annales de Travaux Publiques de Belgique*). Of the many types of Denil fishway studied in the scientific literature, the more commonly used are the simple Denil fishway and the more complex "Alaska Steep-pass".

We deal here with the third type of fishway, that is the more generally adopted for upstream passage of fish in streams obstructions: the vertical slot fishway. It consists of a rectangular channel with a sloping floor that is divided into a number of pools. Water runs downstream in this channel, through a series of vertical slots from one pool to the next one below. The water flow forms a jet at the slot, and the energy is dissipated by mixing in the pool. The fish ascends, using its burst speed, to get past the slot, then it rests in the pool till the next slot is tried [3]. Thus, a fishway can be considered as a water passage around or through an obstruction, so designed as to dissipate the energy in the water in such a manner as to enable the fish to ascend without undue

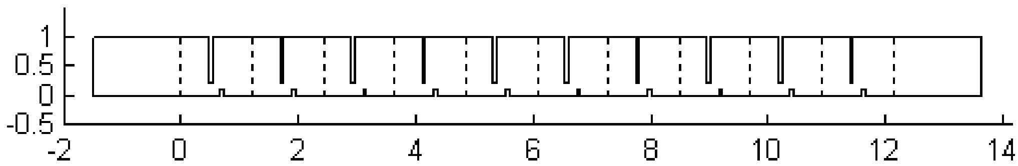

Fig. 1. A ten pool fishway ω.

stress.

Our main aim consists of finding the optimal shape design of the vertical slot fishway so that the higher number of fish can ascend through the obstacle in the river in their best conditions. In this paper we first introduce a mathematical formulation of the optimal design problem for a standard ten pools channel, where the state system is given by the shallow water equations determining the height of water and its velocity (averaged in height), the design variables are the geometry of the slots, and the objective function is related to the existence of rest areas for fish and a water velocity suitable for fish leaping and swimming capabilities. We also obtain an expression for the gradient of the objective function *via* the adjoint system. From the numerical point of view, we present a characteristic-Galerkin method for solving the shallow water (Saint Venant) equations, and a derivative-free algorithm for the computation of the optimal design variables. Finally, we give numerical results obtained for the ten pools channel under study.

2 Mathematical formulation of the problem

We consider a fishway $\omega \subset \mathbb{R}^2$ consisting of ten pools built in a rectangular channel. Each pool has a width of $0.97\,m$ and a length of $1.213\,m$. We also consider two transition pools, one at the beginning and other at the end of the channel, of the same width and a length of $1.5\,m$. The baffles separating the pools have a width of $2r = 0.061\,m$ and are made vertical to a flume bed slope that ranges from 2 to 20%. The fishway is schematized in Fig. 1: water enters by the left side and runs downstream to the right side, and fish ascend in the opposite direction.

Water flow in the channel along the time interval $(0,T)$ is governed by the shallow water (Saint Venant) equations:

$$\left.\begin{array}{ll} \frac{\partial H}{\partial t} + \vec{\nabla}.\vec{Q} = 0 & \text{in } \omega \times (0,T) \\ \frac{\partial \vec{Q}}{\partial t} + \vec{\nabla}.(\vec{u} \otimes \vec{Q}) + gH\vec{\nabla}(H-\eta) = \vec{f} & \text{in } \omega \times (0,T) \end{array}\right\} \quad (1)$$

where $H(x,t)$ is the height of water at point $x = (x_1, x_2) \in \omega$ at time $t \in (0,T)$, $\vec{u}(x,t) = (u_1, u_2)$ is the averaged horizontal velocity of water, $\vec{Q}(x,t) = (Q_1, Q_2) = \vec{u}H$ is the flux, g is the gravity acceleration, $\eta(x)$ represents the bottom geometry of the fishway, and the second member $\vec{f}$ collects all the effects of bottom friction, atmospheric pressure and so on. These equations must be completed with a set of initial and boundary conditions. In order to do that, we need to define three different parts in the boundary of ω: the lateral boundary of the channel, denoted by γ_0, the inflow boundary, denoted by γ_1 and the outflow boundary, denoted by γ_2. We also consider $\vec{n}$ the unit outer normal vector to boundary $\partial\omega = \gamma_0 \cup \gamma_1 \cup \gamma_2$. Thus, the initial and boundary conditions read in the classical form (cf. [10] or [1]):

$$\left.\begin{array}{ll} H(0) = H_0 & \text{in } \omega \\ \vec{Q}(0) = \vec{Q}_0 & \text{in } \omega \\ \vec{Q}.\vec{n} = 0 & \text{on } \gamma_0 \times (0,T) \\ \vec{Q} = Q_1\,\vec{n} & \text{on } \gamma_1 \times (0,T) \\ H = H_2 & \text{on } \gamma_2 \times (0,T) \end{array}\right\} \tag{2}$$

