A problem of uniform exponential decay for a sequence of damped wave equations

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Let Ω be a bounded domain of $\mathbb{R}^d$. We compare the dynamics of the damped wave equation

$$\begin{cases} u_{tt}(x,t) + \gamma_n(x) u_t(x,t) = \Delta u(x,t) + f(x,u(x,t)) & \text{on } \Omega, \\ \frac{\partial u}{\partial \nu} = 0 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

with the dynamics of the wave equation damped on the boundary

$$\begin{cases} u_{tt}(x,t) = \Delta u(x,t) + f(x,u(x,t)) & \text{on } \Omega, \\ \frac{\partial u}{\partial \nu} + \gamma(x) u_t = 0 & \text{on } \partial\Omega, \end{cases} \tag{2}$$

when γ_n converges to $\gamma(x) \otimes \delta_{x\in\partial\Omega}$ in the sense of distributions.
We assume that the nonlinearity f is dissipative. Under natural or generic hypotheses, we are able to prove that, when n goes to $+\infty$, the family of compact global attractors $(\mathcal{A}_n)_{n\in\mathbb{N}\cup\{+\infty\}}$ corresponding to Equations (1) and (2) is lower-semicontinuous in $\mathbb{H}^1(\Omega) \times \mathbb{L}^2(\Omega)$ and upper-semicontinuous in $\mathbb{H}^{1-\varepsilon}(\Omega) \times \mathbb{H}^{-\varepsilon}(\Omega)$, for any $\varepsilon > 0$. To obtain upper-semicontinuity in $X = \mathbb{H}^1(\Omega) \times \mathbb{L}^2(\Omega)$, we have to show the existence of two positive constants M and λ such that

$$\forall n \in \mathbb{N},\ \forall t \geq 0, \quad \|e^{A_n t}\|_{\mathcal{L}(X)} \leq M e^{-\lambda t}\ , \tag{3}$$

where $A_n = \begin{pmatrix} 0 & Id \\ \Delta & -\gamma_n \end{pmatrix}$ is the linear operator associated to Equation (1).
In dimension $d = 1$, using a multiplier method inspired by [2] and [3] or applying a result of [1], we show concrete necessary and sufficient conditions such that (3) holds. In higher dimensions, the question is completely open since we have neither an example satisfying (3), nor a non-trivial counter-example.
For more details about this problem, we refer to [4].

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Fluid-structure interaction models

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General presentation:

Many physical phenomena involve moving or deformable structures interacting with fluids and are of great interest. A special study is necessary when fluid and structure dynamics influence each other: the structure deforms under the action of the fluid stresses and, conversely, the fluid follows the structure displacement.

The structure can be immersed in a fluid (for instance for a plane surrounded by air, a submarine in the sea, wind induced oscillations of a bridge) or the fluid can be contained in the structure (blood in arteries).

Usually the structure is rigid (the structure motion is then composed of a translation part and a rotation part) or deformable (modeled by elasticity equations). If it moves freely, the motion of an elastic structure combines a rigid motion and an elastic motion.

To have an idea of further challenges, we give some open problems.

Open problems

- Global existence of weak solutions.

Global existence of weak solutions is not proved for the interaction problem between an elastic structure and a fluid. For the time being, studies consider regularized elastic motions to avoid problems of instantaneous collisions or interpenetration of the structure.

- Uniqueness and regularity.

Uniqueness has been proved for a rigid structure evolving in a fluid in 2D. A regularity result is proved for this problem in L^2.
Open problems: 3D ? elastic structure ? L^r ?

- The problem of collision.

In 2D, for a rigid structure in a fluid modeled by Navier-Stokes equations, a result obtained in [14] proves that no collision can occur and thus global existence of solution is proved. In the other cases, existence or uniqueness is usually local and is obtained as long as no collisions occur.

- Asymptotic behavior.

The asymptotic behavior of a fluid alone modeled by Navier-Stokes equations is very difficult. A study of [13] deals with the heat equation and a rigid structure in an unbounded domain.

- Controllability of fluid-structure interaction problems.

A one-dimensional result of null controllability is proved by [8] for a fluid modeled by Burgers equation.

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