The design variables will be the two midpoints corresponding to the ends of the baffles (points $a = (a_1, a_2)$ and $b = (b_1, b_2)$ in Fig. 2 representing the first of the ten pools). These two points will configure the shape of the complete fishway ω, since we assume that the structure of the ten pools is the same. We will impose several constraints on the design variables: first, we will assume that point a and b are inside the dashed rectangle of Fig. 2, that is, the following eight relations must be satisfied:

$$\left.\begin{array}{l} \frac{1}{4}\,1.213 \leq a_1, b_1 \leq \frac{3}{4}\,1.213 \\ 0 \leq a_2, b_2 \leq \frac{1}{4}\,0.97 \end{array}\right\} \tag{3}$$

The second type of constraints are related to the fact that the vertical slot must be large enough so that fish can pass comfortably through it. This translates into the two additional relations:

$$\left.\begin{array}{l} b_1 - a_1 \geq 0.1 \\ a_2 - b_2 \geq 0.05 \end{array}\right\} \tag{4}$$

(Taking into account that the width of the baffle is $2r = 0.061\,m$, we are really imposing that the slot width must be, at least of $\sqrt{(0.1 - 2r)^2 + 0.05^2} = 0.063\,m$.)

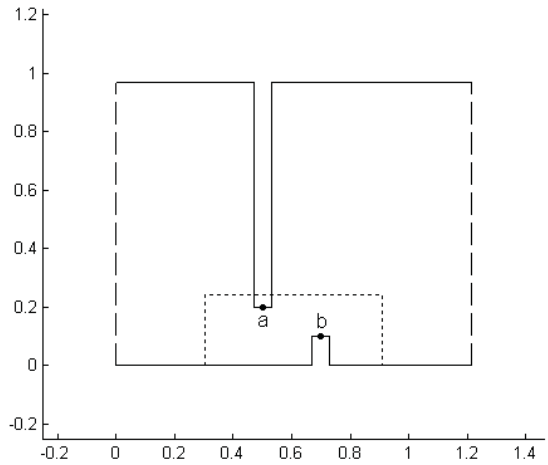

Fig. 2. Scheme of the first pool.

Finally, we introduce the objective function which is intended for obtaining an optimal velocity of water in such a way that in the zone of the channel near the slots (say the lower third) the velocity be as close as possible to a desired velocity $(c, 0)$ suitable for fish leaping and swimming capabilities (and depending on the species of fish). In the remaining of the fishway, the velocity must be very small for making possible the rest of the fish. Moreover, in all the channel, we must minimize the existence of flow turbulence. Thus, if we define the target velocity $\vec{v}$ by:

$$\vec{v}(x_1, x_2) = \begin{cases} (c, 0), & \text{if } \ x_2 \leq \frac{1}{3}\, 0.97 \\ (0, 0), & \text{otherwise} \end{cases} \tag{5}$$

the objective function is given by:

$$j(\omega) = \frac{1}{2} \int_0^T \int_\omega \|\vec{u}^\omega - \vec{v}\|^2 dx\, dt + \frac{\alpha}{2} \int_0^T \int_\omega |curl(\vec{u}^\omega)|^2 dx\, dt \tag{6}$$

where $\alpha \geq 0$ is a weight parameter for the role of the vorticity in the whole cost function, and $\vec{u}^\omega = \frac{\vec{Q}^\omega}{H^\omega}$ for $(H^\omega, \vec{Q}^\omega)$ the solution of the state system (1) with initial and boundary conditions (2).

Then the optimization problem $(\mathcal{P})$ consists of finding the optimal shape ω of the fishway (that is, the optimal points a and b, satisfying the constraints (3) and (4)) such that minimizes the objective function given by (6).

3 Analysis of the optimal shape problem

Let $\mathcal{D}_0$ be a set of topologically admissible domains, that is classically defined as the set of domains ω which are homeomorphic to a reference domain Ω_0 by bijective $W^{1,\infty}$ mappings (a subset of $W^{1,\infty}(\Omega_0;\mathbb{R}^2)$ denoted by $\mathcal{T}_0$), that is, $\omega \in \mathcal{D}_0 \Leftrightarrow \omega = \tau(\Omega_0)$ for any $\tau \in \mathcal{T}_0$. Then, let $\omega = \tau(\Omega_0) \in \mathcal{D}_0$ and let $F \in W^{1,\infty}(\omega;\mathbb{R}^2)$ be an homeomorphism such that $\Omega = F(\omega) \in \mathcal{D}_0$. We define $V \in W^{1,\infty}(\omega;\mathbb{R}^2)$ by $V = F - I$. Let d be any function $d : \Omega \in \mathcal{D}_0 \to d(\Omega) \in \mathbb{R}$. We define the "transported" function $\bar{d}$ by the relation $\bar{d} : F \in W^{1,\infty}(\omega;\mathbb{R}^2) \to \bar{d}(F) = d(F(\omega)) = d(\Omega) \in \mathbb{R}$. So, we can define the domain derivative of d at a given $\omega \in \mathcal{D}_0$ by the expression (cf. [8] for more details):

$$\frac{\partial}{\partial\omega} d(\omega).V = \frac{\partial}{\partial F}\bar{d}(I).V, \qquad \text{for } V \in W^{1,\infty}(\omega;\mathbb{R}^2). \tag{7}$$

If we denote by $A(\omega; H, \vec{Q}; p, \vec{r}) = L(\omega; p, \vec{r}),\ \forall(p,\vec{r})$ the dual formulation of the state system (1), that is,

$$\begin{aligned}\int_0^T\!\!\int_\omega \frac{\partial H}{\partial t} p + \int_0^T\!\!\int_\omega (\vec{\nabla}.\vec{Q})p + \int_0^T\!\!\int_\omega \frac{\partial \vec{Q}}{\partial t}.\vec{r} + \int_0^T\!\!\int_\omega \{\vec{\nabla}.(\vec{u}\otimes\vec{Q})\}.\vec{r} \\ + \int_0^T\!\!\int_\omega gH\vec{\nabla}(H-\eta).\vec{r} = \int_0^T\!\!\int_\omega \vec{f}.\vec{r}, \quad \forall(p,\vec{r})\end{aligned} \tag{8}$$

and we rewrite the objective function in the form $j(\omega) = J(\omega; H, \vec{Q})$, then we can obtain, arguing in the classical manner, the following expression for the "domain derivative" of j:

$$\begin{aligned}\frac{\partial}{\partial\omega} j(\omega).V = & \frac{\partial}{\partial\omega} J(\omega; H^\omega, \vec{Q}^\omega).V \\ & -\frac{\partial}{\partial\omega} A(\omega; H^\omega, \vec{Q}^\omega; p^\omega, \vec{r}^\omega).V + \frac{\partial}{\partial\omega} L(\omega; p^\omega, \vec{r}^\omega).V\end{aligned} \tag{9}$$

where $(H^\omega, \vec{Q}^\omega)$ is the solution of the state system (1) with initial and boundary conditions (2), and $(p^\omega, \vec{r}^\omega)$ is the solution of the adjoint system:

$$\left.\begin{aligned} & -\tfrac{\partial p}{\partial t} + \tfrac{1}{H^2}(\vec{Q}.\vec{\nabla})\vec{r}.\vec{Q} - gH(\vec{\nabla}.\vec{r}) - g\vec{\nabla}H.\vec{r} \\ & \qquad = -(\tfrac{\vec{Q}}{H} - \vec{v}).\tfrac{\vec{Q}}{H^2} - \alpha\, \vec{curl}(curl(\tfrac{\vec{Q}}{H})).\tfrac{\vec{Q}}{H^2} \quad \text{in } \omega\times(0,T) \\ & -\tfrac{\partial \vec{r}}{\partial t} - \vec{\nabla}p - \tfrac{1}{H}(\vec{Q}.\vec{\nabla})\vec{r} - \tfrac{1}{H}(\vec{\nabla}\vec{r})^t\vec{Q} \\ & \qquad = \tfrac{1}{H}(\tfrac{\vec{Q}}{H} - \vec{v}) + \alpha\tfrac{1}{H}\vec{curl}(curl(\tfrac{\vec{Q}}{H})) \qquad \text{in } \omega\times(0,T)\end{aligned}\right\} \tag{10}$$

with final and boundary conditions:

$$\left.\begin{array}{ll} p(T) = 0 & \text{in } \omega \\ \vec{r}(T) = \vec{0} & \text{in } \omega \\ \vec{r}.\vec{n} = 0 & \text{on } \gamma_0 \times (0,T) \\ \vec{r} = \vec{0} & \text{on } \gamma_1 \times (0,T) \\ p = 0 & \text{on } \gamma_2 \times (0,T) \end{array}\right\} \qquad (11)$$

4 Numerical solution

In order to minimize the objective function j the first typical proposal consists of using a gradient-type algorithm, where the gradient of j can be easily obtained from expression (9) *via* the computation of the adjoint system (10). However, due to the essentially geometric nature of the problem $(\mathcal{P})$, we alternatively propose a gradient-free algorithm for solving the discretized optimization problem. In this case, we will change our problem into an unconstrained optimization problem by using a penalty function involving the constraints (3) and (4).

Taking into account that the shape of ω depends only on the two points a and b, if we introduce the variable $y = (a,b) = (a_1, a_2, b_1, b_2) \in \mathbb{R}^4$, we can consider $\omega = \omega(y)$. Then, we can redefine the objective function in the way $\Phi_1 : \mathbb{R}^4 \to \mathbb{R}$, where $\Phi_1(y) = j(\omega(y))$. To evaluate function Φ_1 at each $y = (a,b)$ involved in the process, we first need to solve the shallow water equations (1) in the fishway $\omega = \omega(y)$ and then, once we know the flux $\vec{Q}(x,t)$ and the height of water $H(x,t)$, compute the objective function $\Phi_1(y)$.

In the present paper, the shallow water equations are solved by using an implicit discretization in time, upwinding the convective term by the method of characteristics, and Raviart-Thomas finite elements for the space discretization (the whole details of the numerical scheme can be seen in [2]). So, for the time interval $(0,T)$ we choose an integer number N, consider the time step $\Delta t = \frac{T}{N} > 0$ and define the discrete times $t_n = n\Delta t$ for $n = 0, \ldots, N$. We also consider a Lagrange-Galerkin finite element triangulation $\tau_h(y)$ of the domain $\omega(y)$. (We must remark that the mesh hardly depends on the design variables $y = (a,b)$ and, consequently, for each $\omega(y)$ we have to generate a new triangulation or remesh the previous one). Thus, the numerical scheme provides us, for each discrete time t_n, with an approximated flux $\vec{Q}_h^n$ and an approximated height H_h^n, which are piecewise-linear polynomials and discontinuous piecewise-constant functions, respectively. With these approximated fields we can compute the approximated velocity $\vec{u}_h^n = \frac{\vec{Q}_h^n}{H_h^n}$, and approach the objective function value $\Phi_1(y)$ by the expression:

$$\tilde{\Phi}_1(y) = \frac{\Delta t}{2} \sum_{n=1}^{N} \sum_{E \in \tau_h(y)} \int_E \|\vec{u}_h^n - \vec{v}\|^2 dx\, dt + \alpha \frac{\Delta t}{2} \sum_{n=1}^{N} \sum_{E \in \tau_h(y)} \int_E |curl(\vec{u}_h^n)|^2 dx\, dt \quad (12)$$

We also introduce a function $\vec{\Phi}_2 : \mathbb{R}^4 \to \mathbb{R}^{10}$ collecting all the ten linear constraints on the design variables, i.e., $\vec{\Phi}_2$ is such that $y = (a, b) \in \mathbb{R}^4$ verifies the constraints (3) and (4) if and only if $\vec{\Phi}_2(y) \leq \vec{0}$. Thus, we define the penalty function Φ, which is a combination of the objective function $\tilde{\Phi}_1$ and the function $\vec{\Phi}_2$ representing the control constraints:

$$\Phi(y) = \tilde{\Phi}_1(y) + \beta \| \max\{\vec{\Phi}_2(y), \vec{0}\} \| \quad (13)$$

where the parameter $\beta > 0$ determines the relative contribution of the objective function and the penalty terms. Function Φ is an exact penalty function in the sense that, for sufficiently large β, the solutions of our original constrained problem $(\mathcal{P})$ are equivalent to the minimizers of function Φ.

For computing a minimum of this non-differentiable function Φ we use a direct search algorithm: the Nelder-Mead simplex method [9]. This is a gradient-free method, which merely compares function values; the values of the objective function being taken from a set of sample points (simplex) are used to continue the sampling. We briefly outline the algorithm: the dimension of our optimization problem is 4. A 4-simplex is the convex hull of 5 points in $\mathbb{R}^4$. The method constructs a sequence of simplices as approximations to an optimal solution. The 5 vertices $y_1, y_2, \ldots, y_5$ of each simplex are sorted according to the objective function values: $\Phi(y_1) \leq \Phi(y_2) \leq \ldots \leq \Phi(y_5)$, and the worst vertex y_5 is replaced with a new point $y(\nu) = (1 + \nu)y^* - \nu\, y_5$, where y^* is the centroid of the convex hull of $\{y_1, \ldots, y_4\}$. The value of ν is selected from a sequence $-1 < \nu_\delta < 0 < \nu_\gamma < \nu_\beta < \nu_\alpha$ (typical values are $\nu_\delta = -0.5$, $\nu_\gamma = 0.5$, $\nu_\beta = 1$, $\nu_\alpha = 2$) by rules given in the following algorithm:

While $\Phi(y_5) - \Phi(y_1)$ is not sufficiently small, compute $y(\nu_\beta)$ and $\Phi_\beta = \Phi(y(\nu_\beta))$. Then:

(a) If $\Phi_\beta < \Phi(y_1)$, compute $\Phi_\alpha = \Phi(y(\nu_\alpha))$. If $\Phi_\alpha < \Phi_\beta$, replace y_5 with $y(\nu_\alpha)$; otherwise replace y_5 with $y(\nu_\beta)$. Go to **(f)**.
(b) If $\Phi(y_1) \leq \Phi_\beta < \Phi(y_4)$, replace y_5 with $y(\nu_\beta)$ and go to **(f)**.
(c) If $\Phi(y_4) \leq \Phi_\beta < \Phi(y_5)$, compute $\Phi_\gamma = \Phi(y(\nu_\gamma))$. If $\Phi_\gamma \leq \Phi_\beta$, replace y_5 with $y(\nu_\gamma)$ and go to **(f)**; otherwise go to **(e)**.
(d) If $\Phi(y_5) \leq \Phi_\beta$, compute $\Phi_\delta = \Phi(y(\nu_\delta))$. If $\Phi_\delta < \Phi(y_5)$, replace y_5 with $y(\nu_\delta)$ and go to **(f)**; otherwise go to **(e)**.

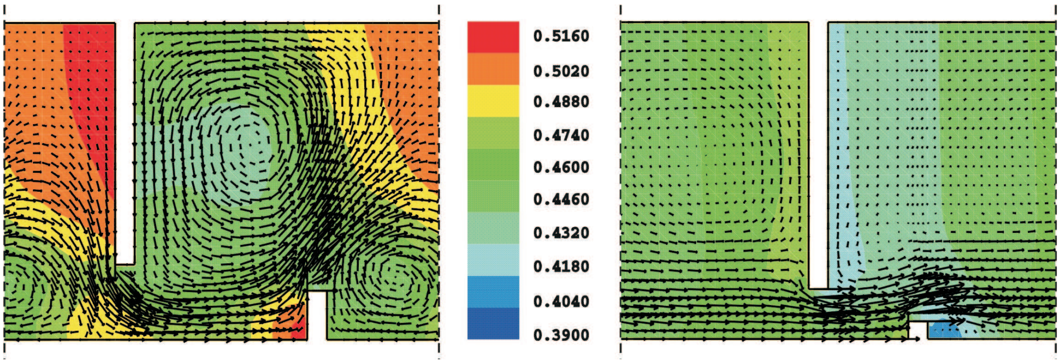

Fig. 3. Initial (left) and optimal (right) heights and velocities for the central pool.

(e) For $k = 2, \ldots, 5$ set $y_k = y_1 + (y_k - y_1)/2$.
(f) Resort the resulting vertices according to Φ values.

Although the Nelder-Mead algorithm is not guaranteed to converge in the general case, it has good convergence properties in low dimensions (cf. [7] for a detailed analysis of its convergence under convexity requirements). Moreover, to prevent stagnation at a non-optimal point, we use a modification proposed by Kelley (cf. [6] for details): when stagnation is detected, we modify the simplex by an oriented restart, replacing it by a new smaller simplex.

In the final part of this section we present the numerical results obtained by using above method to determine the optimal shape of the ten pools channel introduced in Fig. 1, with a slope of 5%. Both initial and boundary conditions were taken as constant, particularly, $\vec{Q}_0 = (0,0)\, m^2 s^{-1}$, $H_0 = 0.5\, m$, $Q_1 = -\frac{0.065}{0.97}\, m^2 s^{-1}$, $H_2 = 0.5\, m$. The time interval for the simulation was $T = 30\, s$. Moreover, for the sake of simplicity, for the second member $\vec{f}$ we have only considered the bottom friction stress for a Chezy coefficient of 57.36. For the objective function we have taken a target velocity value $c = 0.8\, ms^{-1}$, and we have chosen the parameters $\alpha = 0$, $\beta = 500$. For the time discretization we have taken $N = 300$ (that is, a time step of $\Delta t = 0.1\, s$), and for the several space discretizations we have tried regular triangulations of about 9500 elements.

Thus, applying the Nelder-Mead algorithm, we have passed, after 49 function evaluations, from an initial cost $\Phi = 100.69$ for a random simplex, to the minimum cost $\Phi = 26.26$, corresponding to the optimal design variables $a = (0.545, 0.153)$, $b = (0.851, 0.054)$. Fig. 3 shows the water heights (according to given color range) and velocities at final time in the sixth pool, corresponding to the initial random configuration (left), and to the optimal configuration given by a and b (right). It can be seen how, in the latter case, the optimal velocity is close to the target velocity $\vec{v}$, and the two large recirculation regions at both sides of the slot are reduced.

5 Conclusions

In this work the authors have formulated, analyzed and solved an optimal shape problem related to the design of fishways in rivers. Once the physical problem is mathematically well posed in terms of water height and flux, a numerical discretization method is proposed for solving the shallow water equations involved in the modelling. Also a gradient-type algorithm and a direct search method (Nelder-Mead algorithm) are proposed for solving the discrete optimization problem. Finally, the efficiency of the latter algorithm is confirmed by the numerical experiments developed by the authors.

References

[1] V.I. Agoshkov, A. Quarteroni and F. Saleri, Recent developments in the numerical simulation of shallow water equations I: Boundary conditions. *Appl. Numer. Math.*, **15** (1994), 175-200.

[2] A. Bermúdez, C. Rodríguez and M.A. Vilar, Solving shallow water equations by a mixed implicit finite element method. *IMA J. Numer. Anal.*, **11** (1991), 79-97.

[3] R.W. Blake, *Fish locomotion*, (Cambridge University Press, London, 1983).

[4] C.H. Clay, *Design of fishways and other fish facilities*, (Lewis Publishers, CRC Press, Boca Raton, 1995).

[5] C. Katopodis, N. Rajaratnam, S. Wu and D. Towell, Denil fishways of varying geometry. *J. Hydraul. Eng.-ASCE*, **123** (1997), 624-631.

[6] C.T. Kelley, Detection and remediation of stagnation in the Nelder-Mead algorithm using a sufficient decrease condition. *SIAM J. Optim.*, **10** (1999), 43-55.

[7] J.C. Lagarias, J.A. Reeds, M.H. Wright and P.E. Wright, Convergence properties of the Nelder-Mead simplex algorithm in low dimensions. *SIAM J. Optim.*, **9** (1998), 112-147.

[8] F. Murat and J. Simon, *Sur le controle par un domaine géométrique*. Ph.D. Thesis, Univ. Paris VI, France, 1976.

[9] J.A. Nelder and R. Mead, A simplex method for function minimization. *Comput. J.*, **7** (1965), 308-313.

[10] J. Oliger and A. Sundström, Theoretical and practical aspects of some initial boundary value problems in fluid dynamics. *SIAM J. Appl. Math.*, **35** (1978), 419–446.

[11] N. Rajaratnam, G. Van de Vinne and C. Katopodis, Hydraulics of vertical slot fishways. *J. Hydraul. Eng.-ASCE*, **112** (1986), 909-917.